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Adaptive Beamforming with 16 Element Linear Array Using MaxSIR and MMSE Algorithms

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***Abstract*— Smart antenna system is a promising technology to provide higher capacity with more reliability. Adaptive beam forming capability is very significant in smart antenna systems. Maximum Signal-to-Interference Ratio (MaxSIR) and Minimum Mean Square Error (MMSE) beamforming algorithms were investigated using 16 elements linear array antenna. Both algorithms were calculated and simulated using Matlab as well as Zeland's IE3D software. The performance of algorithms for single and multiple beams are presented in this paper.**

***Index Terms*—Adaptive Beamforming, Linear Patch Antenna Arrays, Maximum Signal-to-Interference Ratio (MaxSIR), Minimum Mean Square Error (MMSE), Smart Antenna System.**

I. Introduction

As mobile communication technology evolves, many researches have been conducted to improve the reliability, efficiency and channel capacity of mobile base station system. Capacity and reliability of wireless communication systems are limited by three major impairments; multipath fading, delay spread and co-channel interference [1], [2].

Therefore, researchers began to search for new techniques to improve the capacity and the reliability of mobile base station. One of the promising technologies, which is examined and thought to be able to provide higher capacity, is the *Smart Antenna System*. Smart antenna has the ability to focus its radiation beam towards the desired user whilst reducing the beam pointed towards the undesired user and rejecting interference. In this way, multipath and co-channel interference are effectively reduced.

Basically, smart antenna consists of antenna arrays and Digital Signal Processor (DSP). It is impossible to alter the beam pattern with a single element patch because single element usually produces a fixed radiation pattern and has low gain. A number of elements gathered as an array can produce a directive pattern. In most cases, the elements of an array and its spacing are made to be identical to simplify the design. With the aid of array elements, it is possible to produce radiation patterns which can be controlled electronically. Adaptive array antenna will reduce the problem of limited channel bandwidth, multi path fading, and insufficient range by providing high directivity with tailored beam shapes and suppression of co-channel interference.

In reality, it is the DSP that makes smart antennas a 'smart' system, not the antenna itself [1], [3]. The DSP uses a set of algorithms (MUSIC, ESPRIT and etc) to find the direction-of-arrival (DOA). Meanwhile, another set of beamforming algorithms (MMSE, Max SIR and etc) were used to create directional beam towards active user and null towards interferer. The adaptive antenna array system can form multiple beams at different angles providing a greater coverage area.

This paper focuses on adaptive beam forming and its performance using MMSE and Max SIR algorithms for 16 elements linear array antenna.

II. Beam Forming

A. Linear arrays

Linear array is the most common and most analyzed array structure [4]. The array characteristic can be analyzed by placing M radiating elements side by side horizontally. To simplify the derivation, the element is assumed to be an infinitesimal dipole and no coupling exists between the elements.

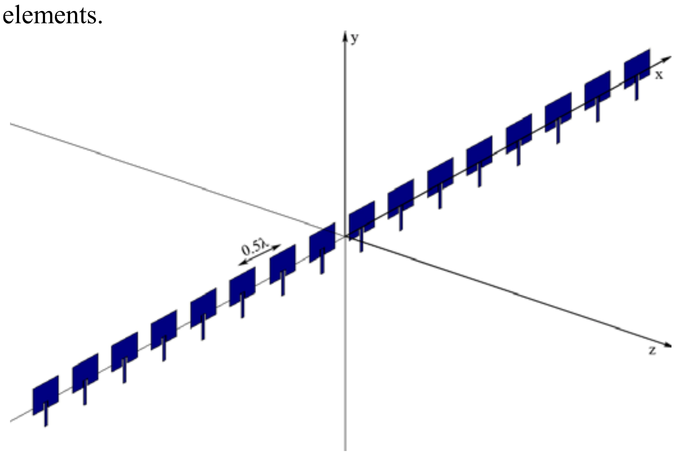


Figure 1: 16-element linear array with 0.5λ spacing

The total field is the vector sum of the field generated by each element as shown by the following equation. The array factor is given by

$$\mathrm{AF} = \sum_{m=1}^{M} w_m e^{+j(m-1)(kd\sin\theta+\beta)} \;\ldots\ldots\ldots\ldots (1)$$

The total field pattern can be varied using the array factor while modifying either the separation, d and/or the excitation weight between the elements. The total field can be obtained by multiplying the array factor with the field of the single element at a selected reference point. This is referred to as pattern multiplication for continuous sources [5].

$$E_{Total} = E_o \times AF \;\ldots\ldots\ldots\ldots (2)$$

Where E_o is the electric field of a single element at a selected reference point.

This equation is general and valid for all arrays. The array factor will be playing a major role in determining the field pattern, while the array factor will be different for different type of array arrangement.

B. *Maximum Signal-to-Interference ratio (MaxSIR)*

Generally, MaxSIR (also known as Applebaum Algorithm) is a beamforming algorithm that maximizes the ratio of desired signal power to the noise (undesired) signal power at the array output [6].

Consider that the signal and the interference are received by an M-element linear array with M potential weight. As stated in [7], additive Gaussian noise is included in each received signal at element *m* and time is represented by the k[th] time sample. The weighted array output *y* is defined as:

$$y(k) = \bar{w}^H \cdot \bar{x}(k) \;\ldots\ldots\ldots\ldots (3)$$

where

$$\bar{x}(k) = \bar{a}_0 s(k) + \begin{bmatrix} \bar{a}_1 & \bar{a}_2 & \ldots & \bar{a}_N \end{bmatrix} \cdot \begin{bmatrix} i_1(k) \\ i_2(k) \\ \vdots \\ i_N(k) \end{bmatrix} + \bar{n}(k)$$

$$= \bar{x}_s(k) + \bar{x}_i(k) + \bar{n}(k) \;\ldots\ldots\ldots\ldots (4)$$

with

$\bar{w} = \begin{bmatrix} w_1 & w_2 & \ldots & w_M \end{bmatrix}^T$ = array weights

$\bar{x}_s(k)$ = desired signal vector

$\bar{x}_i(k)$ = interfering signals vector

$\bar{n}(k)$ = zero mean Gaussian noise for each channel

$\bar{a}_i$ = M-element array steering vector for the θ_i direction of arrival

Then, equation (4) is substituted into equation (3) and thus the weighted array output *y* becomes:

$$y(k) = \bar{w}^H \cdot \left[\bar{x}_s(k) + \bar{x}_i(k) + \bar{n}(k)\right]$$
$$= \bar{w}^H \cdot \left[\bar{x}_s(k) + \bar{u}(k)\right] \;\ldots\ldots\ldots\ldots (5)$$

where

$\bar{u}(k) = \bar{x}_i(k) + \bar{n}(k)$ = undesired signal

The weighted array output power for desired and undesired signals can be determined in terms of array correlation matrices for the desired signal ($\bar{R}_s$) and the undesired signal ($\bar{R}_u$) respectively.

Thus, the weighted array output power for the desired signal is defined as:

$$P_s^2 = E\left[\left|\bar{w}^H \cdot \bar{x}_s\right|^2\right] = \bar{w}^H \cdot \bar{R}_s \cdot \bar{w} \;\ldots\ldots\ldots\ldots (6)$$

where

$\bar{R}_s = E\left[\bar{x}_s \bar{x}_s^H\right]$ = signal correlation matrix

Meanwhile, the weighted array output power for the undesired signal is defined as:

$$P_u^2 = E\left[\left|\bar{w}^H \cdot \bar{u}\right|^2\right] = \bar{w}^H \cdot \bar{R}_u \cdot \bar{w} \;\ldots\ldots\ldots\ldots (7)$$

where

$$\bar{R}_u = \bar{R}_i + \bar{R}_n$$

with

$$\bar{R}_i = \begin{Bmatrix} b_1(\theta_1) & b_1(\theta_2) & \ldots & b_1(\theta_b) \\ b_2(\theta_1) & b_2(\theta_2) & \ldots & b_2(\theta_b) \\ \vdots & \vdots & \ldots & \vdots \\ b_M(\theta_1) & b_M(\theta_2) & \ldots & b_M(\theta_b) \end{Bmatrix}$$

= interferers correlation matrix

$$\bar{R}_n = \sigma^2 \times \bar{I}_M = \sigma^2 \times \begin{Bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & 1 \end{Bmatrix} \ldots\ldots\ldots\ldots (8)$$

= noise correlation matrix

Since signal-to-interference ratio (SIR) is defined as the ratio of the desired signal power to the undesired signal power, SIR can be mathematically defined as:

$$SIR = \frac{P_s^2}{P_u^2} = \frac{\overline{w}^H \cdot \overline{R}_s \cdot \overline{w}}{\overline{w}^H \cdot \overline{R}_u \cdot \overline{w}} \quad \ldots\ldots\ldots\ldots(9)$$

The maximum SIR can be determined by setting the derivative of equation (9) with respect to *w* equal to zero. And the following relationship is derived:

$$\overline{R}_s \cdot \overline{w} = SIR \cdot \overline{R}_u \cdot \overline{w} \ldots\ldots\ldots\ldots(10)$$

or

$$\overline{R}_u^{-1} \cdot \overline{R}_s \cdot \overline{w} = SIR \cdot \overline{w} \quad \ldots\ldots\ldots\ldots(11)$$

According to [7], Equation (11) is an eigenvector equation and SIR is the eigenvalues. The maximum SIR can be determined as the largest eigenvalue (λ_{max}) for the Hermitian matrix $\overline{R}_u^{-1} \cdot \overline{R}_s$. The optimum weight vector w_{opt} is the eigenvector related to the largest eigenvalue.

$$\overline{R}_u^{-1} \cdot \overline{R}_s \cdot \overline{w}_{SIR} = \lambda_{\max} \overline{w}_{opt} = SIR_{\max} \cdot \overline{w}_{SIR} \quad \ldots(12)$$

Rearranging equation (9),

$$w_{SIR} = \frac{\overline{R}_u^{-1} \cdot \overline{R}_s \cdot \overline{w}_{SIR}}{SIR_{\max}} \quad \ldots\ldots\ldots\ldots(13)$$

Since $\overline{R}_s = E\left[|s|^2\right] \overline{a}_0 \cdot \overline{a}_0^H$, the weight vector is defined as:

$$w_{SIR} = \frac{E\left[|s|^2\right] \overline{a} \cdot \overline{a}^H \cdot \overline{R}_u^{-1} \cdot \overline{w}_{SIR}}{SIR_{\max}} \quad \ldots\ldots\ldots\ldots(14)$$

where

$$\overline{a} = [\overline{a}_0 + \overline{a}_1 + \ldots + \overline{a}_b]$$

Equation (14) has been used to generate weight vectors which were used to feed antenna elements for producing adaptive beams towards desired directions.

C. Minimum Mean Square Error

Mean Square Error (MSE) is the difference between the array output and the reference signal [8]. Minimum Mean Square Error (MMSE) minimizes the error while iterating the array weight [7]. Let *d(k)* to be the reference signal sample at the output at instant k and *s(k)* to be the desired signal. *d(k)* must be highly correlated with *s(k)*, otherwise MSE will not be efficient.

The estimated error $\xi(\mathrm{k})$ can be expressed as:

$$\xi(\mathrm{k}) = d(\kappa) - \overline{w}(\kappa)\overline{x}^H(\kappa) \ldots\ldots\ldots\ldots(15)$$

where the $\overline{x}$ is the array vector matrix and the size of the array depend on the number of elements, M in the array and the number of desired steering angles, b.

$$\overline{x} = \begin{Bmatrix} a_1(\theta_1) & a_1(\theta_2) & \ldots & a_1(\theta_b) \\ a_2(\theta_1) & a_2(\theta_2) & \ldots & a_2(\theta_b) \\ \vdots & \vdots & \ldots & \vdots \\ a_M(\theta_1) & a_M(\theta_2) & \ldots & a_M(\theta_b) \end{Bmatrix}$$

According to wiener theory, to cause the output $w(k)x^H(k)$ to correlate as

$$\xi^2(\mathrm{k}) = \left[d - \overline{wx}^H\right]^2 \ldots\ldots\ldots\ldots(16)$$

$$\xi^2 = d^2 - 2d\overline{wx}^H + \overline{wxw}^H \overline{x}^H$$

$$\xi^2 = d^2 - 2\overline{rw} + \overline{ww}^H \overline{R}_x \ldots\ldots\ldots\ldots(17)$$

$$\overline{r} = E[d\overline{x}] = E[d.(\overline{x}_s + \overline{x}_i + \overline{x}_n)] \ldots\ldots\ldots\ldots(18)$$

$$\overline{R}_x = E[\overline{xx}^H] = \overline{R}_s + \overline{R}_u \ldots\ldots\ldots\ldots(19)$$

closely as possible to d(k), the coefficient of the weight are determined using minimum MSE criterion [9]. The MMSE is derived by equating $d\boldsymbol{\varepsilon}^2/d\mathbf{w}$ to zero.

$$\overline{R}_s = E[\overline{x_s x}^H] \ldots\ldots\ldots\ldots(20)$$

$$= \text{signal correlation matrix}$$

$$\overline{R}_u = \overline{R}_i + \overline{R}_n \ldots\ldots\ldots\ldots(21)$$

$$= \text{Undesired correlation matrix}$$

$$\frac{d\xi^2}{d\overline{w}} = -2\overline{r} + 2\overline{wR}_x = 0 \ldots\ldots\ldots\ldots(22)$$

$$\overline{w}_{MMSE} = \overline{rR}_x^{-1} \ldots\ldots\ldots\ldots(23)$$

since *d(k)* and *s(k)* are highly correlated

$$r = E[dx^H] = E[sx^H] = Sa_0 \ldots\ldots\ldots\ldots(24)$$

where

$$S = E[|s|^2] \ldots\ldots\ldots\ldots(25)$$

The optimum weight can be recognized as

$$w_{MMSE} = S\overline{aR}_x^{-1} \ldots\ldots\ldots\ldots(26)$$

where

$$\overline{a} = [\overline{a}_0 + \overline{a}_1 + \ldots + \overline{a}_b]$$

Equation (26) has been used to generate weight vectors which were used to feed antenna elements for producing adaptive beams towards desired directions.

III. SIMULATIONS & RESULTS

The performance evaluation of the proposed algorithm has been accomplished by using Matlab 7.0 and Zeland IE3D simulation.

Microstrip patch antenna was designed and optimized to resonate at 2.0GHz with IE3D simulation. Then a linear array of 16 element patch antenna was designed with 0.5λ spacing between each other. Matlab was used to calculate the weights and plot the array patterns by using array factor theory. Based on equation (14), the max SIR weights are calculated while equation (26) is used for MSE weight calculation. Array factor calculation is used to plot beam patterns. The weights produced from Matlab simulation was used as the input to IE3D simulation.

The IE3D is capable of calculating the array pattern without actually simulating the array. It uses pattern calculation which neglects the effect of mutual coupling between the elements. Most of the analysis is done by using Matlab calculation since the simulation of 16 element linear array with 16 individual port is time consuming. Some of the results were simulated using IE3D to verify the algorithm and it shows similar results compared to Matlab calculated values.

A. Single Beamforming Range

The aim of the single beam range analysis is to check the capability of both algorithms to form a single beam at various angles. In case of single beam, both Maximum SIR and MSE algorithms generate similar weights. Thus the same beam patterns can be obtained at all angles for both algorithms.

Table 1: Single Beam Range

Desired angle (°)	Max SIR/MSE Calculated angle (°)	Max SIR/MSE Error (°)
-90	-90.000	0.000
-80	-80.260	-0.260
-70	-69.950	0.050
-60	-60.210	-0.210
-50	-49.890	0.110
-40	-40.150	-0.150
-30	-29.840	0.160
-20	-20.100	-0.100
-10	-9.786	0.214
0	-0.0456	-0.0456
10	9.695	-0.305
20	20.010	0.010
30	29.750	-0.250
40	40.060	0.060
50	49.800	-0.200
60	60.110	0.110
70	69.86	-0.14
80	80.17	0.17
90	89.91	-0.09

Beams have been generated using both algorithms ranging from -90^0 to $+90^0$ with 10^0 steps and presented in Table 1. The results show that difference between the calculated angles and the desired angles are significantly small, showing high accuracy. The maximum difference is only -0.305°. At -90°, the calculated beam is steered at exactly -90°. Therefore, it is obvious that both algorithms can produce beam accurately at various angles. Weights calculated from Matlab were used to simulate for three desired angles using Zeland IE3D software and shown in Figure 2. Table 2 verifies that the calculated and the simulated angles are similar with a small variation.

Table 2: Matlab Calculation and IE3D Simulation.

Desired Angle (°)	Matlab Calculated (°)	Calculated Error (°)	IE3D Simulated (°)	Simulated Error (°)
20	20.010	0.010	20.0681	0.0681
30	30.320	0.320	30.0293	0.0293
40	39.490	-0.510	39.5287	-0.4713

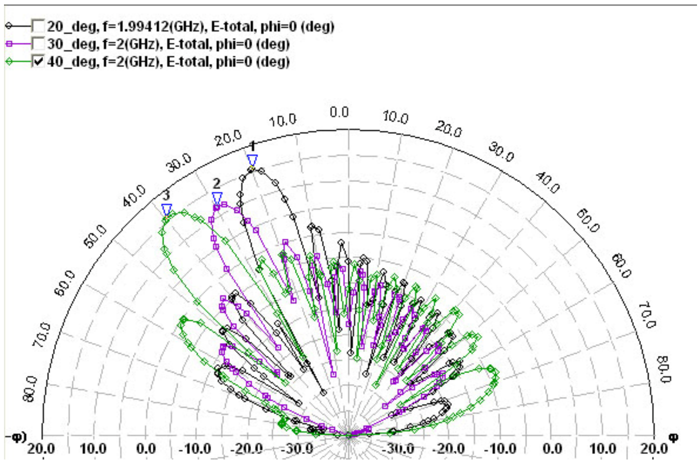


Figure 2: IE3D Simulation results for desired angle 20°, 30° and 40°.

B. Optimum Separation between Two Beams

The theoretical array factor calculation shows that both algorithms have some limitation on producing two beams close together. Both algorithms were analyzed to get a suitable separation range with low error and presented in Table 3. The 1st beam is maintained at 0° while the other is varied from 1° up to 17° to determine a suitable separation which minimizes the error.

The result shows that MaxSIR creates only one beam when the separation angle between two beams is below 4° as shown in Figure 3. Meanwhile, MSE generates two beams with high error relative to the desired beam angle at low separation angles as shown in Figure 4. Both maxSIR and MSE algorithm becomes more reliable (error below 1°) when the separation angle between them is higher than 8 °. MaxSIR could be used to create two beams from 5° with ±1.55° error.

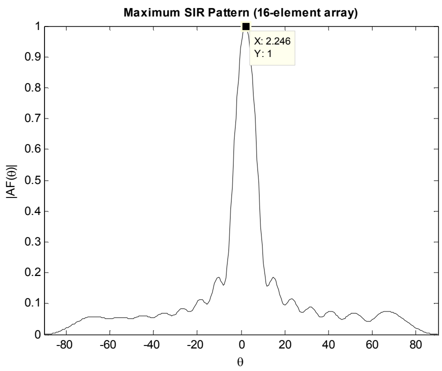


Figure 3: Max SIR calculated Beam Pattern for two beams at 0º and 4º.

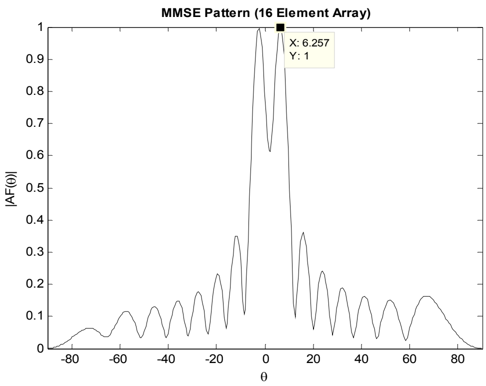


Figure 4: MSE calculated Beam Pattern for two beams at 0º and 4º.

Table 3: Angle variation while two consecutive beams are close together.

1st beam (fixed at 0º)	1st beam error (º)		2nd beam (varied)	2nd beam error (º)	
	MaxSIR	MSE		MaxSIR	MSE
0	0.5273	-3.483	1	-	3.538
0	1.1	-2.91	2	-	3.111
0	1.673	-2.91	3	-	2.684
0	2.246	-2.337	4	-	2.257
0	-0.6186	-2.337	5	0.684	2.403
0	-1.764	-1.764	6	1.403	1.976
0	-1.192	-1.192	7	1.549	1.549
0	-1.192	-1.192	8	1.122	1.122
0	-0.6186	-0.6186	9	0.695	0.695
0	-0.04563	-0.04563	10	0.27	0.27
0	-0.04563	-0.04563	11	-0.16	-0.16
0	-0.04563	-0.6186	12	-0.01	0.56
0	-0.6186	-0.6186	13	0.71	0.71
0	-0.6186	-0.6186	14	0.85	0.85
0	-0.6186	-0.6186	15	1	0.42
0	-0.6186	-0.6186	16	0.57	0.57
0	-0.04563	-0.04563	17	0.14	0.14

C. Optimum Separation between Mainbeam & Interferer

The interference or null placement in both algorithms also has effect on the array factor calculation. Both algorithms showed problems if the interfering signal vectors are correlated with the desired signal vectors. The 1st beam is maintained at 0º while the interferer is varied from 1º up to 11º to determine a suitable separation which minimizes the error. The results are tabulated in Table 4. The analysis shows that both algorithms have similar effect when the null is placed closed to the main beam with a slight variation at 10º separation. Note that the nulls are placed very accurately with the maximum error of 0.27º in MSE and 0.84º in maxSIR. Meanwhile the main beam produces high error below 6 º separation.

Table 4: Angle variation while mainbeam and interferer are close together.

1st beam	1st beam error (º)		Interferer (varied)	Interferer error (º)	
	MaxSIR	MSE		MaxSir	MSE
0	-0.0456	-0.0456	1	-	nil
0	-4.056	-4.056	2	0.1	0.1
0	-2.91	-2.91	3	0.246	0.246
0	-2.337	-2.337	4	-0.181	-0.181
0	-1.764	-1.764	5	-0.035	-0.035
0	-1.192	-1.192	6	0.111	0.111
0	-0.6186	-0.6186	7	0.257	0.257
0	-0.0456	-0.0456	8	-0.17	-0.17
0	-0.0456	-0.0456	9	-0.024	-0.024
0	-0.0456	-0.0456	10	0.122	0.122
0	-0.0456	-0.0456	11	0.84	0.27

D. Two Symmetrical Beams

The performances of both algorithms in forming two beams symmetrically are analyzed and presented in Table 5. For desired angles between -10º/10º and -50º/50º, the calculated angles are very accurate for both algorithms. Nevertheless, at -60º/60º, the difference between calculated angle and desired angles start to increase. The highest difference can be observed when -80º/80º beams are formed where the difference is approximately ±8.3º and ±11.8º for Maximum SIR and MSE respectively. Therefore, it can be proved that both algorithms can form two symmetrical beams accurately at the range of -10º /10º and -60º/60º. This investigation also shows that Maximum SIR and MSE algorithms produce different beam patterns when the number of beam increases.

Table 5: Two Symmetrical Beams Range

1st beam	1st beam error (º)		2nd beam	2nd beam error (º)	
	MaxSIR	MSE		MaxSir	MSE
-10	-0.36	-0.36	10	0.27	0.27
-20	-0.1	0	20	0.01	0.01
-30	0.16	0.16	30	-0.25	-0.25
-40	-0.15	-0.16	40	0.06	0.06
-50	0.11	0.11	50	-0.2	-0.2
-60	1.51	1.51	60	-1.03	-1.03
-70	4.06	4.06	70	-3.58	-3.58
-80	-8.28	11.77	80	8.19	-11.86

E. Maximum Number of Beams

The purpose of this analysis is to find the maximum number of beam that can be produced by both algorithms. This analysis starts with 0º and the number of beam was added until the algorithms produces high error. The step angle of 10º was used since previous analysis shows that both Max SIR and MSE produce low error at 10º separation.

The calculated angles based on array factors generated from both algorithms are shown in Table 6 and Table 7. Both algorithms can produce up to 13 beams as shown in both tables. However, the beam pattern and the angle accuracy for both algorithms are different. The array factor for Max SIR at the extreme end (from -40º/40º until -90º/90º) is lower compared to MSE. High Array factor is desirable since it can give an optimum beam pattern when multiplied with the reference single element beam pattern as discussed in equation (2). Therefore, in this case, the MSE produces an optimum result when 13 beams are formed. This can also be proved when angle accuracy is compared. MSE algorithm gives more accurate calculated angles at the range of -50º and 50º. Meanwhile, MaxSIR can only produce accurate angles between -40º and 40º. Figure 5 shows that the simulated patterns are similar compared to calculated patterns plotted in Matlab.

Table 6: MaxSIR pattern in -60º to 60º Range

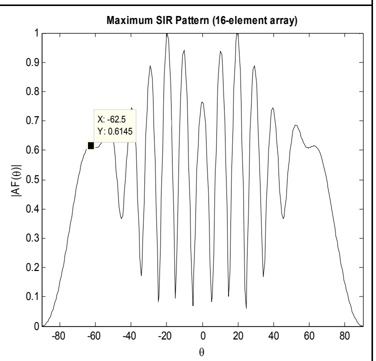
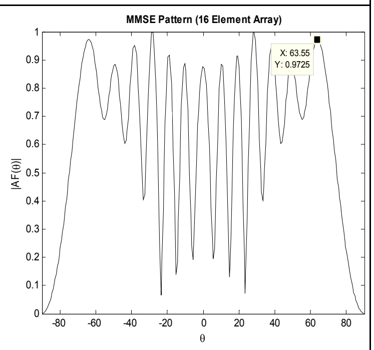
Desired angle	Calculated angle	Beam pattern
-60	-62.5	
-50	-52.18	
-40	-39.58	
-30	-29.27	
-20	-19.53	
-10	-10.36	
0	-0.0456	
10	10.27	
20	19.43	
30	29.18	
40	39.49	
50	52.67	
60	62.41	

Table 7: MSE pattern in -60º to 60º Range

Desired angle	Calculated angle	Beam pattern
-60	-63.64	
-50	-49.32	
-40	-38.43	
-30	-28.12	
-20	-18.95	
-10	-10.36	
0	-0.0456	
10	10.27	
20	18.86	
30	28.6	
40	38.34	
50	49.23	
60	63.55	

IV. CONCLUSION

Two beamforming algorithms such as MaxSIR and MMSE are investigated using 16 elements linear array antenna. Both algorithms produce same weight and beam pattern for single

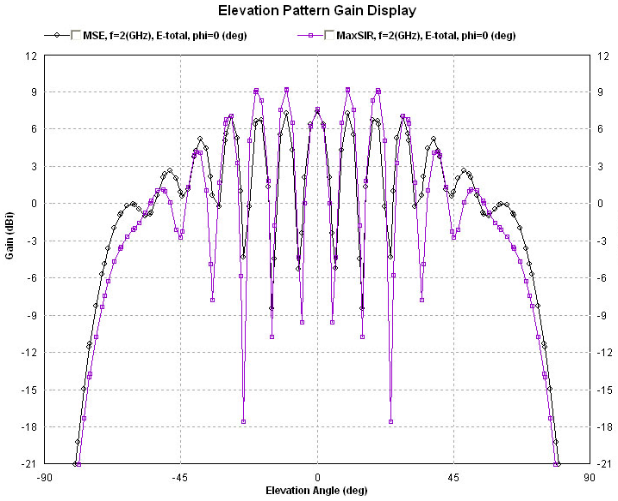


Figure 5: Comparison between MaxSIR and MMSE algorithm using IE3D pattern calculation.

beam, and they are capable of producing a single beam at 180^0 direction. MaxSIR produces only one beam when the separation between two consecutive beams is small. MSE tends to be less reliable in the same environment. Both maxSIR and MSE algorithm becomes more reliable when the separation angle between them is higher than 8º, while the interference can be placed 7º apart from the main beam to minimize the error.

Both algorithms can produce up to 13 beams in the range of -60º to 60º but MaxSIR tends to be less reliable and produces unstable pattern throughout this range. Meanwhile, MSE produces higher array factor (beam pattern) at the higher end (from -40º/40º until -60º/60º). Both algorithms can be improved by increasing the number of elements.

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