

Development of a New Method of Crack Modeling and Prediction Algorithm

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ABSTRACT

In this report, the well known parametric method of signals and systems representation is extended to modeling and prediction of cracks on building and road surfaces. Also, a new algorithm based on complex value autoregressive neural network with split linear activation function for the determination of Complex-Value Autoregressive Moving average (CARMA) coefficients is also proposed in this report. Furthermore, mathematical derivation and detail analysis of the proposed CARMA based Complex-Value Neural Network (CVNN) algorithm is also discussed in this work.

1. INTRODUCTION

A Complex-Value Autoregressive Moving Average (CARMA) models involves representation of Complex Value Data (CVD) by complex: input; output and coefficients. The past output section of CARMA is called Complex-valued Autoregressive (CAR) while the past and the present inputs are called complex-value Moving average (CMA). The general form of a CARMA model is

$$y(n) = -\sum_{k=1}^p a_k y(n-k) + \sum_{j=0}^q b_j x(n-j) \quad (1)$$

where a_k , b_j , $x(n)$, and $y(n)$ are CVD and p and q are real value model order for the CAR and CMA part respectively.

This method has been applied to various fields among which are system identification, linear prediction, communication, seismic signal processing, biomedical image and signal processing [1, 2, 3, 4, 5, 6]. The main aim of this work is to model building and road surface crack pattern using CARMA technique so as to be able to represent the crack pattern an image with few parameters and be able to predict its future direction assuming the circumstances that caused the initial crack remains unchanged. Also, parametric modeling of crack will lead to great reduction in the storage space needed for storing crack images for post filed monitoring and analysis since the original crack pattern (data) can easily be generated from the stored crack model parameters.

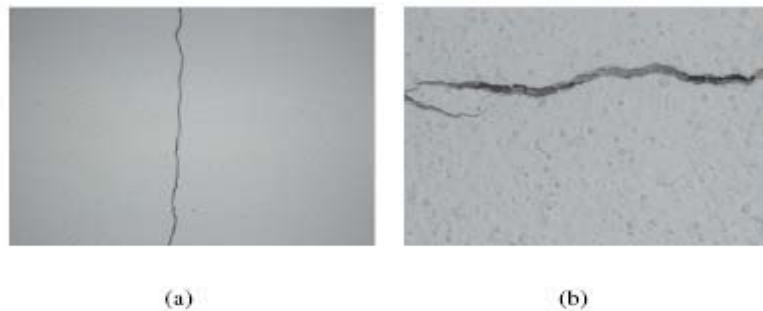




Fig. 1 Building Surface crack Images: (a)-(b) Real Crack Images; (c)-(d) Simulated Binary Crack Images

The organization of this paper is as follows, Section 2 gives a brief and concise introduction to parametric modeling technique. In Section 3, the mathematical modeling and the proposed algorithm (CVNN-CARMA) is presented. Section 4 concludes this report.

2. GENERAL INTRODUCTION TO PARAMETRIC MODELING TECHNIQUE

The use of modeling technique to predict or reconstruct a data sequence is concerned with the representation of data in an efficient technique [1, 3]. Signal modeling involves three major steps, namely;

1. **Model selection:** Model selection involves choosing an appropriate parametric form for the model data. An example of a modeling type is Autoregressive moving Average (ARMA) modeling technique.

The general ARMA equation for a real value data is given by

$$y(n) = -\sum_{k=1}^p a_k y(n-k) + \sum_{k=0}^q b_k x(n-k) \quad (2)$$

taking z-transform of both sides results in

$$Y(z) = -\sum_{k=1}^p a_k Y(z)z^{-k} + \sum_{k=0}^q b_k X(z)z^{-k} \quad (3)$$

and

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^q b_k z^{-k}}{1 + \sum_{k=1}^p a_k z^{-k}} \quad (4)$$

where

$$X(z) = \sum_{k=-\infty}^{\infty} x(n)z^{-n} \quad (5)$$

and

$$Y(z) = \sum_{k=-\infty}^{\infty} y(n)z^{-n} \quad (6)$$

2. **Model Parameter estimation:** This involves estimating the model order and model coefficients from the available data set.

i. **Model Order Determination Methods:** Methods of model order determination includes:

a. **Final Prediction Error (FPE):** FPE is a method of selecting the order of an AR model by minimizing the variance of the prediction error [9]. The function is given by

$$FPE(K) = \sigma^2 \frac{N + (K + 1)}{N - (K + 1)} \quad (7)$$

where K is the model order, N is the number of data points and σ^2 is the total squared error divided by the number of data points, N . It is mathematically express as

$$\sigma^2 = \frac{1}{N} \sum_K^{N-1} \varepsilon^2 \quad (K = 1, 2, 3, \dots, L)$$

where L is the maximum of the order. ε is defined as

$$\varepsilon(n) = x(n) - \bar{x}(n)$$

where $\bar{x}(n)$ is the predicted value of $x(n)$ for order k . By evaluating K from 1 to L the optimal model, K is the one that gives the minimum value of FPE. That is

$$FPE(p) = \min(FPE[k]) \quad (1 \leq K \leq m)$$

b. **Asymptotic Information Criterion (AIC):** The Asymptotic Information Criterion (AIC) normally refer to as Akaike Information criterion is a measure of goodness of fit of an estimated statistic model [9]. AIC reflects the balance between complexity of the model order and goodness of fit. This AIC method of order determination is given by,

$$AIC(K) = N \ln(\text{maximum likelihood}) + 2K$$

the approximate equation function is given as

$$AIC(K) = N \ln \sigma^2 + 2K \quad (8)$$

The term $2K$ represents the penalty for selecting higher order.

c. **Minimum Description Length (MDL)** The MDL is given by

$$MDL(K) = N \ln \sigma^2 + K \ln(N) \quad (9)$$

This increases the penalty factor incur by using higher order as compared to AIC, thus favoring the selection of lower model order.

d. **Residual Variance (RV)** The Residual variance criterion for order determination function is given by

$$RV(K) = \frac{N - K}{N - 2K - 1} \sigma^2 \quad (10)$$

ii. **Model Coefficients Determination Methods:** Some of the known model coefficients determination methods include

- Direct Least Square approach
- Pade Approximation
- Prony method
- Shank method (Modified Prony)
- Autocorrelation method
- Covariance method

3. **Model Validation:** This involve evaluating how well the selected model capture the key features of the data.

3. PROPOSED CRACK MODELING FRAMEWOORK

The proposed parametric crack modeling framework can be divided into three stages namely, Crack Image Pre-Processing (CIPP) stage, Crack Image Modeling (CIM) stage and the Crack Model Output (CMO) stage. The block diagram is as shown in Fig. 2

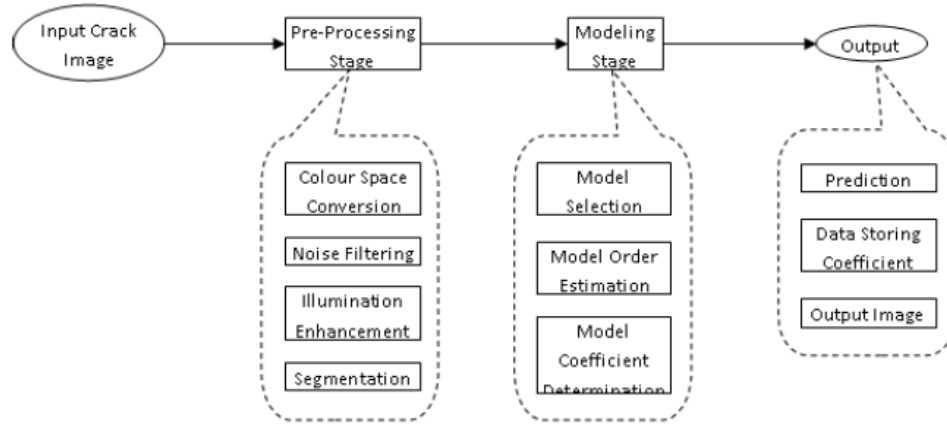


Fig. 2. Framework for Proposed Crack Modeling and Prediction

3.1. Crack Image Pre-Processing (CIPP) Stage

The CIPP stage consists of the following sub-activities:

1. **Color Space Conversion:** This involve converting the input RGB (Red-Green-Blue) image to HSI (Hue Saturation and Intensity) or grey level image.
2. **Median Filtering:** Median filtering operation replaces a pixel by the median of all the neighboring pixels in an NxN window (N is always odd number). This is a nonlinear process and it reduces impulsive noise from an image without distorting the edges too much.
3. **Illumination Equalization:** This involve the application of Global-Local Adaptive Histogram Equalization using Partially-Overlapped Windows (GLAPOW) [7] this is to alleviate the problem of uneven illumination without introducing noise in the crack model image.
4. **Segmentation Stage** The main objective of segmentation stage is to divide the image into regions with same properties or characteristics. Different techniques used for image segmentation include simple thresholds, k-means algorithm and fuzzy c-means. In this work, segmentation by k-means with two non-overlapping clusters is proposed.
5. **Image Co-Ordinates Normalization:** This involve normalizing the crack co-ordinate to complex valued data for modeling purpose.

3.2. Crack Image Modeling (CIM) stage

The CIM stage involves two distinct tasks, namely: model order determination and model coefficients determination. Any of the aforementioned model order determination techniques in subsection 2 can be used to estimate the model order. In determining model coefficients for modeling building surface cracks, a new method of CARMA coefficients determination called Complex- Value Neural Network CARMA (CVNN-CARMA) is hereby proposed.

3.2.1. CVNN-CARMA MATHEMATICAL MODEL

The general CARMA Eqn. 1 can be re-written as

$$y(n) = y_R(n) + iy_I(n)$$

$$= -\sum_{k=1}^p (a_{Rk} + ia_{Ik})(y_{Rk}(n-k) + iy_{Ik}(n-k))$$

$$+ \sum_{j=0}^q (b_{Rj} + ib_{Ij})(x_{Rj}(n-j) + ix_{Ij}(n-j)) \quad (11)$$

where subscript R and I standing for real and imaginary part of a CVD respectively. Decomposing and re-arranging Eqn. 11 gives

$$\begin{aligned} y(n) &= y_R(n) + iy_I(n) \\ &= -\sum_{k=1}^p a_{Rk}y_{Rk}(n-k) + \sum_{k=1}^p a_{Ik}y_{Ik}(n-k) \\ &\quad -i\left[\sum_{k=1}^p a_{Rk}y_{Ik}(n-k) + \sum_{k=1}^p a_{Ik}y_{Rk}(n-k)\right] \\ &\quad + \sum_{j=0}^q b_{Rj}x_{Rj}(n-j) - \sum_{j=0}^q b_{Ij}x_{Ij}(n-j) \\ &\quad + i\left[\sum_{j=0}^q b_{Rj}x_{Ij}(n-j) + \sum_{j=0}^q b_{Ij}x_{Rj}(n-j)\right] \end{aligned}$$

therefore, if

$$y(n) = y_R(n) + iy_I(n) \quad (12)$$

then, the real part of CARMA process is given by

$$\begin{aligned} y_R(n) &= \sum_{k=1}^p a_{Ik}y_{Ik}(n-k) - a_{Rk}y_{Rk}(n-k) \\ &\quad + \sum_{j=0}^q b_{Rj}x_{Rj}(n-j) - b_{Ij}x_{Ij}(n-j) \end{aligned}$$

and the imaginary part can also be expressed as

$$\begin{aligned} y_I(n) &= \sum_{j=0}^q b_{Rj}x_{Ij}(n-j) + b_{Ij}x_{Rj}(n-j) \\ &\quad - \left[\sum_{k=1}^p a_{Rk}y_{Ik}(n-k) + a_{Ik}y_{Rk}(n-k)\right] \end{aligned}$$

3.2.2. CVNN-CARMA MODEL USING ANN

A three layer CVNN-CARMA with split complex-value weight and adaptive linear activation functions for CARMA coefficients determination is hereby proposed. The basic block diagram of the proposed CVNN-CARMA is as given in Fig. 3

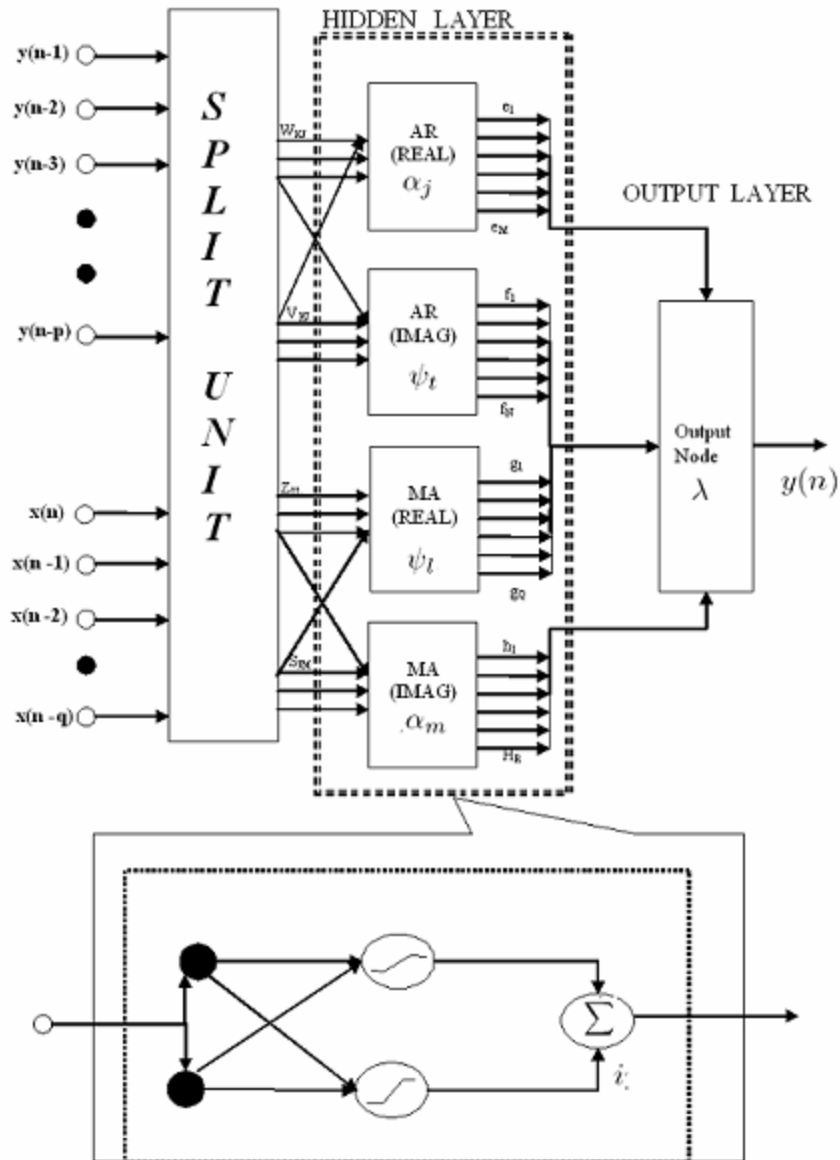


Fig. 3. CVNN-CARMA Block Diagram

In CARMA coefficients determination, either the real part of the output $y_R(n)$ or the imaginary part $y_I(n)$ can be independently used in obtaining the CARMA coefficients. The result obtained using the imaginary part $y_I(n)$ of the output only is hereby presented. Similar result will be obtained using the real part $y_R(n)$ of the CARMA output in determining the CARMA coefficients.

$$\begin{aligned}
 y_I(n) &= \lambda \beta \\
 &= \lambda \left[\sum_{j=1}^M e_j A_j + \sum_{t=1}^N f_t B_t + \sum_{m=1}^Q g_m C_m + \sum_{l=1}^R h_l D_l \right] \quad (13) \\
 &= \lambda \left[\sum_{j=1}^M e_j \alpha_j \sum_{k=1}^p W_{kj} y_R(n-k) + \sum_{t=1}^N f_t \psi_t \sum_{k=1}^p V_{kt} y_I(n-k) \right]
 \end{aligned}$$

$$\begin{aligned}
 & + \sum_{m=1}^Q g_m \mathcal{G}_m \sum_{j=0}^q S_{jm} x_R(n-j) + \sum_{l=1}^R h_l \phi_l \sum_{k=1}^q Z_{jl} x_l(n-j)] \\
 & = \sum_{k=1}^p \sum_{j=1}^M \lambda e_j \alpha_j W_{kj} y_R(n-k) + \sum_{k=1}^p \sum_{t=1}^N \lambda f_t \psi_t V_{kt} y_l(n-k) \\
 & + \sum_{j=0}^q \sum_{m=1}^Q \lambda g_m \alpha_m S_{jm} x_R(n-j) + \sum_{j=0}^q \sum_{l=1}^R \lambda h_l \psi_l Z_{jl} x_l(n-j)
 \end{aligned}$$

Using coefficient matching with Eqn. 13 gives

$$a_{Rk} = - \sum_{t=1}^N \lambda f_t \psi_t V_{kt} \tag{14}$$

$$a_{lk} = - \sum_{j=1}^M \lambda e_j \alpha_j W_{kj} \tag{15}$$

$$b_{Rj} = \sum_{l=1}^R \lambda h_l \psi_l Z_{jl} \tag{16}$$

and

$$b_{lj} = \sum_{m=1}^Q \lambda g_m \alpha_m S_{jm} \tag{17}$$

The CVNN-CARMA algorithm is presented in Table 1.

Table 1. CVNN-CARMA Algorithm

Start
Set Mean Square Error (MSE) value
Initialize all weights, Error(E) and adaptive polynomial coefficients
.....
Step 1: Calculates hidden neurons output (A_j , B_t , C_m , D_l)
Step 2: Calculates the final output ($y_l(n)$ or $y_R(n)$)
Step 3: Calculates the Error value
$E = \frac{1}{2} e(n) e^*(n)$
where
$e(n) = e_R(n) + i e_l(n) = d_R(n) + i d_l(n) - (y_r(n) + i y_l(n))$
Step 4: Back propagate E to calculate weights and coefficients increment
Step 5: Update the Output layer weights and activation function coefficients (λ , e_j , f_t , g_m , h_l)
Step 6: Update hidden layer weights and activation function coefficients (α_j , ψ_t , \mathcal{G}_m , ϕ_l , W_{kj} , V_{kj} , S_{jm} , Z_{jm})
If $MSE \leq E$, repeat step 1 to Step 6
Else
Step 7: Compute CARMA coefficients using Eqn: 14 - 17
End

3.3. Crack Model Output (CMO) stage

The output stage called Crack Model Output gives the output image, the model coefficients from which the crack can be predicted or simulated by driving the assumed system with a white noise. In reconstructing the image from the model coefficients, the complex valued data needs to be de-normalized and re-scaled.

4. CONCLUSION

In this report, a new method of parametric crack modeling and prediction for building and road surface have been proposed. The model is based on CVNN-CARMA technique. The work also shows that the coefficients of a CARMA process can be obtained from the weight and activation function of a trained artificial neural network system. Further work is on the way to validate this model and algorithm with different types of images instead of the simulated images used in developing this model.

REFERENCES

- [1] M. R. Smith, S. T. Nichols, R. M. Henkelman and M. L. Wood, "Application of Autoregressive Moving Average Parametric Modeling in Magnetic Resonance Image Reconstruction", *IEEE Transactions on Medical Imaging*, Vol. M1-5:3, pp 257 - 261, 1986.
- [2] Z. P. Liang, F. E. Boada, R. T. Constable, E. M. Haacke, P. C. Lauterbur, and M. R. Smith, "Constrained Reconstruction Methods in MR Imaging", *Reviews of MRM*, vol. 4, pp.67 - 185, 1992.
- [3] M. R. Smith, S. T. Nichols, R. M. Henkelman and M. L. Wood, "Application of Autoregressive Moving Average Parametric Modeling in Magnetic Resonance Image Reconstruction", *IEEE Transactions on Medical Imaging*, Vol. M1-5:3, pp 257 - 261, 1986.
- [4] R. Palaniappan, "Towards Optimal Model Order Selection for Autoregressive Spectral Analysis of Mental Tasks Using Genetic Algorithm", *IJCSNS International Journal of Computer Science and Network Security*, Vol. 6 No. 1A, January 2006.
- [5] H. Akaike, "Power Spectrum Estimation through Autoregression Model Fitting", *Annals of the Institute of Statistical Mathematics*, vol. 21, pp. 407-419, 1969.
- [6] J. Rissanen, "Modelling by shortest data description", *Automatica*, vol.14, pp.465-471, 1978.
- [7] A. M. Aibinu, M. I. Iqbal, M. Nilsson and M. J. E. Salami, "A New Method of Correcting Uneven Illumination Problem in Fundus Images", *International Conference on Robotics, Vision, Information, And Signal Processing*, Penang, Malaysia, pp. 445 - 449, Nov. 2007.
- [8] A. M. Aibinu, M. J. E. Salami and A. A. Shafie, "A New Complex Valued ARMA Coefficients determination Using Artificial Neural Network", *submitted for publication International Conference on Acoustics, Speech and Signal Processing*, Taiwan, 2009.
- [9] A. M. Aibinu, A. R. Najeeb, A. A. Shafie, and M. J. E. Salami, *Optimal Model Order Selection For Parametric Magnetic Resonance Image Reconstruction*, World Assembly of Engineers, Scientists and Technologists, International Conference on Medical system Engineering (ICMSE), 2008, Singapore, Vol.32, Pg. 191-195, August- September 2008.