

Advances
in
Aircraft Structures

Editor

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Chapter 34

Axial Stiffness Matrix of Non-Uniform Bernoulli-Euler Bar Elements

Erwin Sulaeman

Abstract

This chapter describes the procedure to develop the axial stiffness matrix of beam element having arbitrary variation of axial stiffness distribution along its span. To obtain analytical formulation, the stiffness matrix is constructed from the flexibility matrix. A Bernoulli-Euler differential equation that relates the load and deformation angle derived first. The general rational function resulting from the integration of the Bernoulli-Euler is transformed to a simpler rational function using a minimum denominator rational function procedure.

Keywords: *Axial stiffness matrix, non-prismatic beam, finite element method.*

1. Introduction

The axial stiffness matrix developed in the present Chapter is developed following the derivation for a similar problem presented for bending deformation (Sulaeman, 2011). Consider a tapered axial bar element of length L made of an isotropic elastic material of modulus E as shown in Fig. 1. Assume that the cross-section area $A = A(x)$ about the x axis varies as an arbitrary polynomial function in the form of expansion series in x as follow: