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Traveling Salesman Problem with a Drone Station

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Abstract

Traveling Salesman Problem with a Drone Station

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The importance of drone delivery services is increasing. However, the operational aspects of drone delivery services have not been studied extensively. Specifically, with respect to truck-drone systems, researchers have not given sufficient attention to drone facilities because of the limited drone flight range around a distribution center. In this paper, we propose a truck-drone system to overcome the flight-range limitation. We define a drone station as the facility where drones and charging devices are stored, usually far away from the package distribution center. The traveling salesman problem with a drone station (TSP-DS) is developed based on mixed integer programming. Fundamental features of the TSP-DS are analyzed and route distortion is defined. We show that the model can be divided into independent traveling salesman and parallel identical machine scheduling problems for which we derive two solution approaches. Computational experiments with randomly generated instances show the characteristics of the TSP-DS and suggest that our decomposition approaches effectively deal with TSP-DS complexity problems.

keywords : Drone delivery, Truck-drone service, Drone station, Mixed integer programming

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Chapter 1. Introduction

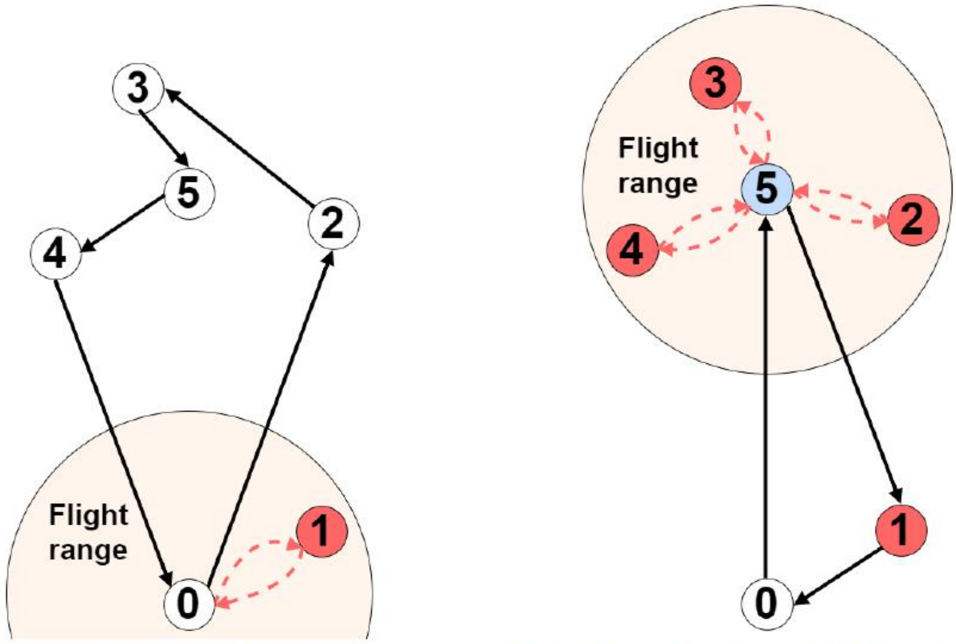
Growing e-commerce and m-commerce increases the importance of efficient logistics. In 2013, Amazon announced drone technology as a future logistic innovation, and many companies have invested into drone research. For example, Amazon unveiled *Amazon Prime Air*, and Google announced *Project Wing* (Grothas 2016, Muoio 2016). Drones have many advantages over the typical truck delivery system (Agatz et al. 2016, Wohlsen 2014). As drones operate independently, they are free from operating labor costs and have relatively unlimited working time. Further, they move through the air and thus avoid the traffic congestion problems of ground transportation. These advantages lead to the highly energy-efficient use of drones. Moreover, the transportation cost per kilometer is much lower than that of other means. However, because of technological limitations, a drone can carry only one parcel of limited weight and volume, and it can deliver to a single customer within a short flight range. To overcome these limitations, drone and truck delivery services can be used such that the characteristics of one complement the other. To demonstrate the combined means of delivery, the HorseFly team at the University of Cincinnati developed a system in which a drone can attach to and launch from a truck (Wohlsen 2014).

The technology needs further development to overcome some realistic problems, and battery capacity is a main concern for drone utilization. As many distribution centers with drone facilities are located far from central cities, relatively few customers are serviceable by drones. For this reason, large retail companies such as Amazon strive to build more distribution centers near major cities, but the expenses of constructing distribution centers are still a huge obstacle to completion. To deal with this logistical problem, a different concept of drone facilities is proposed. Roblin (2015) introduced *Pylons Dronairports*, which contain drone recharge and shelter devices. Designed by Bruni and Sardo, these compact devices can be easily installed any place. In addition, Amazon plans to use street lights and church steeples as drone docking stations (Mogg 2016). Another problem is that the weight and volume capacities of drones are not enough to accommodate commercial delivery services (Gross 2013).

Because many researchers and companies have tried to overcome these problems, some companies have been able to utilize drones for commercial purposes. In contrast, research on the operational aspects of drone delivery has been neglected, and only a handful of papers in drone-truck systems have been presented. One of the initial papers about the traveling salesman problem (TSP) in tandem with drones was conducted by Murray and Chu (2015), who described two different models. The flying sidekick TSP (FSTSP) describes the way a single drone is used with a truck. A drone is attached onto the truck, and a truck driver launches the drone and also retrieves it. The other model is the parallel drone scheduling TSP (PDSTSP) and is the key reference for this paper. Unlike the FSTSP, the PDSTSP can utilize a sufficiently large number of drones. However, drones deliver parcels only within the flight range of the distribution center such that problems arise when the distribution center is far away from a majority of customers.

To overcome the limitations of the PDSTSP, we developed the traveling salesman problem with a drone station (TSP-DS), through which we exploit a drone station, defined as a facility that stores drones and charging devices. The station is ready to launch drones that is, it is “activated” after a truck supplies parcels for drone delivery. We assume that the station can furnish a sufficiently large number of drones and that the location of the station does not depend on that of the distribution center. Specifically, the drone station is located near customer areas and away from the distribution center. The facility can deliver parcels using drones after a truck supplies the deliverables to the drone station, and a truck and a drone station operate independent of the distribution center after the truck supplies parcels for drone delivery. Figure 1 depicts the difference between the PDSTSP and TSP-DS.

We first analyze the fundamental features of the TSP-DS. We define route distortion, and the number of drones to eliminate route distortion is presented. By applying the assumptions of a sufficient number of drones and by considering the distance between the distribution center and a drone station is far enough, we show that the TSP-DS can be divided into the traveling salesman problem (TSP) and the parallel identical machine scheduling problem (PMS). Through this approach, we successfully reduce the complexity of the problem, and obtain the exact solution.



(a) Example of the PDSTSP (b) Example of the TSP-DS

Figure 1. Comparison of the PDSTSP and TSP-DS (red circle: drone-serviceable customer, white circle: truck-only customer, blue circle: a drone station).

The remainder of this paper is composed as follows: Section 2 introduces previous research related to truck-drone systems. Section 3 describes the TSP-DS. Fundamental features of the TSP-DS is presented in Section 4. Section 5 shows the analyses of computational results and discussion, and Section 6 presents conclusions.

2. Literature review

The TSP-DS is one variation of the TSP and the vehicle routing problem. A recent review of the TSP was offered by Applegate et al. (2011) and a review of multiple TSP problems was written by Bektas (2006). Other excellent overviews of the vehicle routing problem were provided by Golden et al. (2008) and Toth and Vigo (2014). Our proposed model is also related to the PMS. Allahverdi et al. (2008), Ruiz and Vazquez-Rodriguez (2010), and Baker and Trietsch (2013) summarized studies of the PMS.

A drone station can operate drones after a truck arrives and supplies parcels. This characteristic is closely related to the PMS with precedence constraints. Tanaka and Sato (2013) studied a single machine scheduling problem with precedence constraints. The objective was to minimize total job completion time, and job idle time was not permitted. A successive sublimation dynamic programming method was applied to find the exact solution. Bilyk et al. (2014) defined a batch scheduling problem with precedence constraints. Identical machines were assumed, and ready time for each job was considered. A variable neighborhood search and a greedy randomized adaptive search procedure were applied to solve the problem. Davari et al. (2016) solved a single machine scheduling problem with time windows and precedence constraints. A branch-and-bound algorithm was proposed to solve the problem. Hassan et al. (2016) studied a PMS with precedence constraints to minimize the makespan. Three valid inequalities were proposed, and their strengths were checked by computational experiments. Nicosia and Pacifici (2016) addressed a multiple machine scheduling problem with precedence constraints. A heuristic method related to the bin packing problem was developed, and a lower bound was proposed. Because traditional studies did not exploit drones, we concentrate on the drone-truck models in this study.

Murray and Chu (2015) offered one of the earliest studies of truck-drone delivery problems and introduced two fundamental models. First, the PDSTSP described a drone facility within a distribution center. To our knowledge, it is the only model in which a drone facility is considered in truck-drone problems. A sufficiently large number of drones can be utilized at the distribution center, but the limited flight range creates practical issues. To alleviate this problem, the FSTSP was developed to describe a truck driver launching and retrieving a drone. This model overcomes the flight range limitation from the distribution center of the PDSTSP, but it only applies to a single drone. Our research is directly related to the PDSTSP and serves as a complementary model applicable to a drone facility separated from the distribution center. To solve the PDSTSP, Murray and Chu (2015) developed a heuristic method based on decomposition of the model into the TSP and PMS. We also used the similar decomposition approach; however, our approach focused on the conditions on the decomposition which guarantees the optimal solution.

Although we take into account a drone facility problem with a truck TSP, a majority of research has concentrated on truck-launch delivery problems, which are intricately related to the FSTSP. In a related study, Agatz et al. (2016) assumed that drones and a truck share the same road network, which allowed them to find the worst-case approximation ratios for the heuristics. However, the assumption fails to take advantage of the drones' capacity to freely move off truck paths and remain unaffected by road conditions.

Ha et al. (2015) introduced the TSP with a drone. They assumed that launching and retrieving a drone is impossible at the same customer node. The mathematical formulation and two heuristic algorithms were developed. Mathew et al. (2015) described the heterogeneous delivery problem by considering a team using a truck and drones with complementary capabilities based on the assumption that drone-serviceable customers can only receive deliveries by drones. The problem can be reduced to the generalized traveling problem, which can be solved with many heuristics methods. In addition, they defined the multiple warehouse delivery problem by showing a special case of the heterogeneous delivery problem and developing two heuristic approaches. Ferrandez et al. (2016) compared the overall travel times and energy consumption of truck-only and truck-drone tandem deliveries. They proposed a clustering-first and routing-second approach. K-means algorithm, used to find an efficient launch location of drones, and genetic algorithms were applied to solve a truck-routing problem.

We introduce several studies not directly related to a truck-drone delivery service; however, these works also showed solutions to drone related problems. Boone et al. (2015) introduced the multiple TSP (MTSP) which can be applied to the drone swarm route plan. They divided the MTSP into two components: clustering and TSP problems. The K-means clustering method was applied to divide cities into multiple clusters, and each drone was allocated to each cluster. A constructive heuristic approach, called 2-opt, was applied to solve the TSP in each cluster. This approach helped reduce significant computation time. Dorling et al. (2017) developed the vehicle routing problem for drone delivery services by deriving an approximated and linearized cost function that accounts for the energy consumption model of multiple drones and by developing mixed integer based programming for the problem. Further, Dorling et al. (2107) built a string-based simulated annealing heuristic. A drone system in an indoor environment was introduced by Khosiawan and Nielsen (2016). The system focused on a scheduling issue, and a system architecture for drone applications in an indoor environment was developed. Furthermore, a framework of scheduler component was presented.

3. Truck-drone Routing Problem

The TSP-DS is an extension of the PDSTSP with the major difference in the location of off-duty drones. In the TSP-DS, drones are stored in and launched from a drone station, not the package distribution center. A drone station can store and utilize a sufficiently large number of drones that deliver drone-fitting parcels with a limited flight range. A large number of drones seems to be vague, therefore, we presented the number of drones which guarantees the minimum makespan of the total delivery time in a latter section. After a truck arrives at the station, drone-fitting parcels are processed for drone delivery and the station is said to be activated. We assume that the location of the station is relatively far from the distribution center; a drone station is farther than the maximum flight distance of a drone launched from the distribution center. Although the decision where to build a drone station can be an important issue, the location of the drone station is assumed to be given. The reason for this assumption is that the location problem should be solved based on the long-term perspective while our topic mainly focuses on the daily delivery service.

Because of safety and weight issues, a single drone cannot carry multiple parcels. Therefore, a drone visits only one customer per sortie while a truck can visit multiple customers in one trip. In addition, some customers order products that exceed the volume and weight capacities of drones. The limited flight range is due to the capacity of drone batteries. We assume that the travel time of vehicles are proportional to distances and drones are faster than a truck because the drones cross air space and the truck must follow ground routes. Because charged batteries are supplied from a drone station, battery charging times for retuned drones are not considered. A truck or a drone delivers an order only once to a customer.

Travel times between nodes are assumed to be symmetric. The truck departs from the distribution center and returns to it after packages are delivered. Likewise, drones return to the station after delivering parcels. The delivery service is considered ended when a truck returns to the distribution center and all drones return to their drone station. We define the last delivery time as the time to finish the total delivery service. The objective of the TSP-DS is to minimize the last delivery time.

3.1 Notation

We regard each customer as a single node and make a network with $N=\{1,...,c\}$ as a node set of customers and $s \in N$ as a drone station node index. In a customer network, we add the distribution center node. We define 0 as the index of the distribution center, and to avoid symmetric problems, we define $c+1$ as the index of the distribution center node for returns. We also define origin set $N_0 = \{0,...,c\}$ and destination set $N_1 = \{1,...,c+1\}$. Multiple drones are located in a drone station, and a set of drones is defined as V.

Customers are sorted by their package information. Weights, volumes, and distances from the drone station are considered to distinguish drone-serviceable customers. We define D as a set of drone-serviceable customers, which is a subset of N. The travel time of a truck between a pair of nodes $(i,j)\,(\forall\, i \in N_0, j \in N_1)$ is defined as $\tau_{i,j}$ and that of the drones is defined as $\tau^d_{i,j}$ $(\forall\, i \in N_0, j \in N_1)$. The binary decision variable $x_{i,j}$ equals 1 if the truck travels from node $i \in N_0$ to node $j \in \{N_1 : j \neq i\}$; it is 0 otherwise. Similarly, the decision variable $x^s_{i,j}$ is defined for the route of a truck until it arrives at a drone station. The binary decision variable $y_{i,v}$ is 1 if customer $i \in D$ is served by drone $v \in V$ launched from a drone station. Variable z refers to the last possible delivery time of a truck and drones. u_i indicates the position of node $i \in N_1$ in the truck's path.

3.2 Mathematical formulation

We can formulate the TSP-DS as follows:

$$Minimize \ \ z \tag{1}$$

$$subject \ to$$

$$\sum_{\substack{i \in N \\ i \neq j}} x_{ij} + \sum_{\substack{v \in V \\ j \in D}} y_{jv} = 1, \forall j \in N \tag{2}$$

$$x^s_{i,j} \leq x_{i,j}, \forall i \in N_0, j \in N_1 \tag{3}$$

$$\sum_{j \in N_2} x^s_{i,j} - \sum_{j \in N_2} x^s_{j,i} = \begin{cases} 1 & \text{if } i = 0 \\ -1 & \text{if } i = s \\ 0 & otherwise \end{cases}, \forall i \in N_0 \cup \{c+1\} \tag{4}$$

$$z \geq \sum_{i \in N_0} \sum_{j \in N_1} \tau_{i,j} x^s_{i,j} + \sum_{i \in D} (\tau^d_{s,i} + \tau^d_{i,s}) y_{i,v}, \ \ v \in V \tag{5}$$

$$z \geq \sum_{i \in N_0} \sum_{\substack{j \in N_1 \\ i \neq j}} \tau_{i,j} x_{i,j} \tag{6}$$

$$\sum_{j \in N_1} x_{0,j} = 1 \tag{7}$$

$$\sum_{i \in N_0} x_{i,c+1} = 1 \tag{8}$$

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{i,j} = \sum_{\substack{i \in N_1 \\ k \neq j}} x_{j,k}, \ \forall j \in N \tag{9}$$

$$u_i - u_j + 1 \leq c + 2\,(1 - x_{i,j}), \ \ \forall i \in N, j \in \{N_1 : j \neq i\} \tag{10}$$

$$1 \leq u_i \leq c+2, \ \forall i \in N_1 \tag{11}$$

$$x_{i,j} \in \{0,1\}, \ \ \forall i \in N_0, j \in \{N_1 : j \neq i\} \tag{12}$$

$$x^s_{i,j} \in R_+, \ \ \forall i \in N_0, j \in \{N_1 : j \neq i\} \tag{13}$$

$$y_{i,v} \in \{0,1\}, \ \ \forall i \in D, v \in V \tag{14}$$

$$u_i \in R_+ \tag{15}$$

The objective function (1) minimizes the delivery time of a truck and drones. Constraint (2) suggests that neither a truck nor a drone can deliver the parcel to a customer more than once. Constraint (3) ensures that $x^s_{i,j}$ follows the path of $x_{i,j}$, and Constraint (4) restricts the route of a truck until it arrives at a drone station. Constraint (5) imposes the criterion that z is greater than or equal to the last delivery time of drone $v \in V$ launched from a drone station. Constraint (6) restricts that z should not less than the last delivery time of a truck. Constraints (7), (8), and (9) specify the flow of the truck. Constraint (7) means that a single truck leaves the distribution center and Constraint (8) denotes that the truck must return to the distribution center. Constraint (9) ensures that the truck leaves customer $j \in N$ to deliver parcels after it arrives to customer node $j \in N$ from customer node $j \in N_0$. Subtours of the truck are eliminated by Constraints (10) and (11). Constraints (12), (13), (14), and (15) define the decision variables.

4. Fundamental features of the TSP-DS

In the TSP-DS, a loaded truck reaches a drone station, and activates the drone delivery process. This activation condition of a drone station has important features, and we demonstrate the main characteristics of the TSP-DS in this section.

Proposition 1.

The activation time of a drone station is always less than or equal to $z/2$.

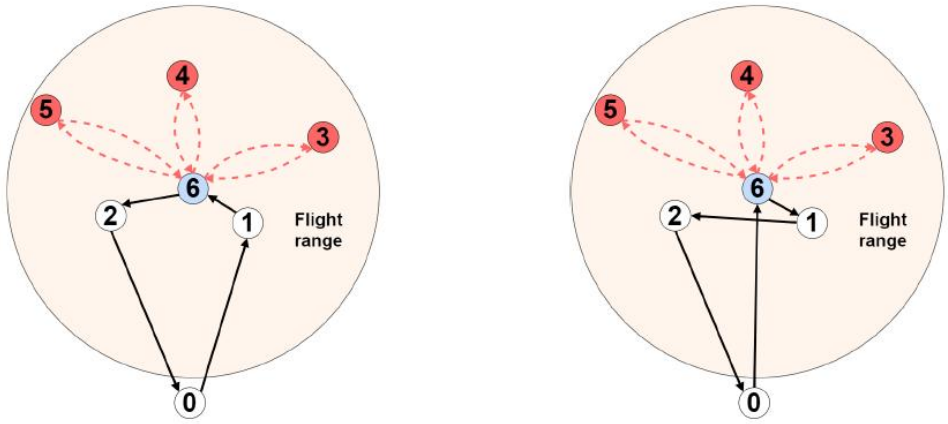
Proof

The travel time matrix of a truck is symmetric, and the total distance of a truck does not change when the travel direction of the truck is reversed on the route. For this reason, when the activation time of a drone station is greater than $z/2$, a truck can be chosen to the same travel route with the reverse direction which activates the drone station before $z/2$. $\square$

4.1 Route distortion

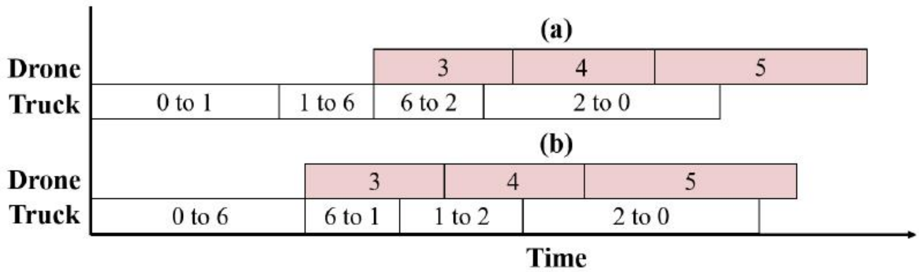
Generally, a drone station is used to maximize the use of drones, and a truck is used on the shortest routes. However, in some cases, a truck driver takes a longer route to activate a drone station earlier which results in the overall reduction in the objective value. Because the proposed model searches the optimal schedule of the global truck-drone system, in which a drone station and a truck interact, we define this case as *route distortion*. In analyzing the fundamental features of the drone–truck system, we do not take into consideration two assumptions: the sufficiently large number of drones in a drone station and the minimum distance between the distribution center and the drone station.

There are two types of the route distortions. In one, a truck delivers parcels to customers who can be serviced by drones. This happens when the last delivery time of a drone is later than that of a truck. In this case, use of a truck to deliver to drone-serviceable customers is more efficient. In the other route distortion case, a truck uses long delivery routes to arrive at a drone station early. A driver would make this decision because drones are only able to deliver to customers after a truck supplies parcels to the station, and thus, an early activation time means an early delivery time by drones (Figure 2).



(a) Optimal schedule of a truck

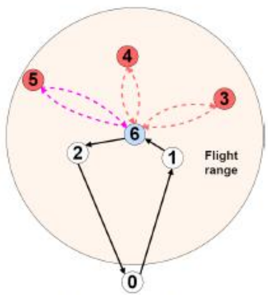
(b) Optimal schedule of a truck and a drone



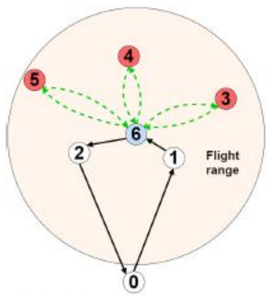
(c) Comparison of schedules between (a) and (b)

Figure 2. The routes of a truck can be influenced by a drone station.

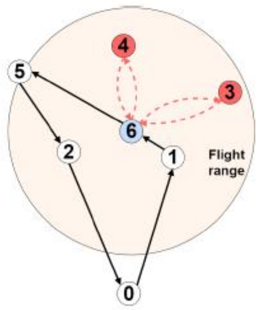
When N is given, three factors affect route distortions. The main factors correspond to the number of drones in a station as well as the velocities and flight ranges of drones (the number of drone-serviceable customers) (Figure 3). When the number of drones increases, a truck takes the shorter routes and the last delivery time from a drone station is earlier than that of the shortest truck routes; further, early activation of a drone station is unnecessary when many drones are available. Likewise, faster drones affect the best route choices for a truck. Decreasing the flight range or the number of potential drone-serviceable customers also offers the same result that a drone station needs not to be activated in the early stage of delivery service. In addition, when the flight range is decreased, one of customers might not be considered a drone-serviceable customer. In this case, the last delivery time from a drone station is earlier than that of a truck from a distribution center. These cases show that the factors related to the drone station workload affect the truck route.



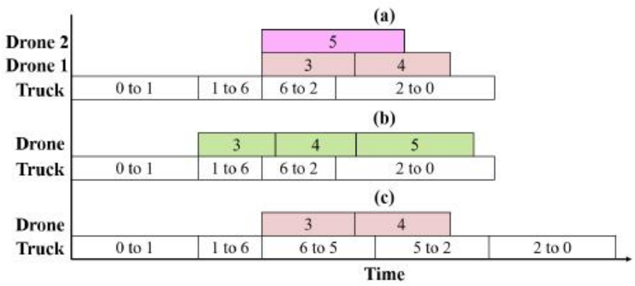
(a) Optimal schedule of a truck and two drones



(b) Optimal schedule of a truck and a high-speed drone



(c) Optimal schedule of a truck and a drone with a smaller flight range



(d) Comparison of schedules between (a), (b), and (c)

Figure 3. Three factors can affect the routes.

4.2 Conditions for the elimination of route distortion

Based on the assumption that a sufficient number of drones is available in a station, we can draw the inequality that eliminates the route distortion.

Proposition 2.

Let $c_{\max}$ be the farthest drone-serviceable customer from the drone station s and α be the travel rate of the drone speed to the truck speed. If the number of drones is sufficient and the problem satisfies $\tau_{s,0} \geq 2\,\tau_{s,c_{\max}}/\alpha$, drones can finish parcel deliveries to all drone-serviceable customers before the truck returns to the distribution center.

Proof

When the number of drones in a station is sufficient, each drone can deliver a parcel to a single customer. In this case, the upper bound for the flight time of a drone from the station (UB_d) is the delivery time of a drone to $c_{\max}$. As the travel time matrix of a truck is symmetric, $\tau_{c_{\max},s} = \tau_{s,c_{\max}}$ and $UB_d = (\tau_{c_{\max},s} + \tau_{s,c_{\max}})/\alpha = 2\,\tau_{s,c_{\max}}/\alpha$. The lower bound of the truck travel time LB_t to return to the distribution center after leaving a drone station s is $\tau_{s,0}$. Therefore, if UB_d is less than LB_t, the last delivery time from a drone station s can be earlier than or the same as the delivery time of the truck. □

In the real world, a sufficiently large number of drones is not needed, and the number of customers is the logical upper bound for drone inventory. However, when many drones are needed, and although we cannot find the minimum number before solving the problem, we can find the bound that likely allows for a sufficient number of drones for delivery services.

Proposition 3.

If the number of drones is $\left\lceil |N| / \left\lfloor \frac{\alpha\,\tau_{o,s}}{2\,\tau_{c_{\max},s}} \right\rfloor \right\rceil$, additional drones are not necessary to shorten the schedule.

Proof

In Proposition 2, UB_d is $2\,\tau_{s,c_{\max}}/\alpha$, and LB_t is $\tau_{s,0}$. Therefore, the lower bound of the maximum number of customers to which a drone can deliver before a truck returns to the distribution center is $\left\lfloor \frac{\alpha\,\tau_{0,s}}{2\,\tau_{c_{\max},s}} \right\rfloor$. The number of customers is $|N|$, and thus, the required number of drones is $\left\lceil |N| / \left\lfloor \frac{\alpha\,\tau_{o,s}}{2\,\tau_{c_{\max},s}} \right\rfloor \right\rceil$. □

Combining Propositions 2 and 3, we can define the following general condition.

Corollary 1

If the number of drones is more than $\left\lceil |N| / \left\lfloor \frac{\alpha\,\tau_{o,s}}{2\,\tau_{c_{\max},s}} \right\rfloor \right\rceil$ and the problem satisfies $\tau_{s,0} \geq 2\,\tau_{s,c_{\max}}/\alpha$, then the route distortion is eliminated.

4.3 Decomposition of the TSP-DS

For our problem, we address the situation in which the majority of customers are located far from the distribution center and the maximum flight distance of a drone from the distribution center is less than the distance between the drone station and the distribution center. It means $\tau_{0,s} \geq R$ (R is the diameter of the flight range). As $c_{\max}$ does not exceed the radius of the flight range, the following inequality holds: $\tau_{0,s} \geq R \geq 2\,\tau_{s,c_{\max}}$. As the drone velocity is the same or exceeds the speed of a truck, our problem always satisfies Proposition 2. we also assume that a drone station can utilize a sufficiently large number of drones, and this assumption satisfies Proposition 3. Therefore, the our problem fulfills the elimination condition of route distortion (Corollary 1).

When the problem satisfies conditions for Corollary 1, a drone station can successfully initiate delivery of all drone-compatible parcels, and a truck does not need to deliver parcels to any customer serviceable by drones. Because the route distortion was eliminated, the model can be divided into two independent problems. The first problem is the TSP through which one finds the shortest truck routes by considering only customers who cannot be serviced by drones. The second problem finds the drone station schedule that minimizes the last delivery time using drones. Because the objective value of the second problem is always less than or equal to the objective value of the first problem (Corollary 1), these two independent problems successfully solve the TSP-DS. We define these two problems as an independent traveling salesman and parallel machine scheduling problem (TSPMS).

However, in terms of a drone station operation, the PMS model can suggest an overuse of drones because the model is not designed to minimize them. Furtherly, it does not exploit the information from the solution of the TSP which provides the arrival time of a truck at the drone station. For this reason, a two-stage traveling salesman and modified parallel machine scheduling problem (TSMPMS) is developed to find a schedule that minimizes the number of drones used at a station by exploiting the solution of the TSP to set the drone station schedule. The first stage is the same as the ordinary TSP. After the TSP is solved, the activation time of a drone station a_s and the last delivery time of a truck z_t are known. As the problem satisfies Corollary 1, $z = z_t$ and the last delivery time of a drone station can be earlier or the same as z. This finding means the upper bound of the drone flight time UB_f is $z - a_s$. Reflecting this information, a modified PMS problem is solved to minimize the number of drones used under the upper bound of the flight time. The process to calculate UB_f is described in Algorithm 1.

Algorithm 1. Calculation for UB_f

```
Initialization : start_node, arrival_station, UB_f = 0
While{i ∈ N_0 − N_d}
{
   While{j ∈ N_0 − N_d}
   {
    if x_{start_node,j} = 1 then
       arrival_station += τ_{start_node,j};
       start_node = j;
       break;
    end-if
   }
   if(start_node = s) then break;
}

if(arrival_station > z/2) then
   UB_f = arrival_station;
else
   UB_f = z − arrival_station;
end-if;
```

The start node (start_node) is initialized as 0 node. The activation time of a drone at a station (active_time) and the upper bound of the flight time (UB_f) are set as 0. The algorithm finds the next node from the start node. When the next node j is found, the activation time and the new start node is updated. The algorithm repeats until the new start node is s. After the activation time is fully updated, the upper bound of the flight time is calculated. Because the problem satisfies Proposition 1, UB_f can be always greater than or equal to $z/2$.

After UB_f is calculated, we can find the schedule of a drone station that utilizes the minimum number of drones without changing the last delivery time. We define a new binary variable d_v; it is 1 if drone $v \in V$ is used for the delivery and 0 otherwise. The mathematical formulation of the modified PMS is as follows:

$$\mathit{Minimize} \sum_{v \in V} d_v \tag{16}$$

subject to

$$\sum_{i \in D} (\tau^d_{s,i} + \tau^d_{i,s})\, y_{i,v} \le UB_f\, d_v, \forall\, v \in V \tag{17}$$

$$\sum_{v \in V} y_{j,v} = 1, \forall\, j \in D \tag{18}$$

$$y_{j,v} \in \{0,1\}, \forall\, j \in D, v \in V \tag{19}$$

$$d_v \in \{0,1\}, \forall\, v \in V \tag{20}$$

The objective function (16) minimizes the required number of drones for delivery service. Constraint (17) suggests that d_v is 1 when drone v is used and the flight time of a drone so it does not exceed the upper bound of the flight time. Constraint (18) shows that each customer serviceable by drones should receive deliveries by a drone. Constraints (19) and (20) define decision variables.

5. Computational experiments

Results of computational experiments and the insight of the developed model are presented in this section. The models were built in XPRESS-IVE 7.9 with the XPRESS-MP mathematical programming solver. Experiments were conducted with an Intel(R) Core(TM) i5-3570 CPU 3.4 GHz with 8.00 GB of RAM in Windows 10.

According to Murray and Chu (2015), the flight range of a commercial drone was approximately 16 km ($\simeq$10 miles). Therefore, we assumed that a circle with 16 km radius is a feasible flight region. To compare the PDSTSP and the TSP-DS, we set two different flight areas. The feasible flight area from drone station s was defined as Region A while that from the distribution center was defined Region B. To avoid overlapping feasible flight regions, we made a gap between them. As a result, the experiments were conducted in a square region of 32 km $\times$ 65 km (Figure 4).

Due to the probabilistic nature of parcel ordering, customers were assigned randomly to specific locations. Furthermore, to concentrate on the effect of a drone station on delivery, we only considered small and light parcels that can be delivered by drones. For this reason, if customers were located in the flight-feasible region, they were assumed to be drone-serviceable customers. When we solved the TSP-DS, customers located in Region A were classified as drone-serviceable customers but others were considered truck-only customers. However, in the PDSTSP, customers in Region B could be serviced by drones while those in Region A could not be serviced by drones. In addition to this, customer locations were restricted to Regions A and B.

The number of drones in a station was calculated using the bound of drones needed to satisfy Proposition 3. In detail, the radius of Region A was 16 km and the distance between the drone station and the distribution center was 33 km. When the travel rate α was set as 2, a drone could deliver parcels to at least two customers before a truck at the drone station returns to the distribution center. Therefore, the minimum number of drones to satisfy the condition for Proposition 3 was no more than $\lceil |N|/2 \rceil$.

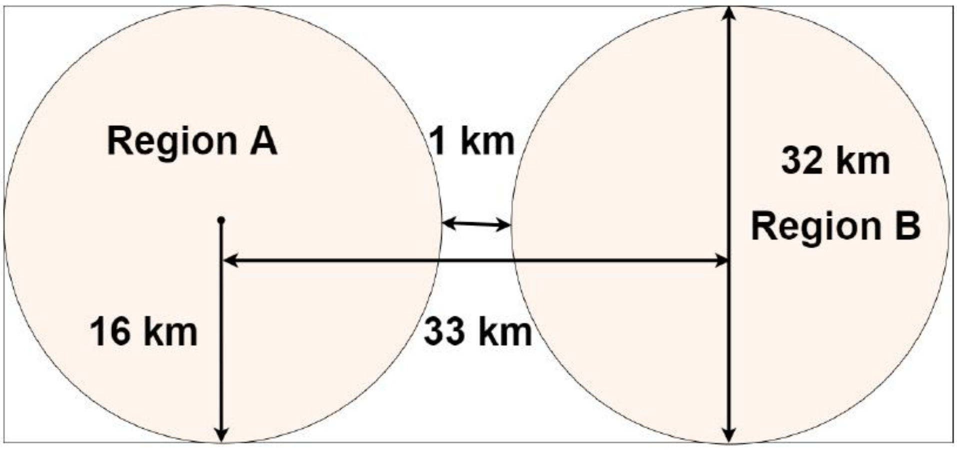


Figure 4. Experimental design.

5.1 Computation times

Two data sets were generated to evaluate the computation times of the models. A small data set was used to compare the performance between the TSP-DS and other models. Due to the complexity characteristic of the TSP-DS, the number of customers was increased from 7 to 11. A large data set was generated to evaluate the performances of the other models, and the number of customers was increased from 20 to 80. In each customer set, 10 random instances were generated. The travel rate α was fixed at 2. We stopped the experiment of each model when it took over 1,800 seconds. The detailed information of the experiments and results are shown in Table 1.

Although three models gave the same objective value, the computation times were distinct between them. With fewer customers, the gaps were small. However, the computation times of the TSP-DS were much greater for the data set with more customers. Although the computation time increases were relatively small, the TSPMS and TSMPMS models were also not free from increased computation times. The computation time difference between the TSPMS and the TSMPMS was negligible in the small problems. However, in the large problems, the TSMPMS was much faster than the TSPMS. The gaps between the computations were increased according to the size of the problem because the second-stage problem of the TSMPMS used bounds derived from the first-stage model while the TSPMS solved two problems independently.

Table 1. Average computation times (seconds) of the TSP-DS, TSPMS, and TSMPMS with respect to the number of customers in the experiment.

Size	Node	TSP-DS	TSPMS	TSMPMS
Small	7	0.255	0.018	0.007
	8	0.647	0.032	0.019
	9	1.598	0.029	0.015
	10	5.360	0.040	0.016
	11	14.021	0.042	0.020
Large	20	-	0.220	0.083
	30	-	0.592	0.397
	40	-	-	0.730
	50	-	-	0.910
	60	-	-	1.978
	70	-	-	9.046
	80	-	-	25.587

To analyse the reason why there are large differences in computation times between models, upper and lower bounds of the models were checked. Solutions of the models were analyzed in 20 and 40 nodes instances. 4 instances were selected among which instances computation times were over 1,000 seconds in each node set.

Optimality gaps between upper and lower bounds and computation times of the two models were summarized in Table 2. In the case of the TSP-DS, both upper and lower bounds improved at the first time; however, upper bounds were not improved later. The optimality gaps were between 14 % to 24 % and they were not tight. Although the number of nodes was small, it took tremendous times to get optimality gaps within 20 %. In the case of the TSPMS, optimality gaps decreased at the initial stage of solving the problem. However, it took a lot of time to find better bounds after the gaps were within 5 %. In general, the PMS part of the TSPMS was the bottleneck to solve the problem. Although the commercial solver could find optimal solutions within a short time, it took enormous times to prove these solutions were optimal. In other words, most of the times were consumed to improve lower bounds. Compared to the TSPMS, the TSMPSM had advantages to solve the PMS which resulted in better computational performances.

Table 2. Optimality gaps between upper and lower bounds and computation times of the TSP-DS and TSPMS for 20 and 40 nodes, respectively.

Model	TSP-DS (20 nodes)		TSPMS (40 nodes)	
Instance	optimality gaps (%)	Computation times (s)	optimality gaps (%)	Computation times (s)
1	14.59	1800.00	0.00	1080.07
2	23.71	1800.00	4.42	1800.00
3	14.16	1800.00	0.98	1800.00
4	22.06	1800.00	2.20	1800.00

5.2 Comparison between the TSP-DS and TSP

We considered the case in which more than one-half of customers are near drone station s. To analyze the characteristics mentioned in Section 4, we conducted experiments by varying the number of customers in Region A, the number of drones, and travel rate α. The total number of customers was set at 10. In each case, 10 experiments were conducted, and the savings between the objective values of the TSP-DS (PDSTSP) and the ordinary TSP was found. Each saving was calculated as (the objective value of the TSP - that of the TSP-DS (PDSTSP)) / the objective value of the TSP. The detailed environment setting and results were shown in Table 3. The results showed that when the number of drones was increased from 1 to 3, the delivery rates were not appreciably changed and the route distortions did not happen.

Table 3. Average savings (%) of the optimal value between the TSP-DS (PDSTSP) and the TSP with respect to the number of customers in Region A, the number of drones, and travel rates.

Number of customers	Number of drones	Travel rate 1.5	2	2.5
6	1	14.25(9.44)	15.48(10.80)	15.54(12.10)
	2	15.54(12.48)	15.54(12.75)	15.54(12.75)
	3	15.54(12.75)	15.54(12.75)	15.54(12.75)
7	1	15.38(7.95)	16.54(10.13)	17.25(11.17)
	2	17.66(11.27)	17.83(11.66)	17.83(11.66)
	3	17.83(11.65)	17.83(11.66)	17.83(11.66)
8	1	19.26(6.35)	22.43(6.71)	25.66(7.01)
	2	27.13(7.01)	28.13(7.01)	28.15(7.01)
	3	28.13(7.01)	28.15(7.01)	28.15(7.01)
9	1	22.85(5.43)	27.21(5.53)	29.69(5.53)
	2	31.02(5.53)	32.33(5.53)	33.28(5.53)
	3	32.81(5.53)	33.34(5.53)	33.34(5.53)

The objective value of the TSP was much later than those of the TSP-DS and PDSTSP, which justifies use of the truck-drone system.

Moreover, the objective value of the TSP-DS was lower than that of the PDSTSP, and the maximum saving of the optimal value was increased according to the number of customers in Region A. It strengthens our argument that utilizing a drone station helps make the last delivery time earlier when the distribution center is far away from a majority of customers. Likewise, the increasing number of drones or increased travel rate α enlarged the saving because releasing the burden of the drone alleviated the burden created by inefficient truck routes.

When the same number of drones was used, the gap was smaller when few customers were in Region A. When 6 customers were in Region A and travel rate α was 1.5, two drones were sufficient to avoid route distortion. However, more than three drones were needed at the same travel rate to serve 9 customers to avoid route distortion. It can be observed that the number of drones in a station has significant impact on the truck route. Because the generated examples satisfied the distance condition of Proposition 3, increasing the number of drones corresponded to the shortened truck route.

5.3 Number of drones in a drone station

To analyze the relationship between the number of customers and number of drones used in the station without route distortion, we calculated the minimum number of drones in the TSMPMS. We varied the number of customers from 10 to 50 to check the trend. Because the ratio of the number of customers serviceable by drones

/ the total number of customers can affect the required number of drones, we varied this ratio from 0.6 to 0.9, and 10 experiments were performed for each ratio. Therefore, 40 experiments were conducted for each number of customer groups. The average and maximum number of drones for each customer group were derived. Moreover, ratio β showed the average number of drones used / total number of customers. Similarly, γ (the maximum number of drones used / total number of customers) was defined. The details of the experiments and results are shown in Table 4.

The average number of drones used was much less than the upper bound derived from Proposition 3. The maximum number of drones used was also smaller than the upper bound. The maximum number of drones was approximately twice the average. Ratios β and γ decreased for increasing number of customers. The decreasing rate of β was higher than γ.

Table 4. Average, maximum, and upper bound for the number of drones used in a station, and ratios β and γ.

Number of customers	Avg	Max	UB	β	γ
10	1.95	3	5	0.20	0.30
20	2.95	6	10	0.15	0.30
30	3.93	7	15	0.13	0.23
40	5.1	10	20	0.13	0.25
50	5.55	12	25	0.11	0.24

5.4 Discussion

We analyzed the flight range (the number of customers serviceable by drones), the velocity of drones, and the number of drones as main factors affecting the route distortion. However, in a realistic-world problem, the drone range and velocity are difficult to control because of safety issues and limited technologies. Fortunately, increasing the number of drones is relatively easy because the sufficient number of drones can be utilized at a drone station which leads to elimination of route distortion.

The other interesting point is that the required number of drones to eliminate route distortion is relatively small. The required number of drones are less than one-third of customers. Moreover, ratio β is negatively affected by the number of customers because increasing the number of customers leads drones to offer more options to deliver parcels to customers in the drone-service area. Therefore, drones deliver more parcels in a given time period if the number of customers increases.

6. Conclusions

We defined a new drone and truck-drone TSP by exploring use of a drone station with three features; 1) It can utilize many drones; 2) it is located far away from the distribution center; and 3) it is activated for delivery after a truck arrives with parcels. The TSP-DS was formulated based on the mixed integer programming and we analyzed characteristics of the TSP-DS. We proved that the mathematical model can be divided into two different mathematical models, and derived the TSPMS and the TSMPMS to give the exact solution of the TSP-DS. Computational experiments showed that the fundamental characteristics of the TSP-DS and the TSMPMS could effectively reduce the complexity problem. Another experiments revealed that the TSP-DS is more effective than the PDSTSP when a majority of customers are located far from the distribution center. We also showed that route distortion can be eliminated with relatively small number of drones. We expect our model can be used as a means to overcome the limits of drone facility problems, and it can be used to establish drone-truck delivery systems in the near future.

In this problem, we assumed that the locations of customers, a drone station, and the distribution center are given, and the results show that the distance between a drone station and the distribution center is an important factor. Therefore, the location problem of a drone station is an extended topic of our problem. Consideration of multiple drone stations may also inform future research. When some of the flight ranges of each drone station overlap, drones could freely move to each station, which would improve the utilization rates of drones.

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초 록

드론을 활용한 서비스의 수요는 계속해서 증가하고 있다. 그러나 드론을 활용한 택배 서비스에서 운용적인 측면에 대한 연구는 제한적으로만 이루어지고 있다. 그중 트럭과 드론을 동시에 이용하는 트럭-드론 배송 시스템의 경우, 드론의 제한적인 가용 범위로 인해 드론 관련 시설을 이용한 운용 방법론에 대한 연구는 더욱 등한시 되고 있다. 본 연구에서는 기존 드론 가용 범위의 한계를 극복하기 위한 새로운 트럭-드론 시스템을 제안한다. 이를 위해 물류 센터와 독립적으로 운용되고 드론 저장 및 드론 충전 설비를 갖춘 시설을 드론 정거장으로 정의한다. 본 연구는 드론 정거장을 활용한 외판원 문제 (TSP-DS)를 제시하고 본질적인 특성을 분석한다. 그리고 TSP-DS가 독립적인 외판원 문제와 평행 머신 스케줄링으로 분해가 가능한 것을 보인다. 실험 결과 및 분석을 통해 TSP-DS의 특징을 재확인할 수 있고, 본 연구에서 제안하는 분해 방법이 효과적으로 TSP-DS의 문제 복잡도를 낮출 수 있음을 보인다.

keywords : Drone delivery, Truck-drone service, Drone station, Mixed integer programming

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