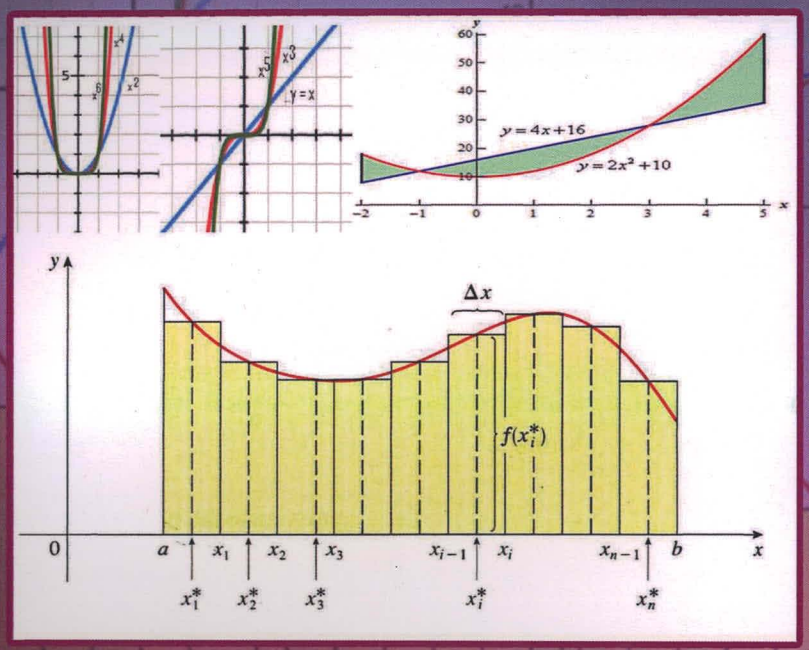
CALCULUS WITH SINGLE VARIABLE



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CALCULUS WITH SINGLE VARIABLE

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CHAPTER 2

THE DERIVATIVES

The derivative is a mathematical tool used to measure rates of change. Basically we are focusing here on the rate of change of functions. The derivative will provide a numerical measure of the steepness of a curve at particular point. By studying the derivative, we will be able to deal numerically with rates of change in applied problems

2.1 The slope of a straight line

As a starting point we will use the slope of a straight line to illustrate a simple understanding of the derivative. All we know that the equations of nonvertical lines can be written as $y = mx + b$,where the number m is called the slope of the line and the point $(0, b)$ is called the y-intercept. Usually we refer to the equation like the above one as slope-intercept equation of the line.

To understand the concept of the slope, imagine walking along a line from left to right. On lines with negative slopes we will be walking downhill; the more negative the slope, the steeper the descent. On lines with positive slopes we will be walking uphill; the greater the slope' the steeper the ascent. Walking on lines of zero slope corresponds to walking on level ground (horizontally), likewise walking on line with ∞ slope corresponds to walking on a wall vertically (try?)

The following graphs are examples of lines with different slopes

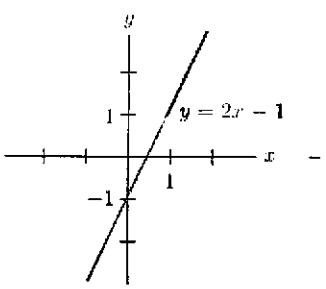
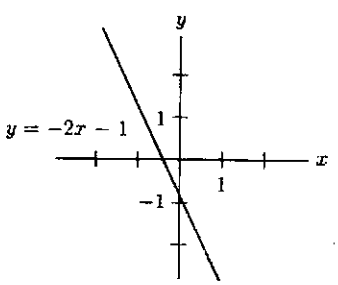


Fig.1: Line with positive slope



Fid.2: Line with negative slope

The slope and y-intercept of a straight line often have physical interpretations, as in the following figure