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## PID-based Control of a Single-Link Flexible Manipulator in Vertical Motion with Genetic Optimisation

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**Abstract**— This paper presents an investigation into dynamic simulation and controller optimization based on genetic algorithms (GAs) for a single-link flexible manipulator system in vertical plane motion. The dynamic model of the system is derived using the Lagrange equation and discretised using the finite difference (FD) method. GA optimization is used to optimize the parameters of the proportional-integral-derivative (PID) based controllers for control of rigid-body and flexible motion dynamics of the system. The important point is to evaluate the range of PID parameter which used in the GAs programmed to find the best value of this parameter. Comparative performance assessment of the control approaches are presented and discussed in the time and the frequency domains.

**Keywords**—PID; genetic algorithm; single-link flexible manipulator

### I. INTRODUCTION

Flexible manipulators are known to offer several advantages in comparison to their rigid counterparts [1], [2], [3]. Lightweight elastic single-link robot manipulators are capable of improving the speed of operation and handling larger payloads in comparison to rigid manipulators with the same actuator capabilities [1], [2]. Therefore, lots of research interest in flexible manipulators for industrial applications. This is due to several advantages associated with flexible manipulators. These include higher gross motion speeds, reduced cost for mechanical subsystem, energy efficiency due to smaller actuators for the same cycle times, portability and improved mobility of manipulator arms, safety due to reduced moving mass, and reduced mounting requirements. However, the control of such systems due to their flexible motion dynamics, with changing payloads, is challenging and requires sophisticated and complex control methods [4]. A simulation environment characterising a single-link flexible manipulator in vertical motion is utilized in this work.

Proportional-integral-derivative (PID) control is used due to its wide-spread use in industrial control applications. The PID controller attempts to correct the error between a measured process variable and a desired set point by calculating and then outputting a corrective action that can adjust the process accordingly.

Moreover genetic algorithm (GA) is used to optimize the parameters of a PID-based controller of the manipulator system in vertical motion. Genetic algorithms (GAs) are used

to develop controllers placed in the feedforward path (PID), feedforward and feedback paths (PIDPID) and with iterative learning control (PIDILC) for control of rigid-body motion and flexible motion dynamics of the system. Input and output data from the simulation are collected and used with GA to obtain suitable controller parameters of the system. Simulation results of the response of the manipulator system with the developed controllers are presented and a comparative assessment of the performance of the developed controllers with PID, PIDPID and PIDILC controller is provided.

### II. THE FLEXIBLE MANIPULATOR SYSTEM

Figure 1 shows an outline description of the single-link flexible manipulator system under consideration in this work, where  $E$ ,  $I$ ,  $\rho$ ,  $M_p$  and  $I_h$  are Young's modulus, area moment of inertia, mass density per unit length, payload mass and hub inertia respectively. This consists of an aluminum cylindrical type flexible manipulator, with characteristic parameters shown in Table I, driven by two motors at the hub, for horizontal and vertical motion, respectively. The measurement sensors consist of an accelerometer at the end-point of the manipulator for measurement of end-point acceleration, shaft encoders and tachometers, at the hub of the manipulator, for measurement of hub angle and hub velocity respectively. The length  $l$  of the manipulator is assumed to be constant, and the effect of axial force is neglected.

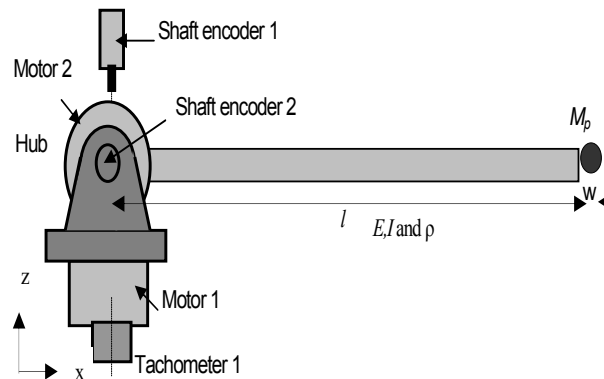


Figure 1. Outline of the flexible manipulator

The flexible manipulator system can be modelled as a pinned-free flexible beam, incorporating inertia at the hub and payload mass at the end-point (see Figure 2). With an angular displacement  $\theta_z$  and an elastic deflection  $u_z$ , the total (net) displacement  $y_z(x,t)$  of a point along the manipulator at a distance  $x$  from the hub can be described as a function of both the rigid body motion  $\theta_z(t)$  and elastic deflection  $u_z(x,t)$  as:

$$y_z(x,t) = x\theta_z(t) + u_z(x,t) \quad (1)$$

Using the Hamilton's extended principle with associated kinetic, potential and dissipated energies of the system, a fourth order partial differential equation (PDE) representing the manipulator motion can be obtained as:

$$EI \frac{\partial^4 u_z}{\partial x^4} + m \frac{\partial^2 u_z}{\partial t^2} - D_s \frac{\partial^3 u_z}{\partial x^2 \partial t} = mg \frac{l}{2} \sin \theta_1 + M_p g \sin \theta_2 \quad (2)$$

where  $m$ =mass per unit length,  $l$ =length of manipulator,  $M_p$ =payload,  $\theta_1 = \theta_z$ = angular displacement,  $\theta_2$ =angle of payload normal to the OX and  $g$ =gravity normal to the manipulator. A simulation algorithm based on finite difference discretisation of equation (2) has been developed [5],[6] and used in this paper as a platform for GA optimisation approach.

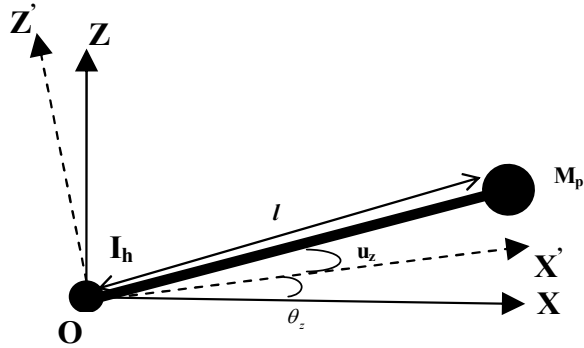


Figure 2. Schematic of the flexible manipulator in XZ plane

TABLE I. PARAMETER OF THE FLEXIBLE MANIPULATOR SYSTEM

Flexible Manipulator		
Parameter	Notation	Value
Length	$L(m)$	1.0
Diameter	$D(m)$	0.0045
Material	-	Aluminium
Cross Sectional Area	$A(m^2)$	$1.5904 \times 10^{-5}$
mass	$m(kg/m)$	$4.3101 \times 10^{-2}$
Hub Inertia	$I_h(kgm^2)$	$5.85 \times 10^{-4}$
Young 's Modulus	$E(N/m^2)$	$71 \times 10^9$

Flexible Manipulator		
Parameter	Notation	Value
Second moment of Inertia	$I(kgm^2)$	$2.0129 \times 10^{-11}$

### III. THE ALGORITHMS

#### A. Genetic Algorithms

Genetic algorithm (GA) optimization is based on natural selection and natural genetics [7]. Genetic algorithms constitute stochastic search methods that have been used in a wide spectrum of applications such as control structure design [8]. GA is used with a randomly initialized population representing parameters of the two PID controllers denoted by  $k1, k2, k3, k4, k5$  and  $k6$  parameter; and ILC controller marked by  $\Gamma, \Phi$  and  $\Psi$  [9]. The fitness function used is based on output error minimisation,  $e(t)$  between the input and feedback hub-angle of the system, and is given as:

$$J(\theta) = \frac{1}{N} \sum_{t=1}^N (e(t))^2 \quad (3)$$

#### B. Control Structure

Proportional-integral-derivative (PID) control is widely used in industrial control applications. The PID algorithm can be described as [10]:

$$u(t) = k_p e(t) + k_i \int e(t) dt + k_d \frac{de}{dt} \quad (4)$$

where  $u(t)$  is the control variable,  $e(t)$  is the control error.

The ILC scheme has previously been used for control of a flexible manipulator [11]. In the current work a PID-type learning scheme is combined with feedforward PID for control of the flexible manipulator system.

$$\eta_{k+1} = \eta_k + (\Phi + \Psi \int dt + \Gamma d/dt) e_k \quad (5)$$

where  $\eta_{k+1}$ =next control signal,  $\eta_k$ = current control signal,  $e_k$ = current error input

Figures 3 and 4 show MATLAB-SIMULINK realisation of the flexible manipulator simulation and control. The simulation of the flexible manipulator is conducted with the four outputs namely hub-angle, hub-velocity, end-point acceleration and end-point residual. Low-pass (LP) filters each with cut-off frequency of 80 Hz is used to band limit the system response to the first three resonance modes for each output. Furthermore, to decouple the flexible motion control loop from the rigid body dynamics a high-pass filter for each output with a cut-off frequency 5 Hz is used. The system is operated with damping ratio,  $D_s=0$  and without payload.

PID controller is the most dominating form of feedback in use today where more than 90% of all control loops are PID [10]. Most loops are in fact PI because derivative action is not used very often. Strength of the PID control is that it also deals with important practical issues such as actuator saturation and integrator windup. Beside the development of design methods for PID control, there are some difficult problems that remain to be solved such as there is no characterization of the process where PID control is useful.

More details about the control strategy of PID, PIDPID and PIDILC scheme of horizontal motion are discussed in [9].

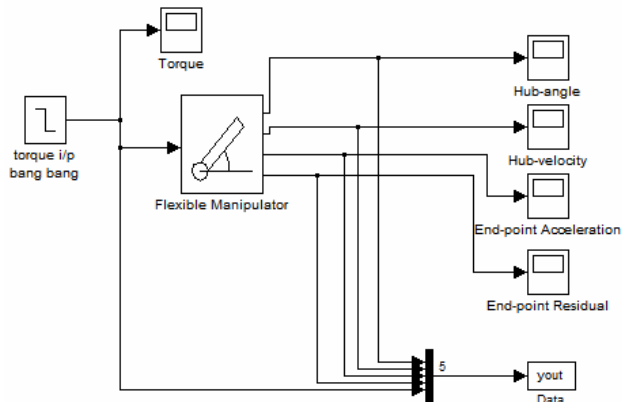


Figure 3. Open loop flexible manipulator simulation with bang-bang torque input [2]

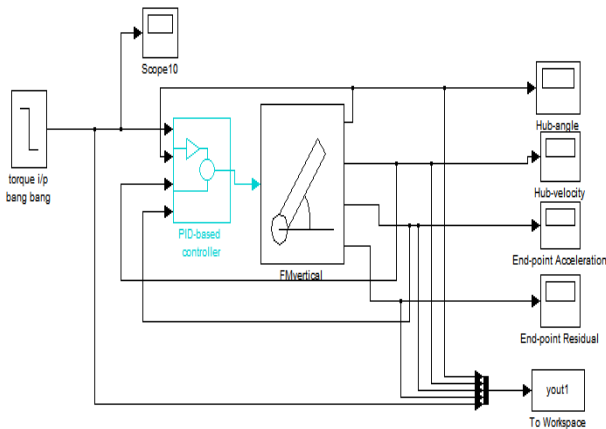
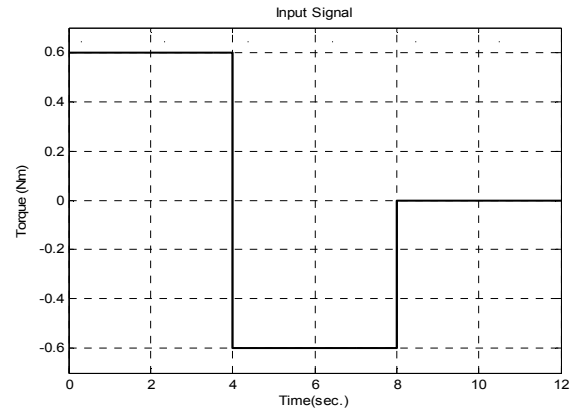


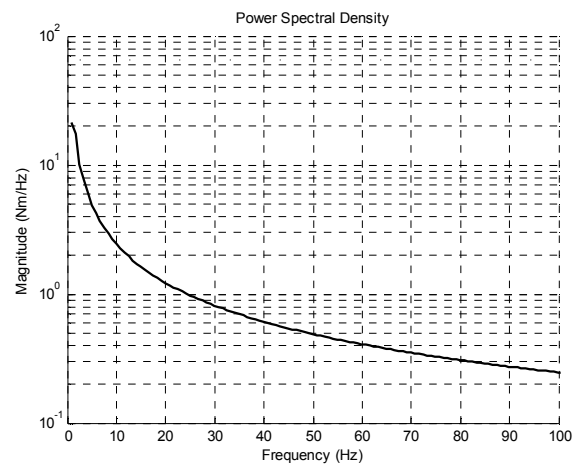
Figure 4. Closed loop control scheme for Flexible Manipulator

### C. Input signal

A bang-bang torque input with amplitude of  $\pm 0.6$  Nm and duration of 4 seconds is used to excite the system. Bang-bang torque is used for positive and negative direction of the flexible manipulator motion. This is shown in Figure 5. The total simulation time is set to 12 seconds and the system behaviour at the hub and end-point observed and recorded.



(a) Time domain



(b) Frequency domain

Figure 5. Bang-bang input signal

## IV. SIMULATION RESULTS

Figure 6 shows the convergence of the GA over 80 generations during optimisation of the PID, PIDPID and PIDILC controllers. The best individual achieved an MSE of  $1.0688 \times 10^{-5}$  with PIDPID control whereas, PID and PIDILC achieved an MSE of  $1.0803 \times 10^{-5}$  after 80 generations. Table II shows the corresponding controller parameters achieved. The performance of the system with the three control schemes is assessed in this section with payloads of  $M_p = 0$  gram. Table III and IV show numerical values of the performance of the system with the three control approaches. It is noted that PID and PIDPID gave the best maximum overshoot output  $30.4423^\circ$  and  $30.4424^\circ$ , respectively. Both controls also gave the best steady-states output  $30.0035^\circ$  and  $30.0037^\circ$ , respectively. PIDILC had the steady-state output  $30.0038^\circ$ . PIDILC gave the maximum overshoot value of 1.484% whereas PID controller gave 1.474% overshoot. PIDPID controller on the other hand lead to overshoot value of 1.475%. The settling time for PID, PIDPID and PIDILC controller were the same 1.949 second, with the rise time of 0.373 second. Moreover, the response of the system was settled in 1.95 seconds for all controls, resulting the good performance of the flexible manipulator. Table IV shows the maximum value of the hub-

velocity, end-point acceleration and end-point residual. It is noted that PID control resulted in the best performance, followed by PIDPID control and PIDILC.

Figures 7-10 show the system response in time and frequency domains with the three controllers, PID, PIDPID and PIDILC. The resonance frequencies and the magnitude of the vibration are depicted in Table V. All controllers gave the same resonance frequency values, which were at 13.645 Hz, 44.834 Hz and 92.5936 Hz, respectively. A similar trend is observed with the PID and PIDPID controllers, where the best reduction in the magnitude of vibration was achieved with PIDPID control compared with PID and PIDILC.

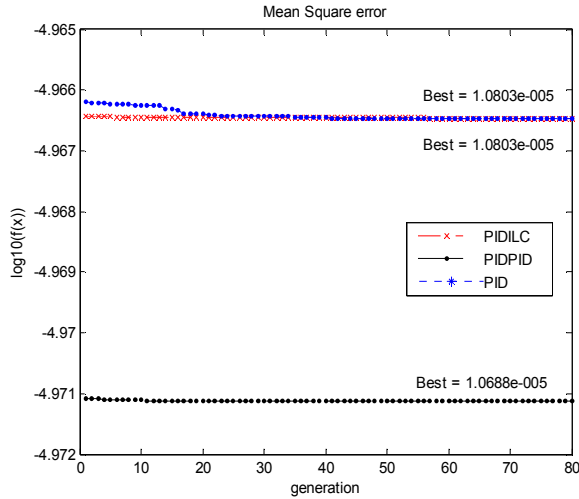


Figure 6. GA convergence over 80 generations

TABLE II. CONTROL PARAMETER AFTER 80 GENERATIONS

Parameters	Controller		
	PID	PIDPID	PIDILC
$K1$	2.6911	2.6911	2.6911
$K2$	$2.2474 \times 10^{-4}$	$2.2474 \times 10^{-4}$	$2.2474 \times 10^{-4}$
$K3$	$1.3499 \times 10^{-2}$	$1.3499 \times 10^{-2}$	$1.3499 \times 10^{-2}$
$K4$	-	$5.5134 \times 10^{-2}$	-
$K5$	-	$1.2527 \times 10^{-4}$	-
$K6$	-	1.75	-
$\Phi$	-	-	$5.2181 \times 10^{-1}$
$\Gamma$	-	-	$6.3225 \times 10^{-4}$
$\Psi$	-	-	$1.1925 \times 10^{-4}$

TABLE III. HUB ANGLE PERFORMANCE OF THE FLEXIBLE MANIPULATOR

Parameters	Controller		
	PID	PIDPID	PIDILC
Rise Time(sec.)	0.373	0.373	0.373
Settling Time(sec.)	1.949	1.949	1.949
% overshoot	1.474	1.475	1.484
% Undershoot	4.205	4.206	4.206

TABLE IV. MAXIMUM VALUE OF THE RESPONSE

Profile	Controller		
	PID	PIDPID	PIDILC
Hub-angle (deg.)	30.4423	30.4424	30.4425
Hub-velocity(deg./sec)	76.7067	76.7079	76.7081
End-point Acceleration (deg./sec <sup>2</sup> )	194.586	191.847	191.535
End-point Residual (m)	$1.045 \times 10^{-2}$	$1.050 \times 10^{-2}$	$1.060 \times 10^{-2}$

TABLE V. RESONANCE FREQUENCY AND MAGNITUDE OF END POINT RESIDUAL RESPONSE

Profile	Controller		
	PID	PIDPID	PIDILC
Mode 1 (Hz)	13.645	13.645	13.645
Magnitude 1 (deg./Hz)	1.1547	$1.36 \times 10^{-2}$	1.189
Mode 2 (Hz)	44.834	44.834	44.834
Magnitude 2 (deg./Hz)	$3.39 \times 10^{-2}$	$2.2275 \times 10^{-4}$	$2.782 \times 10^{-2}$
Mode3 (Hz)	92.593	92.593	92.593
Magnitude 3 (deg./Hz)	$1.09 \times 10^{-3}$	$2.1991 \times 10^{-4}$	$1.111 \times 10^{-3}$

## V. CONCLUSIONS

The development of PID controller with PID and ILC feedback control strategy for vibration reduction based on genetic algorithms for flexible manipulators in vertical plane motion has been presented. A combined feedforward and feedback control structure for control of rigid-body and flexible motion dynamics of a flexible robotic manipulator has been considered. A GA-optimisation strategy for design of PID controllers in the forward and feedback paths of the control structure has been investigated. Furthermore, a GA-optimisation strategy for design of PID controllers in the forward and ILC controller incorporated with end-point acceleration feedback paths of the control structure has been investigated and the performance of the developed control approach has been assessed in comparison to previously reported PIDPID

control, and it has been demonstrated that good performance is achieved with the proposed approach. The control scheme has shown to perform well in reducing the vibration at the end-point of the manipulation.

#### ACKNOWLEDGMENT

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#### REFERENCES

[1] Azad, A. K. M., *Analysis and design of control mechanisms for flexible manipulator systems*, PhD thesis, Department of Automatic Control and Systems Engineering, The University of Sheffield, UK (1994).

[2] H. Poerwanto, *Dynamic Simulation and Control of flexible manipulator systems*, PhD thesis, Department of Automatic Control and Systems Engineering, The University of Sheffield, UK(1998).

[3] R P Sutton, G D Halikias, A R Plummer and D A Wilson, *Modelling and H control of a single-link Flexible Manipulator*. Proceeding of the Instn Mech Engrs Vol 213 Part 1(1998).

[4] W.J. Book, *Modeling, Design, and Control of Flexible Manipulator Arms: A Tutorial Review*, Proceedings of the 29<sup>th</sup> Conference on Decision and Control, Honolulu, Hawaii, 1990.

[5] B.A. Md Zain and M.O.Tokhi, *Modeling and simulation of a single-link flexible manipulator in vertical motion*, Proceeding of CLAWAR-2009, International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines., Intanbul, Turkey, 09-11 September, 2009.

[6] B.A. Md Zain and M.O.Tokhi, *Dynamic modeling of a single-link flexible manipulator in vertical motion using swarm and genetic optimisation*, Proceeding of CLAWAR-2009, International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines., Intanbul, Turkey, 09-11 September, 2009.

[7] Holland, J.H., *Adaptation in natural and artificial systems*. Ann Arbor: The University of Michigan Press, 1975.

[8] Chipperfield A. J., Flemming P.J. and Fonseca C.M., *Genetics algorithm tools for control systems engineering*, Proceedings of Adaptive Computing in Engineering Design and Control, Plymouth Engineering Design Centre, 21-22 September, pp. 128-133, 1994.

[9] B.A. Md Zain and M.O.Tokhi, *Dynamic simulation and control of a flexible manipulator using genetic algorithms*, Proceedings of the Third International Conference on Modeling, Simulation and Applied Optimization, Sharjah, U.A.E. January 20-22, 2009.

[10] Karl J. Astrom and Tore Hagglund, *PID controllers: Theory, Design, and Tuning*, 2nd Edition, The Instrumentation, Systems, and Automation Society, USA, 1995.

[11] M.Z. Md Zain, M.O. Tokhi, Z. Mohamed and M. Mailah, "Hybrid Iterative Learning Control of a Flexible Manipulator". Proceeding of the 23rd IASTED, International Conference Modelling, Identification and Control, Grindelwald, Switzerland, 2004.

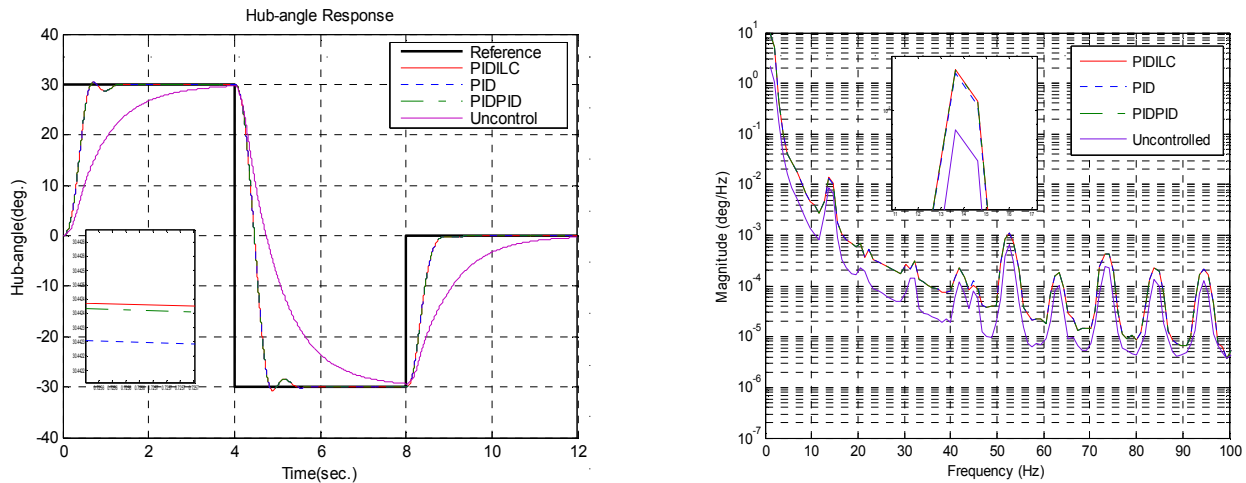


Figure 7. Hub-angle with  $D_s=0$  and without payload

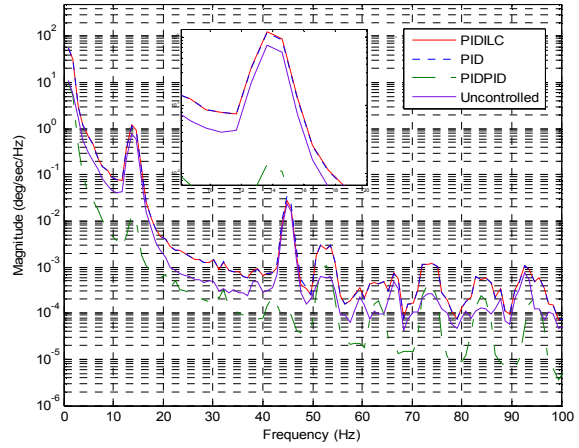
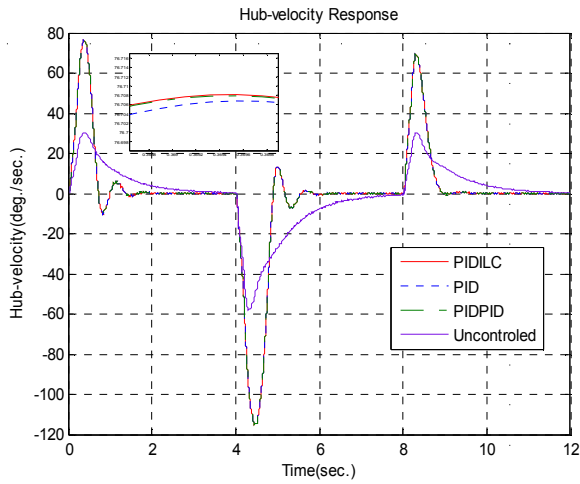


Figure 8. Hub-velocity with  $D_s=0$  and without payload

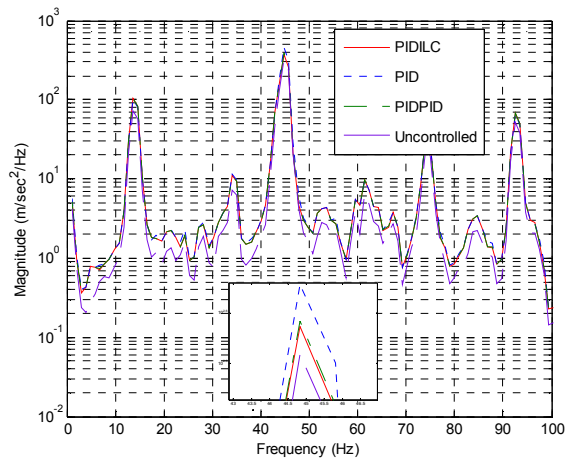
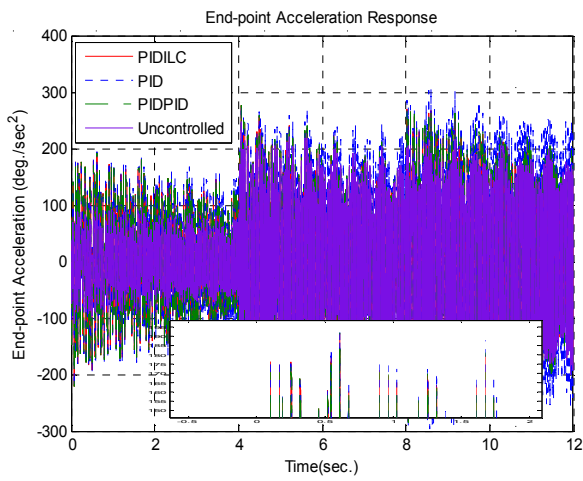


Figure 9. End-point acceleration with  $D_s=0$  and without payload

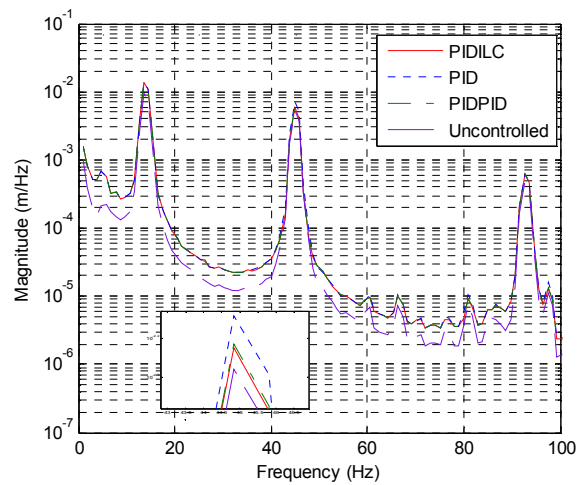
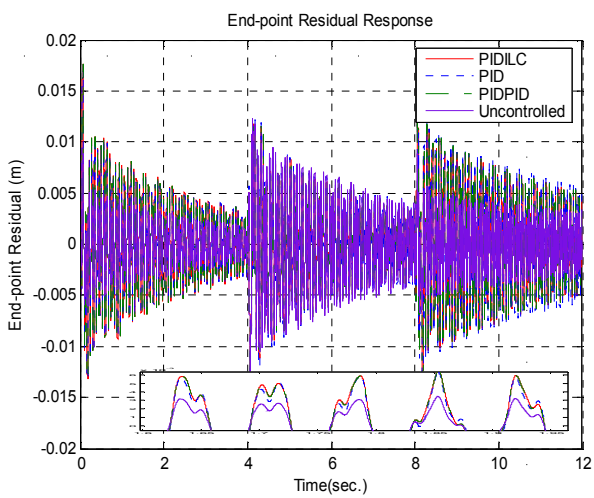


Figure 10. End-point residual with  $D_s=0$  and without payload