Hypersonic Flow Simulation by the Gas-Kinetic Bhatnagar–Gross–Krook Scheme

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The gas-kinetic Bhatnagar–Gross–Krook (BGK) scheme is extended to hypersonic flow simulations and thus shows that the compressible inviscid flow solutions of the simulations are efficiently and accurately obtained from the BGK scheme without the disastrous shock instability phenomenon that occurs in most hypersonic flow simulations involving strong shock waves. For this particular study, the effect of chemistry in hypersonic flows has not been taken into account. Hence, the assumption of calorically perfect gas is imposed in all simulations. The high-order resolution of the scheme is achieved by utilizing monotone upstream-centered schemes for conservation laws-type initial reconstruction. While, an implicit-type time-integration method known as the approximate factorization–alternating direction implicit is adopted for computing both steady and unsteady calculations. The gas-kinetic scheme is tested meticulously in four two-dimensional numerical examples, namely, the blunt-body problem, the double Mach reflection problem, the axisymmetric blunt-body problem, and the flow over a 15-deg ramp. The numerical results of the BGK scheme when compared with the other schemes and experimental data show that this numerical technique is robust, accurate, and stable for hypersonic flow.

Nomenclature

$\hat{A}, \hat{B}$	=	flux Jacobian matrices
C_p	=	pressure coefficient
$d\Xi$	=	volume element
F, G	=	inviscid flux vector
f, g	=	particle distribution function
I	=	identity matrix
J	=	Jacobian of transformation
P	=	pressure
Q	=	primitive variable
R	=	flux residual
t	=	time
U, V	=	macroscopic velocity components
u, v	=	microscopic velocity components
W	=	conservative variable
$\hat{W}, \hat{F}, \hat{G}$	=	transformed vector
x, y	=	Cartesian coordinates
γ	=	specific heat ratio
Δt	=	time step
ε	=	total energy
λ	=	function of pressure and density
ξ, η	=	generalized coordinates
ρ	=	density
ς	=	internal degree of freedom
τ	=	collision time
ϕ	=	van Leer's limiter
φ	=	adaptive parameter
Ψ	=	moment vector

I. Introduction

TREMENDOUS efforts have been devoted and great progress has been achieved in the field of computational fluid dynamics of hypersonic flows in the past decade.[1-4] An evident achievement in this field is the development of the numerical scheme for spatial discretization. Currently, the most notable and successful numerical flux function for high-speed flow computations belongs to the upwind difference schemes because of their superior accuracy. The flux-difference-splitting (FDS) framework is one of the most successful groups among the upwind difference schemes that is widely used and studied. The Roe's FDS scheme,[5] for instance, is the most popular owing to its accuracy for compressible inviscid and viscous flow simulations. However, the occurrence of transverse shock instability and negative internal energy limits its usage in the computation of high-speed flows with strong shock waves and expansion fans.[6] This is supported by the findings of Peery and Imlay[7] for blunt-body computations with Roe's FDS, which produce the carbuncle phenomenon. This phenomenon is a numerical instability that occurs when capturing a strong shock wave in multidimensional computation. In addition to the FDS schemes, another group of upwind difference schemes, namely, the flux-vector-splitting (FVS) schemes can lack the robustness, accuracy, and efficiency in comparison to the FDS schemes. It is well known that FVS schemes have a large numerical dissipation on contact discontinuities, which explains the reason for their poor shock resolution capability at contact discontinuities.[8]

The development of gas-kinetic schemes has attracted much attention in recent years. These schemes are based on the approximate collisional Boltzmann equation.[9,10] A particular strength of the kinetic schemes lies precisely where FDS schemes often fail, such as carbuncle phenomenon, entropy condition, and positivity.[11-13] There are mainly two kinds of gas-kinetic schemes, and the differences lie within the governing equations used in the gas evolution stage. One of the well-known kinetic schemes is called the