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FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE PLATES BASED ON HIGHER ORDER SHEAR DEFORMATION THEORY BY USING FINITE ELEMENT METHOD

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Abstract

The present work deals with free vibration analysis of laminated composite plates using a present higher order displacement model. The present model is tested on the evaluation of free vibration of laminated composite plates in order to illustrate the deficiency of the classical plate theory and the first order shear deformation theory. The present work is also aimed at conducting a parametric study to investigate the frequency response of laminated composite plates for different material properties, number of layers, fiber orientations, boundary conditions and side to thickness ratios. A finite element computer code was developed and implemented to carry out the work.

Introduction

Engineering structures are generally designed on the basis of stress sustaining capacity, that is strength of the structural components. Other than strength of the elements, stability and vibration are also very important, especially when structural elements are thin and subjected to dynamic loads.

Since weight is a crucial factor in aircraft structures, the use of conventional isotropic material gives very little room for weight savings. Unlike the isotropic materials, the properties of composite materials can be tailored to have very high strength and yet being very light. As the strength-to-weight ratio of composite materials is high, structures made of composites often become very thin. In case of a structure made of isotropic material, the natural frequency is high if the element is thin. Adjustment in the natural frequency of such an element can be made either by changing the thickness or adjusting the boundary conditions. But, in case of composite materials there lies an additional feature of adjusting the natural frequency by designing lamination scheme.

Design of suitable lamination scheme can help to optimize the design of engineering structures. But as the kinematics and kinetics of composite materials are very much complicated, it is very difficult to even develop appropriate mathematical model for their analysis. So far some theories and models have been developed by different researchers and being applied for stress and vibration analysis of composite materials but still there lies lot of discrepancies in their results. As such the present work is aimed at applying a new mathematical model, deformation theory, for vibration analysis of laminated composite plates to study the suitability of the new model.

Literature Review

Many theories have been developed over the years to accurately predict the response of laminated composite plates. The earliest plate theory suggested was the Kirchhoff plate theory (CPT). In this theory the normal of the plane is assumed to be straight and normal in the deformed configuration. Such an assumption neglects the transverse shear effects, which have significant impact on the behavior of laminated composite plates. This limits the usage of the theory for thin plates where the transverse shear effects are negligible.

Since there are many structural applications using thick laminated composite plates with very high elasticity modulus to shear modulus ratio, the CPT becomes inadequate. Mindlin [1] further refined the CPT by including the transverse shear effects in his model and this theory is called the first order shear deformation theory (FSDT). In this model, the normal of the plane is assumed to be straight but no longer normal in the deformed configuration. This assumption makes the transverse shear strains and stresses to be constant in the thickness direction of the laminated plate. Since the assumption is very crude, a shear correction factor was introduced for a more realistic prediction of the transverse shear effects. Determining the value of the shear correction factor is problem dependent thus makes it cumbersome. Many improved models have been suggested by various researchers to overcome the deficiency of the FSDT model. In the present work a higher order model (HSDT) suggested by Pervez [2] has been used to investigate the applicability and suitability of the model.

Finite element modeling

The generalized two-dimensional displacement field of a plate at any time, t, is given as:

$$\begin{aligned} u(x,y,z,t) &= u_o(x,y,t) + \alpha_1 z\theta_x + \alpha_2 z^3\zeta_x \\ v(x,y,z,t) &= v_o(x,y,t) + \alpha_1 z\theta_y + \alpha_2 z^3\zeta_y \\ w(x,y,z,t) &= w_o(x,y,t) \qquad (1) \end{aligned}$$

where *(u, v, w)* are the displacement of a generic point *(x, y, z)* in a laminated anisotropic plate at time, *t*, *(u_o, v_o, w_o)* are the displacements of the middle plane of the laminated plate. $\theta_x(x, y, t)$ and $\theta_y(x, y, t)$ are the rotations of the normals of the reference plane about y and x axis respectively. $\zeta_x(x, y, t)$ and $\zeta_y(x, y, t)$ are the additional higher order functions that describes the warping behavior of the plate. Equation (1) corresponds to α_1=1 and α_2=0 for the FSDT model whereas the HSDT model corresponds α_1=1 and α_2=1.

For an undamped free vibration problem of laminated composite plate the equilibrium equation is given as:

$$[M]\{\ddot{D}\} + [K]\{D\} = 0 \qquad (2)$$

where [*K*] is the assembled stiffness matrix, [*M*] is the assembled mass matrix and $\{D\}$ is the assembled degrees of freedom. Equation (2) is solved using the inverse vector iteration scheme [3] to obtain the fundamental frequency and its corresponding mode shapes.

Results and discussion

The dimensionless physical parameters of typical graphite-epoxy are summarized in table 1.

Material Properties	Values
E_1	40.0
E_2	1.0
G_{12}	0.6
G_{13}	0.6
G_{23}	0.5
ν	0.25
ρ	1.0

Table 1: Dimensionless orthotropic property of graphite epoxy

Table 2 and 3 compare the results of the present HSDT model with the results of other models.

Model	Side to thickness ratio				
	5	10	20	50	100
CPT [4]	10.584 (24.12%)	11.011 (6.52%)	11.125 (0.80%)	11.158 (-0.94%)	11.163 (-1.19%)
Wu and Chen [4]	8.527	10.337	11.037	11.264	11.297
Reddy and Phan	9.010 (5.66%)	10.449 (1.80%)	10.968 (-0.63%)	11.132 (-1.17%)	11.156 (-1.25%)

[4]					
FSDT	9.087 (6.57%)	10.582 (2.37%)	11.114 (0.70%)	11.282 (0.16%)	11.311 (0.12%)
HSDT	9.024 (5.83%)	10.552 (2.08%)	11.114 (0.70%)	11.282 (0.16%)	11.311 (0.12%)

Table 2: Dimensionless fundamental frequency of simply supported 2 layered anti-symmetric square cross-ply laminate. Percentage of error in reference to Wu and Chen [4].

Model	Side to thickness ratio				
	4	10	20	50	100
CPT [4]	17.907 (95%)	18.625 (24%)	18.767 (6.4%)	18.799 (0.7%)	18.804 (-0.16%)
Wu and Chen [4]	9.193	15.069	17.636	18.670	18.835
Reddy and Phan [4]	9.497 (3.31%)	15.270 (1.33%)	17.668 (0.18%)	17.606 (-5.7%)	18.755 (-0.42%)
FSDT	9.734 (5.88%)	15.588 (3.44%)	17.848 (1.20%)	18.716 (0.25%)	18.857 (0.12%)
HSDT	9.399 (2.24%)	15.286 (1.44%)	17.736 (0.57%)	18.697 (0.14%)	18.851 (0.08%)

Table 3: Dimensionless fundamental frequency of simply supported 4 layered symmetric square cross-ply laminate. Percentage of error in reference to Wu and Chen [4].

The results clearly show that the present HSDT model predicts the frequency very close to the results of Wu and Chen [4]. In table 1, the CPT was found to predict the fundamental frequency about 24% higher as compared to the solution of Wu and Chen [4], whereas the HSDT model gives percentage error of only 5.83%. In the symmetric lamination, table 3, the CPT was found to predict the fundamental frequency as high as 95% and the present model gives an error of 2.24% which is better than the solution of Reddy and Phan [4].

Conclusions

The complicated behavior of laminated plates has shown the need for further developments of efficient numerical tools for free vibration study of laminated plates. The present work was aimed to apply a new mathematical model to overcome the inadequacy of the CPT and the FSDT. The finite element model developed based on the present HSDT model has proven to give accurate results when compared to other alternative models. Though other higher order models can provide better solutions, it cannot be done without the expense of computational time. The present model not only gives better results but also saves the computational time and effort.

References

[1] Mindlin, R.D. 1951, Influnce Of Rotary
Inertia And Shear Deformation On
Flexural
Motions Of Isotropic Elastic Plates.
ASME J.
Applied Mechanics 18:31-38.

[2] Pervez, T. 1991. Transient Dynamic,
Damping And Elasto-Plastic Analysis Of
Higher Order Laminated Anisotropic
Composite Plates Using Finite Element
Method. Ph.D Thesis. University of
Minnesota, USA.

[3] Bathe, K.J. 1996, Finite Element
Procedures.
Prentice-Hall Inc.

[4] Correia, V.M, Soares, C.M. and Soares,
C.A.
1998, Higher Order Models On The
Eigenfrequencies Analysis And optimal
Design Of Laminated Composite
Structures.
Composite Structures 39:237-253