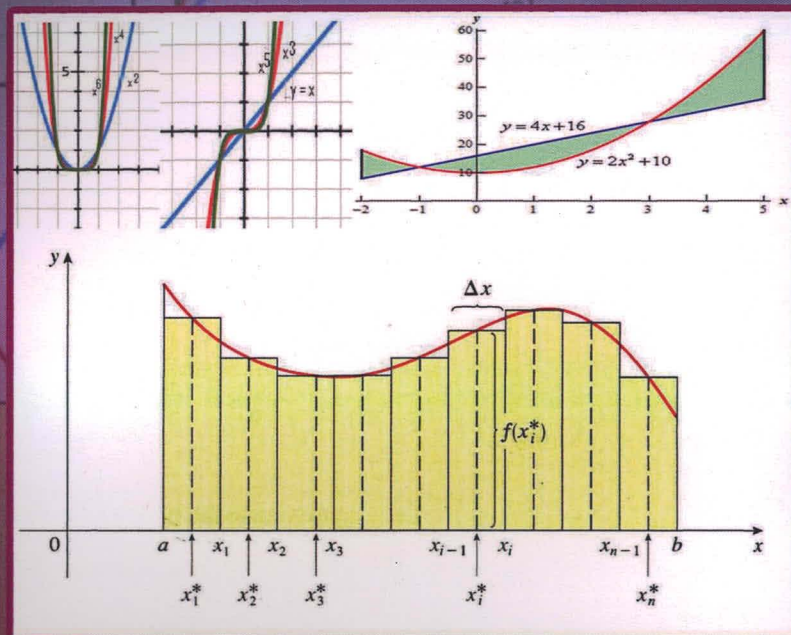


# CALCULUS WITH SINGLE VARIABLE



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# CHAPTER 7

## INFINITE SERIES

Faiz A. Elfaki and M. Azram

If someone asked you to list all natural numbers that are perfect squares, you might begin by writing ; 1, 4, 9, 16, 25, 36, . . . but you would soon realize that it is impossible to actually list all the perfect sequences, since there are infinite number of them. One common method is to write  $1, 4, 9, 16, \dots, n^2, \dots$   $n \in N$  where  $N$  is the set of natural numbers. A list of numbers such as this is generally called a sequence.

### 7.1. Sequences of Real numbers

#### 7.1.1. Definition of Sequence

A sequence is a function with domain a set of successive integers. If we consider the function  $f(n) = 2n - 1$ . The domain of  $f$  is the set of natural numbers  $N$ . The first and second terms are  $f(1) = 2(1) - 1 = 1$ ,  $f(2) = 3$  and the  $n$ th term (general term) is  $a_n = 2n - 1$ . Some time we write the general term of a sequence between braces as  $\{a_n\}$  or  $\{2n - 1\}$ .

**Definition:** If the domain of the function is a finite set of successive integers, then the sequence is called a finite sequence. If the domain is an infinite set of successive integers, then the sequence is called an infinite sequence.

Some sequences are specified by a recursion formula, that is, a formula that defines each term in terms of one or more preceding terms. The sequence we have chosen to illustrate a recursion formula is a very famous sequence in the history of mathematics called the Fibonacci sequence. It named after the most celebrated mathematician of the thirteenth century, Leonardo Fibonacci from Italy (1180-1250).

#### Example 7.1: Finding the General Term of a Sequence

Finding the general term of a sequence whose terms are:

(a) 5, 6, 7, 8, ....

(b) 2, -4, 8, -16, ....

#### Solutions:

(a) Since these terms are consecutive integers, one solution is  $a_n = n$ ,  $n \geq 5$ . if we want the domain of the sequence to be all natural numbers, then another solution is  $b_n = n + 4$ ,  $n \geq 1$

(b) Each of these terms can be written as the product of a power of 2 and a power of  $-1$ :

$$\begin{aligned} 2 &= (-1)^0 2^1 \\ -4 &= (-1)^1 2^2 \\ 8 &= (-1)^2 2^3 \\ -16 &= (-1)^3 2^4 \end{aligned}$$

If we choose the domain to be all natural numbers, then a solution is  $a_n = (-1)^{n-1} 2^n$ .