Some Empirical Evidence on Models of Fisher Relation

Jae-Young Kim and Woong Yong Park

The Fisher relation, describing a one-for-one relation between nominal interest rate and expected inflation, underlies many important results in economics and finance. The Fisher relation is a conceptually simple relation, but the empirical evidence of it is more or less complicated with mixed results. Several alternative models with different implications were proposed in empirical literature for the Fisher relation. We evaluate these alternative models for the Fisher relation based on a post-data model determination method. Our result for data from the U.S. and Korea shows that models with both regimes/periods, a regime with nonstationary fluctuations and the other with stationary fluctuations, fit data best for the Fisher relation.

Keywords: Fisher relation, Nonlinear behavior, Post-data model determination

JEL Classification: C1, C22, C5

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I. Introduction

The Fisher relation underlies many important results in economics and finance. This relation explains that nominal interest rate is determined as the sum of expected inflation and real interest rate which is a constant or a stable variable around a constant. Therefore, the Fisher relation signifies that nominal interest rate has a statistical one-for-one relation with the expected rate of inflation. Although the Fisher relation seems to be simple, several alternative models with different implications were proposed in literature. In this study, we examine the Fisher relation by evaluating the alternative models based on post-data model determination method.

Empirical analysis on the Fisher relation was initiated by Fama (1975). The constancy of real interest was studied by Nelson and Schwert (1977), Garbade and Wachtel (1978), Mishkin (1981, 1984), and Fama and Gibbons (1982). The correlation between nominal interest rate and inflation rate, which is noted as the Fisher effect was studied by Summers (1982), Huizinga and Mishkin (1986), and Mishkin (1990). Rose (1988), Atkins (1989), Mishkin (1992), and Wallace and Warner (1993) studied real interest rate based on concepts of a unit root and co-integration. Huizinga and Mishkin (1986) and Roley (1986) studied the possibility that the change in the U.S. monetary policy in the late 1970s through the mid-1980s affected the dynamics of interest rates and inflation. Evans and Lewis (1995) and Garcia and Perron (1996) used models of regime switch to analyze the behavior of U.S. real interest rate using post-war data, including the period of policy regime change and oil shocks. Kim and Park (2015) studied the possibility of short-run instability of the Fisher relation.

Several Fisher relation approaches and models were proposed in literature, but we do not know which model is the most appropriate for the Fisher relation. This issue is important because each model has a distinct implication for the Fisher relation, which may conflict with each other. In this study, we evaluate those alternative models using data from the U.S. and Korea in the post war period before the 2007-2008 world financial crisis. For this purpose, we employ a post-data model determination method that evaluates the relative probability of each model having generated given data. We use a Markov-chain-Monte-Carlo (MCMC) method to compute the criterion that quantifies the relative probability of each model. The model that yields the highest post-data probability is the one that best fits the data. We use the Gibbs sampler to compute the relative post-data probabilities. Our results show that the best model is not the same for the two countries. However, models with both regimes/periods, a regime with non-stationary fluctuations and the other with stationary fluctuations, seem the most appropriate.

The rest of the paper is organized as follows. Section II introduces the Fisher relation and its related issues. Section III explains several models for the Fisher relation proposed in literature. In Section IV we discuss how to select the most appropriate model for the Fisher relation and provide some empirical results of model selection. Finally, Section V concludes the paper.

II. Fisher Relation and Its Related Issues

The Fisher relation explains how nominal interest rate is determined. Let π^e_{t+1} be the expected rate of inflation from period t to period $t+1$. Let r^*_t and i_t be the *ex ante* real interest rate and the nominal interest rate, respectively, at time t. Nominal interest rate is equal to the real interest rate plus the expected rate of inflation:

$$i_t = r^*_t + \pi^e_{t+1} + \varepsilon_t, \tag{1}$$

allowing for temporary disturbance ε_t.

As explained in Kim and Park (2015) and others, the Fisher relation describes that nominal interest rate has a one-for-one relation with the expected rate of inflation. In other words, the Fisher relation describes that a stable level of the "real interest rate" exists that is equal to the nominal interest rate minus expected inflation, allowing for temporary disturbance. In terms of *ex ante* variables, the relation is written as

$$r^*_t = i_t - \pi^e_{t+1} - \varepsilon_t. \tag{2}$$

We also have an *ex post* version of the relation written as

$$r_t = i_t - \pi_{t+1} - \varepsilon_t, \tag{3}$$

where π_{t+1} and r_t are *ex post* inflation and *ex post* real interest rates, respectively. We use the same notation for disturbance ε_t in *ex ante* and *ex post* cases to save the notation.

Denoting by υ_t the error of inflation expectation, $\upsilon_t = \pi_t^e - \pi_t$, we have that $r_t = r_t^* + \upsilon_t$. If υ_t is stationary, which is the case under rational expectations, then the *ex ante* real interest rate r_t^* and the *ex post* real interest rate r_t have the same statistical properties. In this case, one can analyze the Fisher relation based on *ex post* as well as *ex ante* interest rates. The existence of a stable Fisher relation is, in the statistical sense, the same as that the real interest rate is a constant or a stationary variable fluctuating around a constant. Thus, the Fisher relation is conceptually a simple relation. However, the empirical analysis of the Fisher relation is rather complicated with mixed results.

Several different models are proposed in Fisher relation literature. The issue of which model is the most appropriate for the empirical Fisher relation is very important. We evaluate the alternative models for the Fisher relation based on post-data model determination method.

III. Models for the Real Interest Rate

In the following discussion, we use variable y_t for real interest rate. Let $\mathfrak{T}^T = \{1, \ldots, T\}$ be the sample period. We have four alternative Fisher relation models studied in the literature in the following, M_i, for $i = 0, 1, 2, 3$.

A. Autogression: M_0

The basic model is the p^{th} order autoregression in y_t:

$$(y_t - \mu) = \sum_{s=1}^{p} \phi_s (y_{t-s} - \mu) + \varepsilon_t \tag{4}$$

where ε_t ~ iid $N(0, \sigma^2)$, and all roots of characteristic equation $1 - \varphi_1 z - \ldots - \varphi_p z^p = 0$ lie outside the unit circle.

We can rewrite Model (4) as:

$$y_t = (1 - \textstyle\sum_{s=1}^{p} \phi_s)\mu + \sum_{s=1}^{p} \phi_s\, y_{t-s} + \varepsilon_t, \tag{5}$$

which is the commonly used form in usual time series analysis. We use the mean-deviated form in (4) instead of the common one in (5) because the former is more convenient for adopting standard regime switching models in our study.

B. Model with Partial-sample Instability: M_1

We incorporate the possibility of partial-sample (or short-run) instability in M_0 following the suggestion in Kim (2003), Andrews and Kim (2006), and Kim and Park (2015). Suppose that the process y_t in a relatively short period $\mathfrak{T}_B^T(\subset \mathfrak{T}^T)$ becomes unstable with properties of a non-stationary unit root or of higher volatility. In model M_1, we assume that $\mathfrak{T}_B^T$ is identified a priori, unlike model M_3 below. Then, we obtain the following model for y_t:

$$(y_t - \tau_t) = \sum_{s=1}^{p} [\phi_s \cdot I(t \in \mathfrak{T}_s) + \zeta_s \cdot I(t \in \mathfrak{T}_B)](y_{t-s} - \tau_{t-s}) + \varepsilon_t, \tag{6}$$

where $\mathfrak{T}_S = \mathfrak{T} \setminus \mathfrak{T}_B$; $\varepsilon_t \sim N(0, \upsilon_0^2)$, $I(\cdot)$ is the indicator function, and φ_s and ζ_s are parameters. For $t \in \mathfrak{T}_S$, we assume that the mean of y_t is $\tau_t = \mu_0$ and for $t \in \mathfrak{T}_B$ $\tau_t = \mu_1 + y_{t-1}$. Therefore, in the period $\mathfrak{T}_B$ the process y_t has a unit root, and its first difference Δy_t is a stationary autoregressive process.

C. Markov Regime Switching Model: M_2

We assume that the variable y_t follows regime-switching dynamics across K states $s_t = 1, 2, \ldots, K$:

$$(y_t - \tau_t) = \sum_{s=1}^{p} \phi_s (y_{t-s} - \tau_{t-s}) + \varepsilon_t, \tag{7}$$

for $\varepsilon_t \sim N(0, \upsilon_t^2)$, where $\tau_t = \mu_{s_t}$ and $\upsilon_t^2 = \sigma_{s_t}^2$ in state s_t. We assume that $\mu_1 < \cdots < \mu_K$ for identification. State variable s_t follows the first-order Markov process with transition probability from state i to state j, with $p_{ij} = P[s_t = j | s_{t-1} = i]$ for i, and $j = 1, \ldots, K$. Garcia and Perron (1996) used M_2-type models of regime switch to analyze the behavior of the U.S. real interest rate for post-war data.

D. An Extended Markov Switching Model: M_3

We now consider a Markov switching model that contains a non-stationary state, which is an extension of M_2 that only contains K stationary regimes. In extended model M_3, the last K^{th} state is set to be non-stationary. The extended model M_3 is thus

$$(y_t - \tau_t) = \sum_{s=1}^{p} [\phi_s \cdot I(s_t \neq K) + \zeta s \cdot I(s_t \neq K)](y_{t-s} - \tau_{t-s}) + \varepsilon_t, \tag{8}$$

where $\varepsilon_t \sim N(0, \upsilon_t^2)$. The mean of y_t in a stationary state is $\tau_t = \mu_{s_t}$ and that in the non-stationary state is $\tau_t = \mu_K + y_{t-1}$, which implies that variable y_t has a unit root in the state s_K.

Remark: There is an alternative modeling scheme with the regime switching due to its own lagged variable, known as the self-exciting threshold regression. It was introduced by Tong and Lim (1980) and studied by Seo (2008) in relation to unit root testing for the model. This alternative specification may be a relevant option for modeling the Fisher relation. However, we do not consider this specification in this paper since our objective is to evaluate existing models of the Fisher relation. This modeling scheme may be applied to the Fisher relation in any future work.

IV. Model Selection for the Fisher Relation

In this section, we discuss how to compare different models and select the one that best fits data for the Fisher relation. Then, we provide the results of model selection for models explained in Section III.

A. Post-data Model Selection

In this subsection we explain how to determine the best model for the real interest rate out of several different ones. Our approach is a post data model selection method developed in the Bayesian framework and is a generalized version of the Bayesian information criterion. Through the method, we can evaluate the relative merit of each model and select the model that best fits the data. We use an MCMC method to compute the criterion that quantifies the relative merit of each model.

A sample of T observations for the process y_t is denoted by $Y_T = (y_1, \ldots, y_T)$. A family M consists of candidate models for Y_T in the presence of uncertainty of the true model. A model $m_i \in M$ is associated with a parameter space Θ^i of dimension p_i for $i \in I$, where $I = \{1, \ldots, I\}$. Assume that for each m_i, a family $Q_T^i(\theta^i, Y_T)$ of distribution functions with density $q_T^i(\theta^i, Y_T)$ is defined. Let $\Pr(m_i | Y_T)$ be the post-data (posterior) probability that m_i is true. By Bayes' rule, we obtain:

$$\Pr(m_i \mid Y_T) = \frac{q_T(Y_T \mid m_i)\Pr(m_i)}{\sum_{j \in I} q_T(Y_T \mid m_i)\Pr(m_i)}, \tag{9}$$

where $\Pr(m_i)$ is the prior probability for m_i and $q_T(Y_T | m_j) = q_T^i(Y_T)$. However,

$$q_T(Y_T | m_j) = \int q_T(Y_T | \theta^j, m_j)\phi(\theta^j | m_j)d\theta^i = E_j[q_T(Y_T | \theta^j)], \tag{10}$$

where $\phi(\theta^j | m_j)$ is the prior density associated with the model m_j. If we further assume that $\Pr(m_j)$ is the same for all j, then the model selection rule is to choose m_i for which $E_i[q_T(Y_T | \theta^i)$ is the largest.

The quantity in (10) may be alternatively interpreted as marginal likelihood. The marginal likelihood $q_T(Y_T | m_j) = E_i[q_T(Y_T | \theta^i)]$ may be rewritten as

$$q_T(Y_T) = \frac{q_T(Y_T \mid \theta)\phi(\theta)}{\phi(\theta \mid Y_T)}, \tag{11}$$

where the script j and m_j are omitted for convenience, and $\phi(\theta | Y_T)$ is the posterior density of θ. Equation (11) is a reversed version of Bayes' rule. Given that (11) holds for any θ, we may evaluate $q_T(Y_T)$ for a convenient θ, such as $\theta = \theta^*$ the posterior mean. Taking the logarithm of (11) for $\theta = \theta^*$, we have

$$\ln q_n(Y_T) = \ln q_T(Y_T | \theta^*) + \ln \phi(\theta^*) - \ln \phi(\theta^* | Y_T). \tag{12}$$

Our decision rule is to choose model m_j that yields the highest value of (12). The calculation of log-likelihood and log-prior at $\theta = \theta^*$ is relatively easy unlike the calculation of posterior $\phi(\theta^* | Y_T)$. To compute $\phi(\theta^* | Y_T)$, we can use an MCMC method, such as Gibbs sampling, as in Chib (1995).[1] The computation of marginal likelihood or of posterior $\phi(\theta^* | Y_T)$ for the models in Section III is demanding work with highly sophisticated programming.

We use standard priors for the parameters in literature. That is, the regression coefficients φ and ζ have normal priors, and error variance σ^2

[1] Good references for the MCMC method and the Gibbs sampling are Gelman *et al.* (2000), Chib and Greenberg (1996), and Casella and George (1992), among others.

has an inverted gamma distribution. This normal-inverted gamma prior is a conjugate prior and is commonly used in literature. Mean μ_i has a normal prior, and the transition probability $\{p_{ij}\}$ has a prior of Dirichlet distribution that is conjugate. The prior for the transition probability reflects information on the duration of a state. For example, if the duration of state i is four quarters, then $1/(1 - p_{ii}) = 4$, so that $p_{ii} = 0.75$ for the prior.

B. Data and Empirical Results

Similar to many existing works, we use the three-month Treasury bill rate or equivalence for the nominal interest rate and the consumer price index (CPI) for the price level to compute the inflation rate.[2] We get U.S. data on the Treasury bill rate and CPI from the Federal Reserve Board and the Bureau of Labor Statistics, respectively. The Korean data are obtained from the International Financial Statistics (IFS). All data are seasonally adjusted. The data period is 1953:Q1–2006:Q1 for the U.S. and 1976:Q3–2006:Q1 for Korea. Thus, the data set covers observations in the post war period before the 2007-2008 world financial crisis.

We consider various aspects of each model in the model selection: the number of states K and the order of autoregression running up to 5 for each M_i, $i = 0, 1, 2, 3$. Table 1 shows the results of post-data model selection.

As shown in Tables 1, the best model is different for the two countries: M_1 is the best model for the U.S. and M_3 is that for Korea. However, for data from the two countries, the models with both regimes/periods, a regime with stationary fluctuations and the other with non-stationary/unstable fluctuations, (M_1 and M_3) are selected as the most appropriate models.

These results may seem like evidence against the existence of the Fisher relation as the Fisher relation implies that the real interest rate is stable with stationary fluctuations around a constant. However, our results are not evidence against the Fisher relation. If non-stationary fluctuations, if any, occur only temporarily, then we can say that the Fisher relation prevails in the majority of the sample period. Kim and Park (2015) have shown that the length of the period of non-stationary

[2] The three-month Treasury bill rate for the U.S. and the money market rate for Korea are used.

Table 1
Model Selection Results

	Selected model	Number of states	lag order
Korea	M_3	4	4
U.S.	M_1	-	3

fluctuations in the real interest rate is relatively short for data sets similar to those used in this paper. Nonetheless, it is important to note that the Fisher relation may not be a relation that is stable at all times.

V. Concluding Remarks

We compared several alternative models of the Fisher relation using data from Korea and the U.S based on a Bayesian model selection method. Among the four alternative models for the Fisher relation our results show that models with both regimes/periods of non-stationarity and stationarity seem the most appropriate, although the best model is not the same for the two countries. These results form a new and interesting finding about the Fisher relation that would inspire further investigation of related issues.

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