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Supplementary Material of "Stealthy Adversaries against Uncertain Cyber-Physical Systems: Threat of Robust Zero-Dynamics Attack"

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Abstract

This article is a supplementary material for the submitted paper [1].

I. Derivation of Normal Form Representation of Power Generating System

Let us consider a power generating system with a hybro turbine that consists of (see also [2], [3])

- governor

$$G_{\mathsf{g}}(s) = \frac{1}{T_{\mathsf{g}}s+1}, \tag{1a}$$

- hydro turbine

$$G_{\mathsf{h}}(s) = \frac{-T_{\mathsf{h}}s+1}{0.5T_{\mathsf{h}}s+1} = \frac{-2(0.5T_{\mathsf{h}}s+1)+3}{0.5T_{\mathsf{h}}s+1} = -2 + \frac{3}{0.5T_{\mathsf{h}}s+1} \tag{1b}$$

- load and machine

$$G_{\mathsf{lm}}(s) = \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}s+1}. \tag{1c}$$

where the outputs of the components are given by the incremental frequency deviation Δf (Hz), the change in generator output ΔP (p.u.), and the change in governor valve position ΔX (p.u.). The overall configuration of the power generating system is depicted in [1, Fig. 3]. With a coordinate transformation

$$\xi_1 := \Delta f, \tag{2a}$$
$$\xi_2 := \Delta P + 2\Delta X, \tag{2b}$$
$$\xi_3 := \Delta X, \tag{2c}$$

one can easily represent the plant in the state space as:

$$\dot{\xi}_1 = -\frac{1}{T_{\mathsf{lm}}}\xi_1 + \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\Delta P = -\frac{1}{T_{\mathsf{lm}}}\xi_1 + \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}(\xi_2 - 2\xi_3), \qquad \Delta f = \xi_1, \tag{3a}$$
$$\dot{\xi}_2 = -\frac{2}{T_{\mathsf{h}}}\xi_2 + \frac{6}{T_{\mathsf{h}}}\Delta X = -\frac{2}{T_{\mathsf{h}}}\xi_2 + \frac{6}{T_{\mathsf{h}}}\xi_3, \qquad \Delta P = \xi_2 - 2\xi_3, \tag{3b}$$
$$\dot{\xi}_3 = -\frac{1}{T_{\mathsf{g}}}\xi_3 + \frac{1}{T_{\mathsf{g}}}\left(u - \frac{1}{R}\Delta f\right) = -\frac{1}{T_{\mathsf{g}}}\xi_3 + \frac{1}{T_{\mathsf{g}}}\left(u - \frac{1}{R}\xi_1\right), \qquad \Delta X = \xi_3. \tag{3c}$$

To proceed further, we introduce an additional coordinate change

$$x_1 := \xi_1, \tag{4a}$$
$$x_2 := -\frac{1}{T_{\mathsf{lm}}}\xi_1 + \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\xi_2 - \frac{2K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\xi_3, \tag{4b}$$
$$z := \xi_2 + \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}\xi_1. \tag{4c}$$

It is noted that in this coordinate, the state $\xi = [\xi_1; \xi_2; \xi_3]$ can be expressed as

$$\xi_1 := x_1, \tag{5a}$$
$$\xi_2 := z - \frac{3T_{\mathsf{lm}}}{T_{\mathsf{h}}}\frac{1}{K_{\mathsf{lm}}}\xi_1 = z - \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}x_1, \tag{5b}$$
$$\xi_3 := \frac{T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}\left(-x_2 - \frac{1}{T_{\mathsf{lm}}}\xi_1 + \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\xi_2\right) = -\frac{T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}x_2 - \frac{1}{2K_{\mathsf{lm}}}x_1 + \frac{1}{2}\left(z - \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}x_1\right)$$
$$= \left(-\frac{1}{2K_{\mathsf{lm}}} - \frac{3T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}\right)x_1 - \frac{T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}x_2 + \frac{1}{2}z. \tag{5c}$$

By this, the time derivatives of ξ_i in (3) are rewritten as

$$\begin{aligned}\dot{\xi}_1 &= -\frac{1}{T_{\mathsf{lm}}}\xi_1 + \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\xi_2 - \frac{2K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\xi_3 \\ &= -\frac{1}{T_{\mathsf{lm}}}x_1 + \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\left(z - \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}x_1\right) - \frac{2K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\left(\left(-\frac{1}{2K_{\mathsf{lm}}} - \frac{3T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}\right)x_1 - \frac{T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}x_2 + \frac{1}{2}z\right) \\ &= x_2, \end{aligned} \tag{6a}$$

$$\begin{aligned}\dot{\xi}_2 &= -\frac{2}{T_{\mathsf{h}}}\xi_2 + \frac{6}{T_{\mathsf{h}}}\xi_3 \\ &= -\frac{2}{T_{\mathsf{h}}}\left(z - \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}x_1\right) + \frac{6}{T_{\mathsf{h}}}\left(\left(-\frac{1}{2K_{\mathsf{lm}}} - \frac{3T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}\right)x_1 - \frac{T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}x_2 + \frac{1}{2}z\right) \\ &= \left(-\frac{3}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}} - \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}^2}\right)x_1 - \frac{T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{3}{T_{\mathsf{h}}}x_2 + \frac{1}{T_{\mathsf{h}}}z, \end{aligned} \tag{6b}$$

$$\begin{aligned}\dot{\xi}_3 &= -\frac{1}{T_{\mathsf{g}}}\xi_3 - \frac{1}{R}\frac{1}{T_{\mathsf{g}}}\xi_1 + \frac{1}{T_{\mathsf{g}}}u \\ &= -\frac{1}{T_{\mathsf{g}}}\left(\left(-\frac{1}{2K_{\mathsf{lm}}} - \frac{3T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}\right)x_1 - \frac{T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}x_2 + \frac{1}{2}z\right) - \frac{1}{R}\frac{1}{T_{\mathsf{g}}}x_1 + \frac{1}{T_{\mathsf{g}}}u \\ &= \left(\frac{1}{T_{\mathsf{g}}}\frac{1}{2K_{\mathsf{lm}}} + \frac{1}{T_{\mathsf{g}}}\frac{3T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}} - \frac{1}{R}\frac{1}{T_{\mathsf{g}}}\right)x_1 + \frac{1}{2T_{\mathsf{g}}}\frac{T_{\mathsf{lm}}}{K_{\mathsf{lm}}}x_2 - \frac{1}{2T_{\mathsf{g}}}z + \frac{1}{T_{\mathsf{g}}}u. \end{aligned} \tag{6c}$$

By differentiating the variables in (4) along with the ξ-dynamics (3), we have

$$\dot{x}_1 = \dot{\xi}_1 = x_2 \tag{7a}$$

$$\begin{aligned}\dot{x}_2 &= -\frac{1}{T_{\mathsf{lm}}}\dot{\xi}_1 + \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\dot{\xi}_2 - \frac{2K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\dot{\xi}_3 \\ &= -\frac{1}{T_{\mathsf{lm}}}x_2 + \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\left(\left(-\frac{3}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}} - \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}^2}\right)x_1 - \frac{T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{3}{T_{\mathsf{h}}}x_2 + \frac{1}{T_{\mathsf{h}}}z\right) \\ &\quad - \frac{2K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\left(\left(\frac{1}{T_{\mathsf{g}}}\frac{1}{2K_{\mathsf{lm}}} + \frac{1}{T_{\mathsf{g}}}\frac{3T_{\mathsf{lm}}}{2K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}} - \frac{1}{R}\frac{1}{T_{\mathsf{g}}}\right)x_1 + \frac{1}{2T_{\mathsf{g}}}\frac{T_{\mathsf{lm}}}{K_{\mathsf{lm}}}x_2 - \frac{1}{2T_{\mathsf{g}}}z + \frac{1}{T_{\mathsf{g}}}u\right) \\ &= \phi_1 x_1 + \phi_2 x_2 + \psi z + gu, \end{aligned} \tag{7b}$$

$$\begin{aligned}\dot{z} &= \dot{\xi}_2 + \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}\dot{\xi}_1 \\ &= \left(-\frac{3}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}} - \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}^2}\right)x_1 - \frac{T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{3}{T_{\mathsf{h}}}x_2 + \frac{1}{T_{\mathsf{h}}}z + \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}}x_2 \\ &= \left(-\frac{3}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}} - \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}^2}\right)x_1 + \frac{1}{T_{\mathsf{h}}}z \\ &= Sz + Gx_1 \end{aligned} \tag{7c}$$

where

$$\begin{aligned}\phi_1 &:= -\frac{3}{T_{\mathsf{lm}}T_{\mathsf{h}}} - \frac{3}{T_{\mathsf{h}}^2} - \frac{2}{T_{\mathsf{lm}}T_{\mathsf{g}}} - \frac{3}{T_{\mathsf{lm}}T_{\mathsf{h}}} + \frac{1}{R}\frac{2K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\frac{1}{T_{\mathsf{g}}}, \\ \phi_2 &:= -\frac{1}{T_{\mathsf{lm}}} - \frac{3}{T_{\mathsf{h}}} - \frac{1}{T_{\mathsf{g}}}, \\ \psi &:= \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}} + \frac{K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\frac{1}{T_{\mathsf{g}}}, \\ g &:= -\frac{2K_{\mathsf{lm}}}{T_{\mathsf{lm}}}\frac{1}{T_{\mathsf{g}}}, \\ S &:= \frac{1}{T_{\mathsf{h}}}, \qquad G := -\frac{3}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}} - \frac{3T_{\mathsf{lm}}}{K_{\mathsf{lm}}}\frac{1}{T_{\mathsf{h}}^2}. \end{aligned}$$

REFERENCES

[1] G. Park, C. Lee, H. Shim, Y. Eun, and K. H. Johansson, “Stealthy adversaries against uncertain cyber-physical systems: Threat of robust zero-dynamics attack,” to be submitted.
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