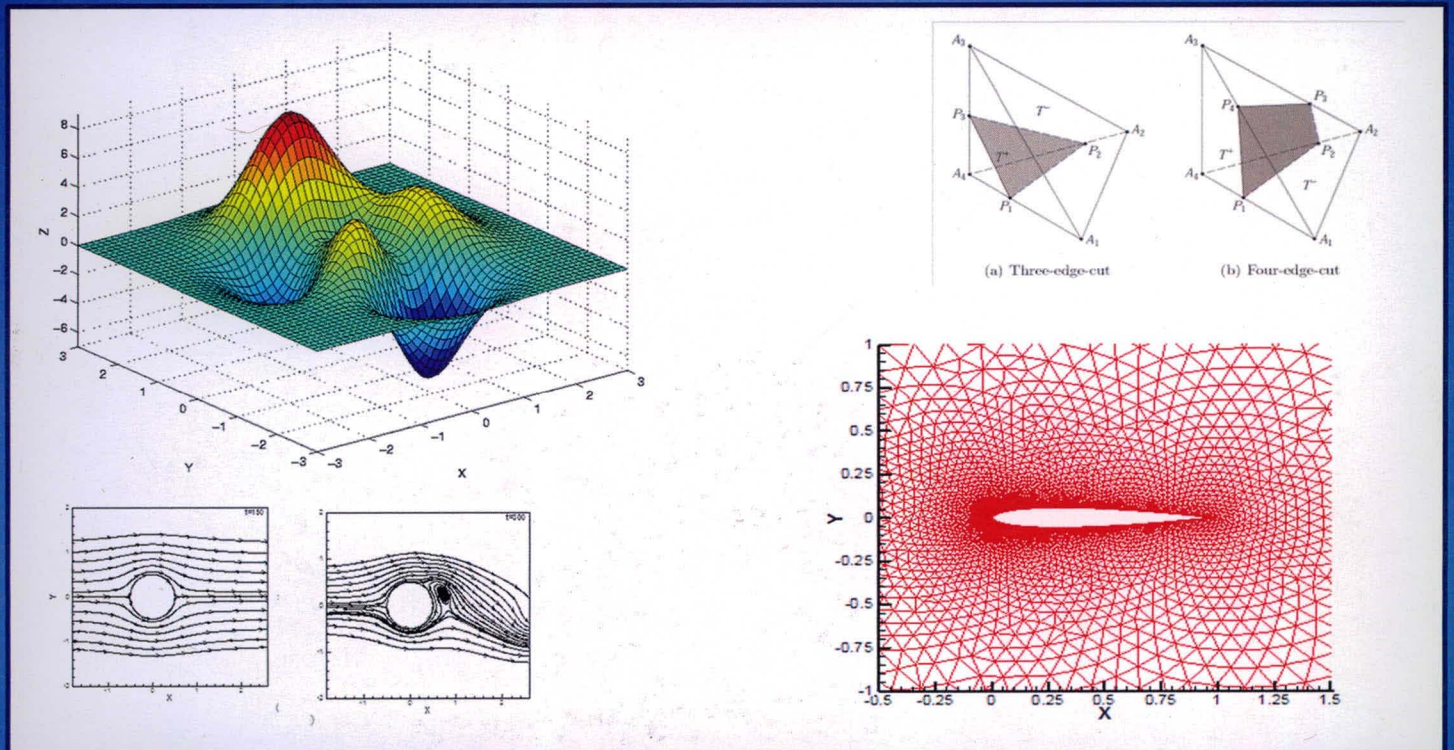


# ADVANCED NUMERICAL TECHNIQUES IN ENGINEERING and SCIENCE



Editors  
AHMAD TARIQ JAMEEL  
WAQAR ASRAR



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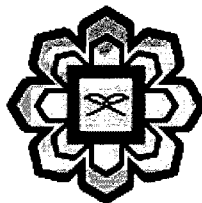
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## CHAPTER 10

# Lower-Upper Symmetric-Gauss-Seidel (LU-SGS) Algorithm for Pseudo Compressibility Method

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### ABSTRACT

A numerical method based on the Pseudo-compressibility is used for solving incompressible Navier-Stokes equations using Lower-Upper Symmetric Gauss-Seidel (LU-SGS) implicit algorithms. The convective flux terms of the Navier-Stokes equations are discretized by a second order central differencing scheme. The resulting inviscid flux terms are presented by central difference scheme with introduction of third order numerical damping. The method is tested and validated on two test cases. The first test case is for the turbulent flow-field around a multi-element airfoil with leading-edge slat and trailing-edge flap in landing configuration. The second is for three dimensional laminar steady flow over a cylinder body at high angle of attack. The computed results for the studied cases are compared with experimental data in terms of surface pressure and lift coefficients and show reasonable agreement. Also the computed flow field over the cylinder body demonstrates the capability of the LU-SGS algorithms in predicting the development of asymmetric vortices.

*Keywords:* Pseudo-compressibility, Navier-Stokes Equations, LU-SGS, finite difference