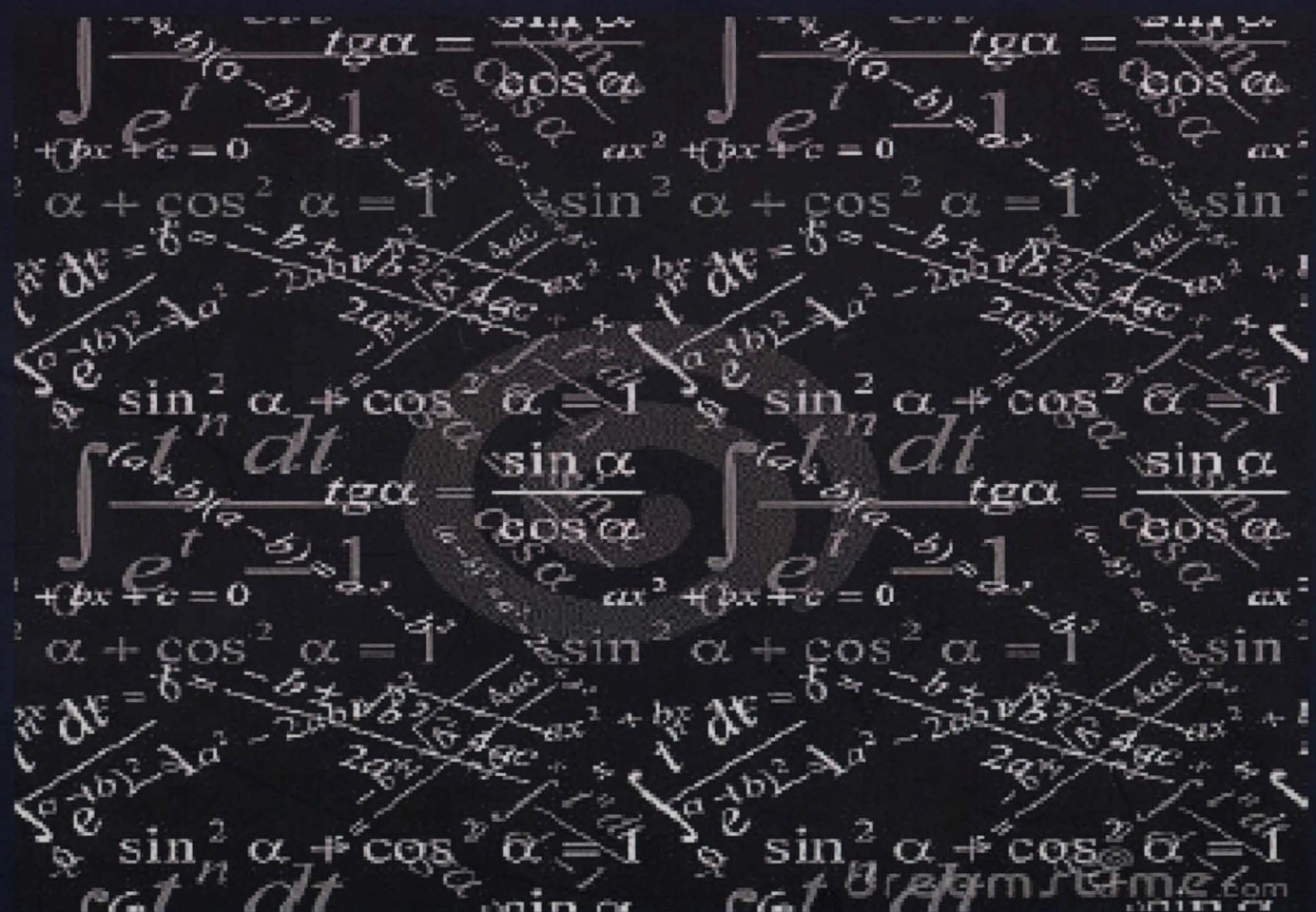




# RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of  
Computational and Theoretical  
Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov

Editors : Nasir Ganikhodjaev

: Mansoor Saburov

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## MODULATIONAL INSTABILITY IN SALERNO MODEL

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### Abstract

We investigate the properties of modulational instability in the Salerno equation in quasi-one dimension in Bose-Einstein condensate (BEC). We analyze the regions of modulational instability of nonlinear plane waves and determine the conditions of its existence in BEC.

**Keywords:** Modulational instability, Salerno model, Bose-Einstein condensate.

### Introduction

The modulational instability is known phenomenon to be fundamental subject of the theory of nonlinear waves. This phenomenon consists of the instability of nonlinear plane waves against weak long-scale modulations with wave numbers (frequency) lower than some critical value. It has been predicted by Banjamen and Feir [3] for waves in deep water and by Bespalov and Talanov [2] for electro-magnetic waves in nonlinear media with cubic nonlinearity. Later, it was observed in nonlinear optics [7, 9, 11], plasma physics [5, 6], and condensate matter (Bose-Einstein Condensate, long Josephson junction,...) [8, 10]. In this work, we determine the regions and conditions of existence of modulational instability in Bose-Einstein condensate in periodic potential trap (optical lattices).

### The Model

It is very important to investigate the modulational instability in different models. We restrict ourselves to the Salerno model in Bose-Einstein condensate in optical lattice in quasi-1D. This model is a combination of the discrete non-linear Schrodinger (DNLS) equation [4] with cubic nonlinearity and Ablowitz-Ladik (AL) equation[1]. It is given by [5]

$$i \dot{\phi} + (1 + \mu |\phi_n|^2) (\phi_{n-1} + \phi_{n+1}) + 2\nu |\phi_n|^2 \phi_n = 0, \quad (1)$$

where  $\phi_n$  is the complex field amplitude at  $n^{\text{th}}$  site of the lattice,  $\mu$  is the nonlinearity of (AL) equation,  $\nu$  is the nonlinearity of (DNLS) equation.