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Equilibrium Linguistic Computation Method for Fuzzy Group Decision-Making

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ABSTRACT

This paper proposes an equilibrium computation method using linguistic variables based on the conflicting bifuzzy sets. The linguistic terms were defined and associated with the triangular fuzzy number as well as labeling system in the early stages. Then, the negation operator was introduced and the bifuzzy approaches were employed to derive the aggregation equilibrium linguistic judgement for evaluation purposes. Some modification of the linguistic geometric averaging operator was also introduced in order to suit our proposed method. Finally, a hypothetical example is given to illustrate the application of our proposed computation method. The results showed that the method is highly beneficial in terms of applicability and offers a new dimension to problem solving technique for the fuzzy group decision-making environment.

Keywords: Computation methods, Conflicting bifuzzy sets, Decision making, Equilibrium linguistic evaluation

1. INTRODUCTION

In today's highly competitive environment, an effective evaluation process is important in any organisation. Group decision makers usually make decisions based on incomplete sources of information and this occurs due to the multiple-factors involved that needs to be considered simultaneously in the required decision-making process. Imprecise sources, ambiguity of the information and uncertain factors are some of the serious threats for the smooth and effective running of any entity. Fuzzy set introduced by Zadeh (1965) is one of the most fundamental and influential tools in the development of the computational intelligence (Herrera *et al.* (2006)). Computing with words (CW) based on the concept of linguistic variable in fuzzy environment is a methodology in which the words are used in place of numbers to accomplish processes of computing and reasoning. Two major imperatives are needed for the CW approach (Pasi and Yager (2006)). First, CW is a necessity when the available information is too ill-defined and imprecise to justify the use of precise numbers, and second, when there is a tolerance for imprecision which can be exploited to achieve tractability, low solution cost, robustness, and better rapport with reality.

The recent evolution of linguistic variable used focuses on two different perspectives,(i) the linguistic preference modelling; and (ii) granular information processing. In the former, the linguistic variables are used to evaluate qualitative aspect; and for preference modelling in information retrieval system as well as decision making related problem. Meanwhile, in the case of the latter, it used to develop inference and reasoning processes in knowledge based-system and intelligent information systems (Herrera *et al.* (2006)).

In general, many research have focused on the linguistic preference modelling. Garcia-Lapresta (2006) introduced CW group decision making model based on simple majority decision rules. Bordogna *et al.* (2006) proposed the linguistic modelling of imperfect spatial information in geographic information systems, while Zadrozny and Kacprzyk (2006) discussed the CW paradigm to the automatic text documents, categorisation problem and information retrieval. The group decision makers with two distinct approach based on the linguistic quantifier of “majority” was also introduced by Pasi and Yager (2006). Other researchers have studied on granular information processing issues. Randon and Lawry (2006) discussed on how a random set based knowledge representation framework can be utilised for evaluating linguistic queries and learning linguistic model. Torra *et al.* (2006) on the other hand, introduced a method for building model for categorical data in ordinal domains.

Presently, most linguistic computation processes are made after considering only ‘one part’ of the aspect without considering at all the other aspects. Although this approach seems perfect, it has certain shortcomings in the evaluation process. To address some of the shortcomings, this research was initiated with the new idea of considering both positive and negative aspects simultaneously in the judgement process. To the best of our knowledge, there is a lack of established research that emphasise this approach, even though this approach is very significant and can be related with daily common experiences. The only research that follows a similar design to our approach is the Yin Yang (2004) concept which emphasises the equilibrium concept (i.e. two sides of a matter) that takes its roots from the traditional Chinese perspective.

Since the nature of the attribute evaluated is subjective and lacks information, this new computation approach is expected to be more precise, comprehensive and efficient in daily procedures. The aim of this paper is to propose the conflicting approach based on the equilibrium linguistic assessment for group decision-making by using triangular fuzzy numbers (TFNs). To do so, this paper is structured as follows: Section 2 briefly discusses the background theory of conflicting bifuzzy sets (CBFS) followed by our proposed linguistic evaluation approach in Section 3. Section 4 and 5 provide an algorithm and hypothetical example to illustrate the proposed method, and finally the conclusion.

2. CONFLICTING BIFUZZY SETS THEORY

The fuzzy sets introduced by Zadeh (1965), and Intuitionistic Fuzzy Sets (IFS) by Atanassov (1986), are well known theories and have been used in various applications. Both theories are very successful and minimise the uncertainty of the initial information which involves human judgement. In IFS, $\mu_A(x)$ is the degree of membership and $\gamma_A(x)$ is the degree of non-membership of x with respect to A. Hence, it is natural to see that $0 < \mu_A(x) + \gamma_A(x) \leq 1$, where $\mu_A(x)$ and $\gamma_A(x)$ cannot both occur at the value 0 or 1 at X.
Abu Osman (2006) in his original work introduced the new theoretical concept of the so-called conflicting bifuzzy sets (CBFS). If $\mu : \boldsymbol{X} \rightarrow \boldsymbol{I}$ and γ : $\boldsymbol{X} \rightarrow \boldsymbol{I}$ are two fuzzy sets, we can define bifuzzy set as (μ,γ): $\boldsymbol{X}$ x $\boldsymbol{X} \rightarrow \boldsymbol{I}$ as in the following structure shown in Figure 1.

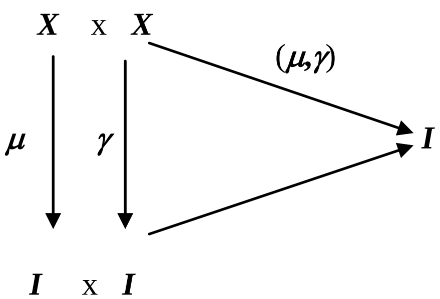

Figure 1: The bifuzzy set structure

Hence, we can see that $0 \le \mu(x) + \gamma(x) \le 2$, where $0 \le \mu(x) \le 1$ and $0 \le \gamma(x) \le 1$. Since the value of fuzzy set are in [0,1], we define 0.5 as the threshold value when the value is greater than 0.5 is said to be dominant. If the value is less than 0.5, it is then said to be under dominant.

Now, if we have two fuzzy sets which are conflicting on the same $\boldsymbol{X}$, we see that the values of $\mu(x)$ and $\gamma(x)$ cannot be both dominant and/or both under dominant, concurrently. Hence, for two conflicting bifuzzy sets (CBFS) μ and γ on the same $\boldsymbol{X}$, if μ is dominant, then γ must be under dominant and its' true conversely. For example, we observe the two fuzzy sets "*good*' and "*bad*" for performance of a candidate. The fuzzy set "*bad*" need not necessarily be the compliment of the fuzzy set "*good*". If the "*good*" performance with $\mu_z(x) = 0.7$, the value for "*bad*" performance need not be 0.3, but may be $\gamma(x) = 0.4$ and $\mu(x) + \gamma(x) = 0.7 + 0.4 > 1$. Thus, we can define the conflicting bifuzzy sets (μ,γ) as $\boldsymbol{X} \text{ x } \boldsymbol{X} \rightarrow \boldsymbol{I}$ such that $0 < \mu(x) + \gamma(x) < 1.5$. Thus, we can consider two conflicting bifuzzy set $\mu : \boldsymbol{X} \rightarrow [0,1]$ and $\gamma : \boldsymbol{X} \rightarrow [0,1]$ defined on the same premises of $\boldsymbol{X}$ to be given as the following definition (Zamali (2009)).

Definition 1 Let a set $\boldsymbol{X}$ be fixed. A *conflicting bifuzzy set A* of $\boldsymbol{X} \text{ x } \boldsymbol{X}$ is an object having the following form:

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in \boldsymbol{X};\ 0 < \mu_A(x) + \gamma_A(x) \le 1.5\} \qquad (1)$$

where the functions $\mu_A : \boldsymbol{X} \rightarrow [0,1]$ represent the *positive degree* of x with respect to A and $x \in \boldsymbol{X}$ such that $\mu_A(x) \in [0,1]$, and the functions $\gamma_A : \boldsymbol{X} \rightarrow [0,1]$ represent the *negative degree* of x with respect to A and $x \in \boldsymbol{X}$ such that $\gamma_A(x) \in [0,1]$, and the $0 < \mu_A(x) + \gamma_A(x) \le 1.5$.

Since the idea is newly introduced, many assumptions and theoretical perceptions can be derived and cultivated from this new concept. Thus, a new concept of evaluation approach is proposed by utilising both the positive and negative aspects, simultaneously. However, the intersection and union of two conflicting bifuzzy sets A and B in $\boldsymbol{X}$ is similar with the IFS. The only difference is that the sum of these two grade degrees can exceed more than 1 (in a logical range based on the Definition 1). Figure 2 shows the chronological years of several set theories. The figure illustrates the progress and evolution of the current CBFS from the first fuzzy set introduction.

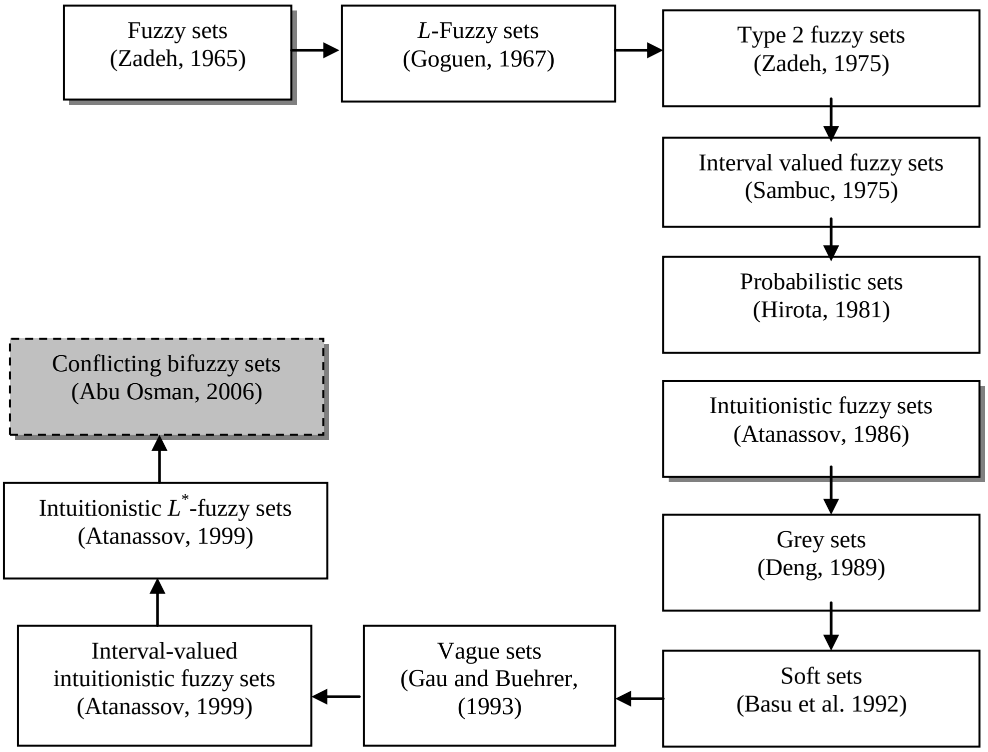

Figure 2: The chronological years of several set theories

3. OUR PROPOSED EVALUATION APPROACH

In this section, we introduce the negation operator and related properties for calculation purposes. Suppose that $L = \{l_\alpha / \alpha = 0.1, 0.2, \ldots, T\}$ is a finite and totally-ordered discrete term set, where l_α represents a possible linguistic variable for a negative aspect and satisfies the following properties as:

i. The set is ordered: $l_\alpha > l_\beta$ if and only if $\alpha > \beta$
ii. There is the negation operator: $Neg\,(l_\alpha) = l_\beta$, such that $\beta = T + 1 - \alpha$
iii. Max operator: $\max(l_\alpha, l_\beta) = l_\alpha$, if $\alpha \geq \beta$
iv. Min operator: $\min\,(l_\alpha, l_\beta) = l_\alpha$, if $\alpha \leq \beta$

TABLE 1: Linguistic variables for ratings and its label

Linguistic variables	Triangular Fuzzy Numbers	Labels (l_α)
Extremely low (EL)	(0,0.05,0.1)	$l_{0.1}$
Very low (VL)	(0,0.1, 0.2)	$l_{0.2}$
Low (L)	(0.1,0.2,0.3)	$l_{0.3}$
Medium low (ML)	(0.2,0.3,0.4)	$l_{0.4}$
Medium (M)	(0.3,0.5,0.7)	$l_{0.5}$
Medium high (MH)	(0.6,0.7,0.8)	$l_{0.6}$
High (H)	(0.7,0.8,0.9)	$l_{0.7}$
Very high (VH)	(0.8,0.9,1.0)	$l_{0.8}$
Extremely high (EH)	(0.9,0.95,1.0)	$l_{0.9}$

In the evaluation process, the linguistic terms of a label system was employed. Hence, the transformation is needed from the linguistic scales into the TFNs (see Table 1 and Figure 3). The set on corresponding transformation of nine linguistic labels is given by:

$$L = \{l_{0.1} = \textit{extremely low},\ l_{0.2} = \textit{very low (VL)},\ l_{0.3} = \textit{low (L)},\ l_{0.4} = \textit{medium low (ML)},\ l_{0.5} = \textit{medium (M)},\ l_{0.6} = \textit{medium high (MH)},\ l_{0.7} = \textit{high (H)},\ l_{0.8} = \textit{very high (VH)},\ l_{0.9} = \textit{extremely high}\} \quad (2)$$

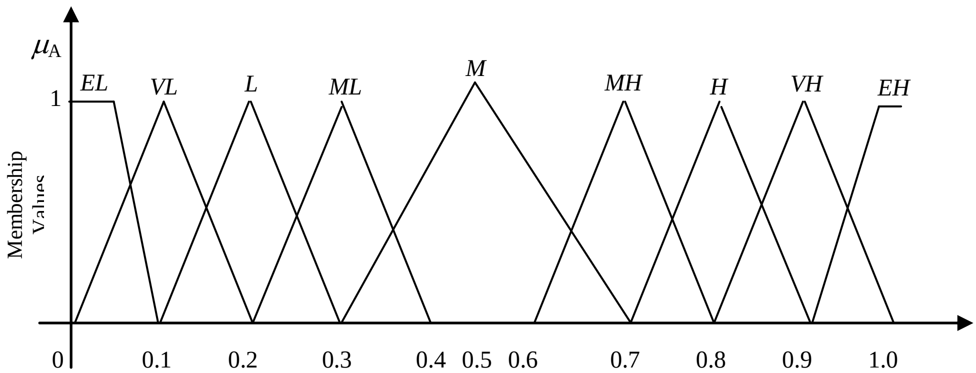

Figure 3: Membership functions of linguistic values for criteria ratings

Next, the best with '*positive*' and '*non-negative*' aspect was needed; where the '*non-negative'* values can be derived by the above negation operator. There are several useful combination operator methods which can be utilised.

The most commonly used combination operator method is the geometric mean, multiplicative operator and *t*-norm operators. In this paper, the geometric mean is employed as a combination operator for evaluating the positive and '*non-negative'* values given as follows.

$$M_A(\mu(x),\upsilon(x)) = \sqrt{\mu(x) \times Neg(l_\alpha)} \quad (3)$$

where $\mu(x)$ is a *positive* value, and $Neg(l_\alpha)$ is a negation operator of negative value (i.e., $Neg(l_\alpha) = l_\beta$ = '*non-negative'* value).

For example, if one attribute is originally assessed as (L,MH) (i.e., '*positive aspect'*, '*negative aspect'*), it should be transformed into (L,ML) (henceforth called linguistic conflicting bifuzzy preference relations), where ML is 'non-negative aspect' derived from the negation operator (i.e., $Neg(l_{MH}) = Neg\ (l_{0.6}) = l_{0.9+0.1-0.6} = l_{0.4} = l_{ML}$). Thus, the linguistic conflicting bifuzzy preference relations can be represented in TFN as (0.1,0.2,0.3;0.2,0.3,0.4), and the combination operation for both aspects can be derived as (0.141,0.245,0.346). The combination using geometric mean operator is called as 'equilibrium linguistic judgment'. Note that the geometric mean was employed due to two reasons; i) the simplest and efficient technique, and ii) it represents a more precise combination value because no extreme conflicting linguistic expressions are allowed in a judgment process. For simplification purpose, the above explanation can be illustrated in Figure 4-6.

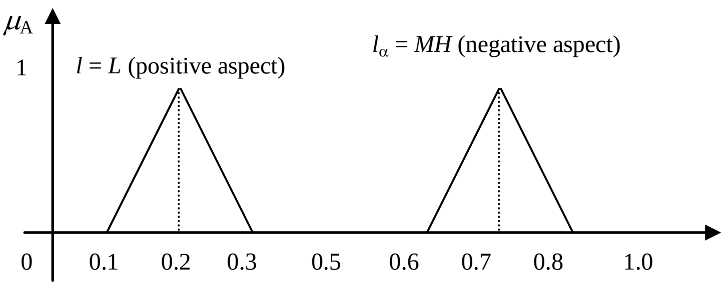

Figure 4: The original conflicting bifuzzy linguistic (l, l_α) evaluation

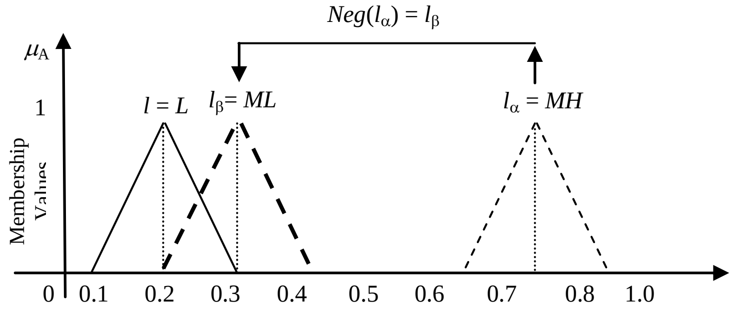

Figure 5: The transformation from original evaluation intolinguistic conflicting bifuzzy preference relations (l, l_β) using negation operator

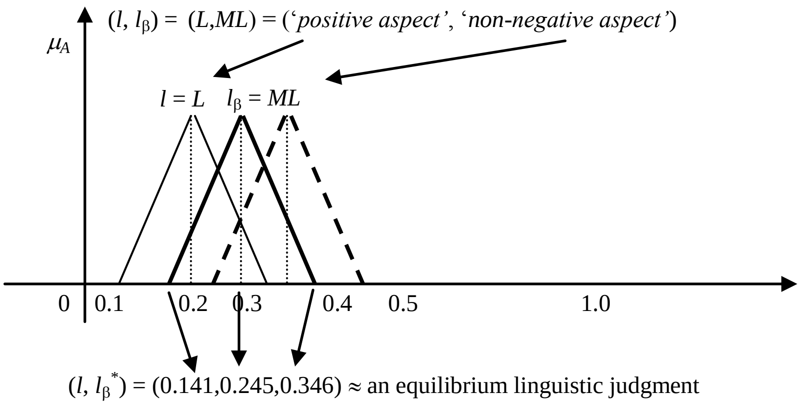

Figure 6: An equilibrium linguistic judgment derived from Figure 5

For the purpose of reference, two aggregation operator definitions from Xu (2004) were reviewed, namely the linguistic ordered weighted geometric averaging (LOWGA), and linguistic geometric averaging (LGA). All these definitions are related to the linguistic approach that has been utilised for aggregating the group decision maker's evaluation throughout this paper.

Definition 2 A LOWGA operator of the dimension n is a mapping LOWGA: $\bar{L}^n \rightarrow \bar{L}$, which has associated with an exponential weighting vector $w = (w_1, w_2, \ldots, w_n)^T$, with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that

$$\text{LOWGA}_w\left(l_{\alpha 1}, l_{\alpha 2}, ..., l_{\alpha_n}\right) = \left(l_{\beta_1}\right)^{w_1} \otimes \left(l_{\beta 2}\right)^{w_2} \otimes ... \otimes \left(l_{\beta n}\right)^{w_n}$$
$$= \left(l_{\beta_1^{w_1}}\right) \otimes \left(l_{\beta_2^{w_2}}\right) \otimes ... \otimes \left(l_{\beta_n^{w_n}}\right) = l_{\beta} \quad (4)$$

where $\beta = \prod_{j=1}^{n} \beta_j^{w_j}$, l_{β_j} is the jth largest of the l_{α_i}.

Definition 3 If the exponential weighting vector $w = (1/n, 1/n, \ldots, 1/n)^T$, then the LOWGA operator is reduced to the LGA operator, i.e.,

$$\text{LOWGA}_w\left(l_{\alpha 1}, l_{\alpha 2}, ..., l_{\alpha_n}\right) = l_{\alpha} \quad (5)$$

4. THE ALGORITHM

From Section 3, the algorithm of the proposed computation method is summarised in five steps as follows.

Step 1: Assign the conflicting linguistic preference relations s^1 denoted by (l_{1j}^{+}, l_{1j}^{-}) and equilibrium linguistic preference relations denoted by (l_{1j}^{+}, l_{1j}^{*}).

Step 2: Employ the modified LGA operator to aggregate the combining *non-negative* equilibrium linguistic preference information to get the fuzzy performance values $(\tilde{P}_1)$ of the i^{th} alternative over all other alternative given as

$$(Ep)_1^{(1)} = \text{LGA}\{(l_{1j}^{+} \otimes l_{1j}^{*})^{1/2} \otimes (l_{1j}^{+} \otimes l_{1j}^{*})^{1/2} \otimes .... \otimes (l_{1n}^{+} \otimes l_{1n}^{*})^{1/2}\}^{1/(n-1)} \quad (6)$$
$(j = 1,2,3, \ldots, n)$

where (l_{1j}^{+}, l_{1j}^{*}) means ('linguistic positive labels', 'linguistic non-negative labels), and $\otimes$ is a multiplication operation of fuzzy numbers.

Step 3: Give the weighting vector $w = (\alpha_1, \alpha_2, \alpha_3)^T$ to aggregate $(Ep)_i^{(k)}$ (k = 1, 2, 3,…m; i = 1, 2, 3, …, n) corresponding to the alternative x_i (i =1,2,3,4) , and then get the fuzzy weighted aggregate performance values $(Ep)_i$ of the i^{th} alternative over all the other alternatives.

Step 4: Defuzzification

Let $\mu_{ij} = (x_i, y_i, z_i), (i = 1, 2, 3, ...m; j = 1, 2, 3, ..., n)$ is positive arbitrary of the TFNs. Thus, the best alternative can be determined using the defuzzification process (Chen (1996)) by choosing the maximum of the crisp value (i.e., performance values (P_i)) over all the criteria given as

$$P_i = \frac{(x_i + y_i + y_i + z_i)}{4}; \ (i = 1, 2, \ldots, n) \tag{7}$$

Step 5: Rank the performance values (P_i) (i = 1, 2,3,4) by descending order and identify the best option. Symbolically it can be written as $P_i \rangle P_{i+1} \rangle P_{i+4} \rangle \ldots \rangle P_n$ where the symbol '$\rangle$' means 'is preferred or superior to'.

5. A HYPOTHETICAL EXAMPLE

Suppose that a government-linked company (GLC) decided to invest money for a new municipal solid waste (MSW) disposal system. Four possible disposal systems may be considered given as (i) x_1, sanitary landfilling, (ii) x_2, incineration, (iii) x_3, composting and (iv) x_4, material recycling system. One main criterion used is the environmental impact assessment (EIA). Three DMs (stakeholders) were involved and they gave their assessment based on experience and expertise. The stakeholders are (i) experts (DM^1), (ii) government agencies (DM^2) and (iii) non-government organisations (DM^3), whose weight vector w = (0.3, 0.4, 0.3). The DMs compared these four disposal systems with respect to the criterion of EIA by using the linguistic terms as given by Equation (2). The conflicting linguistic preference relations DM^k (k = 1, 2, 3) denoted by $(l_{1j}^{(1)+}, l_{1j}^{(1)-})$, are shown in Tables 2, 4 and 6, respectively. Meanwhile, the equilibrium linguistic preference relations denoted by $(l_{1j}^{(1)+}, l_{1j}^{(1)*})$, are shown in Tables 3, 5 and 7, respectively. From Section 4, the following algorithm was employed to obtain the best system.

Step 1: Assign the conflicting linguistic preference relations DM^i (i = 1,2,3) denoted by (l_{1j}^{+}, l_{1j}^{-}), are shown in Table 2, 4 and 6, respectively. The equilibrium linguistic preference relations denoted by (l_{1j}^{+}, l_{1j}^{*}), are shown in Table 3, 5 and 7, respectively.

TABLE 2: Conflicting linguistic preference relation DM^1

	x_1	x_2	x_3	x_4
x_1	-	$(l_{0.2}, l_{0.9})$	$(l_{0.4}, l_{0.6})$	$(l_{0.3}, l_{0.8})$
x_2	$(l_{0.8}, l_{0.1})$	-	$(l_{0.5}, l_{0.6})$	$(l_{0.4}, l_{0.5})$
x_3	$(l_{0.6}, l_{0.4})$	$(l_{0.5}, l_{0.4})$	-	$(l_{0.2}, l_{0.9})$
x_4	$(l_{0.7}, l_{0.2})$	$(l_{0.6}, l_{0.5})$	$(l_{0.8}, l_{0.1})$	-

TABLE 3: Equilibrium linguistic preference relation DM^1

	x_1	x_2	x_3	x_4
x_1	-	$(l_{0.2}, l_{0.1})$	$(l_{0.4}, l_{0.4})$	$(l_{0.3}, l_{0.2})$
x_2	$(l_{0.8}, l_{0.9})$	-	$(l_{0.5}, l_{0.4})$	$(l_{0.4}, l_{0.5})$
x_3	$(l_{0.6}, l_{0.6})$	$(l_{0.5}, l_{0.6})$	-	$(l_{0.2}, l_{0.1})$
x_4	$(l_{0.7}, l_{0.8})$	$(l_{0.6}, l_{0.5})$	$(l_{0.8}, l_{0.9})$	-

TABLE 4: Conflicting linguistic preference relation DM^2

	x_1	x_2	x_3	x_4
x_1	-	$(l_{0.3}, l_{0.4})$	$(l_{0.4}, l_{0.6})$	$(l_{0.6}, l_{0.6})$
x_2	$(l_{0.7}, l_{0.6})$	-	$(l_{0.7}, l_{0.7})$	$(l_{0.4}, l_{0.6})$
x_3	$(l_{0.6}, l_{0.4})$	$(l_{0.3}, l_{0.3})$	-	$(l_{0.4}, l_{0.8})$
x_4	$(l_{0.4}, l_{0.4})$	$(l_{0.6}, l_{0.4})$	$(l_{0.6}, l_{0.2})$	-

TABLE 5: Equilibrium linguistic preference relation DM^2

	x_1	x_2	x_3	x_4
x_1	-	$(l_{0.3}, l_{0.6})$	$(l_{0.4}, l_{0.4})$	$(l_{0.6}, l_{0.4})$
x_2	$(l_{0.7}, l_{0.4})$	-	$(l_{0.7}, l_{0.3})$	$(l_{0.4}, l_{0.4})$
x_3	$(l_{0.6}, l_{0.6})$	$(l_{0.3}, l_{0.7})$	-	$(l_{0.4}, l_{0.2})$
x_4	$(l_{0.4}, l_{0.6})$	$(l_{0.6}, l_{0.6})$	$(l_{0.6}, l_{0.8})$	-

TABLE 6: Conflicting linguistic preference relation DM^3

	x_1	x_2	x_3	x_4
x_1	-	$(l_{0.2}, l_{0.8})$	$(l_{0.6}, l_{0.3})$	$(l_{0.4}, l_{0.7})$
x_2	$(l_{0.8}, l_{0.2})$	-	$(l_{0.4}, l_{0.5})$	$(l_{0.3}, l_{0.8})$
x_3	$(l_{0.4}, l_{0.7})$	$(l_{0.6}, l_{0.5})$	-	$(l_{0.5}, l_{0.5})$
x_4	$(l_{0.6}, l_{0.3})$	$(l_{0.7}, l_{0.2})$	$(l_{0.5}, l_{0.5})$	-

TABLE 7: Equilibrium linguistic preference relation DM^3

	x_1	x_2	x_3	x_4
x_1	-	$(l_{0.2}, l_{0.2})$	$(l_{0.6}, l_{0.7})$	$(l_{0.4}, l_{0.3})$
x_2	$(l_{0.8}, l_{0.8})$	-	$(l_{0.4}, l_{0.5})$	$(l_{0.3}, l_{0.2})$
x_3	$(l_{0.4}, l_{0.3})$	$(l_{0.6}, l_{0.5})$	-	$(l_{0.5}, l_{0.5})$
x_4	$(l_{0.6}, l_{0.7})$	$(l_{0.7}, l_{0.8})$	$(l_{0.5}, l_{0.5})$	-

Step 2: Employ the modified LGA operator to aggregate the combining *non-negative* equilibrium linguistic preference information to get the fuzzy performance values $(\tilde{P}_i^{(k)}; i = 1,2,3,...,n; k = 1,2,3)$ of the i^{th} disposal system over all the other systems. For example, the

$\tilde{P}_1^{(k)}; k = 1,2,3)$ can be obtained as follows.

$$\tilde{P}_1^{(1)} = \text{LGA}\{(l_{11}^{+} \otimes l_{11}^{*})^{1/2} \otimes (l_{12}^{+} \otimes l_{12}^{*})^{1/2} \otimes (l_{13}^{+} \otimes l_{13}^{*})^{1/2} \otimes (l_{14}^{+} \otimes l_{14}^{*})^{1/2}\}^{1/(4\text{-}1)}$$

$$= \{\,'\_'\otimes(\text{l}_{0.2} \otimes \text{l}_{0.1})^{1/2} \otimes (\text{l}_{0.4} \otimes \text{l}_{0.4})^{1/2} \otimes (\text{l}_{0.3} \otimes \text{l}_{0.2})^{1/2}\}^{1/(4-1)}$$

$$= \begin{pmatrix} (\sqrt{(0,0.1,0.2) \otimes (0,0.05,0.1)}) \otimes (\sqrt{(0.2,0.3,0.4) \otimes (0.2,0.3,0.4)} \otimes \\ (\sqrt{0.1,0.2,0.3) \otimes (0,0.1,0.2)} \end{pmatrix}^{\frac{1}{3}}$$

$$= \left((0,0.005,0.02)^{\frac{1}{6}} \otimes (0.04,0.09,0.16)^{\frac{1}{6}} \otimes (0,0.02,0.06)^{\frac{1}{6}} \right)$$

$$= (0,0.1442,\ 0.2402)$$

$$\tilde{P}_2^{(1)} = \{ (l_{0.8} \otimes l_{0.9})^{1/2} \otimes '\_' \otimes (l_{0.5} \otimes l_{0.4})^{1/2} \otimes (l_{0.4} \otimes l_{0.5})^{1/2} \}^{1/(4-1)}$$

$$= \begin{pmatrix} (\sqrt{(0.8,0.9,1.0) \otimes (0.9,0.95,1.0)}) \otimes (\sqrt{(0.3,0.5,0.7) \otimes (0.2,0.3,0.4)} \otimes \\ (\sqrt{0.2,0.3,0.4) \otimes (0.3,0.5,0.7)} \end{pmatrix}^{\frac{1}{3}}$$

$$= \left((0.72,0.855,1.0)^{\frac{1}{6}} \otimes (0.06,0.15,0.28)^{\frac{1}{6}} \otimes (0.06,0.15,0.28)^{\frac{1}{6}} \right)$$

$$= (0.3706,0.5176,0.6542)$$

$\ldots = \ldots$

$\ldots = \ldots$

$$\tilde{P}_4^{(1)} = \{ (l_{0.7} \otimes l_{0.8})^{1/2} \otimes (l_{0.6} \otimes l_{0.5})^{1/2} \otimes (l_{0.8} \otimes l_{0.9})^{1/2} \otimes '\_' \}^{1/(4-1)}$$

$$= \begin{pmatrix} (\sqrt{(0.7,0.8,0.9) \otimes (0.8,0.9,1.0)}) \otimes (\sqrt{(0.6,0.7,0.8) \otimes (0.3,0.5,0.7)} \otimes \\ (\sqrt{0.8,\ 0.9,1.0) \otimes (0.9,0.95,1.0)} \end{pmatrix}^{\frac{1}{3}}$$

$$= \left((0.56,0.72,0.9)^{\frac{1}{6}} \otimes (0.18,0.35,0.56)^{\frac{1}{6}} \otimes (0.72,0.855,1.0)^{\frac{1}{6}} \right)$$

$$= (0.6458,\ 0.7742,0.8921)$$

Similarly, the rest of fuzzy performance values ($\tilde{P}_i^{(k)}\ ; i = 1,\ 2,3,4; k = 2,3$) can be obtained as shown in Table 8.

TABLE 8: The fuzzy performance values $(\tilde{P}_i)$ for each DMs

Alternative	*Fuzzy performance value* $\tilde{P}_i\left(i = 1,2,3,4\right)$		
	DM[1]	*DM*[2]	*DM*[3]
x_1	(0.0,0.1442,0.2402)	(0.2570,0.3719,0.4804)	(0,0.2637,0.3888)
x_2	(0.3706,0.5176,0.6542)	(0.2705,0.3888,0.4996)	(0.0,0.3667,0.5061)
x_3	(0.0,0.3082,0.4391)	(0.0,0.3647,0.4899)	(0.2621,0.4169,0.5661)
x_4	(0.6458,0.7742,0.8921)	(0.5241,0.6338,0.7397)	(0.5260,0.6822,0.8260)

Step 3: Give the weighting vector w = (0.3, 0.4, 0.3)^T to aggregate $(Ep)_i^{(k)}$ (k = 1,2, 3; i = 1,2,3,4) corresponding to the alternative x_i, and then get the equilibrium preference degree $(Ep)_i$ of the i^{th} alternative over all the other alternatives. Thus, the $(Ep)_i^w$ (i = 1,2,3,4) can be obtained as shown in Table 9.

TABLE 9: The fuzzy weighted aggregate performance values $(Ep)_i^w$ (i = 1,2,3,4)

Alternative	$(Ep)_i^w$ (i = 1,2,3,4)
x_1	(0.1028,0.2711,0.3809)
x_2	(0.2194,0.4208,0.5480)
x_3	(0.0786,0.3634,0.4975)
x_4	(0.5612,0.6904,0.8113)

Step 4: Defuzzification the fuzzy performance values to obtain the crisp performance values (P_i) using Equation (7). The result is shown in Table 10.

TABLE 10: The performance values and ranking

Alternative	Performance value (P_i ; i = 1,2,3,4)	Ranking
x_1	0.2565	4
x_2	0.4023	2
x_3	0.3257	3
x_4	0.6883	1

Step 5: Rank the performance values (P_i) (i = 1,2,3,4) by descending order and identify the best option. Thus, the ranking order of the four systems is x_4, followed by x_2, x_3, and the last ranking is x_1, or symbolically written as $P_4 = x_4 \rangle P_2 = x_2 \rangle P_3 = x_3 \rangle P_1 = x_1$. Obviously, the best option is system x_4.

For the sake of comparison, the ordinary fuzzy sets (henceforth called fuzzy sets) approach was used to treat the same problem. The procedure for solving this problem is similar by using the algorithm Step 1 – 5 (see Section 4), except without having a negative aspect judgement for the linguistic of input-datasets. The details example of the analysis can be found in Zamali (2009), and here, we only present the final overall performance values for each disposal system, as shown in Table 11.

It is clear that by using this method, minor changes in the second and fourth ranking take place; x_1 is the second choice among four alternatives when compared to the x_2 in our proposed method, while the rest ranking results does not vary between both methods. Therefore, by using the proposed modified method it can generate two possible results either mutual shifted ranking will occurred (i.e., in the case of first and second ranking) or no changing ranking arises in the final results. In the case of different ranking result, it shows that by considering both aspects (i.e., positive and negative aspect), simultaneously, the final ranking was affected; while the performance values were more-or-less significantly influenced for the same results cases. However, the proposed modified method clearly has its advantages. These advantages include (i) it offers the comprehensive evaluation of the so-called ‘equilibrium approach’, which considers both positive and negative aspects, simultaneously in decision processes. In fact, all the performance values obtained is already in the range of [0,1], so that the results are more beneficial to represent the strength of its membership degree, (ii) it allows the DMs to incorporate both ‘hard data’ and less quantifiable elements more reality such as judgments’, feeling and experiences in both aspects, and (iii) it generates a new dimension of evaluation process from the traditional (i.e., single aspect) shifted to the equilibrium approach in the decision-making process.

Table 11: Performance values and ranking of the disposal system with the fuzzy sets method

Alternative	Performance value (P_i : i = 1,2,3,4)	Ranking
x_1	0.5745	2
x_2	0.4740	4
x_3	0.5078	3
x_4	0.6068	1

6. CONCLUSIONS

In this paper, we have proposed the equilibrium linguistic computation method based on the conflicting bifuzzy sets and developed a modified computation algorithm. Since the group decision-making problems generally involve uncertainty, it is important to incorporate an equilibrium approach to derive comprehensively in any proposed aggregated computation method. The proposed computation method is quite different from the conventional points of view. From the example illustrated, it can be clearly seen that the consideration for both positive and negative aspects is more comprehensible in concept and very promising in the final decision perspective. It demonstrates a highly beneficial method to justify the imprecise information from the group decision makers' perspectives. Moreover, it gives a new dimension technique and is more holistic in the evaluation processes. The hypothetical example in this paper can be applied to other situations as well. Although our proposed equilibrium aggregation method gives a new dimension to the computation perspective, the issue on how serious the conflicts are allowed along the judgement process still needs further investigation. This problem is left for further research in the near future.

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