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공학박사 학위논문

Empirical Research on the Asymmetric Multifractal Properties in Financial Market Data

금융 시장 데이터의 비대칭 멀티프랙탈 특성에 대한 실증 연구

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이 민 혁

Empirical Research on the Asymmetric Multifractal Properties in Financial Market Data

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Abstract

Empirical Research on the Asymmetric Multifractal Properties in Financial Market Data

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After the recent financial crisis, the importance of financial market analysis for financial risk management has been emphasized. Financial markets have diverse characteristics that are difficult to explain from the traditional models. Therefore, the effort on describing such characteristics is required. Specifically, many researches are actively conducted on the features of multifractal and asymmetric correlation in financial markets. Multifractal features can be characterized by various fractal features with self-similarity that does not change with scale; it is difficult to represent in a single fractal dimension. This feature can explain the complexity of stock market. The asymmetric correlation, depending on the market trend, represents the asymmetric structure of the financial market. In this context, this dissertation focuses on the asymmetric correlation of multifractal characteristics in the financial market data where the asymmetric market efficiency is measured using asymmetric multifractal property. At first, ‘Price-based Asymmetric Multifractal Detrended Fluctuation Analysis (Price-based A-MFDFA)’ model is proposed to measure multifractal characteristics which asymmetrically follow the trend of market price. Given that previous models measure the multifractal characteristics based on the entire market, the price-based A-MFDFA model has its advantage by considering the asymmetrical characteristics according to different market conditions. Furthermore, the methods to investigate the cause of multifractal features and the asymmetry are also suggested based on the proposed model. The empirical results in the U.S. financial market data confirms the presence of asymmetric multifractal characteristic and the autocorrelation of the variance in uptrend market and fat-tailed distribution in downtrend market as the cause of multifractality. The results of time-varying asymmetric multifractality show that the difference between the degree of uptrend and downtrend multifractality increases during the financial crisis period. Secondly, a simulation method is applied to prove the ability of capturing the asymmetric multifractal features of the Price-based A-MFDFA model by examining the factors affecting the asymmetric multifractality. In order to mimic the stock market data, an artificial time series with asymmetric features are constructed using the Monte-Carlo simulation. Then, the asymmetric multifractality is observed for each time series using the proposed model. The results show that the proposed model can detect the artificial asymmetric characteristics. In addition, the effects of autocorrelation of time series, autocorrelation of volatility, the skewness and fat-tailed of distribution on the asymmetric long-range dependence and multifractal features are studied. Lastly, a framework for testing the existence of asymmetric long-range dependence and multifractality is proposed. The source of market inefficiency, which has not been identified in previous models, is examined through the uptrend and downtrend multifractal features. The result of thirty four countries suggests that, in the financial crisis period, the difference in the long-range dependence measure and degree of multifractality between uptrend and downtrend increases, whereas the uptrend degree of multifractality has a strong negative correlation with the stock price in financial crisis period. In addition, the relationship between asymmetric long-range dependence and rate of return is tested. In conclusion, the contribution of this dissertation is to further refine the ability of multifractal analysis on asymmetric characteristics in accordance with market conditions as well as the overall market. While past analysis of the overall market focuses on only the downtrend, it is possible to analyze both uptrend and downtrend market through the segmented asymmetric multifractal characteristics. Hence, the proposed model can provide much useful information to various market participants in the perspective of financial risk management.

Keywords: Multifractal, Generalized Hurst Exponent, Asymmetry, Market Efficiency, Financial Market Data, Simulation, Long-range Dependence

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Chapter 1

Introduction

1.1 Research motivation and purpose

Financial risk management is an important issue and has become more important from the global financial crisis in 2007. The past theories of financial risk management used simple time series models (i.e. Gaussian model) to analyze the financial market. However, a real financial market has many features that cannot be explained by the financial market. However, a real financial market has many features that cannot be explained by the traditional simple time series models. An example of such characteristics is a thick tail of the probability distribution of returns and the thicker asymmetry in the left tail portion of the negative return distribution. In addition, there is a jump phenomenon in which a stock price sharply moves, and a momentum phenomenon that continuously changes in the same direction. Furthermore, the volatility of returns shows heteroscedasticity and mean reversion phenomenon. Because of these characteristics, the Gaussian time series model does not explain the real financial market well, which has been remarkable during the global financial crisis. To cope with this limitation, econophysics theories are applied to explain the realities of financial market. Among the econophysics theories, multifractal feature is one of the characteristics that can be observed in financial time series data.

The multifractal property is a phenomenon in which a fractal dimension of multiple values exists simultaneously in a time series. If the fractal dimension is fixed to one value, then it is called monofractal. Multifractal time series are characterized by heavy-tail of probability distribution, volatility clustering, and long-term memory phenomena. In this context, the Hurst exponent is utilized for measuring the long-term memory, which can be a determinant of multifractality. The multifractal analysis of financial market has been applied to investigate stock market prediction (Selvaratnam and Kirley 2006; Eom et al. 2008; Eom et al. 2008; Domino 2011), market collapse prediction (Grech and Mazur 2004; Grech and Pamuła 2008), financial times series modeling (Tzouras et al. 2015), trading strategy (Dewandaru et al. 2015), market efficiency (Wang et al. 2009), and et cetera.

In recent years, the asymmetric correlations have become important in financial market research (Ding et al. 2011; Cao et al. 2014; Baruník et al. 2016). A study of asymmetric correlations in financial markets identifies the characteristics of asymmetric risks which can be applied to various areas including the risk management and diversified investment. Correlations between international markets are much greater for downside movements than the upside. This phenomenon makes the market crash even more dangerous. The portfolio also should be redistributed if the correlation changes with respect to the financial market situation.

There are many studies to analyze the entire stock market in the perspective of multifractal studies, but there are only few studies to analyze and compare the characteristics of each stock according to market conditions with multifractal theories. Since the asymmetric correlation of the stock market should be treated important as above, it is necessary to analyze the financial market separately in multifractal research as well as the entire market research. In addition, it is necessary to study the methodology that can be applied to various places using asymmetric features.

This dissertation provides the resolution by proposing the model named 'Price-based Asymmetric-Multifractal Detrended Fluctuation Analysis (Price-based A-MFDFA)', which can measure the asymmetric multifractal features. A stock market can be divided into a bull and bear market based on the price trend. Then, the multifractality measurement method is applied to each market asymmetrically. Once the model is proposed, the validity of proposed model is tested for the U.S. financial market data. Furthermore, the scaling asymmetries, source of the multifractality, source of asymmetry and time-varying multifractality features are investigated.

A simulation analysis is conducted to validate the proposed model. Using the Monte-Carlo simulation, an artificial time series are asymmetrically generated with various features that affect the long-range dependence and multifractality. Then, the asymmetric Hurst exponent and degree of multifractality for each time series are measured using the price-based A-MFDFA model. By comparing the time series and measured values, the efficacy of the proposed model and its effects to the asymmetric long-range dependence and multifractality are examined.

The efficient market hypothesis defines the information efficiency in the financial market, and it states that all past information is already reflected in the stock market. The stock market is theoretically unpredictable, but there is a long-range memory in the real stock market. A long-range memory phenomenon can be identified by measuring the generalized Hurst exponent. The previous multifractal theory can only present the overall market efficiency, and there are some limitations in recognizing the source of inefficiency. The source of inefficiency can be the false hope during the excessive bull market or the extreme fears during the crisis-phase bearish market. To challenge this limitation, the asymmetric long-range dependence and multifractality are measured for each trend using asymmetric generalized Hurst exponent based on the price-based A-MFDFA model. It explains the reason of market inefficiency by comparing the result of uptrend and downtrend multifractal properties such as long-range dependence and multifractality. The proposed test for the asymmetric long-range dependence and multifractality are applied to various countries; a moving-window method is used to investigate the effect of financial market crisis to the market asymmetric multifractal features.

1.2 Theoretical background

Fractal theory has appeared by defining fractal dimensions in fractal structures in Mandelbrot (1977). A fractal structure is a structure that has self-similarity and infinitely self-replicating. The self-similarity of the structure is applied to describe not only to the external features but also to the statistical properties. It implies that statistical structures of whole and part are similar or identical in fractal nature. ‘Monofractal’ is defined that fractal structure is persisted regardless of the size of the location and range. If the fractal structure is varied depending on the size of the location and range, this structure refers ‘Multifractal’. Fractal structures are found in various phenomena of reality, and fractal characteristics are observed in financial market data covered in this dissertation. The Brownian motion generated using the normal distribution, which is traditionally assumed in financial time series, has monofractal characteristics. However, various researches have shown that financial time series data have multifractal features since a multifractal model of asset returns was proposed in Mandelbrot et al. (1997).

Self-similar process is stochastic process that is invariant in distribution under scaling of time and space. The definition of self-similar process is as follows in Di Matteo (2007).

A random process $\{X(t)\}$ is called self-similar process if it satisfies

$$\{X(ct_1), \ldots, X(ct_k)\} \stackrel{\mathrm{d}}{=} \{c^H X(t_1), \ldots, c^H X(t_k)\}$$

for some $H > 0$ and all $c, k, t_1, .., t_q \geq 0$.

H is called the self-similar index, scaling exponent of the process or the Hurst exponent. If the random process is stationary, H should be a value between 0 and 1.The Hurst exponent is used for measuring the long-term memory of time series. This is related to the autocorrelation of time series and the rate of autocorrelation decreasing with time lag. When the Hurst exponent is over than 0.5, time series have a long-term positive autocorrelation. In the contrary, when the Hurst exponent is smaller than 0.5, time series is anti-persistent. In other words, if data is a positive number, next data and even long term data can be a positive number when the Hurst exponent is over 0.5. However, if the Hurst exponent is less than 0.5, next data and even long term data could be a negative number. It is likely that the sign of data will continue to be switched over a long period of time. When the Hurst exponent is 0.5, the time series is perfectly uncorrelated. There is no autocorrelation for all time lags theoretically. However, in real model, there is a small autocorrelation, but the absolute value of autocorrelation is exponentially decayed. The Hurst exponent follows this proportional equation.

$$E(|X_{t+\tau} - X_t|^2) \sim \tau^{2*H}$$

where τ is the time lag.

The generalized Hurst exponent, *H(q)*, is general form of the Hurst exponent. The generalized Hurst exponent follows below proportional equation.

$$E(|X_{t+\tau} - X_t|^q) \sim \tau^{q*H(q)}$$

where τ is the time lag.

When q is 1 or 2, the generalized Hurst exponent contains information about averaged volatilities at scale τ. When *H(q)* is a constant regardless of the change of *q*, the time series is a monofractal. If *H(q)* depends on q, then time series has a multifractal property. It implies that the structure of the fractal varies depending on the measurement conditions.

There have been various studies on the causes of the characteristics of multifractal. There are two main factors that affect multifractal. First one is autocorrelation of volatility, and second one is heavy tail shape of probability distribution. To investigate the effect of volatility autocorrelation on multifractal, many researches compared the time series with random shuffling time series that is eliminated autocorrelation of volatility (Kantelhardt et al. 2002; Matia et al. 2003; Jiang and Zhou 2008). In addition, they compared surrogate time series to examine the effect of fat tail on multifractality (Lim et al. 2007; Kumar and Deo 2009; Barunik et al. 2012; Grahovac and Leonenko 2014). There is more detailed explanation in chapter 2.4.3.

There are many methodologies to measure the scale exponent, the Hurst exponent. R/S (rescaled range analysis) method (Hurst 1951), generalized Hurst exponent method(Di Matteo et al. 2003) and MFDFA (multifractal detrended fluctuation analysis) method (Kantelhardt et al. 2002) are widely used methodologies. Recently, MFDFA method has been extensively studied to grasp the long-range correlation of nonstationary time series and multifractal features. The advantage of MFDFA model is that it is easy to implement and can make robust estimates for short time series data. MFDFA model is usually applied to investigate the multifractality of financial time series (Sun et al. 2001; Norouzzadeh and Rahmani 2006; Oh et al. 2012).

Multifractal property is also used to measure the efficiency of the stock market (Cajueiro and Tabak 2004; Wang et al. 2009; Rizvi et al. 2014). According to the Efficient Market Hypothesis (Hayek 1945), it is assumed that the price of the capital market is already reflected in all available information. It implies that future price changes cannot be predicted using past price changes. If there is an autocorrelation in the stock market, stock market does not follow random walk. Market efficiency is measured by observing long-range correlation through the Hurst exponent. According to Yuan et al. (2009), degree of multifractality can be used to measure market efficiency. The larger the degree of multi-fractal implies the more inefficient market. On the other hands, the monofractal indicates the efficient market.

1.3 Organization of the research

The rest of this dissertation is organized as follows. In Chapter 2, the price-based asymmetric multifractal detrended fluctuation analysis method is proposed to explore the asymmetric multifractal scaling behavior with different trends in financial marker. In addition, the validity of model is confirmed by applying this model to the U.S. financial market. In Chapter 3, Simulation analysis is investigated for various generated time series to understand the asymmetric long-range correlation and multifractality. After generating artificial time series data with asymmetric features using Monte-Carlo simulation, the price-based A-MFDFA model is examined whether the asymmetric features have been captured. In addition, how various factors affect asymmetric long-range dependence and multifractality is investigated through simulation analysis. In Chapter 4, the asymmetric market efficiency measure is proposed. The asymmetric market efficiency measure is applied to various countries' stock market using the price-based A-MFDFA model. It is also observed that how the asymmetric market efficiency changes in the financial crisis through time-varying features using moving-window method. Lastly, the summary and contributions of this dissertation are reviewed in Chapter 5 with limitations and possible future work.

Chapter 2

Asymmetric multi-fractality in the U.S. stock indices using the price-based model of A-MFDFA

This chapter is published in Lee et al. (2017).

2.1 Introduction

The multi-fractal analysis has been applied to investigate various stylized facts of the financial market including market efficiency (Cajueiro and Tabak 2004; Wang et al. 2009; Wang et al. 2010; Rizvi et al. 2014), financial crisis (Hasan and Mohammad 2015), risk evaluation (Lee et al. 2016), stock markets (Greene and Fielitz 1977; Sun et al. 2001; Lee et al. 2006) and crash prediction (Grech and Mazur 2004). Specifically, the multi-fractal detrended fluctuation analysis (MF-DFA), a generalization of the detrended fluctuation analysis (DFA) (Peng et al. 1994), is a typical approach to measure the long-range autocorrelations and multi-fractality of a time-series (Kantelhardt et al. 2002). Both DFA and MF-DFA also have been widely applied in various fields such as DNA sequences (Ossadnik et al. 1994), heart rate dynamics (Ashkenazy et al. 2001), long-range weather records (Ivanova and Ausloos 1999; Zheng et al. 2008), exchange rate dynamics (Norouzzadeh and Rahmani 2006) and oil market (He and Chen 2010).

Recently, there have been a number of studies in the asymmetric correlation in the financial market (Longin and Solnik 2001; Ang and Chen 2002; Ding et al. 2011). Longin and Solnik (2001) show that the international market correlation increases at the extreme left-tail event based on extreme value theory. Ang and Chen (2002) detect the strong asymmetric correlations between equity portfolios and the U.S. aggregate market. Ding et al. (2011) examine potential fundamental of asymmetric correlation of stock portfolio. Therefore, the research on the asymmetric correlations within the financial market can provide an understanding of the asymmetric features of risk, which can be applied to enhance the portfolio in terms of diversification and risk management.

It is commonly accepted fact that there are two trends of stock market, namely bullish and bearish markets, and they should be treated differently in analyzing the multi-fractal scaling behavior and correlation. However, there are limited numbers of studies focusing on measuring the asymmetric multi-fractality. Alvarez-Ramirez et al. (2009) introduce the asymmetric DFA (A-DFA) to examine asymmetric correlations in the scaling behavior of time-series. Based on A-DFA, Cao et al. (2013) propose the asymmetric multi-fractal detrended fluctuation analysis (A-MFDFA) method to extend MF-DFA methods, whereas Zhang et al. (2016) introduce the asymmetric multi-fractal detrending moving average analysis (A-MFDMA) method to extend MF-DMA (Gu and Zhou 2010) to quantify the long-term correlations of non-stationary time-series.

Interestingly, A-MFDFA method and A-MFDMA methods demonstrate the distinct scaling properties in two different market trends where the up- and down-trends are distinguished based on the linear regression of return dynamics. However, we claim that the gain of portfolio profit is achieved when the market price moves up and the loss of portfolio profit is realized when the market price moves down. That is, the price dynamics can be a better proxy of market trend. Based on this idea, we provide the new model named ‘price-based A-MFDFA’ which employs the price dynamics as more intuitive criterion for separating the market trends. In addition, to distinguish between our new model and conventional A-MFDFA method, we call conventional model as ‘return-based A-MFDFA’ in this dissertation. We employ ‘price-based A-MFDFA’ method for analyzing the stock indices of the United States so that the existence of asymmetric multi-fractal scaling behavior can be observed. Furthermore, we also analyze the stock indices using ‘return-based A-MFDFA’ to compare with our result as a reference. Based on two models, we discuss the empirical difference of two models and features of scaling behavior. Furthermore, we investigate the scaling asymmetries, source of the multi-fractality and asymmetry. Lastly, we explore the time-varying feature of asymmetric scaling behavior. This research is based on our initial work (Lee et al. 2016) and the contents of this chapter is the upgrade and the completion of our previous work.

This chapter is organized as follows: Chapter 2.2 proposes the definition and step-by-step scenario of the return- and price-based model for A-MFDFA; Chapter 2.3 describes the statistical features of data; Chapter 2.4 discusses the empirical findings; and Chapter 2.5 concludes.

2.2 Price-based A-MFDFA

We can investigate the asymmetric multifractal scaling behavior with different trends using the A-MFDFA method. Cao et al. (2013) proposed the 'return-based A-MFDFA' method. We modify 'return-based A-MFDFA' method and introduce the 'price-based A-MFDFA' method, which use price criterion for dividing the market trend. We have a time-series $\{x_t: t = 1,2,\ldots,N\}$. Our proposed method has the following steps.

Step 1: Define $\mathrm{y_t} = \sum_{j=1}^{t}\left(x_j - \bar{x}\right)$, $t = 1,2,\ldots,N$ where $\bar{x} = \sum_{j=1}^{N} x_j/N$.

Step 2: Divide time-series into non-overlapping sub-time series

Let $I_t = I_{t-1}\,exp(x_t)$ for $t = 1,2,\ldots,N$, where $I_0 = 1$ and I_t is a price proxy for return time-series. We divide $\{I_t: t = 1,2,\ldots,N\}$ and $\{\mathrm{y_t}: t = 1,2,\ldots,N\}$ into $N_n \equiv \lfloor N/n \rfloor$ non-overlapping sub-time series of equal length n, where $\lfloor x \rfloor$ is the largest integer less than or equal to x. We repeat this procedure from the other end of $\{I_t\}$ and $\{\mathrm{y_t}\}$ respectively, resulting in $2N_n$ sub-time series. Suppose $G_j = \{g_{j,k}, k = 1, 2, \ldots, n\}$ be the length n sub-time series of $\{I_t\}$ in the jth time interval and $H_j = \left\{h_{j,k}, k = 1,2,\ldots,n\right\}$ be the jth sub-time series of $\{\mathrm{y_t}\}$ for $j = 1,2,\ldots,2N_n$. Then, we have $g_{j,k} = I_{(j-1)n+k}$ and $h_{j,k} = y_{(j-1)n+k}$ for $j = 1, 2, \ldots, N_n$, and for $j = N_n + 1, \ldots, 2N_n$ $g_{j,k} = I_{N-(j-N_n)n+k}$ and $h_{j,k} = y_{N-(j-N_n)n+k}$. Peng et al. (1994) suggests that $5 \leq n \leq N/4$.

Step 3: Construct the fluctuation function

For each sub-time series G_j and H_j, we calculate the local trend by least-squares fits $L_{G_j}(k) = a_{G_j} + b_{G_j}k$ and $L_{H_j}(k) = a_{H_j} + b_{H_j}k$, where k is for the horizontal coordinate. The slope of $L_{G_j}(k)$, b_{G_j}, is used to discriminate whether the trend of G_j is positive or negative. The linear fitting equation, L_{H_j} is used to detrend the integrated time-series H_j. We define the fluctuation function as $F_j(n) = \sum_{k=1}^{n}\left(h_{j,k} - L_{H_j}(k)\right)^2/n$ for $j = 1, 2, \ldots, 2N_n$.

Step 4: Identify trend using price dynamics

Assuming that $\{I_t\}$ has piecewise positive and negative linear trends, the asymmetric cross-correlation scaling property of fluctuation functions can be assessed by the sign of the slope, b_{G_j}. When $b_{G_j} > 0$, the sub-time series G_j of $\{I_t\}$ has a positive trend. By contrast, $b_{G_j} < 0$ indicates that the sub-time series G_j of $\{I_t\}$ exhibits a negative trend.

Step 5: Construct q-order average fluctuation functions

The directional q-order average fluctuation functions of price-based model (when $q \neq 0$) is computed by, $F_q^+(n) = \left(\sum_{j=1}^{2N_n}(1 + sgn(b_{G_j}))\left[F_j(n)\right]^{q/2}/M^+\right)^{1/q}$ and $F_q^-(n) = \left(\sum_{j=1}^{2N_n}(1 - sgn(b_{G_j}))\left[F_j(n)\right]^{q/2}/M^-\right)^{1/q}$ where $M^+ = \sum_{j=1}^{2N_n}(1 + sgn(b_{G_j}))$, $M^- = \sum_{j=1}^{2N_n}(1 - sgn(b_{G_j}))$, and $sgn(x)$ is the sign of x. Note that M^+ and M^- are the number of sub-time series with positive and negative trends, respectively. We assume that $b_{G_j} \neq 0$ and $M^+ + M^- = 2N_n$. The average fluctuation function of the traditional MF-DFA model also can be computed as $F_q(n) = \left(\sum_{j=1}^{2N_n}\left[F_j(n)\right]^{q/2}/(2N_n)\right)^{1/q}$.

Step 6: Calculating the generalized Hurst exponent

If a time-series has a long-range correlation, the following power-law relationship is observed. Let $H(q)$, $H^{+}(q)$, and $H^{-}(q)$ denote the overall, upward, and downward scaling exponents, which are called the generalized Hurst exponents, respectively. Specifically, the scaling satisfies, $F_q(n) \sim n^{H(q)}$, $F_q^{+}(n) \sim n^{H^{+}(q)}$, and $F_q^{-}(n) \sim n^{H^{-}(q)}$. Using the logarithmic form, $H(q)$, $H^{+}(q)$, and $H^{-}(q)$ can be obtained by the ordinary least square method. If $H(q)$ is constant for all q, the corresponding time series is mono-fractal. Otherwise, the time-series are multi-fractal. Note that the correlations in the time-series are persistent if $H(2) > 0.5$, whereas the correlations in the time-series are anti-persistent if $H(2) < 0.5$. If $H(2) = 0.5$, time-series follows random walk process (Kantelhardt et al. 2002).

Analogous to $H(q)$, the up-trend (down-trend) time-series are multi-fractal if the time-series shows positive (negative) trend. In addition, the correlations in the time-series are symmetric if $H^{+}(q) = H^{-}(q)$, whereas the correlations are asymmetric if $H^{+}(q) \neq H^{-}(q)$. The asymmetric scaling behavior means that the correlations are different between positive and negative trends.

Note that the 'return-based A-MFDFA' model, which is used as a benchmark for our 'price-based A-MFDFA' model, is construct using $\{x_t\}$ instead of $\{I_t\}$ in step 2 and analyzes the sub-time series trend of $\{x_t\}$ to separate the positive trend and negative trend in step 4.

2.3 Data Description

Our dataset consists of daily closing prices of the United States stock indices including the Dow Jones Industrial Average Index (DJIA), National Association of Securities Dealers Automated Quotations Composite Index (NASDAQ), New York Stock Exchange Composite Index (NYSE), and the Standard & Poor's 500 Index (S&P500). The experimental period of time-series is from 1991-01-01 to 2015-12-31. Then, we transform the price-series to the logarithmic return-series, $r_t = log(P_t) - log(P_{t-1})$, where P_t is the closing price of index at time t. Specifically, the sample sizes of DJIA, NASDAQ, NYSE, and S&P500 are 6290, 6293, 6291 and 6289 trading dates, respectively.

Figure 2.1 and Table 2.1 demonstrate the evolutions of return series and their descriptive statistics for DJIA, NASDAQ, NYSE and S&P500, respectively. As shown in Figure 2.1 and Table 2.1, the skewness of the entire return series is not zero where all series except for the case of NASDAQ are skewed left. Also, all series are fat-tailed and peaked since the kurtosis of them are greater than three. The JB statistics are all significant at 1% level, suggesting that the normality assumption of the distribution of all return series is rejected. Furthermore, the ADF test shows that the absence of unit root is rejected at the 1% significant level.

Table 2.1 : Descriptive statistics for the returns of indices

	Mean	max	min	std	skew	kurt	JB	ADF
DJIA	0.0003	0.1051	-0.0820	0.0108	-0.15	11.32	18174*	-83.4*
NASDAQ	0.0005	0.1720	-0.1111	0.0176	0.07	8.75	8675*	-82.5*
NYSE	0.0003	0.1153	-0.1023	0.0111	-0.39	13.92	31430*	-82.2*
S&P500	0.0003	0.1066	-0.0919	0.0115	-0.19	11.07	17110*	-84.8*

Note: "max", "min", "std.", "skew" and "kurt" denote maximum, minimum, standard deviation, skewness and kurtosis, respectively. "JB" denotes Jarque-Bera statistics for normality test and "ADF" denotes the Augmented Dicky-Fuller(ADF) test for unit root test. * denotes 1% level of significance.

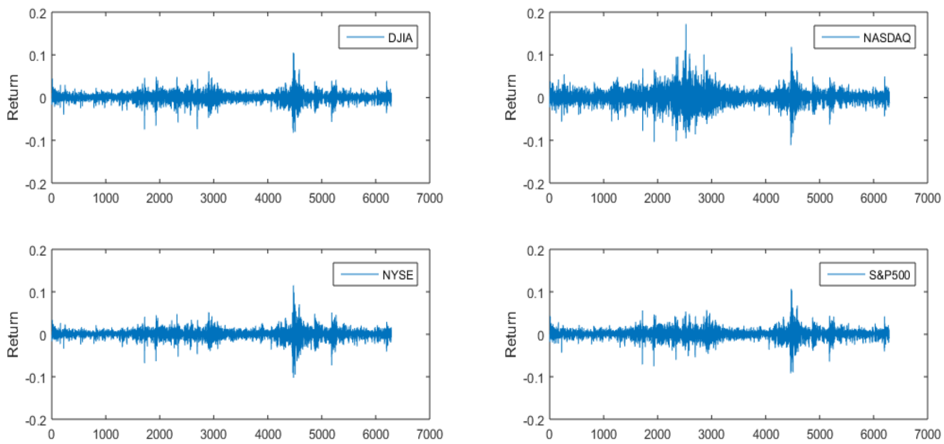

Figure 2.1: Daily return series of indices

2.4 Empirical results of asymmetric scaling behavior

2.4.1 Asymmetric fluctuation functions and their dynamics

Figure 2.2 illustrates the results of both return- (left) and price-based (index-based) (right) models of A-MFDFA when q = 2, which describes how $log_2\left(F_2(n)\right)$ changes with respect to $log_2(n)$. Note that the blue, red, and yellow dots represent the overall, upwards, and downwards, respectively. It is well-known stylized fact that $log_2\left(F_2(n)\right)$ vs. $log_2(n)$ possesses a power-law dependency where the straight dotted line indicates a decent power-law fit. In general, the asymmetry in fluctuation function is discovered within a single unit of time-scale where the distinctions between the values of uptrend and downtrend are observed through most of time-scale. Besides, the dynamics of fluctuation functions exhibit the symmetric evolution in accordance with the time-scale increment. In addition, the newly-suggested approach of the price-based model clearly distinguishes the straight trends of upward and downward pivoting on the overall dots, whereas the conventional approach of return-based model shows the scattered dots with a weak straight trend. Hence, the results suggest that the price-based model provides more robust criterion of detecting the power-law scaling property. In other words, the price-based model performs better clustering of two different trends.

Furthermore, the fluctuation functions of trends show the reverse order of their values between the return- and price-based (index-based) models. All cases of DJIA, NASDAQ, NYSE and S&P500, the descending order of the fluctuation functions of return-based model is upward, overall, and downward, whereas that of price-based model is downward, overall, and upward. Note that the higher value of fluctuation function implies the more volatility of the market.

Figure 2.3 shows the plots of $Df = log_2 F_2^+(n) - log_2 F_2^-(n)$ versus n to visualize the asymmetry of fluctuation function. Based on Fig 2.3, the shape of Df are similar among all indices in each model. Specifically, the return-based model shows many crossovers around zero, whereas the price-based (index-based) model has much less cases of crossovers. Since $Df = 0$ indicates the symmetry between upward and downward, it is clear that the price-based model detects the asymmetry more explicitly than the return-based one. In addition, the mean Df values of return-based model for DJIA, NASDAQ, NYSE, S&P500 are 0.2320, -0.1793, 0.3293, 0.1466, respectively, whereas those of price-based one are -0.8415, -1.0045, -0.9424, -0.9187. Therefore, the downward trend has greater fluctuation function in the price-based model as shown in Figure 2.2.

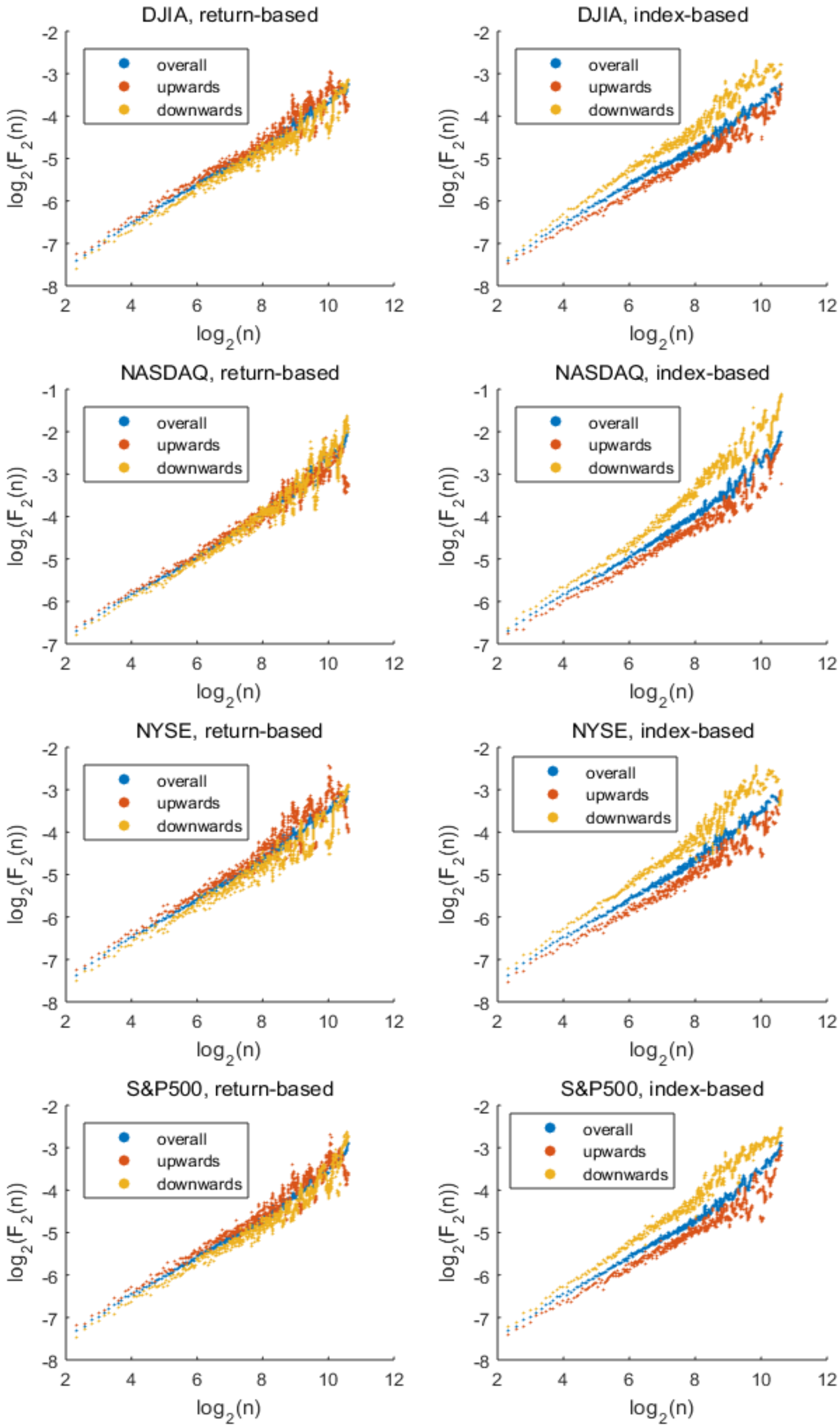

Figure 2.2: Plots of $log_2(F_2(n))$ vs. $log_2(n)$ for DJIA, NASDAQ, NYSE and S&P500

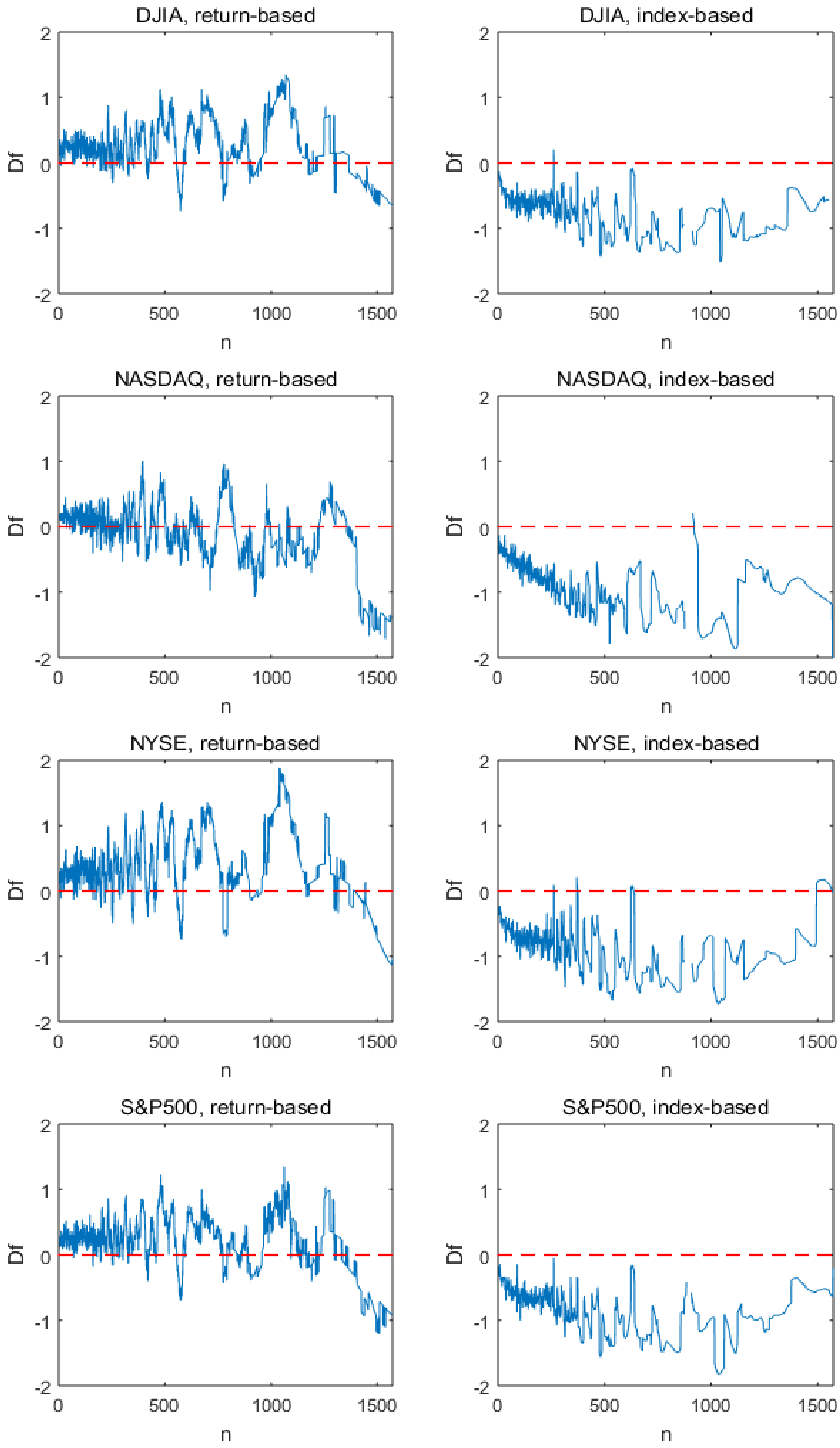

Figure 2.3: Plots of Df for DJIA, NASDAQ, NYSE and S&P500

2.4.2 Estimating the generalized Hurst exponent $H(q)$

Figure 2.4 visualizes the values of generalized Hurst exponents $H(q), H^{+}(q)$, and $H^{-}(q)$ with respect to q varying from -5 to 5 with interval of 0.1. The result shows that the values of $H(q)$, $H^{+}(q)$, and $H^{-}(q)$ decrease when q increases for the most of cases except for NASDAQ, whose values changing regardless of q orders. It refers that each series possess the multi-fractal feature regardless of the trend. In case of the return-based model, the gap between the uptrend and downtrend is small when q is small (i.e. small fluctuation), whereas the gap becomes larger as q increases. The large gap is analogous to the significance of asymmetry. In case of the price-based (index-based) model, the coupling of overall and uptrend is observed, while the downward shows different trend. Furthermore, the gap between the upward and downward decreases as q increases.

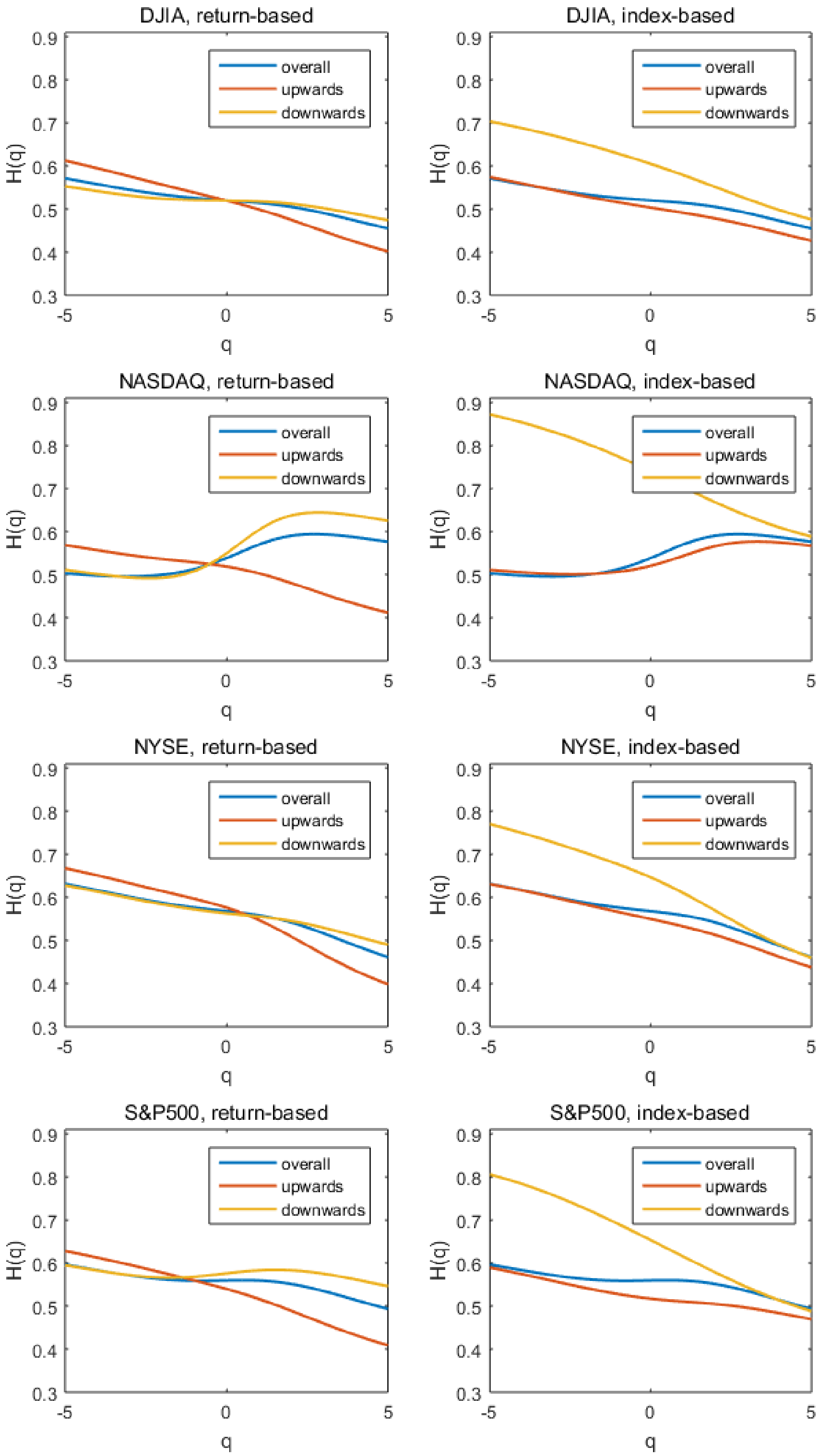

Figure 2.4: Plots of $H(q), H^{+}(q)$, and $H^{-}(q)$ versus q

2.4.3 Source of multi-fractality

In general, there are two major sources of multi-fractality (Kantelhardt et al. 2002; Wei-Xing 2009): (1) different long-range correlations for small and large fluctuations, and (2) fat-tailed probability distributions. It is also well known that the contribution of each source can be evaluated by comparing the multi-fractality of original and modified series. At first, the long-range correlation can be tested by comparing the multi-fractality between the original and randomly shuffled series. The step-by-step scenario of creating the randomly shuffled series is as follows:

(1) Generate pairs (a, b) of random integer numbers with $a, b \leq N$, where N is the length of the time-series.

(2) Change the value in a-th order with b-th order

(3) Repeat (1) and (2) for $20N$ times

Secondly, the fat-tailed distribution can be investigated by comparing the multi-fractality of original and surrogated series (Theiler et al. 1992). The algorithm to create the surrogated series is as follows:

(1) Generate a sequence of random numbers $\{\tilde{x}_t : t = 1,2, \dots, N\}$ with the Gaussian distribution

(2) Rearrange $\{\tilde{x}_t\}$ in the same order of $\{x_t\}$ so that two time series can have the same rank patterns

The degree of multi-fractality can be defined as $\Delta H = max(H(q)) - min(H(q))$ (Yuan et al. 2009). When ΔH is zero, the time-series is called as mono-fractal, and the degree of multi-fractality is stronger as ΔH increases. Let ΔH_{orig}, ΔH_{shuf} and ΔH_{surr} represent the degree of multi-fractality for the original, shuffled, and surrogated series, respectively (Cao et al. 2013). To achieve the robust result, we use the mean of 30 repeated values for ΔH_{shuf}

and $\mathit{\Delta}H_{surr}$.

Tables 2.2 and 2.3 summarize the degree of multi-fractality for the original, shuffled and surrogated series using A-MFDFA. The bold numbers indicate the significance of multi-fractality. Note that either $\mathit{\Delta}H_{\text{shuf}}$ or $\mathit{\Delta}H_{surr}$ is significant if its value is smaller than others given that it is also significantly smaller than $\mathit{\Delta}H_{orig}$. For the most cases of overall, $\mathit{\Delta}H_{shuf}$ and $\mathit{\Delta}H_{surr}$ are smaller than $\mathit{\Delta}H_{orig}$. This implies that the multi-fractality in the U.S. indices is affected by both long-range correlation and fat-tailed distribution. In case of original series, ΔH_{orig} is larger in the upward than downward for the return-based model. However, ΔH_{orig} is significantly larger in the downward than upward for the price-based model. That is, the strong multi-fractality is presented in the downward for the price-based model, while that is observed in the upward for the return-based.

When $\mathit{\Delta}H_{\text{shuf}}$ is much smaller than not only $\mathit{\Delta}H_{orig}$ but also $\mathit{\Delta}H_{surr}$, we can claim that the main source of multi-fractality is long-range correlation. In Table 2.2 for a return-based A-MFDFA model, $\mathit{\Delta}H_{\text{shuf}}$ values for upward stock market are significant smaller in DJIA, NASDAQ, NYSE, and S&P500 so that we can claim that the main source of multi-fractality in these upward stock market is fat-tail distribution.

When $\mathit{\Delta}H_{\text{surr}}$ is much smaller than not only $\mathit{\Delta}H_{orig}$ but also $\mathit{\Delta}H_{shuf}$, we can claim that the main source of multi-fractality is fat-tail distribution. In Table 2.3 for the price-based A-MFDFA model, $\mathit{\Delta}H_{\text{surr}}$ values for downward stock market are significant smaller in DJIA, NASDAQ, NYSE, and S&P500 so that we can claim that the main source of multi-fractality in these downward stock market is long-range correlation.

Table 2.2: ΔH of the original, shuffled, and surrogated series using return-based A-MFDFA model

Return-based A-MFDFA	DJIA			NASDAQ		
	Original series	Shuffled series	Surrogated series	Original series	Shuffled series	Surrogated series
Overall	0.1164	0.0826 (29.00%)	**0.0558 (52.09%)**	0.0983	0.0979 (0.35%)	0.1360 (-38.39%)
Upward	0.2103	**0.1034 (50.82%)**	0.1256 (40.27%)	0.1567	**0.1095 (30.17%)**	0.1490 (4.92%)
Downward	0.0790	0.1069 (-35.42%)	**0.0367 (53.51%)**	0.1527	**0.1152 (24.57%)**	0.2218 (-45.27%)
	NYSE			S&P500		
	Original series	Shuffled series	Surrogated series	Original series	Shuffled series	Surrogated series
Overall	0.1700	0.1247 (26.66%)	**0.1094 (35.69%)**	0.1031	0.0961 (6.79%)	**0.0297 (71.17%)**
Upward	0.2690	**0.1300 (51.67%)**	0.1625 (39.60%)	0.2187	**0.1171 (46.46%)**	0.1291 (40.99%)
Downward	0.1375	0.1431 (-4.11%)	**0.0959 (30.26%)**	0.0487	0.1141 (-134.31%)	0.1008 (-106.98%)

Table 2.3: ΔH of the original, shuffled, and surrogated series using the price-based A-MFDFA model

Price-based A-MFDFA	DJIA			NASDAQ		
	Original series	Shuffled series	Surrogated series	Original series	Shuffled series	Surrogated series
Overall	0.1164	0.0826 (29.00%)	**0.0558 (52.09%)**	0.0983	0.0979 (0.35%)	0.1360 (-38.39%)
Upward	0.1476	**0.0921 (37.60%)**	0.1191 (19.29%)	0.0759	0.1064 (-40.13%)	0.0980 (-29.12%)
Downward	0.2276	0.1533 (32.66%)	**0.0824 (63.82%)**	0.2839	0.1514 (46.69%)	**0.0778 (72.60%)**
	NYSE			S&P500		
	Original series	Shuffled series	Surrogated series	Original series	Shuffled series	Surrogated series
Overall	0.1700	0.1247 (26.66%)	**0.1094 (35.69%)**	0.1031	0.0961 (6.79%)	**0.0297 (71.17%)**
Upward	0.1932	**0.1302 (32.60%)**	0.1527 (20.97%)	0.1199	0.0971 (19.04%)	**0.0385 (67.89%)**
Downward	0.3099	0.1723 (44.41%)	**0.1202 (61.23%)**	0.3175	0.1572 (50.50%)	**0.0963 (69.68%)**

Note for Table 2.2 and 2.3: The value in parentheses is the change in the ΔH value for the shuffled (resp. surrogated) data to that of the original data, (ΔH_{orig}- ΔH_{shuf}) / ΔH_{orig} (resp. (ΔH_{orig}- ΔH_{surr})/ ΔH_{orig})

2.4.4 Source of asymmetry

Alvarez-Ramirez et al. (2009) suggest that the asymmetric scaling behavior also can be produced by the long-range correlation and fat-tailed distribution. Hence, we re-apply the method in section 4.3 to discover the source of the asymmetry scaling behavior. The degree of asymmetric scaling behavior can be defined as $\Delta H^{\pm}(q) = |H^{+}(q) - H^{-}(q)|$ (Rivera-Castro et al. 2012). Note that the time-series has symmetric scaling behavior if $\Delta H^{\pm}(q)$ is close to 0, whereas it has stronger asymmetry as $\Delta H^{\pm}(q)$ increases.

Figure 2.5 shows the value of $\Delta H^{\pm}(q)$ for the original, shuffled and surrogated series. If $\Delta H^{\pm}(q)$ for the shuffled and surrogated series are smaller than those of the original series, then the long-range correlation and fat-tailed distribution can be possible sources of the asymmetric scaling behavior. Analogous to the source of multi-fractality, the smallest value of $\Delta H^{\pm}(q)$ is the main source. In case of return-based model, $\Delta H^{\pm}(q)$ of shuffled and surrogated series are not significantly smaller than that of original series. In other words, the source of asymmetry is not distinguishable. In contrast, the price-based (index-based) model clearly detects the source of asymmetry. The source of asymmetry in DJIA and S&P500 is the long-range correlation based on $\Delta H^{\pm}(q)$ being close to zero for shuffled series. The result of NASDAQ shows that the long-range correlation and fat-tailed distribution are the main sources of asymmetry for $\mathrm{q} < 1$ (small fluctuation) and $1 < \mathrm{q}$ (large fluctuation), respectively. Lastly, the result of NYSE reveals that the fat-tailed distribution and long-range correlation are the main sources of asymmetry for $\mathrm{q} < -2$ and $-2 < \mathrm{q} < 2$, respectively.

2.4.5 Time-varying multi-fractal asymmetry

The time-varying feature of the multi-fractal asymmetry can be studied based on the rolling window method. We set the size of window and slide to be 1000 trading dates (roughly 4 years) and 50 dates, respectively, to achieve the reliable result (Greene and Fielitz 1977; Cristescu et al. 2012). The corresponding results of return- and price-based (index-based) model are illustrated in Figure 2.6 and Figure 2.7, respectively. In general, each model shows the similar evolutionary patterns of ΔH, ΔH^{+}, and ΔH^{-} among most of indices excluding NASDAQ. The difference between two models can be observed when the evolutions of upward and downward are compared. The return-based model shows the similar trend in the evolutions of overall, upward, and downward, whereas the price-based model shows the different evolution between the upward and downward. It is also noticeable that the time-varying ΔHs of upward and downward are correlated in the return-based model, whereas those in the price-based model are uncorrelated. That is, the price-based model is more suitable in discriminating the time-varying multi-fractal asymmetry. Furthermore, the peaks of ΔH, ΔH^{+} and ΔH^{-} in both models are observed in the window period from 1994-1998 to 1996-2000 and from 2005-2009 to 2008-2012, which include the Asian financial crisis in 1997 and the Sub-prime mortgage crisis in 2008, respectively. It refers that the strong multi-fractality is the phenomenon of the financial crisis. As the market efficiency is measured by the degree of multi-fractality (Wang et al. 2010), the result of time-varying multi-fractal asymmetry provide the evidence of inefficient market during the financial crisis.

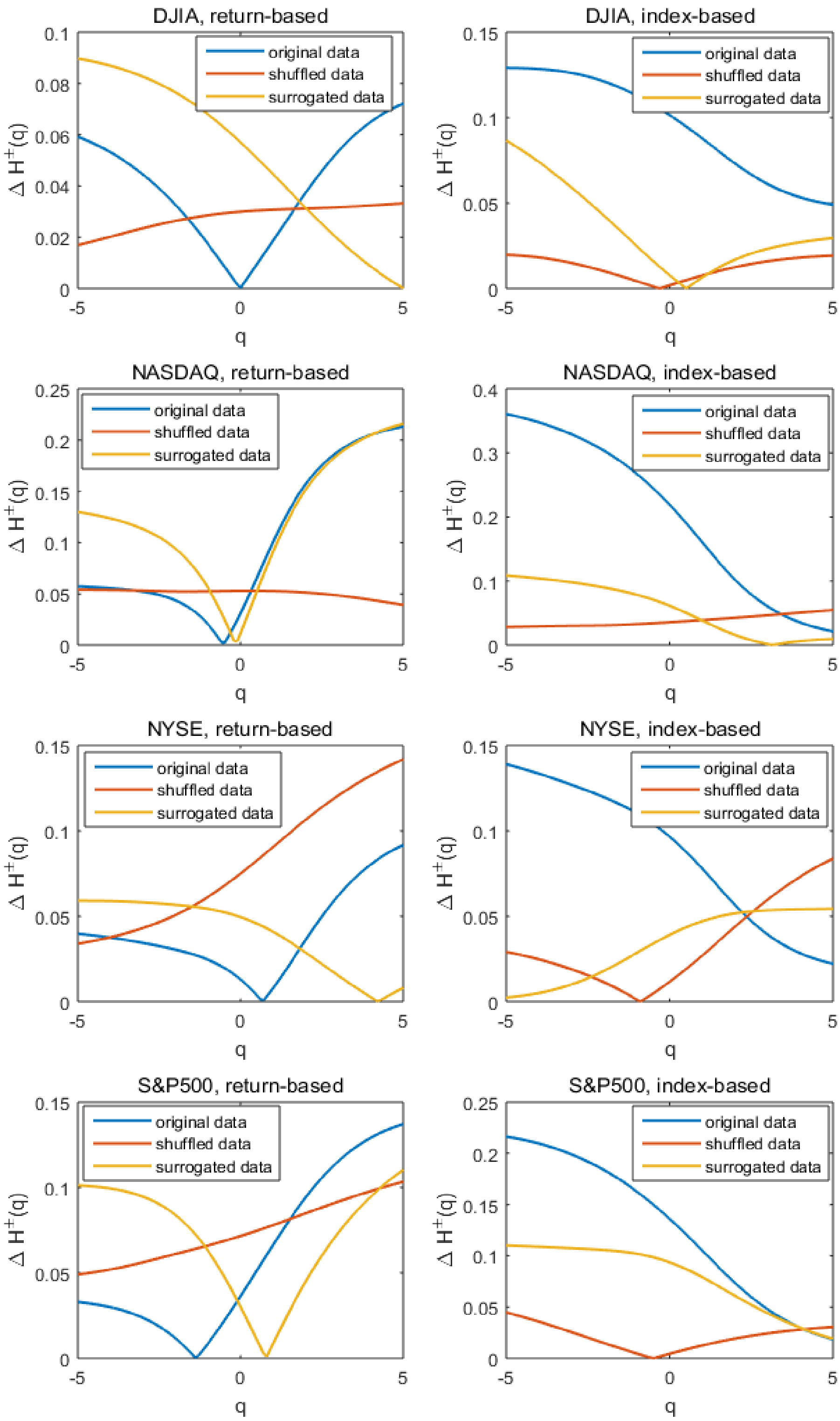

Figure 2.5 Plots of $\Delta H^{\pm}(q)$ for the original, shuffled and surrogated data

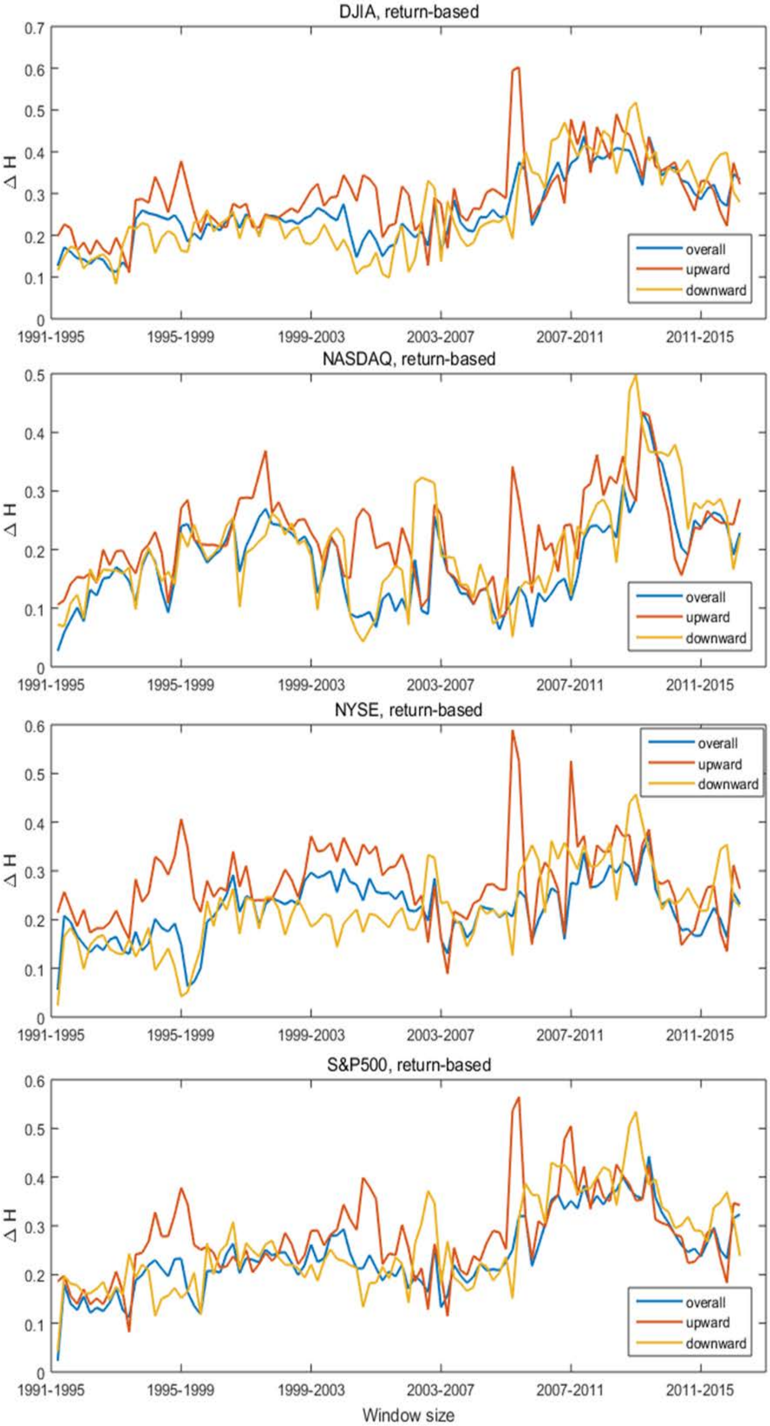

Figure 2.6: Time evolution of ΔH with a slide step of 50 days for the overall, upward and downward for DJIA, NASDAQ, NYSE and S&P500, respectively, using return-based A-MFDFA model

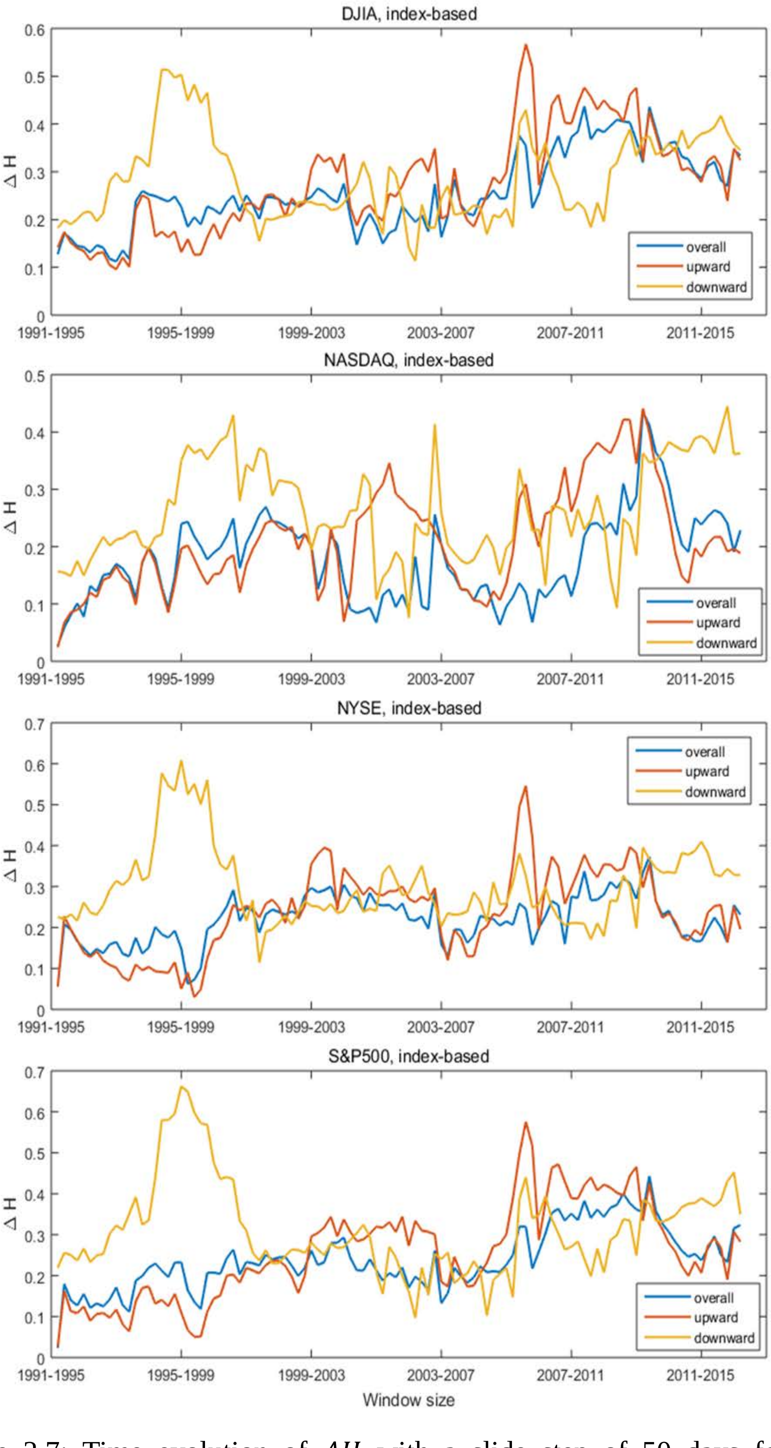

Figure 2.7: Time evolution of ΔH with a slide step of 50 days for the overall, upward and downward for DJIA, NASDAQ, NYSE and S&P500, respectively, using the price-based (index-based) A-MFDFA model

2.5 Conclusion

In this chapter, we propose A-MFDFA with new criterion for separating the market trend. Originally, A-MFDFA method distinguishes the market trend based on the coefficient of regression in the return dynamics. Considering that the coefficient of regression in the price dynamics is more intuitive criterion for the market trend, we provide the price-based model of A-MFDFA so that asymmetric multi-fractal features in the U.S. stock indices can be investigated. At first, we discover that the existence of multi-fractality in all the U.S. stock indices, whose feature is revealed to be asymmetric. Also, the price-based model can detect the asymmetric multi-fractality more distinctly than the return-based model since its fluctuation function shows the clear-cut between the positive and negative trends. Secondly, we find the source of multi-fractality and asymmetry by comparing the original series with the shuffling and surrogated series. Specifically, the long-range correlation is discovered to be the main source for the upward trend, whereas the fat-tailed distribution is the main source for the downward trend. The source of asymmetry is ambiguous in the return-based model, but the price-based model indisputably identifies the source where the source of asymmetry differs in each index and q. Lastly, we explore the time-varying feature of asymmetric multi-fractality based on the moving window method. The time-varying feature of uptrend and downtrend are correlated in the return-based model, whereas the features are clearly distinguished in the price-based model. Furthermore, we detect that the degree of multi-fractality is high during the financial crisis period where the asymmetry between two trends are significantly elevated in the price-based model. Thus, we claim that the price-based model performs better in detecting the asymmetric multi-fractality.

Chapter 3

Study of asymmetric multifractal characteristics through various time series simulations

3.1 Introduction

The price-based A-MFDFA method is introduced in Chapter 2. The price-based A-MFDFA method is methodology for analyzing multifractality by dividing time series based on price criterion for dividing the market trend. If the long-term memory characteristics are different according to the stock market trend, this model can find out the self-similarity features of uptrend and downtrend.

The aim of this chapter is to understand the asymmetric features of multifractality and long-range dependence of various time series. There are various theoretical time series containing the factors affecting the Hurst exponent and multifractality. After generating these time series according to the purpose, using the price-based A-MFDFA model, simulations to know how the asymmetric Hurst exponent and multifractality are formed in time series are progressed. There are three main purposes of this simulation. First, it is to know that A-MFDFA model works properly by comparing the simulation with the time series that are generated differently according to trend and time series that are generated regardless of the trend. That is, once a time series with asymmetric features has been generated, it is checked whether the result of the A-MFDFA model contains that features. Secondly, it is to investigate the changes of asymmetric Hurst exponent according to the changes of autocorrelation and skewness of distribution over time series which affect long-range dependence. Lastly, using the time series which contain the feature of autocorrelation of volatility and heavy-tail which affect the multifractality, it is examined that the change of asymmetric multifractality according to the change of features.

In order to observe the asymmetric features, three methods have been introduced to generate random numbers differently according to the past values, past trends and independent with the past.

3.2 Various probability distribution and time series model

Monte-Carlo simulation is used for obtaining the numerical results by generating random numbers. The Asymmetric Hurst exponent and degree of multifractality are obtained from various generated time series using Monte-Carlo simulation and the price-based A-MFDFA model. With this methodology, the various asymmetric features of variety time series are realized. Skewed distribution and autoregressive model are used to study the features of the asymmetric Hurst exponent. Student's t-distribution, autoregressive conditional heteroscedasticity model and generalized Autoregressive conditional heteroscedasticity model are used to understand the features of asymmetric degree of multifractality. The price-based A-MFDFA model and the asymmetric Hurst exponent are introduced in Chapter 2.2. The degree of multi-fractality is defined as $\Delta H = max(H(q)) - min(H(q))$ in Chapter 2.4.3. The handled time series and probability distribution are as follows.

3.2.1 Normal distribution

When the random number series are generated from normal distribution, it follows Brownian motion. The generalized Hurst exponent of Brownian motion, *H(q),* is 0.5 for all q. It means Brownian motion is random walk and completely uncorrelated series. Therefore, the random number series generated from the normal distribution can be regarded as the basic time series.

Following equation is probability density function of normal distribution.

$$f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is mean and σ is standard deviation.

The standard normal distribution is used for experiments with mean = 0 and sigma = 1.

3.2.2 Skewed distribution

For obtaining the skewed distribution, Pearson system of distribution is used. It uses transformations of various standard random variates for types 0-iii and types v-vii. Pearson distribution has 4 parameter which are mean, standard deviation, skewness and kurtosis. After setting the mean to 0, the variance to 1 and kurtosis to 3, skewed distribution is generated by changing the skewness. Skewness is selected at 0.1 intervals from -0.6 to 0.6 in this research. Skewed distribution is for investigating the asymmetric tail distribution features.

3.2.3 Student's t-distribution

Student's t-distribution (T-dist) is used for estimating the mean of a normally distributed population data where the sample size is small. T-dist is symmetric and bell-shaped like the normal distribution. But T-dist tails are heavier than the normal distribution. T-dist is depending on degrees of freedom, ν. Following equation is probability density function of T-dist.

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)} \left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

where ν is a degree of freedom and should be over than 0.

The smaller value of ν, T-dist(ν) has heavier tails. Therefore, T-dist is used to obtain a time series with heavy tail distribution than normal distribution. T-distributions with the degree of freedom 1, 2, 3, 4, 6, 8 and 10 are explored in this dissertation.

3.2.4 Autoregressive model. (AR model)

The AR(p) model is that the output variable, X_t, depends linearly on its own previous p values. Therefore, generated data have autocorrelation. Following equation is AR(p) model.

$$X_t = c + \sum_{i=1}^{p} a_i X_{t-i} + \epsilon_t$$

where a_i are the parameter of model, c is a constant, and ϵ_t is white noise.

AR(1) model is used in this dissertation. Only past one day data can affect the today's data. Four AR models with AR coefficients 0.2, 0.4, 0.6 and 0.8 are simulated.

3.2.5 Autoregressive conditional heteroscedasticity model (ARCH model)

ARCH(p) model is a stochastic volatility model that the variance of the current error term is a function of the previous time periods' error variance terms. The error variance follows an autoregressive model. This model is usually used for modeling financial time series to capture the time-varying volatility clustering. Following equation is ARCH(p) model.

$$\mathrm{X_t} = \sigma_t \epsilon_t$$

$$\sigma_t^2 = c + \sum_{i=1}^{p} a_i X_{t-i}^2$$

a_i are the parameter of model, c is a constant, and ϵ_t is white noise.

ARCH(1) model is used. Only past one day's variance can affect the today's variance. Four ARCH models with ARCH coefficients 0.2, 0.4, 0.6 and 0.8 and constant 0.8, 0.6, 0.4 and 0.2 are simulated. Constant is chosen so that the total variance is set for 1.

3.2.6 Generalized autoregressive conditional heteroscedasticity model (GARCH model)

GARCH(p,q) model is a stochastic volatility model that variance of the current error term is affected by the past error variance terms and past variance terms. The p is the order of the ARCH terms, X_{t-i}^2 and q is the order of the GARCH terms, σ_{t-i}^2. Following equation is GARCH(p,q) model.

$$X_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = c + \sum_{i=1}^{p} a_i X_{t-i}^2 + \sum_{i=1}^{q} b_i \sigma_{t-i}^2$$

where p is the order parameter of the ARCH terms, X_{t-i}^2 and q is the order parameter of the GARCH terms, σ_{t-i}^2, c is a constant, and ϵ_t is white noise.

GARCH(1,1) model is used for only past one data can affect the today's data. Four GARCH models with GARCH coefficients 0.2, 0.4, 0.6 and 0.8, constant 0.7, 0.5, 0.3 and 0.1 and ARCH coefficient 0.1 are simulated. To investigate the GARCH effect, the GARCH coefficient is changed, the ARCH coefficient is fixed, and the constant is chosen so that the total variance is set for 1.

ARCH model and GARCH model are used to obtain a time series for having autocorrelation of volatility.

3.3 Method to generate time series using Monte-Carlo Simulation

In this chapter, a method of generating time series using the time series model and the distribution described in the above chapter is introduced. Followings are three ways to generate time series.

3.3.1 Homogeneous time series generating

When generating x_t, x_t follows one time series model or probability distribution independently with previous value, x_{t-1}. That is, the generated time series have one features of chosen time series model or distribution. Therefore, this method is named 'Homogeneous time series generating'.

3.3.2 Heterogeneous time series with previous data's sign

When generating x_t, different time series or probability distribution is generated depending on whether x_{t-1} is negative or positive. In other words, the current data is generated from a different distribution depending on previous data's sign. There are two ways to make heterogeneous time series. First one is if the value of x_{t-1} is a positive number, generate next data from the specific time series and generate a random number from standard normal distribution if x_{t-1} is a negative. It is named 'Positive model'. Second is opposite method with first one. If the previous data is negative, next random number is generated from the specific distribution and previous one is positive, then standard normal is generated. It is named 'Negative model'.

3.3.3 Heterogeneous time series with previous data's trend

When generating x_t, different time series or probability distribution is generated depending on the trend from x_{t-d} to x_{t-1} where d is the period length for searching trend. To capture the characteristics of stock market, price trend is used to determine the trend of given period. Price trend is determined by sign of $\sum_{i=1}^{d} x_{t-i}$. When $x_{t-i} = r_{t-i}$ holds (where $r_{t-i} = \ln(I_{t-i}) - \ln(I_{t-i-1})$ is stock logarithm return at time $t-i$) equation $\sum_{i=1}^{d} x_{t-i} = \ln(I_{t-1}) - \ln(I_{t-d-1})$ holds too. That is, $\sum_{i=1}^{d} x_{t-i}$ can be the proxy of price trend. In other words, during that period, the price has risen and the fall has been seen as a trend. As with the chapter 3.3.2, 'Positive model' and 'Negative model' are defined with respect to sign of previous trend. The values of d used in this dissertation are 10, 20, 30, 40 and 50. However, only the d = 20 result is showed in this chapter. The rest results are in Appendix A. Through this method, time series with characteristics of stock market can be generated.

3.4 Simulation results

For Monte-Carlo simulation, 1000 samples of time series are generated with length 1000 for each model. The generalized Hurst exponent values can be calculated using the price-based A-MFDFA model for each one generated time series, then average value of the Hurst exponent, $H(2)$, and the degree of multifractality, ΔH, is obtained for each time series model or probability distribution. Using these values, the long range dependence and multifractality characteristics of the model can be explored by observing the change of the value of model parameters.

3.4.1 Homogeneous time series simulation results

This part contains Monte-Carlo simulation results of generated homogeneous time series. Since the homogeneous time series does not have asymmetric features, it can be seen that there are no asymmetric features through the price-based A-MFDFA. The results from each generated distribution and time series are follows.

3.4.1.1 Skewed distribution

Table 3.1 shows Monte-Carlo simulation results of time series obtained by skewed distribution. It can be seen that all distribution has the mean 0, standard deviation 1 and kurtosis 3, so that the distribution is made to have same with standard normal distribution except skewness. When the variation of the asymmetric Hurst exponent with the change of the skewness is examined, as the skewness increases, the value of the uptrend Hurst exponent decreases and the downtrend one grows larger. This is because if the right skewness increases, the probability that a negative value will be generated increases. Therefore, the probability that positive data will have a negative long-range correlation increases and the probability that negative data will have a long-term positive autocorrelation increases. Proper simulation results are obtained.

3.4.1.2 Autoregressive model

Table 3.2 is Monte-Carlo simulation results of time series obtained by AR model. As the AR coefficient increases, all types of Hurst exponent increase. As the AR coefficient decreases from 0 to negative, the probability that the sign of x_{t-1} and x_t will change becomes higher. It means negative autocorrelation become stronger, so the Hurst exponent value becomes smaller. In contrary, the larger the AR coefficient from 0 to the positive, the greater the probability that x_{t-1} and x_t sign will become the same, so the positive autocorrelation becomes stronger and the Hurst exponent value becomes larger. Since there is little difference between uptrend and downtrend, it is confirmed that there is no asymmetric feature.

3.4.1.3 Normal distribution and student's t-distribution

Table 3.3 shows Monte-Carlo simulation results of time series obtained by repeated generation of normal distribution and Student's t-distribution. The results show that there is little difference between the uptrend Hurst exponent and the downtrend Hurst exponent in all distributions. In addition, the uptrend and downtrend degree of multifractality are not different. This indicates that no asymmetric feature is found. All types of the Hurst exponent values are close to 0.5 and do not change much, so it can be said that there is no long-range correlation. However, it can be seen that as the degree of freedom of student's t-distribution decreases from 10 to 1, as the tail becomes thicker, all types of degree of multifractality become larger. The heavy tail can be noticed by an increase in kurtosis and standard deviation. In other words, as the tail distribution increases, the properties of multifractality become stronger.

3.4.1.4 ARCH model and GARCH model

Table 3.4 contains results of Monte-Carlo simulation about time series generated by ARCH model and GARCH model. It can be seen that the standard deviation converges to 1 since the total variance is set to 1 through constant coefficient adjustment. In case of GARCH model, the ARCH coefficients are all 0.1. In both ARCH model and GARCH model results, all types of Hurst exponent are close to 0.5. In the case of the degree of multifractality, it can be seen that as the ARCH coefficient and GARCH coefficient increase, the multifractality increases. This explains that the autocorrelation of volatility increases the degree of multifractality. When autocorrelation of volatility occurs, kurtosis increases and the tail distribution is thicker. The ARCH coefficient has a greater effect on the change of degree of multifractality than the GARCH coefficient.

Table 3.1: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated homogeneous time series (Skewed distribution)

Skewness	-0.5	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.3	0.4	0.5
$H(2)$	0.4985	0.4997	0.4996	0.4962	0.4941	0.4997	0.4972	0.4937	0.4971	0.5005
$H^{+}(2)$	0.5083	0.5083	0.5020	0.4969	0.4911	0.4923	0.4910	0.4823	0.4811	0.4827
$H^{-}(2)$	0.4796	0.4832	0.4887	0.4871	0.4887	0.4974	0.4963	0.4956	0.5024	0.5095
ΔH	0.0707	0.0709	0.0691	0.0712	0.0740	0.0699	0.0720	0.0696	0.0702	0.0702
ΔH^{+}	0.0922	0.0905	0.0916	0.0955	0.0976	0.0912	0.0915	0.0904	0.0874	0.0855
ΔH^{-}	0.0872	0.0896	0.0888	0.0905	0.0932	0.0924	0.0971	0.0926	0.0922	0.0936
Mean	0.0001	0.0016	-0.0001	-0.0005	-0.0006	-0.0007	0.0006	-0.0004	-0.0007	0.0015
Std	1.0004	0.9991	0.9998	1.0002	0.9996	0.9996	1.0000	0.9992	0.9993	0.9992
Skewness	-0.4945	-0.3966	-0.3022	-0.1987	-0.1003	0.1001	0.2006	0.2972	0.3990	0.4965
Kurtosis	2.9811	2.9898	2.9974	2.9957	2.9997	2.9878	3.0004	2.9897	2.9987	2.9932

Table 3.2: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated homogeneous time series (Autoregressive model)

	AR(-0.8)	AR(-0.6)	AR(-0.4)	AR(-0.2)	AR(0.2)	AR(0.4)	AR(0.6)	AR(0.8)
$H(2)$	0.2542	0.3530	0.4133	0.4592	0.5500	0.6137	0.7094	0.9078
$H^{+}(2)$	0.2520	0.3516	0.4127	0.4557	0.5496	0.6125	0.7076	0.9045
$H^{-}(2)$	0.2548	0.3508	0.4094	0.4575	0.5461	0.6093	0.7046	0.9037
ΔH	0.1272	0.0498	0.0486	0.0579	0.1139	0.1663	0.2172	0.2561
ΔH^{+}	0.1335	0.0580	0.0573	0.0711	0.1289	0.1829	0.2327	0.2762
ΔH^{-}	0.1297	0.0577	0.0586	0.0694	0.1303	0.1793	0.2350	0.2794
Mean	-0.0009	0.0001	0.0000	0.0006	0.0012	0.0001	-0.0020	0.0051
Std	1.6645	1.2510	1.0904	1.0202	1.0197	1.0919	1.2465	1.6565
Skewness	0.0020	0.0017	-0.0007	0.0002	-0.0005	-0.0006	-0.0007	-0.0051
kurtosis	2.986	2.987	2.990	2.995	2.997	2.982	2.989	2.970

Table 3.3: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated homogeneous time series (Normal dist and T-dist)

	Normal dist	T-dist(10)	T-dist(8)	T-dist(6)	T-dist(4)	T-dist(3)	T-dist(2)	T-dist(1)
$H(2)$	0.5016	0.5025	0.5011	0.5030	0.5038	0.5018	0.5031	0.5083
$H^+(2)$	0.4981	0.5030	0.4968	0.4996	0.4981	0.5024	0.5006	0.4992
$H^-(2)$	0.4997	0.4967	0.4992	0.5010	0.5041	0.4958	0.4980	0.4978
ΔH	0.0831	0.1056	0.1163	0.1353	0.1983	0.2730	0.4586	0.9401
ΔH^+	0.0990	0.1193	0.1317	0.1517	0.2061	0.2704	0.4450	0.9798
ΔH^-	0.0954	0.1220	0.1287	0.1438	0.2029	0.2763	0.4502	0.9842
Mean	0.0010	-0.0030	0.0009	0.0006	0.0018	0.0006	-0.0003	2.7533
Std	1.0004	1.1180	1.1543	1.2234	1.4106	1.7058	3.2059	206.6421
Skewness	-0.0007	-0.0052	-0.0048	-0.0029	0.0614	-0.0919	-0.0540	0.3293
Kurtosis	2.995	3.999	4.479	5.635	12.080	29.281	130.092	496.410

Table 3.4: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated homogeneous time series (ARCH model and GARCH model)

	ARCH(0.2)	ARCH(0.4)	ARCH(0.6)	ARCH(0.8)	GARCH(0.2)	GARCH(0.4)	GARCH(0.6)	GARCH(0.8)
$H(2)$	0.5015	0.5022	0.5009	0.4965	0.5026	0.5029	0.5034	0.5050
$H^{+}(2)$	0.4990	0.4987	0.4942	0.4909	0.4994	0.5001	0.4994	0.5008
$H^{-}(2)$	0.4990	0.5006	0.4999	0.4900	0.5006	0.4997	0.5019	0.5025
ΔH	0.1132	0.1641	0.2478	0.3488	0.0981	0.1016	0.1111	0.1367
ΔH^{+}	0.1266	0.1793	0.2580	0.3603	0.1105	0.1165	0.1253	0.1552
ΔH^{-}	0.1282	0.1745	0.2523	0.3567	0.1142	0.1175	0.1264	0.1512
Mean	0.0008	0.0007	-0.0013	-0.0004	-0.0006	-0.0016	-0.0007	-0.0006
Std	0.9992	0.9962	0.9882	0.9485	0.9996	0.9992	0.9997	0.9995
Skewness	-0.0018	0.0094	0.0245	-0.0439	0.0017	-0.0014	0.0011	-0.0073
kurtosis	3.255	4.395	8.807	21.576	3.048	3.073	3.115	3.307

3.4.2 Heterogeneous time series simulation with previous data’s sign results

In this part, Monte-Carlo simulation result of heterogeneous time series with previous data's sign that x_t is generated different distribution according to sign of previous data x_{t-1} is included. Heterogeneous time series should have asymmetric features. If the value of x_{t-1} is positive, next data is generated from the corresponding time series, otherwise generated from the standard normal distribution, this method is named positive model. The negative model is that data is generated from specific distribution when x_{t-1} is negative and from standard normal distribution when x_{t-1} is not negative. The results of the two methods are as follows.

3.4.2.1 Skewed distribution

Table 3.5 is Monte-Carlo simulation results of heterogeneous time series obtained from skewed distribution depending on previous data’s sign. For example, Positive Skew(-0.6) means that if x_{t-1} is a positive number, x_t is generated from the distribution with skewness is 0.6, and if x_{t-1} is not a positive number, x_t is generated from the standard normal distribution. The mean, standard deviation and kurtosis of the skewed distribution are set to 0, 1 and 3, respectively, as normal distribution. As a result, in the case of the positive model, as the skewness increases, the uptrend Hurst exponent decreases and the value of the downtrend Hurst exponent increases. The same result is obtained for the negative model. The reason is that, as the skewness increases, the probability of the negative value being generated increases. It can make the probability of autocorrelation of the negative trend increase. On the other hand, the autocorrelation of the positive trend decreases. When the skewness is reduced, the uptrend Hurst exponent increases and the downtrend Hurst exponent decreases with the above mechanism.

3.4.2.2 AR model

Table 3.6 and Table 3.7 show the results of the heterogeneous time series simulation of the AR model with previous data's sign. Table 3.6 shows the positive model results and Table 3.7 shows the results of the negative model. First, in the case of positive model, all types of Hurst exponent value increases as the AR coefficient increases. When the AR coefficient is over than 0, the Hurst exponent value is more than 0.5, and if the AR coefficient is smaller than 0, the Hurst exponent value is below the 0.5. This is the same reason explained in 3.4.1.2. Because there is a difference between the values of uptrend and downtrend Hurst exponent, it can be confirmed that there is an asymmetric property. When AR coefficient is positive, uptrend Hurst exponent is lower than downtrend Hurst exponent. When AR coefficient is negative, uptrend Hurst exponent is over than downtrend Hurst exponent. In the case of negative model, similar results are obtained with positive model. As AR coefficient increases, all types of Hurst exponent increases. When AR coefficient is positive, downtrend Hurst exponent value is smaller than uptrend. When AR coefficient is negative, downtrend Hurst exponent value is larger than uptrend. This shows that even if the stock market has autocorrelation of return relationship, the opposite effect can be seen when the asymmetric feature is divided by the price trend viewpoint.

3.4.2.3 Student's t-distribution

Table 3.8 shows the results of the heterogeneous time series simulation of the T-dist with previous data's sign. The results show that all types of Hurst exponent are close to 0.5 similar to homogeneous time series simulation of T-dist results. In case of degree of multifractality, as degree of freedom decreases, degree of multifractality increases for both positive and negative models. When degree of freedom is being smaller, the tail distribution is being heavier. That is, more heavy tail makes the degree of multifractality larger. In particular, it is observed that the uptrend degree of multifractality is larger than the downtrend degree of multifractality in positive model, and vice versa in negative model. In the case of positive model, if x_{t-1} is a positive number, next random value is generated from heavy tail distribution. This situation affects upward trend, so the uptrend degree of multifractality gets larger. It is not as much as uptrend degree of multifractality, but downtrend degree of multifractality also increases slightly by that effect. For the same mechanism, downtrend degree of multifractality is observed to be larger in the negative model.

3.4.2.4 ARCH model and GARCH model

Table 3.9 and Table 3.10 are the results of the heterogeneous time series simulation of the ARCH model and GARCH model with previous data's sign, respectively. In the case of ARCH model results, at first, there is a difference in degree of multifractality between uptrend and downtrend, indicating that there is an asymmetric feature. Secondly, as ARCH coefficient increases, all types of the degree of multifractality increases. Lastly, in the case of positive model, uptrend multifractality is larger than downtrend multifractality and vice versa in negative model. That is, if x_{t-1} is a positive number, next random value has autocorrelation of volatility with previous data, so that upward trend is affected by volatility autocorrelation. Negative model has same mechanism, so that downtrend multifractality is larger than uptrend multifractality. GARCH model results are almost same with ARCH model results. Because GARCH effect has less influence on multifractality than ARCH effect, the increased amount of multifractality is less than the ARCH model.

Table 3.5: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous data's sign (Skewed distribution)

	Positive Skew(-0.6)	Positive Skew(-0.3)	Positive Skew(0.3)	Positive Skew(0.6)	Negative Skew(-0.6)	Negative Skew(-0.3)	Negative Skew(0.3)	Negative Skew(0.6)
$H(2)$	0.5023	0.5044	0.5020	0.5030	0.5041	0.5008	0.5030	0.5012
$H^{+}(2)$	0.5090	0.5091	0.4951	0.4891	0.5122	0.5035	0.4970	0.4900
$H^{-}(2)$	0.4912	0.4947	0.5037	0.5126	0.4918	0.4937	0.5047	0.5086
ΔH	0.0757	0.0797	0.0862	0.0874	0.0897	0.0838	0.0794	0.0760
ΔH^{+}	0.0898	0.0939	0.0988	0.0952	0.1078	0.1009	0.0931	0.0854
ΔH^{-}	0.0845	0.0903	0.1019	0.1047	0.0971	0.0955	0.0918	0.0895
Mean	-0.0019	-0.0005	-0.0010	-0.0001	-0.0012	0.0007	-0.0015	-0.0004
Std	0.9995	0.9989	1.0006	0.9986	1.0004	0.9995	0.9997	1.0000
Skewness	-0.3119	-0.1550	0.1458	0.2818	-0.2854	-0.1465	0.1554	0.3137
Kurtosis	3.0021	2.9926	2.9885	2.9911	2.9914	2.9934	3.0009	3.0081

Table 3.6: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous data's sign (Positive model with AR model)

	Positive AR(-0.8)	Positive AR(-0.6)	Positive AR(-0.4)	Positive AR(-0.2)	Positive AR(0.2)	Positive AR(0.4)	Positive AR(0.6)	Positive AR(0.8)
$H(2)$	0.4227	0.4437	0.4630	0.4820	0.5256	0.5608	0.6189	0.7490
$H^{+}(2)$	0.4249	0.4536	0.4657	0.4816	0.5172	0.5474	0.6019	0.7334
$H^{-}(2)$	0.4179	0.4369	0.4561	0.4768	0.5291	0.5742	0.6547	0.8256
ΔH	0.0564	0.0565	0.0592	0.0707	0.1006	0.1284	0.1632	0.1956
ΔH^{+}	0.2530	0.1849	0.1458	0.1077	0.1061	0.1287	0.1604	0.1908
ΔH^{-}	0.0541	0.0550	0.0593	0.0753	0.1365	0.2203	0.3201	0.4354
Mean	-0.2529	-0.1954	-0.1381	-0.0730	0.0883	0.2097	0.4002	0.8056
Std	1.0868	1.0518	1.0240	1.0057	1.0070	1.0351	1.1061	1.3074
Skewness	-0.1177	-0.0573	-0.0195	0.0008	0.0045	0.0241	0.0939	0.2807
Kurtosis	3.0803	3.0280	3.0034	3.0036	3.0005	3.0033	3.0280	3.1143

Table 3.7: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous data's sign (Negative model with AR model)

	Negative AR(-0.8)	Negative AR(-0.6)	Negative AR(-0.4)	Negative AR(-0.2)	Negative AR(0.2)	Negative AR(0.4)	Negative AR(0.6)	Negative AR(0.8)
$H(2)$	0.4228	0.4422	0.4609	0.4846	0.5263	0.5625	0.6180	0.7503
$H^{+}(2)$	0.4177	0.4354	0.4527	0.4770	0.5325	0.5737	0.6511	0.8215
$H^{-}(2)$	0.4320	0.4578	0.4706	0.4880	0.5171	0.5496	0.6012	0.7335
ΔH	0.0556	0.0582	0.0620	0.0704	0.1015	0.1274	0.1650	0.1968
ΔH^{+}	0.0536	0.0555	0.0622	0.0748	0.1411	0.2207	0.3228	0.4505
ΔH^{-}	0.2431	0.1903	0.1458	0.1055	0.1070	0.1288	0.1620	0.1919
Mean	-0.7963	-0.3978	-0.2094	-0.0887	0.0726	0.1384	0.1946	0.2519
Std	1.3027	1.1048	1.0360	1.0067	1.0052	1.0231	1.0510	1.0868
Skewness	-0.2870	-0.0877	-0.0199	-0.0063	0.0069	0.0187	0.0577	0.1178
Kurtosis	3.1216	3.0241	3.0023	3.0018	2.9928	3.0062	3.0218	3.0682

Table 3.8: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous data's sign (T-dist.)

	Positive T-dist(10)	Positive T-dist(8)	Positive T-dist(6)	Positive T-dist(4)	Negative T-dist(10)	Negative T-dist(8)	Negative T-dist(6)	Negative T-dist(4)
$H(2)$	0.5036	0.5038	0.5028	0.5019	0.5026	0.5041	0.5010	0.5039
$H^{+}(2)$	0.4986	0.4961	0.4944	0.4913	0.5031	0.5069	0.5038	0.5075
$H^{-}(2)$	0.5049	0.5069	0.5062	0.5080	0.4967	0.4964	0.4924	0.4951
ΔH	0.0954	0.1010	0.1168	0.1671	0.0963	0.1014	0.1156	0.1635
ΔH^{+}	0.1152	0.1211	0.1404	0.1872	0.1039	0.1071	0.1199	0.1580
ΔH^{-}	0.1021	0.1082	0.1171	0.1618	0.1180	0.1217	0.1371	0.1823
Mean	-0.0004	-0.0012	0.0003	-0.0003	-0.0016	0.0012	-0.0013	-0.0003
Std	1.0606	1.0777	1.1170	1.2236	1.0593	1.0792	1.1175	1.2246
Skewness	0.0029	0.0035	-0.0005	-0.0121	0.0036	-0.0079	-0.0094	-0.0301
Kurtosis	3.6298	3.9373	4.9311	10.3474	3.6116	3.9538	5.0213	11.2458

Table 3.9: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous data's sign (ARCH model)

	Positive ARCH(0.2)	Positive ARCH(0.4)	Positive ARCH(0.6)	Positive ARCH(0.8)	Negative ARCH(0.2)	Negative ARCH(0.4)	Negative ARCH(0.6)	Negative ARCH(0.8)
$H(2)$	0.5003	0.5022	0.5012	0.5004	0.5033	0.5021	0.5001	0.4979
$H^{+}(2)$	0.4949	0.4986	0.4987	0.5027	0.5024	0.5032	0.4997	0.4961
$H^{-}(2)$	0.5011	0.5000	0.4966	0.4917	0.5000	0.4959	0.4940	0.4937
ΔH	0.0978	0.1209	0.1516	0.2107	0.0965	0.1199	0.1572	0.2116
ΔH^{+}	0.1225	0.1568	0.2008	0.2698	0.0994	0.1094	0.1237	0.1460
ΔH^{-}	0.1016	0.1102	0.1255	0.1602	0.1221	0.1569	0.2065	0.2759
Mean	0.0000	0.0001	0.0003	-0.0005	-0.0009	0.0010	-0.0011	0.0003
Std	0.9990	0.9998	1.0009	0.9942	0.9985	0.9996	0.9988	0.9959
Skewness	-0.0006	-0.0034	0.0004	-0.0401	0.0026	0.0073	0.0049	0.0330
Kurtosis	3.1105	3.6004	4.8134	7.2144	3.1172	3.5695	4.7290	7.0189

Table 3.10: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous data's sign (GARCH model)

	Positive GARCH(0.2)	Positive GARCH(0.4)	Positive GARCH(0.6)	Positive GARCH(0.8)	Negative GARCH(0.2)	Negative GARCH(0.4)	Negative GARCH(0.6)	Negative GARCH(0.8)
$H(2)$	0.5035	0.5032	0.5035	0.5034	0.5033	0.4995	0.5032	0.5032
$H^{+}(2)$	0.5019	0.4985	0.5010	0.5008	0.5006	0.4980	0.5041	0.5006
$H^{-}(2)$	0.5002	0.5032	0.5014	0.5012	0.5009	0.4965	0.4980	0.5002
ΔH	0.0884	0.0902	0.0894	0.0928	0.0916	0.0906	0.0912	0.0906
ΔH^{+}	0.1072	0.1107	0.1103	0.1155	0.1002	0.0994	0.0965	0.0952
ΔH^{-}	0.0990	0.0985	0.0980	0.0991	0.1105	0.1101	0.1133	0.1162
Mean	-0.0008	0.0005	0.0004	0.0011	-0.0016	0.0004	-0.0015	-0.0011
Std	0.9996	1.0007	1.0001	0.9984	1.0000	0.9996	1.0014	0.9986
Skewness	0.0003	0.0063	0.0026	-0.0016	0.0010	0.0007	0.0018	0.0010
Kurtosis	3.0320	3.0227	3.0327	3.0398	3.0280	3.0297	3.0260	3.0382

3.4.3 Heterogeneous time series simulation with previous data's trend results

In the last part, Monte-Carlo simulation results of heterogeneous time series with previous data trend are shown which generates x_t differently depending on the trend of previous data series. Trend from x_{t-d} to x_{t-1} is determined by the sign of $\sum_{i=1}^{d} x_{t-i}$ to capture the stock market's price trend. In the case of the positive model, if the sign of the previous trend is a positive number, then next number is generated from specific distribution, and if the sign is a negative, the next data is generated from standard normal distribution. Negative model is generated reversed with positive model. Since the trend cannot be measured if the past data has no d data, first d data is generated from the standard normal distribution. This implied that heterogeneous data is generated from *d+1* data. The results for *d = 20* are represented in this chapter. Other results for *d=10, 30, 40, 50* are in Appendix A.

3.4.3.1 Skewed distribution

Table 3.11 shows the results of the heterogeneous time series simulation of the skewed distribution with the previous data trend. The results are similar with heterogeneous time series simulation of the skewed distribution with previous data sign in chapter 3.4.2.1. In both positive and negative models, as the skewness increases, the uptrend Hurst exponent decreases and the downtrend Hurst exponent increases. The smaller the skewness affects the higher the probability that a positive value will be generated. It increases the long-range dependence of upward trend. On the contrary, the larger the skewness affects the higher the probability that the negative value will be generated. It increases the long-range dependence of downward trend.

3.4.3.2 AR model

Table 3.12 and Table 3.13 show the results of the heterogeneous time series simulation of the AR model with previous data trends. Table 3.12 shows the positive model results and Table 3.13 shows the results of the negative model. In the case of positive model, all types of Hurst exponent increases as AR coefficient increases. When AR coefficient is over than zero, Hurst exponent is over than 0.5 and when AR coefficient is below than zero, Hurst exponent is less than 0.5. Since there is a difference between uptrend and downtrend Hurst exponent, this time series has asymmetric feature. When AR coefficient is positive, uptrend Hurst exponent value is over than downtrend one. When AR coefficient is negative, uptrend Hurst exponent value is smaller than downtrend one. In case that AR coefficient is positive, since the AR model is generated only when the previous trend is positive, the autocorrelation of the uptrend increases and the uptrend Hurst exponent value is larger. If the AR coefficient is negative, the uptrend Hurst exponent value is lowered because data with negative autocorrelation is generated when trend is positive.

In case of negative model, opposite results of positive model are obtained. As AR coefficient increases, all types of the Hurst exponent value increase. In case that AR coefficient is positive, downtrend Hurst exponent is over than uptrend because generating a data with a positive autocorrelation when the trend is negative makes downtrend long-range dependence larger. This result is opposite result of heterogeneous time series simulation of AR model with previous data's sign in 3.4.2.3.

3.4.3.3 Student's t-distribution

Table 3.14 shows the results of the heterogeneous time series simulation of the T-dist with previous data trends. In case of the positive model results, as degree of freedom decreases, uptrend Hurst exponent increases and over than 0.5 and downtrend Hurst exponent decreases and lower than 0.5. When previous trend is positive, next data is generated from heavy-tailed distribution. It makes the variance of uptrend long term period large. That is why uptrend Hurst exponent increases as the tail distribution is heavier. In the case of the downtrend Hurst exponent, the variance becomes smaller in the downtrend long period due to the reflection effect. In case of negative model, opposite results of positive model are obtained.

In case of multifractality, all types of degree of multifractality increases as tail distribution is heavier. Uptrend multifractality is largest one in positive model and downtrend multifractality is largest one in negative model. The reason is that positive model generate fat-tailed data when trend is only positive, so uptrend multifractality is larger, and negative model generate fat-tailed data when trend is only negative, so downtrend multifractality is larger.

3.4.3.4 ARCH model and GARCH model

Table 3.15 and Table 3.16 show the results of the heterogeneous time series simulation of the ARCH model and GARCH model with previous data trends, respectively. Table 3.15 shows the ARCH model results and Table 3.16 shows the results of the GARCH model. In case of ARCH model results, this generated time series has asymmetric feature since there is a difference between uptrend and downtrend degree of multifractality. Also, all types of the degree of multifractality increases as ARCH coefficient increases. In case of positive model, uptrend multifractality is little larger than negative multifractality. In case of negative model, opposite phenomenon is occurred. When previous trend is positive, the data which is autocorrelated with past data’s volatility is generated, so uptrend multifractality is being larger.

Negative model has same mechanism. In case of GARCH model, there are similar results with ARCH model but increasing rate of multifractality is smaller than ARCH model.

In Appendix A, the results of d=10, 30, 40, 50 are existed. When d is 30 or more, in case of positive model, uptrend degree of multifractality is lower than downtrend one, and in case of negative model, downtrend degree of multifractality is lower than uptrend one.

Table 3.11: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous 20 data's trend (Skewed dist.)

Day=20	Positive Skew(-0.6)	Positive Skew(-0.3)	Positive Skew(0.3)	Positive Skew(0.6)	Negative Skew(-0.6)	Negative Skew(-0.3)	Negative Skew(0.3)	Negative Skew(0.6)
$H(2)$	0.5025	0.5052	0.5028	0.5028	0.5053	0.5018	0.5007	0.5046
$H^{+}(2)$	0.5123	0.5080	0.4959	0.4891	0.5152	0.5041	0.4920	0.4907
$H^{-}(2)$	0.4883	0.4974	0.5042	0.5119	0.4920	0.4945	0.5050	0.5138
ΔH	0.0907	0.0861	0.0762	0.0746	0.0731	0.0796	0.0883	0.0913
ΔH^{+}	0.1075	0.1016	0.0899	0.0851	0.0884	0.0941	0.0995	0.0982
ΔH^{-}	0.0970	0.0966	0.0902	0.0909	0.0826	0.0944	0.1037	0.1061
Mean	-0.0001	-0.0003	-0.0001	0.0012	-0.0007	0.0005	-0.0002	-0.0007
Std	1.0000	1.0003	0.9989	1.0004	1.0001	1.0002	1.0007	1.0001
Skewness	-0.2952	-0.1427	0.1478	0.2908	-0.2913	-0.1497	0.1481	0.2936
Kurtosis	2.9906	2.9929	2.9939	2.9897	2.9946	2.9993	2.9973	2.9984

Table 3.12: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous 20 data's trend (Positive model with AR model)

Day=20	Positive AR(-0.8)	Positive AR(-0.6)	Positive AR(-0.4)	Positive AR(-0.2)	Positive AR(0.2)	Positive AR(0.4)	Positive AR(0.6)	Positive AR(0.8)
$H(2)$	0.4172	0.4456	0.4661	0.4826	0.5267	0.5685	0.6416	0.8013
$H^{+}(2)$	0.3988	0.4250	0.4467	0.4709	0.5318	0.5822	0.6591	0.8199
$H^{-}(2)$	0.4166	0.4482	0.4717	0.4863	0.5132	0.5329	0.5779	0.6918
ΔH	0.0866	0.0642	0.0648	0.0726	0.0999	0.1253	0.1446	0.1461
ΔH^{+}	0.1192	0.0860	0.0813	0.0847	0.1208	0.1609	0.2115	0.2371
ΔH^{-}	0.0930	0.0783	0.0798	0.0897	0.1108	0.1371	0.1578	0.1616
Mean	-0.0646	-0.0455	-0.0318	-0.0159	0.0188	0.0483	0.1015	0.2531
Std	1.2028	1.0978	1.0391	1.0090	1.0096	1.0452	1.1298	1.3504
Skewness	-0.0496	-0.0239	-0.0095	0.0016	0.0033	0.0117	0.0403	0.1714
Kurtosis	3.4947	3.1108	3.0128	3.0016	3.0008	3.0148	3.1054	3.4202

Table 3.13: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous 20 data's trend (Negative model with AR model)

Day=20	Negative AR(-0.8)	Negative AR(-0.6)	Negative AR(-0.4)	Negative AR(-0.2)	Negative AR(0.2)	Negative AR(0.4)	Negative AR(0.6)	Negative AR(0.8)
$H(2)$	0.4153	0.4447	0.4629	0.4849	0.5282	0.5702	0.6417	0.7992
$H^{+}(2)$	0.4126	0.4456	0.4647	0.4861	0.5155	0.5309	0.5531	0.6111
$H^{-}(2)$	0.4057	0.4285	0.4496	0.4766	0.5336	0.5857	0.6679	0.8319
ΔH	0.0866	0.0669	0.0678	0.0721	0.1002	0.1247	0.1459	0.1479
ΔH^{+}	0.0917	0.0783	0.0836	0.0878	0.1117	0.1414	0.1827	0.2213
ΔH^{-}	0.1161	0.0854	0.0792	0.0836	0.1211	0.1595	0.2087	0.2464
Mean	0.0634	0.0446	0.0322	0.0157	-0.0190	-0.0485	-0.0996	-0.2455
Std	1.2055	1.0965	1.0378	1.0086	1.0091	1.0460	1.1283	1.3435
Skewness	0.0490	0.0246	0.0081	0.0052	-0.0046	-0.0071	-0.0391	-0.1843
Kurtosis	3.5104	3.0985	3.0156	2.9914	3.0030	3.0183	3.0984	3.4380

Table 3.14: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous 20 data's trend (T-dist.)

Day=20	Positive T-dist(10)	Positive T-dist(8)	Positive T-dist(6)	Positive T-dist(4)	Negative T-dist(10)	Negative T-dist(8)	Negative T-dist(6)	Negative T-dist(4)
$H(2)$	0.5014	0.5031	0.5034	0.5037	0.5019	0.5024	0.5050	0.5036
$H^{+}(2)$	0.5089	0.5136	0.5193	0.5282	0.4896	0.4857	0.4853	0.4722
$H^{-}(2)$	0.4889	0.4883	0.4833	0.4739	0.5097	0.5143	0.5195	0.5307
ΔH	0.0976	0.1049	0.1127	0.1562	0.0955	0.1039	0.1122	0.1549
ΔH^{+}	0.1150	0.1233	0.1396	0.1837	0.1060	0.1148	0.1204	0.1630
ΔH^{-}	0.1099	0.1164	0.1212	0.1633	0.1152	0.1264	0.1385	0.1838
Mean	-0.0024	-0.0001	-0.0001	-0.0001	-0.0014	0.0012	-0.0021	0.0001
Std	1.0570	1.0755	1.1100	1.2042	1.0575	1.0751	1.1117	1.2041
Skewness	-0.0023	0.0035	0.0052	-0.0397	-0.0002	-0.0056	0.0096	-0.0595
Kurtosis	3.6311	3.9820	4.8155	10.6141	3.6375	3.9856	4.8259	10.4612

Table 3.15: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous 20 data's trend (ARCH model)

Day=20	Positive ARCH(0.2)	Positive ARCH(0.4)	Positive ARCH(0.6)	Positive ARCH(0.8)	Negative ARCH(0.2)	Negative ARCH(0.4)	Negative ARCH(0.6)	Negative ARCH(0.8)
$H(2)$	0.5002	0.5016	0.5016	0.4992	0.5035	0.5025	0.5002	0.4979
$H^{+}(2)$	0.4966	0.4979	0.4924	0.4792	0.5007	0.5028	0.5027	0.5090
$H^{-}(2)$	0.4991	0.5003	0.5042	0.5132	0.5011	0.4971	0.4916	0.4808
ΔH	0.0983	0.1267	0.1723	0.2686	0.0962	0.1242	0.1722	0.2649
ΔH^{+}	0.1166	0.1467	0.1912	0.2794	0.1057	0.1304	0.1748	0.2753
ΔH^{-}	0.1089	0.1306	0.1748	0.2760	0.1150	0.1444	0.1877	0.2764
Mean	-0.0002	-0.0003	0.0004	-0.0006	-0.0008	0.0010	-0.0009	0.0002
Std	0.9984	0.9980	0.9918	0.9597	0.9984	0.9985	0.9884	0.9624
Skewness	-0.0010	-0.0068	0.0043	-0.0935	0.0007	0.0084	0.0036	0.0491
Kurtosis	3.1126	3.6405	5.1270	8.6073	3.1183	3.6123	4.9705	8.8644

Table 3.16: Average of asymmetric Hurst exponent, asymmetric degree of multifractality, mean, standard deviation (Std), Skewness and Kurtosis for each simulated heterogeneous time series with previous 20 data's trend (GARCH model)

Day=20	Positive GARCH(0.2)	Positive GARCH(0.4)	Positive GARCH(0.6)	Positive GARCH(0.8)	Negative GARCH(0.2)	Negative GARCH(0.4)	Negative GARCH(0.6)	Negative GARCH(0.8)
$H(2)$	0.5037	0.5034	0.5035	0.5030	0.5026	0.4997	0.5030	0.5027
$H^{+}(2)$	0.5011	0.4996	0.5018	0.5004	0.4994	0.4970	0.5030	0.5001
$H^{-}(2)$	0.5015	0.5026	0.5004	0.5001	0.5013	0.4969	0.4973	0.4997
ΔH	0.0885	0.0914	0.0950	0.1083	0.0915	0.0918	0.0967	0.1083
ΔH^{+}	0.1053	0.1077	0.1104	0.1247	0.1031	0.1043	0.1076	0.1163
ΔH^{-}	0.1030	0.1045	0.1075	0.1193	0.1068	0.1075	0.1150	0.1283
Mean	-0.0005	0.0005	0.0004	0.0013	-0.0016	0.0004	-0.0017	-0.0012
Std	0.9996	1.0011	1.0005	1.0004	1.0003	0.9994	1.0019	1.0003
Skewness	0.0018	0.0061	0.0034	-0.0021	0.0000	0.0011	0.0006	0.0026
Kurtosis	3.0320	3.0277	3.0425	3.0954	3.0296	3.0343	3.0396	3.1070

3.5 Conclusion

In this chapter, the asymmetric features of long-range dependence and multifractality are examined by simulation analysis. To change autocorrelation and skewness of data to affect long-range dependence, AR model and Skewed distribution is used to generate time series. Multifractality is affected by heavy-tail distribution and autocorrelation of volatility. Therefore, student's t-distribution and ARCH, GARCH model are used to generate time series, too. In addition, in order to incorporate asymmetric characteristics into the time series, there are three ways to generate time series. The first method is to generate data regardless of past values named 'homogeneous time series'. The second and third are to generate random numbers from different models depending on past values and past trend values named 'heterogeneous time series'. This implies that the time series generated through the second and third method has asymmetric features. Using the price-based A-MFDFA model, asymmetric Hurst exponent and degree of multifractality are obtained for various generated time series, and characteristics of the asymmetric long-range dependence and asymmetric multifractality are examined.

The results are summarized as follows. At first, the simulation results of the homogeneous model are symmetric except skewed distribution. In other words, there is no difference between uptrend and downtrend results. Because time series generated by skewed distribution has autocorrelation, heterogeneous nature, asymmetric results are obtained. In case of heterogeneous model, asymmetric results are obtained in all generated time series. This means that the price-based A-MFDFA model captures asymmetric properties well because time series with artificially asymmetric features are examined. Second, the results of the simulated heterogeneous time series with the previous data and the previous data trend are as expected from the positive model and the negative model, respectively. In the case of the positive model, uptrend feature is observed more than downtrend feature according to desired effect. In case of the negative model, down trend feature is larger than uptrend feature. In case of AR model with negative AR coefficient, negative autocorrelation effect is applied, so that result is reversed as expected. The most interesting case is the AR model of the simulated heterogeneous time series with previous data. The downtrend Hurst exponent value is higher than the uptrend one when the AR coefficient is positive in the positive model. This result means that the autocorrelation feature of the return series has the opposite effect when the trend is divided with price.

Chapter 4

Evaluating the asymmetric long-range dependence and multifractality of financial markets

4.1 Introduction

Since the work of Fama (1970), the theory of Efficient Market Hypothesis (EMH) has been popular in the field of Finance. The term, *Efficient*, implies the condition when the market reflects all available information. The weak-form of EMH states that the past information is reflected in the current asset price. Therefore, the exceed return cannot be obtained based on its past price. In other words, the price moves a random walk with a zero mean or a positive drift. However, it is now widely known that the price does not follow the random walk when the market is inefficient. Furthermore, the long memory exists in the market when the market does not follow the random walk. For instance, Kim and Shamsuddin (2008) test for the martingale hypothesis in the stock prices of Asian markets using multiple variance ratios test, whereas Rounaghi and Zadeh (2016) measure the market efficiency using ARMA model and predict stock returns.

In this context, many studies in Econophysics have focused on measuring the market efficiency by analyzing the long memory based on the multifractal theory. Multifractality in financial markets has been studied for discovering various stylized facts and applications including stock markets (Sun et al.

2001; Lee and Chang 2015; Stošić et al. 2015), foreign exchange market (Oh et al. 2012; Stošić et al. 2015), market crash prediction (Grech and Mazur 2004) and market risk (Lee et al. 2016). The Multi-Fractal Detrended Fluctuation analysis (MFDFA) method (Kantelhardt et al. 2002) is a typical approach to measure the multifractal features of a time series. The main approaches that have been utilized for the market efficiency include the Hurst Exponent (Liu et al. 2010; Wang et al. 2010; Horta et al. 2014; Arshad et al. 2016; Sensoy and Tabak 2016; Mensi et al. 2017; Shahzad et al. 2017), the Generalized Hurst exponent approach (Di Matteo et al. 2003), the rescaled range statistical analysis (Cajueiro and Tabak 2004; Cajueiro and Tabak 2004; Hull and McGroarty 2014), the Detrended Fluctuation analysis(DFA) (Wang et al. 2009; Wang et al. 2010), the Multifractal Detrended Cross-Correlation Analysis (Shahzad et al. 2017) and the mixture of DFA, Detrended Moving Average and Height-height correlation analysis (Kristoufek and Vosvrda 2013). The efficient market implies the expectation of rationality among investors. Thus, the inefficient market is realized when the investor expresses the irrationality due to a specific incident. The previous studies based on the multifractal theory can provide the overall market efficiency, but they also have limitations in discovering the source of inefficiency.

The possible sources of market inefficiency are the overheated bull market with false hope and the crisis-phase bearish market with excessive fears. The source in bull market is caused by the irrationality in long position, whereas the source in bear market is caused by the irrationality in short position. Therefore, in this chapter, asymmetric long-range dependence and degree of multifractality is measured to investigate the source of market inefficiency. Asymmetric long-range dependence and multifractality are defined as measured long-range dependence and multifractality by dividing the stock market according to the market trends in which the stock price rises or falls. Using the price-based Asymmetric-MFDFA method introduced in Chapter 2 (Lee et al. 2017), we compute the generalized Hurst exponent by distinguishing the long-range correlation for overall, up-trend and down-trend, simultaneously. Then, we analyze the asymmetric long-range dependence and degree of multifractality by discussing the difference among the asymmetric generalized Hurst exponent for different trends to access the market efficiency according to market trends. If there is an up-trend long-range dependence, the source of market inefficiency in the stock market is from the bull-market. In contrary, if down-trend long-range dependence is detected, then bear market is the source of market inefficiency.

We mainly focus on three different approaches for the analysis. The first approach is generating the theoretical values of the generalized Hurst exponent and asymmetric one based on the Monte Carlo simulation with the time series following the Brownian motion. Furthermore, we suggest the nonparametric significance level as a proxy to test if the empirical values of the Hurst exponent and degree of multifractality are in the range of the random walk. The second approach is measuring the cross-sectional asymmetric generalized Hurst exponent for 34 countries and four different sub-periods to discover the impact of the financial crisis to the market long-range dependence and multifractality. The last approach is analyzing the time-varying aspects of the asymmetric generalized Hurst exponent and its relation to the stock market indices based on the moving window method. In previous studies, it is discovered that the moving window method in Econophysics can provide the time-varying aspects of financial market in details (Carbone et al. 2004; Jang et al. 2011; Song et al. 2016).

This chapter is organized as follows: Section 4.2 explains the mathematical background of the Asymmetric-MFDFA method; Section 4.3 describes the stock market data; Section 4.4 discusses the results of the empirical findings; and Section 4.5 concludes.

4.2 Methodology

4.2.1 Price-based A-MFDFA

The asymmetric multifractal scaling behavior can be discovered using the price-based A-MFDFA method. Let $\{x_t: t = 1,2,\ldots,N\}$ be a return time series, then the price-based A-MFDFA can be computed in following steps (Lee et al. 2017).

Step 1: Define $\mathrm{y_t} = \sum_{j=1}^{t}(x_j - \bar{x})$, $t = 1,2,\ldots,N$ where $\bar{x} = \sum_{j=1}^{N} x_j/N$.

Step 2: Divide the time series into non-overlapping sub-time series

Let $I_t = I_{t-1}\, exp(x_t)$ for $t = 1,2,\ldots,N$, where $I_0 = 1$ and I_t is a price proxy for return time series. Then, $\{I_t: t = 1,2,\ldots,N\}$ and $\{\mathrm{y_t}: t = 1,2,\ldots,N\}$ can be divided into $N_n \equiv \lfloor N/n \rfloor$ non-overlapping sub-time series of equal length n. Note that $\lfloor x \rfloor$ is the largest integer less than or equal to x. This procedure is repeated from the other end of $\{I_t\}$ and $\{\mathrm{y_t}\}$, respectively, which yields $2N_n$ sub-time series. Suppose $G_j = \{g_{j,k}, k = 1, 2, \ldots, n\}$ be the n-length sub-time series of $\{I_t\}$ in the j^{th} time interval and $H_j = \{h_{j,k}, k = 1,2,\ldots,n\}$ be the j^{th} sub-time series of $\{\mathrm{y_t}\}$ for $j = 1,2,\ldots,2N_n$. Then, $g_{j,k} = I_{(j-1)n+k}$ and $h_{j,k} = y_{(j-1)n+k}$ represent $j = 1, 2,\ldots, N_n$, whereas $g_{j,k} = I_{N-(j-N_n)n+k}$ and $h_{j,k} = y_{N-(j-N_n)n+k}$ represent $j = N_n + 1,\ldots, 2N_n$. Note that $10 \leq n \leq N/4$.

Step 3: Define the fluctuation function

For each sub-time series G_j and H_j, the local trend can be calculated based on least-squares fits $L_{G_j}(k) = a_{G_j} + b_{G_j}k$ and $L_{H_j}(k) = a_{H_j} + b_{H_j}k$, where k refers to the horizontal coordinate. That is, the positive or negative trend of G_j is depending on the slope of $L_{G_j}(k)$, b_{G_j}. Note that the integrated time series H_j is detrended by the linear fitting equation, L_{H_j}. The fluctuation function is then defined as $F_j(n) = \sum_{k=1}^{n} \left(h_{j,k} - L_{H_j}(k)\right)^2 /n$ for $j = 1, 2, \ldots, 2N_n$.

Step 4: Determine the trend

Let $\{I_t\}$ has a piecewise positive and negative linear trend, then the asymmetric cross-correlation scaling property of fluctuation functions can be determined by the sign of the slope, b_{G_j}. Therefore, $b_{G_j} \geq 0$ indicates a positive trend of sub-time series G_j of $\{I_t\}$, whereas $b_{G_j} < 0$ indicates the negative trend.

Step 5: Define the q-order average fluctuation functions

Let $M^+ = \sum_{j=1}^{2N_n}(1 + sgn(b_{G_j}))$, $M^- = \sum_{j=1}^{2N_n}(1 - sgn(b_{G_j}))$, and $sgn(x)$ denotes the sign of x, then the directional q-order average fluctuation functions of the price-based model (when $q \neq 0$) can be defined as $F_q^+(n) = \left(\sum_{j=1}^{2N_n}(1 + sgn(b_{G_j}))\left[F_j(n)\right]^{q/2}/M^+\right)^{1/q}$ and $F_q^-(n) = \left(\sum_{j=1}^{2N_n}(1 - sgn(b_{G_j}))\left[F_j(n)\right]^{q/2}/M^-\right)^{1/q}$. Note that $b_{G_j} \neq 0$ and $M^+ + M^- = 4N_n$. In addition, the average fluctuation function of MF-DFA model is $F_q(n) = \left(\sum_{j=1}^{2N_n}\left[F_j(n)\right]^{q/2}/(2N_n)\right)^{1/q}$.

Step 6: Calculate the generalized Hurst exponent

The power-law relationship is observed in a time series which possess the long-range correlation. Let $H(q)$, $H^+(q)$, and $H^-(q)$ refers to the overall, up-trend, and down-trend scaling exponents, which are called the generalized Hurst exponents, respectively. Note that the scaling satisfies, $F_q(n) \sim n^{H(q)}$, $F_q^+(n) \sim n^{H^+(q)}$, and $F_q^-(n) \sim n^{H^-(q)}$. $H(q)$, $H^+(q)$, and $H^-(q)$ can be obtained by the ordinary least square method based on the logarithmic form.

The time series is mono-fractal when its $H(q)$ is constant for all q. Otherwise, the time series is multi-fractal. Furthermore, the correlation in the time series is persistent when $H(2) > 0.5$, whereas the correlation is anti-persistent when $H(2) < 0.5$. The time series follows random walk process when $H(2) = 0.5$ (Kantelhardt et al. 2002). In the same context of $H(q)$, the up-trend or down-trend time series are multi-fractal when its $H^{+}(q)$ or $H^{-}(q)$ depends on q, respectively. Specifically, the correlation in the time series is symmetric if $H^{+}(q) = H^{-}(q)$, whereas the correlation is asymmetric if $H^{+}(q) \neq H^{-}(q)$. The asymmetric scaling behavior indicates that the correlation is different between positive and negative trends.

4.2.2 Evaluating the existence of asymmetric long-range dependence and multifractality

In this study, the existence of asymmetric long-range dependence and multifractality are conducted to access the source of market efficiency using the asymmetric generalized Hurst exponent. Basically, the efficient market shows that its generalized Hurst exponents for all q's are equal to 0.5. Therefore, long-range dependence is evaluated based on how much $H(q)$ is far from 0.5. In addition, degree of multifractality is the difference of maximum value and minimum value of $H(q)$.

4.2.2.1 Testing the existence of asymmetric long-range dependence

The first test is evaluating the existence of long-range dependence based on how much the Hurst exponent, $H(2)$, is far from the 0.5. $H(2)$ is the Hurst exponent corresponding to the scaling of the second moment which indicates the autocorrelation (q=2) with the time horizon. The time invariant scaling of variance is studied in Peng et al. (1994) by analyzing the multifractality in q=2. When the Hurst exponent is not 0.5, the market can be considered as inefficient due to the existence of the long-range correlation.

The Hypothesis is as follows;

$$\mathrm{H}_0: H(2) = 0.5 \ \ vs\ \mathrm{H}_1: H(2) \neq 0.5$$

To test the existence of asymmetric long-range dependence, we use $H^{+}(2)$ and $H^{-}(2)$ instead of $H(2)$.

4.2.2.2 Testing the existence of asymmetric multifractality

The second test is focusing on the extreme value of the generalized Hurst exponent. Wang et al. (2009) uses the degree of multifractality to test the market inefficiency. The scale exponents $\max_q H(q)$ and $\min_q H(q)$ are considered as the extreme cases of the variations. Therefore, the degree of multifractality can be measured using the following equation,

$$\text{Degree of multifractality}(\Delta H) = \max_q H(q) - \min_q H(q)$$

Hence, the hypothesis based on the degree of multifractality is,

$$\mathrm{H}_0: \Delta H = 0 \ \ vs\ \mathrm{H}_1: \Delta H > 0$$

To test the existence of asymmetric multifractality, $H^{+}(q)$ and $H^{-}(q)$ are utilized instead of $H(q)$.

4.2.2.3 Grouping with respect to the existence of long-range dependence and multifractality

Stock markets in each country can be classified into eight different groups since the asymmetric long-range dependence and multifractality are analyzed based on overall, up-trend, and down-trend trends.

As illustrated in Table 4.1, the mark “O” refers to the no long-range dependence or no multifractality, whereas "X" refers that market has long-range dependence or multifractality. By classifying the groups, we can investigate the sources of market inefficiency in details.

Table 4.1: Groups with respect to asymmetric long-range dependence and multifractality

	Overall market	Up-trend market	Down-trend market
Group 1	O	O	O
Group 2	O	O	X
Group 3	O	X	O
Group 4	O	X	X
Group 5	X	O	O
Group 6	X	O	X
Group 7	X	X	O
Group 8	X	X	X

4.3 Data description

We analyze the existence of asymmetric long-range dependence and multifractality of stock indices in 34 countries. The daily indices are obtained from the Thomson DataStream in their local currency. Table 4.2 shows the description of the selected countries and the tick information in the DataStream. The total period for analysis is considered from 2003-01-01 to 2016-12-31. The total period is then divided into four sub-periods: Pre-crisis (from 2005-01-03 to 2007-07-31), Subprime-crisis (from 2007-08-01 to 2009-12-07), European-crisis (from 2009-12-08 to 2012-04-27), and Post-crisis (2012-04-28 to 2016-12-31). Note that this sub-periods are also considered in Horta et al. (2014). We transform the indices into the logarithmic return-series, $r_t = \log(P_t) - \log(P_{t-1})$, where P_t is the closing price of each index at time t.

Table 4.2: List of selected country ticker, country name, index name, stock exchange market name and DataStream code

	Ticker	Country	Index	Stock Exchange Market	DataStream code
1	ARG	Argentina	Buenos Aires Stock Exchange Merval Index	Buenos Aires Stock Exchange	ARGMERV
2	AUS	Australia	S&P/ASX 200 Index	Australian Securities Exchange	ASX200I
3	BEL	Belgium	BEL 20 Index	Brussels Stock Exchange	BGBEL20
4	BRA	Brazil	Ibovespa Brasil Sao Paulo Stock Exchange Index	Sao Paulo Stock Exchange	BRBOVES
5	CAN	Canada	S&P/TSX Composite Index	Toronto Stock Exchange	TTOCOMP
6	CHL	Chile	Santiago Stock Exchange IGPA Index	Santiago Stock Exchange	IGPAGEN
7	CHN	China	Shanghai Stock Exchange Composite Index	Shanghai Stock Exchange	CHSASHR
8	CZE	Czech Republic	Prague Stock Exchange Index	Prague Stock Exchange	CZPXIDX
9	DNK	Denmark	OMX Copenhagen 20 Index	Copenhagen Stock Exchange	DKKFXIN
10	FIN	Finland	OMX Helsinki Index	Helsinki Stock Exchange	HEXINDX
11	FRA	France	CAC 40 Index	Paris Stock Exchange	FRCAC40
12	DEU	Germany	Deutsche Boerse AG German Stock DAX Index	Frankfurt Stock Exchange	DAXINDX
13	GRC	Greece	Athens Stock Exchange General Index	Athens Stock Exchange	GRAGENL
14	HKG	Hong Kong	Hong Kong Hang Seng Index	Hong Kong Stock Exchange	HNGKNGI
15	IND	India	S&P BSE SENSEX Index	Bombay Stock Exchange	IBOMSEN
16	IDN	Indonesia	Jakarta Stock Exchange Composite Index	Jakarta Stock Exchange	JAKCOMP
17	IRL	Ireland	Irish Stock Exchange Overall Index	Irish Stock Exchange	ISEQUIT
18	ISR	Israel	Tel Aviv Stock Exchange 125 Index	Tel Aviv Stock Exchange	ISTA100
19	ITA	Italy	FTSE MIB Index	Milan Stock Exchange	FTSEMIB

Table 4.2(continue)

	Ticker	Country	Index	Stock Exchange Market	DataStream code
20	JPN	Japan	Nikkei 225 Index	Tokyo Stock Exchange	JAPDOWA
21	KOR	Korea	Korea Stock Exchange KOSPI Index	Korea Stock Exchange	KORCOMP
22	MEX	Mexico	Mexican Stock Exchange Mexican Bolsa IPC Index	Mexican Stock Exchange	MXIPC35
23	NLD	Netherlands	AEX-Index	Amsterdam Stock Exchange	AMSTEOE
24	NZL	New Zealand	S&P/NZX 10 Index	New Zealand Stock Exchange	NZ10CAP
25	PAK	Pakistan	Karachi Stock Exchange KSE100 Index	Karachi Stock Exchange	PKSE100
26	PHL	Philippines	Philippines Stock Exchange PSEi Index	Philippine Stock Exchange	PSECOMP
27	PRT	Portugal	PSI 20 Index	Lisbon Stock Exchange	POPSI20
28	RUS	Russia	MICEX Index	Moscow Exchange	RSMICEX
29	ZAF	South Africa	FTSE/JSE Africa All Share Index	Johannesburg Stock Exchange	JSEOVER
30	ESP	Spain	IBEX 35 Index	Madrid Stock Exchange	IBEX35I
31	SWE	Sweden	OMX Stockholm 30 Index	Stockholm Stock Exchange	SWEDOMX
32	CHE	Switzerland	Swiss Market Index	SIX Swiss Exchange	SWISSMI
33	GBR	United Kingdom	FTSE 100 Index	London Stock Exchange	FTSE100
34	USA	United States	S&P 500 Index	New York Stock Exchange / Nasdaq Exchange	S&PCOMP

4.4 Results and Discussions

We analyze the asymmetric long-range dependence and multifractality in stock markets based on three different results. At first, we compute the theoretical values and their confidence intervals of the Hurst exponent and the degree of multifractality using the standard normal distribution generated by the Monte Carlo simulation. Secondly, we test the existence of asymmetric long-range dependence and multifractality in the stock indices of 34 countries for different sub-periods. Lastly, we discover the time-varying characteristics of asymmetric long-range dependence and degree of multifractality based on the moving window method, which reveals the association with the stock indices.

4.4.1 Monte Carlo Simulation

When $H(q) = 0.5$, a return time series of stock index price follows the random walk, and the underlying market is considered as an efficient market. Otherwise, the underlying market is inefficient and presents the long-term dependence. Therefore, we need to set the statistical criterion for discriminating whether the generalized Hurst exponent is 0.5 or not.

Since $H(q) = 0.5$ can be obtained in the Brownian motion, we compute the theoretical values of $H(2)$ using the Monte Carlo simulation as follows. At first, we generate the values of standard normal distribution for the length of 250×2, 250×3, 250×4, 250×5, 250×12, which are corresponding to the 2, 3, 4, 5, and 12 years of trading days, respectively. Using 5000 iterations, we create the 5000 generalized Hurst exponents for each time length. Then, we calculate the mean, standard deviation, and nonparametric confidence intervals based on the theoretical Hurst exponents. Furthermore, we compute degree of multifractality $\Delta H = \max_q H(q) - \min_q H(q)$ so that the mean, standard deviation, and nonparametric confidence intervals for degree of multifractality can be obtained for each length. Note that the confidence interval of the Hurst exponent, $H(2)$, is used as the two-sided test, whereas that of degree of multifractality is used as the one-sided test.

4.4.1.1 $\boldsymbol{H(2)}$ simulation for long-range dependence

Table 4.3 shows the Monte Carlo simulation results for various time lengths and trends regarding the Hurst exponent, $H(2)$. Comparing the result among trends, the overall trend shows the mean of $H(2)$ closer to 0.5 and smaller standard deviation then those of up-trend and down-trend. The result relies on the fact that the time length is smaller in the up-trend and down-trend since the time series is divided into two sub-time series, which reduces the robustness of simulation. However, we observe that as the time length increases the mean of $H(2)$ and standard deviation develops into 0.5 and smaller value for all trends. Based on this simulation result, we test the hypothesis regarding the empirical values of $H(2)$ using the nonparametric 95% confidence intervals.

4.4.1.2 Degree of multifractality simulation

Table 4.4 shows the results of Monte Carlo simulation for various time lengths and trends regarding degree of multifractality. Analogous to the result in Table 4.3, as time length increases, the mean of degree of multifractality, standard deviation, and the confidence interval converges to 0, smaller value, and smaller interval, respectively. Furthermore, degree of multifractality also shows more robust result in overall than the up-trend and down-trend. Note that the form of confidence interval for degree of multifractality is different to that of $H(2)$. $H(2)$ requires the two-sided test, which indicates the quantile interval 0.025 – 0.975 to compute the confidence interval. In contrast, degree of multifractality requires the one-sided test, which indicates the quantile interval 0 - 0.950.

Table 4.3: Results of Monte Carlo simulation with various time lengths and trend about $H(2)$

Trend	Time length	Mean	Standard Deviation	Quantile					
				0.995	0.975	0.950	0.050	0.025	0.005
Overall $H(2)$	250×2	0.4970	0.0610	0.6444	0.6171	0.5993	0.3983	0.3798	0.3482
	250×3	0.4971	0.0531	0.6303	0.6031	0.5869	0.4120	0.3948	0.3683
	250×4	0.4973	0.0491	0.6208	0.5953	0.5803	0.4199	0.4027	0.3740
	250×5	0.4973	0.0462	0.6169	0.5899	0.5735	0.4227	0.4097	0.3793
	250×12	0.4967	0.0384	0.5923	0.5709	0.5593	0.4336	0.4190	0.3972
Up-trend $H^{+}(2)$	250×2	0.4906	0.0776	0.6943	0.6452	0.6191	0.3656	0.3407	0.2837
	250×3	0.4918	0.0670	0.6689	0.6248	0.6041	0.3819	0.3634	0.3256
	250×4	0.4922	0.0612	0.6617	0.6143	0.5952	0.3951	0.3761	0.3310
	250×5	0.4931	0.0577	0.6508	0.6112	0.5887	0.3997	0.3835	0.3434
	250×12	0.4942	0.0468	0.6097	0.5861	0.5703	0.4162	0.4028	0.3683
Down-trend $H^{-}(2)$	250×2	0.4914	0.0789	0.7035	0.6492	0.6259	0.3642	0.3402	0.2995
	250×3	0.4924	0.0670	0.6680	0.6269	0.6044	0.3846	0.3649	0.3251
	250×4	0.4934	0.0620	0.6560	0.6167	0.5953	0.3926	0.3749	0.3357
	250×5	0.4934	0.0576	0.6493	0.6077	0.5900	0.4023	0.3811	0.3488
	250×12	0.4923	0.0468	0.6144	0.5861	0.5692	0.4161	0.4009	0.3711

Table 4.4: Results of Monte Carlo simulation with various time lengths and trend about degree of multifractality

Trend	Time length	Mean	Standard Deviation	Quantile					
				0.995	0.990	0.975	0.950	0.925	0.900
Overall ΔH	250×2	0.0968	0.0533	0.2507	0.2367	0.2111	0.1889	0.1783	0.1700
	250×3	0.0806	0.0439	0.2007	0.1882	0.1721	0.1576	0.1482	0.1408
	250×4	0.0711	0.0389	0.1738	0.1665	0.1506	0.1400	0.1319	0.1254
	250×5	0.0648	0.0360	0.1572	0.1496	0.1397	0.1284	0.1202	0.1144
	250×12	0.0492	0.0271	0.1190	0.1145	0.1046	0.0970	0.0909	0.0864
Up-trend ΔH^+	250×2	0.1233	0.0644	0.3027	0.2822	0.2543	0.2332	0.2200	0.2095
	250×3	0.1042	0.0525	0.2448	0.2295	0.2103	0.1938	0.1828	0.1746
	250×4	0.0935	0.0471	0.2203	0.2059	0.1867	0.1736	0.1638	0.1546
	250×5	0.0856	0.0432	0.1968	0.1882	0.1711	0.1586	0.1499	0.1432
	250×12	0.0665	0.0323	0.1448	0.1389	0.1288	0.1191	0.1137	0.1088
Down-trend ΔH^-	250×2	0.1228	0.0635	0.2976	0.2797	0.2540	0.2305	0.2167	0.2069
	250×3	0.1037	0.0516	0.2431	0.2241	0.2054	0.1900	0.1801	0.1725
	250×4	0.0935	0.0464	0.2178	0.2037	0.1855	0.1705	0.1615	0.1532
	250×5	0.0865	0.0426	0.1946	0.1863	0.1706	0.1569	0.1484	0.1429
	250×12	0.0673	0.0326	0.1468	0.1395	0.1291	0.1202	0.1144	0.1104

4.4.2 The results for testing the existence of asymmetric long-range dependence and multifractality in each period

We measure the cross-sectional asymmetric long-range dependence and multifractality for 34 countries and sub-periods. The point of interest is focused on how two financial crises affect the asymmetric long-range dependence and multifractality. As described in Chapter 4.3, the total period is divided into the cross-sectional periods, namely the Pre-crisis, Subprime-crisis, European-crisis and Post-crisis. Note that we use the results of Monte Carlo simulation in Chapter 4.4.1 as the nonparametric confidence interval for testing the existence of long-range dependence and degree of multifractality with 5% significance level. The time lengths of total period and sub-periods are 3130, 672, 614, 624 and 1220 trading dates, respectively. Therefore, the test for total period uses the result of 250 × 12 time length; the Pre-crisis, Subprime-crisis and European-crisis use the 250 × 3 time length result; and the Post-crisis uses the 250 × 5 time length.

4.4.2.1 Results of testing the existence of asymmetric long-range dependence using the Hurst exponent

Table B.1, B.2 and B.3, in Appendix B, show the result of the existence of asymmetric long-range dependence test for different asymmetric Hurst exponents, $\mathrm{H}(2)$, $\mathrm{H}^{+}(2)$ and $\mathrm{H}^{-}(2)$, respectively. Note that the bolded numbers are countries that belong to either side at 5% significance level. It refers that the rejection of null hypothesis, stating the asymmetric Hurst exponent is 0.5, indicates the presence of long-range correlation in the stock market. If the stock market has long-range dependence, the stock market is treated as the inefficient market. The results show that the stock markets of the most countries have no long-range dependence in each period. The following stock markets has long-range dependence in overall trend: CHN, IDN, ISR, PAK in Total, ZAF in Pre-crisis, PAK in Subprime-crisis, CAN, BEL in European-crisis, and CAN, ZAF, NLD, SWE, FRA, IRL, USA, ESP, BEL, GBR, CHE in Post-crisis. The countries showing the long-range dependence in up-trend market are CHN, CHL, ZAF in Total, PRT in Pre-crisis, PAK, CHN, JPN, CHE, PRT in Subprime-crisis, AUS, CHE, PRT in European-crisis, and ESP, GBR in Post-crisis, whereas the countries showing the long-range dependence in down-trend market are PAK, IDN, USA, ZAF, DNK, RUS, HKG, IRL, CAN, FIN, NLD in Total, GRC, USA in Pre-crisis, PAK in Subprime-crisis, DEU, CAN in European-crisis, DNK, IRL, CZE, GRC, CHE.

In summary, the long-range dependence of down-trend is majority in Total period, whereas the most numbers of countries regarding the long-range dependence in overall and down-trend are detected in the Post-crisis where the long-range dependence is caused by the Hurst exponent being less than 0.5. In addition, the strongest long-range dependence of up-trend are observed during the Subprime-crisis (5 countries) and the European-crisis (3 countries).

4.4.2.2 Results of testing the existence of asymmetric long-range dependence using the degree of multifractality

Table B.4, B.5 and B.6, in Appendix B, show the existence of asymmetric multifractality test for ΔH, ΔH^{+} and ΔH^{-}, respectively. Note that the bolded country names indicate the rejection of one-sided efficiency test whose null hypothesis suggests the value of ΔH is 0. That is, the extreme value of generalized Hurst exponent is spreading up and down from 0.5. The results suggest that the number of stock markets which has overall, uptrend and downtrend multifractal properties in Total period are 18, 26, 28; 19, 12, 29 countries in Pre-crisis; 33, 31, 30 countries in Subprime-crisis; 29, 25, 29 countries in European-crisis; and 33, 24, 31 countries in Post-crisis. Interestingly the largest numbers of countries showing the multifractal market regarding the overall and up-trend tests is the Subprime-crisis, whereas the numbers of multifractal market regarding the down-trend are similar in all periods.

4.4.2.3 Group distribution

We divide each country into eight different groups as defined in Table 4.1 based on the results of the Hurst exponents or the degree of multifractality in Table 4.5 and 4.6, respectively. The result of the Hurst exponents, Table 4.5, shows that the Group 1, whose market has not long-range dependence for all trends of overall, up-trend, and down-trend, is the majority. The Group 2, whose market has long-range dependence for down-trend only, is the second majority in Total period. The Group 3, whose market has long-range dependence for up-trend, is the second largest in Subprime-crisis and European-crisis periods. The Group 4, whose market has long-range dependence for up-trend and down-trend, has one country, ZAF, in Total period. The Group 5, whose market has long-range dependence for overall only, is the second majority in Post-crisis. The Group 6, whose market has long-range dependence for overall and down-trend, has few countries in Total, European-crisis, and Post-crisis. The Group 7, whose market has long-range dependence for overall and up-trend, has few countries in Total and Post-crisis.

Lastly, the Group 8, whose market has long-range dependence for all trends, has only one country, PAK, in Subprime-crisis. Following the result that Group 1 is major; the market has generally no long-range dependence. In Subprime-crisis and European-crisis, Group 3 is the second major group, so long-position may be irrational.

Table 4.6 shows the grouping results of the test of the existence of asymmetric multifractality. It shows a different pattern from the group distribution divided by the result of the test of the existence of Hurst exponent. That is, when $q = 2$, stock return has no long-range dependence, but when q is varied from -5 to 5, extreme values of q have long-range dependence. Results are as follow. The Group 1 has few countries in Total, Pre-crisis, European-crisis and Post-crisis. The Group 2 is the second majority in Pre-crisis and the third majority in Total. The Group 3 has few countries in Total, Subprime-crisis and European-crisis. The Group 4 is the second largest in Total. The Group 5 has one country for each Pre-crisis and European-crisis. The Group 6 is the second majority in Subprime-crisis, European-crisis and Post-crisis. Finally, the Group 7 has few countries in all periods. Subprime-crisis, European-crisis and Post-crisis periods have a similar group distribution. Prior to the crisis, Group 2 is the major group and Group 8 is the second, but after the crisis, Group 8 is main group and Group 6 and Group 7 are the next most. That is, down-trend multifractality is the major source of multifractality and irrationality is coming from short-position.

Table 4.5: Groups of countries based on the test results of the existence of long-range dependence

	Total	Pre-crisis	Subprime - crisis	European - crisis	Post-crisis
Group 1	ARG AUS BEL BRA CHE CZE DEU ESP FRA GBR GRC IND ITA JPN KOR MEX NZL PHL PRT SWE	ARG AUS BEL BRA CAN CHE CHL CHN CZE DEU DNK ESP FIN FRA GBR HKG IDN IND IRL ISR ITA JPN KOR MEX NLD NZL PAK PHL RUS WER	ARG AUS BEL BRA CAN CHL CZE DEU DNK ESP FIN FRA GBR GRC HKG IDN IND IRL ISR ITA KOR MEX NLD NZL PHL RUS SWE USA ZAF	ARG BRA CHL CHN CZE DNK ESP FIN FRA GBR GRC HKG IDN IND IRL ISR ITA JPN KOR MEX NLD NZL PAK PHL RUS SWE USA ZAF	ARG AUS BRA CHL CHN DEU FIN HKG IDN IND ISR ITA JPN KOR MEX NZL PAK PHL PRT RUS
Group 2	CAN DNK FIN HKG IRL NLD RUS USA	GRC USA		DEU	CZE DNK GRC
Group 3	CHL	PRT	CHE CHN JPN PRT	AUS CHE PRT	
Group 4	ZAF				
Group 5	ISR	ZAF		BEL	BEL CAN FRA NLD SWE USA ZAF
Group 6	IDN PAK			CAN	CHE IRL
Group 7	CHN				ESP GBR
Group 8			PAK		

Table 4.6: Groups of countries based on the test results of the existence of multifractality

	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
Group 1	IND RUS	BRA NZL PRT		CHN GRC	NZL
Group 2	BRA ISR KOR NZL SWE	BEL CHE DEU FIN FRA GRC HKG IDN IND ITA MEX USA			
Group 3	AUS CHN FIN		NZL	DNK	
Group 4	BEL CAN HKG NLD USA ZAF				
Group 5		CAN		DEU	
Group 6	ARG	ARG AUS DNK ESP GBR ISR	DNK FIN HKG	CAN CZE FIN PAK PHL RUS	ARG BRA CHN IRL ISR KOR PRT RUS SWE
Group 7	CZE	KOR	ARG CAN CHN	ARG GBR IRL	HKG ZAF
Group 8	CHE CHL DEU DNK ESP FRA GBR GRC IDN IRL ITA JPN MEX PAK PHL PRT	CHL CHN CZE IRL JPN NLD PAK PHL RUS SWE ZAF	AUS BEL BRA CHE CHL CZE DEU ESP FRA GBR GRC IDN IND IRL ISR ITA JPN KOR MEX NLD PAK PHL PRT RUS SWE USA ZAF	AUS BEL BRA CHE CHL ESP FRA HKG IDN IND ISR ITA JPN KOR MEX NLD NZL PRT SWE USA ZAF	AUS BEL CAN CHE CHL CZE DEU DNK ESP FIN FRA GBR GRC IDN IND ITA JPN MEX NLD PAK PHL USA

4.4.3 Time-varying asymmetric Hurst exponent and multifractality

We discover the time-varying asymmetric Hurst exponent and multifractality using the moving window method whose window size and moving day are set to be 250 days and one trading day, respectively. Figure 4.1 demonstrates the time-varying $H(2)$ and ΔH of the United States as representative with the evolution of its stock index price. The red solid, yellow and purple dashed lines represent the up-trend, down-trend and stock index price, respectively. Also, the black vertical lines divide the total period into the sub-periods as explained in Chapter 4.3. Note that the time-varying $H(2)$ and ΔH of the remaining countries can be found in Appendix C. Analyzing the result of $H(2)$, we observe the large gap between the $H^{+}(2)$ and $H^{-}(2)$ during the Subprime-crisis. Specifically, the up-trend, $H^{+}(2)$, develops into zero, whereas the down-trend evolves around 0.5. The similar phenomenon is also found in the other countries in Appendix C. Furthermore, the result of ΔH shows the large gap between the ΔH^{+} and ΔH^{-} during the Subprime-crisis. That is, the Hurst exponent in up-trend shows the anti-persistent and strong asymmetry during the outbreak of financial crisis.

We also visually detect the positive correlation between the evolution of stock index price and the up-trend Hurst exponent, $H^{+}(2)$ and the negative correlation between the stock index price and ΔH^{+}. Hence, we calculate the mean of correlation of the entire countries between $H(2)$ or ΔH with stock index price for different sub-periods. Note that the correlation result for each country can be found in Table D.1 to D.6 in Appendix D. The mean correlation in Table 4.7 shows the evidence of the strong positive correlation (0.6302) between $H^{+}(2)$ and stock index price and the strong negative correlation (-0.6091) between the ΔH^{+} and stock index price during the

Subprime-crisis. It implies that the crisis delivers the decrement of $H^{+}(2)$ and the increment of ΔH^{+}. In general, the asymmetric Hurst exponent shows the positive correlation for total period against the stock index price, whereas asymmetric ΔH shows the positive correlation in down-trend and negative correlation in overall and up-trend.

In order to examine the direct relationship between asymmetric long-range dependence and returns, daily average returns of 34 countries are obtained when the daily asymmetric Hurst exponent value satisfies a certain condition. The rate of return referred in here is the rate of annual return converted from the rate of daily return. When the daily uptrend Hurst exponent is greater than 0.6, the average daily annual return for 34 countries is 8.08%. On the other hand, when the uptrend Hurst exponent is less than 0.4, the average daily return is -2.52% based on the annual return. In other words, the return of the day with uptrend long-term positive autocorrelation is higher than the return of the day with uptrend long-term negative autocorrelation. In particular, the average return with the uptrend long-term negative autocorrelation is negative. In case of downtrend long-range dependence, the average return is 5.52% when downtrend Hurst exponent is greater than 0.6 and 17.76% when it is less than 0.4 based on annual rate of return. That is, return on day with downtrend anti-persistent is over than return on day with downtrend persistent as opposed to uptrend. In conclusion, the asymmetric Hurst exponent is associated with the rate of return such that the return is higher when the time series has the features of uptrend persistent and downtrend anti-persistent. In the case of forecasting the return after one day, realized return is 4.42% when the uptrend Hurst exponent is over than 0.6 and 3.72% when the uptrend Hurst exponent is less than 0.4. In case of downtrend, the realized return after one day is 8.00% when the downtrend Hurst exponent is over than 0.6 and 10.60% when the downtrend Hurst exponent is less than 0.4. The prediction results are similar to results of the direct relationship between asymmetric long-range dependence and daily return.

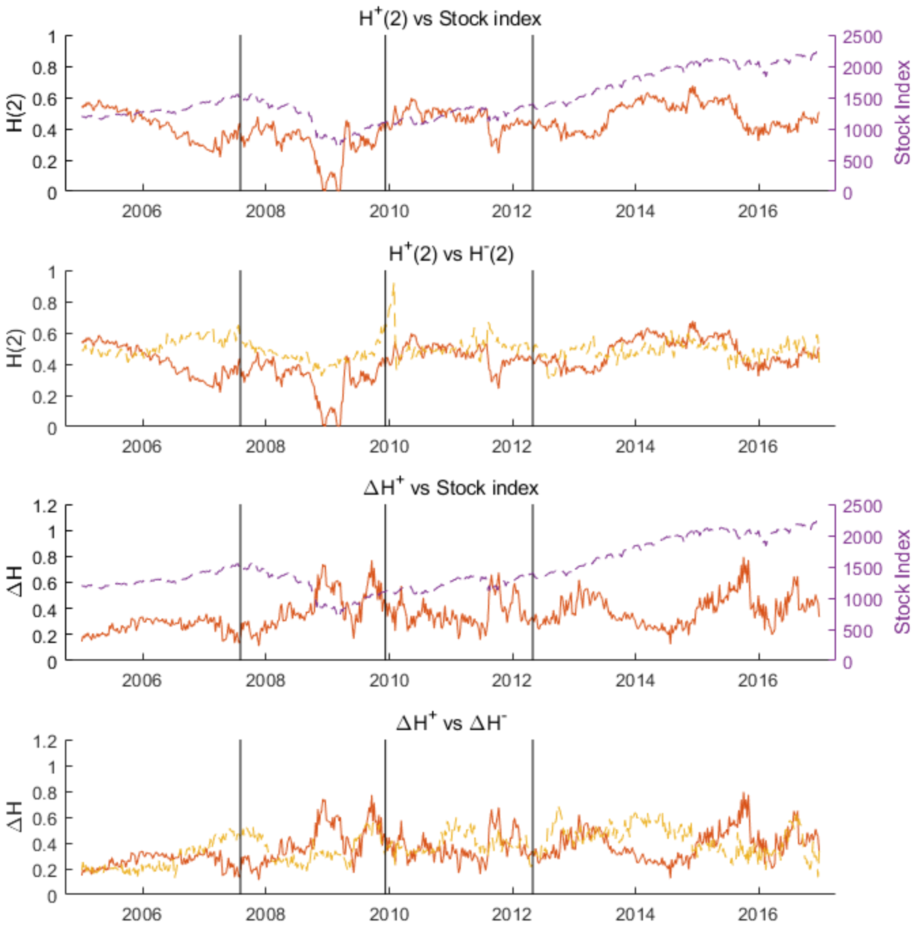

Figure 4.1: Time-varying asymmetric Hurst exponent and degree of multifractality with stock index price about United States. (Red solid line is up-trend, yellow dashed line is down-trend and purple dashed line is stock index price. Vertical black solid lines are for dividing the period. Other countries' figures are in Appendix C)

Table 4.7: Summary of correlation between the asymmetric Hurst exponent or the degree of multifractality and stock index

	Total	Pre-crisis	Subprime -crisis	European -crisis	Post-crisis
Mean of correlation ($H(2)$, stock index)	0.1135	-0.3457*	0.3307*	-0.0003	0.3134*
Mean of correlation ($H^{+}(2)$, stock index)	0.2362	-0.2992	0.6302**	0.1640	0.3341*
Mean of correlation ($H^{-}(2)$, stock index)	0.0478	0.1285	0.1624	0.1117	0.1732
Mean of correlation (ΔH, stock index)	-0.0150	-0.0241	-0.2899	-0.0500	0.0891
Mean of correlation (ΔH^{+}, stock index)	-0.1634	0.0786	-0.6091**	-0.2797	0.0037
Mean of correlation (ΔH^{-}, stock index)	0.1088	0.3402*	0.1471	0.1761	0.0582

Note: * and ** indicate the absolute correlations greater than 0.3 and 0.6, respectively

4.5 Conclusion

In this chapter, we propose a framework for analyzing the asymmetric long-range dependence and multifractality. We apply the framework to 34 of world stock indices using the price-based Asymmetric-MFDFA, which analyzes the multifractality of stock returns for overall, up-trend, and down-trend. Our research is novel for its trial to understand the source of market inefficiency by studying the asymmetric long-range dependence and multifractality given that the focus of previous researches is limited in the market inefficiency of overall market.

The main results of this study are as follows. At first, we provide the criteria for testing the existence of asymmetric Hurst exponent and multifractality based on the asymmetric generalized Hurst exponents. The theoretical asymmetric Hurst exponent and degree of multifractality are generated based on the Monte Carlo simulation using the time series following the Brownian motion. Then, the 5% confidence interval is suggested to test the existence of long-range dependence and multifractality. The test is valid since the generalized Hurst exponents of empirical stock indices must not possess the long-range correlation. Secondly, we measure the asymmetric Hurst exponent and degree of multifractality for 34 countries to test the existence of asymmetric long-range dependence and multifractality. The result implies that the financial markets of most countries have no asymmetric long-range dependence in terms of the Hurst exponent. However, the asymmetric multifractality are observed when extreme values of generalized Hurst exponents are evaluated. Also, we classify the countries into eight groups based on their existence of asymmetric long-range dependence and multifractality. Interestingly, the Group 8, which refers to the group possessing the multifractality in overall, up-trend, and down-trend, has the greatest numbers of countries during the Subprime-crisis. Lastly, we analyze the time-varying aspects of the asymmetric Hurst exponent and degree of multifractality. In general, the result shows the existence of positive correlation between the stock index price and the Hurst exponent and the negative correlation between the stock index price and the degree of multifractality. Interestingly, the financial crisis reveals the strong positive correlation between the stock indices and up-trend Hurst exponent and the strong negative correlation between the stock indices and the up-trend degree of multifractality. Also, there is a large gap between $H^{+}(2)$ and $H^{-}(2)$, and between ΔH^{+} and ΔH^{-} during the Subprime-crisis. In addition, as a result of examining the relationship between the rate of return and asymmetric long-range dependence, the return on day with uptrend persistent is higher than the return on day with uptrend anti-persistent. In case of downtrend, conversely, the rate of return on day with downtrend long-range negative autocorrelation is above than that with downtrend long-range positive autocorrelation.

The contributions of this chapter are as follows. In the past, obtaining long-range dependence and multifractality for the overall market was used to predict the market crisis or to prepare for a decline. This methodology was useful for finding the point at which the decline began or ended. However, there is little research on when the stock market will go up or the uptrend market will end if the market is booming. Therefore, this research has made it possible to study the beginning and end of the upsurge by obtaining not only the long-range dependence and multifractality of the entire stock market, but also the uptrend and downtrend stock markets segmentalized. In addition, asymmetric long-range dependence and multifractality can be used to observe the asymmetric market efficiency. If the market is inefficient, there is asymmetric long-range dependence. It means that the stock market is able to predict the future by looking at the pattern since it does not follow the random walk. Therefore, as the rate of return and asymmetric long-range dependence are directly related, this study enables research on investment strategies that predicts the future by finding repeated patterns in an asymmetric inefficient market.

Our research has limitation in its price-based model whose main method only considers the linear regression to divide market trend. Since there are many further researches about dividing market regimes (Maheu and McCurdy 2000; Pagan and Sossounov 2003), the regime detection methods can be useful for distinguishing the asymmetric long-range dependence and multifractality. Despite this weakness, our research is novel in providing a simple approach to explore the asymmetric Hurst exponent and multifractality to analyze the market's irrationality depending on its trends.

Chapter 5

Concluding Remarks

5.1 Summary and contributions

Recently, the structure of financial markets and assets becomes more complicated, and it becomes more difficult to explain the real market phenomenon using the traditional models. Consequently, many researchers have devoted their efforts in developing models that can explain the characteristics of the financial markets. In this context, the importance of studying the asymmetric characteristics with different features according to the market trend has been increased. Therefore, this dissertation focuses on identifying and applying the asymmetric long-range dependence and asymmetric multifractal characteristics in financial market data. The results are summarized as follows.

At first, the price-based A-MFDFA model is proposed with more definite criterion for separating the market price trend. It is used to explore the asymmetric long-range dependence and asymmetric multifractality. In addition, the proposed model is applied to the U.S. financial market to validate the efficacy of the model. As a result, it is discovered that the U.S. stock market has multifractal features and asymmetric characteristics. In addition, the source of multifractality is inspected which discovers that the fat-tailed distribution and long-range dependence are the source of downtrend multifractality and uptrend multifractality, respectively. The fat-tail distribution and long-range dependence can be the source of asymmetry. Also, the time-varying features of asymmetric multifractality are explored, which finds that the multifractality between uptrend and downtrend increases during the financial crisis in the U.S market.

Secondly, the simulation analysis is applied to generate the time series to verify the usability of the proposed model and the effect of various factors to the asymmetric characteristics. The heterogeneous time series with asymmetric features and homogeneous with symmetric features are artificially generated and analyzed with the price-based A-MFDFA model. The results show that the price-based A-MFDFA model can capture the asymmetric properties well. In addition, it is observed that the asymmetric Hurst exponent and asymmetric multifractality change with respect to the adjustment of skewness, autocorrelation, fat-tailed and volatility autocorrelation affecting long-range dependence and multifractality.

Lastly, a framework for testing the existence of asymmetric long-range dependence and multifractality are proposed using asymmetric generalized Hurst exponent derived from the price-based AMDFA model. Using this framework, the source of market inefficiency in the uptrend and downtrend market is tested by investigating the asymmetric long-range dependence and multifractality of the stock markets in thirty four countries. The result shows that the thirty four countries are classified into eight groups based on their multifractal properties. The empirical results indicate the degree of changes in asymmetric long-range dependence and multifractality with respect to the crisis, whereas the existence of strong negative correlation between the stock index price and the uptrend degree of multifractality in crisis periods. In addition, the gaps between the uptrend and downtrend multifractality become larger during the Subprime-crisis. Finally, the direct relationship between the rate of return and asymmetric long-range dependence is examined. The result shows that the return on day with positive persistent and negative anti-persistent features is over than that of the return on day with positive anti-persistent and negative persistent.

The contribution of this dissertation is as follows. A proper methodology to measure the asymmetric Hurst exponent and multifractality in stock market is proposed, namely the price-based A-MFDFA. In addition, it is confirmed that the price-based A-MFDFA model has better performance to capture the asymmetric characteristics of the stock market than the return-based A-MFDFA model. The asymmetric Hurst exponent and multifractality can be used for other researches to study asymmetric long-range dependence and the stock market time series data. Other contribution lies on providing information of asymmetric market inefficiency using the test of existence of asymmetric long-range dependence and multifractality. In other words, the criteria for testing the existence of asymmetric long-range dependence and multifractality are provided. In the past, the limited view on the multifractal characteristics in the overall market are studied for the purpose of analyzing the financial market crisis or falling. That is, those studies were focused on only the downtrend market. However, this dissertation extends the view by separately analyzing the beginning and end of the uptrend market with not only the long-range dependence and multifractality of the overall market, but also those of uptrend and downtrend markets. In addition, if the market has asymmetric long-range dependence or multifractality, the market is inefficient and does not follow the random walk. This dissertation studies the investment strategies by finding the repetitive patterns in an asymmetric inefficient market. Also, by comparing the asymmetric multifractal properties, the source of market efficiency is analyzed in terms of uptrend or downtrend market. It also stresses the understanding of market crash, the biggest concern in the financial market, and its affect to the asymmetric multifractal properties. The result shows that the uptrend multifractal property and the stock index price are related to each other. The relationship between the rate of return and asymmetric long-range dependence is also examined. Therefore, it is possible to study the financial market crisis using the uptrend property. Obviously, these implications can be useful information for the market participants and policy makers when they make related decision.

5.2 Limitations and future work

This study can be developed in the following way. The price-based a-MFDFA model divides the stock market using only the linear trend of the stock price. In addition to the linear trend, the model can be improved by applying other methods of dividing the stock market regimes. Since there are many further researches about dividing market regimes (Maheu and McCurdy 2000; Pagan and Sossounov 2003), the regime detection methods can be useful for distinguishing the asymmetric long-range dependence and multifractality. Non-linear market trend also can be treated to divide market regimes. Secondly, in this dissertation, only fat-tailed distribution and long-range dependence are considered as factors affecting asymmetry. In addition to the factors related to the distribution of stock returns, asymmetry can also be induced in relation to stock market sentiment, trading volume or investment amount by investment group. Research on finding other factors that affect asymmetry is needed to analyze. Lastly, there are various methodologies for measuring market efficiency, but there is no standardized and formalized methodology. Therefore, only way to verify market efficiency is to compare the results of various methodologies of measuring market efficiency. To cope with this problem, it is necessary to combine various methodologies into a single unified methodology to verify the method. Asymmetric market efficiency discussed in this dissertation is also the first attempted method to measure asymmetric efficiency, so a methodology for verifying this is also needed.

The proposed price-based A-MFDFA model in dissertation can be applied to develop in various fields. The first is the development of a time series model containing the asymmetric Hurst exponent properties.

Multifractal model of asset returns (MMAR)(Mandelbrot et al. 1997), multifractal random walk (MRW)(Bacry et al. 2001) and other multifractal time series models can be improved to a time series model considering asymmetric multifractal features to better reflect the stock market. Second, portfolio selection and investment strategy considering the asymmetric long-range dependence can be researched. In this dissertation, only direct relationship between the rate of return and asymmetric long-range dependence is examined. If asymmetric long-range dependence exists in the stock market, there are repeated patterns that help predict the future. Studying the investment strategy that finds the pattern of the stock market and uses it to develop the portfolio theory to hedge the risk or raises the stock return is remained research. Lastly, asymmetric multifractality can be applied to predict market crash. Grech and Mazur (2004) and Grech and Pamuła (2008) refer that it is possible to predict market crisis through a local Hurst exponent. In this dissertation, asymmetric Hurst exponent, which is a result of asymmetrically dividing the Hurst exponent, is associated with stock price and market crash, especially with uptrend multifractal feature. Based on these results, modeling that predict market crisis rather than just past analysis is the remaining task.

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Appendix A

Table A.1: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 10 data's trend (Skewed dist. and T-dist.)

Day=10	Positive Skew(-0.6)	Positive Skew(-0.3)	Positive Skew(0.3)	Positive Skew(0.6)	Negative Skew(-0.6)	Negative Skew(-0.3)	Negative Skew(0.3)	Negative Skew(0.6)
$H(2)$	0.5024	0.5029	0.5007	0.5033	0.5007	0.5034	0.5024	0.5022
$H^{+}(2)$	0.5087	0.5039	0.4926	0.4904	0.5088	0.5065	0.4937	0.4875
$H^{-}(2)$	0.4915	0.4979	0.5041	0.5109	0.4878	0.4951	0.5060	0.5139
ΔH	0.0935	0.0868	0.0754	0.0702	0.0708	0.0748	0.0862	0.0941
ΔH^{+}	0.1116	0.1020	0.0890	0.0806	0.0876	0.0884	0.0990	0.0999
ΔH^{-}	0.0992	0.0993	0.0932	0.0868	0.0780	0.0895	0.0992	0.1093
	Positive T-dist(10)	Positive T-dist(8)	Positive T-dist(6)	Positive T-dist(4)	Negative T-dist(10)	Negative T-dist(8)	Negative T-dist(6)	Negative T-dist(4)
$H(2)$	0.5007	0.5008	0.5051	0.5032	0.5017	0.5029	0.5059	0.5007
$H^{+}(2)$	0.5046	0.5052	0.5137	0.5183	0.4925	0.4933	0.4943	0.4818
$H^{-}(2)$	0.4915	0.4907	0.4902	0.4814	0.5056	0.5078	0.5109	0.5124
ΔH	0.0995	0.1036	0.1102	0.1518	0.0955	0.1014	0.1094	0.1573
ΔH^{+}	0.1146	0.1242	0.1341	0.1775	0.1077	0.1145	0.1179	0.1578
H^{-}	0.1113	0.1126	0.1194	0.1591	0.1119	0.1180	0.1332	0.1852

Table A.2: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 10 data's trend (AR model)

Day=10	Positive AR(-0.8)	Positive AR(-0.6)	Positive AR(-0.4)	Positive AR(-0.2)	Positive AR(0.2)	Positive AR(0.4)	Positive AR(0.6)	Positive AR(0.8)
$H(2)$	0.4053	0.4387	0.4621	0.4815	0.5262	0.5661	0.6348	0.7904
$H^{+}(2)$	0.3941	0.4317	0.4550	0.4768	0.5241	0.5640	0.6294	0.7733
$H^{-}(2)$	0.4000	0.4319	0.4577	0.4798	0.5204	0.5464	0.5867	0.6898
ΔH	0.0845	0.0686	0.0665	0.0716	0.1017	0.1298	0.1552	0.1589
ΔH^{+}	0.1348	0.0950	0.0832	0.0840	0.1194	0.1528	0.1951	0.2114
ΔH^{-}	0.0872	0.0782	0.0783	0.0867	0.1144	0.1435	0.1803	0.2136
	Negative AR(-0.8)	Negative AR(-0.6)	Negative AR(-0.4)	Negative AR(-0.2)	Negative AR(0.2)	Negative AR(0.4)	Negative AR(0.6)	Negative AR(0.8)
$H(2)$	0.4035	0.4383	0.4595	0.4840	0.5275	0.5666	0.6338	0.7890
$H^{+}(2)$	0.3985	0.4326	0.4546	0.4803	0.5236	0.5472	0.5866	0.6776
$H^{-}(2)$	0.3945	0.4306	0.4544	0.4813	0.5241	0.5642	0.6251	0.7660
ΔH	0.0869	0.0689	0.0690	0.0717	0.1019	0.1281	0.1564	0.1579
ΔH^{+}	0.0872	0.0754	0.0805	0.0848	0.1134	0.1389	0.1717	0.2045
ΔH^{-}	0.1396	0.0964	0.0859	0.0847	0.1214	0.1558	0.2002	0.2184

Table A.3: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 10 data's trend (ARCH and GARCH model)

Day=10	Positive ARCH(0.2)	Positive ARCH(0.4)	Positive ARCH(0.6)	Positive ARCH(0.8)	Negative ARCH(0.2)	Negative ARCH(0.4)	Negative ARCH(0.6)	Negative ARCH(0.8)
$H(2)$	0.5007	0.5018	0.5008	0.5003	0.5031	0.5026	0.4997	0.4989
$H^{+}(2)$	0.4970	0.4992	0.4966	0.4937	0.5003	0.5012	0.4983	0.4970
$H^{-}(2)$	0.4995	0.4988	0.4986	0.4999	0.5014	0.4989	0.4954	0.4928
ΔH	0.0985	0.1239	0.1699	0.2550	0.0962	0.1247	0.1694	0.2558
ΔH^{+}	0.1204	0.1538	0.2058	0.2905	0.1025	0.1213	0.1508	0.2062
ΔH^{-}	0.1050	0.1200	0.1500	0.2084	0.1181	0.1533	0.2013	0.2877
	Positive GARCH(0.2)	Positive GARCH(0.4)	Positive GARCH(0.6)	Positive GARCH(0.8)	Negative GARCH(0.2)	Negative GARCH(0.4)	Negative GARCH(0.6)	Negative GARCH(0.8)
$H(2)$	0.5037	0.5030	0.5034	0.5032	0.5027	0.4997	0.5029	0.5023
$H^{+}(2)$	0.5011	0.4992	0.5022	0.5026	0.4989	0.4969	0.5020	0.4986
$H^{-}(2)$	0.5010	0.5020	0.4992	0.4982	0.5012	0.4972	0.4991	0.4999
ΔH	0.0901	0.0920	0.0940	0.1054	0.0917	0.0939	0.0953	0.1046
ΔH^{+}	0.1087	0.1107	0.1149	0.1285	0.1015	0.1033	0.1031	0.1059
ΔH^{-}	0.1013	0.1019	0.1032	0.1104	0.1086	0.1129	0.1178	0.1309

Table A.4: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 30 data's trend (Skewed dist. and T-dist.)

Day=30	Positive Skew(-0.6)	Positive Skew(-0.3)	Positive Skew(0.3)	Positive Skew(0.6)	Negative Skew(-0.6)	Negative Skew(-0.3)	Negative Skew(0.3)	Negative Skew(0.6)
$H(2)$	0.5031	0.5034	0.5038	0.5040	0.5034	0.5022	0.5030	0.5046
$H^+(2)$	0.5116	0.5058	0.4957	0.4921	0.5112	0.5049	0.4957	0.4911
$H^-(2)$	0.4901	0.4962	0.5067	0.5112	0.4916	0.4949	0.5058	0.5137
ΔH	0.0848	0.0854	0.0808	0.0762	0.0772	0.0793	0.0835	0.0856
ΔH^+	0.1027	0.1007	0.0951	0.0849	0.0921	0.0947	0.0964	0.0947
ΔH^-	0.0931	0.0968	0.0944	0.0935	0.0862	0.0936	0.0969	0.1016
	Positive T-dist(10)	Positive T-dist(8)	Positive T-dist(6)	Positive T-dist(4)	Negative T-dist(10)	Negative T-dist(8)	Negative T-dist(6)	Negative T-dist(4)
$H(2)$	0.5020	0.5034	0.5024	0.5028	0.5005	0.5013	0.5025	0.5032
$H^+(2)$	0.5093	0.5154	0.5203	0.5299	0.4878	0.4844	0.4812	0.4697
$H^-(2)$	0.4899	0.4864	0.4798	0.4718	0.5087	0.5135	0.5191	0.5338
ΔH	0.0972	0.1001	0.1164	0.1536	0.0953	0.1014	0.1114	0.1522
ΔH^+	0.1169	0.1216	0.1361	0.1829	0.1069	0.1154	0.1231	0.1624
ΔH^-	0.1061	0.1100	0.1281	0.1616	0.1159	0.1176	0.1362	0.1816

Table A.5: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 30 data's trend (AR model)

Day=30	Positive AR(-0.8)	Positive AR(-0.6)	Positive AR(-0.4)	Positive AR(-0.2)	Positive AR(0.2)	Positive AR(0.4)	Positive AR(0.6)	Positive AR(0.8)
$H(2)$	0.4248	0.4492	0.4675	0.4830	0.5264	0.5691	0.6430	0.8053
$H^{+}(2)$	0.3929	0.4167	0.4415	0.4674	0.5362	0.5891	0.6709	0.8374
$H^{-}(2)$	0.4319	0.4589	0.4783	0.4902	0.5086	0.5310	0.5753	0.7039
ΔH	0.0837	0.0647	0.0646	0.0712	0.1007	0.1210	0.1331	0.1430
ΔH^{+}	0.1124	0.0830	0.0805	0.0845	0.1204	0.1598	0.2119	0.2341
ΔH^{-}	0.0928	0.0818	0.0803	0.0887	0.1132	0.1305	0.1419	0.1446
	Negative AR(-0.8)	Negative AR(-0.6)	Negative AR(-0.4)	Negative AR(-0.2)	Negative AR(0.2)	Negative AR(0.4)	Negative AR(0.6)	Negative AR(0.8)
$H(2)$	0.4232	0.4488	0.4648	0.4853	0.5291	0.5700	0.6417	0.8059
$H^{+}(2)$	0.4259	0.4552	0.4720	0.4896	0.5131	0.5221	0.5432	0.6073
$H^{-}(2)$	0.4061	0.4265	0.4460	0.4740	0.5380	0.5949	0.6829	0.8651
ΔH	0.0815	0.0645	0.0676	0.0708	0.0970	0.1192	0.1366	0.1410
ΔH^{+}	0.0921	0.0794	0.0843	0.0859	0.1096	0.1368	0.1739	0.1955
ΔH^{-}	0.0996	0.0787	0.0785	0.0832	0.1208	0.1592	0.2111	0.2389

Table A.6: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 30 data's trend (ARCH and GARCH model)

Day=30	Positive ARCH(0.2)	Positive ARCH(0.4)	Positive ARCH(0.6)	Positive ARCH(0.8)	Negative ARCH(0.2)	Negative ARCH(0.4)	Negative ARCH(0.6)	Negative ARCH(0.8)
$H(2)$	0.5006	0.5017	0.5008	0.5003	0.5030	0.5019	0.4999	0.4992
$H^{+}(2)$	0.4967	0.4989	0.4911	0.4771	0.5007	0.5024	0.5041	0.5107
$H^{-}(2)$	0.4999	0.4997	0.5038	0.5162	0.5009	0.4956	0.4903	0.4807
ΔH	0.0982	0.1263	0.1748	0.2662	0.0962	0.1252	0.1767	0.2744
ΔH^{+}	0.1160	0.1421	0.1884	0.2677	0.1074	0.1364	0.1844	0.2998
ΔH^{-}	0.1106	0.1360	0.1837	0.2945	0.1134	0.1416	0.1850	0.2682
	Positive GARCH(0.2)	Positive GARCH(0.4)	Positive GARCH(0.6)	Positive GARCH(0.8)	Negative GARCH(0.2)	Negative GARCH(0.4)	Negative GARCH(0.6)	Negative GARCH(0.8)
$H(2)$	0.5037	0.5027	0.5031	0.5030	0.5031	0.5001	0.5033	0.5028
$H^{+}(2)$	0.5015	0.4989	0.5014	0.4995	0.4999	0.4984	0.5031	0.5009
$H^{-}(2)$	0.5011	0.5019	0.4998	0.5011	0.5014	0.4973	0.4981	0.4991
ΔH	0.0886	0.0920	0.0963	0.1086	0.0917	0.0930	0.0969	0.1084
ΔH^{+}	0.1034	0.1064	0.1117	0.1247	0.1040	0.1061	0.1094	0.1193
ΔH^{-}	0.1033	0.1059	0.1098	0.1227	0.1073	0.1076	0.1126	0.1257

Table A.7: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 40 data's trend (Skewed dist. and T-dist.)

Day=40	Positive Skew(-0.6)	Positive Skew(-0.3)	Positive Skew(0.3)	Positive Skew(0.6)	Negative Skew(-0.6)	Negative Skew(-0.3)	Negative Skew(0.3)	Negative Skew(0.6)
$H(2)$	0.5036	0.5027	0.5051	0.5038	0.5040	0.5027	0.5022	0.5024
$H^{+}(2)$	0.5142	0.5054	0.4981	0.4927	0.5128	0.5034	0.4953	0.4886
$H^{-}(2)$	0.4887	0.4954	0.5068	0.5107	0.4912	0.4978	0.5042	0.5111
ΔH	0.0869	0.0823	0.0809	0.0778	0.0794	0.0799	0.0816	0.0883
ΔH^{+}	0.1021	0.0981	0.0931	0.0890	0.0965	0.0957	0.0961	0.0938
ΔH^{-}	0.0934	0.0945	0.0956	0.0924	0.0881	0.0913	0.0969	0.1040
	Positive T-dist(10)	Positive T-dist(8)	Positive T-dist(6)	Positive T-dist(4)	Negative T-dist(10)	Negative T-dist(8)	Negative T-dist(6)	Negative T-dist(4)
$H(2)$	0.5035	0.5035	0.5051	0.5038	0.5053	0.5013	0.5043	0.5053
$H^{+}(2)$	0.5118	0.5139	0.5206	0.5290	0.4937	0.4865	0.4845	0.4741
$H^{-}(2)$	0.4911	0.4887	0.4856	0.4748	0.5129	0.5117	0.5192	0.5312
ΔH	0.0969	0.1009	0.1135	0.1595	0.0950	0.1035	0.1118	0.1513
ΔH^{+}	0.1143	0.1207	0.1370	0.1879	0.1059	0.1134	0.1226	0.1614
ΔH^{-}	0.1070	0.1110	0.1236	0.1625	0.1151	0.1258	0.1350	0.1787

Table A.8: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 40 data's trend (AR model)

Day=40	Positive AR(-0.8)	Positive AR(-0.6)	Positive AR(-0.4)	Positive AR(-0.2)	Positive AR(0.2)	Positive AR(0.4)	Positive AR(0.6)	Positive AR(0.8)
$H(2)$	0.4290	0.4506	0.4682	0.4832	0.5266	0.5690	0.6445	0.8090
$H^{+}(2)$	0.3873	0.4127	0.4381	0.4660	0.5370	0.5918	0.6750	0.8472
$H^{-}(2)$	0.4416	0.4657	0.4828	0.4928	0.5083	0.5300	0.5812	0.7109
ΔH	0.0788	0.0614	0.0643	0.0720	0.0971	0.1175	0.1244	0.1385
ΔH^{+}	0.1093	0.0816	0.0784	0.0837	0.1190	0.1565	0.1985	0.2051
ΔH^{-}	0.0892	0.0787	0.0799	0.0892	0.1079	0.1282	0.1323	0.1377
	Negative AR(-0.8)	Negative AR(-0.6)	Negative AR(-0.4)	Negative AR(-0.2)	Negative AR(0.2)	Negative AR(0.4)	Negative AR(0.6)	Negative AR(0.8)
$H(2)$	0.4264	0.4505	0.4664	0.4860	0.5288	0.5706	0.6421	0.8112
$H^{+}(2)$	0.4340	0.4608	0.4769	0.4919	0.5118	0.5210	0.5442	0.6230
$H^{-}(2)$	0.4023	0.4248	0.4445	0.4736	0.5383	0.5990	0.6892	0.8786
ΔH	0.0786	0.0628	0.0665	0.0697	0.0982	0.1146	0.1287	0.1372
ΔH^{+}	0.0905	0.0789	0.0841	0.0862	0.1095	0.1357	0.1637	0.1761
ΔH^{-}	0.0952	0.0755	0.0774	0.0807	0.1212	0.1532	0.2009	0.2108

Table A.9: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 40 data's trend (ARCH and GARCH model)

Day=40	Positive ARCH(0.2)	Positive ARCH(0.4)	Positive ARCH(0.6)	Positive ARCH(0.8)	Negative ARCH(0.2)	Negative ARCH(0.4)	Negative ARCH(0.6)	Negative ARCH(0.8)
$H(2)$	0.5010	0.5026	0.5011	0.5006	0.5032	0.5024	0.4997	0.4991
$H^{+}(2)$	0.4975	0.4992	0.4918	0.4796	0.5002	0.5029	0.5019	0.5121
$H^{-}(2)$	0.4995	0.5007	0.5040	0.5153	0.5013	0.4979	0.4919	0.4795
ΔH	0.0982	0.1255	0.1779	0.2720	0.0953	0.1253	0.1784	0.2747
ΔH^{+}	0.1135	0.1397	0.1875	0.2702	0.1073	0.1389	0.1911	0.3034
ΔH^{-}	0.1113	0.1372	0.1895	0.2982	0.1121	0.1377	0.1842	0.2671
	Positive GARCH(0.2)	Positive GARCH(0.4)	Positive GARCH(0.6)	Positive GARCH(0.8)	Negative GARCH(0.2)	Negative GARCH(0.4)	Negative GARCH(0.6)	Negative GARCH(0.8)
$H(2)$	0.5037	0.5025	0.5037	0.5030	0.5031	0.4996	0.5033	0.5020
$H^{+}(2)$	0.5023	0.4981	0.5029	0.4997	0.5003	0.4971	0.5023	0.4995
$H^{-}(2)$	0.5001	0.5019	0.4992	0.5010	0.5012	0.4970	0.4993	0.4986
ΔH	0.0884	0.0932	0.0945	0.1096	0.0903	0.0923	0.0974	0.1091
ΔH^{+}	0.1035	0.1071	0.1078	0.1244	0.1032	0.1081	0.1099	0.1232
ΔH^{-}	0.1036	0.1059	0.1099	0.1246	0.1061	0.1060	0.1130	0.1255

Table A.10: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 50 data's trend (Skewed dist. and T-dist.)

Day=50	Positive Skew(-0.6)	Positive Skew(-0.3)	Positive Skew(0.3)	Positive Skew(0.6)	Negative Skew(-0.6)	Negative Skew(-0.3)	Negative Skew(0.3)	Negative Skew(0.6)
$H(2)$	0.5047	0.5040	0.5037	0.5034	0.5012	0.5030	0.5016	0.5019
$H^{+}(2)$	0.5133	0.5059	0.4955	0.4912	0.5090	0.5048	0.4922	0.4883
$H^{-}(2)$	0.4914	0.4974	0.5066	0.5118	0.4885	0.4961	0.5067	0.5104
ΔH	0.0820	0.0806	0.0806	0.0787	0.0788	0.0809	0.0832	0.0851
ΔH^{+}	0.0981	0.0953	0.0929	0.0896	0.0958	0.0957	0.0974	0.0927
ΔH^{-}	0.0893	0.0928	0.0961	0.0941	0.0872	0.0925	0.0957	0.1021
	Positive T-dist(10)	Positive T-dist(8)	Positive T-dist(6)	Positive T-dist(4)	Negative T-dist(10)	Negative T-dist(8)	Negative T-dist(6)	Negative T-dist(4)
$H(2)$	0.5044	0.5029	0.5037	0.5031	0.5023	0.5028	0.5030	0.5022
$H^{+}(2)$	0.5104	0.5095	0.5171	0.5270	0.4929	0.4876	0.4874	0.4744
$H^{-}(2)$	0.4933	0.4906	0.4864	0.4753	0.5073	0.5125	0.5148	0.5257
ΔH	0.0935	0.1003	0.1131	0.1510	0.0967	0.0990	0.1160	0.1546
ΔH^{+}	0.1133	0.1221	0.1339	0.1779	0.1086	0.1122	0.1274	0.1617
ΔH^{-}	0.1032	0.1088	0.1205	0.1574	0.1145	0.1171	0.1375	0.1815

Table A.11: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 50 data's trend (AR model)

Day=50	Positive AR(-0.8)	Positive AR(-0.6)	Positive AR(-0.4)	Positive AR(-0.2)	Positive AR(0.2)	Positive AR(0.4)	Positive AR(0.6)	Positive AR(0.8)
$H(2)$	0.4311	0.4524	0.4681	0.4833	0.5269	0.5698	0.6433	0.8107
$H^+(2)$	0.3900	0.4118	0.4391	0.4660	0.5375	0.5927	0.6763	0.8502
$H^-(2)$	0.4457	0.4708	0.4832	0.4930	0.5100	0.5308	0.5809	0.7154
ΔH	0.0743	0.0603	0.0634	0.0710	0.0988	0.1125	0.1197	0.1401
ΔH^+	0.1016	0.0832	0.0792	0.0832	0.1192	0.1477	0.1841	0.1765
ΔH^-	0.0857	0.0755	0.0788	0.0881	0.1090	0.1247	0.1315	0.1444
	Negative AR(-0.8)	Negative AR(-0.6)	Negative AR(-0.4)	Negative AR(-0.2)	Negative AR(0.2)	Negative AR(0.4)	Negative AR(0.6)	Negative AR(0.8)
$H(2)$	0.4286	0.4526	0.4662	0.4861	0.5282	0.5706	0.6435	0.8140
$H^+(2)$	0.4393	0.4650	0.4773	0.4922	0.5114	0.5229	0.5532	0.6457
$H^-(2)$	0.4011	0.4248	0.4448	0.4737	0.5382	0.5991	0.6895	0.8792
ΔH	0.0778	0.0614	0.0652	0.0702	0.0980	0.1117	0.1183	0.1400
ΔH^+	0.0915	0.0761	0.0813	0.0866	0.1117	0.1342	0.1536	0.1567
ΔH^-	0.0926	0.0743	0.0776	0.0824	0.1203	0.1475	0.1818	0.1789

Table A.12: Average of asymmetric Hurst exponent and asymmetric degree of multifractality for each simulated heterogeneous time series with previous 50 data's trend (ARCH and GARCH model)

Day=50	Positive ARCH(0.2)	Positive ARCH(0.4)	Positive ARCH(0.6)	Positive ARCH(0.8)	Negative ARCH(0.2)	Negative ARCH(0.4)	Negative ARCH(0.6)	Negative ARCH(0.8)
$H(2)$	0.5008	0.5022	0.5016	0.5002	0.5032	0.5024	0.4999	0.4997
$H^{+}(2)$	0.4967	0.4988	0.4923	0.4797	0.5008	0.5025	0.5022	0.5087
$H^{-}(2)$	0.5003	0.4996	0.5049	0.5157	0.5015	0.4969	0.4919	0.4826
ΔH	0.0974	0.1249	0.1721	0.2710	0.0972	0.1238	0.1787	0.2701
ΔH^{+}	0.1133	0.1392	0.1833	0.2685	0.1085	0.1384	0.1912	0.2993
ΔH^{-}	0.1105	0.1350	0.1826	0.2966	0.1142	0.1382	0.1850	0.2635
	Positive GARCH(0.2)	Positive GARCH(0.4)	Positive GARCH(0.6)	Positive GARCH(0.8)	Negative GARCH(0.2)	Negative GARCH(0.4)	Negative GARCH(0.6)	Negative GARCH(0.8)
$H(2)$	0.5025	0.5027	0.5028	0.5029	0.5028	0.4991	0.5027	0.5017
$H^{+}(2)$	0.5002	0.4990	0.5014	0.4989	0.4996	0.4970	0.5019	0.5004
$H^{-}(2)$	0.5001	0.5018	0.4993	0.5014	0.5018	0.4965	0.4978	0.4983
ΔH	0.0911	0.0919	0.0953	0.1088	0.0923	0.0921	0.0964	0.1089
ΔH^{+}	0.1050	0.1058	0.1094	0.1221	0.1058	0.1056	0.1094	0.1211
ΔH^{-}	0.1061	0.1065	0.1098	0.1248	0.1062	0.1059	0.1128	0.1247

Appendix B

Table B.1 Results of $H(2)$ of all countries with various periods.

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	0.5416	0.4912	0.5319	0.4927	0.5322
AUS	0.5130	0.4582	0.4642	0.4165	0.4321
BEL	0.5414	0.4909	0.5517	**0.3693**	**0.3833**
BRA	0.5278	0.4381	0.4220	0.4516	0.5096
CAN	0.5414	0.4968	0.4465	**0.3714**	**0.4093**
CHE	0.4705	0.4797	0.4213	0.4436	**0.3449**
CHL	0.5481	0.5123	0.5033	0.5395	0.4824
CHN	**0.6327**	0.5304	0.4789	0.5371	0.5634
CZE	0.5574	0.4820	0.5369	0.4780	0.4136
DEU	0.4927	0.4434	0.4915	0.5154	0.4464
DNK	0.5417	0.4600	0.5175	0.4398	0.4254
ESP	0.4863	0.4772	0.4787	0.4545	**0.3843**
FIN	0.5322	0.4644	0.4846	0.4594	0.4119
FRA	0.4718	0.4506	0.4601	0.4399	**0.3970**
GBR	0.4514	0.4419	0.4768	0.4410	**0.3626**
GRC	0.5516	0.5167	0.5663	0.4920	0.4168
HKG	0.5500	0.4643	0.4693	0.4579	0.5198
IDN	**0.5943**	0.5264	0.5055	0.4604	0.4602
IND	0.5546	0.5409	0.5145	0.5097	0.4243
IRL	0.5523	0.4847	0.4512	0.4512	**0.3882**
ISR	**0.5906**	0.4660	0.4926	0.4907	0.4571
ITA	0.5030	0.4846	0.5545	0.4787	0.4279
JPN	0.5143	0.4886	0.4690	0.4848	0.4416
KOR	0.5136	0.5028	0.4946	0.4712	0.4264
MEX	0.4988	0.4671	0.4793	0.4230	0.4236
NLD	0.5282	0.5268	0.5468	0.4501	**0.4038**
NZL	0.5030	0.5330	0.5473	0.4260	0.4699
PAK	**0.5798**	0.5481	**0.6670**	0.5836	0.4799
PHL	0.5295	0.4627	0.4743	0.5054	0.4701
PRT	0.5554	0.6028	0.5173	0.3992	0.4774
RUS	0.5670	0.4514	0.4391	0.4445	0.4385
SWE	0.4973	0.4658	0.4265	0.4190	**0.3991**
USA	0.5062	0.4132	0.4616	0.4358	**0.3881**
ZAF	0.4609	**0.3840**	0.4533	0.4020	**0.4083**

Note: The bolded numbers indicate that market has long-range dependence with 5% statistical significance

Table B.2 Results of $H^{+}(2)$ of all countries with various periods

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	0.5380	0.5551	0.4892	0.4769	0.5739
AUS	0.4602	0.5285	0.3645	**0.3552**	0.4501
BEL	0.4547	0.5187	0.3859	0.3901	0.3861
BRA	0.5323	0.4424	0.5002	0.4816	0.5029
CAN	0.4631	0.5532	0.4272	0.4030	0.4054
CHE	0.4716	0.5053	**0.3403**	**0.3320**	0.3858
CHL	**0.6063**	0.5741	0.5294	0.5354	0.4730
CHN	**0.6902**	0.5651	**0.6331**	0.5886	0.6105
CZE	0.4883	0.5362	0.5194	0.5143	0.4617
DEU	0.4728	0.4732	0.4554	0.4266	0.4336
DNK	0.5200	0.5088	0.4179	0.4010	0.4730
ESP	0.4580	0.5468	0.4155	0.3980	**0.3737**
FIN	0.4540	0.4896	0.4913	0.4775	0.4259
FRA	0.4146	0.5034	0.4039	0.3933	0.4029
GBR	0.4148	0.4873	0.4284	0.4124	**0.3468**
GRC	0.5013	0.5241	0.5553	0.5409	0.4777
HKG	0.4654	0.4950	0.4879	0.4655	0.4828
IDN	0.5765	0.5727	0.5190	0.5024	0.4423
IND	0.5304	0.5627	0.5288	0.5262	0.4312
IRL	0.4916	0.5303	0.4707	0.4513	0.4341
ISR	0.5451	0.4929	0.4527	0.4207	0.4817
ITA	0.4274	0.5392	0.4675	0.4434	0.4380
JPN	0.5355	0.4701	**0.6262**	0.5624	0.4749
KOR	0.4914	0.5223	0.4714	0.4564	0.4452
MEX	0.4707	0.4818	0.4751	0.4565	0.4470
NLD	0.4356	0.5558	0.4564	0.4362	0.3875
NZL	0.4559	0.5605	0.3755	0.3815	0.4695
PAK	0.5071	0.5875	**0.6530**	0.6023	0.5152
PHL	0.5324	0.4723	0.5701	0.5452	0.4996
PRT	0.5537	**0.6398**	**0.3032**	**0.2712**	0.5060
RUS	0.4781	0.5394	0.4353	0.4150	0.4470
SWE	0.4638	0.4934	0.4396	0.4204	0.4029
USA	0.4442	0.4785	0.4624	0.4395	0.3960
ZAF	**0.3779**	0.4583	0.4561	0.4342	0.4224

Note: The bolded numbers indicate that market has long-range dependence with 5% statistical significance

Table B.3 Results of $H^{-}(2)$ of all countries with various periods

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	0.5666	0.4824	0.5834	0.4990	0.5478
AUS	0.5323	0.4309	0.4535	0.4603	0.4321
BEL	0.5823	0.5550	0.5282	0.3749	0.4274
BRA	0.4798	0.4744	0.4664	0.4115	0.5034
CAN	**0.5923**	0.5060	0.4641	**0.3483**	0.4254
CHE	0.4485	0.5371	0.4231	0.4836	**0.3439**
CHL	0.4546	0.4479	0.4739	0.5554	0.4987
CHN	0.5575	0.4117	0.4782	0.5052	0.4854
CZE	0.5759	0.5549	0.5187	0.4595	**0.3640**
DEU	0.5428	0.4762	0.4889	**0.6858**	0.5001
DNK	**0.6058**	0.4848	0.4963	0.5053	**0.3722**
ESP	0.4819	0.4657	0.4734	0.4677	0.4187
FIN	**0.5921**	0.5770	0.4444	0.4494	0.4236
FRA	0.4868	0.4862	0.4621	0.4698	0.4137
GBR	0.4847	0.5092	0.4886	0.4692	0.4124
GRC	0.5443	**0.6429**	0.5257	0.4676	**0.3474**
HKG	**0.5957**	0.4829	0.4836	0.4474	0.5927
IDN	**0.6280**	0.4791	0.5132	0.4657	0.4959
IND	0.5402	0.6192	0.5058	0.4905	0.4060
IRL	**0.5956**	0.4554	0.4181	0.4467	**0.3659**
ISR	0.5797	0.5230	0.5005	0.5235	0.4551
ITA	0.5219	0.4642	0.4657	0.4975	0.4260
JPN	0.4940	0.5576	0.4916	0.4250	0.4077
KOR	0.5260	0.5717	0.5249	0.5873	0.4141
MEX	0.5859	0.5748	0.5071	0.3711	0.4043
NLD	**0.5862**	0.5703	0.5448	0.4732	0.5025
NZL	0.5857	0.4783	0.5523	0.4479	0.4695
PAK	**0.7372**	0.5547	**0.6921**	0.5543	0.4054
PHL	0.5558	0.5094	0.4885	0.4739	0.4226
PRT	0.5340	0.5036	0.5184	0.4175	0.4429
RUS	**0.6007**	0.5090	0.5048	0.4486	0.4566
SWE	0.5023	0.6066	0.5008	0.4475	0.4295
USA	**0.6263**	**0.3419**	0.4567	0.4593	0.4624
ZAF	**0.6258**	0.5040	0.5319	0.3687	0.4035

Note: The bolded numbers indicate that market has long-range dependence with 5% statistical significance

Table B.4 Results of overall *degree of multifractality* of all countries with various periods

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	**0.1086**	**0.1847**	**0.2081**	**0.2852**	**0.1655**
AUS	0.0597	**0.2176**	**0.3677**	**0.1818**	**0.2037**
BEL	0.0380	0.1497	**0.2738**	**0.3545**	**0.2189**
BRA	0.0851	0.1530	**0.1947**	**0.2166**	**0.1383**
CAN	0.0941	**0.1961**	**0.3256**	**0.2096**	**0.1877**
CHE	**0.1657**	0.1341	**0.2987**	**0.2931**	**0.3486**
CHL	**0.2216**	**0.3385**	**0.5475**	**0.3328**	**0.2108**
CHN	0.0320	**0.3745**	**0.2283**	0.1364	**0.1452**
CZE	**0.1138**	**0.4421**	**0.3633**	**0.2553**	**0.2203**
DEU	**0.1463**	0.1524	**0.3411**	0.1295	**0.2006**
DNK	**0.1139**	**0.1783**	**0.2058**	0.1133	**0.2828**
ESP	**0.1594**	**0.2105**	**0.3534**	**0.2535**	**0.2655**
FIN	0.0342	0.1155	**0.1900**	**0.2262**	**0.2061**
FRA	**0.1073**	0.1321	**0.3458**	**0.2407**	**0.2434**
GBR	**0.1246**	**0.1740**	**0.2714**	**0.2740**	**0.2937**
GRC	**0.1351**	0.0746	**0.4975**	0.1559	**0.3283**
HKG	0.0448	0.1448	**0.1873**	**0.2937**	**0.1619**
IDN	**0.1097**	0.1347	**0.3353**	**0.2854**	**0.2621**
IND	0.0541	0.1289	**0.2368**	**0.2610**	**0.1907**
IRL	**0.0986**	**0.2811**	**0.2355**	0.1372	**0.2392**
ISR	0.0570	**0.2355**	**0.2748**	**0.1954**	**0.1565**
ITA	**0.1492**	0.0855	**0.1950**	**0.2632**	**0.2377**
JPN	**0.2565**	**0.1813**	**0.3202**	**0.3934**	**0.2394**
KOR	0.0852	**0.1854**	**0.3701**	**0.2533**	**0.1803**
MEX	**0.1394**	0.1235	**0.2633**	**0.3020**	**0.2780**
NLD	0.0288	**0.2204**	**0.2641**	**0.2348**	**0.2211**
NZL	0.0397	0.0753	0.1570	**0.2287**	0.0472
PAK	**0.2554**	**0.2590**	**0.9456**	**0.1816**	**0.2546**
PHL	**0.1482**	**0.2490**	**0.2452**	**0.1590**	**0.2110**
PRT	**0.2895**	0.1108	**0.4958**	**0.2724**	**0.1616**
RUS	0.0660	**0.3634**	**0.3392**	**0.1707**	**0.1988**
SWE	0.0747	**0.2035**	**0.1766**	**0.2330**	**0.1453**
USA	0.0563	0.1031	**0.2216**	**0.3387**	**0.2475**
ZAF	0.0816	**0.4064**	**0.1713**	**0.2698**	**0.1799**

Note: The bolded numbers indicate that market has multifractal feature with 5% statistical significance

Table B.5 Results of uptrend *degree of multifractality* of all countries with various periods

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	0.1147	0.1081	**0.2796**	**0.4335**	0.1246
AUS	**0.1285**	0.1441	**0.3193**	**0.2502**	**0.2093**
BEL	**0.1320**	0.1254	**0.2285**	**0.2986**	**0.2027**
BRA	0.1052	0.1637	**0.2988**	**0.2152**	0.1202
CAN	**0.2172**	0.1554	**0.3918**	0.1725	**0.1772**
CHE	**0.1590**	0.1008	**0.3169**	**0.3929**	**0.2113**
CHL	**0.1703**	**0.1967**	**0.4158**	**0.3093**	**0.2359**
CHN	**0.1916**	**0.3779**	**0.2602**	0.1871	0.0665
CZE	**0.2203**	**0.4299**	**0.3504**	0.0928	**0.1914**
DEU	**0.1724**	0.1431	**0.3062**	**0.3238**	**0.2146**
DNK	**0.1429**	0.1188	0.1598	**0.2033**	**0.2209**
ESP	**0.1689**	0.1459	**0.2884**	**0.3502**	**0.2745**
FIN	**0.1553**	0.0942	0.1185	0.1795	**0.2006**
FRA	**0.1745**	0.0905	**0.3234**	**0.3366**	**0.2455**
GBR	**0.1738**	0.1307	**0.3230**	**0.3550**	**0.3137**
GRC	**0.1838**	0.0539	**0.3992**	0.1884	**0.2794**
HKG	**0.1342**	0.1173	0.0938	**0.3269**	**0.2437**
IDN	**0.1397**	0.0839	**0.3025**	**0.2588**	**0.2486**
IND	0.1076	0.1158	**0.1957**	**0.2653**	**0.1800**
IRL	**0.2122**	**0.2089**	**0.3206**	0.1119	0.1020
ISR	0.0341	0.1861	**0.2681**	**0.2538**	0.1114
ITA	**0.2337**	0.0378	**0.3064**	**0.3748**	**0.2273**
JPN	**0.2716**	**0.2405**	**0.4109**	**0.2220**	**0.2208**
KOR	0.0904	**0.2182**	**0.4418**	**0.3361**	0.1374
MEX	**0.2008**	0.1336	**0.2292**	**0.2601**	**0.2772**
NLD	**0.1423**	**0.2040**	**0.2927**	**0.2957**	**0.2678**
NZL	0.0895	0.0677	**0.3021**	**0.1978**	0.0320
PAK	**0.3228**	**0.2280**	**0.4184**	0.1792	**0.2388**
PHL	**0.1593**	**0.2232**	**0.2171**	0.0703	**0.1608**
PRT	**0.3137**	0.1114	**0.3863**	**0.3948**	0.1462
RUS	0.1079	**0.2632**	**0.4292**	0.1652	0.1446
SWE	0.1122	**0.2013**	**0.2703**	**0.2959**	0.1567
USA	**0.1598**	0.0248	**0.2387**	**0.3658**	**0.2302**
ZAF	**0.2385**	**0.3387**	**0.2538**	**0.2724**	**0.1982**

Note: The bolded numbers indicate that market has multifractal feature with 5% statistical significance

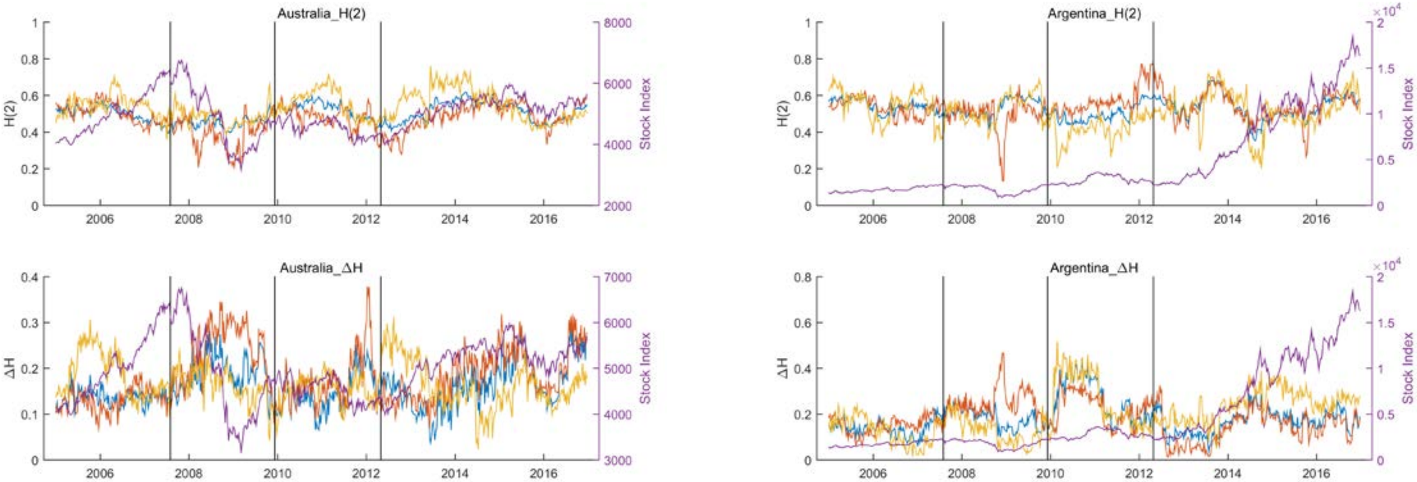
Table B.6 Results of downtrend *degree of multifractality* of all countries with various periods

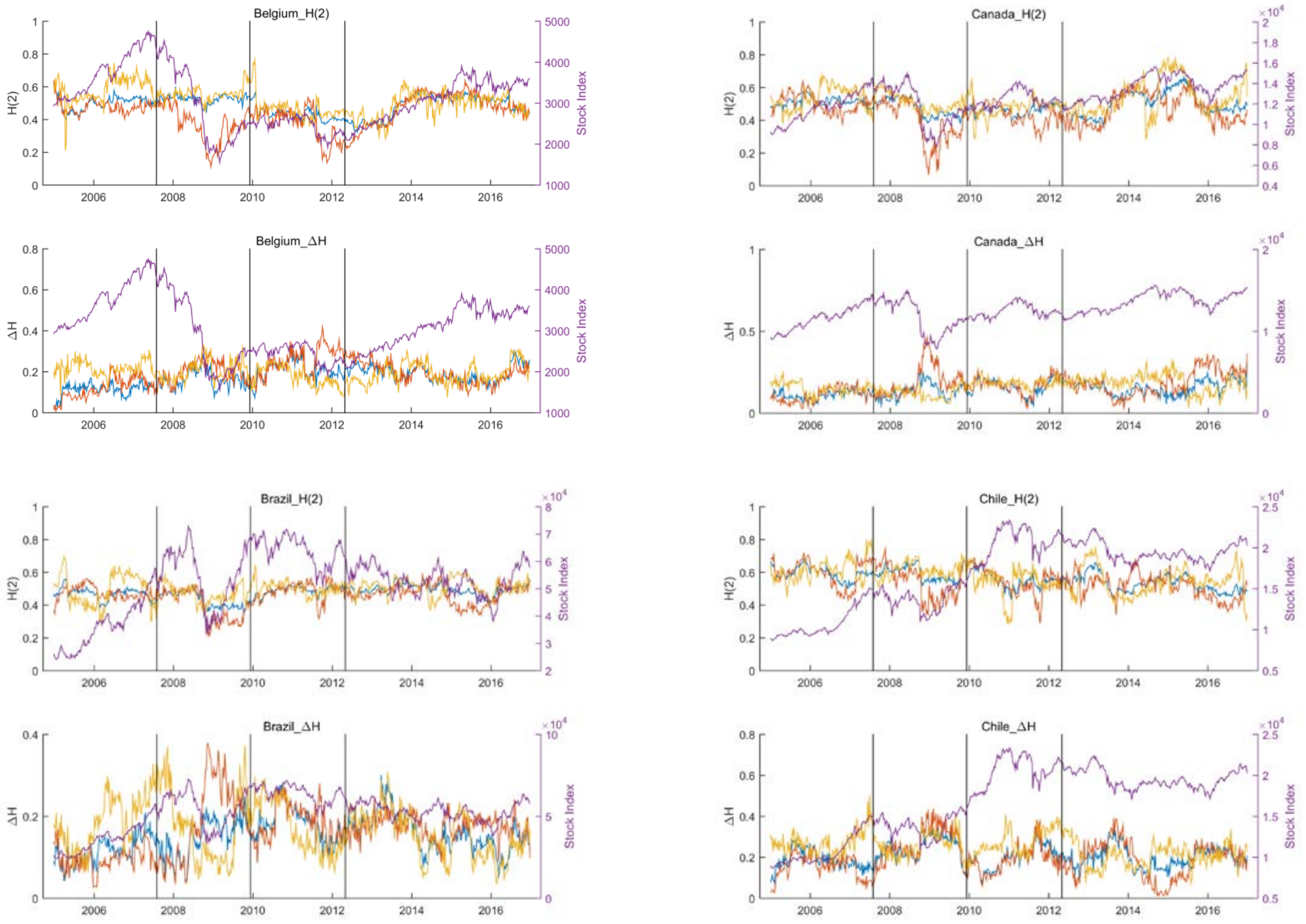
Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	**0.2269**	**0.2757**	0.1323	0.1255	**0.2315**
AUS	0.0678	**0.2952**	**0.4046**	**0.2708**	**0.2297**
BEL	**0.1254**	**0.3189**	**0.3479**	**0.4482**	**0.3346**
BRA	**0.1439**	0.1585	**0.2381**	**0.2007**	**0.1936**
CAN	**0.1529**	0.1847	0.1688	**0.2580**	**0.3554**
CHE	**0.2051**	**0.3611**	**0.4512**	**0.3898**	**0.4919**
CHL	**0.2993**	**0.5276**	**0.6003**	**0.4713**	**0.2184**
CHN	0.0451	**0.3552**	0.1617	0.1159	**0.2534**
CZE	0.0954	**0.4607**	**0.3574**	**0.3445**	**0.2211**
DEU	**0.2463**	**0.2736**	**0.3724**	**0.3565**	**0.2658**
DNK	**0.2148**	**0.3061**	**0.2261**	0.1627	**0.3334**
ESP	**0.2067**	**0.2934**	**0.4682**	**0.2594**	**0.2936**
FIN	0.1127	**0.3500**	**0.2602**	**0.2783**	**0.2584**
FRA	**0.1534**	**0.2489**	**0.4147**	**0.2144**	**0.2544**
GBR	**0.1967**	**0.3814**	**0.3087**	0.1761	**0.3160**
GRC	**0.1533**	**0.3862**	**0.3544**	0.1653	**0.3508**
HKG	**0.1641**	**0.2229**	**0.3103**	**0.2734**	0.1508
IDN	**0.2895**	**0.3660**	**0.4023**	**0.3698**	**0.4462**
IND	0.0749	**0.2498**	**0.2561**	**0.2062**	**0.2239**
IRL	**0.1448**	**0.4746**	**0.2307**	**0.2132**	**0.3746**
ISR	**0.1580**	**0.3347**	**0.3129**	**0.2980**	**0.2939**
ITA	**0.1856**	**0.2395**	**0.2016**	**0.2170**	**0.2315**
JPN	**0.1500**	**0.2525**	**0.2795**	**0.3649**	**0.2550**
KOR	**0.1606**	0.1213	**0.2641**	**0.4596**	**0.2568**
MEX	**0.2852**	**0.3548**	**0.2893**	**0.3505**	**0.2899**
NLD	**0.1474**	**0.3519**	**0.3498**	**0.1978**	**0.2757**
NZL	**0.1871**	0.0866	0.1497	**0.3561**	0.0607
PAK	**0.5165**	**0.4606**	**1.1116**	**0.2627**	**0.2815**
PHL	**0.2379**	**0.4399**	**0.3022**	**0.3647**	**0.3171**
PRT	**0.2128**	0.0812	**0.6992**	**0.3143**	**0.1852**
RUS	0.1092	**0.3416**	**0.4693**	**0.3253**	**0.3426**
SWE	**0.2030**	**0.3784**	**0.2093**	**0.2128**	**0.2523**
USA	**0.3140**	**0.2910**	**0.2505**	**0.3143**	**0.3574**
ZAF	**0.2260**	**0.4704**	**0.2836**	**0.2365**	0.1113

Note: The bolded numbers indicate that market has multifractal feature with 5% statistical significance

Appendix C

Figure C. Time-varying asymmetric $H(2)$ and *degree of multifractality* with stock index. Blue line is overall, red line is up-trend, yellow line is down-trend and purple line is stock index. Vertical lines are for dividing the period.

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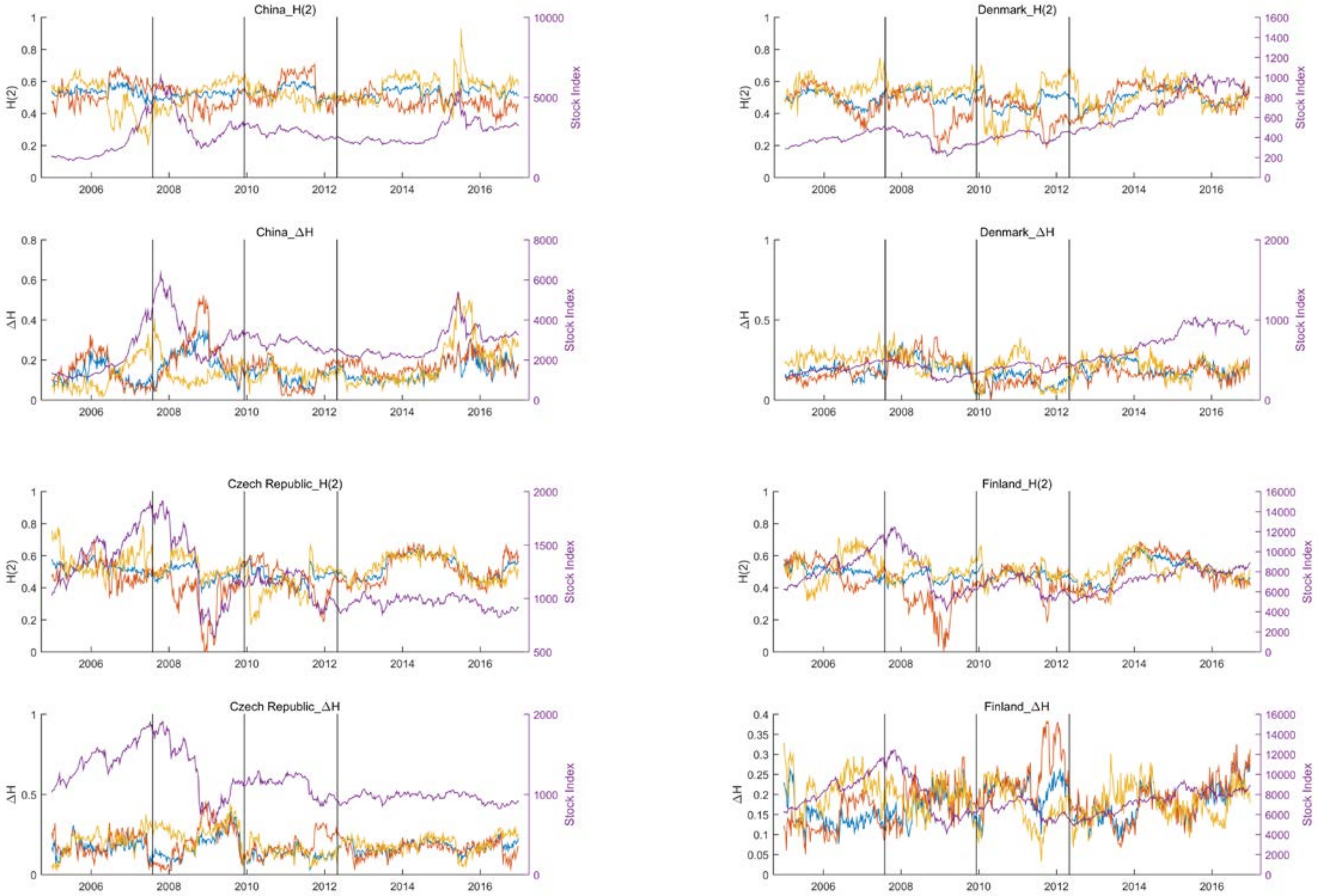
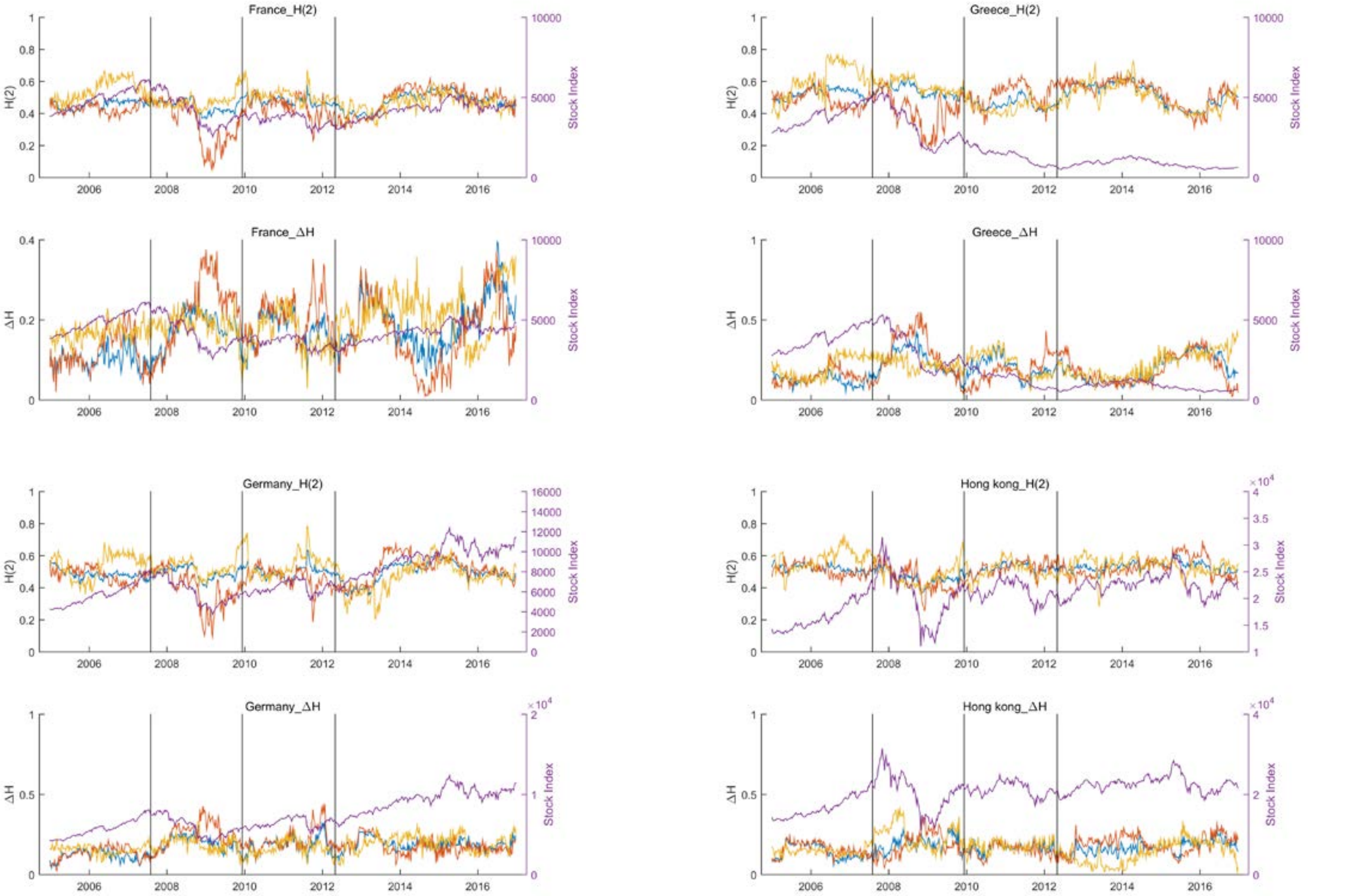
Canada_H(2)
Belgium_ΔH
Canada_ΔH
Brazil_H(2)
Chile_H(2)
Brazil_ΔH
Chile_ΔH
H(2)
ΔH
Stock Index
2006
2008
2010
2012
2014
2016



China_H(2)
Denmark_H(2)
China_ΔH
Denmark_ΔH
Czech Republic_H(2)
Finland_H(2)
Czech Republic_ΔH
Finland_ΔH
H(2)
ΔH
Stock Index
2006
2008
2010
2012
2014
2016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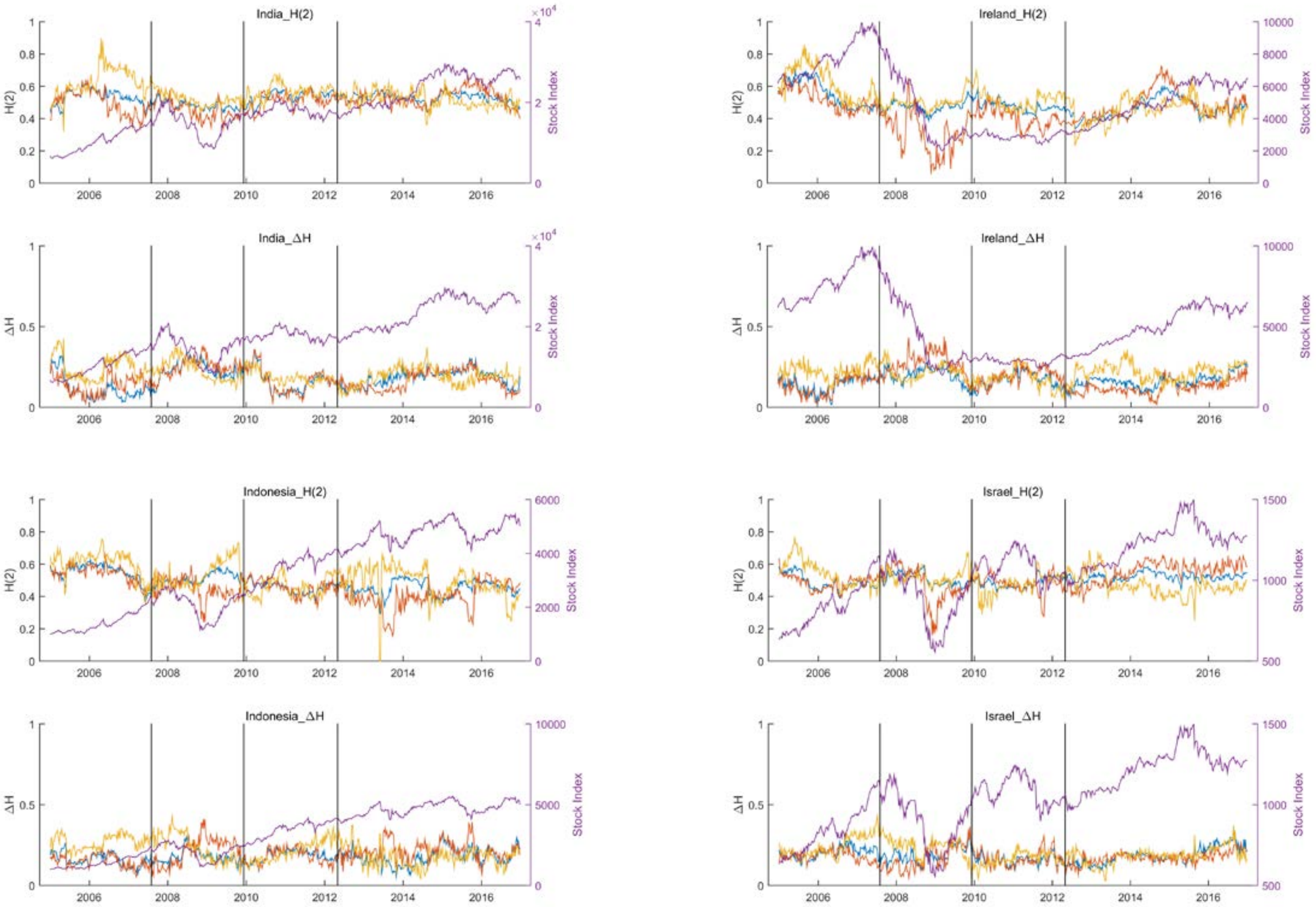
Greece_H(2)
France_ΔH
Greece_ΔH
Germany_H(2)
Hong kong_H(2)
Germany_ΔH
Hong kong_ΔH
H(2)
ΔH
Stock Index
2006
2008
2010
2012
2014
2016


India_H(2)
Ireland_H(2)
India_ΔH
Ireland_ΔH
Indonesia_H(2)
Israel_H(2)
Indonesia_ΔH
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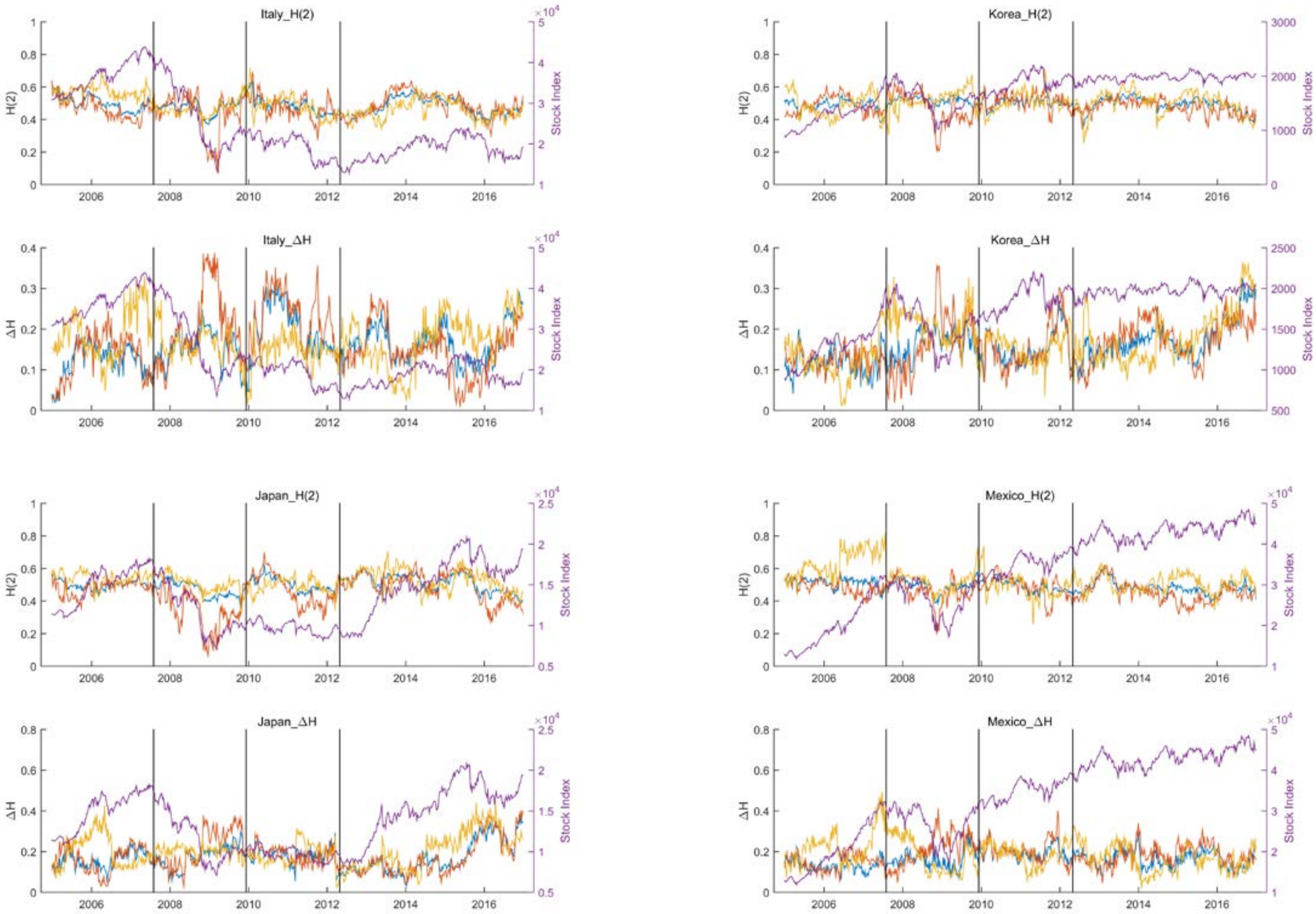
H(2)
ΔH
Stock Index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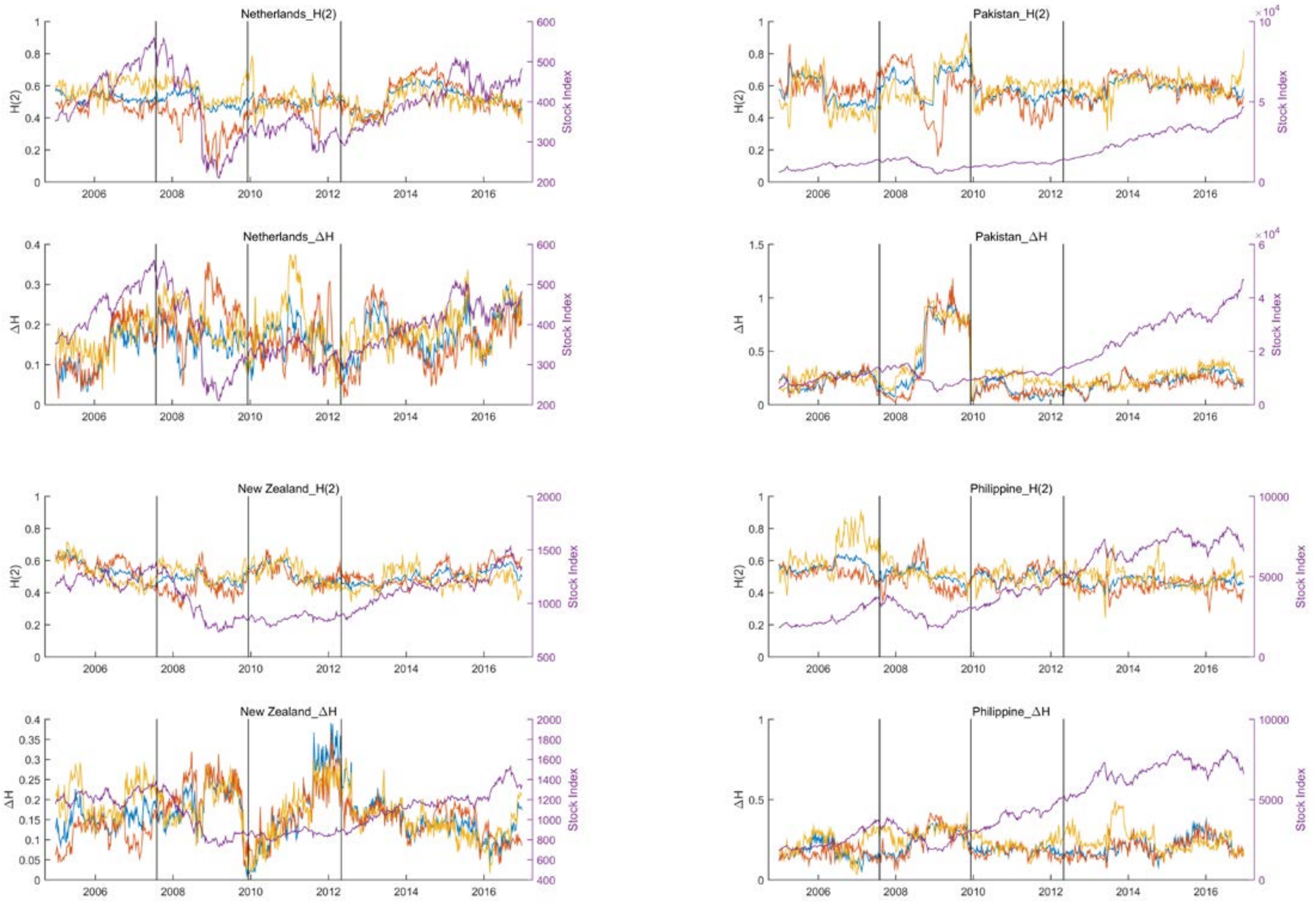
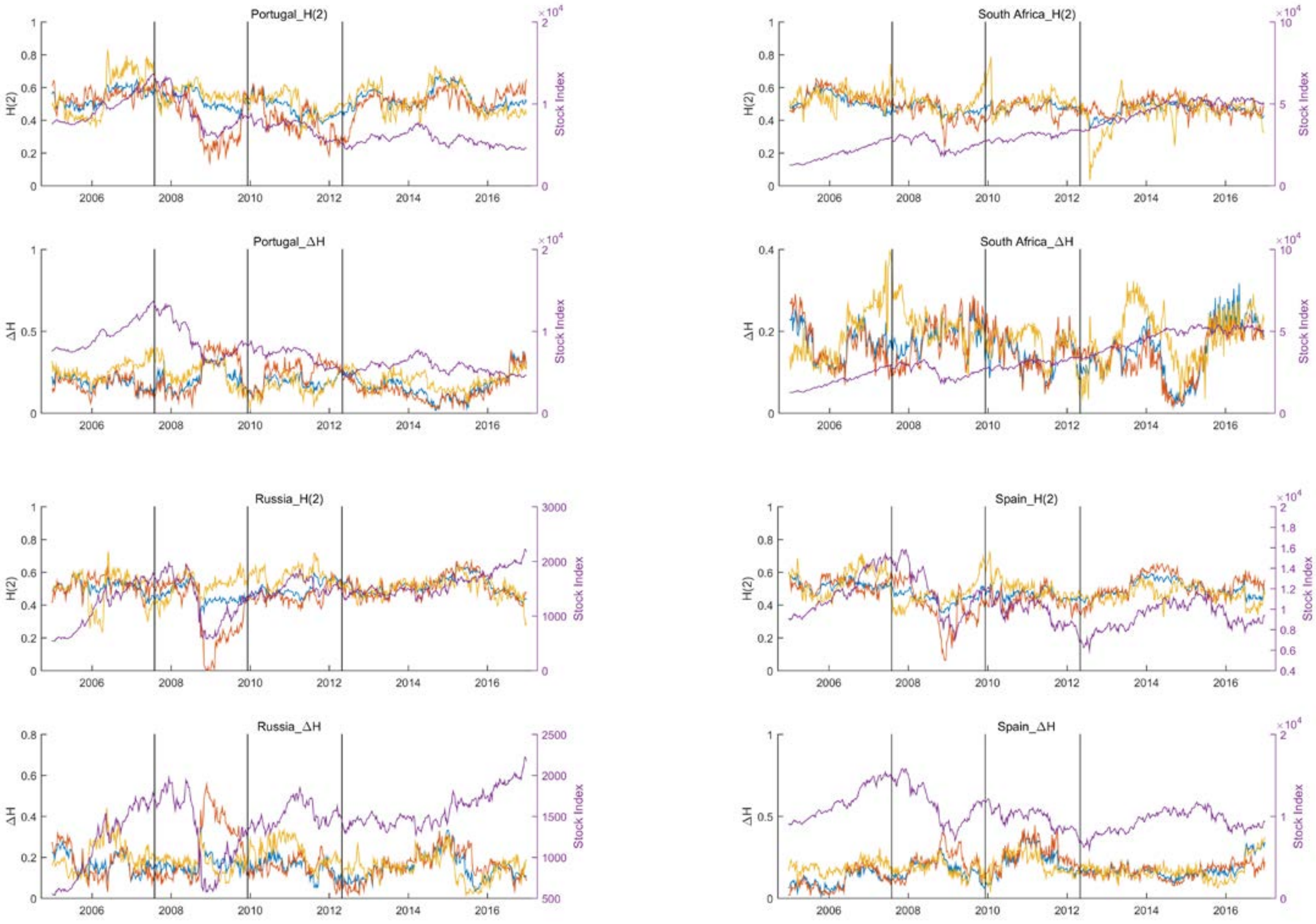
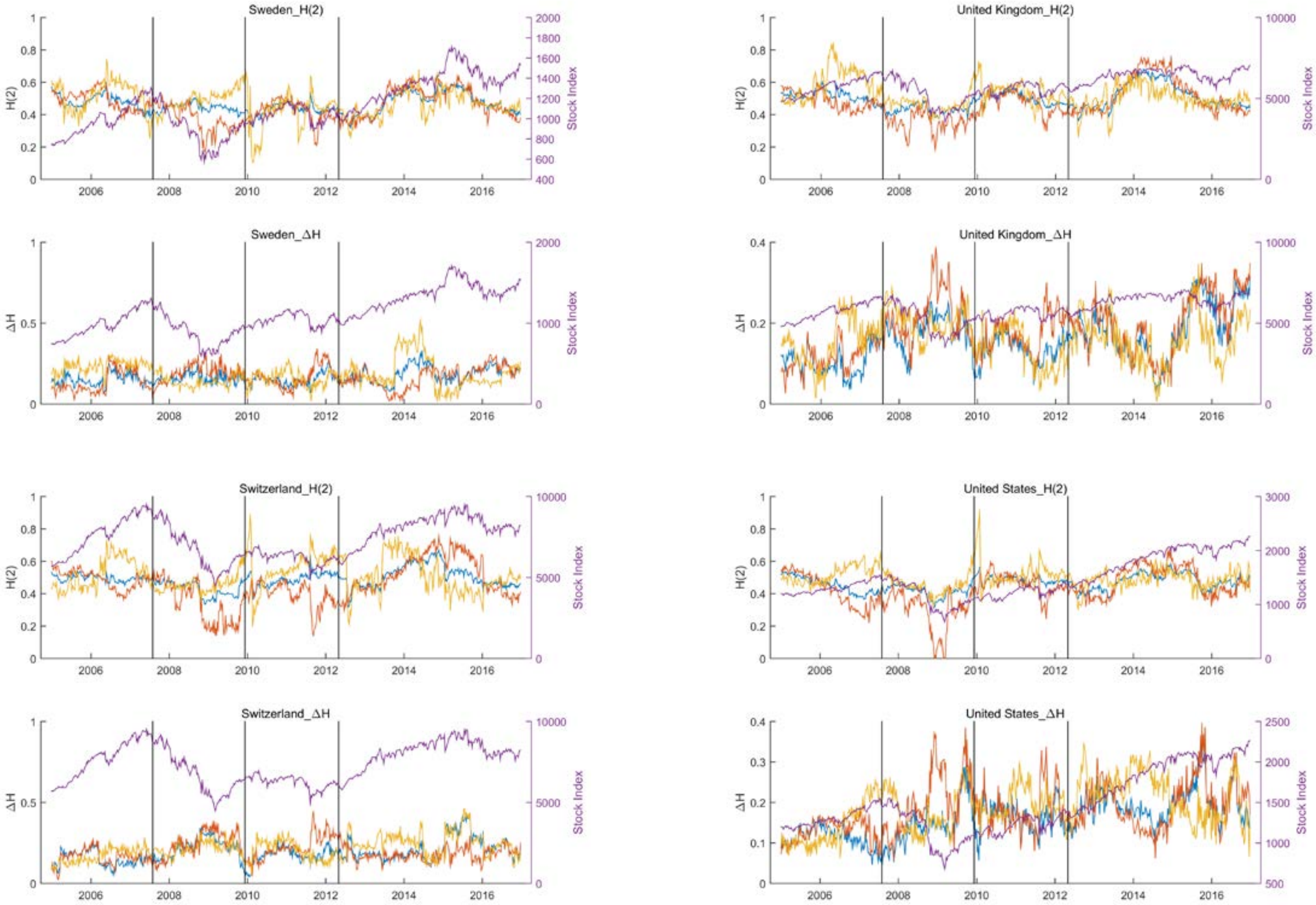
Italy_ΔH
Korea_ΔH
Japan_H(2)
Mexico_H(2)
Japan_ΔH
Mexico_ΔH
H(2)
ΔH
Stock Index
$\times 10^4$
2006
2008
2010
2012
2014
2016


Netherlands_H(2)
Pakistan_H(2)
Netherlands_ΔH
Pakistan_ΔH
New Zealand_H(2)
Philippine_H(2)
New Zealand_ΔH
Philippine_ΔH
H(2)
ΔH
Stock Index
×10^4
2006
2008
2010
2012
2014
2016


Portugal_H(2)
South Africa_H(2)
Portugal_ΔH
South Africa_ΔH
Russia_H(2)
Spain_H(2)
Russia_ΔH
Spain_ΔH
H(2)
ΔH
Stock Index
×10^4
2006
2008
2010
2012
2014
2016


Sweden_H(2)
United Kingdom_H(2)
Sweden_ΔH
United Kingdom_ΔH
Switzerland_H(2)
United States_H(2)
Switzerland_ΔH
United States_ΔH
H(2)
ΔH
Stock Index
2006
2008
2010
2012
2014
2016

Appendix D

Table D.1 Correlation between stock index and the overall Hurst exponent

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	0.0167	-0.5068*	-0.5415*	0.0848	-0.0916
AUS	0.1122	-0.6857**	0.0944	0.6176**	0.6199**
BEL	0.4348*	0.4130*	0.0995	0.4377*	0.6906**
BRA	0.1447	-0.4939*	0.6568**	-0.0916	0.2528
CAN	0.4410*	0.0567	0.8422**	-0.4934*	0.6185**
CHE	0.4309*	-0.2326	0.7236**	-0.5816*	0.5406*
CHL	-0.4435*	-0.3081*	0.2156	-0.7193**	0.4126*
CHN	-0.2472	-0.4292*	-0.5861*	0.4140*	-0.0649
CZE	0.0209	-0.4038*	0.4101*	-0.3574*	0.3725*
DEU	0.2638	-0.5704*	0.5771*	-0.3535*	0.5925*
DNK	0.2002	-0.3379*	0.6420**	-0.5697*	0.3282*
ESP	0.2453	-0.0875	0.5669*	0.4492*	0.6455**
FIN	0.1151	-0.6883**	-0.0981	-0.0447	0.5013*
FRA	0.3478*	0.4049*	0.6900**	0.3480*	0.6853**
GBR	0.3712*	-0.4715*	0.1447	-0.0022	0.5072*
GRC	0.2753	0.1282	0.4656*	0.3139*	0.6720**
HKG	0.2862	-0.5816*	0.7550**	0.4898*	0.4289*
IDN	-0.7144**	-0.7149**	-0.4551*	-0.4112*	-0.4204*
IND	0.0980	-0.0618	0.4046*	0.2595	-0.2823
IRL	0.2598	-0.8108**	0.0539	0.1337	0.3935*
ISR	0.2122	-0.4847*	0.5503*	-0.7363**	0.2583
ITA	0.1847	-0.6396**	0.1270	0.6978**	0.4495*
JPN	0.0854	-0.1931	0.7966**	0.2189	-0.2421
KOR	0.0269	-0.2889	0.1564	0.2097	-0.0072
MEX	-0.2755	-0.4716*	0.6354**	-0.5066*	0.0371
NLD	0.3960*	-0.2676	0.7509**	-0.1730	0.3078*
NZL	0.2407	-0.5408*	0.2726	-0.5425*	0.8279**
PAK	0.0224	-0.6361**	-0.2368	-0.0517	0.1694
PHL	-0.6128**	-0.1617	-0.0393	0.2575	-0.2562
PRT	0.3464*	0.6463**	0.5316*	0.5329*	0.1307
RUS	0.1065	-0.4748*	0.6781**	-0.1621	-0.1161
SWE	0.3387*	-0.4283*	-0.0278	0.4763*	0.6536**
USA	0.3573*	-0.8474**	0.5576*	-0.1612	0.5169*
ZAF	-0.2276	-0.5840*	0.8297**	0.0053	0.5250*
Mean	0.1135	-0.3457*	0.3307*	-0.0003	0.3134*

*Note: * and ** indicate the absolute correlations greater than 0.3 and 0.6, respectively*

Table D.2 Correlation between stock index and the up-trend Hurst exponent

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	0.0538	-0.3097*	0.4913*	0.0825	-0.0838
AUS	0.3129*	-0.5569*	0.4868*	0.3241*	0.8028**
BEL	0.6238**	-0.0083	0.7648**	0.8079**	0.6783**
BRA	0.2003	-0.4738*	0.6552**	0.2031	0.2785
CAN	0.4415*	-0.2239	0.8813**	-0.2697	0.5116*
CHE	0.6399**	-0.6304**	0.8162**	0.6678**	0.7477**
CHL	-0.2737	-0.0695	0.4575*	-0.4643*	-0.0231
CHN	0.1676	0.4353*	0.7416**	0.2951	0.0333
CZE	0.1430	-0.1543	0.6556**	0.5850*	0.2048
DEU	0.5542*	-0.3374*	0.8490**	0.0894	0.5482*
DNK	0.3599*	-0.5291*	0.8676**	0.2784	0.1627
ESP	0.2344	-0.1112	0.7371**	0.3425*	0.5343*
FIN	0.2928	-0.5547*	0.6256**	0.6581**	0.5784*
FRA	0.4461*	0.1799	0.8409**	0.5456*	0.6945**
GBR	0.4639*	-0.6889**	0.2940	0.2244	0.5280*
GRC	-0.1674	-0.2347	0.7050**	-0.1436	0.5132*
HKG	0.2821	-0.4856*	0.6913**	0.1815	0.2497
IDN	-0.5618*	-0.7427**	0.5451*	-0.4846*	0.2542
IND	0.2722	-0.5727*	0.5495*	0.1811	0.1823
IRL	0.4500*	-0.6717**	0.5585*	0.2904	0.3359*
ISR	0.6401**	0.1499	0.8504**	-0.1328	0.6482**
ITA	0.0327	-0.6241**	0.4709*	0.4626*	0.4140*
JPN	0.3057*	0.5056*	0.6551**	0.3682*	-0.0838
KOR	0.2370	0.0868	0.7769**	0.1936	0.0262
MEX	-0.1878	-0.5600*	0.4615*	-0.3315*	-0.1309
NLD	0.4397*	-0.1403	0.7794**	0.2593	0.2999
NZL	0.3103*	-0.7192**	-0.2894	-0.4442*	0.6800**
PAK	0.2098	-0.3798*	0.7603**	-0.4429*	0.1491
PHL	-0.4775*	-0.5141*	-0.0589	0.1916	0.0717
PRT	0.2994	0.3648*	0.8715**	0.6521**	0.1367
RUS	0.3944*	-0.1007	0.9005**	-0.0846	0.0686
SWE	0.4882*	-0.4940*	0.7614**	0.6937**	0.5666*
USA	0.4813*	-0.8067**	0.6427**	-0.1543	0.4462*
ZAF	-0.0774	-0.2019	0.6307**	-0.0492	0.3349*
Mean	0.2362	-0.2992	0.6302**	0.1640	0.3341*

*Note: * and ** indicate the absolute correlations greater than 0.3 and 0.6, respectively*

Table D.3 Correlation between stock index and the down-trend Hurst exponent

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	0.0053	-0.4156*	-0.4253*	0.1102	-0.0237
AUS	0.1165	-0.1824	0.2945	0.6882**	0.1091
BEL	0.3977*	0.3862*	-0.1470	0.0870	0.5058*
BRA	0.0384	-0.0288	0.5391*	0.2001	0.0133
CAN	0.3970*	0.3031*	0.6686**	0.1880	0.5071*
CHE	-0.0100	0.5647*	-0.0332	-0.5064*	-0.1403
CHL	-0.2866	0.4535*	0.1405	-0.3943*	0.4490*
CHN	-0.2871	-0.8096**	-0.7774**	0.4170*	0.0053
CZE	0.1455	0.0253	0.2948	-0.6779**	0.4518*
DEU	-0.1223	0.3226*	0.1851	-0.0165	0.5032*
DNK	-0.0195	0.3854*	0.2586	-0.2055	0.2248
ESP	0.2858	0.4613*	-0.1436	0.5747*	0.4585*
FIN	0.2019	0.4117*	-0.1151	-0.2356	0.2137
FRA	0.3728*	0.5060*	0.1559	0.2456	0.4517*
GBR	0.1904	0.5044*	0.3189*	-0.2260	0.3178*
GRC	0.6045**	0.5217*	0.5442*	0.0342	0.6586**
HKG	0.0685	0.5245*	0.3627*	0.5871*	0.2167
IDN	-0.4687*	-0.4783*	-0.2434	0.3878*	-0.4673*
IND	-0.3263*	0.4975*	0.5786*	0.4795*	-0.3948*
IRL	0.2172	-0.6719**	-0.1152	0.2003	0.3882*
ISR	-0.5331*	-0.7223**	0.5389*	-0.2777	-0.6366**
ITA	0.5425*	-0.1299	-0.1813	0.6325**	0.3568*
JPN	0.1686	-0.1374	0.7400**	-0.1327	-0.1458
KOR	-0.0043	-0.2658	-0.4143*	0.4845*	0.0264
MEX	-0.3469*	0.6714**	0.6927**	-0.1305	0.2747
NLD	0.3680*	0.2699	0.7345**	0.0967	0.2201
NZL	0.0048	-0.2658	0.6872**	-0.2585	0.3264*
PAK	0.1083	-0.4902*	-0.6448**	0.4712*	0.2021
PHL	-0.3158*	0.4833*	0.3888*	0.3115*	-0.2427
PRT	0.4313*	0.7607**	0.1183	0.4276*	0.2126
RUS	-0.0690	-0.1733	-0.1179	0.3163*	-0.2858
SWE	-0.0180	0.0044	-0.4280*	0.0127	0.5001*
USA	0.0696	0.7780**	0.4316*	0.0069	0.2081
ZAF	-0.3003*	0.3058*	0.6359**	-0.0994	0.4237*
Mean	0.0478	0.1285	0.1624	0.1117	0.1732

*Note: * and ** indicate the absolute correlations greater than 0.3 and 0.6, respectively*

Table D.4 Correlation between stock index and overall *degree of multifractality*

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	-0.0130	0.1576	0.5230*	-0.2210	0.2855
AUS	0.1310	-0.2250	-0.1029	-0.6195**	0.5535*
BEL	-0.3167*	0.1441	-0.0461	0.2011	-0.0716
BRA	0.2787	0.4604*	-0.4882*	0.3059*	0.1694
CAN	0.0595	0.0714	-0.2918	-0.4169*	-0.1169
CHE	0.1219	0.2116	-0.5615*	0.1273	0.4021*
CHL	-0.0811	-0.2354	-0.4797*	0.2432	-0.0542
CHN	0.1464	-0.4622*	-0.3058*	-0.1769	0.6837**
CZE	-0.2905	0.0659	-0.6807**	0.2823	-0.4258*
DEU	0.3244*	0.0605	-0.0562	0.1333	0.1841
DNK	0.1034	-0.3729*	0.2364	0.5772*	-0.1858
ESP	-0.1354	0.5999*	-0.1528	0.1110	-0.1735
FIN	-0.0992	-0.3222*	-0.2920	0.1078	0.4866*
FRA	-0.3113*	-0.2662	-0.5646*	0.0467	-0.0416
GBR	0.0284	0.0912	-0.3283*	-0.0896	-0.0919
GRC	-0.2341	-0.0070	0.1513	0.2318	-0.5998*
HKG	-0.0764	-0.5607*	-0.3702*	-0.1015	0.0267
IDN	0.0647	-0.3321*	-0.0693	0.2425	0.3577*
IND	0.1485	-0.5876*	-0.1979	-0.6201**	0.4120*
IRL	0.0024	0.3162*	0.0203	-0.2909	0.2744
ISR	0.0615	0.4137*	0.0683	-0.1646	0.2830
ITA	-0.3573*	0.2442	-0.3571*	0.2406	-0.1869
JPN	0.1878	0.2969	-0.4194*	-0.2894	0.4849*
KOR	0.2356	0.2218	-0.3502*	-0.1750	0.2764
MEX	0.2098	-0.0302	0.1504	-0.3947*	-0.3674*
NLD	0.0435	0.6287**	-0.2757	0.1767	0.4050*
NZL	-0.3507*	0.1968	-0.4283*	0.2449	-0.7010**
PAK	-0.1356	0.5693*	-0.9134**	-0.6121**	0.6309**
PHL	0.0353	-0.4633*	-0.8357**	0.1421	0.2473
PRT	-0.2251	-0.6300**	-0.6894**	-0.3427*	-0.5530*
RUS	-0.3618*	-0.4943*	-0.5544*	-0.1316	-0.0599
SWE	0.1403	0.0829	-0.1938	-0.1824	0.2142
USA	0.3133*	-0.4278*	-0.4480*	-0.0891	0.0898
ZAF	-0.1573	-0.2373	-0.5529*	-0.1970	0.1919
Mean	-0.0150	-0.0241	-0.2899	-0.0500	0.0891

*Note: * and ** indicate the absolute correlations greater than 0.3 and 0.6, respectively*

Table D.5 Correlation between stock index and up-trend *degree of multifractality*

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	-0.2117	0.2451	-0.5687*	-0.2186	0.2471
AUS	0.0219	0.0280	-0.4890*	-0.5281*	0.7124**
BEL	-0.5106*	0.5586*	-0.4630*	-0.6066**	-0.4916*
BRA	0.0397	-0.0260	-0.7239**	-0.1795	-0.2841
CAN	-0.0744	0.3773*	-0.7523**	-0.4406*	0.0057
CHE	-0.4138*	0.5027*	-0.7389**	-0.5837*	-0.3244*
CHL	-0.0629	-0.2659	-0.5188*	0.2160	-0.0306
CHN	-0.1193	-0.6527**	-0.6068**	-0.3093*	0.4575*
CZE	-0.2979	0.0569	-0.8384**	-0.7163**	-0.3721*
DEU	-0.1571	0.2205	-0.6208**	-0.1615	-0.2207
DNK	-0.0474	0.0177	-0.3558*	-0.1475	0.0548
ESP	-0.1593	0.6332**	-0.6172**	-0.1026	-0.1933
FIN	-0.1615	0.2795	-0.3315*	-0.6524**	0.3915*
FRA	-0.4446*	0.0466	-0.8266**	-0.4330*	-0.1836
GBR	-0.1013	0.3741*	-0.7021**	-0.2940	-0.1416
GRC	-0.0668	0.3044*	-0.1467	-0.7596**	-0.5239*
HKG	-0.2733	-0.5383*	-0.7938**	0.1118	-0.1819
IDN	0.1574	-0.1471	-0.7225**	0.1422	0.0465
IND	-0.0326	-0.2126	-0.5256*	-0.5825*	0.4747*
IRL	-0.2073	0.4131*	-0.3074*	-0.4004*	0.2810
ISR	-0.1757	-0.3950*	-0.3013*	-0.4287*	0.1206
ITA	-0.2661	0.3878*	-0.6671**	-0.1185	-0.4355*
JPN	0.0041	0.0518	-0.6435**	-0.0749	0.3844*
KOR	0.0371	0.1789	-0.7410**	-0.1002	0.1807
MEX	0.0115	0.1273	-0.7000**	-0.3633*	-0.0823
NLD	-0.2488	0.6080**	-0.7253**	-0.2958	0.1536
NZL	-0.3700*	0.1158	-0.0567	0.2783	-0.6254**
PAK	-0.2237	0.4539*	-0.9220**	-0.3142*	0.4806*
PHL	-0.0537	-0.1911	-0.8223**	0.1676	0.2622
PRT	-0.1951	-0.1941	-0.7913**	-0.6067**	-0.6065**
RUS	-0.5510*	-0.5836*	-0.8990**	0.0313	0.0815
SWE	-0.0742	0.2797	-0.6073**	-0.6282**	0.3149*
USA	-0.0488	0.0730	-0.6988**	-0.1865	0.0758
ZAF	-0.2766	-0.4541*	-0.4855*	-0.2227	0.0975
Mean	-0.1634	0.0786	-0.6091**	-0.2797	0.0037

*Note: * and ** indicate the absolute correlations greater than 0.3 and 0.6, respectively*

Table D.6 Correlation between stock index and down-trend *degree of multifractality*

Countries	Total	Pre-crisis	Subprime-crisis	European-crisis	Post-crisis
ARG	0.4090*	-0.4144*	0.6307**	-0.2323	0.4745*
AUS	-0.1837	-0.5001*	0.3003*	0.1757	-0.5416*
BEL	0.1200	0.1200	-0.3051*	0.4701*	0.2218
BRA	0.2380	0.5887*	0.4930*	0.5152*	0.2749
CAN	0.3259*	-0.1392	0.4252*	0.0390	0.1989
CHE	0.3082*	0.6981**	-0.3585*	0.4691*	0.3505*
CHL	-0.1553	0.7220**	-0.3215*	0.2613	0.1354
CHN	0.6101**	0.8791**	0.5479*	0.0726	0.8784**
CZE	0.1646	0.6860**	-0.3574*	0.4411*	-0.3173*
DEU	0.3012*	0.1882	0.2908	0.4098*	0.3050*
DNK	-0.1405	0.5334*	0.3352*	0.5391*	-0.3809*
ESP	0.1198	0.4995*	0.0507	0.3752*	-0.1931
FIN	0.2412	0.0051	-0.1026	0.3981*	0.2578
FRA	0.0588	0.1564	-0.0405	0.3340*	0.1476
GBR	0.0457	0.3952*	0.3172*	0.0583	-0.0254
GRC	-0.1039	0.5235*	0.4350*	0.6292**	-0.6331**
HKG	0.1226	-0.0216	0.5122*	-0.0769	0.1684
IDN	-0.3682*	0.1623	0.5467*	0.7336**	-0.1973
IND	-0.1256	-0.0121	0.4537*	-0.2634	0.2016
IRL	0.2764	0.0962	0.0157	-0.1983	0.0398
ISR	-0.1828	0.6466**	-0.0485	0.1982	0.2830
ITA	0.1422	0.3125*	0.0202	0.1899	0.1336
JPN	0.4691*	0.2156	0.3400*	-0.2514	0.7187**
KOR	0.2292	0.0868	0.4313*	-0.2270	0.2483
MEX	-0.1072	0.5909*	0.7081**	-0.3049*	-0.3147*
NLD	0.3478*	0.5512*	0.6169**	0.5959*	0.5218*
NZL	-0.1474	0.2556	-0.4308*	0.3055*	-0.3322*
PAK	-0.1430	0.4263*	-0.8904**	-0.2969	0.5763*
PHL	0.0109	0.2123	-0.3359*	-0.2373	-0.3974*
PRT	0.2901	0.5868*	-0.0737	-0.0774	-0.3016*
RUS	0.0554	0.5440*	0.2310	0.2229	-0.1731
SWE	0.0623	0.3594*	0.2852	0.3187*	-0.1962
USA	0.3383*	0.8730**	0.3353*	0.4456*	-0.2727
ZAF	0.0709	0.7410**	-0.0558	-0.0457	0.1186
Mean	0.1088	0.3402*	0.1471	0.1761	0.0582

*Note: * and ** indicate the absolute correlations greater than 0.3 and 0.6, respectively*

초 록

최근 다양한 금융위기 이후, 금융위험관리를 위한 금융시장 분석의 중요성은 더욱 강조되고 있다. 금융시장은 과거의 모형으로 설명하기 어려운 다양한 특성들을 가지고 있기 때문에 이를 설명하기 위한 노력이 필요하다. 특히, 금융시장에서 나타나는 멀티프랙탈 특성과 비대칭 상관관계에 대한 연구가 활발히 진행되고 있다. 멀티프랙탈 특성은 스케일에 따라 변하지 않는 자기유사성을 가진 프랙탈 특징이 다양하게 나타나는 것으로 프랙탈 차원을 하나로 나타내기 어려운 구조이다. 이를 통해 주식시장에서 나타나는 복잡성을 설명할 수 있다. 비대칭 상관관계는 시황에 따라 달라지는 특성으로 금융시장의 비대칭 구조를 나타낸다. 따라서 본 학위논문은 주식시장 데이터에서 나타나는 멀티프랙탈 특성의 비대칭 상관관계에 대한 연구를 진행하였다. 더불어, 비대칭 멀티프랙탈 특성을 이용하여 주식시장의 비대칭 효율성을 측정해보았다. 먼저, 본 학위논문은 멀티프랙탈 특성을 주가 지수의 추세에 따라 비대칭적으로 측정하는 'Price-based Asymmetric Multifractal Detrended Fluctuation Analysis (A-MFDFA)' 모형을 제시하였다. 기존의 모형이 전체시장에 대해서만 멀티프랙탈 특성을 측정하였다면, 본 모형은 주식시장을 지수의 추세를 기준으로 나누어 비대칭적인 특성을 고려한 멀티프랙탈 특성을 측정하였다는 데에 강점이 있다. 또한 제시된 모형을 이용하여 멀티프랙탈 특성의 원인, 비대칭 멀티프랙탈 특성이 나타나는 원인을 알아보는 방법을 제시하였다. 본 모형을 미국 금융시장 데이터에 적용한 결과, 미국 금융 시장에

비대칭 멀티프랙탈 특성이 있는 것을 확인하였고, 비대칭 멀티프랙탈 특성의 원인이 상승국면일 경우 변동성의 자기상관성이, 하강국면일 경우 확률 분포의 두꺼운 꼬리분포임을 밝혔다. 시간변화에 따른 비대칭 멀티프랙탈 특성의 변화를 관찰한 결과, 금융위기 기간에 상승국면 멀티프랙탈과 하강국면 멀티프랙탈의 수치의 차이가 증가함을 보였다. 두 번째로, 본 학위논문은 시뮬레이션 방법을 이용하여 제시한 'Price-based A-MFDFA' 모형이 비대칭 멀티프랙탈 특성을 성공적으로 잡아내는지 확인하고, 어떠한 특성이 비대칭 멀티프랙탈 특성에 영향을 주는지 알아보았다. 주식시장을 모방하기 위해 인위적으로 비대칭적인 특성을 가지는 시계열을 몬테카를로(Monte-Carlo) 시뮬레이션을 통해 만들어 낸 후, 제시된 모델을 이용하여 각 시계열의 비대칭 멀티프랙탈 특성을 관찰하였다. 그 결과 제시된 모형은 인위적으로 만들어진 비대칭 특성을 잘 나타내었다. 또한, 시계열의 자기상관성, 시계열 분포의 왜도, 두꺼운 꼬리분포와 변동성의 자기상관성이 비대칭 장기적 의존성과 멀티프랙탈 특성에 어떻게 영향을 주는지 밝혔다. 마지막으로, 비대칭 장기 기억 현상과 멀티프랙탈 특성이 존재하는지 알아보는 실험 방법론을 제시하였다. 기존의 시장 효율성 측정값이 나타내지 못했던 시장 비효율성의 원인을 상승 국면과 하강 국면의 멀티프랙탈 특성을 통해 알아보았다. 이를 34개 국가의 금융 시장에 적용해 본 결과, 금융위기기간에 상승 국면과 하강 국면의 장기 기억 현상 측정값과 멀티프랙탈 측정값의 차이가 커지는 현상과 상승 국면의 멀티프랙탈 측정값이 주가 지수와 강한 음의 상관관계를 가지는 현상을 관찰하였다. 비대칭 장기 기억 현상과 수익률 간의 관계에 대해서도 관찰하였다. 결론적으로 본 학위 논문은 전체 시장에 대해 사용했던 멀티프랙탈 특성 분석을

더 세분화하여 시황에 따른 비대칭 멀티프랙탈 특성 분석이 가능하게 했다는 점에 의의가 있다. 과거의 전체 시장에 대한 분석이 주식시장의 하락에 초점을 두었다면, 세분화된 비대칭 멀티프랙탈 특성을 통해 주식시장이 상승할 때와 하락할 때 각각에 대한 분석이 가능하게 된다. 따라서 금융 위험 관리에 대한 유용한 정보가 시장 참여자들에게 제공될 것이다.

주요어: 멀티프랙탈, 일반화된 허스트 지수, 비대칭성, 시장효율성, 금융 시장 데이터, 시뮬레이션, 장기 기억 현상

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