Stress Analysis of Conical Pipe-reducers

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This paper deals with the stresses in the conical pipe reducers, used for connecting pipes of unequal diameters, under uniform internal pressure. Computer program for stress analysis of general composite shells, with necessary modifications, has been used for the present analysis. In this analysis, governing non-linear equations for axisymmetric deformation of conical reducers are solved by the method of multisegment integration, developed by Kalnins and Lestingi. The discrepancy between the linear and the non-linear theories in predicting the stresses in the reducers is discussed in the analysis. The effect of variation of the apex angle and the thickness ratio of the conical reducers on the stress distribution is also discussed in this paper.

Keywords : Stress analysis, Pipes, Conical reducers

NOTATION

C, D	: extensional rigidity, Eh; bending rigidity $Eh^3/[12(1-\nu^2)]$
$\overline{C}, \overline{D}$	: $(1-\nu^2)\,x_e/r$, $1/\{12\,\overline{P}\,\overline{T}^2\,\overline{R}(1-\nu^2)\}$
E, ν	: Young's modulus, Poisson's ratio, respectively
$h, \overline{T}$	: reducer wall thickness, R/h
H, V	: radial and axial components of stress resultants, respectively
$\overline{H}, \overline{V}$	: H/PR, V/PR
k_θ, k_i	: circumferential and meridional curvature changes, respectively
$\overline{K}_\theta, \overline{K}_x$	: $k_\theta x_e$, $k_x x_e$
M_θ, M_x	: circumferential and meridional couple resultants, respectively
$\overline{M}_\theta, \overline{M}_x$	: M_θ/PRh, M_x/PRh
N_x, N_θ	: meridional and circumferential stress resultants, respectively
$\overline{N}_x, \overline{N}_\theta$	: N_x/PR, N_θ/PR
$P, \overline{P}, \overline{R}$	: external pressure, P/E, x_e/R
R_1, R	: radii of the smaller and larger pipes respectively, connected by the reducer
$r_0, \overline{r}_0$	: radial distance of points on the undeformed middle surface from axis of symmetry, r_0/x_e
u, w	: radial and axial displacements, respectively
$\overline{u}, \overline{w}$	: uEh/PR^2, wEh/PR^2
$x, \overline{x}$	: distance measured along meridian, x/x_e
x_e	: total meridional length of the reducer measured from its apex.
α, β, r	: shell parameter, $\phi_0 - \phi$, $r_0 + u$
γ	: semi-apex angle of the reducer
$\in_x, \in_\theta$	: middle surface strains
$\overline{\in}_x, \overline{\in}_\theta$	: $\in_x Ehx_e/PR^2$, $\in_\theta Ehx_e/PR^2$
σ_c, σ_a	: circumferential and meridional stresses respectively
$\sigma_{ci}, \sigma_{co}, \sigma_{ai}, \sigma_{ao}$	: circumferential and meridional stresses at the inner and outer fibres, respectively.
ϕ_0, ϕ	: angle between axis of symmetry and normal to undeformed and deformed middle surfaces, respectively

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INTRODUCTION

Stresses in the pipe reducers, used for connecting pipes of unequal diameters, are completely different from those in the pipes under the same internal pressure. It is, thus, essential to analyze the stresses in the reducers separately from those in the pipes. Further, use of reducers always involves the use of flanges for their connection with the pipes. These flanges always introduce additional stresses in the pipes as well as in the transitional reducer elements. The present analysis thus investigates the stresses in the pipe reducers for varying parameters.

It has been shown[1,3-11] that the non-linear theory of shells are essential for analyzing the distribution of stresses in the pressure vessel problems. This is due to the fact that, at the junctions of two different segments of shells or at points having small meridional radius of curvature, the linear theory fails to account for the pronounced change in curvature and consequently predicts unrealistic solutions. This fact becomes more evident at higher loading and for thinner shells[3,4]. Solutions are, thus, obtained for reducers of different thicknesses, using both linear and the non-linear theories in the present analysis, so that the shortcomings of the linear theories in case of reducers are verified and noted.

The multisegment method of integration developed by Kalnins and Lestingi[1] is used here to solve the non-linear shell equations for the conical reducers. This multisegment integration technique is a very powerful tool for handling nonlinear shell equations where both the finite element and finite difference techniques usually fail due to nonconvergence in the iteration process of these techniques in case of boundary value problems. This method can be used for shell meridian of any length with discontinuity in slope or thickness where method like direct integration fails.

GOVERNING EQUATIONS

Reissner's[2] nonlinear equations for axisymmetric deformation of shells of revolution, as modified by Uddin[3], is used in this analysis. The governing equations are given under:

$$\overline{\epsilon}_\theta = \frac{\overline{u}}{\overline{r}_0} \quad (1)$$

$$\phi = \phi_0 - \beta \quad (2)$$

$$\overline{k}_\theta = (\sin\phi_0 - \sin\phi)/\overline{r}_0 \quad (3)$$

$$\overline{N}_x = \overline{H}\cos\phi + \overline{V}\sin\phi \quad (4)$$

$$\overline{\epsilon}_x = \overline{C}\,\overline{N}_x - \nu\,\overline{\epsilon}_\theta \quad (5)$$

$$\overline{k}_x = \overline{M}_x/\overline{D} - \overline{k}_\theta\,\nu \quad (6)$$

$$\overline{N}_\theta = \left(\overline{\epsilon}_\theta = \nu\overline{\epsilon}_x\right)/\overline{C} \quad (7)$$

$$\overline{M}_\theta = \overline{D}\,(\overline{k}_\theta + \nu\,\overline{k}_x) \quad (8)$$

$$\overline{\alpha} = \overline{L} + \overline{\epsilon}_x \quad (9)$$

$$\overline{r} = \overline{L}\,\overline{r}_0 + \overline{u} \quad (10)$$

$$\frac{d\overline{w}}{d\overline{x}} = \overline{\alpha}\sin\phi - \overline{L}\sin\phi_0 \quad (11)$$

$$\frac{d\overline{u}}{d\overline{x}} = \overline{\alpha}\cos\phi - \overline{L}\cos\phi_0 \quad (12)$$

$$\frac{d\overline{\beta}}{d\overline{x}} = \overline{k}_x \quad (13)$$

$$\frac{d\overline{V}}{d\overline{x}} = -\,\overline{\alpha}\cos\phi\,(\overline{V}/\overline{r} - \overline{P}\,\overline{T}) \quad (14)$$

$$\frac{d\overline{H}}{d\overline{x}} = -\,\overline{\alpha}\left\{(\overline{H}\cos\phi - \overline{N}_\theta)/\overline{r} + \overline{P}\,\overline{T}\sin\phi\right\} \quad (15)$$

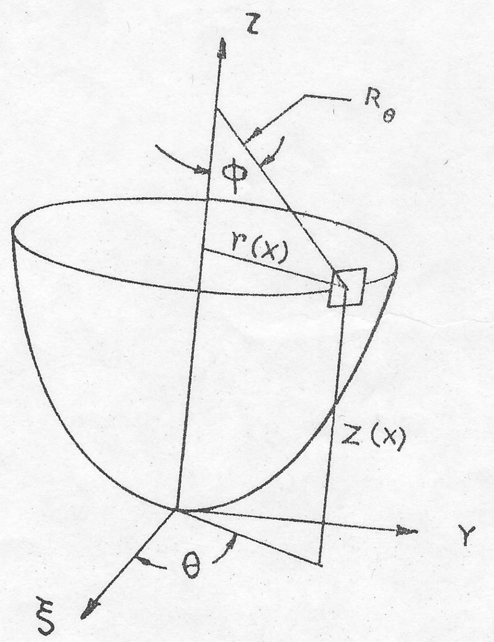

Fig 1 Middle surface of shell

$$\frac{d\overline{M}}{d\overline{x}} = \alpha\cos\phi\,(\overline{M}_\theta - \overline{M}_x)/\overline{r} - \overline{\alpha}\,\overline{P}\,\overline{T}^2\,(\overline{H}\sin\phi - \overline{V}\cos\phi) \quad (16)$$

The above mentioned governing equations include approximations of the Kirchoff hypothesis and other plausible assumptions, like the middle surface strains are negligible compared to unity, leading to the dropping of terms of the order of *h/R* in the constitutive equations. However, these assumptions do not introduce any significant error in the solutions of most practical thin shell problems.

BOUNDARY CONDITIONS

For the general case of axisymmetric deformations of shells of revolution, it was shown[3] that the boundary conditions at the edges require specification of :

$$\overline{H} \text{ or } \overline{u},\ \overline{M}_x \text{ or } \overline{\beta},\ \text{and } \overline{V} \text{ or } \overline{w} \quad (17)$$

Considering both the larger and the smaller diameters of the reducer sufficiently large and connected to the reducers through flanges, it is quite justified to assume that both the

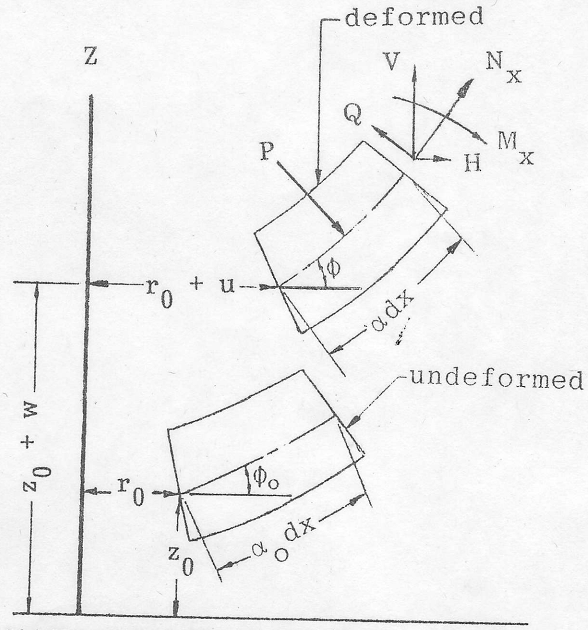

Fig 2(a) Side view of element of shell in deformed and undeformed state

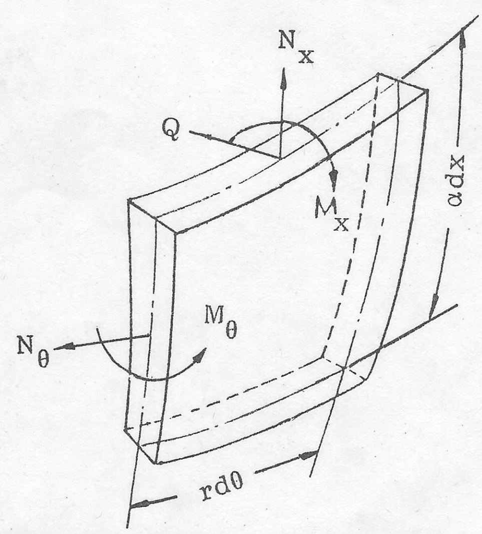

Fig 2(b) Element of shell showing stress resultants and couples

ends of the reducer are fixed or clamped. Hence, conditions at both the larger and smaller ends of the reducer are specified as

$$\bar{u} = 0,\ \bar{\beta} = 0,\ \bar{w} = 0 \qquad (18)$$

SOLUTION

The same method of multisegment integration as used by Uddin[3] for nonlinear analysis of pressure vessels has been employed with boundary conditions given in equations (18). The program starts with an initial arbitrary load $\bar{P}$ and an incremental load step $\Delta \bar{P}$, and then solves the nonlinear governing equation for load $\bar{P}$ with a preassigned convergence criterion. The load $\bar{P}$ is then increased by the incremental load step $\Delta \bar{P}$ and solution is then obtained for this new load, through iteration, with solutions of immediate previous load as initial values. If the solution fails to converge at any load $\bar{P}$,

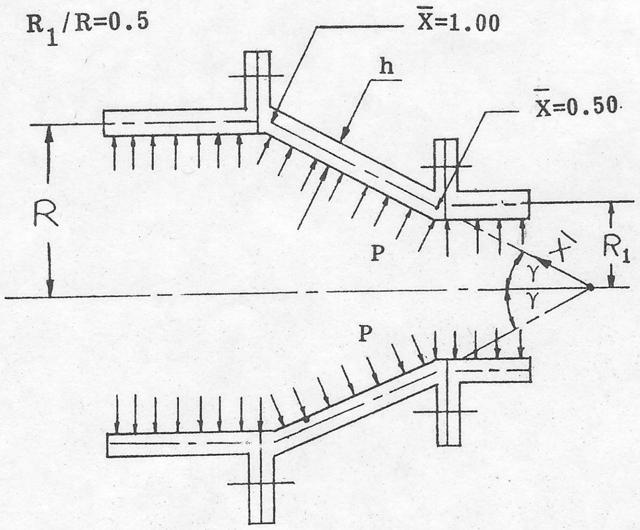

Fig 3(a) Conical reducer (parameters : R/h = thickness ratio, R_1/R = reduction ratio, γ = semi-apex angle of the conical reducer)

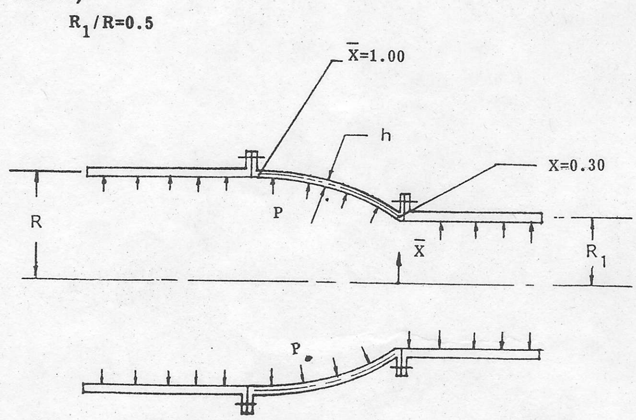

Fig 3(b) Parabolic reducer (parameters : R/h = thickness ratio, R_1/R = reduction ratio[11])

the load step is halved and the solution is again attempted. In this way, nonlinear solutions are obtained at increased values of the loading parameter upto the desired level of loading.

RESULTS AND DISCUSSION

In order to point out the deficiencies of the results of the linear theories, the results of both the linear and the nonlinear theories are shown in the same figures. It should be mentioned here that a diameter reduction ratio of 0.5 is considered in this analysis, *ie* $\bar{x}$ = 1.0 corresponds to the larger end of the reducer while $\bar{x}$ = 0.5 corresponds to the smaller end (Fig 3). From the results as shown in Figs (4 - 8), it is evident that the linear theory is very conservative in predicting the results. It is also noted that the conservativeness in estimating the stresses, stress resultants and the moment resultants increases with the increase of thinness. Uddin[3] pointed out that for shells subjected to internal pressure this conservativeness of the linear theory also increased for increasing values of loading parameters, although, for avoiding the crowding of too many results, it is not shown in this analysis. However, it is observed that the prediction of nonlinear theory about the meridional stress resultants, $\bar{N}_x$, is higher than that of the linear theory, which is quite contrary to the observations in other composite shell problems.

It should be pointed out here that the stress parameters are all normalized in terms of the loading parameter. As a result, there is no variation of the normalized parameters with the loading for linear theory [Figs (4 - 8)].

The most interesting observation in the present analysis is the effect of the apex angle on the predicted results. It can be observed from the Figs (4 - 8) that the discrepancy between the linear and the nonlinear theories in predicting the results becomes more prominent for reducers of higher apex angle (90°) than those of smaller apex angle (60°).

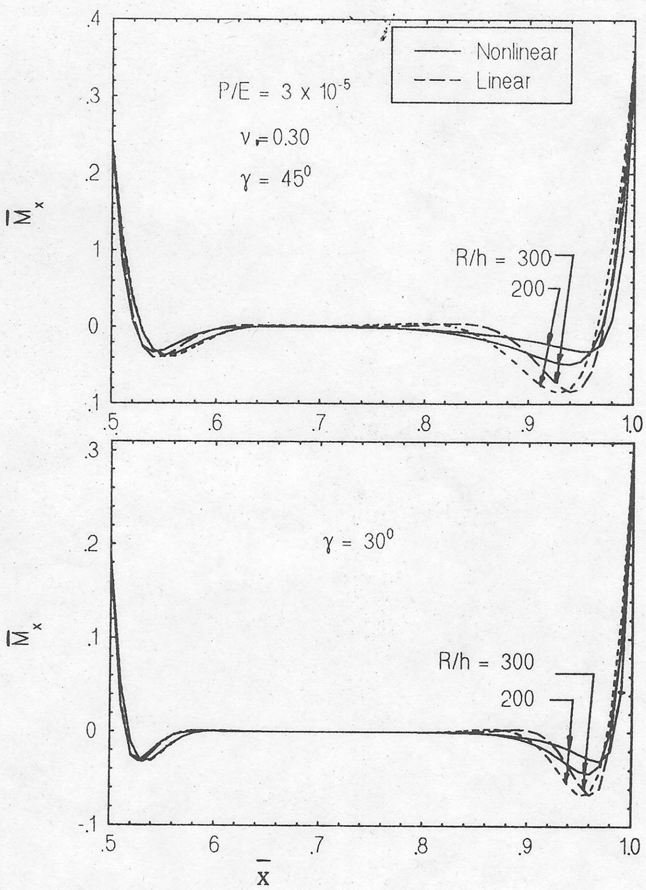

Fig 4 Effect of apex angle and thickness ratio on meridional bending moments in the reducer

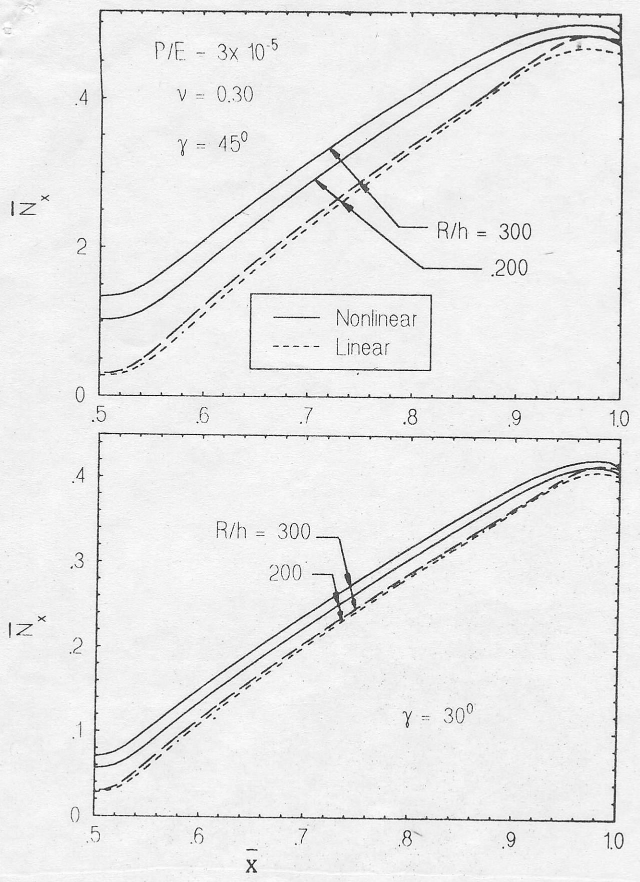

Fig 5 Effect of apex angle and thickness ratio on meridional stress resultants in the reducer

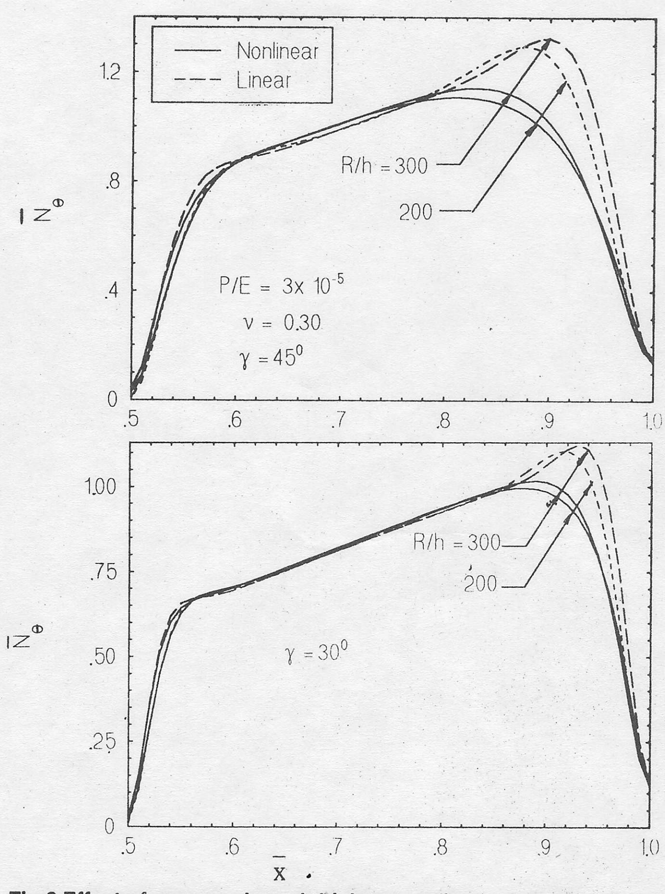

Fig 6 Effect of apex angle and thickness ratio on circumferential stress resultants in the reducer

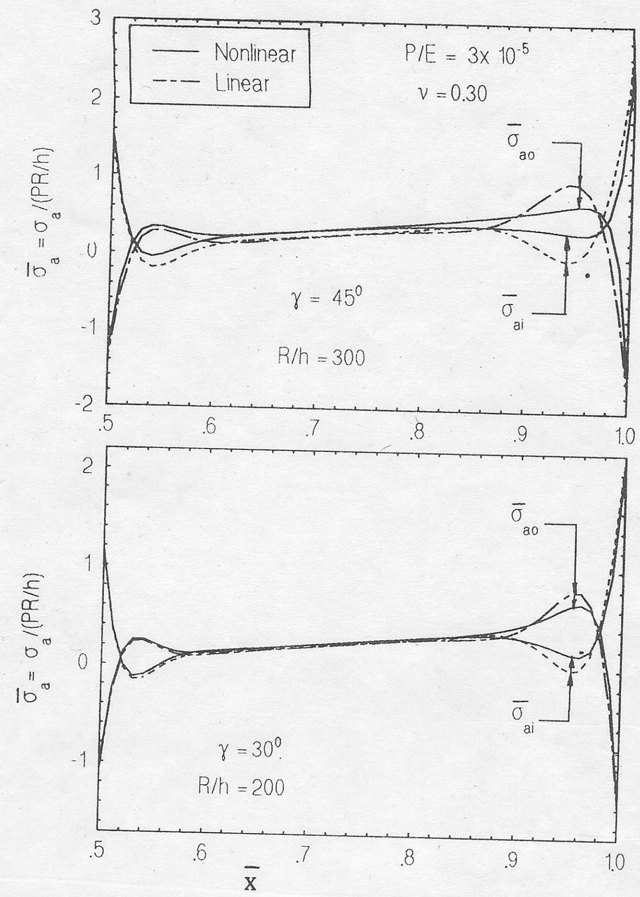

Fig 7 Effect of apex angle and thickness ratio on meridional stresses in the reducer

This is because of the fact that the cone approaches to a plate as the apex angle is increased, losing its membrane stiffness. Consequently, conical reducers with higher apex angle deforms substantially under load, resulting in noticeable discrepancies between the two theories in predicting the results.

Fig 4 shows the variation of the bending moments along the shell meridian. The presence of bending moment, in effect, shows the disturbance in the membrane solutions of the reducer. In other words, the magnitude of the bending moment is a measure of the deviation from the characteristic membrane behaviour of shells. The discrepancy between the linear and the nonlinear theories in predicting the solutions is observed here to increase with increasing thinness and increasing apex angle.

The meridional stress resultants are shown in Fig 5. As an exception, it is observed that the linear theory predicts lower values of the stress resultants in comparison to that of the nonlinear theory, quite contrary to the normal expectations.

The circumferential stress resultants are shown in Fig 6. It is clear from this figure that the linear theory overestimates the results compared to nonlinear theory. Further, it is observed that this stress resultant increases as apex angle of the reducer increases and its maximum value occurs in a zone very close to the flange at its larger end.

The variations of the meridional stresses at the inner and the outer surfaces along the reducer meridian are shown in Fig 7. The presence of bending moments at and near the two fixed ends are responsible for the wavy distribution of the stresses near the two ends. The maximum meridional stress in the reducer occurs at the inner face of its junction with the larger-end flange and its value is about 5.2 times the membrane meridional stress in a cylindrical shell of radius R and thickness h.

The circumferential stresses at the inner and the outer surfaces of the reducer are shown in Fig 8. Here also, it is observed that the prediction of the stresses by the linear theory is much higher than that by the nonlinear theory.

Observation from the stress curves shown in Figs 7 and 8, reveals that the meridional stresses at the inner surface (σ_{ai}) is the most critical stress. It is also seen that the reducer is critically stressed near the two ends.

Rahman[11] studied the stresses in a parabolic reducer [Fig (3b)] subjected to internal pressure. The result of the analysis has

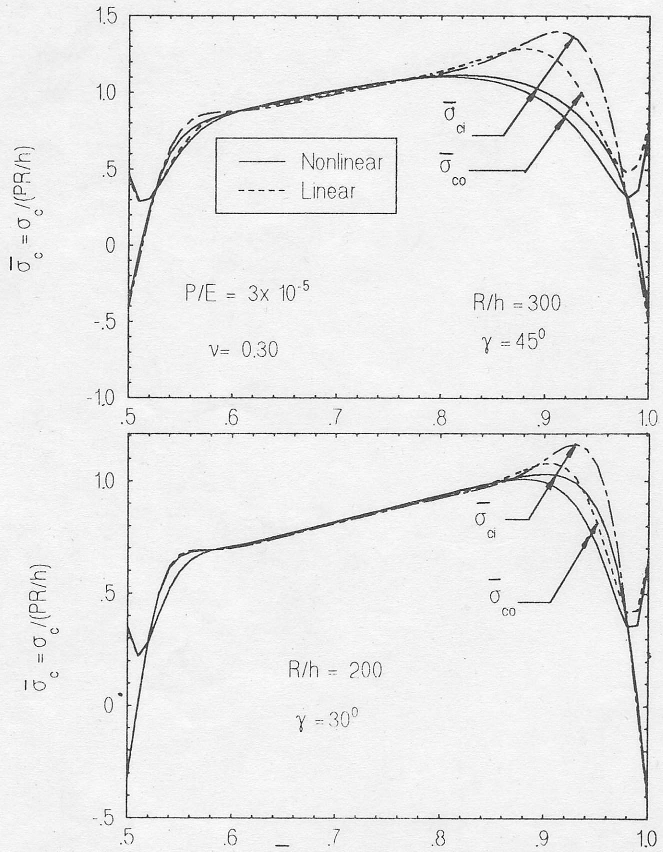

Fig 8 Effect of apex angle and thickness ratio on circumferential stresses in the reducer

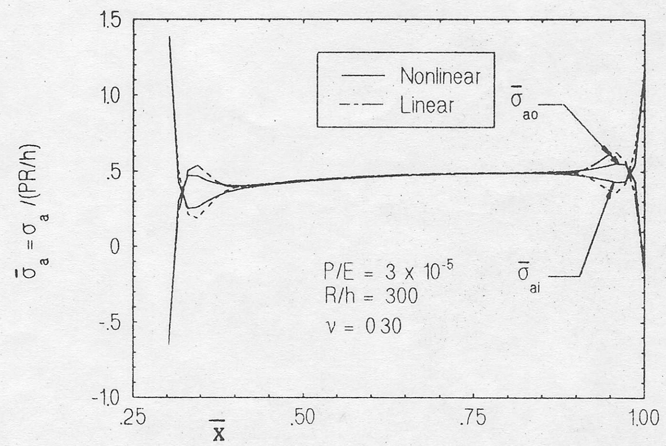

Fig 9 Meridional stresses for a parabolic reducer[11]

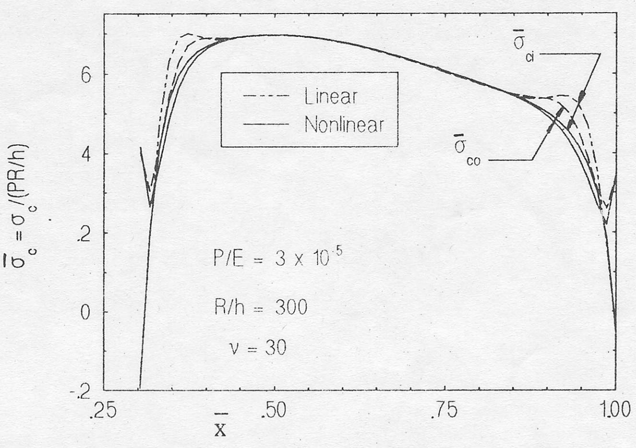

Fig 10 Circumferential stresses for a parabolic reducer[11]

shown that, for the same diameter reduction ratio (0.50) and thickness ratio (300), the stresses in a parabolic reducer are much lower in magnitude than that in a conical reducer [Figs (9 and 10)] at the same intensity of loading. From Fig 9, the maximum non-dimensional meridional stress at the inner surface is only 1.4 in case of a parabolic reducer, whereas it is as high as 2.6 in the identical conical reducer (Fig 7). Similarly, the circumferential stresses are also of much smaller magnitude in a parabolic reducer (Fig 10) than that in an identical conical reducer (Fig 8).

Comparing the conical reducers with their counterpart parabolic reducers, it is observed that the parabolic reducers are far superior to their conical counterparts in respect of stresses in them for the same internal pressure. Of course, stress should not be the only factor in concluding the superiority of the parabolic reducers over their conical counterparts. For example, factors like ease of fabrication and loss of energy by fluid flowing through them should also be taken into consideration in evaluating the ultimate superiority of one kind of reducer over the other.

CONCLUSIONS

It is concluded that the nonlinear theory is essential for analyzing the stress problems of reducers, specially of thinner reducers. In most of the cases, the linear theory fails to account for the effect of change in curvature and consequently overestimates the stresses developed in the reducers under uniform internal pressure. The pipe reducers are critically stressed at and near the flange connected ends because of the presence of the bending moments. An important observation of this work is that the more the apex angle the more is the discrepancy between the linear and the nonlinear theories in predicting the results. The result of this analysis has also revealed the fact that, between a conical and an identical parabolic reducer, the latter one can withstand much hither internal pressure.

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