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ANALYSIS OF MECHANICAL DRAFT WET COOLING TOWERS

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ABSTRACT

The objective of the paper is to present a review on the analysis of mechanical-draft wet cooling towers. Starting with the basic fundamentals of a cooling tower, an attempt is made here to present an analysis of the important computational models available. The physical situation within a cooling tower is very complex (films and droplets of water in air are in a constantly changing configuration). There is no mathematical model which is capable of simulating every detail of simultaneous heat and mass transfer process occurring within the tower. Consequently, simplifying assumptions must be made for the analysis. A comprehensive list of assumptions is provided which are used for the different models.

Eight computational models are analyzed here,namely (a) ESC code, (b) FACTS, (c) VERAZD, (d) STAR, (e) Sutherland's Model, (f) Model by Fujita and Tezuka, (g) Webb's Model, (h) Model by Jaber and Webb. Each model makes use of somewhat different set of assumptions. So, the results of the calculations of heat/mass transfer coefficients also differ. Analysis of the above models gives us an idea about different numerical solutions of cooling tower design. It is difficult to draw general conclusions concerning the comparative merits of the correlations, or of the codes. Yet it is attempted here to compare the different models from the view point of design, computational error, computational time, simplicity of usage and practicability.

INTRODUCTION

Cooling tower performance has not been regarded by many as important an issue as the performance level of the turbine or boiler in the past. Because the cooling tower is generally removed from the main unit, its existence was not necessarily acknowledged. Much attention is being paid now-a-days on cooling tower design. A study by North American Electric Reliability Council in 1986 estimated that about US $20 million per year in heat rate alone are lost because of cooling tower efficiency below design [12].

The function of a wet cooling tower is to reduce the temperature of circulating water by bring it into direct contact with air. This cooling is achieved partly by the evaporation of a fraction of the circulating water, and partly by a transfer of sensible heat.

A mechanical draft cooling tower is one which utilizes fans to move ambient air through the tower. Mechanical-draft cooling towers are classified into two different types: induced and forced draft towers. In induced draft cooling towers, air is drawn through the tower and in forced draft cooling towers it is forced through the tower. At the same time these towers can be grouped in crossflow or counterflow, depending on the relative movement of air and water. In the crossflow type, air generally travels horizontally across the falling water, while in the counterflow, it travels vertically upward through the falling water.

The basic theory of cooling tower operation was first proposed by Walker et al [23], who developed the basic equations for total mass and energy transfer, and considered each process separately. Merkel [19]combined the coefficients of sensible heat and mass transfer into a single over-all coefficient based upon enthalpy-potential as the driving force. The theory proposed by Merkel requires a few simplifying assumptions which have been almost universally adopted for the calculation of cooling tower performance.

SIMPLIFYING ASSUMPTIONS FOR ANALYSIS

The physical situation within a wet cooling tower is very complex where the films and droplets of water in air are in a constantly changing configuration. No mathematical model, no matter how sophisticated, is capable of simulating every detail in the simultaneous heat and mass transfer process occurring within the tower. Consequently, simplifying assumptions must be made to analyze the combined heat and mass transfer that takes place. Different computational models make use of somewhat different sets of assumptions. So, the results of the calculations of heat/mass transfer coefficients also differ in different models. A comprehensive list of assumptions for different models follows [12].

1. Merkel's assumptions:
   a. Use of the enthalpy driving force
   b. Lewis number(Le) is equal to unity
   c. Driving force for mass transfer is the difference in absolute humidity at the water surface and in the air next to the surface
   d. Quantity of heat transferred by evaporation is represented by the heat of vaporization
   e. Loss of water due to evaporation is neglected.
2. Separate effects of heat and mass transfer are neglected
3. Possible fogging in the air within the tower is neglected
4. Airflow is constant throughout the tower
5. Inlet conditions (i.e., m_w, m_a, T, i) are uniform in the cooling tower. (Water loading and air temperature are probably nonuniform).
6. Uniform fill characteristics and uniform pressure drop throughout the tower.
7. The surface temperature of water is same as the bulk temperature.
8. Heat transfer through the spray zone and rain zones of crossflow towers is neglected.
9. Backmixing of airflow in spray and rain zones of counterflow towers is neglected.
10. Redistribution of water due to mixing (entrance effects), segregation (air/water segregation to lower energy flow configurations), and air drag are neglected.
11. Changing rate of heat and mass transfer with time is neglected since phases are mixed.

ANALYSIS

The generally accepted concept of cooling tower performance was developed by Merkel (19). Assumptions 1-11 are applicable to simplify the development of the final equation. The analysis combines the sensible and latent heat transfer into an over-all process based on enthalpy potential as the driving force. The two processes are combined, into a single equation:

$$\frac{d\,i_a}{d\,T_w} = \frac{\dot{m}_w c_w}{\dot{m}_a} \tag{1}$$

$$\dot{m}_a\, d\,i_a = \dot{m}_w c_w\, d\,T_w = K_w a\; d\,V\,(i'_s - i_a) \tag{2}$$

From which tower volume is given by

$$V = \frac{\dot{m}_w c_w}{K_w a} \int_{T_{w2}}^{T_{wi}} \frac{d\,T_w}{i'_s - i_a} \tag{3}$$

$$\frac{d\,i}{d\,T} = \frac{i'_s - i_a}{T_w - T_a} \tag{4}$$

To determine the air and water conditions throughout the cooling tower, Eqs. (1) and (4) must be solved. Different numerical solutions of Eqs. (1) and (4) are available.

General one-dimensional and two-dimensional models developed by Electric Power Research Institute (EPRI) [13] are discussed below.

ONE-DIMENSIONAL MODEL

Consider first the configuration in which air and water are in counterflow (Figures 1 and 2). Assume that the airflow and water flow are uniform across a cross section of the cooling tower and the water temperature is uniform through the film, i.e., $T_s = T_w$.

The mass balance for the control volume gives

$$- d\dot{m}_w = d\dot{m}_v = \dot{m}_a \, dw_a \tag{5}$$

and an energy balance for the water film and the air gives

$$-d(\dot{m}_w i_w) \equiv d\dot{q}_{total} = d\dot{q}_a + i_v d\dot{m}_v = \dot{m}_a \, d i_a \tag{6}$$

$$or, \dot{m}_w c_w T_w - c_w (T_w - T_o) \, d\dot{m} = d\dot{q}_{total} = d\dot{q}_a + d\dot{m}_v (i_v - i_w) + i_w d\dot{m}_v \tag{7}$$

The rate of sensible heat transfer may be expressed as

$$d\dot{q}_a = h(T_s - T_a) \, dA_s = ha(T_s - T_a) \, dV \; or, \; \dot{q}'''_a = ha(T_s - T_a) \tag{8}$$

where $q''' = d\dot{q}/dV$

dA_s = the differential surface area for heat transfer, and

a = the surface area per unit volume.

Different codes use different mass transfer equations. For example, ESC code uses the equation

$$\frac{d\dot{m}_v}{dA_s} = K_w (w'_s - w_a) \tag{9a}$$

whereas VERA2D code uses

$$\frac{d\dot{m}_v}{dA_s} = K_f \frac{f_s - f_a}{1 - f_s} \tag{9b}$$

This expression is obtained by using a control volume assessment of the mass transfer through a film of saturated air next to the interface rather than solving the differential mass balance. It neglects the variation in the total density across the film. In terms of the absolute humidity, Eq.(9) may be written as

$$\frac{d\dot{m}_v}{dA_s} = K_f \frac{w'_s - w_a}{r + w_a} \tag{9c}$$

The FACTS code uses Eq.(9c). The rate of enthalpy change of water is equal to the combined sensible heat transfer and evaporative heat transfer. For a unit volume,

$$q'''_{total} = d(\dot{m}_w i_w)/dV = ha(T_w - T_a) + K_w a(w'_s - w_a)(\lambda_s + i_w) \tag{10}$$

Where $\lambda_s = i_v - i_w$ is the enthalpy of vaporization at Ts.

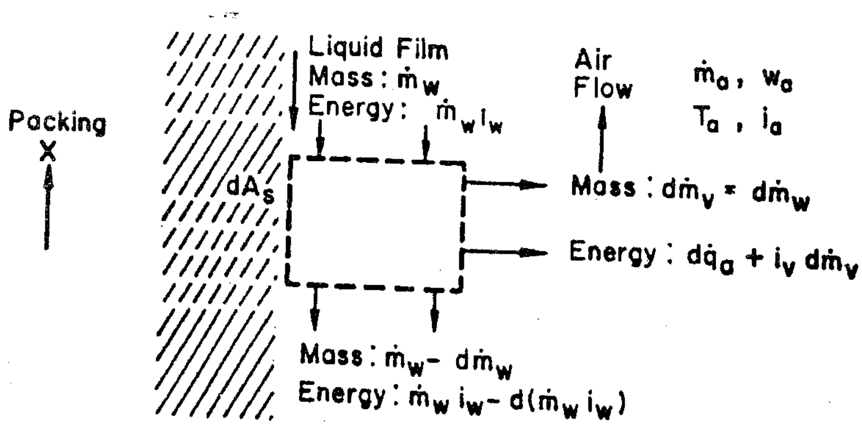

Fig. 1: Schematic of Heat/Mass Transfer Model (Counterflow)

Eq. (9) can be written in a form that emphasizes the similarity of the enthalpy driving force and the combined temperature and mass-difference driving force or

$$\dot{q}'''_{total}\, K_w\, a[c_h\, (T_w - T_a)\, (\frac{h}{c_h K_w}) + (\lambda_s + i_w)\, (w'_s - w_a)] \tag{11}$$

where $c_h = c_a + wc_v$ i.e., is the average moist air heat capacity across the film.

The dimensionless parameter h/c_hK_w is referred to as the Lewis number.

$$Le = \frac{h}{c_h K_w} \tag{12}$$

The value of Le for water vapor in air is taken approximately as 0.9 in the temperature range of interest.

The enthalpy difference between air at the interface conditions and air outside of the boundary layer is

$$i'_s - i_a = c_a\,(T_s - T_a) = w'_s\,[\lambda_s + c_v\,(T_s - T_o)] - w_a\,[\lambda_s + c_v\,(T_a - T_o)] \tag{13}$$

where λ_o is the enthalpy of vaporization at T_o, the reference temperature. Noting that $\lambda_s = \lambda_o + (c_{pv} - c_{pw})$ $(T_s - T_o)$, and that we have assumed $T_s = T_w$, we can combine Eqs. (9a) through (13) to yield

$$\dot{q}'''_{total} = K_w\, a[(i'_s - i_a) + (T_s - T_a)\left\{(Le - 1)c_h + (w'_s - w_a)\frac{c_v}{2}\right\}] \tag{14}$$

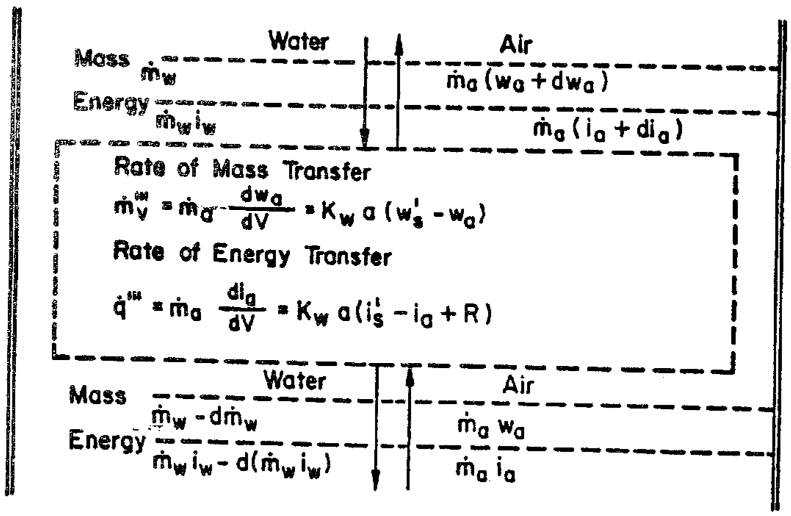

Fig. 2: Schematic of Mass and Energy Relationships

The following summarizes the mass and energy balances and rate relationships.

Mass Balance

$$-d\dot{m}_w = d\dot{m}_v = \dot{m}_a \, dw_a \tag{15}$$

Energy Balance

$$-d(\dot{m}_w i_w) = \dot{m}_a \, di_a = -[\dot{m}_w c_w \, dT_w + c_w (T_w - T_o) d\dot{m}_w]$$

Hence,

$$-dT_w = \frac{\dot{m}_a}{c_w \dot{m}_w} [di_a - c_w (T_w - T_o) dw_a] \tag{16}$$

Mass Transfer Rate

$$\dot{m}''' = \frac{d\dot{m}_v}{dV} = \dot{m}_a \frac{dw_a}{dV} = K_w a (w'_s - w_a) \tag{17a}$$

There fore,

$$dw_a = \frac{K_w a \, dV}{\dot{m}_a} (w'_s - w_a) \tag{17b}$$

Energy Transfer Rate

$$\dot{q}'''_{total} = \dot{m}_a \frac{di_a}{dV} = K_w a(i'_s - i_a + R) \tag{18a}$$

Therefore,

$$di_a = \frac{K_w a\, dV}{\dot{m}_a}(i'_s - i_a + R) \tag{18b}$$

where from Eq.(13),

$$R = (T_w - T_a)\left\{(Le - 1)c_h + (w'_s - w_a)\frac{c_v}{2}\right\} \tag{19}$$

In order to calculate the cooling experienced by the water, Eqs.(15),(16),(17b) and (18b) must be solved simultaneously to obtain the point values.

Fahim et al. [9] by taking the saturation humidity as a quadratic function of temperature, obtain an analytical equation which is capable of calculating the change of water temperature in an adiabatic air-water contact tower. Following assumptions are considered in formulating the basic equations:

a. Air-water contact is made under adiabatic conditions while the two fluids flow countercurrently in plug flow mode.
b. Packings are completely wetted by the water so that heat and mass transfer takes place at the same area of air-water interface.
c. Specific heat of humid air, enthalpy of vaporization of water, and water flow rate are assumed to be constant in the air-water contact tower.
d. The Lewis relation holds between the heat and mass transfer coefficients on the air side at air-water interface.

The heat and mass balances based on the assumptions (a) through (c) give on the

air side

$$G''c_p \frac{dT_a}{dx} + ha(T_a - T_s) = 0 \tag{20}$$

$$G''\frac{dw}{dx} - K_w a(W_i - W_s) = 0 \tag{21}$$

water side

$$L'' c_w \frac{dT_w}{dx} - h_w a(T_w - T_s) = 0 \tag{22}$$

At the air-water interface

$$ha(T_a - T_s) + h_w a(T_w - T_s) = K_w a \lambda (w_s - w) \tag{23}$$

They also obtain analytical equations to predict changes of water and air temperatures and the air-water interface temperature in the tower by approximating the saturation humidity as a linear function of temperature.

TWO-DIMENSIONAL MODEL

The simplest analyses of crossflow towers use a two-dimensional grid which incorporates variations in water temperature and air enthalpy at both horizontal and vertical cross sections. But these calculations are essentially one-dimensional in that they assume that the flows of the two phases, air and water, are each one-dimensional, i.e., there is no flow normal to the principal direction of flow for either phase.

In the two-dimensional flow of air, the expression for the mass flow rate of water vapor is not expressed as Eq.(17a)

$$\dot{m}''' = \frac{d\dot{m}_v}{dV} = \dot{m}_a \frac{dw_a}{dV} = \dot{m}_a \frac{dw_a}{dx} \quad (in\ one-dimension)$$

but is rather

$$\dot{m}''' = \underset{(a)}{\frac{\partial}{\partial x}(\rho\, u\, w_a)} + \underset{(b)}{\frac{\partial}{\partial y}(\rho\, v\, w_a)} - \underset{(c)}{\frac{\partial}{\partial x}(\Gamma_t \frac{\partial w_a}{\partial x})} - \underset{(d)}{\frac{\partial}{\partial y}(\Gamma_t \frac{\partial w_a}{\partial y})} \tag{24}$$

which shows the contributions to the flux of water vapor from the component of air velocity normal to the main stream (term b), and from turbulent mixing in both x and y directions (terms c and d).

Similarly, the expressions for the energy flux must consider the contribution of convective and turbulent mixing from the orthogonal directions. No codes have been developed which handle two-dimensional flow of water, as the accurate modelling of the turbulence terms is very difficult.

DISCUSSION OF THE EXISTING MODELS

Eight important mathematical and numerical models are analyzed in this paper, namely (i) ESC code, (ii) FACTS, (iii) VERA2D, (iv) STAR, (v) Sutherland's Model, (vi) Model by Fujita and Tezuka, (vii) Webb's Model, (viii) Model by Jaber and Webb.

ESG CODE

The ESC code developed by the Environmental Systems Corporation is based on the classical Merkel Model [2] for counterflow and Zivi-Brand Model [26] for crossflow. The ESC code is a one dimensional one though, for crossflow configurations, it uses a two-dimensional matrix of air and water flow, but treats the flow as one-dimensional (uncoupled). Thus, it is appropriate to classify this code as one-dimensional for both counterflow and crossflow. All assumptions of the Merkel model are incorporated into the formulation and the contribution of R in Eq.(18b) is considered negligibly small.

Under these conditions the air enthalpy and water temperature need to be calculated on a point basis and Eqs.(16) and (18b) can be combined to yield the familiar Merkel equation.

$$-dT_s = \frac{\dot{m}_a}{c_w \dot{m}_w}[di_a] = \frac{G}{c_w L} di_a \tag{25}$$

$$di_a = \frac{K_w \bar{a}\, dV}{\dot{m}_a}(i'_s - i_a) \tag{26}$$

$$\frac{L}{G} c_w\, dT_s = -di_a = -\frac{K\ a\ dV}{G}(i'_s - i_a) \tag{27}$$

But,

$$dV = A_c \, dY$$

and

$$\frac{L}{A_c} = L''$$

$$-\frac{L}{G} c_w \, dT_s = \frac{K a A_c \, dY}{G} (i'_s - i_a)$$

$$-\frac{c_w \, dT_s}{i'_s - i_a} = \frac{K a A_c \, dY}{L} = \frac{K a \, dY}{L''} \tag{28}$$

$$or, \frac{K a Y}{L''} = -\int_{T_{w2}}^{T_{w1}} \frac{c_w \, dT_s}{i'_s - i_a} \tag{29}$$

The objective of the computation is to determine the conditions of air enthalpy and water temperature through the tower that result in the right hand side of the Eq.(29) being equal to the value KAY/L''. Determination of mass transfer coefficients for crossflow operation is not significantly more difficult than for counterflow. The expression of the relationship between the changes in air enthalpy and water temperature, comparable to Eq.(25) is

$$c_w L'' \, dT \, dx = G'' \, di_a \, dy \tag{30}$$

and Eq.(28) is replaced by two equations

$$\frac{KAY}{L''} = -\int_{T_{w2}}^{T_{wi}} \frac{c_w \, dT}{i'_s - i_a} \text{ and } \frac{KAX}{G''} = -\int_{i_{a1}}^{i_{a2}} \frac{di}{i'_s - i_a} \tag{31}$$

FACTS

It is developed by the Tennessee Valley Authority [4]. This code is more sophisticated than a one-dimensional model, yet it contains simplifications which prevent it from being classified as a true two-dimensional code. An integral formulation of the conservation equations (conservation of mass and energy for both air and water) is applied, in conjunction with the Bernoulli equation (with head loss included). Assumptions 3 and 7 through 11 are applicable in the formulation of this code. Other assumptions include the following:

a. Evaporation loss is neglected in the water mass balance.

b The flow of air is two-dimensional in the fill region of a cross flow tower, and one-dimensional in the fill region of a counterflow tower.

c. For counterflow towers, the air is assumed to flow between co-linear hyperboloid pathlines. The fraction of air mass flow between each pathline is computed from the Bernoulli equation and reflects flow resistance in both the fill and the rain zones. The pressure drop and transfer characteristics of the fill are integrated in the radial direction to obtain average values. These are weighted by the velocity head, airflow, and waterflow.

d. With crossflow towers, the airflow distribution is evaluated using the Bernoulli equation (with head loss) and the conservation of mass for air. The heat and mass transfer model used in FACTS is given below:

Mass transfer

$$\frac{d\dot{m}_v}{dA_s} = K_f \frac{w'_s - w_a}{1 + w_a}$$

Heat and mass transfer

$$\dot{q}''' = K_f\, a \frac{(w'_s - w_a)}{(1 - w_a)}\lambda_s + h\, a\, (T_s - T_a)$$

In the absence of heat transfer coefficient correlations, h and K_w are related through a calculated local Lewis number $Le = h/c_h K_f$. FACTS has the capability to model towers containing hybrid fill or fills that have voids or obstructions. To a limited extent, FACTS provides with capabilities to account for flow nonuniformities for which FACTS offers the option of specifying a flow distribution of water at the tower inlet. FACTS allows for the input of separate heat/mass transfer and pressure drop correlations for spray and rain regions in counterflow towers.

FACTS code package calculates the outlet conditions of the cooling tower using operating parameters L'' and G'' and known (or assumed) values of Ka.

VERA2D

VERA2D, developed for Electric Power Research Institute by CHAM of North America [18] treats the flow of water in the cooling tower as one-dimensional and the flow of air as two-dimensional and steady. Assumptions 7 through 11 are applicable in the formulation of this code. Two-dimensional, partial differential equations are solved for the conservation of mass and energy for both air and water and the conservation of momentum for moist air. These equations are written in terms of the local values. Following are the governing conservation equations in Cartesian coordinates [17].

Mass of air

$$\frac{\partial}{\partial x}(\rho_a u) = \frac{\partial}{\partial y}(\rho_a v) = \dot{m}''' \tag{32}$$

Mass of water

$$\frac{\partial}{\partial x}(\rho_w u_w) = \dot{m}''' \tag{33}$$

x-direction momentum

$$\frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho v u) u - \frac{\partial}{\partial x}(\mu_{eff}\frac{\partial v}{\partial x}) - \frac{\partial}{\partial y}(\mu_{eff}\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial x} - f_x - (\rho - \rho_{amb})g \tag{34}$$

y-direction momentum

$$\frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(\rho v^2) - \frac{\partial}{\partial x}(\mu_{eff}\frac{\partial v}{\partial x}) - \frac{\partial}{\partial y}(\mu_{eff}\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y} - f_y \tag{35}$$

Air enthalpy

$$\frac{\partial}{\partial x}(\rho u i_a) + \frac{\partial}{\partial y}(\rho v i_a) - \frac{\partial}{\partial x}(\Gamma_{eff}\frac{\partial i_a}{\partial x}) - \frac{\partial}{\partial y}(\Gamma_{eff}\frac{\partial i_a}{\partial y}) = \dot{q}''' \tag{36}$$

Water enthalpy

$$\frac{\partial}{\partial x}(\rho_w u_w i_w) = -\dot{q}''' \tag{37}$$

Momentum fraction of air

$$\frac{\partial}{\partial x}(\rho u f_a) + \frac{\partial}{\partial y}(\rho v f_a) - \frac{\partial}{\partial x}(\Gamma_{eff}\frac{\partial f_a}{\partial x}) - \frac{\partial}{\partial y}(\Gamma_{eff}\frac{\partial f_a}{\partial y}) = \dot{m}''' \quad (38)$$

Equation of state

$$\rho = \frac{pM_a}{R(T_a + 273)} \quad (39)$$

All the equations for air are coupled through convective fluxes (ρu and ρv); the momemtum equations are coupled through pressure as well.

The VERA2D code, because of the two-dimensional flow calculation, includes following generalities:

a. Nonuniform inlet air and water temperatures and flow rates may be specified
b. Variation of air density through the tower is included as a function of T and P.
c. Evaporation of water (which leads to nonuniform water distribution) is modeled.
d. Heat transfer is related to both water temperature and ambient pressure.
e. Turbulence is simulated by a local equilibrium model

$$\dot{q}''' = K_f\, a \frac{(f'_s - f_a)}{(1 - f'_s)}(\lambda_s + i_w) + ha(T_s - T_a)$$

By virtue of a momentum balance, VERA2D calculates the distribution of airflow throughout the tower. The calculation procedure of VERA2D employs the latest form of the iterative, successive-substitution, finite difference method of Patankar and Spalding [20].

STAR

It was developed by Electric de France [6]. It is applicable to counterflow and crossflow natural and mechanical draft cooling towers. STAR solves the two-dimensional differential equations of fluid dynamics and thermodynamics by applying a method of finite differences to a grid of rectangular mesh using a fractional step algorithm. The primary modeling assumptions are as follows:

a. Relative variations of density are below 0.1, so that the Boussinesq approximation can be used.
b. Moist air, even if saturated and loaded with water in liquid form, behaves, from a dynamic point of view, like a perfect gas.
c. In the fill, water flows vertically at constant velocity, either by running off along the packing surface, or as rain in a dispersion, i.e., no change of water momentum.
d. In the rain zone below the packing of a counterflow cooling tower, the diameter of water droplets is assumed to be constant.
e. Exchange of heat in the field of dispersion is governed by the difference in air and water temperature at their surface of separation. (Water temperature is assumed to be constant locally).
f. The driving force involved in mass exchange is generated by the difference in the concentration of water vapor between the surface of the water droplets and air. Vapour concentration at the surface of the liquid is assumed equal to the water content at the saturation point corresponding to the temperature of the interface.
g. The mixture of dry air, water vapour, and liquid water in suspension is assumed to be in thermodynamic equilibrium at all times.
h. Turbulence is simulated by a local equilibrium model.

Input data for STAR are as follows:

i) Vertical profile of temperature and moisture content of ambient air, ii) horizontal profile of hot water flow rate and temperature at the outlet of the fill, iii) local coefficients of drag for the fill, drift eliminators and struts, iv) local exchange coefficient for the fill, v) diameter of droplets in the rain zone.

Output is as follows:

i) Velocity, temperature, water content and pressure fields of the moist air, ii) water flow rate and temperature fields which give the mean cold water temperature.

SUTHERLAND'S MODEL (1983)

This model is developed for mechanical draft counterflow cooling towers. This is a one-dimensional model. A computer program was developed for an accurate analysis of the tower. Computer program was also developed for the approximate Markel's analysis. A comparison between both the accurate and the approximate analyses was made.

Assumptions 2 through 11 are applicable in the formulation of this model. Other assumptions include the following:

a. Water loss by drift and heat transfer through the walls of the tower are negligible
b. Steady flow conditions
c. Lewis number $Le = h/(c_h K_w) = 0.9$

Final form of the equation is

$$\frac{di}{dw} = Le\left(\frac{i'_s - i_a}{w'_s - w_a}\right) + i_v(1 - Le) \tag{40}$$

$$\frac{dT_w}{dw} = \frac{\frac{d_i}{d_w} - i_w}{c_w\left[(m_{w1}/\dot{m}_a) - (w_1 - w_a)\right]} \tag{41}$$

In order to determine the air and water conditions throughout the cooling tower, Eqs.(40) and (41) must be solved.

Thus Eq.(40)

$$\frac{di}{dw} = f_1(w, i, T_w) \tag{42}$$

and Eq.(41)

$$\frac{dT_w}{dw} = f_2(w, i, T_w) \tag{43}$$

These two linear simultaneous differential equations are solved by a fourth order Runge-Kutta method. To calculate the tower volume

$$[\, V = \frac{\dot{m}_a}{K_w a} \int_{w_2}^{w_1} \frac{dw}{w'_s - w_a} \,]$$

the integral is solved by a four point Gaussian quadrature technique Lowan et al. [16].

MODEL BY FUJITA AND TEZUKA (1984)

This model calculates the thermal performance of counterflow and crossflow mechanical draft cooling towers using the enthalpy potential theory. The method recommends calculation of NTU (number of transfer units) $= \frac{KaV}{L}$ for counterflow towers by CTI (Cooling Towers Institute) method [7]. Then the NTU for crossflow towers can be calculated from the NTU for the counterflow tower by using the correction factor F_o, given by Eq.(45). Assumptions 1-11 are applicable in the formulation of this model.

The CTI code recommends a simple integration method like that of Tchevycheff for

$$\frac{KaV}{L} = c_w \int_{T_{w2}}^{T_{w1}} \frac{dT}{i'_s - i_a}$$

In this method, the NTU is approximated by the product of the cooling range $(T_{w1} - T_{w2}) = \Delta T_L$, c_w and the arithmetic mean of the reciprocals $1/(i_s' - i_a)$, the latter being evaluated at the four points corresponding to the water temperatures of $T_{w2} + 0.1\Delta T_L$, $T_{w2} + 0.4\Delta T_L$, $T_{w1} - 0.4\Delta T_L$ and $T_{hw} - 0.1\Delta T_L$.

The crossflow NTU is given by:

$$(CrossflowNTU) = \frac{(CounterflowNTU)}{(CorrectionFactor)} \tag{44}$$

The value of the correction factor in eq. (44) is given by

$$F_o = 1 - 0.106(1 - S)^{3.5} \tag{45}$$

where

$$S = \frac{(i_{w2} - i_{a1})}{(i_{w1} - i_{a2})}$$

WEBB'S MODEL (1988)

This model outlines an exact design procedure for cooling towers. This is a one-dimensional model, which considers water loss by evaporation. Lewis no. is taken to be 0.87. Assumptions 2-11 are applicable in this model. Following are the equations solved in the model for an exact solution:

$$q'' = K_w [F_1 F_2 (T_s - T_a) + i'_s (w_s - w_a)] \tag{46}$$

where

$$F_1 = \frac{b}{1 - e^{-b}} \tag{47}$$

$$b = \frac{c_{pm} K_w (w_s - w_a)}{h} \tag{48a}$$

$$c_{pm} = c_p + w_a c_v \tag{48b}$$

The analogy between heat and mass transfer gives

$$F_2 = \frac{h}{K_w} = c_{pm}\ Le^{2/3}\frac{M_m}{M_a} \tag{49}$$

$$\dot{m}_a\ dw = K_w\ (w_s - w_a)\ aSdx \tag{50a}$$

$$\frac{K_w\, a\, S\, X}{\dot{m}_a} = \frac{K_w\, a\, X}{G_a} = \int_{w_2}^{w_1} \frac{dw}{w_w - w_a} \tag{50b}$$

The heat flux across the water film is given by

$$\dot{q}'' = h_w\ (T_w - T_s) \tag{51}$$

mass balance

$$d\dot{m}_w = \dot{m}_a\ dw \tag{52}$$

Heat transfer rate from the water film over dx is

$$d\dot{q} = d(m_w\ i_w) = \dot{m}_w\, di_w + i_w\, d\dot{m}_w \tag{53}$$

Air temperature change is given by

$$dT = d\dot{q} - i_v\ d\dot{m}_w \tag{54}$$

Increment,

$$dA_s = \frac{d\dot{q}}{\dot{q}''} \tag{55}$$

For an exact solution, one must solve eq.(46) for T_s. This is accomplished by setting Eq.(46) equal to Eq.(51) and solving for T_s. With T_s known, one directly solves eq.(46) or (51) for the heat flux, q''.

Let dq be taken as the independent variable. One must solve for the change of the variables dw, di_w, and dm_w over the increment, and calculate the required incremental area, dA_s. With T_s and q'' known, and dq as the independent variable, one proceeds as follows:

a. Solve eq.(55) for dA_s
b. Equate eq.(50a) and (52) and solve for dw
c. Solve eq.(52) for dm_w
d. Solve eq.(53) for di_w
e. Solve eq.(54) for dT.

The above solution procedure is not used in practice due to the difficulty experienced with unknown h_w. The following approximate solution procedure is employed. Assume $T_s = T_w$, which implies $h_w = \alpha$, and proceed as follows:

a. Solve Eq.(46) for q''
b. Proceed with steps (1) through (5) of the earlier procedure

Alternately, one may choose di_w as the independent variable and simultaneously solve Eqs.(50a), (52) and (53) for dw, dm_w and dq respectively.

Then

$$dA_s = \frac{d\dot{q}}{q''}$$

MODEL BY JABER AND WEBB (1989)

This model gives an effectiveness-NTU design method for both counterflow and crossflow cooling towers. The definitions of effectiveness and NTU are totally consistent with the fundamental definitions used in heat exchanger design.

Earlier, Berman(1961) described how the "log-mean enthalpy method" (LMED) could be applied to cooling tower design. London et al(1940) introduced definitions of ε and NTU to use in plotting cooling tower test data. However, these definitions were not generally consistent with the basic definitions used today in heat exchanger design.

Attempts to apply the F-LMED or ε-NTU methods to cooling tower design use the enthalpy driving potential. Thus, the 'log-mean enthalpy difference (LMED)' corresponds to the 'log-mean temperature difference (LMED)' of heat exchanger design. One problem associated with use of the F-LMED or ε-NTU methods for cooling tower design is that the slope of the saturated air enthalpy curve versus temperature is a curved line. So, the use of the F-LMED method involves errors associated with approximating this curve with a straight line. Berman [15] rigorously applied the F-LMED method to cooling towers, and defined a correction factor (δ) to correct for the curvature of the i versus T curve. Traditional cooling tower design methods typically use an incremental method, which approximates the i versus T curve into N segments, where N may be in the range of 4 or more. Each segment is a straight line approximation to the i versus T curve.

Webb [24, 25] defined the enthalpy driving potential as

$$d\dot{q} = K_w \, (i'_s - i_a) \, dA_s \tag{56}$$

In this model NTU formulation uses $(i'_s - i_a)$ as the driving potential. This potential corresponds to $(T_{w1}-T_{w2})$ used in heat exchanger design. Figure 3 shows a plot of air enthalpy versus water temperature for a counterflow cooling tower. The curved line is the enthalpy of saturated air (i's)and the straight line is the water operating line.

Assuming a linear variation of i versus T, one may define the log-mean enthalpy (LMED) for the cooling tower process illustrated in Fig. 3 as

$$\Delta I_m = \frac{\Delta I_2 - \Delta I_1}{\ln\,(\Delta I_2/\Delta I_1)} \tag{57}$$

where

$$\Delta I_1 = i'_{s1} - i'_{a2} \quad \text{and} \quad \Delta I_2 = i'_{s2} - i_{a1}$$

The F-LMTD method of heat exchanger design uses the UA_s value of the heat exchanger. The corresponding value for cooling tower design is K_wA_s.

The correction factor is given by

$$\delta = (i'_{s1} + i'_{s2} - 2i_{a_{av}})/4 \tag{58}$$

The enthalpy correction factor is independent of T_{WB}, approach and $\dot{m}_w/\dot{m}_a$. It is a function of cooling range alone.

Introducing Eq.(58) in Eq.(57) gives the corrected LMED as

$$\Delta I_m = \frac{\Delta I_2 - \Delta I_1}{\ln[(\Delta I_2 - \delta)/(\Delta I_1 - \delta)]} \tag{59}$$

Energy balance on the water film and the air gives

$$d\dot{q} = \dot{m}_w c_w dT_w = \dot{m}_a di_a \tag{60}$$

The slope of the i versus T curve is defined as

$$f' = di'_s / dT_w \tag{61}$$

Therefore,

$$d\dot{q} = (\dot{m}_w c_w/f')\, di'_s \tag{62}$$

From Eqs. (60) and (62) one may write $di'_s - di_a = d(i'_s - i_a)$ as

$$d(i'_s - i_a) = d\dot{q}\ [(f'/\dot{m}_w c_w) - 1/\dot{m}_a] \tag{63}$$

Solving Eq.(63) for dq and substituting the result in Eq.(56), we get

$$\frac{d(i'_s - i_a)}{(i'_s - i_a)} = K_w\ [(f'/\dot{m}_w c_w) - 1/\dot{m}_a]\ dA_s \tag{64}$$

The corresponding equation that occurs in the ε - NTU development for a heat exchanger with $C_1 = C_{min}$ is

$$\frac{d(T_{w1} - T_{w2})}{(T_{w1} - T_{w2})} = U\ [1/\dot{m}_{w1} c_{w1} - 1/\dot{m}_{w2} c_{w2}]\ dA_s \tag{65}$$

Eq.(64) contains the term m_a as compared to the "capacity rate" $\dot{m}_{w2} c_{w2}$ in eq.(65). By analogy with eq.(65), we can define m_a as the air capacity rate for a cooling tower, and the water capacity rate as

$$\dot{m}_w^* = \dot{m}_w c_w/f' \tag{66}$$

Capacity rate ratio

$$C_R = \dot{m}_{min}/\dot{m}_{max} \tag{67}$$

There are two possible cases: $\dot{m}_w^* < \dot{m}_a$ and $\dot{m}_w^* > \dot{m}_a$

Case 1: $\dot{m}_w^* < \dot{m}_a$. After substituting $\dot{m}_w^* = \dot{m}_{min}$ and $\dot{m}_a = \dot{m}_{max}$ in Eq.(64) one obtains

$$\frac{d(i'_s - i_a)}{(i'_s - i_a)} = \frac{K_w dA_s}{\dot{m}_{min}} (1 - C_R) \tag{68}$$

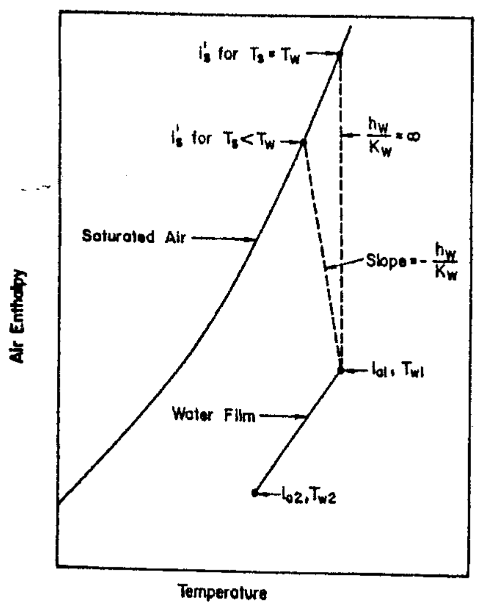

Fig.3: Water Operating Line on Enthalpy - Temperature Diagram

Eq.(68) corresponds to the heat exchanger ε -NTU equation

$$\frac{d(T_{w1} - T_{w2})}{(T_{w1} - T_{w2})} = \frac{UdA_s}{C_{min}} (1 - C_R) \tag{69}$$

In heat exchanger design, the term UA_s/C_{min} is defined as the "number of transfer units", NTU. The analogous definition for the NTU of a cooling tower is

$$NTU = \frac{K_w A_s}{\dot{m}_{min}} \tag{70}$$

Effectiveness ε is defined identically to that used in heat exchanger design

$$\varepsilon = \dot{q}_{act} / \dot{q}_{max} \tag{71}$$

where

$$\dot{q}_{max} = \dot{m}_{min} (i'_{s2} - i_{a2}) \tag{72}$$

Referring to Fig.3 and integrating Eq.(68) between the entering and leaving air states, i_{a2} and i_{a1} respectively, gives

$$\frac{i'_{s1} - i'_{a2}}{i'_{s2} - i'_{a1}} = \exp\left[-NTU(1 - C_R)\right] \tag{73}$$

It can be shown that

$$\frac{i'_{s1} - i_{a2}}{i'_{s2} - i_{a1}} = \frac{\varepsilon - 1}{\varepsilon C_R - 1} \quad (74)$$

Equating Eqs.(73) and (74) give the final ε - NTU relationship for a counterflow cooling tower

$$\varepsilon = \frac{1 - \exp[-NTU(1 - C_R)]}{1 - C_R \exp[-NTU(1 - C_R)]} \quad (75)$$

which is identical to the ε - NTU expression for a counterflow heat exchanger.

CASE 2: $\dot{m}_a < \dot{m}_w^*$. For this case $\dot{m}_a = \dot{m}_{min}$ and $\dot{m}_w^* = \dot{m}_{max}$. Substitution of these expressions into Eq.(64) gives Eq.(68). Continuing as for Case 1 leads to Eq.(75).

Using the definitions for effectiveness and NTU described above, the resulting ε - NTU equations for a counterflow cooling tower are shown to be identical to those for a counterflow heat exchanger design. It can also be shown that the ε - NTU equations for crossflow heat exchangers are also applicable to crossflow cooling towers.

A simple procedure for a one-increment design using the enthalpy correction factor is outlined below:

1. Calculate the slope of the saturation line, $f' = \Delta i'_s / \Delta T_L$
2. Calculate $\dot{m}_w^* = \dot{m}_w c_w / f'$ and compare to $\dot{m}_a$ to determine $C_R = \dot{m}_{min}/\dot{m}_{max}$
3. Find $\Delta i_a = (\dot{m}_w/\dot{m}_a)\ c_w \Delta T_L)$
4. Calculate the effectiveness $\varepsilon = (\dot{m}_a \Delta i_a) / [\dot{m}_{min}(i'_{s2} - \delta - i_{a2})]$.
5. Read(or calculate) NTU from the ε - NTU chart (or equation).

CONCLUSIONS

The operating efficiency of each code is treated as a combination of computational efficiency and the "user friendliness". Computational efficiency involves a comparison of CPU time requirement and run-time memory requirements for each computer model. A comparison of the operating efficiency of all codes with each other has not been made. But some available information is provided.

Convergence is not a problem with FACTS. However, convergence with VERA2D is very sensitive to the value of a user specified relaxation parameter that governs convergence time of the solution. The differences in run-time and memory requirements between FACTS and VERA2D are substantial. On average, VERA2D requires eight times CPU time and nine times the memory capacity of FACTS. These differences are attributable primarily to the two-dimensional momentum equations solved in VERA2D. In a validation study [12], STAR required 0.8 hours per case on a CRAY 1 computer for a computational grid of 7000 points.

The average value of Ka/L'' for crossflow fills calculated by the FACTS code was 1.05 times the value calculated by the ESC code. For the counterflow fills, the comparable ratio was 0.89. Using the best predictions of ESC, FACTS and VERA2D codes, on the average, the codes were able to predict cold water temperatures within 0.39C for all towers considered. Overall, the VERA2D predictions provided the closest match to the experimental data, with an average discrepancy of 0.29C. The VERA2D results were followed by FACTS, with an overall average of 0.38C, then the ESC code, with an overall average of 0.42C.

In Sutherland's model, two computer programs were developed. Computer program 'TOWER A' for accurate analysis and computer program 'TOWER B' for approximate analysis were developed. Following conclusions can be drawn from Sutherland's model at a fixed value of

i) For the same range and approach, the higher the wet- bulb temperature, smaller is the tower.
ii) For the same wet-bulb temperature and range, the larger the approach, smaller is the tower.
iii) For the same wet-bulb temperature and approach, the large the range, larger is the tower.
iv) For the same initial and final water temperatures, the lower the inlet air wet-bulb temperature, smaller is the tower.

As a check on the accuracy of the computer predictions, values of NTU calculated by computer program 'TOWER B' are in excellent agreement with those given by Baker and Shryock [3]. Values of NTU calculated by Threlkeld [22] are within approximately 2 percent of the value determined by program 'TOWER A'. Substantial underestimates of tower volume from 5 to 15 percent are obtained when the approximate analysis is used.

From the model of Fujita and Tezuka [10], it can be concluded that the counterflow NTU can be calculated by the CTI method, and the crossflow NTU can be calculated easily from this counterflow NTU by using the correction factor F_o with sufficient accuracy. Errors involved in the calculation of crossflow NTU by using the correction factor is depicted in Figure 4.

In Webb's model, counterflow calculations are performed using the Tchevycheff's method as proposed by the CTI and crossflow calculations are based on the operating conditions provided by the user [1].

Error associated with four frequently used approximations for the driving potential is compared with the exact driving potential. The expression within the brackets of eq.(46) is defined as exact driving potential DP_E. Thus,

$$DP_E = F_1 F_2 \, (T_s - T_a) + i'_s \, (w_s - w_a) \qquad (76)$$

Assuming all of the components of F_1F_2 in eqs.(47) and (49) equal to 1.0, except $Le^{2/3}$, which is equal to 0.87, eq.(56) can be written in a near exact form as

$$DP_{NE} = 0.87 \, (T_s - T_a) + i'_s \, (w_s - w_a) \qquad (77)$$

and in approximate form by assuming $F_1F_2 = 1$ as

$$DP_{AP} = (T_s - T_a) + i'_s \, (w_s - w_a) \qquad (78)$$

Approximate forms of three equations for the Merkel enthalpy potential are

$$DP_{M1} = c_p \, (T_s - T_a) + i_v \, (w_s - w_a) \qquad (79)$$

$$DP_{M2} = c_{pm} \, (T_s - T_a) + i_v \, (w_s - w_a) \qquad (80)$$

$$DP_{m3} = c_{pm} \, (T_s - T_a) + \lambda \, (w_s - w_a) \qquad (81)$$

The error associated with Eqs.(77) through (81) is calculated by

$$ED_{app} = 100 \, (DP_{app} - DP_E) / DP_E \qquad (82)$$

where DP_{app} refers to the approximate forms. The errors associated with eqs.(77) through (81) are ED_{NE}, ED_{AP}, ED_{M1}, ED_{m2} and ED_{MS} respectively. Table 1 lists errors for a practical range of T_s, T_a and relative humidity (ψ).

It may be noted from Table 1 that

a. The "near exact" equation (Eq.77) predicts DP_E within -1.5 to +0.2%.

b. The errors associated with Eqs.(78) and (79) generally agree within 1%. Eq.(79) predicts DP_F within 0.0 to 4.3%, and Eq.(78) within -0.2 to 3.8%.

c. Eq.(80) overpredicts DP_E 2.9 to 6.1%

d. Eq.(81) predicts DP_E within -6.0 to 2.0%

The model of Jaber and Webb [11], by using proper and consistent definitions of ε and NTU clearly illustrates as to how the ε - NTU or the standard LMED methods for the design of heat exchangers can be used to design cooling towers. The definitions of ε and NTU given by Eqs. (71) and (70) respectively are applicable to all cooling tower operating conditions. One-increment sizing calculations may be quickly performed for any flow configuration. The calculations are improved by using multi-increments and/or the enthalpy correction factor. Use of the enthalpy correction factor reduces the error associated with a lower number of increments. Moreover, a one-increment design with this correction factor is equivalent to a two-increment design without the correction. Using ε NTU curve for the appropriate flow configuration, one may quickly calculate the required NTU for specified operating conditions. This negates the need for the extensive sets of curves given by Kelly [14] and the Cooling Tower Institute [8].

Table 1: Error of Approximations to Driving Potential

(Heat Transfer to Water)**

		$T_s = 30^oC$		$T_s = 50^oC$		$T_s = 70^oC$	
T)oC)	Error	$\psi = 0.2$	$\psi = 1$	$\psi = 0.2$	$\psi = 1$	$\psi = 0.2$	$\psi = 1$
2	ED_{NE}	-0.5	-0.6	-0.7	-0.8	-1.1	-1.1
	ED_{AP}	3.5	3.8	1.6	1.7	0.0	0.0
	ED_{M1}	3.7	4.3	1.7	1.9	0.0	1.1
	ED_{M2}	5.3	6.1	4.8	5.1	4.6	4.7
	ED_{M3}	1.6	2.0	-2.1	-2.0	-5.7	-5.7
26	ED_{NE}	-0.1	**	-0.4	-0.6	-0.8	-0.8
	ED_{AP}	0.7	**	0.9	1.0	0.0	0.0
	ED_{M1}	0.8	**	1.0	1.6	0.1	0.2
	ED_{M2}	2.9	**	4.4	5.2	4.8	5.1
	ED_{M3}	1.8	**	-3.2	-3.1	-5.9	-5.9
46	ED_{NE}	**	**	-0.1	**	-0.4	-0.7
	ED_{AP}	**	**	0.2	**	0.0	0.2
	ED_{M1}	**	**	0.2	**	0.1	0.4
	ED_{M2}	**	**	3.8	**	5.0	5.6
	ED_{M3}	**	**	-4.3	**	-6.0	-6.3

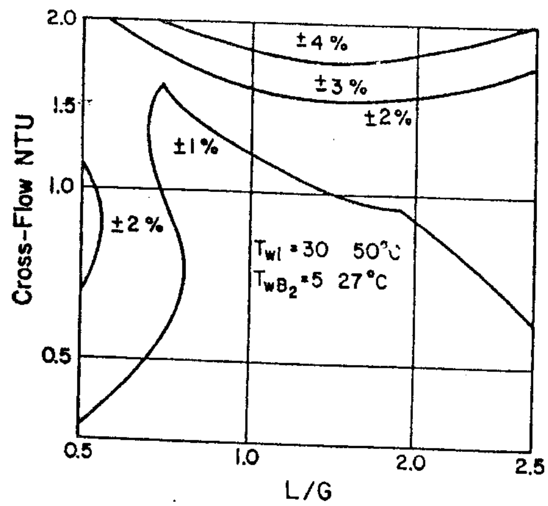

Fig. 4: Error Map for Crossflow NTU Calculated Using Counterflow NTU and the Correction Factor F_o

NOMENCLATURE

A	=	Cross-sectional area of the counterflow tower, m^2
A_s	=	Total surface area at the air/water interface, m^2
a	=	Surface area of water droplets per unit volume of tower, m^2/m^3
C	=	Fluid capacity rate = mc, J/s C
C_R	=	Capacity rate ratio = m_{min}/m_{max}
c	=	Heat capacity, J/kgC
c_h	=	Heat capacity of humid air, J/kgC
c_p	=	Heat capacity of air, J/kgC
c_{pm}	=	Heat capacity of steam-air mixture, J/kgC
F_o	=	Correction factor defined by Eq.(45)
f	=	Mass fraction of water vapor in moist air [kg vapor/kg(air + water)
f'	=	Slope of saturated air enthalpy versus temperature curve, J/kg_o C
f_x,f_y	=	Resistance to air flow in x and y directions respectively, N/m^3
G	=	Total flow rate of air, kg/s
G_a	=	Mass velocity of air, kg/s.m^2

g	=	Gravitational acceleration, m/s^2
h	=	Convective heat transfer coefficient, W/m^2C
h_W	=	Water film heat transfer coefficient, W/m^{2C}
ΔI	=	Enthalpy difference between air at interface and local bulk air, J/kg
i	=	Enthalpy, J/kg
i'_s	=	Enthalpy of saturated moist air at the water temperature T_W, J/kg dry air
K	=	Mass transfer coefficient, $kg/s.m^2$
L	=	Total flow rate of water, kg/s
L_e	=	Lewis number (h/c_pK), (dimensionless)
M	=	Molecular weight, kg/mol
$\dot{m}$	=	Mass flow rate, kg/s
$\dot{m}_a$	=	Mass flow rate of dry air, kg/s
$\dot{m}W^*$	=	Water side capacity rate, kg/s
N	=	Number of increments
NTU	=	Number of transfer units as defined by Eq.(70)
P	=	Pressure, Pa
q	=	Rate of heat transfer, J/s
q_a	=	Sensible heat transfer rate, J/s
R	=	Universal gas constant, J/kg-mol.K
S	=	Cross-section for air flow, m^2
T	=	Temperature, C
T_B	=	Dry-bulb temperature of moist air, C
T_{WB}	=	Wet-bulb temperature of air, C
ΔT_L	=	Cooling range = $T_{W1} - T_{W2}$, C
U	=	Overall heat transfer coefficient, W/m^2 C
u,v	=	x' and y' components of velocity respectively, m/s
V	=	Volume of cooling tower, m^3
w	=	Specific humidity, kg vapor/kg dry air
$\overline{w}$	=	Average specific humidity in the film, kg vapor/kg dry air
X	=	Fill depth, m
x	=	Vertical Cartesian coordinate, m
Y	=	Fill height, m
y	=	Horizontal Cartesian coordinate, m
δ	=	Enthalpy correction factor, J/kg
ε	=	Thermal effectiveness = q_{act}/q_{max}
Γt	=	Effective turbulent diffusivity, m^2/s
Γeff	=	Effective exchange coefficient, kg/m.s
λ	=	Enthalpy of vaporization, J/kg
μeff	=	Effective viscosity, kg/m.s

ρ = Density, kg/m^3

ψ = Relative humidity, (dimensionless)

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