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OPTIMIZATION OF PRODUCT MIX IN A TYPICAL YARN MANUFACTURING INDUSTRY

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ABSTRACT

In a typical yarn manufacturing company different types of yarns are produced and usually a schedule of production is prepared and followed for smoother production. However, company's taking decision regarding order receiving and scheduling become tougher when it gets simultaneous orders of different yarns. A compromise is required between profit and capacity. An LP model has been formulated for some selected yarns and solved and this paper presents an analysis to find the optimum product mix indicating the quantity (tons) of each category of yarns to be produced and the duration of machine hours to be allocated. It would also help to identify the abundant or scarce resources. Moreover, through the analysis it is possible to prioritize the expansion of the scarce resources in allocation of additional investment, if required. A sensitivity analysis in this regard provides the information of increment of the scarce resources and maximum change in marginal profit

Keywords: LP, Product-mix, Sensitivity

1.INTRODUCTION

Linear programming problems are concerned with the efficient use/or allocation of limited resources to meet the desired objectives. These problems are characterized by a large number of solutions that satisfy the basic conditions of each problem. Selection of a particular solution as the best solution to a problem depends on some aim or overall objective that is implied in the statement of the problem. A solution that satisfies both the conditions of the problem and the given objective is termed as an optimum solution. A typical example is that of the manufacturing company that must determine what combination of available resources will enable it to manufacture products in a way which not only satisfies its production schedule, but also maximizes its profit.

A linear programming problem differs from the general variety in that a mathematical model or description of the problem can be stated, using relationship, which are called 'straight line' or 'linear' equations. Mathematically, the relationships can be expressed in the form

$$a_1x_1 + a_2x_2 + \ldots\ldots + a_jx_j \ldots\ldots + a_nx_n = b$$

where, a_j's and b are known coefficients and the x_j's are unknown variables

The complete mathematical statement of a linear programming problem includes a set of simultaneous linear equations representing the conditions of the problem. Thus an LP problem describes a valid, practical programming problem usually as a nonnegative solution with a corresponding finite value of the objective function. [1]

The yarn manufacturing company under study has varieties of products/yarns. But to start the problem with simplicity only four major products/yarns [Combed Yarn (100% Cotton with long fiber), Carded Yarn(100% Cotton with medium/short fiber), Blended Yarn (35% Cotton & 65% polyester), and Polyester Yarn (100% polyester)] has been considered for study and subsequent analysis. Combing, Carding, Simplex, and Ring machines are primarily used in processing the yarns. A particular set of machines may be used for a particular type of yarn production. Except Combed yarn all other yarns passes through the same Combing, Carding and Simplex machines. The Combed yarn passes through all three machines as well as combing machine. The constraints for LP formulation are taken as available non-stop machine hours per day corresponding to different machines. The capacity constraint in terms of available machine hours seems to be most significant since time management is a vulnerable part for any production organization. However, the company receives a lot of orders for different products/yarns. This situation provides an opportunity for the company to look for an optimum product-mix (the quantity of each product per day) with view to ensuring maximum profit.

It is more important to get analytical result from linear programming instead of merely numerical answers. Someone may be interested to know how an answer depends on the input specifications or how sensitive the solution is to the original data. The importance of the sensitivity analysis becomes apparent since often the technological specifications are based on estimates and the constraints included may only be approximate. Further, a number of real constraints may be promotionally left absent from the model [2]. And the objective function may not completely exhaust the factors of relevance in evaluating a solution.

In this paper the optimum product mix has been determined as well as the sensitivity analysis was carried out in order to get relevant management decisions.

1. MATHEMATICAL FORMULATION OF THE PROBLEM

The method of solution of a linear programming problem by evaluating all basic feasible solution is not efficient enough. The simplex method of solving linear programming problems is a method, which does not ordinarily require evaluation of all basic feasible solutions. The simplex method, in addition to giving the optimal solution give 'shadow price' [3] of the limited sources, information that are very useful for management planning.

It has already been pointed out that the company under study is a textile industry that produces different varieties of yarns. All varieties are consumed in local markets. There is a steady demand of the company's products because of consistent quality of the yarn. Again garments sector in Bangladesh is an emerging sector and the demand for quality yarns is increasing day by day. Management in such companies is conscious enough for quality product. However, attention in the solution of industrial operational problems (specifically decision making problem) appears to be inadequate. A recent study shows that a local dyeing industry could increase its productivity upto 42% using same facilities if optimal production planning was maintained [4].

Formulation of a decision problem into mathematical form is called mathematical programming. The mathematical programming of the problem is presented below sequentially.

1.1 Products Varieties

It is already stated that too many products/yarns of different grades the company is currently manufacturing. But some yarns are manufactured on large scale and some are on medium scale. Again some yarns are manufactured upon only getting large order. Therefore, the yarns that seem to be produced/manufactured all the year round (four major products/yarns) were taken into account for LP analysis. The combed yarn is the most expensive yarn since it is manufactured only from fine graded long fibers, which are picked by combing long fibers from raw cotton. Long fiber needs few turns of twisting compared to short fiber. Therefore, a garment made from combed yarn gives more comfort absorbing more sweat from human body. In addition to combed yarn, three more yarns such as carded, blended and polyester yarns are considered for analysis and numbered below as Combed yarn (100% cotton) – Product 1, Carded yarn (100% cotton) – Product 2, Blended yarn (35% cotton & 65% polyester) – Product 3, Polyester yarn (100% polyester) – Product 4

1.2 Machine's Capacities

It was found many machines were involved in operation for production in the factory. But for yarn manufacturing only the machines that are indispensable for production system were taken into consideration for LP formulation. The machines are classified according to their operation. a) Carding Machine (No of Machine 35): 40 kg per hr (product 1 & 2), 45 kg per hr (product 3 & 4); b) Combing Machine (No of Machine 20): 25 kg per hr (product 1 only); c) Simplex Machine (No of Machine 13): 110 kg per hr (product 1 & 2), 192 kg per hr (product 3) 41.5 kg per hr (product 4); d) Ring Machine (No of Machine 144): 6.8 kg per hr (product 1), 6.3 kg per hr (product 2), 15.43 kg per hr (product 3), 9.25 kg per hr (product 4). Machine capacities in terms of hour required per ton of yarn production (instead of Kg/hr or ton/hr) was determined and shown in Table 1

Table 1: Capacities of different machines and processing time

Stage	Time/unit (hour per ton)				Stage capacity (hr/day)
	Product 1	Product 2	Product 3	Product 4	
Carding	0.71	0.71	0.63	0.63	22
Combing	2.0	-	-	-	21
Simplex	0.70	0.70	0.40	0.84	19
Ring	1.02	1.10	0.45	0.75	23

2.3 Sample Calculation For Processing Time

This sample calculation is made for the carding machine for product 1. Reciprocal of 40 kg per hr = hr/40 kg = 1000 hr/ (40x1000 kg) = 1000/40 hr/ton = (1000/40)/35 hr/ton [for all 35 machine] = 0.71 hr /ton

2.4 Objective Function and Restriction Equation

Profit data for per ton production of each yarn is required in determining the objective function. In any organization the management become reluctant in disclosing sales/production cost related data for confidential reasons. The industry in which the study was carried out was not something different. However, the cost/sales related data was assumed as that of Table 2.

Table 2: Sales Information of different yarns

Count	Sa. Price Tk./lb	Var. Cost Tk./ lb	Fix. Cost Tk./lb	Profit Tk./lb	Profit Tk./ ton
Combed	96	45	36	15	33000
Carded	72	35	34	3	6600
Blended	60	30	28.75	1.25	2750
Polyester	44	22	24	2	4400

Usually for a company five sets of restriction equation [5] can be imparted: i) Plant Capacity ii) labour hours iii) machine hours iv) ingredients and v) maximum level of production. The company under study was found highly automated and least use of manpower. Labour cost is cheap enough therefore; the restriction for labour hours is insignificant. Primarily the company is backward linkage industry for textile industry, which produces different graded yarn. Though there exist varieties in different graded yarn, no of raw material/ingredients required for yarn production are very limited only cotton of different fiber length: long fiber, medium long fiber and short fiber. Therefore, the restriction in this area is also insignificant to us. Again, the study is carried out to find the exact situation of production therefore; no binding/restriction regarding maximum level of production was primarily imposed. The restriction in plant capacity (kg/hr) and machine hours (hr/day) are seems to too important. A combined restriction formula was developed where plant capacity was used as coefficient for variables in the restriction equation. Let Z be the profit and x_i be the production rate (ton/day) of yarn. Subscripts 1, 2, 3 and 4 represent Combed, Carded, Blended and Polyester yarn respectively.

Therefore the objective function is to maximize $Z= 33000x_1+6600x_2+2750x_3+4400x_4$, which is subjected to the constraints

$$0.71x_1+0.71x_2+0.63x_3+0.64x_4<=22$$
$$2x_1<=21$$
$$0.70x_1+0.70x_2+0.40x_3+0.84x_4<=19$$
$$1.02x_1+1.10x_2+0.45x_3+0.75x_4<=23$$

After the construction of objective function and determining the constraints the problem was run by education version TORA/POM software. Final table of LP solution is presented in Table 3.

Table 3: Final Table of the LP solution

Basic	Z	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	Soltn
Z	1	0	0	0	121	147	13436	0	5904.9	421213
x_3	0	0	0	1	0.42	2.94	-0.07	0	-1.9	19.4
x_1	0	1	0	0	0	0	0.50	0	0	10.5
S_3	0	0	0	0	0.31	-0.3	-0.01	1	-1.42	1.61
x[illegible]	0	0	1	0	0.50	-1.2	-0.43	0	1.68	3.20

2. SENSITIVITY ANALYSIS

By examining the outcome of the simplex method computation, the following information can be made available:

a) Optimum solution indicating the desired quantity of production per day to ensure maximum profit.
b) Status of resources identifying whether they are scarce or abundant.
c) The dual price i.e. the unit worth of resources or shadow price.
d) The sensitivity of the optimum solution under the changes in availability of resources and fluctuation of marginal profit/cost

2.1 Optimum Solution

From the standpoint of implementing the LP solution, the mathematical classification of the variables as basic and non-basic is of no importance and should be totally ignored in reading the optimum solution. From the optimum Table we have the following summary.

Decision Variable	Optimum value	Decision Ton/day
x_1	10.5	Produce 10.5 ton of combed yarn
x_2	3.206	Produce 3.206 ton of carded yarn
x_3	19.47	Produce 19.47 ton of blended yarn
z	421213	Resulting a profit of Tk. 4.21 Lacs

Therefore, profit $Z = 33000x_1+6600x_2+2750x_3 = 33000x10.5+6600x3.206+2750x19.47$ = Tk. 4.21 Lacs. Again it was stated in abstract that duration of machine hours to be allocated, will be determined which would facilitate management to meet optimum product mix. The following Table depicts the distribution of machine hours required to follow for each product/yarn production.

Stage	Time (hours)			
	Product 1	Product 2	Product 3	Product 4
Carding	7.45	2.27	12.26	0
Combing	21	-	-	-
Simplex	7.35	2.24	7.78	0
Ring	10.71	3.52	8.76	0

3.2 Status Of Resources

A constraint is classified as scarce or abundant depending respectively on whether or not the optimum solution "consumes" the entire available amount of the associated resources.

The status of the resources (abundant or scares) in any LP model can be secured directly from the optimum table by observing the value of slack variables. A positive slack means that the resource is not used completely, thus is abundant (Simplex machine), whereas a zero slack indicates that the entire amount of the resource is consumed by the activities of the model [6]

Resources	Slack	Status of rescues
Carding m/c capacity	S_1=0	Scarce
Combing m/c capacity	S_2=0	Scarce
Simplex m/c capacity	S_3=1.61	Abundant
Ring m/c capacity	S_4=0	Scarce

The slacks for Carding, Combing and Ring machines are zero therefore, machine hours for these machines consumed entirely for the level of optimum production. In case of Simplex machines machine hours are of yet to consumed entirely. The management if now want to expand resources (machine hours) can pay attention for Carding, Combing or Ring machines not in any circumstances for Simplex machine.

3.3 Dual Price (Unit Worth Of Resource Or Shadow Price)

Dual price or shadow price actually indicates the worth of the resource. The dual price of the resources 1, 2, 3 and 4 can be summarized as:

Y_1 =Tk. 147 per hour Carding machine
Y_2 = Tk. 13436 per hour of Combing machine
Y_3 = Tk. 0 per hour of Simplex machine
Y_4 = Tk. 5904 per hr of Ring machine

If s_1 is changed from its current zero level, the value of z will change @ Tk. 147 per hour. But a change in s_1 is actually equivalent to changing resource 1- Carding m/c capacity. From the above analysis Combing machine should be given priority in the allocation of additional machine hours or funding for new machine set-up since an increase in the operation of one hour Combing machine hour would increase the value of Z by Tk. 13436. Next priority deserves for Ring machines since its contribution to objective function is @ Tk. 5904 per machine hour compared to @ Tk. 147 per hour of Carded machine.

3.4 Maximum Change In Resource Availability

To determine the range of variation in the availability of a resource, for which the dual prices remain applicable, need to perform additional computations. In case the first resource in the model is changed by an amount D_1, it means the available working period will be $22+D_1$ hours. If D_1 is positive, the resources increase and vice versa.

Equation	Right side elements in iteration			
	0 (starting)	1	2	3(Final)
Z	0	346500	367620	421213+147 D_1
1	$22+D_1$	$14.54+D_1$	$6.61+D_1$	$19.47+2.94D_1$
[illegible]	21	10.5	10.5	$10.5+0D_1$
3	19	11.65	3.82	$1.61-0.33D_1$
4	23	12.29	11.17	$3.206-1.20D_1$

Therefore,

$x_3=19.47+2.94\ D_1\geq0$ ....(1)
$x_1=10.5+0.\ D_1\geq0$ ....(2)
$x_4=1.61-0.33D_1\geq0$ ....(3)
$x_{[?]}=3.20-120\ D_1\geq0$ ....(4)

Case 1

If $D_1>0$, the relations (1) and (2) are always satisfied for $D_1>0$ whereas the relations (3) and (4) impart $D_1\leq4.78$ and $D_1\leq2.66$ respectively. Therefore, D_1 has to be less than the numerical value of 2.66 i.e. $D_1\leq2.66$.

Case 2

If $D_1<0$, the relations (3) and (4) are always satisfied for $D_1<0$. The equation (1) imparts $D_1\geqslant-6.62$ and the equation (2) imparts $D_1\geqslant-\infty$. Therefore, D_1 has to be greater than the numerical value of 6.62 i.e. $D_1\geqslant-6.62$. By combining the cases 1 and 2, the range of D_1 can be written as $-6.62\leq D_1\leq2.66$. This means that the minimum and the maximum hours of carding machine capacity for which the dual price (per unit contribution of resource 1) y_1=147 remains valid are 22-6.62=15.38 hours and 22+2.66=24.66 hours respectively. Similarly, for other machines these values have been computed and presented in Table 4. Combing machine seems to be more flexible/tolerable in respect of dual value which remain unchanged for the range of 0~28 machine hours. Upper bound should not confuse to some one since we have not restricted/imposed 24 hrs as a limiting boundary for non-stop useful machine hours.

Table 4: Ranges of the RHS of the constraints (Resources)

Constraints for machine	Dual Value	Slack/ Surplus	Original Value	Lower Bound	Upper Bound
Carding	147.26	0	22	15.39	24.66
Combing	13436.2	0	21	0	28.41
Simplex	0	1.62	19	17.38	Infinity
Ring	5904.95	0	23	21.10	26.84

3.5 Maximum Change In Marginal Profit

To find the permissible ranges for change in marginal profit or cost these analyses are important. Any change in the coefficients of objective function will affect only the objective equation in the optimum tableau. This means that such changes can have the affect of making the solution non-optimal. Our goal is determine the range of variation for the objective coefficients (one at a time) for which the current optimum $Z = 33000x_1+6600x_2+2750x_3+4400x_4$ remains unchanged. The final iteration would be

Basic	Z	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	Soltn
Z	1	0	0	0	121+ 0.50 d_2	147- 1.20 d_2	13436- 0.43 d_2	0	5904.9+ 1.68 d_2	421213
x_3	0	0	0	1	0.42	2.94	-0.07	0	-1.9	19.4
x_1	0	1	0	0	0	0	0.50	0	0	10.5
S_3	0	0	0	0	0.31	-0.33	-0.01	1	-	1.61
x▯	0	0	1	0	0.50	-1.20	-0.43	0	1.42 1.68	3.20

The only change occurs in the non-basic coefficient in the z row. The changes can be obtained from the original tableau by multiplying the non-basic coefficient and the right hand side in the x_2 row by d_2 and then adding original optimum Z row.

Table 5: Ranging for variable's coefficient

Variable	Value	Reduced Cost	Original Value	Lower Bound	Upper Bound
x_1	10.5	0	33000	6127.604	Infinity
x_2	3.20	0	6600	6359.921	6722.223
x_3	19.47	0	2750	2700	5856.338
x_4	0	121.48	4400	-6127.604	4521.486

$121.48+.50d_2 \geqslant 0$ or $d_2 \geqslant -240.07$, $147.25-1.20d_{\square} \geqslant 0$ or $d_{\square} \leq 122.7$, $13436.2-0.43d_{\square} \geqslant 0$ or $d_2 \leq 31246$, $5904.9+1.68d_2 \geqslant 0$ or $d_2 \geqslant -3645$ therefore, $-240.07 \leq d_2 \leq 122.7$ impart $6600-240.07 \leq c_2 \leq 6600+122.7$ or $6359.93 \leq c_2 \leq 6722.7$ satisfying that the current optimum remain unchanged for the range of [6359.93, 6722.7]. Similar calculations for other variables were carried out and shown in tabular form in Table 5. Combed yarn seems to be more flexible/adopting yarn in case of market fluctuation since current optimum remain unchanged for the lowest contribution to profit/objective function @ Tk. 6127/ton compared to its current value @ Tk. 33000/ton.

4. DISCUSSION

Textile is a very potential sector in Bangladesh depending on which various backward linkage industries are growing. Yarn manufacturing industries can be identified as the resultant effect of the growth of indigenous textile industries. The companies usually run production on the basis of thumb rule instead of applying state-of-the-art operation management techniques. The company under study is a leading manufacturing company for quality yarn production and currently it receive huge orders. To be more competitive in the global market the company has a scope of utilizing LP and other operations management techniques. From the LP solution it can be recommended to stop the production of polyester yarn if there is not any other binding condition. Again from sensitivity analysis it is evident that management can prioritize Combing machine, in allocation of additional fund for its expansion. The problem of chaos that management often faces regarding compromise between capacity and profit during simultaneous orders can now be mitigated following optimum product-mix solved by LP. Management should prioritize optimum product-mix in receiving fresh orders. Again production time for each machine for each product can also be followed as it is shown in Article 3.1. The problem could be more widely articulated or more result oriented if other constraints arising out of various limiting conditions could be taken into account.

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