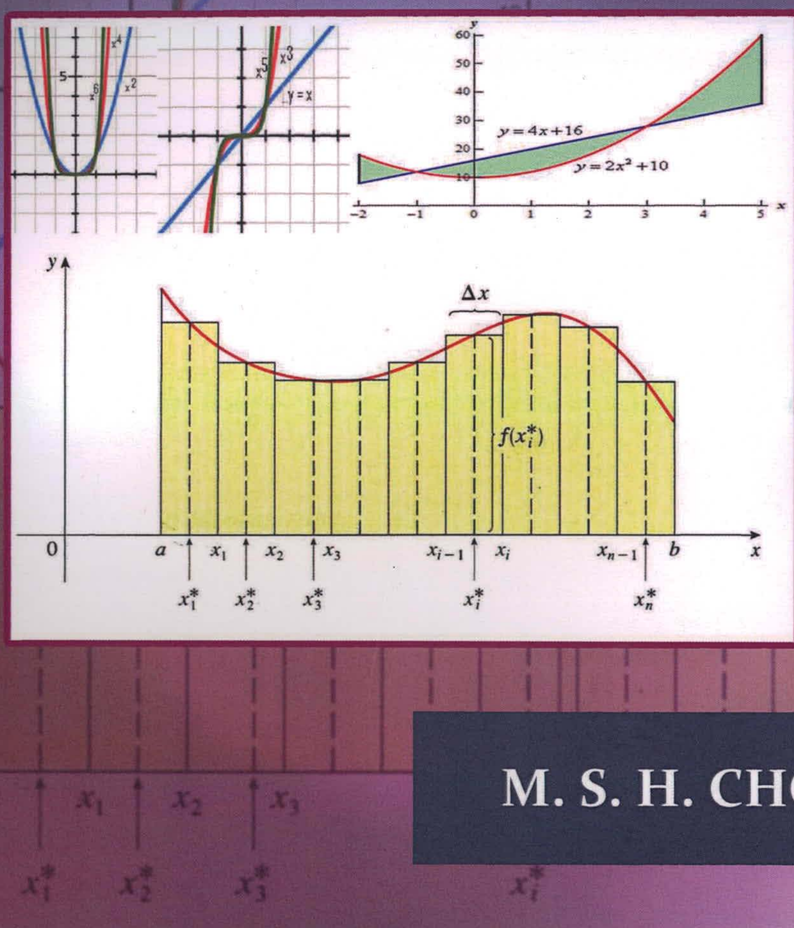
CALCULUS WITH SINGLE VARIABLE



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CHAPTER 7

INFINITE SERIES

Faiz A. Elfaki and M. Azram

If someone asked you to list all natural numbers that are perfect squares, you might begin by writing ; 1, 4, 9, 16, 25, 36, . . . but you would soon realize that it is impossible to actually list all the perfect sequences, since there are infinite number of them. One common method is to write $1,4,9,16,...,n^2,...\quad n \in N$ where N is the set of natural numbers. A list of numbers such as this is generally called a sequence.

7.1. Sequences of Real numbers

7.1.1. Definition of Sequence

A sequence is a function with domain a set of successive integers. If we consider the function $f(n) = 2n - 1$. The domain of f is the set of natural numbers N. The first and second terms are $f(1) = 2(1) - 1 = 1$, $f(2) = 3$ and the nth term (general term) is $a_n = 2n - 1$. Some time we write the general term of a sequence between braces as $\{a_n\}$ or $\{2n - 1\}$.

Definition: If the domain of the function is a finite set of successive integers, then the sequence is called a finite sequence. If the domain is an infinite set of successive integers, then the sequence is called an infinite sequence.
Some sequences are specified by a recursion formula, that is, a formula that defines each term in terms of one or more preceding terms. The sequence we have chosen to illustrate a recursion formula is a very famous sequence in the history of mathematics called the Fibonacci sequence. It named after the most celebrated mathematician of the thirteenth century, Leonardo Fibonacci from Italy (1180-1250).

Example 7.1: Finding the General Term of a Sequence
Finding the general term of a sequence whose terms are:
(a) 5, 6, 7, 8, ... **(b)** 2, -4, 8, -16, ...

Solutions:
(a) Since these terms are consecutive integers, one solution is $a_n = n$, $n \geq 5$. if we want the domain of the sequence to be all natural numbers, then another solution is $b_n = n + 4$, $n \geq 1$
(b) Each of these terms can be written as the product of a power of 2 and a power of -1:

$$2 = (-1)^0\, 2^1$$
$$-4 = (-1)^1\, 2^2$$
$$8 = (-1)^2\, 2^3$$
$$-16 = (-1)^3\, 2^4$$

If we choose the domain to be all natural numbers, then a solution is $a_n = (-1)^{n-1}\, 2^n$.