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THRESHOLD MODELLING OF NONLINEAR DYNAMIC RELATIONSHIPS: AN APPLICATION TO A DAILY SERIES OF ECONOMIC ACTIVITY

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Abstract

This paper develops a methodology to model non-linear dynamic relationships. The non-linear functions are approached by the inclusion of threshold variables in an iterative forward search process which allows for different lenghts of the response functions to impulses at different intervals of the explanatory variables. The paper includes an application of these methods to the forecasting of the daily consumption of electricity as a function of temperature in Spain.

Key words

ARMAX models; model specification; electricity consumption.

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1. INTRODUCTION

This paper presents a methodology for modelling nonlinear dynamic relationships.

The aim of this paper is to propose an operational procedure for searching for specification in those problems where the only extra-sample information available indicates that the dependent variable, Y_t is generated by

$$Y_t = w^{(x)}(L)\ f(X_t) + \frac{\theta(L)}{\phi(L)} a_t$$

where

- $f(X_t)$ is a nonlinear function, of an unknown type, on the explanatory variable X_t.
- $w^{(x)}(L) = w^{(x)}_0 + w^{(x)}_1 L + \ldots + w^{(x)}_{s(x)} L^{s(x)}$, is a finite order polynomial in the lag operator L; the order of the polynomial may be different on the basis of the value of the variable X_t.
- $\theta(L)$ and $\phi(L)$ are finite polynomials in L, so that the roots of $\phi(L)$ fall on or outside the unit circle and the roots of $\theta(L)$ outside it.

At no time are restrictions imposed either on the form of f(.), or on the dynamics of the response. Interaction is allowed between the dynamics of the relationship and the value of X_t, so that different values of the explanatory variable can carry in association different dynamic effects on Y_t.

Emphasis is placed on the selection of the most suitable specification for the available sample. The procedure is esentially data-based, so no special type of extra-sample information is needed. Naturally, if available, it would be used, restricting the search process to the direction indicated by the a priori information.

In section 2 there is a discussion of the hypotheses on which the procedure proposed in this paper for dynamic relationships is based, and the way to define the variables used to approach the unknown function.

In section 3 the methodology for the specification and estimation of dynamic, nonlinear models is presented, with an initial comment on the approach underlying it, followed by a discussion of the successive stages it consists of.

The asymptotic behaviour of the estimators of the previous section are analysed in section 4, by discussing the conditions which must be met for the resulting specification to be consistent. Given that in practice we find ourselves working with finite samples, section 5 discusses the criteria proposed for the selection and validation of specifications in finite samples.

In point 6 an application of this methodology is presented: with it a model is made of the daily relationship between temperature and electricity demand in Spain, for the period 1983-1989.

The main conclusions of the work are summarised in section 7.

2. PROCEDURE FOR THE APPROXIMATION OF NON-LINEAR DYNAMIC RELATIONSHIPS: REQUISITES FOR ITS APPLICATION AND USE OF A PRIORI INFORMATION

Spline functions are an useful tool for solving the problem of the approximation of nonlinear functions of an unknown form; Wegman and Wright (1983) provide a thorough review of their application in statistics.

However, when we extend the usual framework to include the modelling of a changing dynamic dependence, the problem is complicated considerably. For this reason, in this paper we restrict ourselves to piecewise linear approximations.

The main characteristics of the problem we are dealing with, as well as the basic assumptions that have to be met, are the following:

2.1 Assumptions on the Dynamics of the Dependent Variable

The dependent variable, henceforth called Y_t, follows a stochastic process which allows a univariate representation given by

$$Y_t = \frac{\theta^u (L)}{\phi^u (L)} e_t \qquad (2.1)$$

where the MA part is necessarily invertible and $\phi^u (L)$ can have roots on the unit circle.

2.2 Assumptions on the Input-Output Relationship

An explanatory variable, denoted by X, is available, which meets:

1º) X is, at least, weakly exogenous with regard to Y.

2º) The number of real positive unit roots in the ARIMA representation of variable X is zero or one; there are no restrictions imposed on the number of complex or negative roots.

3º) There is a causal relationship from X to Y, so that this relationship can be exploited for, for example, improving the forecast of Y compared to what can be obtained from the model (2.1).

4º) The full effect of a change in X need not be restricted to one moment in time; rather a variation in X can unleash a dynamic adjustment process in Y. Nor is the existence of a contemporary effect excluded.

5º) X does not necessarily have to be able to explain in itself the whole dynamic of Y, so the residual term is allowed not to be white noise and even to follow a general ARIMA process.

6º) The relationship between X and Y is not linear; moreover, the order of the dynamic structure may be different according to the value taken by X.

2.3 Linearisation of the Relationship and Use of a Priori Information

The nonlinear function f(.) is approximated by linearising it by intervals; this can be done in a purely empirical way, though what is recommended is to be able to put together as much extra-sample information as possible. The steps to follow are:

1º) Two values are determined in the range of variation of X between which the influence of X on Y is nonlinear. Thus X^1 and X^n ($X^1 < X^n$) are specified so that

$$f\,(X^*_t) \approx a + bX^*_t$$

for all X^*_t less than X^1 or greater than X_n. It is desirable to be able to fix these values from a priori information; but if this is not available,

$$X^1 = \min_t X_t \qquad\qquad X^n = \max_t X_t$$

can be taken, which implies supposing that the relationship is nonlinear for all the observed values of X.

2º) The interval (X^1, X^n) is divided into n-1 subintervals; if we call the values of X (Knots) which determine the intervals $X^1, X^2, \ldots, X^{n-1}, X^n$

$$X^i = X^1 + \left(\frac{X^n - X^1}{n - 1} \right) (i - 1) \qquad i = 1,2,\ldots,n$$

are the values of X which the search procedure will consider as possible thresholds.

The value of n will depend both on the a priori information that we may be able to collect on the degree of nonlinearity of f(.), and the type of data we are using. In principle, the more thresholds are considered the better the approximation to the unknown linear function will be, provided that all the intervals (x^i, X^{i+1}) that are formed contain a sufficient number of observations.

In absence of specific information, it is reasonable to share out the values $X^2, X^3, \ldots, X^{n-1}$ in an uniform manner throughout the search interval. When concrete information is available to make it advisable, the most worthwhile thing is to intensify the search for concrete subintervals.

As can be observed, we do not start from the assumption that each value of X constitutes a possible knot, but rather that a wide set of candidates is fixed a priori, on which the search process is carried out.

3º) To determine whether the effects of X on Y are always of the same type, either positive or negative, for all the possible values of X; or whether, on the contrary, the relationship between X and Y is growing for some values of X and decreasing for others.

Although the type of relationship can be determined in an empirical way, for most problems to which this technique can be applied enough a priori information is available to clarify the question beforehand. In those cases where this is not the case, the analyst will have to proceed as if both types of effects existed, acting in the way indicated in the second part of the description of stage 1 (section 3).

4º) The threshold variables on which the approximation to f (.) are to be based are constructed.

These variables can be of the type:

$$Z^i_t = \begin{cases} X_t - X^i & \text{if } X_t \geq X^i \\ 0 & \text{if } X_t < X^i \end{cases}$$

in this case a threshold is imposed on each of the values X^i in such a way that only those values of X higher than X^i will have effects on Y.

The other possibility is:

$$Z^i_t = \begin{cases} X^i - X_t & \text{if } X_t \leq X^i \\ 0 & \text{if } X_t > X^i \end{cases}$$

where now the values of X less than X^i are the ones affecting Y.

When the variable X always has an effect of only one type (either positive or negative), the analyst must choose which of both types of variables Z^i_t is more suitable for his problem. On the contrary, if the response of Y is growing or decreasing on the basis of the concrete values of X, it is useful to combine both types of threshold variables.

Mathematically it is unimportant to use one possibility or another, since a trivial reparameterisation of either of them allows the desired response to be expressed. Nevertheless, from the point of view of the interpretation and presentation of the results, it can be useful to use both, since it enables the coefficients to be estimated directly with sign which makes their interpretation easier. In the example in section 6 it will influence this point.

Once these variables have been defined, it is a question of trying to approximate the model

$$Y_t = w^{(x)}(L)\ f(X_t) + \frac{\theta(L)}{\phi(L)} a_t$$

by

$$Y_t = w(L)\ X_t + \sum_{i=1}^{n} w_i(L)\ Z^i{}_t + \frac{\theta(L)}{\phi(L)} a_t \quad .$$

Thus, the term $w^{(x)}(L)\ f(X_t)$, which represents the nonlinear contribution of X, is approximated by

$$w(L)\ X_t + \sum_{i=1}^{n} w_i(L)\ Z^i{}_t \quad ,$$

that is, the sum of the dynamic (linear) contributions of the variables $Z^i{}_t$.

3. METHODOLOGY FOR THE SPECIFICATION OF DYNAMIC THRESHOLD MODELS

In this section we present in outline form the metodology we propose for modelling the type of relationships we are considering, leaving the details for the following sections.

The approach of the specification search process is different depending on whether we are talking of dynamic or functional specification.

In dynamic specification a general formulation is opted for, with the idea that at any time one can operate with a good approximation of the dynamic structures contained in the model. The particular details that the dynamic formulation may require will be worked out only at the end of the process, when an approximation of the nonlinear function is available. For that purpose the following considerations must be taken into account:

1) The starting point is the univariate model of Y; this model is consistent, though inefficient if the variable X is included in the set of available information.

2) This univariate model is imposed on the disturbance throughout the whole search process of the relationship between X and Y.

3) Each candidate to threshold Z^i_t is allocated a polynomial w_i (L) with sufficient length to register all the dynamics that may exist.

In the search for functional specification we act in the opposite way, from the particular to the general; that is, from the specification of the univariate model, we go on to a forward search, where at each stage the inclusion of an additional threshold is considered. Thus, and once the iterative procedure of threshold selection begins, the dynamic is hardly an object of attention, since the study centres on the approximation to the nonlinear relationship between the variables.

From many viewpoints, it would be better to propose an over-parameterised model both in the dynamic and in the nonlinearity, in order subsequently to eliminate nonsignificant variables. Unfortunately, the complexity which the general model may have does not make this method recommendable.

The procedure proposed consists of the following stages:

STAGE 0) A univariate model is obtained -denoted by S0- for the dependent variable Y_t, which admits this type of representation as we have supposed.

STAGE 1) If the effect of X on Y is always of the same sign, a linear response function is tried.

Thus we have a model of the form

$$Y_t = w\ (L)\ X_t + \frac{\theta\ (L)}{\phi\ (L)}\ a_t \qquad (3.1)$$

where θ (L) and ϕ (L) have the same form as the univariate model of the previous stage and w (L) is a sufficiently general polynomial in the lag operator L. We will call this model S1.

Then we check whether S1 improves the adjustment that has been reached with S0.

If S1 improves the adjustment, the models of the following stages will be compared with S1, and, if the opposite is the case, with S0. Nevertheless, due to the mere fact that S1 does not improve the S0 adjustment, it cannot be concluded that no relationship exists between X and Y. Given that the a priori information shows that a highly nonlinear relationship exists, a model like (3.1) may be unable to incorporate the effect of X on Y.

Thus, the model (3.1) has a mainly informative nature, and the procedure is not halted by the fact of preferring S0 to S1, unlike what will happen in the next stages.

If the response function presents positive or negative effects according to the values of X, an approximate determination is made by means of a previous analysis of the data of the zones where the response is positive and the zones where the response is negative, by adjusting a linear function to each of them. Unlike the previous case, here there already is a treatment, albeit a simple one, of the nonlinear relationship as such.

It is important to point out that the aim of this stage is not to begin to obtain definitive results. For example, it is not necessary to determine exactly where the zone with negative effects begins in order to adjust the corresponding linear function: a simple approximation based on some type of a priori information or on a previous analysis of the sample is enough.

With no loss of generality, in what follows we will assume that the response function is a sole effects one, though, as we shall see in section 6, the application refers to an example where the response function contains effects of both types.

STAGE 2)

2.1.- n models of the type

$$Y_t = w(L)\ X_t + w^i\ (L)\ Z^i_t + \frac{\theta\ (L)}{\phi\ (L)}\ a^i_t \quad , \ i = 1,\dots,n \tag{3.2}$$

are estimated, where w^i (L) is a polynomial in the lag operator L of constant length s, whatever the threshold X^i that is considered.

Normally, s will be equal to the length of w(L), but it is not necessary.

2.2.- In each case the residual standard deviation $\hat{\sigma}_a(i)$ is calculated, choosing as the first threshold the value of X^i so that

$$\min_i \ \hat{\sigma}_a\ (i)$$

Let us call the chosen model S2, and the explanatory variable included in it Z^{s2}.

2.3.- A decision is made between the models S1 (or S0, according to the result of stage 1) and S2, by using some of the criteria of section 5. If S0 or S1 is chosen, the selection process is considered to be over, and we conclude that there is no relationship between X and Y (if the comparison is between S0 and S2), or that there is no nonlinear relationship (if we have compared S1 with S2).

Normally, this will not occur, since it is to be assumed that extra-sample information is available to justify the need to establish the search process on the basis of the existence of a strong nonlinear relationship between X and Y.

When at this stage we approach the comparison between S0 and S2 and we opt for S0, it is essential to check that the variable X really does not help to explain Y.

For this, stage 2 must be reproduced, without including the term $w(L)\ X_t$ in the models (3.2). The reason is as follows: if the unknown nonlinear function combines an interval in which X does not affect Y with an interval in which a relationship does exist, the coefficients of w(L) may be zero or almost zero, and their contribution to the explanation of Y nil. Consequently, we are weighting a marginal gain achieved exclusively with the inclusion of Z^{s2}_t by the number of parameters associated both with X_t and Z^{s2}_t. Thus, the elimination of $w(L)X_t$ from (3.2) may give a different solution in the comparison of the resulting new S2 model with the univariate model.

If S2 is chosen, the variable Z^{s2} is included in all subsequent stages.

STAGE 3)

3.1. - n-1 models are estimated given by

$$Y_t = w(L)\ X_t + w_{s2}(L)\ Z^{s2}_t + w_i(L)\ Z^i_t + \frac{\theta\ (L)}{\phi\ (L)}\ a^i_t$$

$$i = 1,2,\ldots,n$$
$$i \neq s2$$

where now Z^{s2}_t is also included in all expressions. In these models the order of the polynomials w_i (L) is always the same, though they do not necesarily have to coincide with those of w_{s2} (L)or w(L).

3.2. - The model with minimum residual variance is chosen - a model called S3 - which includes Z^{s2} and

Z^{s3} as explanatory variables.

3.3. - A choice is made between S2 and S3. If we decide on S2, the selection process is over. If we choose S3, the process continues with the search for a third variable, and so on. The procedure ends when, at the H+1 stage, the SH model becomes preferable when compared with S(H+1), or when all the $Z^i{}_t$ variables have already been included. Afterwards, the next step is:

STAGE H+2)

At this stage the chosen model,

$$Y_t = w(L)\ X_t + w_{S2}(L)\ Z^{S2}{}_t + \ldots + w_{SH}(L)\ Z^{SH}{}_t + \frac{\theta\ (L)}{\phi\ (L)}\ a_t,$$

is reestimated, eliminating those coefficients which may be nonsignificant at the H+1 stage. Thus, when the functional form of the relationship between X and Y has been determined, we return to the general dynamic specification, to analyse whether any simplification is allowed.

The previous procedure can be summarised in four main points;

1º) The obtaining of a univariate model which registers the main dynamic aspects of Y.

2º) Combination of this dynamic residual term with a first linear approximation to the functional relationship between X and Y with a general dynamic structure.

3º) Iterative approximation procedure of the functional relationship by means of the inclusion of thresholds variables, all of them with general dynamic structures.

4º) Estimation of the resulting specification by the elimination of nonsignificant parameters in the dynamic specifications.

As we said at the beginning of the section, the procedure has as its starting point a consistent model - SO - and incorporates extra information which enables uncertainty in the forecasting of Y to be reduced.

4. CONSISTENCY PROPERTIES OF THE SEARCH PROCEDURE

One of the properties that any dynamic model has to fulfil is that of consistency. For this the model must be clearly specified, which in this case implies that when the size of the sample goes to infinite, the linear approximation tends to the true functional form. This is what we now go on to discuss.

4.1 Consistency of the Approximation Process of the Functional Form

For the piecewise linear approximation to be consistent, as $T \to \infty$ it is necessary that $k \to \infty$ and $k=o(T)$, where k is the number of intervals considered. Consequently, the number of thresholds must grow at a speed less than that of the sample size.

With $k \to \infty$ it is imposed that the discretisation implicit in the approximation of a continuous function by piecewise linear functions is less and less restrictive, up to the point where in the limit the continuous function and its discrete approximation are confused.

With $k/T \to 0$ it is imposed that the number of observations available to estimate the effect on each interval (x^i, x^{i+1}) tends to infinite, guaranteeing consistent estimators of this effect.

Altogether, both requisites ensure that the estimation of f(.) is the result of estimating the effect on arbitrarily small intervals with a number of observations which tend to infinite in each one of them.

Therefore, in principle, any approximation to f(.) is consistent if the number of thresholds is allowed to increase without limit.

This type of consistency requisite appears in several statistical problems which apply discrete approximation techniques to continuous functions.

Thus, Hannan (1963) proposed a semiparametric estimation method for multiple systems of regressions with stationary residuals. To introduce into the analysis the type of stationary process that follows the term of error, a discrete approximation of its spectrum is made.

In Espasa and Sargan (1977) and Espasa (1977) these results were extended to Structural Dynamic Econometric Models, approaching their estimation by full information maximum likelihood in the frequency domain, and approximating the spectrum of each residual term by a succession of piecewise constants.

4.2 Strategy in Dynamic Specification

In a stable world, the dynamics are fixed and finite or approximately finite, and from the very beginning we can have a specification which registers its main characteristics. For this a global vision of the dynamic that one is trying to model must be held, and this can be achieved by starting out from a sufficiently general specification.

4.2.1 The univariate model as an initial condition

The starting point for the procedure is the univariate model of the dependent variable. In the whole process of subsequent search this model is imposed on the disturbance, even though the parameters may be estimated jointly with the dynamic structure of the input.

This ensures that the following properties are fulfilled:

1º) The residual term is white noise, so the statistical techniques based on this assumption have asymptotical validity.

2º) The search process centres on the contribution of the explanatory variables, since the residual dynamic always takes the same form. As well as simplifying the analysis considerably, it has the advantage of facilitating the development of the sequential process.

3º) It is to be expected that the inclusion of explanatory variables may affect the univariate model's residual dynamic structure, as certain factors which were previously recorded in an indirect form from the past of the series may become explicit in the relationship linking X to Y.

As a result, the values of the parameters of the disturbance model must not remain fixed in the estimates obtained for the univariate model. Therefore, it is the data which provide the best combination of residual and systematic dynamics.

This is particularly important if the univariate model includes some unit root. In this case, when an explanatory variable is included it is important not to impose the corresponding difference directly, and to check if this root is still necessary: Dolado, Jenkinson and Sosvilla-Rivero (1990) provide an extensive panorama of this type of tests.

4º) In accordance with the previous point, it may no longer be possible to reject the hypothesis that some parameters of the disturbance model are zero.

This is no problem, since in overparameterised models all the estimators are consistent, including the second moments. Let us remember that the proposed procedure combines a forward search for thresholds with a possibly overparameterised dynamic specification.

Once the approximation to the nonlinear relationship between X and Y has been determined, this possible overparameterisation is corrected by reestimating the model without including those non-significant parameters.

4.2.2 Determination of the length of the polynomials associated to X and to its transformations

In accordance with the treatment given to the dynamic part of the model, in all stages the polynomials w(L) and $w_i(L)$ must be large enough to register all the relevant dynamics. Thus deliberately overparameterised response functions are searched for, in order to keep within reduced limits the probability of not including some relevant lag, even at the expense of possibly including some superfluous lags.

Nevertheless, the decision to include an extra threshold or not depends on the marginal gain provided by the explanation of Y weighted by the number of parameters added to the model. As a result, if we specify polynomials which are too long we may not include relevant thresholds.

To reduce this effect the following must be carried out:

a) To make an initial trial to detect the right length before proceeding to the search for the first threshold.

b) To check, as different thresholds are tried out, that the length of the lag polynomials is not too long.

In any case, at the stage where the choice made is not to include an extra threshold, thus bringing the search process to an end, we must take special care to make sure that this decision is not due to an excessively wide specification of the corresponding w_i (L).

5. CRITERIA FOR CHOOSING BETWEEN MODELS WHEN DEALING WITH FINITE SAMPLES

In the previous section we discussed the conditions for the consistency of the procedure. When these have been established, we must still solve a selection problem among alternative specifications with finite samples. This is the aim of this section.

The problem of choosing the final model cannot be solved by means of the rigorous application of statistical tests, since their derivation for the case of finite sample is very complex. Thus, in some cases a choice must be made among nested models, in others, among non nested models and, in general, among specifications combining the inclusion of superfluous lags with imperfect approximations of the functional form. Thus, the rules to follow in deciding in each case deserve a detailed commentary.

5.1 Proposed Criteria for Choosing Among Nested Models

In order to come to a decision about the inclusion of an extra threshold we propose that the following criteria should be used (Amemiya, 1980):

1) R^2 corrected ($\bar{R}^2$)

2) Prediction criterion (PC)

3) Akaike's Information Criterion (AIC)

In principle, it is better to decide on the basis of the information provided by the three together, though just one may be chosen, in which case, we would use PC or AIC.

With these three criteria we do not intend to give an exhaustive list; rather, our aim is to select a reasonable number of complementary criteria. For our purposes, the three proposed are characterised by:

1º) All are suitable for a forward specification search.

2º) They have different bases, so that in finite samples their joint use provides greater information on the selection problem.

3º) For the reasons mentioned in the previous section, all the selection techniques designed for models with white noise disturbance have an asymptotical validity.

4º) In not making formal tests we do not have a level of significance associated to each decision, so that in some way there is a masking of the problem of the real size of the tests which give rise to whatever model we may finally obtain. Nevertheless, the problem is still latent, and must be borne in mind when analysing the final results. To alleviate this problem post-sample stability tests will be used.

5.2 Proposed Criteria for Choosing Threshold Candidates

In each of the stages 2, ..., H, H+1, we must have a rule for choosing the threshold, from those remaining, whose inclusion in the model will be considered. With no loss of generality, we are going to assume that we are in the second stage: n non nested models like (3.2) have been estimated and it is now a question of choosing which of the variables Z^i_t is the candidate for consideration.

Since these models are not nested, such a rule would have to be based on the Theory of Statistical Testing of non nested hypotheses. Nonetheless, its application in our case comes up against two types of obstacles:

a) The above-mentioned one regarding the distribution of the statistics.

b) These tests are based on a limited number of alternative models. Nevertheless, in most of the cases considered in this paper, the number of possible thresholds n is large; thus a Cox-Atkinson-type contrast requires creating the mixture

$$f_1(Y,X,Z^1{}_t/\delta_1)^{\tau 1} \quad f_2(Y,X,Z^2{}_t/\delta_2)^{\tau 2} \ \ldots \ f_n(Y,X,Z^n{}_t/\delta_n)^{\tau n}$$

with the restriction $\tau 1 + \tau 2 + \ldots . \tau n = 1$, and δ_i is the set of parameters of the model related to $Z^i{}_t$. Other tests present similar problems.

Consequently, the choice of the variable $Z^i{}_t$ to be considered will have to be made on the basis of discrimination criteria between alternative models. Now, since the only difference between the different models is in the variable $Z^i{}_t$ being considered in each case, all criteria are reduced to minimising the sum of squares of the residuals or any monotonous transformation of it. That is why in section 3 we proposed the residual standard deviation as the criterion to be used in selecting among alternative candidates.

There is an additional reason for proposing this criterion. At least in the early stages, the model will be infraparameterised, so that (Hocking 1976, p.6) $E(\hat{\sigma}^{2\,(i)}_{a}) = \sigma^2_a + k^{(i)}$, where $\hat{\sigma}^{2\,(i)}_{a}$ is the estimator of σ^2 in stage i and $k^{(i)}$ a positive constant which is a function of the sample. Thus, the choice of the model with minimum residual variance on average tends to bring us close to the right direction.

5.3 Diagnostic Checking

Once the model has been specified and estimated in its final version, we must go on to validate it. To do this, as well as applying the tests normally used in dynamic modelling, two specific types of analysis must be made:

1º) Sensitivity analysis of the chosen thresholds

To confirm that the iterative procedure has led to a suitable approximation of the non observable relationship, it is worthwhile comparing the final model with slightly different specifications which are non nested with the proposed one, and in which the chosen thresholds have been slightly varied.

To accept as valid the model achieved in the H+2 stage it is essential that this model should not be worse than any of the alternatives from the previous paragraph.

This comparison could be made from a test of non-nested hypothesis. Nevertheless, for reasons which we shall see below, a test based on the asymptotic distribution of the residual variance must produce similar results, and be much easier to implement.

We know that in a model of the form

$$Y_t = \sum_{i=1}^{n} w_i(L)\ Z^i_t + n_t$$

where $n_t = \mu(L)\ a_t$ is a stationary disturbance, the maximum-likelihood estimator of σ^2_a achieves that

$$\hat{\sigma}^2_a \underset{a}{\approx} N\ [\sigma^2_a,\ (2\sigma^4_a\ /\ T)^{1/2}] \quad .$$

Therefore, given a probability $\beta = 1 - \alpha$, we have that asymptotically

$$P\ [\hat{\sigma}^2_a < \sigma^2_a - c\ (2\sigma^4_a\ /\ T)^{1/2}] = \alpha$$

with c a value such that $P(N(0,1) > -c) = \beta$. In this sense,

$$K = \sigma^2_a - c\ (2\sigma^4_a\ /\ T)^{1/2}$$

is a lower quota of the values of σ^2_a. As we do not know σ^2_a, we can substitute its value by a consistent estimator, so that

$$\hat{K} = \hat{\sigma}^2_a - c \quad (2\hat{\sigma}^4_a / T)^{1/2}$$

If for a specification different to the one proposed the estimator of the corresponding residual variance, denoted by $\hat{\sigma}^2_a(A)$, is less than $\hat{K}$, we may assume that the alternative specification is better than the proposed one. Otherwise, we conclude that the proposed one is not worse than the alternative one.

This test, of asymptotical validity, has less power than the usual tests of non nested hypothesis for small samples. But, bearing in mind the sample sizes needed for applying the procedure of this paper, the distribution for finite samples is thought to be sufficiently close to the asymptotical for the loss of power to be made up for by its greater simplicity.

2º) Prediction tests

Given that the specification search procedure is based on maximum exploitation, by means of iterative methods, of the information contained in a specific sample, we must consider the problem of data mining.

It follows that to be able to guarantee having achieved a model that reproduces the theoretical data generating process, and not the characteristics of a particular sample, the results must be supported by rigorous post-sample prediction tests.

Regardless of whether the final aim of the model is forecasting or some other function, these tests are both structural change and misspecification tests. In any case, surmounting them is essential for validating the model.

To implement it, the usual chi-square test based on the prediction errors variance is completely valid. To increase its power it must be designed as a one tail test, in such a way that the rejection area is exclusively associated with too high values of the variances of forecasting errors.

A crucial point in the methodology proposed here is that of the availability of sample information extensive enough for exhaustive tests of this type to be carried out. Though at first sight this requisite limits its field of application, this is a simple consequence of the task proposed: as a counterweight to not having imposed any type of a priori restrictions either on the dynamics or on the type of nonlinearity existing in the relationship, we have to have extensive sample information.

6. APPLICATION TO THE FORECASTING OF DAILY ELECTRICITY DEMAND

This specification procedure has been applied to model the relationship between temperature and the daily electricity demand. Given that daily demand is highly sensitive to alterations in working conditions (bank holidays, summer, Easter and Christmas holidays,etc), first of all a univariate model with intervention analysis was constructed in the form

$$LD_t = \delta ' I_t + \frac{\theta^u(L)}{\blacktriangle\blacktriangle_7} e_t$$

where LD_t is the logarithm of daily electricity demand in Spain (except for the Canary Islands, the Balearics, Ceuta and Melilla), $\delta ' I_t$ summarises a set of intervention variables for modelling the alterations in working conditions, and

$$\hat{\theta}^u(L) = (1 + 0'05L - 0'04L^2) \# (1 - 0'87L^7 - 0'05L^{14}) \# (1 + 0'05L^{357} + 0'10L^{364} + 0'05L^{365} + 0'04L^{728} + 0'04L^{731} + 0'10L^{735}) \quad .$$

The residual standard deviation equals 0.015738. For the construction of this model the 2557 observations included between January 1, 1983 and December 31, 1989 have been used.

It is important to note that the nonparametric and semiparametric procedures proposed in Electric Power Research Institute (1983) and Engle, Granger, Rice and Weiss (1986) can not be applied in this case, due to the complexity of the daily dynamics of LD.

This model enables us to use a corrected electricity demand such as $LD^c_t = LD_t - \hat{\delta}' I_t$, a variable which is supposed to be related to temperature (T_t) by means of a dynamic and nonlinear function. Ignoring the dynamics, the a priori information suggests that the relationship is in the form described in Figure 1.

HERE COMES FIGURE 1

There is a neutral zone, corresponding to temperatures between T* and T** in figure 1, in which the effect of temperature on demand is nil. For temperatures below T* we enter the so called cold zone, where demand reacts positively to falls in temperature, while above T** we are in the hot zone, where demand reacts positively to increases in temperature. The more we penetrate each of these zones, the greater is the response of demand.

The problem posed consists of determining the values of T* and T**, as well as analysing the degree of nonlinearity of the response in each of the hot and cold zones.

The temperature variable which has been used is a national index of maximum daily temperature, obtained as a weighted average of the maximum temperatures of a series of representative observatories.

Bearing in mind the particular aspects of the forecasting problem that is posed, we can add to the theoretical information of a general nature:

- The climatological conditions in Spain are such that index values below 8° C(46.4° F) or above 32° C (89.6° F) are rare. As a result, we have not considered thresholds for temperature values below 8° C or above 32° C.

Neither did it appear reasonable to have cold zones extending beyond 22° C(71.6° F) or hot zones beginning before 22° C. In addition, the theoretical information also imposes on us the threshold marking the beginning of the cold zone not being higher than the threshold marking the beginning of the hot zone.

- We have not found any reason to intensify the search in a particular interval, and it has been judged satisfactory to use thresholds separate among themselves at 1° C (1.8° F). Therefore, possible thresholds were defined for the cold zone at 8°C, 9°C, 10°C, 11°C, 12°C, 13°C, 14°C, 15°C, 16°C, 17°C, 18°C, 19°C, 20°C, 21°C, and 22°C; for the hot zone,the candidates were 22°C, 23°C, 24°C, 25°C, 26°C, 27°C, 28°C, 29°C, 30°C, 31°C and 32°C.

- As far as the dynamics between temperature and electricity demand is concerned,the available information indicated that a polynomial of order 10 (registering possible contemporary effects, one day ahead,..., nine days ahead), was long enough.

The following step was to construct the variables associated to each cold threshold

$$C^i_t = \begin{cases} i - T_t & \text{if} \quad T_t \leq i \\ 0 & \text{if} \quad T_t > i \end{cases}$$

(with i = 8°C, 9°C,..., 22°C)

and to each heat threshold

$$H^i_t = \begin{cases} T_t - i & \text{if} \quad T_t \geq i \\ 0 & \text{if} \quad T_t < i \end{cases}$$

(with i = 22°C, 23°C,..., 32°C),

and then to apply the selection procedure step by step.

To avoid the problem of data mining we have reserved the first six years (1983 to 1988) for specification and estimation, and the seventh (1989) to make prediction tests. Furthermore, given the characteristics of the problem, we have proceeded in an independent way with the cold and hot zones: thus, instead of deriving threshold by threshold, we have dealt with each zone separately, without combining thresholds of cold and heat in the same model till the last stage.

Beginning the development for the cold zone, in the first stage we construct models in the form

$$LD^c_t = w^i \; (L) \; C^i_t + \frac{\theta(L)}{\nabla\nabla_7} a^i_t$$

where i = 8° C, 9° C...., 22° C; $\theta(L)$ is a polynomial similar to $\hat{\theta}^u(L)$ though with coefficients which are jointly estimated with those of w^i (L); the latter is a polynomial of the form $w_i(L) = w^i_0 + w^i_1 L + \ldots + w^i_9 L^9$ and in all cases the lack of structure in a^i_t is shown. We also have checked that the regular and weekly MAs of the residual term do not contain any unit root which is cancelled with the difference operators.

The relevant results for the choice of the first threshold are registered in the first column of results of Table 1: the minimum is produced for $C^{20}{}_t$, so that the candidate value to the first threshold is 20° C (68° F).

HERE COMES TABLE 1

The residual standard deviation for the univariate model, calculated with the 1983-1988 sample, is equal to 1.5868%; the values of the chosen criteria both for the univariate model and that including $C^{20}{}_t$ are registered in Table 2.

HERE COMES TABLE 2

In the next stage we construct models of the form

$$LD^c{}_t = w^{20}(L)\ C^{20}{}_t + w^i(L)\ C^i{}_t + \frac{\theta(L)}{\blacktriangle\blacktriangle_7}\ a^i{}_t$$

with i = 8°C, 9°C, ..., 19°C, 21°c and 22°C. We first tried out polynomials of length ten, and did not obtained improvements. But by revising the individual coefficients we observed that this may be due to our imposing responses that are too long.

Consequently, we repeated the process by trying out response functions of the form $w^i(L) = w^i_0 + w^i_1 L + w^i_2 L^2 + w^i_3 L^3$ for all possible thresholds, except for C^{20}_t, which still had a polynomial of length ten. The residual standard deviation of the resulting models appears in the column labelled as "second threshold" in Table 1.

Everything points to a possible second threshold at 8° C. To confirm it, the three criteria for this new model were calculated; after having compared the corresponding columns in Table 2, we decided to add a new threshold at 8° C.

Even though there are few temperatures below 8°C, this second knot enables a much better understanding of the response of the demand to extreme values of T in the cold zone. The extended model has much smaller residuals for the dates affected by these extreme values.

The next step is to estimate expressions of the form

$$LD^c_t = w^{20}(L)\ C^{20}_t + w^8(L)\ C^8_t + w^i(L)\ C^i_t + \frac{\theta(L)}{\blacktriangle\blacktriangle_7} a^i_t \quad ,$$

where $w^{20}(L)$ has length 10, $w^{8}(L)$ length 4 and the previous results recommend trying out $w^{i}(L) = w^{i}_{0} + w^{i}_{1}L$, $i = 9,10,\ldots, 19,21,22$.

The last column of Table 1 registers the residual standard deviations of all possible specifications. It can be inferred from it that all candidates produce very similar values, and that, unlike before, there is no global minimum, but various local minima. The minimum residual standard deviation occurs for the model with a threshold at 18° C (64.4° F).

When comparing the two last columns of Table 2, it is seen that, though marginally, $\bar{R}^2$ suggests the inclusion of the new threshold, while PC and AIC lead to stop the search process. This is to be expected, since in general PC and AIC are more restrictive than $\bar{R}^2$ when it comes to including a new variable.

Given that the final aim of this model is to forecast, we have opted for not including the new threshold, since the gain in terms of $\hat{\sigma}_a$ is practically nil. Of course, an alternative attitude would be to include this threshold, search for a fourth candidate and so on, checking in the efficient estimation of the resulting specification whether the threshold at 18° C is really significant.

Let us now go on to detail the search process for the hot zone; we begin by trying out specifications of the type

$$LD^c_t = w^i(L)\ H^i_t + \frac{\theta(L)}{\Delta\Delta_7}\ a^i_t$$

where i = 22° C, 23° C, ..., 32° C. Table 3 presents the residual standard deviations of the models obtained, and points to a threshold at 24° C (75.2° F). The corresponding values of the criteria are shown in Table 4, and all of them point towards the existence of a hot zone.

HERE COME TABLES 3 AND 4

Afterwards, we pose

$$LD^c_t = w^{24}(L)\ H^{24}_t + w^i(L)\ H^i_t + \frac{\theta(L)}{\Delta\Delta_7}\ a^i_t$$

for i = 22° C, 23° C, 25° C, ..., 32° C. Here a problem arises similar to the one experienced in the cold zone: if for the first threshold polynomials of length ten are a good starting point, for this second one they are not suitable since the responses of new thresholds, when they exist, are shorter.

It follows that, once this has been checked, we should repeat the previous ten models considering $w^i(L) = w^i_0 + w^i_1 L + w^i_2 L^2$ for all the heat variables except H^{24}_t. The new results appear in the third column of Table 3, which advises the choice of H^{29}_t as the possible second threshold. The resulting values for the three criteria, indicate that this second threshold must be included in the model.

In the following stage, we estimate specifications of the form

$$LD^c_t = w^{24}(L)\ H^{24}_t + w^{29}(L)\ H^{29}_t + w^i(L)\ H^i_t + \frac{\theta(L)}{\blacktriangle\blacktriangle_7}\ a^i_t$$

with i = 22°C, 23°C, 25°C, 26°C, 27°C, 28°C, 30°C, 31°C and 32°C, and $w^i(L) = w^i_0 + w^i_1 L$.

The last column of Table 3 registers the resulting standard deviations which show a minimum al 30°C. After calculating the new values of the criteria, we are faced with the same situation that occured when considering a third threshold for the cold zone, since $\bar{R}^2$ suggests one action and PC and AIC another.

For the same reasons already mentioned, we have decided to stop the process and proceed to the joint estimation of a specification including the variables C^{20}_t, C^8_t, H^{24}_t and H^{29}_t. All the thresholds turn out to be significant, though the length of the dynamic response is reduced due to the existence of nonsignificant coefficients.

The final model is presented in the Appendix; Figures 2 and 3 summarize the relationship between temperature and electricity demand. The main characteristics of the model are:

HERE COME FIGURES 2 AND 3

1) The relationship is nonlinear, with knots at 8°C (46°F), 20°C (68°F), 24°C (75°F) and 29°C (84°F); there is a temperature interval, between 20°C and 24°C, so that there is no relationship between T and D.

2) Temperatures below 20°C form the cold zone and temperatures above 24°C the hot zone. In the cold zone the relationship can be satisfactorily approached by a linear function between 8°C and 20°C. For temperatures below 8°C, the slope of the response function becomes less, which indicates less elasticity of demand compared to temperatures above 8°C.

3) In the hot zone the relationship is similar. There is a response function that is linear from 24°C to 29°C. Upwards of 29°C, the slope is less, so that

the marginal effect of temperatures above 29°C is less than for temperatures between 24°C and 29°C.

4) The gain of the transfer function associated to the cold zone is much greater than the gain of the function of the hot zone.

5) The dynamics between T and D is concentrated in the main thresholds, 20°C for the cold zone and 24°C for the hot zone. The relevant lags go from 0 to 7 in both cases, though the lags 0, 1 and 2 concentrate a large part of the gain.

6) The auxiliary thresholds, 8°C and 29°C, have associated negative coefficients, which provoke demand into being more inelastic for extreme temperatures. The dynamic of the polynomial linked to the knot at 8°C registers effects in the lags 0, 1 and 2; the knot at 29°C only affects D with a lag at one day.

7) The residual term shows dynamic dependence, which is modelled by means of an ARIMA process of the MA form (1,2) (7,14) (357, 364, 365, 728, 731, 735) and (1-L) $(1-L^7)$ for the AR part. This dynamic structure is almost the same as that of the univariate model, with some changes in the coefficents of the lags one and two.

8) The residual standard deviation goes from 1.57% in the univariate model to 1.33% in the model with a temperature effect. This means a reduction of the residual variance of more than 28%.

9) This reduction of the variance is in part achieved by almost eliminating the big errors of the univariate model. This last model is unable to capture the variations in the demand related to a sudden change of temperature, which entires bad predictions for certain dates. Our final model permits to handle these variations in such a way that extreme errors are much less frequent. As the cost associated with a prediction error grows more than proportional in the electric sector, this is a major improvement.

As for the predictions tests which we have made before accepting the final model as valid, these are summarised in Table 5.

HERE COMES TABLE 5

The test used is the usual one, given by

$$\sum_{i=1}^{n} \frac{e_i^2}{\hat{\sigma}_a^2} \underset{A}{\sim} X_n^2$$

where e_i is the one-period prediction error for the day i and $\hat{\sigma}_a^2$ the estimation of the residual standard deviation obtained in the efficient estimation of the model for the period 1983 to 1988.

The test has been made for the year as a whole and by half years, always with a size of 5% on the right tail. The results validate the specification proposed.

A final note concerning heteroskedasticity: when dealing with daily series of economic activity, heteroskedasticity is a much smaller problem than when analyzing daily financial data. Moreover, its main determinants in our problem are related to temperature variation: the final model residuals seem much more homoskedastic than the univariate model innovations. Even if formal hypothesis testing still indicates a slightly monthly-varying variance for the final model, its effect on forecasting is negligible; and everything point that the best way to handle it is by including additional metheorological variables instead of adding a second equation for the variance.

7. <u>CONCLUSIONS</u>

With this paper we aim to have offered an operative procedure for the specification and validation of nonlinear dynamic relationships.

This procedure, based on starting from the univariate model of the series to be explained and adding at successive stages degrees of nonlinearity in the relationship with the explanatory variable, is essentially data-based.

Therefore, it is particularly recommendable in those cases where the problem centres on the choice of specification to be used. When the available a priori information allows the search process to be limited, a suitable modification of it in the indicated direction continues to provide good results.

Since it is a data-based procedure there is a risk of finally modelling the characteristics of a particular sample, rather than the true data generating process (DGP). Thus, it is vital to reserve part of the available sample for carrying out post-sample tests, which requires large enough sample sizes.

We have discussed the consistency of the process, by which we understand that as the size of the sample tends to infinite, the specification proposed tends to the true DGP. However, since it is a process specifically thought out for being applied in real data analysis, an attempt has been made to provide a number of criteria and tests which enable it to be applied in finite samples.

Finally the procedure has been applied to model the daily relationship between electricity demand and temperature in Spain and the results have been proved very useful in forecasting the demand.

APPENDIX: FINAL MODEL FOR THE DEMAND OF ELECTRICITY

The daily relationship between electricity demand and temperature is given by

$$LD^{c}_{t} = w^{8}(L)\ C^{8}_{t} + w^{20}(L)\ C^{20}_{t} + w^{24}(L)\ H^{24}_{t} + w^{29}(L)\ H^{29}_{t} + \frac{\hat{\theta}(L)}{\Delta\Delta_{7}}\ a_{t}$$

where the impulse response functions are summarized in Table A.1, and the MA part is

$$\hat{\theta}(L) = (\ 1 - \underset{(8.1)}{0.17L} - \underset{(8.1)}{0.17L^{2}}\ )\ \#\ (\ 1 - \underset{(38.6)}{0.84L^{7}} - \underset{(3.4)}{0.08L^{14}})\ \#\ (\ 1 + \underset{(3.3)}{0.08L^{357}} + \underset{(6.5)}{0.15L^{364}} + \underset{(1.9)}{0.04L^{365}} + \underset{(3.5)}{0.09L^{728}} + \underset{(3.2)}{0.08L^{731}} + \underset{(4.5)}{0.11L^{735}})$$

The final residual standard deviation is equal to 0.0132981.

HERE COMES TABLE A.1

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TABLE 1. Residual Standard Deviation for the Choice of Thresholds; Cold Zone

POSSIBLE THRESHOLD (°C)	FIRST THRESHOLD	SECOND THRESHOLD	THIRD THRESHOLD
8	1.5791	1.3768	--
9	1.5749	1.3776	1.3768
10	1.5695	1.3783	1.3766
11	1.5577	1.3809	1.3762
12	1.5408	1.3824	1.3765
13	1.5188	1.3840	1.3767
14	1.4931	1.3847	1.3766
15	1.4660	1.3850	1.3764
16	1.4393	1.3859	1.3761
17	1.4207	1.3864	1.3764
18	1.4058	1.3863	1.3760
19	1.3939	1.3866	1.3761
20	1.3878	--	--
21	1.3908	1.3872	1.3763
22	1.4012	1.3876	1.3766

NOTE: All residual standard deviations have been multiplied by 100.

TABLE 2. Criteria for the Selection of Thresholds; Cold Zone

CRITERION	UNIVARIATE MODEL	MODEL with C^{20}_t	MODEL with C^{20}_t, C^8_t	MODEL with $C^{20}_t, C^8_t, C^{18}_t$
$\bar{R}^2$	0.9875851	0.9905040	0.9906361	0.9906383
PC	2.55325E-04	1.96190E-04	1.93817E-04	1.93951E-04
AIC	2.55325E-04	1.96190E-04	1.93817E-04	1.93951E-04

TABLE 3. Residual Standard Deviations for the Choice of Thresholds; Hot Zone

POSSIBLE THRESHOLD	FIRST THRESHOLD	SECOND THRESHOLD	THIRD THRESHOLD
22	1.5631	1.5582	1.5558
23	1.5602	1.5590	1.5567
24	1.5590	--	--
25	1.5603	1.5582	1.5565
26	1.5644	1.5578	1.5564
27	1.5691	1.5571	1.5566
28	1.5733	1.5570	1.5567
29	1.5781	1.5568	--
30	1.5802	1.5578	1.5555
31	1.5807	1.5582	1.5563
32	1.5806	1.5586	1.5567

NOTE: All residual standard deviations have been multiplied by 100.

TABLE 4. Criteria for the Selection of Thresholds; Hot Zone

CRITERION	UNIVARIATE MODEL	MODEL with H^{24}_t	MODEL with $H^{24}_t H^{29}_t$	MODEL with $H^{24}_t H^{29}_t H^{30}_t$
$\bar{R}^2$	0.9875851	0.9880165	0.9880334	0.9880422
PC	2.55325E-04	2.47591E-04	2.47579E-04	2.47624E-04
AIC	2.55325E-04	2.47590E-04	2.47579E-04	2.47624E-04

TABLE 5. Prediction Tests; 1989

PERIOD	No OF OBSERV.	STATISTIC	CRITICAL VALUE AT 5 %
JAN-DEC	365	313.6	410.1
JAN-JUNE	181	114.8	213.0
JULY-DEC	184	199.5	216.3

NOTE: We used the approximation $(2X^2_n)^{1/2} - (2n - 1)^{1/2} \underset{A}{\sim} N(0,1)$

TABLE A.1 Impulse Response Functions for the Final Model

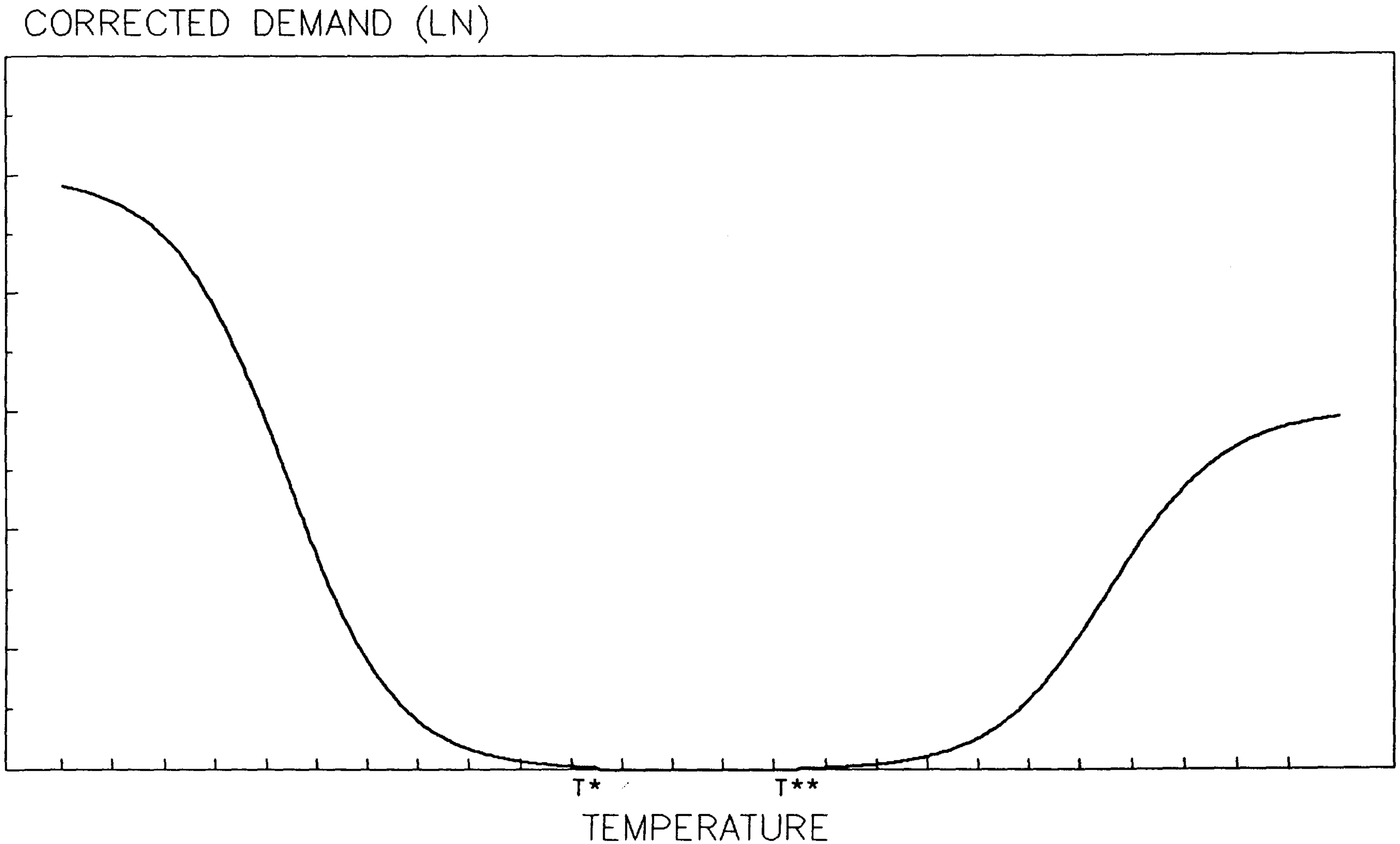
LAG	C^8_t	C^{20}_t	H^{24}_t	H^{29}_t
	VARIABLE			
0	-.592 (5.4)	.538 (22.8)	.174 (5.5)	
1	-.184 (1.6)	.304 (12.9)	.390 (9.2)	-.216 (3.0)
2	-.244 (2.2)	.273 (11.7)	.104 (3.3)	
3		.147 (7.1)	.065 (2.1)	
4		.129 (6.2)	.013 (.4)	
5		.086 (4.2)	.095 (3.0)	
6		.059 (2.9)	.087 (2.8)	
7		.058 (2.8)	.093 (3.0)	
gain	-1.020	1.594	1.021	-.216

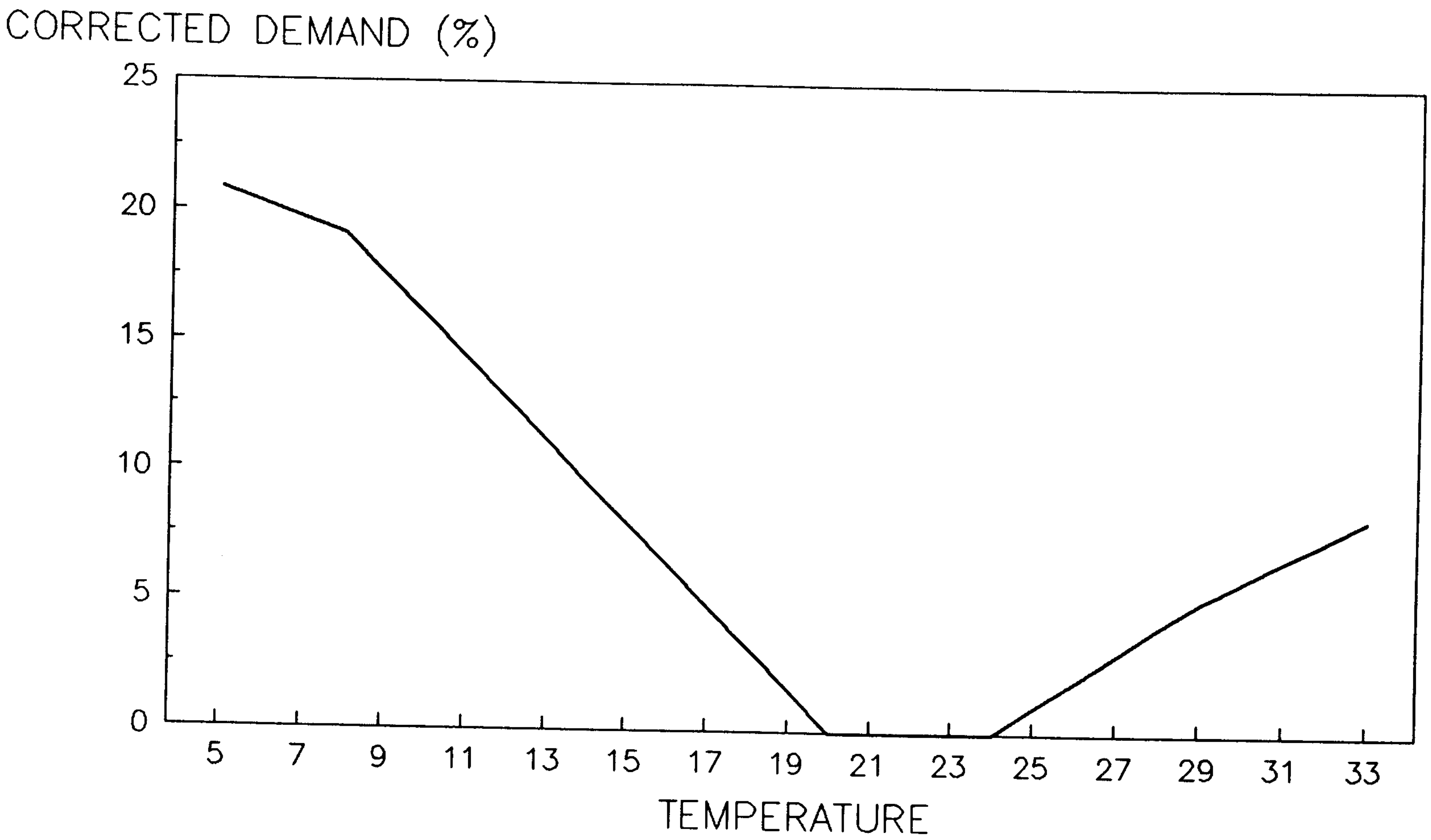
NOTE: The coefficients may be interpreted as semielasticities, as they show the effect (multiplied by 100) that a change of 1°C has on LD^c

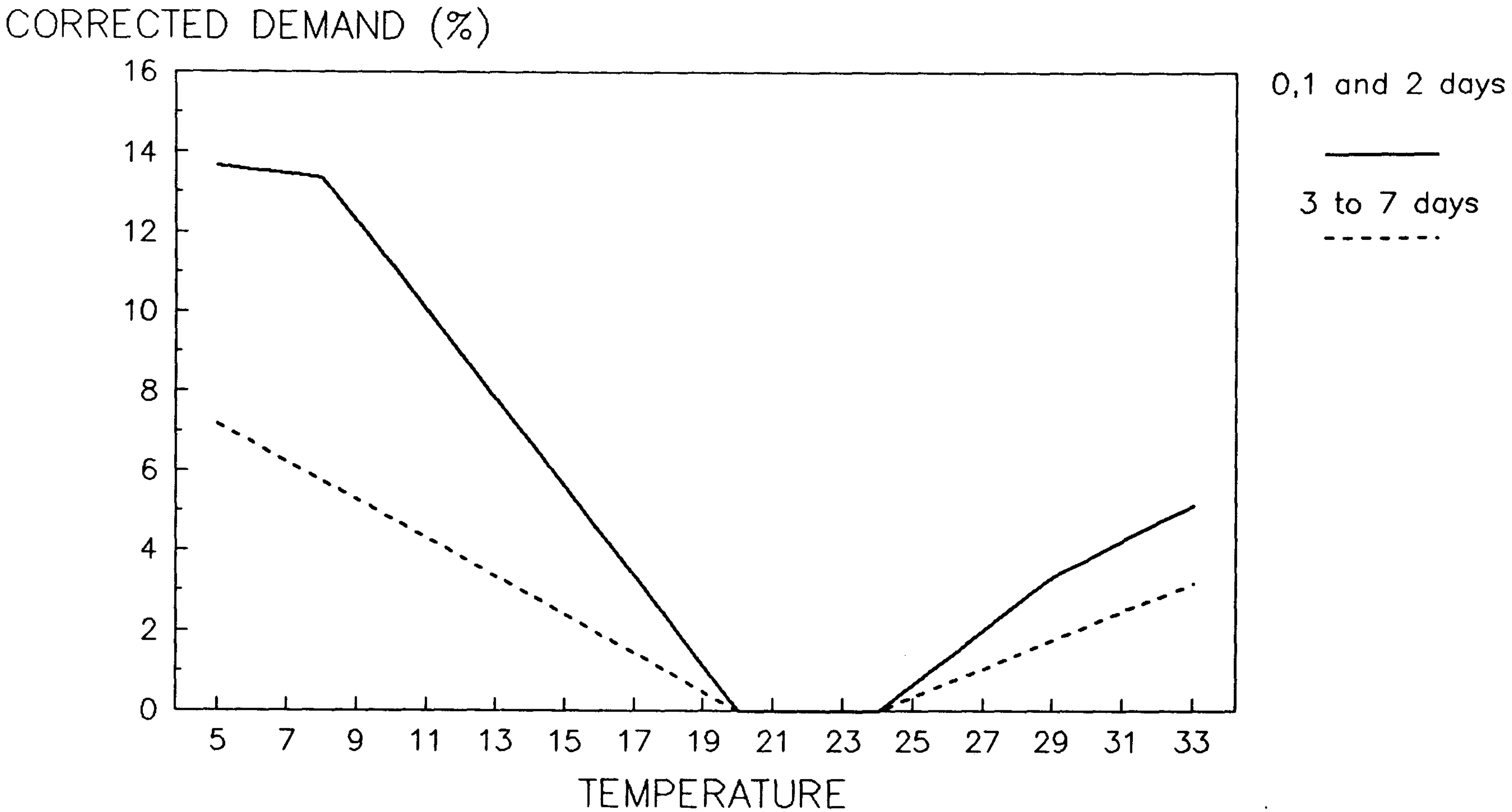
Figure 1. Theoretical Form of the Relationship Between Temperature and Electricity Demand

Figure 2. Estimated Relationship: Total Effect of a Given Temperature on the Corrected Demand

Figure 3. Summary of the Dynamics


CORRECTED DEMAND (LN)
T*
T**
TEMPERATURE


CORRECTED DEMAND (%)
25
20
15
10
5
0
5 7 9 11 13 15 17 19 21 23 25 27 29 31 33
TEMPERATURE


CORRECTED DEMAND (%)
16
14
12
10
8
6
4
2
0
5 7 9 11 13 15 17 19 21 23 25 27 29 31 33
TEMPERATURE
0,1 and 2 days
3 to 7 days