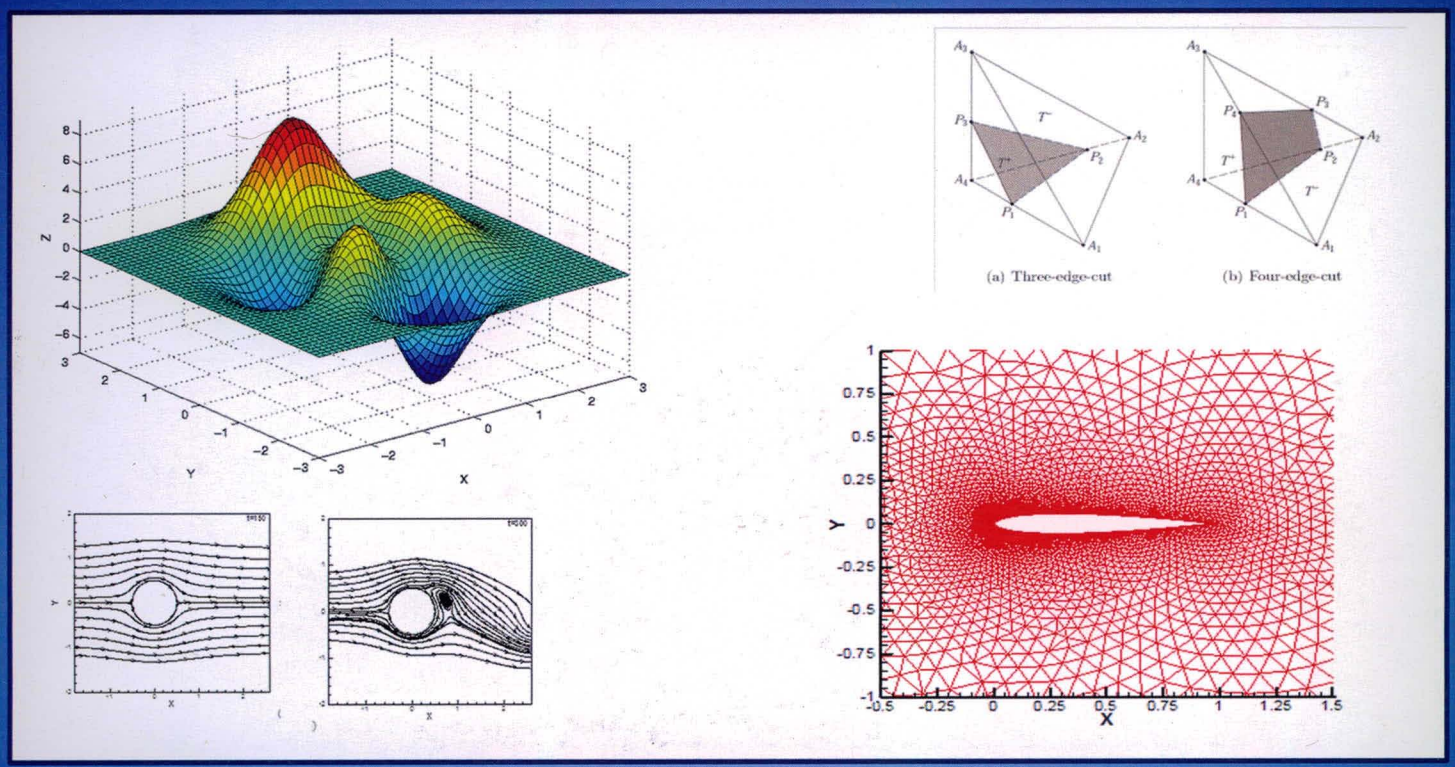
ADVANCED NUMERICAL TECHNIQUES IN ENGINEERING and SCIENCE



Editors
AHMAD TARIQ JAMEEL
WAQAR ASRAR

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ADVANCED NUMERICAL TECHNIQUES IN ENGINEERING AND SCIENCE

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CHAPTER 2

Numerical Simulation of a Simple Couette Flow in MATLAB using Explicit and Implicit Finite Difference Schemes

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ABSTRACT

Numerical simulation in Engineering and Science disciplines is fast emerging as a viable and relatively inexpensive alternative to experimental testing which can often involve very elaborate setups and prohibitively expensive equipment. Numerical simulations generally involve solving single or multiple differential equations of varying complexity, ranging from simple ordinary differential equations to multiple, coupled, partial differential equations. The results obtained thereby are then visualized using appropriate visual rendering tools to get a representation of the actual problem being investigated. The aim of this chapter is to provide the reader with a simple, step-by-step guide to carry out a numerical simulation of a simple Couette flow problem using the MATLAB platform. The fundamentals of the finite difference methodology of numerical simulations are illustrated in this simplistic example, which can easily be extended to simulating more complex scenarios involving differential equations of higher complexity. It is also hoped that this exercise will encourage the reader to explore MATLAB capabilities for research and development work.

Keywords: Numerical simulation, Couette flow, Finite difference, Explicit and implicit implicit schemes, MATLAB, Instability