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The Epistemic Structure of Economics Models and the Problem of Confirmation

A Dissertation
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Abstract

Epistemic Structure of Economics Models and the Problem of Confirmation

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We propose that confirmation on economics models has a distinctive two-layered espistemic structure. That is, we analyze that the econometrician as the outside observer interferes with the very model she is observing. We further argue that, due to this structure and the impossibility of knowing the true probability distribution, the Market Selection Hypothesis (MSH) is not directly confirmable from data. We provide a formal proof of this result in the framework of a Bayesian confirmation model.

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Chapter 1

Introduction

In this thesis, we investigate the problem of confirmation on economics models with a particular focus on the Market Selection Hypothesis (MSH). Ever since Hempel's (1943, 1945) early studies on the logic of confirmation, most philosophical literature on confirmation has been devoted to natural science models where the scientist, the outside observer, does not interfere in the very model she is observing. Such literature includes Carnap (1962), Earman and Salmon (1992), Maher(1996), Christensen (1997), Hajek and Joyce (2008), and Fitelson (2013). Here, we extend the scope of discussion to economics models. We suggest that, unlike the case with natural science models, confirmation on economics model has a distinctive epistemic structure: econometrician, the outside observer, can interfere in the very model she is observing. We argue that, due to this epistemic structure and the impossibility of knowing the true probability distribution, the MSH is not directly confirmable with data.

As M. Friedman (1970) points out, economics has two main tasks: (i) to build theoretical models to explain some existing phenomena and make meaningful predictions and (ii) to evaluate the performances of such theoretical models by the precision, scope, and *conformity with experience* of the predictions they yield. There are various ways to evaluate the performances of economics models in light of their conformity with reality. Among them, two most important methods are *significant test* and *falsification*. Various economics literature on these two empirical methods includes Foster, D. and Vohra, R. (1998), Fortnow, L. and Vohra, R. (2007, 2009), and Olszewski, W. and Sandroni, A (2008, 2009a, 2009b and 2011). As Hempel (1965) notes, however, confirmation is one of the most fundamental ways to evaluate such significance to reality, other than these two empirical methods. In this thesis, we extend the scope of discussion to confirmation.

Roughly speaking, the MSH is an economic hypothesis that the market selects for rational agents and thus that only rational agents can survive from the market competition.[1] Here, the notions of "rationality" and "direct confirmability" are defined as follows. First, agent is defined as *rational* when she holds the *true* belief. When belief is formally described as a sequence of subjective probability distributions, the true belief comes to be defined as the sequence of subjective probability distributions which coincide with true objective ones. A rational agent is then defined as the one whose beliefs are congruent to the sequence of the true distributions.

Modern economics builds on the hypothesis that an individual economic agent's behavior is rational in the sense that the agent's choice decision is made from her optimizing behavior. The standard model to formalize such an optimizing behavior is the one that maximizes expected utility. Intuitively, however, it is not always optimal to maximize one's own expectation unless the agent's subjective expectation is based on the *true* probability distribution on the market states. If her subjective

[1] Various attempts have been made in the economics literature to develop formal models for this selection hypothesis. Blume and Easley (2006) constructed a model to show that, under standard assumptions, rational agents survive over irrational agents when markets are complete, while they may not be able to survive when markets are incomplete. Other literature relaxes one or other assumptions to arrive at different survival results. It deserves to note, however, that we do not depend on specific forms of selection models to show that the MSH is not directly confirmable with data.

expectation is not from the true market condition, the maximizing behavior of the agent would not be *truly* optimal from the market point of view. It would only be what is *believed* to be optimal from the agent's point of view. Thus, in order to be truly rational, an agent must hold the true probability distribution. Note that this concept of rationality is different from the standard understanding of rationality. The standard view only requires that the rational agents should act according to their *perceived* best interest. Since what is perceived as best does not have to be the truly best, the rationality in the standard sense has nothing to do with the true probability. However, the standard understanding cannot exclude strange cases where agents, who are behaving based on their "irrationally" rosy expectations, are regarded as rational as long as they perceive such behaviors as best. In contrast, the Rational Expectations Hypothesis stipulates that the rational agent's perception does meet the objective standard of correctness.[2] Economic models for the MSH aim to provide one formal justification for the concept of rationality in this objective sense.

Second, *direct confirmability* is defined as the possibility of an inductive support from empirical data.[3] In a Bayesian model, such inductive support is assumed to be measured by agent's degree of belief updated by the evidence known from data.

[2]Blume and Easley (2015) argue that:

> Rational expectations is a misuse of the adjective. Unfortunately it is probably too late to abandon the term. There is no connection between the rationality principle, which claims that individuals act in their perceived best interest, and the rational expectations hypothesis, which claims that those perceptions meet some ex ante standard of correctness.

In our context, however, the adjective of the Rational Expectations Hypothesis conveys the most appropriate meaning of rationality.

[3]Here, *direct* confirmation by data is the concept in the comparision with *indirect* comfirmation by computer simulation. In this thesis, we discuss the problem of direct confirmation only. Therefore, in what follows, we will use the term of "confirmation" instead of direct confirmation, for simplification. Also, we will briefly sketch the future research plan on indirect confirmation by computer simulation in conclusion.

If the agent's system of degrees of beliefs is coherent, it satisfies the laws of probability. Thus, such inductive support in the Bayesian system is formally described by the incremental subjective probability conditional on the evidence from data. A hypothesis is *confirmed* when the incremental probability conditional on the evidence known from data becomes high enough with its affirmation.

We argue for the impossibility of direct confirmation by showing that the epistemic structure of confirming the MSH does not allow evaluation on such probabilistic increment. By dealing with the problem of confirmation on the MSH, we aim to shed light on the practical significance of rationality in the objective sense to the economics model. In general, this conclusion is applied to the economics models that are based on the above two definitions of rationality and direct confirmability.

Under this goal, the organization of this thesis is as follows: In chapter 2, we discuss three requirements for the concept of confirmation. We argue that a hypothesis H is confirmed by evidences E when the conditional probability of H given E satisfies (i) the incremental and (ii) the absolute requirements and the evidences meet (iii) the epistemic requirement. The epistemic requirement is that evidence should be what is *known* to be true by experience. Also, we discuss two issues regarding conditional probability in a confirmation model: what kind of probability this conditional probability is and what mode of supposition is considered when the probability is taken "conditional" on evidence. We argue that the Bayesian confirmation model is more apt to represent the *epistemic* structure of confirmation on the economics model, and thus that the appropriate interpretation of conditional probability in the confirmation is *subjective* one. We also argue that the appropriate mode of supposition is *indicative*.

In chapter 3, we review the Rational Expectations Hypothesis (REH) and two famous economic theses from the REH, namely the Lucas Critique and the Time

Inconsistency. We suggest that evaluations on economics models have epistemic structure. We analyze this epistemic structure of the two economic theses. We then review the MSH based on the model developed by Blume and Easley (2006), focusing on its theoretical implication to the REH. When the MSH holds, the REH also holds, but not vice versa.

In chapter 4, then we investigate the two-layered epistemic structure of confirming the MSH. We argue that, due to the impossibility of knowing the true probability distribution and the two-layered epistemic structure, the MSH is not directly confirmable. We build a formal Bayesian model to show this.

In chapter 5, we briefly discuss the implications of the confirmation impossibility. Chapter 6 is a conclusion.

Chapter 2

Confirmation Theory: Conditional Probability and Evidence

In confirmation theory, a hypothesis is confirmed when it is supported by evidence. The degree of such support from evidence is assumed to have numerical representation as conditional probability of the hypothesis given evidence. Here, the concepts of confirmation and evidence are tied together. Confirmation is the relation that holds between E and H when E is evidence for hypothesis H. Also, in order for E to serve as evidence for H, E must confirm H. Therefore, E cannot confirm H when there is no evidence available for H, or vice versa. In general, confirmation theorists approve the following two schemata as characteristics for the relation of confirmation:

(i) *incremental* : E increases the evidential support for H

(ii) *absolute* : H is highly supported given evidence E[1]

In the vein of these schemata, we suggest that E confirms hypothesis H when

[1] A. Hajek & J. Joyce, (2008), "Confirmation", in S. Psillos & M. Curd (eds.), *The Routledge Companion to the Philosophy of Science,* Routledge.

(i^*) *incremental* : the conditional probability of H given E is comparatively greater.

(ii^*) *absolute* : the conditional probability of H given E exceeds some threshold θ.

(iii^*) *epistemic* : evidence E is known to be true by experience.

Before we discuss these three conditions in detail, let us note that at least two important questions are raised here: First, what kind of probability is this conditional probability of H given E when it is used in the incremental and absolute conditions of confirmation? In particular, is it subjective or objective? Second, what exactly do we mean by "conditional" on evidence when the conditional probability matters with respect to confirmation?

Regarding the first question, two influential theories are suggested: the Carnapian confirmation theory[2] and the Bayesian one[3], respectively. The Carnapian confirmation theory argues that the inductive support from evidence is something objective and thus can be measured by using objective conditional probability, while the Bayesian confirmation argues that the inductive support from the evidence is subjective to the agent who is measuring such support. The objective conditional probability has its own name as epistemic probability. Also, it is called logical or inductive probability. The subjective conditional probability in Bayesian theory is called posterior probability. We prefer the Bayesian confirmation model because it is particularly apt to represent the epistemic structure of confirmation on the economics models.

Second, provided that we adopt the Bayesian confirmation model, the next question is what is the relation between the two events or propositions whose evidential

[2]Carnap, R. 1962 *Logical Foundations of Probability*, Chicago: The University of Chicago Press.

[3]Howson and Urbach, 2005. *Scientific Reasoning: The Bayesian Approach,* Chicago : Open Court.

relation is reflected on the subjective conditional probability in confirmation? In particular, when we understand $p(H|E)$ as the conditional probability of H given E, what do we mean by "given"? For our purposes two important modes of the supposition to consider are *indicative* and *subjunctive*. We argue that the appropriate mode of supposition is indicative when E is provisionally added to the agent's belief system in confirmation. We also argue that this indicative mode is closely related to the third requirement of confirmation that evidence should be what is known to be true by experience.

In the following sections, we discuss these two issues respectively, and then discuss the three requirements for evidence to confirm hypothesis.

2.1 Carnapian vs. Bayesian Confirmation Theory

Carnapian analysis of confirmation assumes that there exists a unique probability function which represents the confirmation relation between evidence E and hypothesis H. Such probability is objective and logically derived from a priori system on state and structure descriptions once evidence is obtained. State descriptions are the maximally consistent statements that describe the properties of all the individuals. A structure description is the maximal set of state descriptions each of which is homogeneous. For instance, let us consider a language consisting of two names a and b for individuals and one predicate F. Then, there are four state descriptions and three structure descriptions as follows:

State Descriptions	**Structure Descriptions**	**Weight**	**Measure**
F_a & F_b	everything is F	$\frac{1}{3}$	$\frac{1}{3}$
$\neg F_a$ & F_b	one F and one $\neg F$	$\frac{1}{3}$	$\frac{1}{6}$
F_a & $\neg F_b$			$\frac{1}{6}$
$\neg F_a$ & $\neg F_b$	everything is $\neg F$	$\frac{1}{3}$	$\frac{1}{3}$

Suppose now that we consider a hypothesis H that the individual b has property F. Also, suppose that we observe the individual a having property F. Let us denote this observational statement as E. Then, from this system, the prior probability is calculated as $\frac{1}{2}$, while the inductive probability is calculated as $\frac{2}{3}$. Intuitively, E inductively supports H, which is captured in this system by increase in the probability of H conditioning on E greater than the prior probability.

Although the Carnapian system captures such intuitive notions of objective evidential support, however, it has at least two serious problems. First, it can be shown that there are infinitely many probability functions with some suitable initial measures on descriptions which can represent such inductive support[4]. Thus, it fails to provide non-arbitrary unique weight system on descriptions. Later, Carnap relaxed the idea of determining unique confirmation function, allowing a continuum of functions to accommodate various degree of inductive cautiousness. However, critics doubt that Carnap succeeded in finding the "correct" confirmation function. Second, the inductive probability is logically calculated from the system once the system is a priori given. Therefore, it does not represent empirical relation between a hypothesis H and the evidence E. However, if the logical relationship is not empirically measurable, how can we know a specific value of epistemic probability from the observational evidence in the specific cases? In particular, from the first point, it is already clear that no unique system can be a priori determined. Thus, experience

[4] Carnap, R. 1962 *Logical Foundations of Probability*, Chicago: The University of Chicago Press.

should provide some clue on how to figure out specific value of probability, if it is not given a priori. One way to answer this question would be that we obtain such probability value by statistical law which says that the objective chance of H conditional on E is p. Unfortunately, however, statistical law cannot be known *a priori* either. To learn the value of the epistemic probability, we must discover the chance p first, but then the epistemic probability becomes at most redundant.

Because of these two defects, we adopt the Bayesian confirmation model over the Carnapian logical model. Note that the first defect uniquely belongs to the Carnapian confirmation model but that the second defect is a general problem to epistemic probability. In addition, we find one more convincing reason in particular with confirmation model on economic hypothesis: Bayesianism is apt to display well the subjective epistemic structure of confirmation on economics hypothesis. This point will become clearer in chapter 3 and 4, where we plan to discuss the epistemic structure of the MSH in terms of the Bayesian model in detail.

One last comment is that we do not bind ourselves to a specific interpretation on probability even though we adopt the Bayesian confirmation model. Indeed, it is still controversial whether there exists a true objective probability or probability is merely subjective. In this thesis, we do not seek for the answer on what kind of thing probability is per se. Instead, we pursue the question of whether an agent, say an econometrician, can know the true probability if there exists any such thing.

2.2 Conditional Probability and Modes of Supposition

2.2.1 The Primitive: Conditional vs. Unconditional Probability

Conditional probability in confirmation is in general understood as probability conditioning on some body of evidence or information, a probability "relativised to" a specified set of events or propositions. In mathematical practice, this relativisation can be formally captured by division function. Accordingly, conditional probability is defined as a ratio of two unconditional probabilities. A simple example of such ratio formula is the probability of getting face value of 6 with a fair die, conditioning that the value is of even number. Then, we divide the unconditional probability of getting $\{6\}$, $\frac{1}{6}$, by the unconditional probability of getting $\{2,4,6\}$, $\frac{1}{2}$, to obtain the conditional probability $\frac{1}{3} = \frac{1}{6}/\frac{1}{2}$. On the other hand, when "conditioning" is practically interpreted as "relativised to" a special set of outcomes, this set does not exhaust all the possible outcomes. Only unconditional probability is relativised to the set of all possible outcomes, which returns no information. For instance, tautology is treated as containing no information and thus unconditional probability can be considered as the probability conditional on tautology. Combining these two practices, however, causes the following problem: Conditional probability is defined as a ratio by unconditional probabilities, but at the same time, unconditional probability is defined as a special case of conditional probability, if conditioning on no information is regarded as a special case of conditioning. However, this seems at least philosophically problematic, for the conditional probability and unconditional probability are defined in a circular way. Let us discuss this point more in the Kolmogorov axiomatic system.

Defining on σ-field, Kolmogorov established the axiomatic probability system in the infinite space. This is the so-called *measure-theoretic* probability system. Let Ω be a non-empty set and $\mathcal{F}$ be a collection of subsets of Ω which is closed under complement and countable unions. Here, Ω is called a set of "outcomes," and $\mathcal{F}$ a set of "events." Note that $\mathcal{F}$ may be smaller than $\wp(\Omega)$, which can be interpreted that probability may not be defined with some events. Under the subjectivist interpretation of probability, this can be interpreted as that the agent cannot make likelihood judgement on some events. Then, probability is defined as a function $p : \mathcal{F} \to [0, 1]$ that assigns probability measures to events and satisfies the following three axioms.

(i) $p(A) \geq 0$, for all $A \in \mathcal{F}$

(ii) $p(\Omega) = 1$

(iii) if $\{A_i \in \mathcal{F}\}_{i=1}^{\infty}$ is a countable sequence of disjoint sets, then $p(\cup_i A_i) = \sum_i p(A_i)$

Once the unconditional probability is defined by axioms $(i) - (iii)$, the conditional probability in the Kolmogorov system is defined as the ratio formula:

$p(A|B) = \frac{p(A \cap B)}{p(B)}$ where $p(B) > 0$.

One problem with the ratio formula, however, is that it does not allow conditioning on probability zero event, while it seems conceptually plausible to ask for such conditioning. For example, McGee (1994) considers the following case. Suppose that a fair coin is going to be tossed infinitely many times and that the outcomes are independent. Then, unless the probability of tails on the first toss is either exactly 1 or 0, every particular infinite sequence of heads and tails will have probability 0. Suppose, for reductio, that some particular sequence, say, the infinite sequence consisting of all tails, has probability $\varepsilon > 0$. Also, let the event of the ith toss with tails be denoted by t_i. Then, the event of first n tosses being tails becomes $\bigcap\limits_{i=1}^{n} t_i$. Since the coin is fair and outcomes are independent, $\Pr(\bigcap\limits_{i=1}^{n} t_i) = \prod\limits_{i=1}^{n} \Pr(t_i) = \frac{1}{2}^n$. Now, for any given ε, we can always find some k such that $\frac{1}{2}^k < \varepsilon$. However, if $T = \bigcap\limits_{i=1}^{\infty} t_i$ is the event that all the infinite outcomes of tossing are tails, for this k, $\varepsilon = \Pr(T) \leqslant \Pr(\bigcap\limits_{i=1}^{k} t_i) < \varepsilon$, which is contradictory. On the other hand, since it is certain that at least one of such particular events will occur, it is certain that some probability zero event is going to happen. Thus, if a proposition with zero probability is discovered to be true, the agent shall revise her system of belief accordingly. This is the case where the conditional probability conditioning on zero event is asked for.

Two techniques are suggested to solve this problem: one approach is to assign infinitesimal but still positive probabilities to the improbable events. The other approach is to take conditional probability as more primitive and construct axiomatic systems based on the conditional probability. The first approach was beginning to be developed by Skyrms (1980) and Lewis (1980). The second was by Reyni (1955) and Popper (1959). The literature on Reyni-Popper measure is vast. The important research includes Harper (1976), McGee (1994), Van Fraassen (1996) and Hammond (1996).

It deserves to note, however, that the problem of conditioning on probability zero event does not necessarily force us to take the approach that conditional probability must be more primitive. In particular, we can accommodate conditioning on probability zero into the Kolmogorovian system as $p(A \cap B) =_{df} p(A|B) \times p(B)$ with $p(A \cap B) = 0$ once $p(B) = 0$. Therefore, the Kolmogorov system itself is neutral about the debate on the zero-conditioning problem. Then, perhaps a new Kolmogorovian measure theoretic approach may start from conditional probability to set up an axiomatic system, while deriving the above three Kolmogorov axioms from this new system. The standard Kolmogorov system starts from unconditional probability, deriving conditional probability from there.

We emphasize this point, because our main topic in this paper is Bayesian confirmation on economics model. Bayesian confirmation has conditional probability as its essential component and thus cannot avoid the zero-conditioning problem, while many economic models are mostly built up based on the Kolmogorovian measure-theoretic system. As it will be clear in later chapters, we argue that the true probability, if any, should be the same to both of the econometrician outside the economics model and the agent inside the economics model. Then, there should be no conflict between two probability systems,one of which represents the beliefs of econometrician in the Bayesian confirmation model and the other of which represents the beliefs of agents inside the economic model. Thus, we wish to ensure that Kolmogorov system does not have to exclusively support one specific form of probability system to avoid the problem of conditioning on improbable event. We do not want to worry about mixing any potentially conflicting probability systems when we take Bayesian method in confirming an economics model.

2.2.2 The Modes of Supposition in Bayesian Confirmation

Provided that the Bayesian confirmation model is considered, its subjective conditional probability is interpreted as how an agent's system of beliefs on hypothesis is revised when evidence is provisionally added to her existing belief set. According to Joyce (1999), such supposition of evidence is a form of provisional belief revision in which agent accepts some proposition E on evidence as true and makes the minimum changes in her other opinions needed to accommodate this modification. For our purpose, two important modes of supposition are the indicative mode and the subjunctive mode. When you suppose evidence indicatively in confirmation, you ask how things regarding the hypothesis *will be* if, as matter of fact, the evidence *is* true as it happens to be. On the other hand, when you suppose evidence subjunctively, you ask how things regarding the hypothesis *would have been* in a counterfactual situation if the evidence *had been* true as perhaps it happened not to be.

For instance, let us consider the following sentences from Adams (1970): "If Oswald did not kill Kennedy, someone else did", which is an example of indicative supposition, and "If Oswald had not killed Kennedy, someone else would have.", which is an example of subjunctive one. You can accept the first, because Kennedy was killed as a matter of fact and so you can conclude that someone must have done it. But you can reject the second, because in some counterfactual situation Kennedy might not have been killed although he happened to be killed in the actual world. Of course, such difference may not be huge, and it is controversial how to classify conditionals in a clear way. Moreover, it is beyond the scope of current thesis to settle the problem of which mode of supposition is appropriate to interpret conditional probability in general. At the risk of oversimplification, however, we accept that the conditional probability under the indicative supposition is tied to the *epistemical* possibility, while the one under the subjunctive supposition is tied to the *metaphysical* possibility. Epistemic possibility is affected by the current state of our knowledge about the actual world, while metaphysical possibility is based on the usual possible worlds in Lewis (1976). We call the probability representing the epistemic possibility *indicative probability* and the one representing the metaphysical possibility *subjunctive or counterfactual probability.* Based on this understanding, we argue for the following two things.

First, we agree with Joyce (2009) that at least the supposition involved in the confirmation theory is indicative, although not all the conditional probabilities in general might be so. However, we argue that Joyce has to face extra burden to explain further when he combines this indicative mode with his support for the *probative* measure of confirmation. Probative measure is the form of measuring the degree of confirmation by comparing $p(H|E)$ and $p(H|\neg E)$. Instead, incremental measure is comparing $p(H|E)$ and $p(H)$.

Suppose that $E = \{E_1, \ldots, E_n\}$ is a partition which consists of the set of all the disjoint alternatives of evidences that you might obtain during the confirmation process. Then, let p_j be the coherent conditional belief functions that you have on a hypothesis H when you obtain the evidence E_j. Now, suppose that you actually obtain E_k. In other words, E_k is true and all $E_j's$ are false for $j \neq k$ in the actual world. Then, all the probabilities which are conditioning on any E_j with $j \neq k$ must be the counterfactual probability, not indicative one. For E_j with $j \neq k$ would be true only in the counterfactual world once E_k is true in the actual world. Therefore, at least one conditional probability, either $p(H|E_k)$ or $p(H|\neg E_k)$, must be counterfactual probability while the other is indicative probability. If so, however, we will end up mixing the counterfactual probability and indicative probability at the same time in one confirmation model if we adopt the probative measure.[5] Therefore, Joyce faces burden to explain more why it is safe to mix two kinds of conditional probabilities in one confirmation equation. In particular, Joyce admits that indicative supposition obeys Bayes' Theorem but that subjunctive supposition is not so bound. Thus, mixing these two probabilities may cause a problem especially when we allow the Bayesian confirmation model to follow Bayes' Theorem.

Of course, this problem can be easily removed when we choose the alternative

[5] Even though probative confirmation equation always ends up mixing these two kinds of probabilities altogether when one conditional probability is indicative, it may not be always the case that mixing the two kinds of probabilities happens. For suppose that only E_k is true in this actual world but probability is conditioning on the set of $\{E_k, E_j\}$ where E_j is false for all $j \neq k$. Then, $p(H|E)$ and $p(H|\neg E)$ are both counterfactual but none of them is indicative.

measure where we compare a posterior and a prior probabilities, $p(H|E)$ and $p(H)$, for $p(H)$ is not involved with any supposition. Admittedly, Joyce argues that various confirmation measures have their own merits and thus that he does not support exclusively one particular form of measure. Therefore, it is not fatal problem to Joyce but at least he needs to assure us that mixing two probabilities is safe to do.

Second, we argue that the indicative supposition in confirmation is closely related to our third knowledge requirement for confirmation. We will discuss this more in later chapter.

2.3 Confirmation and Evidence

Recall that, according to our definition, evidence E confirms hypothesis H when (i^*) the subjective probability of H conditional on E becomes comparatively greater and (ii^*) it is high enough to exceed some threshold, and (iii^*) the evidence is what is known to be true by experience. We call (i^*) the incremental requirement and (ii^*) the absolute requirement. Lastly, we call (iii^*) the epistemic requirement.

Achinstein (2001) argues that the first condition (i^*) is neither sufficient nor necessary for E to confirm H. For there are counterexamples which can be intuitively accepted as the case of confirmation but their conditional probabilities are not comparatively greater, and vice versa. Achinstein also argues that the second condition (ii^*) is only necessary, for there are counterexamples which cannot be intuitively accepted as the case of confirmation although the conditional probability is high enough. We find some of his counterexamples convincing. But we disagree with his analysis. Instead of concluding that neither can define the concept of confirmation, we argue that both of the two conditions are essentially required for confirmation. We argue for the epistemic condition separately. The third epistemic requirement is in the same vein as the thesis that knowledge and only knowledge constitutes the evidence. Assuming the uncontroversial thesis that knowledge is evidence, we discuss only the thesis that, if an agent counts proposition E as her evidence, she knows it. In other words, if an agent does not know some proposition E, she cannot count it as evidence to confirm hypothesis.

2.3.1 Two Probabilistic Conditions for Confirmation

Suppose that a scientist is wondering about a hypothesis that there is intelligent life on Mars, and thus gathers some evidence to support it. If such collected evidences indeed confirm the hypothesis, her degree of confidence on the hypothesis must increase comparatively and become strong enough after obtaining evidence. However, in order to count as confirmed, how drastically such belief must be revised or how strong such revised belief must be, when the evidence is newly added? In other words, provided that her coherent degree of belief has numerical representation as probability, how much should her subjective conditional probability be comparatively greater and how high should it be?

Recall that confirmation theorists approve the two schemata, incremental and absolute, as characteristics for the relation of confirmation:

(i) *incremental* : E increases the evidential support for H

(ii) *absolute* : H is highly supported given evidence E

The incremental relation is reflected in our first requirement (i^*) for confirmation and the absolute relation is reflected in the second requirement (ii^*) for confirmation. The intuition behind these requirements are as follows: First, regarding the incremental condition, the hypothesis can count as confirmed by the evidences, when we become more confident of a hypothesis after obtaining some evidences. Here the important problem is how to measure the degree of "more confident". Vast literature discusses various functional forms of probabilities to measure such degree. Suggested functional forms include difference function $f(h,e) = \Pr(h|e) - \Pr(h)$ or $f(h,e) = \Pr(h|e) - \Pr(h|\neg e)$, log-ratio function. $f(h,e) = \log(\Pr(h|e)/\Pr(h))$ or log-likelihood ratio function $f(h,e) = \log(\Pr(e|h)/\Pr(e|\neg h))$.

Second, regarding the absolute condition, the hypothesis can count as confirmed, when we become confident enough given evidences. For instance, suppose that there are only two alternative hypotheses, H and $\neg H$. Then, if E supports $\neg H$ more than H, H cannot count as confirmed by E, which returns the threshold of $\frac{1}{2}$. For when $p(H|E) > p(\neg H|E)$ it follows that $p(H|E) > \frac{1}{2}$. However, if there are fairly many alternative hypotheses, $H_1, ..., H_n$, the threshold of $\frac{1}{2}$ may be too high. It depends on the context in each case what specific numerical value each scientist decides to accept as the threshold.

One thing to emphasize here is that both probabilistic conditions, incremental and absolute, are required to define the concept of confirmation. To see this point, let us consider the following examples from Achinstein (2001).

Lottery Counterexample

K : On Monday all 1000 tickets in a lottery sold, of which John bought 100 and Bill bought 1. One ticket was drawn at random on Wednesday.

E : On Tuesday all the lottery tickets except those of John and Bill were destroyed, and on Wednesday one of the remaining tickets was drawn at random.

H : Bill won.

A reasonable assignment of probabilities in this case is this:

$\Pr(H|K) = \frac{1}{1000}$, since given just K, Bill has just one of the 1000 tickets

$\Pr(H|E\&K) = \frac{1}{101}$, since given the additional information E, Bill now has 1 of the 101 tickets remaining.

Irrelevant information counterexample

K : Michael Jordan is a male basketball star

E : Michael Jordan eats Wheaties.

H : Michael Jordan will not become pregnant.

A reasonable assignment of probabilities in this case is this:

$\Pr(H|K) = 1$, since Michael Jordan's gender entails that he cannot be pregnant.

$\Pr(H|E\&K) = \Pr(H|K)$, since E is irrelevant information and thus does not affect the probability of H.

In the lottery counterexample, although $\Pr(H|E\&K) > \Pr(H|K)$, E given the background information K does not confirm H. For E intuitively supports that John won more than that Bill won. Therefore, Achinstein argues that the incremental requirement is not sufficient for confirmation. However, we diagnose that this lottery example only shows that the incremental requirement alone is not enough to constitutes the concept of confirmation. In this case, E supports more $\neg H$ than H and thus its conditional probability does not exceed a threshold $\frac{1}{2}$. It does not work for confirmation, because it does not satisfy the absolute requirement although it satisfies the incremental one.

In the irrelevant information counterexample, although $\Pr(H|E\&K)$ is very high given K, E does not confirm H. For the degree of confidence on H given K does not change with additional information on E. Therefore, Achinstein argues that the absolute requirement is not sufficient for confirmation. However, we also diagnose in this case that this irrelevant information example only shows that the absolute requirement alone is not enough to constitutes the concept of confirmation. In this case, it does not work for confirmation, because it does not satisfy the incremental requirement although it satisfies the absolute one. Therefore, our lessons from these counterexamples are that both conditions are essentially required to define what confirmation is. Neither of them alone cannot satisfy the concept of confirmation.

2.3.2 Epistemic Condition for Confirmation

Let us return to the case where a scientist considers the hypothesis H_1 that there exists intelligent life at Mars. Suppose in addition that the scientist runs across the following statements and considers whether to accept them as evidences which confirm H_1:

E_1 : There is intelligent life on Mercury.

E_2 : There is intelligent life on Venus.

E_3 : There is intelligent life on Jupiter

E_4 : There is intelligent life on Saturn

E_5 : There is intelligent life on Uranus

E_6 : There is intelligent life on Pluto[6]

Obviously, $E_1, ..., E_6$ altogether, if they were indeed true, will make the hypothesis H_1 fairly probable and increase the conditional probability higher than without them. $E_1, ..., E_6$ and H_1 together constitute a strong inductive argument. Therefore, both the incremental and absolute requirements are satisfied for confirmation in this case. Notwithstanding, it is intuitively plausible that no scientist is willing to accept $E_1, ...E_6$ as scientific evidences which confirm H_1. How can we then justify this

[6]Skyrms. B. 2000. *Choice and Chance: An Introduction to Inductive Logic.* Canada: Wadsworth/Thomson Learning.

intuition? One possible suggestion is that this is so because $E_1, ...E_6$ have very low probability.

In contrast, let us consider the hypothesis H_2 that a US infant, say Joe, was born with some fatal disease and a proposition E that Joe died right after birth in 2014. Then, provided that E is true, it makes the hypothesis H_2 fairly probable and also makes the conditional probability comparatively greater. Therefore, this example also satisfies both the incremental and absolute requirements for confirmation. Unlike the former case, however, clearly scientist will accept E as evidence that confirms H_2. According to the United States Center for Disease Control, the infant mortality rate reaches historic low at 0.0058, as of 2014. Therefore, provided that the probability of Joe's death can be equated to the death of any infant who was born in US in 2014, E has very low probability.

In both cases, $E_1, ..., E_6$ and E are very improbable, but H_2 is confirmed while H_1 is not. Therefore, the low probability of $E_1, ...E_6$ cannot be blamed for being unqualified for evidence.[7] We diagnose that such difference in confirmation results comes from the fact that only E can be *known* to be true in this actual world and thus serve as evidence to confirm H. $E_1, ..., E_6$ cannot become factual *knowledge* in light of our current epistemic state. From here we derive the reason why we should require additional epistemic condition for confirmation. This requirement is in line with Williamson's thesis that evidence is knowledge. Also, Maher(1996) argues for this requirement for confirmation. On the other hand, Hempel (1965) and Carnap (1962) do not require even the truth of E in order for it to confirm H, while Achinstein (2001) admits that E should be true in order to confirm H.

[7]Someone might argue that $E_1, \ldots, E_6$ are very improbable while E is only fairly improbable. Therefore, it might be argued that this difference in the degree of improbableness must be counted as the reason for the opposite intuition for confirmation results. However, there is no clear-cut standard on how to distinguish between very improbable and fairly probable. The degree of improbableness is too vague to serve as any criterion in this case.

It deserves to note that we do not require the probability of E to be one when we add the epistemic condition that evidence must be knowledge. Therefore, we do not answer how to deal with so-called old evidence problem even though we require evidence to be knowledge. Some philosophers like Dretske (1971) argue that knowledge must be certain, namely that the probability of E must be 1 if E is knowledge. However, there are many practical context that knowledge does not have to count as full certainty. For instance, even if you know that there exists a bottle of water in your refrigerator, you would not stake your entire fortune to bet on it. Your practical knowledge does not completely eliminate your uncertainty. Hawthorne and Stanley (2008) is in the similar vein of supporting this practical sense of knowledge. Also, Lewis (1996) can be interpreted in this line. For Lewis allows the case where the subject S knows that A but S does not have to eliminate by the evidence every possibility that A does not hold, as long as S can properly ignore such possibility in some context.

One more thing to note is that , according to this requirement, the probability in confirmation is conditional on knowledge and not just on true belief, or on anything falling short of knowledge. It is widely accepted that knowledge is not just true belief, although some philosophers refuse to analyze knowledge further into true belief plus something else. Then, this epistemic requirement entails that we cannot confirm hypothesis when its confirmation requires some *knowledge* to provide evidence but it is not possible to do so. It will be clear in chapter 4 why this point matters.

Lastly, it deserves to note that, in later chapters, we use the term "true belief" to mean a special subjective *partial* belief (subjective probability) which is congruent to the true objective probability. This way of using the terms "true belief" is originated from economists. "True belief" in this sense, however, might sound strange to philosophers. For philosophers normally combine *truth* with *full* belief while *ac-*

curacy with *partial* belief. Thus, "true (partial) belief" does not seem to be a right combination of concepts. On the other hand, it is not clear how to make a clear distinction between full and partial belief. A naive semantic comparison between "full" and "partial" may promptly lead to the argument that we reach full belief as we increase the degree of partiality in belief up to the full level. This implies, however, that a full belief is the belief whose degree is 1, meaning that the subjective probability is 1, which cannot be accepted. For only those beliefs that are *certain* can have probability 1 but not all full beliefs are certain. Nevertheless, we stick to these terms, because these are useful to draw an analogy to the famous distinction between knowledge and the true belief in full sense. We will argue later that it is not possible to do conditioning on knowledge of the objective true probability, although it might be possible to conditionalize on the true belief on such probability.

Chapter 3

The Market Selection Hypothesis (MSH) and the Rational Expectations Hypothesis (REH)

Modern economics is founded on the rationality principle of individual economic agents, or of market, the aggregate unit of individuals. However, the concept of rationality itself is very controversial. As Blue and Easley (2015) point out, rationality is for economist undefinable but nonetheless easily identified; and yet no two economists share a common definition. In this chapter, we argue that the appropriate concept of rationality should be based on the objective true probability distribution. This concept of rationality is in the objective sense, compared to the standard concept which is in the subjective sense. Evolutionary models such as the MSH model are proposed as one way to analyze the market rationality in terms of individual rationality based on this objective sense of rationality. The goal of this chapter is to make it clear why we focus particularly on the MSH in this thesis. We note that the MSH aims to provide a fundamental justification on analyzing the economic agent's decision-making in terms of this appropriate concept of objective rationality.[1] Because of this foundational importance, we focus on the MSH.

The rationality principle is that individual economic agent's behavior is rational in the sense that the agent's choice decision is made from her optimizing behavior. Here, we mean that the agent's optimizing behavior is to maximize her own expectation on the objectives which she wants to accomplish. The typical examples in economic models are behaviors of maximizing the expected present value of utility on consumptions or maximizing the expected present value of profit on output. The former is the typical setting on consumer theory and the latter is on producers theory.

Intuitively, however, it is not always optimal to maximize one's own expectation unless the subjective expectation is based on the *true* probability distribution on the state variables. If the subjective expectation is not from the true market state, the maximizing behavior would not be a *truly* optimal one from the market point of view, but would only be what is *believed* to be optimal from the agent's point of view. For example, suppose that Jonathan considers buying an one-million-dollar prize lottery ticket whose winning chance is one out of million. The ticket costs 2 dollars. To make life simple, let us assume that Jonathan enjoys uniform utility on every dollar-value and that he enjoys zero utility on no consumption. Then, the truly optimizing behavior is not to buy the ticket, because the expected utility from consuming this lottery is negative. Now, suppose that Jonathan is considering buying a ticket from a special lottery store where the last winning ticket was sold. Jonathan believes that this store has some secret power to increase the winning probability up to 10 times greater, according to which the losing probability is implied. In this case, Jonathan should buy a lottery ticket, because buying is now believed to be an

[1] It deserves to note that the MSH is not the only way to conduct such foundational analysis. We do not intend to meet such reckless challenge to cover all the justification analyses on the general decision rules. We just hope to shed some light on how to evaluate the significance of one convincing alaysis on the foundation of economics.

optimizing behavior to himself. Obviously, however, it is not truly optimizing one from the lottery market point of view.

Thus, to behave in a truly optimal way, it is important that the agent should have subjective probability distribution which coincides with the true objective one, if any. In other words, in order for any individual agent to be genuinely rational, it is required for her to hold the true probability distribution, if any. Within a standard framework on economics models, one paradigmatic claim is suggested in this vein. That is, the *Rational Expectations Hypothesis* (REH) stipulates that the representative agent ends up behaving according to the predictions of the objective rationality in equilibrium. That is, under the REH, the rational agents have the objective distribution and the market represents this objective true distribution because all the agents in the market end up sharing the common distribution with the rational agents. But what does guarantee that the individual rationality carries over to the aggregate market rationality? One way to justify the REH is suggested by the MSH. According to the MSH, the market selects for rationality, and so eventually the market will become dominated by rational agents at equilibrium. Recently, some economists tried to do more formal investigation on this market-selection idea. Among them, we investigate Blume and Easley's works in more depth.

Blume and Easley (2006) constructed a model to show whether the rational trader survives from market competition. With the help of the concept of entropy, they prove that the rational trader with (more) correct belief survives other agents with (worse) wrong belief in the complete market if both kinds of agents have the same patience. Here, *survival* is defined to be a positive consumption in the long run, while *vanishing* is defined to be zero consumption.[2] Therefore, the trader who forecasts

[2] Technically, a survival is defined as a positive lim sup, while a vanishing is defined as a zero limit. According to this mathematical distinction between lim sup and limit, it is still regarded as survival that the consumption drops to arbitrarily small level for infinitely many times, as long as the

future (more) correctly will eventually survive in the market, while those whose forecasts are persistently (worse) wrong will be driven out of the market. Just as the nature selects for the agents with features better-fitting to the natural environment according to the Natural Selection model in biology, so does the market selects for the rational agents with true belief. On the other hand, Blume and Easley argue that it may not be the case with the incomplete market that the rational agents survive over the irrational agents. If the rational agent cannot utilize over the wrong belief of the irrational agent because of the incomplete market structure, she is not guaranteed to survive. In this thesis, however, we focus mainly on the discussions in the complete market, because the basic logic of the confirmation impossibility does not depend on the types of market, complete or incomplete.

In the following sections, we review the REH briefly. After then, we briefly review the epistemic structure of economics models, and apply the analysis on this structure to two specific examples from the economics models. Such examples are two famous economic theses, the Lucas Critique and the Time Inconsistency from the REH. Studying these specific examples, we aim to shed some light on how to understand the epistemic structure of economics models and how to understand the econometrician's double roles as an outside observer and an inside agent while evaluating the economics models that she belongs to. Lastly, we review the Blume and Easley model for the MSH, which will be used to formalize the confirmation model for the MSH in chapter 4.

consumption is recovered to be big enough infinitely often. In contrast, for vaishing, consumption must stay arbitrarily small for all but finitely many times.

3.1 The Concept of Rational Expectations

The concept of Rational Expectations (RE) was first proposed by John Muth (1930∼2005), and later revised and refined further by a number of economists. Among them are Robert Lucas, Edward Prescott and Thomas Sargent. The main idea of the Muthian RE is that we can replace the representative agent's subjective expectation on the key variables by the objective mathematical expectation. Here, it is presumed that there exists some objective distribution which is possibly different from the subjective distribution. This objective distribution is assumed to be represented by some statistical distribution.[3] Let us formalize this idea in a simple setting by the following single equation on, say, the price variable:

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[3] It deserves to note that the objective distribution is conceptually different from the statistical distribution. A statistical distribution is derived from some statistical model which consists of some stated assumptions and some known basic statistical distributions such as normal distribution or poisson distribution, etc. For example, in case of Muth (1961), model stipulates some statistical distribution of u_t which is derived from the assumptions on its linear relation to ε_t and some known basic statistical distribution on ε_t.

Provided that there exists such thing as objective distribution according to which any event is happening in the physical world, it is not necessary that the objective distribution coincides with any such statistical distributions.

First, let us consider the following pharagraph from Kahneman and Tversky (1982) which emphasizes a careful distinction on the concepts of probability.

> We use the term "subjective probability" to denote any estimate of the probability of an event, which is given by a subject, or inferred from his behavior. These estimates are not assumed to satisfy any axioms or consistency requirements. We use the term "objective probability" to denote values calculated, on the basis of stated assumptions, according to the law of the probability calculus. It should be evident that this terminology is noncommittal with respect to any philosophical view of probability.

Here what Kahneman and Tversky mean by objective distribution is rather close to the statistical distribution. When Muth equated the mathematical prediction of the economic theory with an objective expectation, he does not make a sharp distiction between an objective and statistical distribution. Under the Rational Expectations Hypothesis, Muth derives an optimal expectation solution from the model. To regard such RE equilibrium solution as a true prediction on the future movement of the economy, a double coincidence is required, i.e. the coincidence of statistical distribution with an objective one as well as the coincidence of subjective distribution with an objective one. An objective distribution is owned by nature and a statistical distribution is by econometrician, while a subjective distribution is by economic agent inside the model. In this respect, a statistial distribution is a special kind of subjective distribution. However, it also has some objective character in the sense that the basic statistical distribution is selected among the known distributions from analysis on the objective data.

$$E_{t-1}(p_t) = p_t^e$$

where $E_{t-1}(p_t)$ represents the mathematical prediction of the relevant economic theory given the information through the $(t-1)$'st period. Also, p_t^e represents the subjective expectation of the economic agent inside the model. Provided that there exists a unique true objective probability distribution in the nature, this equation implies that all the economic agents in the model indeed come to hold a common expectation same as that from the nature. This apparently simple idea however provides powerful tools for econometrician to solve the optimal prediction problem. For example, suppose that, following Muth (1961), we consider short-period price changes in an isolated market with a fixed production lag of a non-storable commodity. Then the market equations take the form,

(Demand) $$C_t = c - \beta p_t$$

(Supply) $$P_t = s + \gamma p_t^e + u_t$$

(Market Equilibrium) $$P_t = C_t$$

where

P_t represents the number of units produced in a production period,

C_t is the amount consumed,

p_t is the market price in the tth period,

p_t^e is the market price expected to prevail during the tth period on the basis of information available through the $(t-1)$'st period,

u_t is an error term.

Just as in typical economic settings, the equations mean that the consumers demand more according as the price of the product is lower, while producers supply more according as they expect the higher price. A product market will be on equilibrium once its demand and supply become equal. From the market equilibrium condition, then the mathematical prediction of the theory is derived as:

$$E_{t-1}(p_t) = \alpha_0 - \tfrac{\gamma}{\beta} p_t^e - \tfrac{1}{\beta} E_{t-1}(u_t).$$

Therefore, under the REH, an econometrician obtains the following reduced form solution :

$$E_{t-1}(p_t) = \alpha_1 - \tfrac{1}{\beta+\gamma} E_{t-1}(u_t)$$

In virtue of this rational expectations assumption, explicit optimal expectations can be derived as solutions, if any, within a model. With explicit solutions, the model becomes practically valuable, because it can then provide a scientific basis for policy evaluation as well as forecast for the future movement of the economy. However, a practical value from any assumption does not justify the assumption itself. The REH provides powerful empirical value to the model but its practical value does not justify the hypothesis. We need a separate justification for it. As it becomes clear in the next section, the MSH offers one justification for the REH.

In general, consider any dynamic economics model for some key dependent variables y_t, which leads to the following reduced form,

$$y_t = h(x_t, x_{t-1}, \ldots, u_t, u_{t-1}, \ldots, y_{t+1}^e) \quad \cdots\cdots \quad (1)$$

where x_t's are observations on some determinant variables, u_t's are disturbances terms and y_{t+1}^e is unobservable subjective expectations on y_{t+1} based on the information up through time t. Then, under Rational Expectations, this reduced form becomes,

$$y_t = h(x_t, x_{t-1}, \ldots, u_t, u_{t-1}, \ldots, E_t y_{t+1}) \quad \cdots\cdots \quad (2)$$

where $E_t y_{t+1}$ is the objective expectation of y_{t+1} conditional on the information observed up to time t. If we assume that y's have some relation with the past history of x's and that x's are distributed according to some probability function dF,[4] then a final form of the expectation solution for the model is

$$E_t y_{t+1} = \int \phi(x_{t+1}, x_t, \ldots, u_{t+1}, u_t, \ldots | g) dF \quad \cdots\cdots \quad (3)$$

where g represents the parameters in the relation between y's and x's.

One thing to note here is that, in any single commodity market level or at a macroeconomic level, there are thousands or millions of economic agents. Therefore, we must ask whose expectations in the real world we are considering when we deal with y^e_{t+1} in the model. Muth argues that the subjective expectation in the model represents the average of the expectations from the individual agents in the real world. In early 60s when Muth constructed the Rational Expectations model, one of the major findings from expectations data was that the average of expectations in an industry is closer to the elaborate equations system like RE model, although there are considerable differences in the expectation opinions across people. Muth wanted to reflect this finding on his RE model where economic agents' subjective expectations are on average equal to the conditional expectation from the model.

This average expectation idea on the RE, however, is different from the common distribution idea which is discussed under current macroeconomic paradigm. In order to see the difference clearly, let us consider the following case where the subjective probability distribution represents the weighted averages of the probability measures

[4] Theoretically the probability function dF is treated as a true objective one, while econometrically it is identified as a statistical distribution. Again, conceptually an objective distribution is different from a statistical one, whose bifurcation matters when we try to empirically confirm the model. We will discuss this more as we proceed.

from individual agents in the industry, while the objective one represents the true probability distribution of the random variable X_t. For example, suppose that we are considering a random variable X_t whose possible values are the face values from rolling a dice. Then, the true probability distribution of X_t is uniform $(\frac{1}{6})$, while the subjective distribution of X_t is the weighted averages of the probability measures which individual agents assign on each possible state of X_t according to their beliefs.[5] Although Muth (1961) mentioned the distributional coincidence as well, however, his actual formula was based on the expectational coincidence on a single variable. It was not until 70s that the Muthian Rational Expectations ideas in the distributional sense were fully developed on the functional variables.

But as we already saw it, the REH, whether it stipulates expectational coincidence or distributional coincidence, requires a separate justification for itself. How can, say, those two subjective and objective probability distributions come to be equal to each other under RE? The Muthian idea about RE reflects the insight that if there are patterns among the empirical frequencies in the data, people will quickly learn the patterns and start using them to forecast a future event correctly, provided that the statistical pattern derived from the past data sets can keep representing the

[5] Let $p^j_{i,t}$ represent the probability measure which jth agent assigns on the ith possible value of random variable X_t at time t. Then, provided that there are N number of agents in one industry and M different possible states for X_t, the weighted average expectation of the industry on X_{t+1} can be expressed as follows:

$$E_t(X_{t+1}) = \sum_{j=1}^{N} \beta_j E^j_t(X_{t+1}) = \sum_{j=1}^{N} \beta_j \sum_{i=1}^{M} \{p^j_{t,i} \times X^i_{t+1}\} = \sum_{i=1}^{M} \{\{\sum_{j=1}^{N} \beta_j p^j_{t,i}\} \times X^i_{t+1}\}$$

where β_j represents the weight of jth agent with $\sum_{j=1}^{N} \beta_j = 1$.

According to this expression, the industry subjective probability measure is the weighted arithmetic mean of the individual subjective measures. But the individual subjective probability distribution represents each agent's partial belief on the state variable in the industry. Therefore, under the Muthian RE equilibrium, the individual agents in the model can have heterogeneous subjective beliefs but their average constitutes the industry subjective distribution.

In contrast, under the modern RE equilibrium, all the agents in the model come to share a common subjective distribution. There is no room for individual heterogeniety. MST justifies this second version of the modern RE equilibrium.

true distribution on the possible future values of the variable. During the learning procedure, some agents who are doing better can profit over the agents who know less, but in the end, all the agents remaining in the market must have learned the true pattern in the data history by arbitrage. However, unless the data history comes from a stable underlying structure, the agent cannot use past information to forecast future, because the unstable structure does not guarantee the recurring future to be the same as the past which the agents just learned. We discuss this issue of the unstable underlying structure more in the later chapters.

The Muthian rationale behind the REH is closely related to Friedman's market selection idea. Such rationale has remained as an informal idea until recently when some economists construct formal models for the selection idea. Among them, we review the MSH model of Blume and Easley (2006) in the following section. Before doing that, we investigate the epistemic structure of economics models under the REH.

3.2 The Epistemic Structure of Economics Models under the Rational Expectations

3.2.1 The Epistemic Structure of Economics Models

In economics, how to formalize the individual agent's decision-making constitutes a crucial part of the model. For example, a model on the car industry is based on the individual agents' decisions on purchasing or producing cars. However, when agents make such decisions as of now, they also consider what the future economy will evolve like. Therefore, it is essential part of economics model to formalize the agents' forming expectations on the future economy.

Such agent-based model has two distinctive characteristics: (i) the common inside-outside agent feature and (ii) the self-referential feature. The *common agents feature* of economics model is that the outside observer, say econometrician, is also working as an agent inside the model. The *self-referential feature* of economics model is that, the inside agent's belief can affect the very model which she belongs to, because her beliefs on the future values of variables, which are determined in the model, can affect the equilibrium of those variables in that model.

We call the variables whose values are determined in the model "endogenous variables". For instance, in the model on the car industry, the equilibrium price of car is the endogenous variable determined in that model. This equilibrium price reflects the agents' expectations on the future movements of the price. Therefore, the agents' beliefs on the future affect the current equilibrium of the model. Due to these two features, the confirmation on economics model has a distinctive epistemic structure that the outside observer, who conducts confirmation, can interfere in the very model she is observing, while updating the evidence.

In the Bayesian confirmation of the economics model, such epistemic structure comes into existence for the following two reasons. The *outside agents' beliefs*, which are acquired from the evidence during the confirmation process, may be *inherited to the agents inside* the model through the common inside-outside agent mechanism. Then, these *possibly* inherited beliefs to the inside agents can be *reflected on* the equilibrium of those variables in the *very model* through the self-referential mechanism.

For instance, let us recall the following reduced form equation from Muth (1961):

$$p_t = \alpha_0 - \tfrac{\gamma}{\beta} p_t^e - \tfrac{1}{\beta} u_t.$$

where p_t represents the equilibrium price for, say car and p_t^e represents the expected price by the agents inside the model. This equation clearly shows the self-referential mechanism that the endogenous variable, p_t, determined in the model is influenced by the beliefs on the future values on the variable, p_t^e at equilibrium. Also, when the outside observer, say econometrician, derives the expectation solutions from the above equation under the REH, she is assumed to equate the objective true expectation $E_{t-1}(p_t)$ to the subjective expectation p_t^e of the agents inside the model. This equalization reflects the common inside-outside agent structure. For the common agent structure means that, whatever the true probability is, if one probability distribution is true to the agents inside the model, it is also true to the econometrician. Because the econometrician is also residing in the same model as the other agents do, the true distribution, if any, should be the same to both of them.

It deserves to note the difference between the REH and the common inside-outside agent structure. The common agent structure only means that the econometrician and the agents inside the model *potentially* share the same true probability distribution if any, while the REH assumes that they *actually* shares the common distribution which happens to be true in the very model where they reside. The REH is a much stronger stipulation than the common agent structure. One more thing to note is that the REH is conventionally described as the stipulation that all the economic agents in the model, including the econometrician, come to *know* the true probability distributions at equilibrium. This description, however, is not correct. For holding the true belief in common does not necessarily imply knowing the truth. Notwithstanding, in the next section where we deal with two theses from the RE, we just follow the conventional expression of the economists.

We plan to argue later in detail that these epistemic features of economics model bring about a complicated problem in confirmation. In the next section, we show how two famous theses from the RE model, i.e. Lucas Critique and Time Inconsistency, can be analyzed in terms of these two epistemic characteristics. The simultaneous role of econometrician as an outside observer as well as inside agent of the model and her belief affecting the model itself lie at the center of the analysis.

3.2.2 Applications of the Analysis on the Epistemic Structure to Two Theses under Rational Expectations

The 'Lucas Critique' is a criticism of econometric policy evaluation procedures that fail to recognize the following economic logic:[6]

> [G]iven that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any changes in policy will systematically alter the structure of econometric models. (Lucas, 1976, p. 41)

Therefore, if the policy evaluation is performed under the assumption that the policy change does not alter the structure of the model, the evaluation leads to a wrong conclusion. Formally speaking, let s_t be the vector of state variables describing all the aspects of history relevant to the future evolution of the economy and x_t represent the vector of government policy variables. Then, a typical macroeconomic theory tries to estimate the parameter values in the following equilibrium law of motion:

$$s_{t+1} = F(\theta, s_t, x_t, \mu_t) \quad \cdots\cdots \quad (6)$$

where F is specified in advance, θ is a parameter vector to be estimated and μ_t is a vector of random disturbances. Lucas Critique criticizes the general practice that

[6] Ljungqvist, L. 2008. "Lucas Critique." *The New Palgrave Dictionary of Economics.* Second Edition. Eds. S. Durlauf. and L. Blume. Palgrave Macmillan. p. 1.

when a policy evaluation is performed, θ is treated as a fixed vector as if it is not influenced by that policy change. Lucas Critique continues to argue that the policy evaluation based on this wrong practice results in the wrong evaluation outcomes. But under what process are the parameter values of the equilibrium equation affected by policy changes? Here, the two epistemic features of economics model are working as crucial elements. First, with a new policy executed, there arrives a new relationship between the state variables and the policy variables. The econometrician, an outside observer of this model, obtains new estimation on such parametric relationship from new data sets and through her understanding on the structure of the economy. Since this econometrician also resides inside the model, the economic agents inside the model come to share such estimation results with the econometrician. Second, as this model is self-referential, then this new estimation result shared by all the economic agents changes the way how they form their expectations on the future movement of the key variables in the model, which again changes the derived equilibrium path. Only under a correct equilibrium path considered, the consequence of a new policy can be properly evaluated. As a result, different policy change will induce different equilibrium path and so the consequence of each of the policy changes must be evaluated separately under each of the equilibrium paths respectively. The assumption of the fixed θ amounts to that of the fixed equilibrium path throughout the policy changes.

For example, suppose now that, instead of the general form of the reduced equation in (1), we have the following linear model for the equilibrium law of motion from Sargent (1976),

$$y_t = \alpha + \lambda y_{t-1} + \beta m_t + u_t \quad \cdots\cdots \quad (7)$$

Just as the form of F in (6) is specified in advance, so is (7) specified as linear function from the following structure:

$$y_t = \xi_0 + \xi_1(m_t - m_t^e) + \xi_2 y_{t-1} + u_t \quad \cdots\cdots \quad (8)$$

$$m_t = g_0 + g_1 y_{t-1} + \varepsilon_t \quad \cdots\cdots \quad (9)$$

$$E_{t-1} m_t = g_0 + g_1 y_{t-1} \quad \cdots\cdots \quad (10)$$

where (8) says that the endogenous variable y_t moves only by the unanticipated movements of money m_t, and (9) represents the government's monetary policy rule. The econometrician's job is to estimate the parameter values in (7), using data on (y_t, m_t). If we can safely assume that the parameter values α, λ and β are fixed just as estimated by the past data regardless of the policy changes, we can infer the optimal feedback rule from those parameter values. Also, under the Rational Expectation, economic agent "knows" the consequences of the government policy and takes this into account in forming her expectation. Therefore, under the RE, (8), (9) and (10) return the following equation:

$$y_t = (\xi_0 - \xi_1 g_0) + (\xi_2 - \xi_1 g_1) y_{t-1} + \xi_1 m_t + u_t \quad \cdots\cdots \quad (11)$$

The parameters of (11) clearly include parameters of policy rule, g_0 and g_1 which continued in effect during the estimation period. Provided that the economic agent approaches closely to the true economics environment through various estimation technique, the parameters of (11) changes once the government policy changes. Therefore, in this simple RE model, all the policy changes will be exactly offset by these changes in parameter. Any policy argument based on the assumption that the parameter values are fixed as estimated by previous data will be fallacious.

Next, let us consider Time Inconsistency Thesis. A policy π_t is consistent if the optimality is attained when the policy is selected as best with the current situation given as fixed. Time Inconsistency occurred when the best chosen policy is not optimal any more. For example, suppose that for some historical reasons a big habitat happen to have been established in a particular flood plain. Now, the government is about to decide whether to take a flood control measures in this region. Given that many populations already reside there, the best policy is to construct a strong flood control system. However, it would not be an optimal policy, because people would not have kept the habitat if people had expected that the government would never take any flood control measures in this region. The socially optimal outcome is to construct a village in safer place so that the government saves the budget for any flood control system. In this case, the flood control policy is inconsistent, because it is the best policy given the situation as fixed but the optimality will be attained if such policy is never taken. For example, let us consider a two-period optimal control problem. For $T = 2$, π_2 is the best policy plan to maximize the following objective function under some constraints:

$$S(x_1, x_2, \pi_1, \pi_2),$$

subject to

$$x_1 = X_1(\pi_1, \pi_2) \text{ and } x_2 = X_2(\pi_1, \pi_2)$$

where $S(\cdot)$ is a social objective function and $X_t(\cdot)$ is an economic agent's decision function at time t.

Note that the policy decision is given as an argument to the individual decision function but that this policy decision is made under the constraint of the individual decision. The outside observer, i.e. econometrician, understands this complicated system. Since the econometrician is also the agent residing in the system, if her understanding is shared by the agent inside the model, then this shared understanding contributes to agent's forming the expectations on the future movement of the system under the RE, which in turn affects her current decision through the self-referential mechanism.

Kydland and Prescott (1977) suggested the following as reasons for this inconsistent result:

> Current decisions of economic agents depend in part upon their expectations of future policy actions. Only if these expectations were invariant to the future policy plan selected would optimal control theory be appropriate. In situations in which the structure is well understood, agents will surely surmise the way policy will be selected in the future. Changes in the social objective function reflected in, say, a change of administration do have an immediate effect upon agents' expectations of future policies and affect their current decisions.

The econometrician evaluates the entire economic model as if she observes the model standing outside. But at the same time she also influences the model itself while residing inside the model. We reviewed how these two features work in the Lucas Critique and the Time Inconsistency under the RE.

3.3 The Market Selection Model

The REH stipulates that in equilibrium all the economic agents in the market come to "know" the true probability distributions of market states. However, how can this hypothesis be justified? One suggestion is offered by the survival argument that only the rational agents are able to survive in the long run. During the adjustment process to the long-run equilibrium, the agents with systematic wrong belief lose constantly and so they are eventually driven out of the market. As a result, the market is dominated by only rational traders in equilibrium. Blume and Easley formally investigate this survival argument as follows.

3.3.1 Notations and Basics

Following Blume and Easley (2006), it is assumed that time is discrete and begins at date 0. The possible states at each date form a finite set $\{1, \ldots, S\}$. Σ is the set of all sequences of states with a representative sequence $\sigma = (\sigma_0, \ldots)$. Then, $\sigma^t = (\sigma_0, \ldots, \sigma_t)$ denotes the partial history through date t and $1_t^s(\sigma)$ is the indicator function which takes on the value 1 if $\sigma_t = s$ and 0 otherwise. For any probability measure q on Σ, $q_t(\sigma) = q(\{\sigma_0 \times \cdots \times \sigma_t\} \times S \times S \times \cdots)$ is the (marginal) probability of the partial history. Each $q_t(\sigma)$ is assumed to be F_t-measurable; that is their value depends only on the realization of states through date t.

An economy contains I consumers, each with consumption set $\mathbf{R}_+$. A *consumption plan* $c : \Sigma \longrightarrow \prod_{t=0}^{\infty} \mathbf{R}_+$ is a sequence of $\mathbf{R}_+$-valued functions $\{c_t(\sigma)\}_{t=0}^{\infty}$ in which each c_t is F_t-measurable. Each consumer i is endowed with a particular consumption plan, an endowment stream e^i. Consumer i has a utility function $U^i : c \longrightarrow [-\infty, \infty)$ which is the expected present discounted value of some payoff stream with respect to some probability measure, i.e. some beliefs. Consumer i's beliefs are represented by a probability distribution p^i on Σ. She also has a payoff function $u^i : \mathbf{R}_+ \longrightarrow [-\infty, \infty)$ on consumptions. We denote true probability distribution as p. Now, suppose that the planner attains Pareto optima allocations $c^* = (c^{1*}, \cdots, c^{I*})$. Then, there is a vector of welfare weights $(\lambda_1, \cdots, \lambda_I) \gg 0$ such that c^* solves the problem

$$\max \sum_i \lambda^i U^i(c) = \sum_i \lambda^i (E_{p^i}\{\sum_{t=0}^{\infty} \beta_i^t u^i(c_t(\sigma))\})$$
$$\text{such that } \sum_i c^i - e \leqslant 0$$
$$\forall t, \sigma, \;\; c_t^i(\sigma) \geqslant 0$$

The first order conditions are:

For all σ and t,

(1) there is a number $\eta_t(\sigma) > 0$ such that if $p^i(\sigma) > 0$, then

$$\lambda^i \beta_i^t u^{i\prime}(c_t^i(\sigma)) p_t^i(\sigma) - \eta_t(\sigma) = 0 \quad \cdots \quad (1)$$

(2) If $p^i(\sigma) = 0$, then $c_t^i(\sigma) = 0$.

Now, from (1) we obtain the following equation:

For any two traders i and j, and for any path and date t such that $p_t^i(\sigma), p_t^j(\sigma) \neq 0$,

$$\log \frac{u^{i\prime}(c_t^i(\sigma))}{u^{j\prime}(c_t^j(\sigma))} = \log \frac{\lambda_j}{\lambda_i} + t \log \frac{\beta_j}{\beta_i} + \sum_{\tau=0}^{t} (Y_\tau^j - Y_\tau^i) \quad \cdots \quad (2)$$

where $Y_\tau^i = \sum_{s \in S} (1_\tau^s(\sigma)(\log p^i(s|\mathcal{F}_{\tau-1}) - \log p(s|\mathcal{F}_{\tau-1})))$ and $1_\tau^s(\sigma) = 1$ if $\sigma_t = s$ and 0 otherwise.

3.3.2 Belief Selection in the Complete Market

In this section, we will discuss two examples of belief selection in complete market, i.e. i.i.d. beliefs case and Rational Expectations case with identical discount factors. The results on the equilibrium allocation in the complete market comes from the Fundamental Theorem of Welfare Economics. Let us define first what we mean by survival and vanish in a formal way.

Definition 1. Trader i vanishes on path σ iff $\lim c_t^i(\sigma) = 0$, while she survives iff $\limsup c_t^i(\sigma) > 0$.

Theorem 1 The agent i with more correct belief survives, while the agent with less belief vanishes, p^i-almost surely.

Proof of Theorem 1 Let us consider a simple economy where the true distribution of states and the forecast distributions are all i.i.d.. Other various cases are discussed in Blume and Easily (2006). Since the distributions are from independent draws, the current state is independent from the past histories, i.e. $p_t^i(\sigma_t|\mathcal{F}_{\tau-1}) = p_t^i(\sigma_t), \forall i, t$. Therefore, $p_t^i(\sigma) = \prod_{\tau=0}^{t} p_t^i(\sigma_t|\mathcal{F}_{\tau-1}) = \prod_{\tau=0}^{t} p_t^i(\sigma_t)$. Also, since the distributions are identical as ρ^i at each date t, $p_t^i(\sigma_t) = \rho^i(\sigma_t)$. Therefore, summing them up, $p_t^i(\sigma) = \prod_{\tau=0}^{t} \rho_t^i(\sigma_t)$. Note then that $\frac{1}{t}\sum_{\tau=0}^{t} Y_\tau^i = \frac{1}{t}\sum_{\tau=0}^{t}\sum_{s\in S}(1_\tau^s(\sigma)(\log\frac{\rho^i(s)}{\rho(s)}))$. Since $\frac{1}{t}\sum_{\tau=0}^{t} 1_\tau^s(\sigma)$ converges to $\rho(s)$ almost surely for each s by law of large numbers, $\frac{1}{t}\sum_{\tau=0}^{t} Y_\tau^i$ converges to $\sum_{s\in S}\rho(s)(\log\frac{\rho^i(s)}{\rho(s)}))$ almost surely. Here, $\sum_{s\in S}\rho(s)(\log\frac{\rho^i(s)}{\rho(s)}))$ is the relative entropy of probability distribution ρ^i on S with respect to the true distribution ρ. Let us denote this relative entropy as $I_\rho(\rho^i)$. Then, provided that the discount factors are identical for all the agents, it follows from (2) that

$$\frac{1}{t}\log\frac{u^{i\prime}(c_t^i(\sigma))}{u^{j\prime}(c_t^i(\sigma))} \longrightarrow I_\rho(\rho^i) - I_\rho(\rho^j) \qquad a.s \qquad \cdots \qquad (3)$$

Note that $I_\rho(\rho^i)$ measures how far agent i's probability distribution ρ^i is away from the true distribution ρ. If the rhs of (3) is positive, then it means that ρ^i is more away from the truth than ρ^j; which implies that trader j's belief is more correct than that of trader i. But when the rhs of (3) is positive, $\log\frac{u^{i\prime}(c_t^i(\sigma))}{u^{j\prime}(c_t^i(\sigma))}$ must diverge to infinity, which implies that $u^{i\prime}(c_t^i(\sigma)) \longrightarrow \infty$.Since $u^{i\prime}$ is a strictly decreasing function staring from the infinity at around zero, it must be that $c_t^i(\sigma) \longrightarrow 0$.

Second, let us consider an economy where traders i and j have identical discount factors, but only i has true probability distribution.

Definition 2. Agent i has rational expectations if $p^i = p$.

Theorem 2. An agent with rational expectations survives p-almost surely.

Proof of Theorem 2 For any p^i, p is more accurate than p^i from the perspective of p. Then, it immediately follows from Theorem 1 that the rational agent survives p-almost surely.

So far, we have reviewed the REH which stipulates the relationship between individual rationality and market rationality. We have also analyzed the epistemic structure of economics model in two macroeconomic theses from the REH. In the analysis of such epistemic structure, the economic agent's epistemic state, belief, works as a crucial factor. We note that, when belief is represented as subjective probability, the concept of rationality must be tied to the true objective probability. In the next chapter, focusing on the espistemic structure, we investigate the problem of confirmation on the economics model where market rationality is analyzed in terms of individual rationality in this sense.

Chapter 4

The Epistemic Structure of Economics Models and the Problem of Confirmation

4.1 The Outline of the Argument

First, we argue that the confirmation on the MSH has a distinctive epistemic structure which is shared by the confirmation on all economics models. Recall that models in economics are crucially different from models in natural sciences in the following two respects. The first is that the *outside observer* of the model must be at the same time the *inside agent* residing in the model. The second is that the inside agent's belief can *affect* the very model in such a way that some key variables in the model are influenced by the inside agent's beliefs on the future values of those variables. We will call the first aspect of economics the "common inside-outside agents structure". Also, following Thomas Sargent, we will call the second aspect of economics the "self-referential structure".[1]. Because of these two distinctive characteristics of economics model, the confirmation on an economics model comes to have the following interesting structure: In the confirmation of the economics model, the outside observer, say an econometrician, plays the role of evaluating the degree of inductive support of the evidence on the model. But since this outside observer is also working as an agent inside the model, her beliefs acquired from the evidence during the confirmation process *may* affect the evaluated model itself. The reason is that those beliefs of the inside agents, which are possibly inherited from the beliefs of the outside agents through the common inside-outside agent mechanism, may be reflected on the values of some key variables which are determined in the model through the self-referential mechanism. We regard this feature of the outside agent's belief *possibly* having an influence on the evaluated model itself as a distinctive epistemic structure of the confirmation on economics model. Confirmation on the MSH has this epistemic structure, too.

Second, we argue that the two-layered beliefs, the belief of the agent inside the model and the belief of the agent outside the model, are involved in the Bayesian confirmation model and that they become tied together during the confirmation procedure. We adopt the Bayesian confirmation model in particular to investigate the distinctive structure of confirmation on the MSH. By the "Bayesian model", we mean that the inductive support of evidence on a hypothesis is supposed to be measured by the econometrician's subjective probability on that hypothesis given evidence. Using Bayesianism, we intend to disclose the two-layered epistemic structure of confirmation on the economics model, that is, the structure where the subjective belief of the econometrician outside the model is involved in the process of confirming

[1] Sargent, T. 2008. "Rational Expectations." *The New Palgrave Dictionary of Economics.* Second Edition. Eds. S. Durlauf. and L. Blume. Palgrave Macmillan.

an economic hypothesis at the higher level and the subjective belief of the economic agents inside the model is involved in such an economic hypothesis at the lower level. What is crucial about the MSH, however, is that the two levels of subjective beliefs inside and outside the model turn out to become tied together during the confirmation procedure, because of the distinctive epistemic structure of confirmation on the MSH.

Third, we argue that the agents suffer from some epistemic limitation to know certain true features of her economic environment. In this thesis, we discuss the *empirical* nature of the MSH confirmation and its limits. In so-called empirical sciences such as physics, chemistry or economics, scientists not only build a theoretical model to explain the physical, chemical or economic phenomena but also conduct an empirical confirmation on the predictions from the theoretical model to see whether the theoretical model matches reality. However, it must be noted that when the conformity of the MSH model with reality is evaluated during the confirmation procedure, the reality is what is experienced by the econometrician, the very scientist who judges its conformity. Therefore, because of the distinctive epistemic structure of the confirmation on the economics model, if the econometrician suffers from any experiential limitation to capture some true features of the economic environment, then the inside agent *may* also suffer from the same kind of epistemic limitation to capture such true features, which in turn may affect the economics model itself whose conformity with experience is under evaluation.[2] Based on *the Frequentist Theorem*

[2]In this chapter, we follow the typical statistical decision settings, which does not necessarily reflect the psychological characters of decision-making. Accordingly, our empirical limitation comes into existence even under the consideration of a more strict sense of the rational decision in this statistical setting, not only under a more relaxed and realistic consideration on the human nature. We will not discuss which assumptions on the economic agents, statistical or psychological, are more reasonable approximations on the human decision process. It would not be necessary to cite Robert Lucas' skeptical comments on the behavioral economics in contrast with Herbert Simon's positive emphasis on it, in order to show how difficult it is to derive any definite conclusion on such issue.

Rather, we accept the general statististical settings on the economic agent's decision process as as well as *the Calibrationist Theorem* in statistics, we argue that the econometrician suffers from such epistemic deficiency: she cannot *know* the true distribution, provided that there exists any such thing as the true distribution. Accordingly, such statistical theorems lead to the epistemic insufficiency of the agents, which in turn may affect the very model under the evaluation of confirmation.

Lastly, combining these three points, we conclude that the MSH is not directly confirmable. From the third point, we argue that the econometrician cannot know the true distribution. From the first and the second point, however, if the econometrician does not know the true distribution, this epistemic deficiency may be reflected on the model itself during the confirmation procedures. Then, the original true MSH model undergoes a transformation into the new epistemically restricted MSH model. Now that the econometrician believes that these two model may be different from each other, she cannot confirm the original model, using the restricted one, unless she is sure that the confirmation results from these two different models will always be the same, which we prove is not the case. Therefore, we conclude that the MSH is not empirically confirmable to the econometrician.

With this introductory sketch on the argument of this chapter, we proceed to a more detailed argument in the following three sections. In Section 2, we briefly review the general discussion on the kinds of probability and then discuss what kind of probability the *true probability* is and how this true probability is formally treated in economics by focusing on the specific proposals in Cogley, et. al. (2009, 2012a, 2012b and 2013) In Section 3, we discuss how to *know* the specific value of the true probability under the supposition on its kinds as understood in Section 2. The goal

much as we can: We will assume that an economic agent can process the infinite sequence of data sets to calculate the maximum of the expected value. We will then argue that, even under this extreme circumstance, there still remain some empirical limitations in the econometrician's access to the true world. From there, we derive a further conclusion that it is not easy for economics to sustain a stable status as an empirical science because its epistemic structure does not allow an empirical confirmation on its own fundamental theories.

of this section is not to argue for which kind of probability is the true probability per se or whether there exists any such thing as the true probability. Rather, our goal is to focus on whether we can know it empirically given that there exists such thing as the objective true probability. If there exists no such thing, we simply accept that we cannot know it because there exists nothing to know. In Section 4, based on the discussion in the previous sections, we construct a formal model to show the confirmation impossibility of the MSH. The result from this formal discussion also has a more general implication, namely that no economic theories will be confirmable either if its definition of the rationality is based on the true probability distribution and if what we call the "Principle of Epistemic Effect" applies.

4.2 True Belief, True Probability, and the True Data-Generating Process

A true belief in the economics model is usually defined as a subjective probability which is congruent to the true data-generating process (DGP).[3] The true DGP is a process governing the infinite sequence of events in the order of time, according to which the observations of its results are represented in the actual data. For example, Cogley, et. al. (2012a, 2012b and 2013) takes an income growth rate as the true market state variable and assume that the Markov and the Bernoulli processes are combined to govern the time series of income growth rate as the true underlying data-generating processes. Accordingly, the econometrician observes the results of

[3] "Process" is a technical term which mathematically denotes a collection of random variables in the order of time whose domains share a common probability space. Therefore, strictly speaking, a process does not match a probability measure, given that random variable is a function and thus that a process is a collection of functions while probability measure is just a function. Then, the congruency of subjective probability and the true data-generating process may not look appropriate. Accordingly, we define here the data-generating process as a probability measure on the inifnite product of the common probability space.

the postulated Markov and Bernoulli processes through the sequence of income data and aims to retrieve those true processes inversely from the income data. If a sequence of data is indeed generated according to the true process, the econometrician must be able to track its traces from the empirical frequencies in this sequence. In the following, we review how the true DGP is treated in the economics model and then argue that the kind of probability involved in the true DGP is not a subjective but an objective one which the econometricians seek to identify from the relative frequencies of data. Based on the discussion on this point, we further investigate how to know the true DGP in the next section.

4.2.1 Kinds of Probabilities and their Representations

Probabilities are often classified into three basic kinds: physical, epistemic and subjective. Physical probabilities are also called "chances", and subjective probabilities "credences." Chance and epistemic probability are related to the real features of the world, i.e. objective, while credence is related to the features of the human agent whose degree of belief in the objective states or in propositions is supposed to be measured by that credence, i.e. subjective. For example, the probability of getting the face value 6 from rolling a die is a chance. And is the epistemic one the probability of our economy undergoing a financial crisis that is logically entailed by some evidences on the collapsing housing market. On the other hand, my degree of belief that it will rain tomorrow is a credence. The physical probability and the epistemic probability are all determined by the real features of our world, but my degree of belief is merely a matter of my opinion and need not be justified by any other real factors than myself.

Subjective Probability

My subjective degree of belief is quantified as subjective probability when a numerical representation is uniquely assigned to it and such representation is rationalized to satisfy the so-called three probability axioms. The rationalization of credence to satisfy the probability axioms is called *probabilism*. Since my belief is subjective in its nature in the sense that it can be whatever I believe so, it needs further rationalization to require for my degree of belief to follow any quantitative rule if it does follow. Thus, two issues are involved in the subjective probability theory: how to obtain a certain numerical representation and how to rationalize such representation to satisfy the probabilism. Of course, these two issues are not separate from each other. Nonetheless, we answer for the issues respectively in the following.

A standard way to respond to the first issue is to construct an axiomatic system of my likelihood judgments in the form of binary relations on the states of the world or propositions. This system of my likelihood judgments can be revealed as a phenomenon through my axiomatic system of preferences. Then, the existence of these systems serves as a necessary and sufficient condition for the existence of unique probability measures. I quantify my own degree of belief as probability measure by representing it through the system of my preference ordering on the choice-actions under uncertainty.

Regarding the second issue, there are two ways to respond, the pragmatic one and the non-pragmatic one. A standard pragmatic justification results in the Dutchbook argument, while a non-pragmatic one results in the Accuracy argument based on the pure epistemic motivation. The Representation argument for probabilism can be classified as in the line of pragmatic justification, while Calibration argument can be classified in the line of non-pragmatic one. For Representation argument relies on the rationality of my preference ordering on choice actions that are pragmatically motivated, while the Calibration argument relies on the good match with reality that is epistemically motivated to reflect the world correctly. In this section, the first point is more relevant, for the topic of this chapter is mainly on how to know the true value of probability when it is identified as a degree of belief, not how to justify its specific numerical properties. Although my subjective probability can be represented by my preference system in an ideal choice setting, it still remains as something just in my mind. Therefore, how to elicit such inner thought out truly as an explicit public opinion is a separate issue from the representation theory.

Objective Probability

Just as a human agent represents her degree of belief as her subjective probability through preference system, the nature analogously represents its objective possibility as physical probability or epistemic probability through empirical frequency or information.

In spite of this distinction, however, the three kinds of probabilities are somehow related to each other. For any proposition A, for example, A's chance might just be A's epistemic probability relative to *all* the relevant evidence we have or could obtain. A's epistemic probability might in turn just be the credence in A that we *ought* to have given our evidence about it. And my credence in A might be just what I *think* A's chance or epistemic probability is. Naturally, there is no necessary connection among these relations. My credence in A does not necessarily have the value which the A's epistemic probability guides that we ought to have. Also, my credence in A might come from my thought about A's chance, not necessarily bounded by A's objective chance itself.

4.2.2 Cogley, et. al. Model and the True Data-Generating Process

Provided that the true belief is interpreted as what is congruent to a true data-generating process (TDGP), let us discuss how the TDGP is formally treated in economics model, first in a particular example and then in a more general setting. Following Blume and Easley (2006) and Cogley, et. al. (2013), we assume that time is discrete and begins at date 0. The possible states at each date form a finite set, say $S = \{g_l, g_m, g_h\}$. In particular, let us suppose that S represents the set of all possible realizations of the aggregate income growth rate where the high-growth state g_h represents an economic boom, the medium-growth state g_m represents a mild contraction, and the low-growth state g_l represents an economic depression. These outcomes on the growth rate are assumed to depend on the realization of two independent random variables, s and d: The random variable s is following a Markov-switching process with a transition matrix,

$$\Pi_s = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}$$

The transition matrix Π_s shows the probabilities of s's transiting from one state to another. For example, $\Pi_s(1,1)$ represents the probability of s's keeping occurring, while $\Pi_s(1,2)$ represents the probability of s variable's moving from occurring at the previous period to not occurring in the next period. The random variable d is an **i**ndependent and **i**dentically **d**istributed (**i.i.d.**) Bernoulli variate with success probability p_d , i.e. $B(p_d)$. The mechanism of how the income growth rate g is determined by the two independent random variables (s, d) is as follows: The high-growth state g_h occurs when s occurs independently of the outcome for d, a mild contraction state g_m occurs when neither s nor d occurs, and an economic depression,

g_l occurs when s does not occur but d occurs. The resulting transition matrix for growth states is:

$$\Pi_g = \begin{bmatrix} p_{11} & (1-p_{11})(1-p_d) & (1-p_{11})p_d \\ 1-p_{22} & p_{22}(1-p_d) & p_{22}p_d \\ 1-p_{22} & p_{22}(1-p_d) & p_{22}p_d \end{bmatrix}$$

The transition matrix Π_g shows the probabilities of the income growth variable's transiting from one state to another. For example, $\Pi_g(1,1)$ represents the probability of the growth variable currently staying in the high state from the previous period, (the economy keeps in a good standing) while $\Pi_g(1,3)$ represents the probability of moving from the high-growth state to the low-growth state. (The economy suddenly plunges from boom into depression.) Since the economy stays in a good standing only when the random variable s keeps occurring independently of random variable d, its transition probability is p_{11}. On the other hand, since the economy moves from a high state to a low state only when the random variable s moves from s's occurring to not occurring and the random variable d is realized, its probability is $(1-p_{11})p_d$. Note that, in this model, neither s nor d is observable but that only g can be observed by an econometrician. Also, note that g_h occurs no matter whether d occurs or not, but that g_m occurs only when d occurs while g_l occurs only when d does not occur. Therefore, in this example, the realization of the disaster-state variable d can be noticed by an econometrician only when the mild-contraction occurs while the unrealization of d can be noticed only when the deep contraction occurs. The information on the probability of the disaster's occurring is revealed only under the states g_m and g_l.

So far, we have looked at what the TDGP is in a particular example, i.e. the income growth rate model. Now let us discuss it in a more general setting. Let Σ

be the set of all sequences of states with a representative sequence $g = (g_0, \ldots)$. For instance, in Cogley, et. al. example, g_t takes as value one of $\{g_l, g_m, g_h\}$ at each time t. Also, let $g^t = (g_0, \ldots, g_t) \in \Sigma$ denote the partial history through the date t. The set Σ together with its product sigma-field, $\mathcal{F}$, is the measurable space on which everything will be built. Then, for any probability measure p on Σ, $p_t(g) = p(\{g_0 \times \cdots \times g_t\} \times S \times S \times \cdots)$ becomes the (marginal) probability of the partial history. Each g^t is assumed to be $\mathcal{F}_t$-measurable, that is, their value depends only on the realization of states through the date t. Now, each agent i's beliefs are assumed to be represented by this probability distribution p^i on Σ. Then, letting the true probability distribution be denoted as p, $p_t(g)$ is defined to be determined by the true data-generating processes which are, in Cogley et. al. example, the two processes of s and d. In other words, provided that $p_t(g)$ is determined by n number of true processes, $p_t(\sigma)$, $p_t(\sigma')$, $p_t(\sigma'')$, $\cdots$, $p_t(\sigma^{(n)})$, for any function $f : [0,1] \times \cdots \times [0,1] \to [0,1]$, $p_t(g) = f(p_t(\sigma), \ldots, p_t(\sigma^{(n)}))$. Note then that $p_t(\sigma) = \prod_{\tau=0}^{t} p(\sigma_\tau | F_{\tau-1})$ for any t and so $p_t(\sigma) = p(\sigma_t | F_{t-1}) p_{t-1}(\sigma)$. In Cogley, et. al. case, s follows the Markov process and so the one-step transition matrix Π_s consists of $p(s_t | F_{t-1})$, while $p(d_\tau | F_{\tau-1})$ is constant as $\rho(d)$, for all τ and for i.i.d. bivariate, d. Cogley, et. al. specify the one-step transition matrix Π_g of $p(g_t | F_{t-1})$ which is determined by Π_s of $p(s_t | F_{t-1})$ and $B(p_d) = \rho(d_t)$ for the recursive version of the true process. Since only g, not d or s, is assumed to be observable in their model, the ultimate goal of the econometrician in Cogley et. al. is to derive the underlying true data-generating process $B(p_d)$, from the data on g, assuming that Π_s is already known. In general, in order to identify the TDGP, it is required to specify how to derive such an underlying true process from observable data.

4.2.3 What Kinds of Probabilities are Involved in the TDGP?

The true data-generating process is the process that truly governs the actual data of the important variable in the model. Since the true process, if any, is supposed to govern the real data, it cannot be just someone's subjective opinion. It must be objective, and hence be either epistemic or physical probability. If the true process is epistemic probability, then unless the information from data entails a unique logical probability, we cannot know it from data analysis. For the purpose of the current research, however, let us emphasize that in most practical cases such logical relation between hypothesis and evidence is not uniquely determined upon data analysis in non-arbitrary way.[4] Since the relevant question here is how to know the true process from data, let us then set the epistemic probability aside as a kind of probability involved in the TDGP and focus on the physical probability instead.

It deserves to note, however, that, while interpreting the true probability involved in the TDGP, we do not dispute whether there exists such thing as the TDGP. There might be no such thing as the true objective process; every probability might be simply subjective. If that is the case, then the problem of knowing the true probability will turn out to be the problem of identifying the general rule to elicit sincerely one's own inner thought out as public announcement. However, provided that every agent's true inner thought is beyond the realm of data, no general rule on this sincere relationship between true inner thought and its public announcement is attainable by data analysis. Therefore, we should conclude that there is nothing to know purely from data if there exists only subjective probability.

It also deserves to note that we do not dispute whether the true probability

[4] For instance, in Carnap system, it can be shown that there are infinitely many probability functions with some suitable initial measures which represent inductive support. Since no convincing way has been suggested to choose which function is the "correct" one among those infinitely many probability functions, critics argue that Carnap fails to provide non-arbitrary unique system.

in economics model is the true data-generating process or not. In this chapter, our question is whether we can *know* by data the true probability, if any. We just accept that, in the economics model where the econometrician can approach the true probability, if any, only by data, it must be that the TDGP is the true probability. As long as the TDGP is what makes the true probability knowable out of data, it suffices for our purpose.

4.3 Do We Know the True Process?

Now that only physical probabilities are relevant with regard to knowing the true probability in the economics model, let us discuss how to know the true process from data when it is physical probability. Before we move on, however, it deserves to note that, from now on, we will treat subjective probability as what amounts to the *probability quoted in forecast*. There can be objection to this way of treating subjective probability. For forecast is an act, while subjective probability is supposed to represent belief, and act is not belief. We will consider this objection briefly at the end of this section.

Suppose that a weather forecaster predicted that, with 30 percent, an event, say hurricane, would hit on some specific season. Then, observing whether this particular event happened or not on that season does not provide us reason to judge whether such probabilistic forecast to be correct as true one. For what happened on that particular season alone does not make the forecast correct or incorrect. However, if these probabilities have been announced for a long time, then observing what have happened during that period can provide a reliable reason for such judgment. For what have happened may make the forecasts correct as long as those observations have been accumulated long enough. This is indeed the case, if a sequence of data is generated according to the true process. For the true process must have left its trace on the sequence of data and thus we must be able to track its trace from the sequence as long as it is long enough. This trace is represented as relative frequency among those data.

Thus, observing the sequence of data generated by the true process, and seeing whether its relative frequency matches the probabilistic forecast, must provide a reliable reason to judge whether such forecast is correct as true one. If such match is not obtained, the forecasts fall under discredit and the true probability is not said to be known from those data. In other words, in order for the forecasts to be known as reliably true one, they must *not* be at least always *mis-calibrated*. Let us call this the Calibrationist Position. Further, if the forecasters can form a unique group of events whose members are **i**ndependently and **i**dentically **d**istributed (i.i.d.), knowing the relative frequency in this group will suffice to know the true probability. Let us call this the Frequentist Position. In the following, we discuss these two positions respectively in detail. We argue that the Frequentist Theorem shows that an econometrician cannot know the truth from data because she cannot form such i.i.d. group from data, while the Calibrationist Theorem shows that an econometrician cannot know the truth from data because for any arbitrary test set she cannot exclude the possibility that her forecast is mis-calibrated.

4.3.1 DGP and the Frequentist Position

Behind the idea that there is a true process which generates some data on a certain variable and that we can discover this underlying process reversely from the frequencies of data, there exists the Frequentist Perspective. The Frequentist Perspective is the idea that the true process may be identified from empirical frequency because either true probability is identical with relative frequency in the certain domain or it is reducible to that frequency. This Frequentist Perspective is reflected on the frequentist or propensity interpretation on probability.[5]

Obviously, however, the true process cannot be exactly identifiable from any sequence of actual frequencies. For example, the outcomes of flipping a fair coin can be thought of as being generated according to the Bernoulli process with the success probability of $\frac{1}{2}$. But any frequency from the actual coin-tossing experiments hardly is exactly $\frac{1}{2}$. Rather, the true success probability $\frac{1}{2}$ must be regarded as the frequency that would be approached as the number of repeated coin-flips goes to infinity.

However, if the true process is approached only in the limit, then how can we know it from the finite series of data? This is exactly what the Law of Large Numbers is all about. For instance, the Strong Law of Large Numbers tells us that the empirical relative frequency of fair-coin tossing will converge to the true probability distribution with probability one. Accordingly, we can approximate the true probability of getting head from the relative frequency of occurring heads against tails, if we repeat the artificial experiment of tossing a fair coin sufficiently large number of times. Statistically, this amounts to assuming that our experimental world is independently and identically distributed. However, unlike in such simple cases as a fair-coin tossing where the artificial experiments can be manipulated as i.i.d., it is generally impossible for an econometrician to set up any controlled experiment in the history of economic events. One way for an econometrician to circumvent this experimental limitation is to find a set of similar kind of events to a particular case, i.e. the so-called reference class, and measure its relative frequency to this reference set.

[5] Although frequency theorists who argue for the frequency interpretation on probability are in the frequentist perspective, not all propensity theorists are in this perspective. For example, Popper (1959) argues that probability has the property of propensity to produce sequences whose frequencies are equal to the probabilities, and so he can be classified in the frequentist perspective. However, Giere (1971) clearly denies that propensities are reducible to less theoretial concepts like relative frequency, and so cannot be in the frequentist perspective.

For example, suppose that we draw a red ball from an urn with *constant* $\frac{1-p}{p}$ ratio of red and white balls. If we repeat these ball-drawings many times, say 10,000 times, we will observe red balls approximately p times out of the total drawings. We then regard this stable value of p as a true success probability of getting a red ball. In contrast, when we have a record of hurricane-hitting New York p times out of the past 100 seasons, we would not hastily conclude that p is the underlying true probability of experiencing hurricane in NY for some seasons. For the value of p is not even stable, first of all. Obviously, as we collect more and more records on the hurricane-hitting times in the future seasons, we expect the values of p to fluctuate more.

However, let us suppose, for the time being, that this fluctuation in p of the actual hurricane-hitting times reflects only a short-run randomness. A record of 100 seasons simply may not be long enough. Even in the case of urn-drawing, didn't we have fluctuations in the empirical frequency of drawing red balls before the records reached a stable limit? Therefore, it might be that if we accumulate a large amount of hurricane records, the long-run relative frequencies would reach some stable convergent limit, which would reveal a true underlying structure of our world. However, a probability theorem can show that a stable relative frequency does not necessarily reveal the true underlying probability if the underlying structure itself is not stable.

Let A_t be the event that a hurricane hits NY in season t and let X_t be a random variable whose value is 1 if the event A_t occurs at t and 0 otherwise. Then, $S_n = \sum_{k=1}^{n} X_n$ will be the number of events that have occurred up to season n. Now suppose that the true probability of hurricane-hitting NY, i.e. $P(A_t)$ is changing across the seasons.

Borel-Cantelli lemma. if $\sum_{t=1}^{\infty} P(A_t) < \infty$ then $P(A_t \ i.o.) = 0$.

Proof. Let $N = \sum_{k=1}^{\infty} X_n$ be the number of events that ever occur. Fubini's theorem implies $EN = \sum_{t=1}^{\infty} P(A_t)$ which is finite by assumption. Thus, $N < \infty \ \ a.s.$

Therefore, unless the hurricane completely stops hitting NY in the near future almost surely, which is fairly unrealistic assumption, the sum of probabilities of long-run events A_t $'s$ will be infinite.

Theorem 3. if $A_1, A_2, ...$are pairwise independent and $\sum_{t=1}^{\infty} P(A_t) = \infty$ then as $n \to \infty$

$$\tfrac{1}{n} S_n \to \lim \tfrac{1}{n} \sum_{t=1}^{n} P(A_t) \quad a.s.$$ [6]

The relative frequency of the actual events will converge to a stable average of the true probabilities at each time. In case where the true probability that hurricane occurs at NY changes across seasons, which is quite plausible, the long-run relative frequency will only reveal the average of the changing probabilities, not the each true probability. To retrieve an underlying true structure from the long-run behaviors of data sets, the assumption of constant probability over the period is crucial. For example, suppose that the true probability of hurricane-hitting is alternating between $\frac{1}{2}$ and $\frac{1}{3}$. In odd periods, the objective probability of having hurricane is $\frac{1}{2}$, while the true probability of experiencing hurricane at even periods is $\frac{1}{3}$. According to the Theorem 3, then the long-run relative frequency of actual happening of hurricane will converge to $\frac{5}{12}$ which is neither $\frac{1}{2}$ nor $\frac{1}{3}$. Indeed, if there exists a limit in the average of true probabilities, the long-run sequence of empirical frequencies converges to this limit but it does not reveal any true probability. Therefore, any stable convergence of empirical frequency does not necessarily imply that we can retrieve a true probability

[6] This is the Theorem 2.3.8 in Durrett (2010). A detailed proof is given in Durrett (2010) *Probability : Theory and Examples. p. 68.*

out of it. In order to obtain the true probability from data, we must first be able to separate the events whose true probability is 1/2 from the events whose true probability is 1/3 and group them apart as two different reference classes. However, this would only mean to know the true probabilities a priori so that we are able to classify events according to their true probabilities even before measuring their empirical frequencies. This then does not make sense because it is circular: it was required to classify events into relevant reference classes in order to know the true probabilities, but it is now required to know the true probabilities in order to classify events into relevant reference classes.

To sum up, the difference between the statistical patterns in urn-drawings and hurricane-hitting lies in that the former displays some convergent frequencies which manifest a stable underlying structure of the world, while the latter does not. Since the urn keeps a *constant structure* of red and white ball composition, the long-run history of urn-drawings reveals a stable underlying process, say Bernoulli (p).[7] In contrast, it is generally agreed that there is no constant structure under the weather environment across the time. Therefore, the historical data of hurricane records does not reveal the true probability. The upshot is that the value of the true probability cannot be known as relative frequency from data when there is no stable underlying process.

[7]It deserves to note when any repeated independent trials are called Bernoulli. According to Feller (1968, p.146), it is only when there are just two possible outcomes for each trial, and when their success probabilities remain the same throughout the trials. Thus, only the repeated random drawings from an urn of *constant composition* represent Bernoulli trails. Note here the italics *constant composition.* In case of typical economics model like Cogley, et. al. (2013), the probability of occurrences of disasters is not constant every time when they occur, even if each occurrence of disaster might be independent each other. In the artificially manipulated experiment such as urn-drawing, the composition of the urn can be controlled to be constant, while no such controlled experiment is possible in the history of our real world. Thus, the identical distribution condition to apply SLLN cannot be easily satisfied even when economists accumulate the large number of historical sample data on disaster. This is one of the difficulties that social scientists are destined to face. One way to circumvent this difficulty may be to consider computer simulation, which we will discuss in a separate paper.

It deserves to note that we cannot yet argue for the impossibility of probabilistic knowledge *in general* because we have not yet proved that there cannot be a stable underlying process in any arbitrary case. Therefore, we need to show further that the econometrician cannot know the true distribution in any arbitrary events without assuming no stable process. This is what the Calibrationist Theorem tells us in the next section. According to the Calibrationist Theorem, we cannot know the true distribution in arbitrary events whose occurrences do not necessarily follow or deviate from stable process.

4.3.2 DGP and the Calibrationist Position

In the previous section, we have shown that we cannot group i.i.d. Bernoulli sets apart to satisfy the condition to know the true probability from data. Now, let us extend our discussion from the Bernoulli distribution to any arbitrary probability distribution. For example, when the true state variable in the MSH model is a growth rate of GDP, the true data-generating process of the growth rate may follow a Markov process. This is the case with Cogley. et. al. models. Thus, in order to know the true probability, the econometrician must figure out the Markov transition matrix of GDP growth rate that does not necessarily come from the Bernoulli process.

A forecasting system is *well-calibrated* if it assigns a probability, say 30 percent, on average to the events whose long-run proportion that actually occurs is 30 percent. According to Dawid (1982), a forecasting agent expects her fairly arbitrary test set of forecasts to be well-calibrated in the long-run. In particular, if the forecasting agent holds the true probability as her subjective one, she is truly guaranteed to be well-calibrated as long as she is coherent. However, this does not guarantee that she can *know* the true probability from the frequencies of the well-calibrated test set. Let us discuss this in a more detail.

Following Dawid (1982), let us suppose that the forecaster has an arbitrary subjective probability distribution Π defined over $ß_\infty = \bigvee_{i=0}^{\infty} ß_i$, where $ß_i$ is denoted by the totality of events known to the forecaster on day i. The probability forecasts she makes on day i are for events or quantities in $ß_{i+1}$, and are calculated from her current conditional distribution $\Pi(\cdot|ß_i)$. For each day i we have an arbitrary associated event $S_i \in ß_i$, for example the event of precipitation on day i. We denote the indicator of S_i by Y_i, and introduce $\hat{Y}_i = \Pi(S_i|ß_{i-1}) = E(Y_i|ß_{i-1})$, the probabilistic forecast of S_i on day $i-1$. One way to compare forecasts with reality is to pick out some fairly arbitrary *test set* of days, and in that set compare (a) the proportion p of days whose associated events in fact occur with (b) the average forecast probability π for those days. Formally, we introduce indicator variables $\xi_1, \xi_2, \ldots$, at choice, to denote the inclusion of any particular day i in the test set where $\xi_1 = 1$ if the day i is included in the test set and $\xi_1 = 0$ otherwise, and express $p_k = (\sum_{i=1}^{k} \xi_i)^{-1} \cdot (\sum_{i=1}^{k} \xi_i Y_i)$ and $\pi_k = (\sum_{i=1}^{k} \xi_i)^{-1} \cdot (\sum_{i=1}^{k} \xi_i \hat{Y}_i)$. In other words, p_k indicates the portion of the days when the events in fact occur among those k days which are included in the test set, and π_k indicates the average forecast probability in the test set.

Lemma 1 Suppose that ξ_i is $ß_{i-1}$ measurable. With $\Pi-$probability one, if the number of test days goes to infinity, then $p_k \to \pi_k$.

Proof of Lemma 1 A detailed proof of Lemma 1 is given in Dawid (1982). A simpler one is as follows: Let $X_i = (\sum_{j=1}^{i} \xi_j)^{-1} \cdot \xi_i (Y_i - \hat{Y}_i)$. Since $(\sum_{j=1}^{i} \xi_j)^{-1}, \xi_i$ and $\hat{Y}_i$ are $ß_{i-1}$-measurable, it follows that $E(X_i|ß_{i-1}) = 0$ and so that $\sum_{i=1}^{k} X_i$ is a Martingale adapted to $ß_{k-1}$. Also, $E((\sum_{i=1}^{k} X_i)^2) = \sum_{i=1}^{k} E(X_i^2) \leq \alpha \cdot E\{\sum_{i=1}^{k}((\sum_{i=1}^{k} \xi_i)^{-1} \cdot \xi_i)^2\} \leq \frac{\lambda\pi^2}{6}$, provided that $var(Y_i|ß_{i-1})$ is uniformly bounded above by λ. Then, by the Martingale convergence theorem, $\sum_{i=1}^{k} X_i$ converges, which from Kronecker's lemma implies that

$$p_k - \pi_k = (\sum_{i=1}^{k} \xi_i)^{-1} \cdot \sum_{i=1}^{k} (\sum_{j=1}^{i} \xi_j)^{-1} X_i = (\sum_{i=1}^{k} \xi_i)^{-1} \cdot \sum_{i=1}^{k} \xi_i (Y_i - \hat{Y}_i) \to 0.\ Q.E.D.$$

Lemma 1 shows that an econometrician is sure that her test set will be well-calibrated with her subjective probability one when the test set is sufficiently large. Then, if the agent's subjective probability Π on $ß_\infty$ is congruent to the true probability p, her test set of forecasts will be well-calibrated in the long-run with true probability p one. Thus, if she knows the true probability, her forecasts $\Pi(\cdot|ß_i)$ are congruent to the true one $p(\cdot|ß_i)$, which truly guarantees her forecasts in the test set to be well-calibrated whenever she obtains a sufficiently large and fairly appropriate test set. A true guarantee to be well-calibrated is the necessary condition to know the true probability from the empirical frequencies of this test set. Unfortunately, however, she is not truly guaranteed so. For if the Nature generates data in the way that its true process deviates from the agent's subjective probability, the agent is not expected to be well-calibrated with the true probability one. This is shown in Oakes (1985).

Theorem 4 (Oakes 1985) No statistical analysis, however complex, of sequential data can be guaranteed to provide asymptotically valid forecasts for every possible set of outcomes.[8]

Proof Let p denote an arbitrary true probability distribution which is supposed to generate a sequence $\{Y_1, Y_2, \ldots\}$ where

$p(S_i|ß_{i-1}) = f\{\Pi(S_i|ß_{i-1})\}$, with the function $f([0,1]) \to [0,1]$ being defined by $f(x) = x + \frac{1}{2}$ $(0 \le x \le \frac{1}{2})$, $f(x) = 1 - x$ $(\frac{1}{2} \le x \le 1)$.

Then, under p with $p(Y_{I_k} = 1) = f(\alpha)$ where $\hat{Y}_i = \alpha$ for a sequence $\{i : i = I_1, I_2, \ldots\}$ and Y_{I_k} forms Bernoulli, the calibration property does not hold, because

[8] As it is formulated in Dawid, A. P, 1985, "Self-Calibrating Priors Do Not Exist: Comment", *Journal of the American Statistical Association, Vol. 80*, No. 390, p.340.

the average probability forecast is not the same as the limiting relative frequency among days on which the forecast probability is α. *Q.E.D.*

Theorem 4 implies that the forecaster cannot exclude the possibility that her test set may be mis-calibrated. Lemma 1 guarantees well-calibration with the true probability one only when the forecaster knows the true probability. Now that it cannot be excluded by Theorem 4 that the true process deviates from the subjective forecast and thus that the test set of forecaster can be mis-calibrated, Theorem 4 then implies that the forecaster cannot know the true probability from the empirical frequencies of her test set.

Note here that the calibration criterion does not require a background of repeated trials under constant conditions. We argued in the previous section that the econometrician cannot know the true distribution, if any, when the underlying structure is not stable. Now, we further argue that, even in the case where there is no agreement on whether a stable distribution exists or not, the econometrician cannot know whether her belief is true, because her belief is not guaranteed to be well-calibrated.

4.3.3 Remarks on the Discussions of Probabilistic Knowledge

So far, we have discussed the problem of knowing the true probability in the context of forecasting system. In other words, in order to deal with the problem, we inquired whether the announced forecasts match the empirical frequencies. However, this assumes that the probabilities quoted in the forecasts can be regarded as the sincere representations of the forecaster's coherent subjective degrees of belief. For example, a weather forecaster sequentially assigns probabilities to rainy events, like "the precipitation probability at time t is 30 percent". Here, we regard the announced probability of 30 percent as the forecaster's subjective degree of belief on the rainy event.

However, someone might argue that forecasting is a motivated action while subjective probability is a belief, and thus that the set of quoted forecasts cannot be considered as a set of subjective probabilities. For belief is not considered as a motivated act in general. Indeed, it sounds strange if we come to believe what we believe because we intentionally decide to believe so. Rather, belief is considered as given in advance to do any motivated act. Therefore, assessing whether forecasts match the reality can be different from assessing whether subjective probabilities do so. However, it does not hurt our current research purpose to assume that, in the ideal situation, the forecaster does not have any incentive not to announce her subjective probability sincerely. Thus, under this assumption, we regard the assessment of probabilistic forecast as that of subjective probability.

At this point, someone might argue that the confirmation impossibility of the MSH seems redundant. For it seems that, from the fact that no agent inside the model knows the truth, it might follow that no agent can be rational. If it becomes impossible that agents can be rational, then the MSH seems an impossible hypothesis. Obviously, no impossible hypothesis can be confirmed or disconfirmed.

However, the impossibility of being rational does not follow from the fact that no agent inside the model knows whether her belief is true. For an agent can *hold* the true belief even though she does not *know* whether her belief is true or not. As long as she holds the true belief, she is defined to be rational. It deserves to note here that there is a significant distinction between holding the true distribution and knowing true one. The rational agent inside the model, if any, indeed holds true belief by definition, but she cannot be said to know the truth just because she holds such belief. True belief alone falls short of being knowledge.

Also, someone might argue that even if the econometrician, that is to say, the judge of the MSH, does not know the truth, at least the model-builder of the MSH must have known it. Otherwise, the MSH model would be threatened to be empty, because the model-builder can no longer specify who should count as the rational agent in that model. It seems that an empty model is useless and thus that the problem of confirmation on such model is useless as well. To reply to this objection, it deserves to note that the MSH does not have to tell us *how* the rational agents come to have such true belief, just as the natural selection theory in biology is not required to show how the survivors in the nature manage to obtain those features that enable them to fit better to their natural environment. The MSH only tells us that the rational agents, if any, will survive once they obtain the true belief no matter how they manage to do so. Since the MSH does not have to explain how rational agents manage to hold the subjective beliefs which coincide with the true objective ones, the model-builder of MSH does not have to know the true distribution,in order to construct the MSH model. Even if the model-builder of the MSH does not know the truth, the MSH would work well. Being threatened potentially to be empty does not make the MSH useless.

4.4 The MSH and the Problem of Confirmation

4.4.1 Bayesian Confirmation Model

In the Bayesian confirmation model, we usually refer to $\Pr(h)$ as the prior probability of hypothesis h, and $\Pr(h|e)$ and $\Pr(h|\neg e)$ as the posterior probabilities of h in light of evidence e or $\neg e$, respectively. They are subjective probabilities representing the beliefs of the very agents who are conducting confirmation. In particular, if we regard such agent as an econometrician, we denote p^e as her subjective probability.

We adopt the following equation for the incremental requirement of confirmation:

evidence e *confirms* h if $f(h, e) > 0$

evidence e is *neutral* to h if $f(h, e) = 0$

evidence e *disconfirms* h if $f(h, e) < 0$

where f is a function from $X \times Y$ to $\mathbb{R}$. Here, X is the set of propositions on hypothesis and Y is the set of propositions on evidence and this function f is supposed to measure the degree to which e increases the subjective probability of h. The examples suggested for such function f are difference function $f(h, e) = \Pr(h|e) - \Pr(h)$ or $f(h, e) = \Pr(h|e) - \Pr(h|\neg e)$, log-ratio function $f(h, e) = \log(\Pr(h|e)/\Pr(h))$ or log-likelihood ratio function $f(h, e) = \log(\Pr(e|h)/\Pr(e|\neg h))$.

In the next sub-section, we will argue for the impossibility of confirmation on the MSH under the equations on $f(h, e)$. By impossibility of confirmation, we mean that it is not possible to determine whether $f(h, e) > 0$ or not. One thing to note with these equations is that, in order to argue for the confirmation impossibility, we do not rely on any one specific form of measuring function among the four examples being considered. There has been debate on which measurement method should be used to gauge such degree of confirmation. As a whole, there have been two kinds of discussions: The first is which form of conditional or unconditional probabilities should be compared to measure the comparative increment. The second is which functional form with those probabilities should be considered to measure the degree of confirmation. For example, for the first kind of discussion, Joyce (1999) and Christensen (1999) argue that we should compare the probabilities conditional on the evidence versus those on the negation of the evidence, while arguing against the comparison between posterior versus prior probabilities. The former is called "probative confirmation", while the latter is called "incremental confirmation". Also, for the second kind of discussion, Eells & Fitelson (2000) argues that there is a so-called problem of measure sensitivity so that the Bayesian argument heavily relies on the specific form of function f. However, it will be clear that our argument on the confirmation impossibility is independent of these discussions, for our proof does not rest on which functional form to be adopted to measure the degree of confirmation.

4.4.2 The Model for Confirmation Impossibility of the MSH

The major notations in this section are just the same as those in section 4.2.2. As we did in section 4.2.2, we assume that time is discrete and begins at date 0. Also, we assume that the possible states at each date form a finite set. Then, following Blume & Easley (2006) and Cogley, et. al.(2013), we stipulate that the MSH is the hypothesis that the rational agents survive in the complete market p-almost surely, where p denotes the true probability. Let us call this hypothesis h. Recall that the probability space for p is a triple $(\Sigma, \mathcal{F}, p)$.[9] Then, the hypothesis h can be expressed

[9] Let us suppose that the possible states at each date form a finite set $\{1, \ldots, S\}$. For instance, just as we did in section 4.2.2, when g_t is a radom variable whose values are the income growth rates realized at each t, the set of all possible states can be assumed as $\{g_l, g_m, g_h\}$. Then, $S = 3$ and g_t takes values as one of $\{g_l, g_m, g_h\}$. Now, let us denote by g^t a partial history of income growth rates up to t. Then, $g^t = (g_0, \ldots, g_t)$. For instance, when $t = 3$, we have only three periods, say yesterday, today and tomorrow. Then, an example of g^2 can be (g_l, g_m) which shows that the economy undergoes a smooth progress from yesterday to today and the future is open.

Now, let Σ be the set of all sequences of states with a representative sequence $g = (g_0, \ldots)$ where t can go to infinity. Then, for any t, $g^t = (g_0, \ldots, g_t) \in \Sigma$. Also, the set Σ together with its product sigma-field, $\mathcal{F}$, is the measurable space on which everything will be built in the MSH model. For instance, when $t = 3$ and $S = 3$, Σ has 3^3 number of elements (g_1, g_2, g_3) which have three different values from $\{g_l, g_m, g_h\}$ at each of three dates t.

Now, the product sigma-field is $\bigotimes_{t \in \{1,2,3\}} \Sigma_t$ where Σ_t is a sigma-field on $\{g_l, g_m, g_h\}$ for each $t \in \{1, 2, 3\}$. Then, Σ_1 can be $\{\varnothing, \{g_l\}, \{g_m\}, \{g_h\}, \{g_l, g_m\}, \{g_m, g_h\}, \{g_l, g_h\}, \{g_l, g_m, g_h\}\}$. Now, $\bigotimes_{t \in \{1,2,3\}} \Sigma_t$ is a sigma-field generated by $\{\prod_{t \in \{1,2,3\}} E_t : E_t \in \Sigma_t\}$. For instance, now for simplicity, let us suppose that $t = 2$ and $S = 2$. Thus, g_t takes only two values, either g_l or g_h at each date, today or tomorrow. Otherwise, the example of product sigma-field is too large and comlicated. Moreover, let us suppose that $\Sigma_t = \{\varnothing, \{g_l\}, \{g_h\}, \{g_l, g_h\}\}$ constant for each t. Then, the product sigma-field is the sigma-field generated by $\{(g_l, g_l), (g_l, g_h), (g_h, g_l), (g_h, g_h)\}$ whose cardinality is 2^4.

Then, for any probability measure p on the sigma-field $\mathcal{F}$, $p_t(g) = p(\{g_0 \times \cdots \times g_t\} \times S \times S \times \cdots)$ becomes the (marginal) probability of the partial history. Each g^t is assumed to be $\mathcal{F}_t$-measurable, that is, their value depends only on the realization of states through the date t. Now, each agent in a formal way as follows: Let a trader i have a rational belief when $p^i = p$, where p is the true probability. Also, let $c_t^i(g)$ be the consumption of the agent i at t.

(1) $h : \forall i \; s.t \;\; p^i = p, \;\; \limsup_{t\to\infty} c_t^i(g) > 0$ p-almost surely.[10]

It deserves to note that the measurable spaces for p^i and p are the same. In other words, the spaces, on which the probability measures p and p^i are assigned, are the same as $(\Sigma, \mathcal{F})$ for all the agents i's who reside in the same world. This is virtually what it is implied by the common inside-outside agent structure. Recall that, for any agent i and j, their true probabilities are the same when they reside in the same world. Under the common inside-outside agent structure, whatever is the true probability to one agent i, it is also the very true probability to the other agent j in the same model.[11] In particular, under the common inside-outside agent structure, if the econometrician resides in the same model as any agent i does, the econometrician's true probability is also p, whenever the agent i's true probability is p.

To confirm the MSH in the Bayesian model, an econometrician must obtain the

i's belief is assumed to be represented by this probability distribution p^i on $\mathcal{F}$.

[10] It deserves to note that in Blume and Easley (2006) the almost sure convergence of rational agent's consumption in equilibrium is with respect to the *true* probability, p. Therefore, the probability space of p involved in this almost sure convergence is the same as that of rational agent's probability p on Σ.

On the other hand, it is not clear how Blume and Easley (2006) can defend this almost sure convergence if there exists no such thing as the true probability. For now, we leave this as an open question, because this problem is ciritical not just to Blume and Easley (2006). Indeed, everyone who considers the concept of *almost sure convergence* faces the similar problem.

Recall that almost sure convergence involves measuring the distance between two probability distributions and that this distance is relative to some specific probability measure taken. Therefore, such convergence results depend on with respect to which probability measure we are gauging the distance. Indeed, if there exists the objective true probability, it is reasonable to measure such distance with respect to this objective probability.

[11] Note that this does not require that all the agents inside the model actually come to hold this true probability in common as their beliefs, which maintains under stronger stipulation, i.e. the REH. Under the REH, for any agent i and j with $i \neq j$ in the same model, $p^i(g) = p^j(g) = p(g)$ for any $g \in \mathcal{F}$, if p denotes the true probability distribution in this model.

following formula for confirmation:

(2) $f(MSH,\, E) > 0$,

where f is a function of p^e, p^e is the subjective probability of the econometrician who conducts the confirmation on the MSH and E is the set of her total evidences available at the time of conducting confirmation.[12] For instance, when f is the incremental difference function, the confirmation formula becomes:

(3) $p^e(\forall i \ s.t \ \ p^i = p, \ \ \limsup_{t\to\infty} c_t^i(g) > 0 \ p\text{-almost surely} \mid E) - p^e(\forall i \ s.t \ p^i = p, \ \limsup_{t\to\infty} c_t^i(g) > 0 \ p\text{-almost surely}) > 0.$[13]

We have noted that the measurable space for p^e is the same as that for p^i for any agent i who resides in the same world as the econometrician. This is what the common inside-outside agent structure implies. However, someone might be worried about the fact that the econometrician's probability is on the hypothesis h

[12] Let us denote by E the set of totality of events known to the econometrician at the time of confirmation. Then, it is the case that $E \subset \mathcal{F}$.

For instance, recall that, when $t = 3$ and $S = 2$, $\mathcal{F}$ is the product sigma-field, $\bigotimes_{t\in\{1,2,3\}} \Sigma_t$ where Σ_t is a sigma-field on $\{g_l, g_h\}$ for each $t \in \{1,2,3\}$. We consider three dates, past, present and future. Also, recall that evidence is the set of known events which is assumed to be subset of all the events, $\mathcal{F}$, known or unknown. Then, for instance, among all the possible events, E can be the sigma-field generated by $\{(g_l, g_l),\ (g_l, g_h),\ (g_h, g_l),\ (g_h, g_h)\}$.

Now, note in this example that $\mathcal{F}$ is the sigma-field when $t = 3$ but that E is only the sigma-field when $t = 2$. For E consists only of the known events, while $\mathcal{F}$ consists of all the events, known or unknown. Therefore, E considers only yesterday and today, and leaves tomorrow open. Also, note that when $t = 3$, an element of E, say (g_l, g_l), represents a partial history up to today, which amounts to the set $\{(g_l, g_l, g_l),\ (g_l, g_l, g_h)\}$. Then, obviously, every element of E is included in $\mathcal{F}$.

[13] The left-hand side of equation (3) includes the econometrician's subjective *conditional* probability *on* E. Then, the measurable space of the conditional probability, $p^e(MSH|E)$, consists of E which is a sub-collection of $\mathcal{F}$. Note that we count MSH as a random variable whose measurable map is from $(\Sigma, \mathcal{F})$. (We discuss this in more depth shortly.) Therefore, for any agents i and j inside the MSH model, the measurable spaces of conditional probabilities on the same evidence E, $p^i(MSH|E)$ and $p^j(MSH|E)$ are the same.

For instance, let us consider the conditional probability of getting 6 given the evidence that we obtain even numbers from throwing a die. Then, the measurable space for the probability on the events of getting a number, say 6 from throwing a die includes the measurable space for the probability on the events of getting even numbers. This is in the same line as when Durrett (2010) defines the conditional expectation of X given $\mathcal{F}_Y$, $E(X|\mathcal{F}_Y)$, to be any random variable Y such that $E(X|\mathcal{F}_Y) = Y \in \mathcal{F}_Y \subset \mathcal{F}_X$ where a random variable $X \in \mathcal{F}_X$.

of the model while the inside agent's probability is on the states of the very model. Therefore, the set of possible states of the model, say $\{g_l, g_m, g_h\}$, does not include the set of possible states of the hypothesis, say $\{true, false\}$, and so the two measurable spaces for p^e and p^i might not be the same.

To respond to this worry, let us note that the hypothesis h can be treated as a random variable whose value is 1 when h is true and 0 when it is not true. Then, h is a function from the sigma-field generated by Σ to $\{0, 1\}$. This function is a measurable map from $(\Sigma, \mathcal{F})$ to $(\{0, 1\}, \sigma(\{0, 1\}))$. Now, note then that

$$h^{-1}(B) \equiv \{g : h(g) \in B\} \in \mathcal{F} \text{ for all } B \in \sigma(\{0, 1\}).$$

Therefore, the measurable space for $p^e(h)$ is also $(\Sigma, \mathcal{F})$. For instance, let us consider the hypothesis h that a head occurs when we flip a fair coin. Then, even if the possible states of this hypothesis is either 0 or 1, the measurable space for our subjective probability of h being true, i.e. a head's occurring, consists of $\{H, T\}$ and the sigma-field generated by this set.

Let us now define the *best confirmation model*. Our goal is to show the confirmation impossibility of the MSH by showing that it is not possible to confirm the MSH even under the best confirmation model. For if we cannot confirm the MSH under this best model, we cannot confirm it in any other models, which implies that it is impossible to confirm the MSH.

Definition 2 The *best confirmation model* is the confirmation model conditioning on the best evidence. The *best evidence* is what comparatively increases the probability of h the most. Under the Bayes' theorem, this is equal to the most likely evidence conditioning on that the confirming hypothesis is true, provided that the best evidence and its alternatives are equally surprising.[14]

Lemma 2 The best evidence to confirm the MSH is the data from the agent with the most accurate probability distribution to the true one.

Proof of Lemma 2. Suppose not. Then, with p' being denoted by the most accurate distribution, there exist at least one p'' such that p'' is less accurate than p' but the evidence from the agent with p'' confirms the MSH better than from p'. Given that the MSH is true, then the agent with $p\prime$ survives over the one with p''. Thus, it is more likely to obtain the positive consumption data from the agent with $p\prime$ than the data from the agent with p'',[15] which contradicts that the evidence from p'' confirms better than the evidence from $p\prime$, provided that it is equally surprising to obtain consumption data sets from the agent with p' and from the agent with p''. *Q.E.D.*

Note here that we use the more general version of the MSH, namely that the agents with *more* correct beliefs survive from the market competition when markets are complete. A simpler version of the MSH which we previously introduced is that the agents with correct beliefs survive. Indeed, if there exist rational agents with

[14] It deserves to note that the assumption on the uniform prior is a common practice in statistics, especially when there is no particular reason to assign higher prior probability to some evidence than the other evidences. For instance, see Dawid, A. P 1982, "The Well-Calibrated Bayesian", *Journal of the American Statistical Association,* Vol. 77, No. 379. p. 607.

Now, let us consider the incremental difference formular for confirmation as an example. Then, the best evidence, E^*, is what makes $p(h|E^*)$ greater than $p(h|E)$ for any E. Now, under Bayes' Theorem, $p(h|E) = p(E|h) \cdot \frac{p(h)}{p(E)}$. Thus, for any E, $\frac{p(h|E^*)}{p(h|E)} = \frac{p(E^*|h)}{p(E|h)}$ when $p(E^*) = p(E)$. Therefore, under the assumption that E^* and E are equally likely, the best evidence amounts to what is the most likely evidence when the hypothesis is true.

It deserves to note that the assumption on the equal surprise of obtaining E and E^* does not mean that evidences come from uniform distribution. For instance, even if E and E^* follow, say the normal distributions, it is still possible that $p(E)$ is same as $p(E^*)$.

[15] It deserves to remind that the survival is defined by the positive lim sup of consumption. Therefore, if the more correct agents survive better, which means that their consumptions are more likely to be positive, we are more likely to obtain the consumption data from them.

the true belief on the market, their beliefs are more accurate than any other beliefs. Therefore, the proofs from general version of the MSH also holds for the simpler version of the MSH. However, unless it is essentially required to use the general version in the proof, we will stick to the simpler version to make the argument simpler and clearer.

Obtaining the evidence for the MSH means obtaining consumption data from the rational agents in order to check their long-run behaviors. Provided that the MSH is true, its best evidence should be the data from the agents with the most accurate distribution, possibly the true one itself, because less accurate agent's data cannot serve as better evidence for the MSH. For it is more likely to obtain the supporting consumption data from the agent with the more accurate distribution than from the agent with the less accurate one, given that the agent with the more accurate distribution indeed survives out of competition.

Lemma 3 Let $p : \mathcal{F} \to [0, 1]$ be the true distribution whose existence is assumed, and let P be the set of all the subjective probability distributions on $\mathcal{F}$ which agent i holds as estimations on the true probability distribution p.[16] Let us denote the

[16] It deserves to note that P does not trivially include p. For instance, let us say that a probability distribution is doxastically possible if and only if it is compatible with the total amount of evidence currently given to us. Then, if P must include all such doxastically possible distributions, then it might be the case that, whatever p is, as long as it *is* a probability distribution and does not contradict to any available evidence, it must be included in P. Now, the closest elements of P to p is trivially p itself and the best evidence itself must be the data from the true distribution. Later in this section, however, we will argue for that the best MSH model may be different from the true MSH model to the econometrician even under the best evidence. Unfortunately, this argument would not work if the best evidence is trivially from the true distribution. For the best MSH model becomes then simply the true MSH model.

However, it is obvious that there are (presumably infinitely) many doxastically possible probability distributions among which there are ones that we have not had (or we have not thought of) due to our epistemic limitation. It is also realistic to think that, among doxastically possible probability distributions, there may be the ones that we cannot have or think of, for they are beyond our cognitive capacity even with the maximal epistemic developments we are able to achieve – the ones that are, we may say, subjectively unavailable to us. Thus, let P be the set of all doxastically possible and subjectively accessible probability distributions on $\mathcal{F}$ which agent i holds as estimations on the true probability distribution p. Then there is no reason to think that p is trivially a member of P.

element in P by p^i. Also, let ρ be a distance function from the functions in P to p, i.e. $\rho : p \times P \to [0, \infty)$ such that $\rho(p, p^i) \geq 0$, with $\rho(p, p^i) = 0$ for all $p^i \in P$ such that $p^i = p$. Then, provided that P is compact and ρ is lower semi-continuous, the set $D = \{\rho(p, p^i) | p^i \in P\}$ attains the minimal value.

Proof of Lemma 3 Before we prove Lemma 3, it deserves to mention that the distance function ρ is not necessarily a metric,[17] because neither symmetricity nor triangle inequality is required. For instance, ρ can be a relative entropy function to the true process p such that $\rho(p, p^i) = \sum_{g \in \mathcal{F}} p(g) \cdot (\log p(g) - \log p^i(g))$, provided that p is absolutely continuous with respect to p^i. Indeed, a relative entropy satisfies neither symmetricity nor triangle inequality.

Now, let us denote by P the set $\{p^i(g) : g \in \mathcal{F}$, p^i is the doxastically possible and subjectively accessible probability distribution of an agent i in the model$\}$. Then, suppose that, for each elementary event $\sigma^* \in \mathcal{F}$, the belief set $BF = \{p^i(\sigma^*) : p^i(\sigma^*) \in P\}$ is closed. For if agents have all but finitely many estimations which are arbitrarily close to a particular limit, it is reasonable to include this limit in their belief sets as well. Now that $p^i(g)$ is bounded in $[0, 1]$, BF is compact. Note then that the set of all functions $\{p_i : \mathcal{F} \to BF\}$ can be identified with the product space $BF^{\text{card}(\mathcal{F})}$, where $\text{card}(\mathcal{F})$ denotes the cardinality of $\mathcal{F}$. Then, by Tychonoff's theorem,[18] P is also compact. Now, by Weierstrass theorem,[19] letting that ρ be a lower semi-continuous function, $\rho(p, p^i)$ on P attains the minimum value. $Q.E.D$

It deserves to note that the lower semi-continuity of ρ is satisfied by the various existing ways of measuring distance between probability functions in the following:[20]

[17]Folland, G. 1999. *Real Analysis.* New York: John Wiley & Sons, p. 13.

[18]Folland, G. 1999. *Real Analysis.* New York: John Wiley & Sons.

[19]Barbu, V. and Precupanu, T. 2012, *Convexity and Optimization in Banach Space.* New York: Springer.

[20]It deserves to note that $V_P(Q)$, $B_P(Q)$ and $H_P(Q)$ are all continuous and so lower semi-continuous, but that $D_{KL}(P||Q)$ is just lower semi-continuous. For the lower semi-continuity of the

Variational Distance: $V_P(Q) = sup\{|P(A) - Q(A)| : A \in \sigma(\Sigma)\}$

Brier Score: $B_P(Q) = \sum\limits_{\sigma\in\Sigma}(P(\sigma) - Q(\sigma))^2$

Hellinger Distance: $H_P(Q) = \sum\limits_{\sigma\in\Sigma}(P(\sigma) + Q(\sigma) - 2(P(\sigma)\cdot Q(\sigma))^{\frac{1}{2}})$

Kullback-Leibler Divergence: $D_{KL}(P||Q) = \sum\limits_{\sigma\in\Sigma}(\log P(\sigma)\cdot \log\frac{P(\sigma)}{Q(\sigma)})$

Corollary 1 The most accurate probability distribution to the true one is well defined, provided that the true distribution, if any, exists. In other words, suppose that $P^c \subset P$ represents the set of the most accurate subjective probability distributions to the true one p. Also, let ρ be a distance function defined as in Lemma 3. Then, when $P^c = \{p^i : \rho(p, p^i) = \inf_{p^i\in P}\rho(p, p^i)\}$, P^c is well-defined.

Proof of Corollary 1. If $p \in P$, $\inf_{p^i\in P}\rho(p, p^i) = 0$ at $p^i = p$. Thus, it is well-defined. Suppose now that $p \notin P$. Then, by Lemma 3, $\inf(D) \in D$, which means that P^c is well-defined. $Q.E.D.$

It deserves to note that P^c is not always well-defined even if the true probability distribution exists.[21] For instance, let us consider the following case of flipping a coin: Suppose that the true probability of getting a head (H) is 0 and that of getting a tail (T) is 1. Also, let us suppose further that P does not include p. Then, P can be a set $\{p^i : p^i(H) = \frac{1}{r}, p^i(T) = 1 - \frac{1}{r}$ for any $r \in \mathbb{R}[1, \infty)\}$. Now, $\rho(p, p^i) = \frac{2}{r^2} \rightarrow 0$ as $r \rightarrow \infty$. Thus, $\inf \rho(p, p^i)$ is attained at 0 when $p^i = p$. However, by assumption, $p \notin P$. Therefore, $\inf(D) \notin D$. In this case, all the estimations are approaching the true distribution arbitrarily close, but nonetheless they never reach it. However, excluding such cases, Lemma 3 and Corollary 1 guarantee that P^c exists whenever p exists. It also deserves to note that P^c is well-defined even when p is not included in P. Therefore, obtaining P^c does not guarantee that $p \in P$.

Kullback-Leibler function, see John C. Baez and Tobias Fritz, 2014, A Bayesian Characterization of Relative Entropy, *Theory and Applications of Categories*, Vol. 29, No. 16.

[21] Here, by P^c being well-defined, we mean that the set P^c is not empty.

Now that the best confirmation model for the MSH is well-defined from Definition 2, Lemma 2, Lemma 3, and Corollary 1, we are ready to show that we cannot confirm the MSH even under this best model. From Lemma 3 and Corollary 1, it follows that if the true probability distribution indeed exists, its closest distributions should be able to be determined, so far as it is well-defined how to measure such distance between any two probability distributions. Then, from Definition 2 and Lemma 2, it follows that the best confirmation model for the MSH is defined by the best evidence which is well-defined by the closest distributions.[22]

From Lemma 3 and Corollary 1, then we show that there indeed exist such most accurate distributions provided that there exists such thing as the true distribution. Recall that we do not commit ourselves to inquiring what the true probability is per se or whether there exists such thing as the true probability. Rather, we intend to inquire what we can do, assuming that there may exist such thing as the true distribution. Lemma 3 and Corollary 1 justifies that we can indeed pursue such inquiry. Now, let us proceed to prove that the MSH is not confirmable from data under this best model. Let us first introduce the following principle.

Principle of the Epistemic Effect Given the two-layered epistemic structure

[22] For instance, when we consider the absolute difference function, the best confirmation model becomes $p^e(MSH|E^*) - p^e(MSH)$. Recall then that, when we consider MSH as a random variable whose measurable map is from $(\Sigma, \mathcal{F})$, the measurable space of conditional probability of MSH on E consists of $E \subset \mathcal{F}$.

In the confirmation of MSH, E is the set of *known* events of consumption $c^i(g)$ occurring to the agent i in the model. Recall from section 3.3 that $c^{i^*}(g)$ is assumed to be $\mathcal{F}$- measurable; that is their values only depend on the realization of states g. Therefore, although the intutive description on evidence is that it is the set of consumption data, we formally express the evidence as the set of g. It thus turns out that $E \subset \mathcal{F}$. Now that E^* is the "best" among E, it is the case that $E^* \subset E \subset \mathcal{F}$.

Now, let us discuss E^* more with the consumption data. Recall that the best evidence for the MSH is the consumption data of the agents with the closest beliefs (the closest probability distributions) to the true distribution. Then, for all the $g \in E$, let us denote by $p^{i^*}(g)$ the closest distribution such that $p^{i^*}(g) \in P^c$ where P^c is defined as in Lemma 3 except that P is now a set of p^i on E, not on $\mathcal{F}$. It was shown in the Lemma 2 that the consumption data of the agent i^* with the closest distributions is the best evidence E^* for the MSH. Therefore, $E^* = \{g \in E : c^{i^*-1}(B)$ $\in E$ for some $B \in \sigma(\mathbb{R}[0,\infty))$ and $p^{i^*}(g) \in P^c\}$.

of confirmation on the MSH, the epistemic limitation of the econometrician on the true distribution, while updating the evidence, is directly reflected on the very MSH model whose evaluation is underway in the best confirmation model.

First of all, it deserves to note that the MSH "model" must be distinguished from the best "model" for the confirmation on the MSH. The model for the confirmation on the MSH is at the higher level than the MSH model, in the sense that the MSH model is a model of some economic phenomena but the confirmation model is a meta-model, i.e., a model about the conformity of the MSH model to reality.

The Principle of the Epistemic Effect comes from the two distinctive epistemic features of the confirmation on the MSH. Recall that the two distinctive features are (1) the common inside-outside agent structure and (2) the self-referential structure. Also, remember that the best confirmation model for the MSH is the model in which the econometrician holds the most accurate distribution on the purpose of obtaining her best evidence. Then, due to the common inside-outside agent structure, the rational agent in the MSH model under the best confirmation model is *at best* the one whose belief is congruent to the econometrician's most accurate distribution. For whatever is the most accurate to the econometrician must also be the most accurate to the agents inside the model, when the econometrician is also one of the inside agents. Note that the econometrician performs confirmation under the best model. Then, by definition, the econometrician must be the agent who possesses the best evidence under such best confirmation model. Therefore, by Lemma 2, her belief must be the most accurate one in the model. Note that the objective true distribution, if any, is common to both of the inside and the outside agents, and thus that the set of the closest distributions must be the same as well, when they reside in the same world. But the statistical theorem shows that even the econometrician's *most accurate* estimation cannot be *known* to be true even when it happens to *be* the true one. This epistemic limitation of the econometrician is also applied to the rational agent inside the model for the MSH. Then, due to the self-referential structure, the epistemic limit of the inside agent's belief must be reflected on the very MSH model, because this epistemic limit fetters her beliefs on the future values of the endogenous variables in the MSH model.

Let us denote by the MSH^* the hypothesis that the agent with the *best but only possibly true* distribution survives, and by the MSH the hypothesis that the agent with the *true* distribution survives. The MSH^* is the epistemically restricted MSH in the sense that all the agents inside the MSH^* model is fettered by the epistemic limit on the true distribution so that their beliefs are at best only *possibly* true. Under the Principle of Epistemic Effects, the econometrician obtains only $p^e(MSH^*|E^*)$ at most, no matter how hard she strives for $p^e(MSH|E^*)$, where p^e represents her subjective conditional probability given the best possible evidence E^*. Recall from Lemma 2 that the best evidence of the MSH is the data from the most accurate beliefs. However, this most accurate belief is not necessarily the true one.

Lemma 4 Under the Principle of Epistemic Effect, the econometrician is not sure that the best MSH^* model is the same as the true MSH model under the best confirmation model.

Proof of Lemma 4 Without loss of generality, let us assume that econometrician tries to confirm the MSH. In other words, she tries to show that

$f(\forall i \ s.t \ p^i = p, \ \limsup_{t\to\infty} c_t^i(g) > 0 \ p\text{-almost surely}, E) > 0.$

However, under the Principle of Epistemic Effect, the best confirmation model consists of $p^e(\forall i \ s.t \ \ p^i \in P^c, \limsup_{t\to\infty} c_t^i(g) > 0 \ p\text{-almost surely}|E^*)$.

Note that, since the econometrician cannot know the true distribution, the following holds:

$\forall \varepsilon > 0$, $p^e(\forall i\ s.t\ p^i \in P^c,\ \rho(p, p^i) > \varepsilon | E) \geqslant \kappa(\varepsilon)$ for $\kappa(\varepsilon) > 0$, even when $p \in P^c$.

Here, $\kappa(\varepsilon)$ denotes some positive threshold which depends on ε. It deserves to note here that $\kappa(\varepsilon)$ is strictly positive, not zero, by which we intend to exclude the case that the econometrician is certain with probability one that her best estimation is arbitrarily close to the true one. Obviously, this case is contradictory to the fact that she does not know the truth. Even if the econometrician forms the closest belief to the true distribution, her subjective probability that such best belief may not be true is greater than zero, even when her closest belief is indeed the true distribution itself. She would be fairly certain that her best belief is arbitrarily close to the true one, if she knew the truth.

Thus, now that the econometrician does not know the truth, she believes even under the best evidence that her *best* MSH model may be distinguished from the *true* MSH model. Then, we want to show that $q^e(p^e(MSH^*|E^*) \neq p^e(MSH|E^*)) > 0$ for the econometrician's arbitrary subjective probability function q^e[23]. The basic idea is that the econometrician is not sure that her conditional belief on MSH^* and her conditional belief on MSH are the same even under the best evidence.

First, under the assumption that logically equivalent statements have the same probability for any arbitrary probability function p^e, we will prove the following two inequalities:

(1) $p^e(\forall i\ s.t\ \ p^i \in P^c, \limsup_{t\to\infty} c_t^i(g) > 0$ p-almost surely $| E^*) \geqq p^e(\forall i\ s.t\ \ p^i \in P^c\ \&\ p^i = p, \limsup_{t\to\infty} c_t^i(g) > 0$ p-almost surely $|E^*) - p^e(\exists i\ s.t\ \ p^i \neq p\ \&\ p^i \in P^c$ $\&\ p(\limsup_{t\to\infty} c_t^i(g) > 0) \neq 1 \mid E^*)$.

(2) $p^e(\forall i\ s.t\ \ p^i = p,\ p(\limsup_{t\to\infty} c_t^i(g) > 0) = 1 \mid E^*) \geqq p^e(\forall i\ s.t\ \ p^i \in P^c \&\ p^i =$

[23] It deserves to note that q^e is a typical higher order probability. Therefore, the measurable space for q^e consists of $\mathbb{R}[0,1] \times \mathbb{R}[0,1]$ and the sigma-field generated by this cartesian product. For instance, q^e assigns probability measure to the set of events that, for any r_1, $r_2 \in \mathbb{R}[0,1]$, $p^e(MSH^*|E^*) = r_1$ and $p^e(MSH|E^*) = r_2$, and $r_1 \neq r_2$.

p, $p(\limsup_{t\to\infty} c_t^i(g) > 0) = 1|E^*) - p^e(\exists i \ s.t \ \ p^i = p \ \& \ p^i \notin P^c \ \& \ p(\limsup_{t\to\infty} c_t^i(g) > 0) \neq 1 \mid E^*)$.

Note that formula (1) amounts to $p^e(MSH^*|E^*) \geqslant A - C_1$ while formula (2) amounts to $p^e(MSH|E^*) \geqslant A - C_2$, where

$A = p^e(\forall i \ s.t \ p^i \in P^c \ \& \ p^i = p,\ p(\limsup_{t\to\infty} c_t^i(g) > 0) = 1|E^*)$,

$C_1 = p^e(\exists i \ s.t \ p^i \neq p \ \& \ p^i \in P^c \ \& \ p(\limsup_{t\to\infty} c_t^i(g) > 0) \neq 1 \mid E^*)$.

$C_2 = p^e(\exists i \ s.t \ p^i = p \ \& \ p^i \notin P^c \ \& \ p(\limsup_{t\to\infty} c_t^i(g) > 0) \neq 1 \mid E^*)$.

Our strategy is to show that the probability of $p^e(MSH^*|E^*) \neq p^e(MSH|E^*)$ is positive if $|C_1 - C_2| > 0$.

For when $|C_1 - C_2| > 0$, $A - C_1 > A - C_2$ or vice versa. Without loss of generality, let us suppose that $A - C_1 > A - C_2$.

Then, for any $p^e(MSH^*|E^*) \geqslant A - C_1$, if $A - C_1 > p^e(MSH|E^*) \geqslant A - C_2$, it is possible that $p^e(MSH^*|E^*) \neq p^e(MSH|E^*)$.

Now, to derive the equalities (1) and (2), let us first adopt the following notational simplifications:

let's denote $p^i = p$ by $T(i)$, and $p^i \in P^c$ by $B(i)$, and

$p\{\limsup_{t\to\infty} c_t^i(g) > 0\} = 1$ by $S(g, i)$.

Then, MSH is $\forall i(T(i) \to S(i))$ and MSH^* is $\forall i(B(i) \to S(i))$.

Lemma 5 is then the thesis that

$q^e(p^e(\forall i(T(i) \to S(i))|E^*) \neq p^e(\forall i(B(i) \to S(i))|E^*)) > 0$.

Now, note that $\forall i(T(i) \to S(i)) \equiv \forall i((T(i)\&B(i)) \to S(i)) \ \& \ \forall i((T(i)\&\neg B(i)) \to S(i))$ and that

$\forall i(B(i) \to S(i)) \equiv \forall i((T(i)\&B(i)) \to S(i)) \ \& \ \forall i((\neg T(i)\&B(i)) \to S(i))$.

Thus, $p^e(\forall i(T(i) \to S(i))|E^*) =$

$p^e(\forall i((T(i)\&B(i)) \to S(i)) \ \& \ \forall i((T(i)\&\neg B(i)) \to S(i))|E^*)$.

However, $p^e(\forall i((T(i)\&B(i)) \to S(i)) \ \& \ \forall i((T(i)\&\neg B(i)) \to S(i))|E^*) = p^e(\forall i((T(i)\&B(i)) \to$

$S(i))|E^*) + p^e(\forall i((T(i)\&\neg B(i)) \rightarrow S(i))|E^*) - p^e(\forall i((T(i)\&B(i)) \rightarrow S(i)) \text{ V } \forall i((T(i)\&\neg B(i)) \rightarrow S(i)))|E^*)$.

For note that $p^e(A\&B|E^*) = p^e(A|E^*) + p^e(B|E^*) - p^e(A \text{ V } B|E^*)$.

Now, whatever $p^e(A \text{ V } B|E^*)$ is, it is smaller than or equal to 1.

Thus,

$p^e(\forall i((T(i)\&B(i)) \rightarrow S(i)) \; \& \; \forall i((T(i)\&\neg B(i)) \rightarrow S(i))|E^*) \geqq p^e(\forall i((T(i)\&B(i)) \rightarrow S(i))|E^*) + p^e(\forall i((T(i)\&\neg B(i)) \rightarrow S(i))|E^*) - 1$.

Now, note that

$p^e(\forall i((T(i)\&\neg B(i)) \rightarrow S(i))|E^*) - 1 = -p^e(\neg\forall i((T(i)\&\neg B(i)) \rightarrow S(i))|E^*) = -p^e(\exists i(T(i)\&\neg B(i)\&\neg S(i))|E^*)$.

Therefore, $p^e(MSH|E^*) = p^e(\forall i((T(i)\&B(i)) \rightarrow S(i)) \; \& \; \forall i((T(i)\&\neg B(i)) \rightarrow S(i))|E^*) \geqq p^e(\forall i((T(i)\&B(i)) \rightarrow S(i))|E^*) - p^e(\exists i(T(i)\&\neg B(i)\&\neg S(i))|E^*)$.

In the same way,

$p^e(MSH^*|E^*) = p^e(\forall i((T(i)\&B(i)) \rightarrow S(i)) \; \& \; \forall i((\neg T(i)\&B(i)) \rightarrow S(i))|E^*) \geqq p^e(\forall i((T(i)\&B(i)) \rightarrow S(i))|E^*) - p^e(\exists i(\neg T(i)\&B(i)\&\neg S(i))|E^*)$.

Then, $q^e(p^e(MSH|E^*) \neq p^e(MSH^*|E^*)) > 0$ if $|p^e(\exists i(T(i)\&\neg B(i)\&\neg S(i))|E^*) - p^e(\exists i(\neg T(i)\&B(i)\&\neg S(i))|E^*)| > 0$.

Therefore, her probability that $p^e(MSH|E^*) \neq p^e(MSH^*|E^*)$ becomes positive if $|p_e(\exists i \; s.t \; p^i = p \; \& \; p^i \notin P^c \; \& \; p(\limsup_{t\rightarrow\infty} c_t^i(g) > 0) \neq 1 \mid E^*) - p^e(\exists i \; s.t \; p^i \neq p \; \& \; p^i \in P^c \; \& \; p(\limsup_{t\rightarrow\infty} c_t^i(g) > 0) \neq 1 \mid E^*)| > 0$.

We now need to show that this absolute difference is not zero, in order to finish Lemma 5.

First, let us note that

$p_e(\exists i \; s.t \; p^i = p \; \& \; p^i \notin P^c \; \& \; p(\limsup_{t\rightarrow\infty} c_t^i(g) > 0) \neq 1 \mid E^*) = 0$,

for she is sure that the true belief, if any, must be included in the best set.

Also, note that $p^e(\exists i \; s.t \; p^i \in P^c \; \& \; p^i \neq p \; \& \; p(\limsup_{t\rightarrow\infty} c_t^i(g) > 0) \neq 1 \mid E^*) \geqslant$

$p^e(\exists j \neq i \ s.t \ p^j = p \neq p^i$ for some $p^i, p^j \in P^c|E^*)$ which is not zero.

It is not zero, because the econometrician is not sure even under the best evidence that her best belief sets P^c include only true beliefs. She is not sure even when p actually happens to be in P^c. Indeed, since $\forall \varepsilon > 0$, $p^e(\forall i \ s.t \ p^i \in P^c, \ \rho(p, \ p^i) > \varepsilon|E^*) > 0$ even when $p \in P^c$, $p^e(\rho(p^j, p^i) > \varepsilon_0) > 0$, for some $\varepsilon_0 > 0$ and some $p^i, p^j \in P^c$ with $p^j = p$. *Q.E.D.*

According to Lemma 4, the econometrician is not sure that the MSH^* model and the MSH model are the same ones given her best evidence. For she is not sure that her best estimation is the true one. Formally speaking, for any arbitrary subjective function q^e and p^e, $q^e(p^e(MSH^*|E^*) \neq p^e(MSH|E^*)) > 0$. She is not sure even if her best estimation is actually the true one.

It deserves to emphasize again that even if the outside and the inside agent happen to *hold* the best distribution under the best confirmation model, neither of them *knows* whether their best distribution is actually the best. For they do not know what the true distribution is, and thus they do not know whether their distribution is the closest to the true one. If the econometrician could have known whose belief is the best, it would have been good enough for the purpose in hand just to confirm that the best agent survives. The econometrician would not have had to strive for the perfect result, for no one can do so in this imperfect world.

Now that the econometrician is not sure that the MSH and the MSH^* are the same hypotheses, she cannot confirm the original MSH by using the epistemically restricted MSH^*, unless she is sure that those two hypotheses always return the same confirmation results. However, Theorem 5 shows that this is not the case.

Theorem 5 The econometrician is not sure that the two hypotheses in Lemma 5 return the same confirmation result. Therefore, the econometrician cannot confirm the MSH directly from data.

Proof of Theorem 5 By Lemma 4, we show that $q^e(p^e(MSH^*|E^*) \neq p^e(MSH|E^*)) > 0$. Thus, unless the econometrician is sure that the MSH model and the MSH^* model always return the same confirmation result even though they may be different, she cannot confirm the MSH. Note that two hypothesis, the MSH and the MSH^*, do not return the same confirmation result when they assign different conditional probabilities given the same best evidence. For recall from the discussion in chapter 2 that a hypothesis is confirmed when it satisfies three requirements. In particular, note that the incremental requirement is the most relevant in this proof. For instance, when the econometrician adopts the difference function for confirmation, she must determine whether $p^e(MSH|E^*) > p^e(MSH)$ or not. However, note from the Principle of Epistemic Effect that the econometrician cannot but obtain $p^e(MSH^*|E^*)$ no matter how hard she strives for $p^e(MSH|E^*)$. Therefore, unless she is sure that $p^e(MSH|E^*) > p^e(MSH)$ whenever she obtains $p^e(MSH^*|E^*) > p^e(MSH)$, she cannot evaluate whether $p^e(MSH|E^*) > p^e(MSH)$ or not. Note however that $p^e(MSH^*|E^*)$ and $p^e(MSH|E^*)$ are two different real numbers when $p^e(MSH^*|E^*) \neq p^e(MSH|E^*)$. Then, since a real number system is dense and thus there always exists a real number between any two real numbers, there can exist another real number, say $p^e(MSH)$ between $p^e(MSH^*|E^*)$ and $p^e(MSH|E^*)$. Therefore, the probability of getting opposite confirmation results from the MSH model and the MSH^* model is positive. $Q.E.D.$

For instance, let us consider again the absolute difference function for confirmation. Since $q(p^e(MSH^*|E^*) \neq p^e(MSH|E^*)) > 0$ for any arbitrary p^e, then by Theorem 5, it is always possible for the econometrician to find some value for $p^e(MSH)$ which is between $p^e(MSH^*|E^*)$ and $p^e(MSH|E^*)$. Thus, for instance,

$q^e(p^e(MSH^*|E^*) > p^e(MSH) > p^e(MSH|E^*)) > 0$. In case then where $p^e(MSH^*|E^*) > p^e(MSH) > p^e(MSH|E^*)$, the econometrician faces the following problem: She may conjecture the affirmative confirmation result on MSH from E^*, for she obtains that $p^e(MSH^*|E^*) > p^e(MSH)$. But, at the same time, the true result is that MSH is not confirmed to her, for she also obtains that $p^e(MSH) > p^e(MSH|E^*)$. With her positive subjective probability, she is not sure that the MSH model can be confirmed via the MSH^* model. Even if she obtains that $p^e(MSH^*|E^*) > p^e(MSH)$ for instance, she cannot determine from there whether $p^e(MSH|E^*) > p^e(MSH)$. Therefore, she cannot confirm the MSH model by using the MSH^* model directly even by the best data.

Note that, in Theorem 5, we do not require the probability of opposite confirmation, q^e, to be 1. In other words, the econometrician is not required to be sure whether the confirmation results from the MSH and the MSH^* are always opposite. For suppose instead that the econometrician is sure that the confirmation results are always opposite. Then, she would be able to confirm the MSH whenever she obtains $p^e(MSH^*|E^*) < p^e(MSH)$ or vice and versa.

The idea behind Theorem 5 is as follows: In order to confirm the MSH, the econometrician tries to find some inductive support from evidence that the consumption data of rational and irrational agents inside the model would converge respectively to certain limits in the long run. To obtain such evidence in the first place, the econometrician must tell which consumption data come from rational agents, which again requires her to know which belief is the rational one. Now, since the rational belief is defined as the true probability distribution, obtaining relevant evidence on the MSH requires for the econometrician to know the true distribution. Accordingly, the econometrician does some statistical estimations to know the true distribution. Now, suppose that, among those estimations, there exists the best one in the sense that it is the closest to the true distribution. Such best estimation may happen to be the true distribution itself, but the point is that no one is guaranteed to know that even her best estimation is the true one even if it happens to be so. Nevertheless, the best way for the econometrician to confirm the MSH is at most to use such best estimation. Then, it follows that the econometrician cannot confirm the MSH, because she does not have the relevant evidence for the MSH even in the case where she uses the best estimation for the confirmation on the MSH. If the econometrician does not obtain the relevant evidence to determine whether to confirm the MSH even with the best estimation, we conclude that no evidence really serves to her for the confirmation on the MSH.

It deserves to note that the two distinctive features in the epistemic structure of the confirmation on the MSH play crucial roles in the proof. One feature is that the econometrician is also an agent inside the MSH model. Thus, in the best confirmation model on the MSH, the rational agent is at best the one whose belief is the econometrician's best estimation. For if they reside in the same world, whatever is the best for the econometrician must also be the best to the inside agent. But the statistical theorems show that even the econometrician's best estimation cannot be known to be true. This epistemic limit of the econometrician then directly fetters the rational agent inside the very model under the best confirmation model.

The other feature is that, through the self-referential mechanism, such epistemic limitation of the inside agent leads to the major change in the very evaluated model. For the limitation in the inside agent's belief must be reflected on the very model as long as such a limited belief is regarding the future values of some key variables determined in the model. The result of confirmation impossibility then follows from the fact that the evaluated model becomes the epistemically restricted one during the confirmation procedure, and thus that the econometrician always has to use the epistemically restricted MSH^* model in order to confirm the original MSH model.

It must be clear here that the confirmation impossibility is not just a personal problem for the econometrician, but a *structural* problem for the confirmation of the MSH, a problem that inevitably follows from its epistemic structure.

Chapter 5

Knowledge and the Implications of the Confirmation Impossibility of the MSH

In the previous section, we argue that confirmation on economics model has characteristic epistemic structure and further argue that due to this epistemic structure economic agent cannot confirm the MSH. It would be unfortunate, however, if what we have done is only something destructive. In this section, our goal is to sketch briefly that we can extend the impossibility thesis further into two constructive directions. First, we plan to argue that the confirmation impossibility actually saves the MSH from its own internal contradictory problem of rationality. Second, we plan to argue that survival argument can give us some hint to construct a new model on how to evaluate accuracy of partial belief. We plan to argue further that the confirmation impossibility shows why knowledge and rationality should split in this accuracy model.

5.1 A Solution to the Internal Problem of the MSH

In this section, we show that with the result of confirmation impossibility we can give a better answer to the question which is originally raised by Blume and Easley. In addition to the behaviors of rational agents in the complete market, the MSH also argues that, in some incomplete market, rational agents may not be able to survive. This result in the incomplete market leads to some puzzling question on why rational traders then do not change their investment behaviors in order not to be driven out from the market if they know that they will vanish when they stick to their original investment decision.

This question is puzzling, because even in cases where the rational traders vanish in the incomplete market, it is still optimal for those rational traders to make such investment decisions that will eventually drive themselves out from the market. How is it possible then that some decision to kill herself is eventually optimal to herself? What is worse is that, according to their interpretation, rational agent knows from the beginning that her decision will kill herself but that notwithstanding it is optimal for her to make such a decision. Then, what does it mean by "optimal"? How can we justify the behaviors of vanishing rational agents as optimal? Here, Blume and Easley conclude that what they proved is not normative, but just descriptive. In other words, their proof does not imply what the rational agent should do. It just describes what she does, which looks puzzling. While just describing what it is, B&E seem to think that they do not have to answer why it is so, or how to resolve such puzzle.

Instead, we argue that the rational agent has a true belief but that such belief alone does not constitute knowledge. Since she makes such investment decision without knowing that she is destined to be driven out from the market by that decision, she does not contradict herself by her decision. The rational traders hold the subjective probability distribution which at best happens to coincide with a true objective probability distribution. However, since the agents are not god, they do not know whether their beliefs are true or not, even though they are indeed true. In order to validate their subjective belief as true one, the best thing they can do is to match it with reality from past data sets, but it only implies that the coherent rational traders are not guaranteed to treat their belief as the true probability distribution. Accordingly, rational traders do not know the true state of the market and, they are competing, without knowing who will be destined to vanish in that market. Since they do not know their fate, they do not have reason to change their investment behavior.

Someone might argue that vanishing does not necessarily implies killing oneself as long as the consumption of vanishing agent is defined by *lim sup*. For no matter how small it is, *lim sup* is something positive and so she can manage to maintain her lifeline although she looks jeopardized. As long as vanishing agent lives on positive consumption, it does not necessarily imply that she kills herself and so that, under this definition, there is no contradiction in her consumption behavior. However, it must be noted that *lim sup* must be smaller than any arbitrary *epsilon* although it remains something positive. Therefore, vanishing agent must live on whatever is smaller than any positive consumption which she barely maintains her life. There is no reason to believe that she is still alive at this arbitrary small consumption.

5.2 The MSH and Accuracy Model

In this section, we distinguish the epistemic rationality and the practical rationality and sketch how to construct an accuracy model which reflects both of the kinds of rationality. Joyce (1998) argues that the epistemically rational agent must strive to hold a system of partial beliefs which represents the world accurately and thus that the accuracy model to gauge how to represent the world must be epistemically motivated. Also, Gibbard (2006) argues that, in order for accuracy to serve as a norm for epistemic rationality, the inaccuracy function of partial beliefs must measure the practical guidance value of having such beliefs. Therefore, it is pertinent to build accuracy model which is epistemically motivated but measures the practical guidance value at the same time. From the hint of the MSH, we plan to suggest a model where the epistemically rational belief is the one that brings forth the practically rational act in the evolutionary perspective. Further, we show that this accuracy model has a significant implication to economics, namely that it will restore the true meaning of rationality for the most influential paradigm in macroeconomics, the REH.

When an agent chooses an action among the possible options, her belief guides her which action to take. For example, an agent chooses to take a bottle of water in the refrigerator, when she fully believes that a sip of water will relieve her thirsty. On the other hand, it seems clear that even when she does not fully believe some proposition, her belief can lead her to take an action. For example, an agent chooses to bring an umbrella even when she believes partially that it will rain in the afternoon. This partial idea is formalized in a model where agent chooses an action which maximizes the expected utility and the expectation is calculated from the probabilities over the possible states. Admittedly, the utility-maximization is not the only model for the decision-rule. Notwithstanding, it is worth to note that it is not the goal of this paper to evaluate which decision-rule is the best, i.e. whether the maximization rule of the expected utility or the likelihood estimate of the utility, or one of any other rules is the "rational" one for any decision-maker to follow. Rather, provided that the decision-maker is practically "rational" in a sense that she follows the expected utility maximization rule, we try to answer which probability distribution is the best one for any practically rational agent to take into account when she makes any decision. For doing this, we consider a future uncertainty, not past uncertainty. In other words, we do not consider the uncertain situation where a specific state is already realized but we do not know which state has been realized. For example, we do not ask whether the skin of dinosaur was colorful or not. Rather, we focus on uncertain situations where no possible states is yet realized, like whether it will rain tomorrow or not. For only the latter kind of uncertainty is relevant to deal with forecast which is apt to our discussion.

We often find ourselves in the situation where we have to make a decision under uncertainty. Jane decides to buy a lottery ticket under uncertainty whether she will win or not. Robert, a president of the outdoor equipment company, makes an investment decision under uncertainty whether there will be a severe cold winter this coming season or not. And we often evaluate such decision as rational or imprudent. A representative theory to explain a decision under uncertainty is the Subjective Expected Utility (SEU) Theory. Under SEU, a rational decision rule is to make a choice which returns the maximum of expected utility calculated by a utility function and a subjective probability distribution on the states. For instance, if we fix a utility function as a specific functional form, there corresponds a maximizer to each subjective distribution and so theoretically there can be many different optimal decisions as subjective distribution varies. However, SEU does not tell you which maximizer is the best. Under SEU, all the decisions are equally rational as long as they are made coherently.

However, there are many situations where we cannot evaluate a decision as rational even though it is made from maximized expected utility calculation. We often blame an investment decision as irrational when it is made under a rosy expectation. We often blame a gambling addicts as irrational, who expects excessively optimistic winning chance. It seems that, for any decision under uncertainty to be rational, it must be not just from a coherent rule. The decision also must be calculated under the basis of a reasonable subjective probability distribution. Let us call this reasonable credence the epistemically rational one. Then, under what criterion can we evaluate any subjective probability distribution as epistemically rational? Here we have to distinguish a practical rationality of a optimal action from an epistemic rationality of a reasonable belief. To obtain epistemic rationality, we have to isolate belief from action and then evaluate the beliefs separately. How can we do this?

In this section, we argue that a survival argument from the MSH provides one criterion to decide which belief is epistemically rational. The MSH shows that, in the evolutionary perspective, the partial belief which coincides with the objective probability, if any, beats over all the other partial beliefs in a sense that the objective partial belief brings the most success, i.e. survival, to the agent whose decision is based on this belief. In terms of survival, the objective probability is the best partial belief and so epistemically rational. Note that all the agents in the MSH are maximizing their expected utility under the same utility function. Therefore, they all share the same procedure to attain the practical rationality in the light of SEU theory, except that they have different beliefs. To this extent, the MSH isolates the effects of partial beliefs and selects the best one among them.

According to the survival argument, the objective probability is the credence which brings the most practically rational action. Since the objective probability seems to be some fact about our world, obtaining the objective probability may constitute knowledge. Then, this seems to enable us to extend the standard Reason-Knowledge principle to the knowledge on the objective probability. However, one concern with this approach is that it is controversial what the objective probability is. Moreover, in the previous section, I argue that the knowledge on the objective probability is not empirically possible. Then, this seems to imply that, while knowledge on the objective probability is not possible, we can rationally act the most on the credence which coincides with the objective probability, which implies why knowledge and rationality should split in this accuracy model.

Chapter 6

Conclusion

We have so far investigated the problem of confirmation on the MSH and constructed a system of formal proofs to show that the MSH is not directly confirmable. At this point, however, it deserves to ask why the problem of confirmation on economics model matters. In particular, noting that vast research has been already performed on how to evaluate the empirical significance of natural science models, we can ask what we contribute further by dealing with (i) the problem of confirmation and in particular (ii) the problem of confirmation on economics models. We answer these two inquiries in the following way: Regarding (i), we explain what distinguishes the problem of confirmation from the problem of empirical test or verification particularly in the Quine-Duhem Thesis. Regarding (ii), we explain what distinguishes the confirmation on economics models from the confirmation on natural science models.

First, the problem of confirmation is different from the problem of empirical test or verification in the Quine-Duhem Thesis. Roughly speaking, the Quine-Duhem Thesis is about the underdetermination of scientific hypothesis *as true* by empirical evidence, while the problem of confirmation is that it is impossible to determine how *probable* a hypothesis becomes by evidence. The former concept of determination as true does not allow degree, while the latter concept of confirmation does. In this respect, the probabilistic confirmation is more fundamental way of evaluating the empirical significance of a hypothesis. For even if we cannot determine a hypothesis as true definitely according to the Quine-Duhem Thesis, we can still meaningfully discuss whether it is more probable or not with evidence. It is widely accepted that at least some physics/chemistry/biology hypotheses are directly well confirmed even though they are underdetermined according to the Quine-Duhem thesis. Therefore, it is important to discuss the problem of confirmation separately.

Second, the confirmation on economics model is different from the confirmation on natural science model in the following respect. Let us first remind that the confirmation impossibility of economic hypothesis is the structural problem of the economics model, not just the epistemic problem of the econometrician. Note that the econometrician suffers from the epistemic limitation on knowing the true probability distribution, which leads us to the problem of confirmation impossibility. Arguably, the physicist may not suffer from such epistemic deficiency, circumventing it by performing some controlled experiments. However, even if we assume that the physicist also undergoes the same epistemic deficiency as the economist does, the physicist does not necessarily face the problem of confirmation impossibility. For example, even if the physicist does not know the true distributions on the potential positions of subatomic particles, she can still evaluate whether evidences, say from double-slit experiments, make Quantum Mechanics more probable or not. However, in the case of economics model, the epistemic problem inevitably leads us to the problem of confirmation impossibility due to the epistemic structure of economics models. Therefore, it is important to discuss the problem of confirmation on economics models separately from that of natural science models.

Milton Friedman argues that it is a strong indirect evidence of a good theory that we have been able to successfully apply the theory to specific problems but could not have refuted the implication of the theory with evidence for a long time. Let us consider the following passages from Friedman (1970):

> The evidence for a hypothesis always consists of its repeated failure to be contradicted, continues to accumulate so long as the hypothesis is used. $\cdots\cdots$ Yet the continued use and acceptance of the hypothesis over a long period, and the failure of any coherent, self-consistent alternative to be developed and be widely accepted, is strong indirect testimony to its worth[1].

However, as we have seen, it is impossible to confirm the MSH directly with data. It is not because the MSH is empirically correct in light of evidence, but rather because the MSH is not confirmable, that the Market Selection idea has been used without being contradicted by evidence for a long time. Pace Friedman, the repeated failure of contradicting the MSH cannot be regarded as a strong support for the MSH. The impossibility of confirmation already implies the repeated failure of contradicting, but confirmation impossibility of the MSH cannot be regarded as a strong indirect evidence for it.

Although we have discussed only rational traders with *exactly correct belief* so far, all of the results can be applied to a more general case of the agent with *more correct belief*, because the concept of more correct belief eventually depends on the concept of exactly correct belief. For in order to determine which belief is more correct we need to measure how closer any belief is to the given correct distribution, and so the knowledge on the true belief is necessary to measure the relative closeness.

[1] Friedman, M. 1970. *Essays in Positive Economics.* Chicago: University of Chicago Press, p. 14.

Lastly, we would like to briefly discuss an alternative way of confirming the MSH *indirectly* by computer simulation. Although the true distribution, if any, cannot be known to the agents in this actual world, an econometrician may circumvent this problem of ignorance by constructing a simulation world. In the simulated world, a true distribution is artificially generated and thus some agents are legitimately conferred with rationality true to that world. Then the econometrician may be able to confirm the MSH indirectly by checking whether the artificially designed rational agent survives in the simulated world. And this is exactly what Cogley, et. al. (2009, 2012a, 2012b and 2013) do.

Indeed, each simulated world, once designed so, is endowed with its own true distribution. Hence, an econometrician, who is the outside simulator, comes to hold a god's perspective to know the truth in each world. However, each generated distribution is true only to *its own world*, not necessarily to this actual world. Therefore, the simulation result can be significant as indirect confirmation only when the artificially designed true distribution makes sense to this actual world. We conjecture that the indirect confirmation is not possible either, for such significance is limited when arbitrary two simulated worlds are observationally equivalent to the econometrician in the actual world.

For instance, Cogley, et. al. artificially generated data sets in a simulated world and accordingly obtains a position to know what the true data generating process would be in their model. This then legitimately allow them to let some agents inside that world equipped with true belief. By doing simulation, the simulator outside the model obtains a God's perspective. The problem, however, is that there can be as many God's perspectives as the simulated worlds. One particular God's perspective belongs only to that simulated world. It does not necessarily make sense to this actual world. Therefore, to make their simulation results have some *real world* implication, Cogley, et. al. did the calibration with data from this actual world. However, such calibration does not guarantee to identify one particular probability distribution true to this actual world. Therefore, in case where there is observational equivalence between the two simulated worlds and so two different true beliefs are calibrated with the same data sets, the simulation result will return only a limited real-world implication.

Moreover, in principle, we cannot observationally distinguish between the belief and the preference which an agent possesses. For we can always find two observationally equivalent agents whose beliefs are absolutely continuous to each other and one of whose utility functions is state-dependent with Radon-Nikodym derivative while the other one is state-independent. Then, against any particular simulated world, we can always construct another simulated world where smart agents with absolutely continuous true belief and state-dependent preference behave in the observationally same way as those smart agents do in that particular world. Then, both agents will survive as rational ones in both simulated worlds, but at least one of the agents is irrational in terms of view from our real world. Therefore, just as in the observationally equivalent calibration case, the simulation result will return a limited real-world implication.

The discussion on the indirect confirmation by computer simulation can be extended further into the realm of the Artificial Intelligence. Suppose that a computer programmer designs a program to let a machine perform some intelligent tasks. A typical way of architecting such program is to design the machine to make an optimal decision while performing each task. Here, "optimality" means that the machine maximizes (minimizes) the expected gain (loss). However, in order to let the machine make such optimal decisions, the programmer must design the general decision rule by providing the *true* probability distributions and the *true* functional forms to measure the right gain/loss.

Here come the analogies between the econometrician and the computer programmer as outside agents who design the world, and between the rational agent and the machine as inside agents who make optimal decision in each world according as the outside agents direct. We can then argue that the machine cannot act as if it is intelligent unless it learns the optimal decision rules all by itself, which includes learning the true probability distribution and the true functional form for the gain/loss. We can treat many related issues in this framework, including the issue about who should be truly responsible for the consequences from the optimal decisions given that the programmer only designs a general rule on how to decide optimally, not directly dictating what to do in each specific case. We hope to discuss more about related issues in the future research.

References

Achinstein, P. (2000), "Why Philosophical Theories of Evidence Are (and Ought To Be) Ignored", *Philosophy of Science* 67, Supplement, pp. S180-S192.

————————-. (2001), *The Book of Evidence.* New York: Oxford University Press.

Arntznius, F. 2007. Rationality and Self-Confidence. in Tamar Szabo Gendler & John Hawthorne (eds.), *Oxford Studies in Epistemology: Volume 2*, Oxford: Oxford University Press.

Barbu, V. and Precupanu, T. 2012, *Convexity and Optimization in Banach Space.* New York: Springer.

Berger, O. J. 1985. *Statistical Decision Theory and Bayesian Analysis.* New York: Springer-Verlag.

Blume, L and Easley, D. 1993. Economic Natural Selection. *Economics Letters 42:* 281-289.

———————— 2006. If You're so Smart, Why Aren't You Rich? Belief Selection in Complete and Incomplete Markets. *Econometrica, vol. 74*: 929-966.

———————— 2008. Market Selection and Asset Pricing. In *the Handbook of Financial Markets: Dynamics and Evolution.*

———————— 2015 "Rationality." (ed.) Durlauf, S. and Blume, L. The New Palgrave Dictionary of Economics Online. Palgrave Macmillan.

Brier, G. 1950, “Verification of Forecasts Expressed in Terms of Probability”, *Monthly Weather Review*, 78(1): 1–3.

Carnap, R. 1955 "Statistical and Inductive Probability" In Antony Eagle (ed.), *Philosophy of Probability: Contemporary Readings.* Routledge

———— 1962 *Logical Foundations of Probability*, Chicago: The University of Chicago Press.

Casella, G. and Berger, R. 2002. *Statistical Inference,* CA: Duxbury.

Chow, G. 1980. Estimation of Rational Expectations Models. *Journal of Economic Dynamics and Control, Vol. 2.*

Cogley, T. and Sargent, T. 2008. The Market Price of Risk and the Equity Premium: A Legacy of the Great Depression, *Journal of Monetary Economics. Vol. 55*: 454-476.

————- 2009. Diverse Belief, Survival and the Market Price of Risk, *Economic Journal. vol. 119*: 354-376.

Cogley, T., Sargent, T. and Tsyrennikov, V. 2012. Market Prices of Risk with Diverse Beliefs, Learning and Catastrophes, *American Economic Review, Papers and Proceedings, vol. 102*: 141-46.

————- 2012b, Speculation And Wealth Dynamics When Investors Have Diverse Beliefs And Financial Markets Are Incomplete, unpublished manuscript: Cornell University and NYU.

————- 2013, Wealth Dynamics in an Bond Economy with Heterogeneous Beliefs, unpublished manuscript: Cornell University and NYU.

Richard M. Cyert and Morris H. DeGroot 1974, Rational Expectations and Bayesian Analysis, *Journal of Political Economy, Vol. 82, No. 3* (May - Jun., 1974), pp. 521-536

Dawid, A. P 1982, The Well-Calibrated Bayesian, *Journal of the American Sta-*

tistical Association, Vol. 77, No. 379.

———————- 1985, Calibration-Based Empirical Probability, *The Annals of Statistics, vol. 13 No. 4.*

———————- 1985, Self-Calibrating Priors Do Not Exist: Comment, *Journal of the American Statistical Association, Vol. 80, No. 390.*

DeLong, J. B., Shleifer, A., Summers, L. and Waldmann, R. 1990. Noise Trader Risk in Financial Markets, *Journal of Political Economy 98*: 703-38.

——————— 1991. The Survival of Noise Traders in Financial Markets, *Journal of Business 64:* 1-19.

Dretske, F. 1971. Reaons, Knowledge and Probability, *Philosophy of Science 38*: 216-220.

Durrett, R. 2010. *Probability : Theory and Examples.* Cambridge: Cambridge University Press.

Eagle, Antony. 2011. *Philosophy of Probability: Contemporary Readings.* New York: Routledge.

Elga, Adam. 2010. Subjective Probabilities should be sharp. *Philosopher's Imprint,* 10(5).

Epstein, E. 1962. "A Bayesian Approach to Decision Making in Applied Meteorology". *Journal of Applied Meteorology, vol. 1, Issue 2*: 169-177.

Epstein, E. and Murphy, A. 1967. "Verification of Probabilistic Predictions : A Brief Review," *Journal of Applied Meteorology 6*, p. 748-755.

Feller, W. 1968. *An Introduction to Probability Theory and its Applications.* New York: John Wiley & Sons.

Fitelson, B. and Hawthorne, J. 2010 How Bayesian Confirmation Theory Handles the Paradox of the Ravens in Eells and Fetzer (ed.) *The Place of Probability in Science, Boston Studies in the Philosophy of Science.* Springer.

Folland, G. 1999. *Real Analysis.* New York: John Wiley & Sons.

Fortnow, L. and R.V. Vohra (2008) The Complexity of Forecast Testing, *Econometrica, Vol. 77, Issue 1.*

Foster, D.P. and R.V. Vohra (1993) "Asymptotic Calibration," *Biometrika, 85-2*, 379-390.

Friedman, M. 1970. *Essays in Positive Economics.* Chicago: University of Chicago Press.

Gailmard, S. 2014. *Statistical Modeling and Inference for Social Science.* New York: Cambridge University Press.

Giere, N. 1971. "Objective Single-Case Probabilities and the Foundations of Statistics" In Antony Eagle (ed.), *Philosophy of Probability: Contemporary Readings.* Routledge.

Gibbard, A. 2007 "A Rational Credence and the Value of Truth" In Tamar Szabo Gendler & John Hawthorne (eds.), Oxford Studies in Epistemology. OUP Oxford.

Gilboa, I. 2010. *Theory of Decision under Uncertainty.* Cambridge: Cambridge University Press.

Gilboa, I., Postlewaite, A. and Schmeidler, D. 2012. Rationality of Belief or: Why Savage's Axioms Are Neither Necessary Nor Sufficient For Rationality, *Synthese, 187*: 11-31.

Gillies, D. 2000. *Philosophical Theories of Probability*, London: Routledge.

Goldman, A. 2009. "Williamson on Knowledge and Evidence" in D. Pritchard (ed), *Williamson on Knowledge,* Oxford University Press.

Good, I. J. 1952. Rational Decisions, *Journal of Royal Statistical Society 14*, 107-114.

——— 1962. Subjective Probability as the Measure of a Non-measurable Set. In *Logic, Methodology, and Philosophy of Science: Proc. Of the 1960 International*

Congress. Stanford: Stanford Univ. Press.

——————— 1965 *The Estimation of Probabilities: An Essay on Modern Bayesian Methods.* Cambridge: M.I.T Press.

Hajek, A. 2007, The Reference Class Problem is Your Problem Too, *Synthese, Vol.156(3)*, pp.563-585

————— 2008, Arguments for–or against–Probabilism? *British Journal for the Philosophy of Science, Vol. 59 (4)*: 793-819.

————— 2009, Fifteen Arguments Against Hypothetical Frequentism. *Erkenntnis* 70: 211-235.

————— A Puzzle about a Degree of Belief, *Unpublished Manuscript.*

Hansen, L. P. and Sargent, T. 2006. *Robustness.* Princeton: Princeton University Press.

Hawthorne, J and Stanley, J. 2006. "Knowledge and Action", *Journal of Philosophy.*

Hempel, C. G. 1962, "Deductive-Nomological vs. Statistical Explanation", *Minnesota Studies in the Philosophy of Science. V. III*, pp. 98-169.

————————- 1965, *Aspects of Scientific Explanation.* New York: Free Press.

Hoff, P. D. 2009. *A First Course in Bayesian Statistical Methods.* NY: Springer.

Hoover, K. and Young, W. 2013. Rational Expectations: Retrospect and Prospect. *Macroeconomic Dynamics, 17*: 1169-1192.

Howson and Urbach, 2005. *Scientific Reasoning: The Bayesian Approach,* Chicago : Open Court.

Jeffrey, R. 1977 "Mises Redux" In Antony Eagle (ed.), *Philosophy of Probability: Contemporary Readings.* Routledge.

—————- 1983. *The Logic of Decision.* Chicago: University of Chicago Press.

Joyce, James. 1998. A Nonpragmatic Vindication of Probabilism, *Philosophy of*

Science 65, pp. 575-603.

————————. 1999. *The Foundations of Causal Decision Theory.* Cambridge: Cambridge University Press.

————————. 2004. "Bayesianism" In Alfred R. Mele and Piers Rawlings, *The Oxford Handbook of Rationality*, Oxford: Oxford University Press.

————————. 2004. "The Development of Subjective Bayesianism", In Dov M. Gabbay, John Woods & Akihiro Kanamori (eds.), *Handbook of the History of Logic.* Elsevier.

————————. 2005. "How Probabilities Reflect Evidence" *Philosophical Perspectives, 19, Epistemology.*

Kahneman, D. and Tversky, A. 1982. Subjective Probability: A Judgement of Representativeness, in D. Kahneman, P. Slovic and A. Tversky (ed.) Judgment under uncertainty: Heuristics and biases, New York: Cambridge University Press.

Kogan, L, Ross, S, Wang, J and Westerfield, M. 2006. The price impact and survival of irrational traders The Journal of Finance, Vol. 61: 195-229.

———— 2009 Market Selection. *NBER Working Paper No.* 15189.

Kreps, D. 1988. *Notes on the Theory of Choice.* Colorado: Westview Press.

Kyburg, H. 1978. Subjective Probability: Criticisms, Reflections and Problems. *Journal of Philosophical Logic, pp. 157-180.*

Kydland and Prescott. 1977. Rules Rather than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy. vol. 85.*

Levi, Isaac. 1982. Ignorance, Probability and Rational Choice. *Synthese, Vol. 53*: 387-417.

————— 1985. Imprecision and Indeterminacy in Probability Judgment. *Philosophy of Science, 52:* 390-409.

Lewis, D. 1980. A subjectivist's guide to objective chance. In R. Jeffrey (ed.), *Studies in Inductive Logic and Probability, vol. 2.* Berkeley: University of California Press.

Ljungqvist, L. 2008 "Lucas Critique" *The New Palgrave Dictionary of Economics.* Second Edition. Eds. S. Durlauf. and L. Blume. Palgrave Macmillan.

Ljungqvist, L and Sargent, T. 2004. *Recursive Macroeconomic Theory*, Cambridge: MIT Press.

Lucas, R. 1972. Expectations and the Neutrality of Money. *Journal of Economic Theory 4*: 103-124.

———— 1970 "Econometric Policy Evaluation: A Critique." (1976). In Brunner, K.; Meltzer, A. *The Phillips Curve and Labor Markets. Carnegie-Rochester Conference Series on Public Policy 1.* New York: American Elsevier. pp. 19–46.

Maher, P. 1996 Subjective and Objective Confirmation, *Philosophy of Science Vol. 63, No. 2: 149-174*

————. 2006 The Concept of Inductive Probability, *Erkenntnis 65 (2)*:185-206.

Mellor, D. H. 2005. *Probability: A Philosophical Introduction*, New York: Routledge.

Miller, D. 1966. A Paradox of Information. *The British Journal for the Philosophy of Science, Vol. 17*, pp. 59-61.

Muth, J. 1961. Rational Expectations and the Theory of Price Movements. *Econometrica, Vol. 29, No. 3*: 315-335.

Oakes, D. 1985. Self-Calibrating Priors Do Not Exist. *Journal of the American Statistical Association, Vol. 80, No. 390.*

Olszewski, W. and Sandroni, A.2008. "Manipulability of Future-Independent Tests." *Econometrica Volume 77, Issue 1*

———————————————— 2009. "A Nonmanipulable Test." *Annals of Statistics. 37(2)*: 1013-1039.

———————————— 2009 "Strategic Manipulation of Empirical Tests." *Mathematics of Operations Research. 34*: 57-70.

———————————- 2011. "Falsifiability." *American Economic Review. 101(2)*: 788-818.

Piazzesi, M. 2007. Estimating Rational Expectations Model, *The New Palgrave Dictionary of Economics.*

Popper, K. 1959. "A Propensity Interpretation of Probability" In Antony Eagle (ed.), *Philosophy of Probability: Contemporary Readings.* Routledge.

Pritchard, Duncan, 2007. *Epistemic Luck.* New York : Oxford University Press.

Ramsey, F. P. 1931. Truth and Probability, in D. Mellor (ed.) *Philosophical Papers*, Cambridge: Cambridge University Press.

Roberts, H.V. 1968. "On the Meaning of the Probability of Rain," Paper presented to First National Conference on Statistical Meteorology.

Roush, S. 2010. *The Recalibrating Bayesian.* unpublished manuscript: King's College London.

Salmon, W. 1975. *The Foundations of Scientific Inference.* University of Pittsburgh Press.

———— 1975. "Confirmation and Relevance", *Minnesota Studies in the Philosophy of Science*, v. 6 pp.3-36.

———— 1984. *Scientific Explanation and the Causal Structure of the World.* Princeton: Princeton University Press.

———— 1989. Four Decades of Scientific Explanation. Pittsburgh: University of Pittsburgh Press.

———— 1990. "Causal Propensities: Statistical Causality vs. Aleatory Causality" *Topoi* 9, pp. 95-100.

Sargent, T. 1987. "Lucas's Critique". *Macroeconomic Theory* (Second ed.). Orlando: Academic Press. p. 397–98.

——— 2008. "Rational Expectations." *The New Palgrave Dictionary of Economics.* Second Edition. Eds. S. Durlauf. and L. Blume. Palgrave Macmillan.

Sargent, T. and Wallace, N. 1976. Rational Expectations and the Theory of Economic Policy. *Journal of Monetary Economics, Vol. 2.*

Savage, L. J. 1971 Elicitation of Personal Probabilities and Expectations, *Journal of the American Statistical Association, Vol. 66, No. 336.*

——— 1972. *The Foundations of Statistics*, New York: Dover Publications.

Schervish, M. 1985. Self-Calibrating Priors Do Not Exist: Comment, *Journal of the American Statistical Association, Vol. 80, No. 390.*

Sheffrin, S. 1996. *Rational Expectations.* Cambridge: Cambridge University Press.

Simon, H. 1959. Theories of Decision-Making in Economics. *American Economic Review, Vol. 49*: 223-283.

Skyrms, B. 1980. Higher Order Degrees of Belief, in D. Mellor (ed.) *"Prospects for Pragmatism" Essays in honor of F. P. Ramsey.*

——— 2000. *Choice and Chance: An Introduction to Inductive Logic.* Canada: Wadsworth/Thomson Learning.

Strevens, M. 2011. Probability out of Determinism, in Claus Beisbart & Stephan Hartmann (eds.), *Probabilities in Physics.* Oxford University Press. 339–364.

van Fraassen, 1985 Calibration: A Frequency Justification for Personal Probability, in R. Cohen & L. Laudan (eds.), *Physics, Philosophy, and Psychoanalysis.* D. Reidel

——— 2008. Rescher on Explanation and Prediction, in A. Robert (ed.) *Rescher Studies.*

von Mises, 1957. "The Definition of Probability" In Antony Eagle (ed.), *Philosophy of Probability: Contemporary Readings.* Routledge.

Walley, P. 1991. *Statistical Reasoning with Imprecise Probabilities.* Chapman and Hall.

Wallis, K. 1980. Econometric Implications of the Rational Expectations Hypothesis. *Econometrica, Vol. 48, No. 1.*

Whiteman, C.H. 1983. Linear Rational Expectations Models: A Users Guide. Minneapolis: University of Minnesota Press.

Williams, G. 1992. *Natural Selection: domains, levels, and challenges.* Oxford: Oxford University Press.

Williamson, T. 2000. *Knowledge and its Limits.* Oxford: Oxford University Press.

경제모형의 인식적 구조와 입증 문제

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본 논문에서 우리는 시장 선택 이론 (Market Selection Theory, MST)을 중심으로 경제이론의 입증 (confirmation) 문제에 관해 탐구한다. 입증 논리에 관한 헴펠 (Hempel 1943)의 초기 연구이래로 입증에 관한 대부분의 철학 논의들은 자연과학 이론에 집중되어 왔다. Carnap (1962), Earman & Salmon (1992), Maher(1996), Christensen (1997), Hajek & Joyce (2008), 그리고 Fitelson (2013)의 논의들이 자연과학을 암묵적으로 다룬 그러한 사례에 해당한다. 자연과학 이론은 외부 관찰자가 자신이 관찰하는 모형에 직접 개입하지 않는다는 특징을 가지고 있다. 본 논문에서 우리는 입증의 논의 영역을 경제 모형으로 확대하고자 한다. 우리는 경제 모형에 관한 입증은, 입증을 하려는 모형 외부의 경제학자가 자신이 입증하는 모형에 직접 개입하게 되는 독특한 인식적 구조를 가진다는 점을 보이고, 나아가 이러한 구조적 특징으로 인해 시장 선택 이론이 데이터에 의해 직접적으로 입증 가능하지 않다는 것을 보이고자 한다.

대략적으로 시장 선택 이론은 시장이 합리적 행위자를 선택하기 때문에 오직 합리적 행위자만이 시장 경쟁에서 살아남을 수 있다고 주장하는 이론이다. 이러한 시장 선택 이론은 시장 경제를 근본적으로 지지하는 강력한 직관을 담고 있기 때문에 이를 형식화하려는 시도 하에 다양한 모형이 개발되었다. 본 논문에서는 여러 가지 모형 중에서 Blume & Easley (2006)의 모형을 그 주요 사례로 살펴본다. 하지만 여기에서 한 가지 주의해야 할 점은, 입증 불가능성에 관한 우리의 논의가 특정의 시장 선택 모형에 의존하지 않는다는 점이다. Blume & Easley (2006)는 경제 행위자들의 의사 결정 모형에 대한 표준적인 가정 하에, 시장이 완전할 때 합리적 행위자들이 살아남는 반면 시장이 불완전하면 살아남지 못할 수도 있다는 것을 보였다. 하지만 이러한 표준적 가정을 완화할 경우 시장이 완전할 경우에라도 합리적 행위자뿐만 아니라 비합리적 행위자들도 같이 살아남는 것이 가능하다는 것을 보이는 모형들 또한 개발되었다. 우리의 입증 불가능성 논거는, 어떤 가설이 합리적 행위자만이 살아남는다는 것을 주장하든 그렇지 않다고 주장하든 개인의 합리성에 기반하여 시장의 합리성을 분석하려는 모든 가설에 적용된다.

시장 선택 이론이 직접 입증 가능하지 않다고 할 때, "합리성"과 "직접 입증 가능성"이라는 용어는 다음과 같이 정의된다.

첫째, 경제 행위자는 참인 믿음을 가질 경우에 합리적이라고 정의된다. 형식적으로, 믿음이 주관적 확률 분포들의 시퀀스(sequence)라고 기술될 때, 참인 믿음이란 참인 객관적 확률 분포와

일치하는 주관적 확률 분포들의 시퀀스로 정의된다. 따라서 합리적 행위자란 참인 분포와 일치하는 믿음을 가진 행위자로 정의된다.

현대 경제학은 개별 행위자의 선택 결정이 최적화 행위로부터 얻어진다는 의미에서 합리적이라는 가설에 기초하여 성립된다. 그러한 최적화 행위를 형식화하는 표준적 모델은 기대 효용 극대화 (Expected Utility Maximization) 모형이다. 하지만 직관적으로 행위자의 주관적 예상이 시장에 대한 참인 확률 분포에 기반하지 않는 한 자기 자신의 예상을 극대화하는 것이 항상 최적 행위라고는 할 수 없다. 왜냐하면 그러한 주관적 예상이 참인 확률분포에서 나온 것이 아니라면 극대화 행위가 시장의 관점에서 보았을 때 정말로 최적화되었다고 보기 어렵기 때문이다. 이는 단지 개인적인 관점에서 보았을 때 최적화되었다고 믿어지는 행위일 뿐이다. 따라서 진정으로 합리적이기 위해서는 행위자는 참인 확률분포를 가져야만 한다.

참고로 합리성에 대한 이러한 정의는 표준적인 이해와는 다르다. 표준적인 견해는 합리적 행위자는 자신에게 최선이라고 믿는 이해 관계에 따라 행동해야 한다고 요구할 뿐이다. 하지만 최선이라고 믿은 것이 반드시 실제로 최선일 필요는 없기 때문에, 표준적인 의미에서의 합리성 개념은 참인 확률과는 무관하다. 이와는 대조적으로, 합리적 기대 가설은 (Rational Expectation Hypothesis) 합리적 행위자의 인식이 참인 믿음에 대한 객관적 기준을 만족시킨다고 주장한다. 시장 선택 이론은 객관적 의미에서의 합리성 개념을 정당화하는 모형을 구성하려는 시도에서 제안되었다.

둘째, 직접 입증 가능성이란 이론이 경험적 데이터로부터 귀납적으로 지지 가능하다는 것으로 정의된다. 베이지안 (Bayesian) 모형에서 그러한 귀납적 지지란 데이터에 의해 알려진 증거로부터 형성된 행위자의 주관적 믿음 정도로 측정된다고 가정된다. 만일 행위자의 믿음 체계가 일관적이라면 그러한 믿음 체계는 확률 법칙을 만족한다. 따라서 베이지안 체계에서 그러한 귀납적 지지는 형식적으로 증거를 조건화 하여 얻어진 주관적 조건부 확률로 (subjective conditional probability) 기술된다. 가설은 데이터에 의해 알려진 증거를 조건부로 하는 증가적 확률이 (incremental probability) 충분히 높을 경우 입증된다고 한다.

본 논문에서 우리는 비록 이론적 모형을 구성하는 것이 가능하다고 하더라도 그러한 시장 선택 이론을 데이터를 가지고 직접 입증하는 것은 가능하지 않다고 주장한다. 프리드먼이 (Friedman 1970) 지적하듯이, 경제학의 두 가지 주요 과제는 첫째, 기존의 경제 현상들을 설명하고 올바른 예측을 하는 이론적 모형을 구성한 후, 둘째 그러한 이론의 예측이 현실과 얼마나 일치하는지 그 범위와 정확성의 관점에서 실천적인 평가를 내리는 데에 있다. 그런데 현실과의 적합성 여부의 관점에서 경제 모형의 실천적 함축을 평가하는 다양한 방법이 있는데, 그 가운데 입증은 그러한 적합성을 평가하는 가장 근본적인 방식에 속한다. 우리는 시장 선택이론의 인식적 구조가 입증의 증가 확률을 평가할 수 있도록 허용하지 않기 때문에 직접적으로 입증 가능하지 않다고 주장한다. 시장 선택 이론의 입증 문제를 다루면서, 우리는

객관적 의미에서의 합리성 개념이 경제 모형에서 갖는 실천적 함의에 대해 새롭게 주목하고자 한다.

이러한 목표 하에 본 논문은 서론과 결론을 포함한 6개의 장으로 구성된다. 2장에서 우리는 입증 개념이 성립하기 위한 세 가지 필수요건에 대해 논의한다. 우리는 어떤 가설이 증거에 대해 갖는 조건부 확률이 (i) 증가적 요건과 (ii) 절대적 요건을 만족시키고 이에 더해 증거가 (iii) 인식적 요건을 만족시킬 때 우리는 그러한 가설이 증거에 의해 입증된다고 본다. 또한, 우리는 입증의 조건부 확률이 갖는 두 가지 특징에 관해 논의한다: 첫째, 우리는 베이지안 입증 모형이 경제 이론의 입증이 갖는 인식적 구조를 보다 잘 드러내기 때문에 입증의 조건부 확률에 대한 적절한 해석은 주관적 확률 해석이라고 주장한다. 둘째, 우리는 조건부 확률이 의미하는 "조건"에 적절한 양태는 직설법적 조건 (Indicative Conditional)이라고 주장한다. 3장에서 우리는 합리적 기대 가설과 이로부터 파생된 두 가지 논제인 루카스 비판 (Lucas Critique)과 동태적 비일관성 (Time Inconsistency) 논제를 검토한다. 이 장에서 우리는 경제 모형이 갖는 두 가지 특징에 기반해서 이들 논제의 인식적 구조에 관해 분석한다. 그리고 나서 우리는 시장 선택 이론이 합리적 기대 가설에 주는 이론적 함축에 관해 다룬다. 4장에서 우리는 시장 선택 이론의 입증이 갖는 2층적 인식 구조에 관해 탐구한다. 우리는 (i) 참인 확률 분포를 알 수 없고 (ii) 시장 선택 이론의 입증이 2층적 인식적 구조를 갖는 경우, 시장 선택 이론을 직접적으로 입증하는 것은 가능하지 않다고 주장한다. 우리는 이를 보이기 위해 형식적인 베이지안 모형을 구성한다. 5장에서 우리는 입증 불가능성이 주는 건설적인 함축에 관해 간략히 논한다. 1장은 서론이고 6장은 결론이다.

2장에서 우리는 입증 모형이 가져야 하는 세 가지 요건에 관해 논한다. 어떤 이론 혹은 가설은 증거(Evidence)에 의해 지지될 때 입증된다고 한다. 증거로부터 받는 그러한 지지의 정도는 조건부 확률이라는 수적인 (numerical) 표현을 갖는다고 가정된다. 여기에서 입증과 증거라는 개념은 서로 밀접히 연관되어 있다. 즉, 입증은 E가 H의 증거가 될 때 E와 H 사이에 성립하는 관계이다. 또한 E가 H의 증거가 되려면 E는 H를 입증해야 한다. 일반적으로 철학자들 사이에서는 다음의 두 가지 도식이 입증 관계의 특성을 나타내는 것으로 받아들여 진다:

(i) 증가적 관계 : E는 H에 대한 증거적 지지를 증가시킨다.

(ii) 절대적 관계 : H는 주어진 증거 E에 의해 높은 정도로 지지된다.

이러한 두 가지 도식에 드러난 직관을 반영해서 우리는 입증에 관한 다음의 세 가지 형식적 조건을 제안한다.

(i) 증가적 조건 : E가 주어졌을 때 H의 조건부 확률은 상대적으로 증가해야 한다.

(ii) 절대적 조건 : E가 주어졌을 때 H의 조건부 확률은 일정한 한계점을 넘어야 한다.

(iii) 인식적 조건 : 증거 E는 경험에 의해 참임이 알려져야 한다.

이러한 세 가지 조건에 관해 논의하기 전에 우선 최소한 두 가지 중요한 논의가 선행되어야

한다. 첫째, 증거적 지지의 정도를 측정한다고 가정되는 조건부 확률은 어떤 종류의 확률인가? 특별히 이러한 확률은 주관적인가 아니면 객관적인가? 둘째, 입증과 관련해서 조건부 확률이 중요하다고 할 때 증거에 "조건"한다는 것은 정확히 무엇을 의미하는가?

첫 번째 질문과 관련해 우리가 고려하는 두 가지 대표적 이론은 카르납의 입증론과 베이즈주의 입증론이다. 카르납 이론은 증거로부터의 귀납적 지지는 객관적인 것이며 따라서 객관적 확률로 측정 가능하다는 이론인 반면, 베이즈주의 이론은 증거가 주는 귀납적 지지는 그러한 지지를 측정하는 행위자에 의존하는 주관적인 것이라는 주장이다. 객관적 조건부 확률은 인식적 확률이라는 이름으로 불리기도 하고 논리적 혹은 귀납적 확률로 불리기도 한다. 주관적 조건부 확률은 사후확률로 불리기도 한다. 우리는 베이즈주의 입증 모형이 경제 모형에 관한 입증의 인식적 구조를 잘 드러내는데 특히 적합하기 때문에 카르납 모형보다는 주관적 모형을 선택한다.

둘째, 우리가 베이즈 입증 모형을 취한다는 전제 하에, 다음 질문은 주관적 조건부 확률에 반영되는 증거적 관계란 어떤 관계인가 하는 물음이다. 특히 우리가 E가 주어졌을 때 H의 확률이라고 할 때, "주어진"이라는 것은 무엇을 의미하는가? 이러한 질문에 답하기 위해 본 논문에서 중요하게 고려하는 두 가지 가정 양태는 직설법적 가정과 반사실적 가정이다. 우리는 입증에서 E가 행위자의 믿음 체계에 잠정적으로 추가되었다고 할 때 적절한 가정 양태는 직설법적 가정이라고 주장한다. 우리는 또한 이러한 직설법적 양태는 입증에 대한 세 번째 인식적 조건과 밀접히 연관되었다고 주장한다.

3장에서 우리는 경제 이론이 자연과학의 이론과는 달리 (i) 외부 관찰자가 또한 내부 행위자이기도 하다는 특성과 (ii) 경제 모형 안의 행위자가 갖는 주관적 믿음이 경제모형 그 자체에 영향을 끼친다는 두 가지 인식적 특성을 갖는다고 제안하고, 이러한 인식적 특성에 기반하여 루카스 비판과 동태적 시간 비일관성 논제를 분석한다. 대략적으로 루카스 비판이란 경제 정책이 변하면 경제 행위자의 믿음도 변하므로 경제의 균형 경로도 변하기 때문에 정책 결과들 간의 비교가 어렵다는 비판이다. 동태적 시간 비일관성은, 어떤 시점에 최선이라고 선택된 정책이 그 정책을 실제로 실행하는 시점에 가면 더 이상 최적이 아닌 상황에서 발생한다. 이러한 루카스 논제와 동태적 시간 비일관성 논제가 옳은지 혹은 어떠한 경제학적 의의를 가지는지를 논하는 것은 본 논문에서 다루고자 하는 바가 아니다. 다만 우리는 경제 모형의 인식적 구조를 논하면서 이러한 루카스 비판과 동태적 비일관성 논제에는 인식적 구조에 대한 기본적인 가정이 기초하고 있다는 점을 지적하고자 한다. 가령, 루카스 비판의 경우 경제 정책이 변하면 그러한 정책적 변화를 인지하는 경제학자가 있고 이러한 인지로 인한 경제학자의 변화된 믿음은 행위자 공통 구조와 자기 지시적 구조에 의해 균형 경로의 변화로 이어진다. 그리고 나서 우리는 시장 선택 모형의 한 가지 사례로 Blume & Easley의 모형을 검토한다. 본 논문에서 우리는 합리성 개념의 객관적 의미에 주목하기 때문에 이를 주요 내용으로 삼고 있는 합리적 기대 가설을 살펴

보고, 합리적 기대 가설이 어떻게 가능한지를 정당화해주는 한 가지 모형으로 제안된 Blume & Easley의 이론에 대해 살펴 본다.

4장에서 우리는 시장 선택 이론의 입증 불가능성에 관해 구체적으로 논의한다. 우리의 논증은 다음과 같다. 첫째, 우리는 시장 선택 이론의 입증은 경제 모형의 입증이 갖는 독특한 인식적 구조를 갖는다고 주장한다. 즉, 경제 모형의 입증에서 외부 관찰자인 경제학자는 증거의 귀납적 지지 정도를 평가하는 역할을 하는데, 입증 모형이 가지는 인식적 구조로 인해 입증 과정에서 증거로부터 얻은 경제학자의 믿음은 입증 대상이 되는 경제 모형 자체에 영향을 끼치는 것이 가능하게 된다. 즉, 경제학자가 증거로부터 얻은 믿음은 외부자-내부자 공통 구조를 통해 내부 행위자에로 전이되는 경우가 생기고, 그러한 믿음이 모형의 내생 변수에 영향을 끼치는 한 다시 경제 모형의 자기 지시적 구조를 통해 경제 모형 자체에 영향을 끼치게 되는 것이다.

둘째 우리는 베이즈주의 (Bayesian) 입증 모형 하에서는 모형 외부의 높은 차원의 믿음과 모형 내부의 낮은 차원의 믿음이라는 중층적 (two-layered) 믿음이 관련된다고 주장한다. 이에 더해 우리는 입증 과정을 겪으면서 이러한 중층적 믿음은 서로 밀접하게 연관된다고 주장한다. 베이즈주의를 통해 우리는 경제 모형의 입증이 가지는 중층적 인식 구조, 즉 경제 모형을 입증하는 과정에서 모형 외부의 경제학자가 가지는 높은 차원의 주관적 믿음과, 입증의 대상이 되는 모형의 내부에서 경제 행위자가 가지는 낮은 차원의 주관적 믿음이 밀접하게 연관되는 그러한 구조를 드러내고자 한다. 이점과 관련해 시장 선택 이론의 입증에서 핵심적인 사항은 이러한 모형 내부와 외부의 중층적인 주관적 믿음들이 입증을 거치며 서로 밀접하게 연관된다는 점이다.

셋째, 우리는 경제 행위자들이 자신의 경제 환경의 참인 특성을 아는데 있어 인식적 제한에 직면한다고 주장한다. 물리학이나 화학 혹은 경제학과 같은 소위 경험과학에서 과학자들은 관련된 현상을 설명하고 예측하는 이론적 모형을 구성할 뿐 아니라 그러한 이론적 모형이 현실과 일치하는지를 경험적 입증을 통해 평가한다. 하지만 우리가 시장 선택 모형의 현실 일치성을 평가할 때 현실이란, 그러한 일치성을 평가하는 경제학자 자신이 경험한 현실이라는 점에 주목해야 한다. 따라서 입증 모형이 갖는 중층적 인식 구조 때문에 만일 경제학자가 경제 환경의 참인 특성을 이해하는데 어떤 경험적 한계를 겪는다면 경제 행위자 역시 같은 종류의 경험적 한계를 겪을 수 있고 이는 다시 현실과의 일치성을 평가 받고 있는 경제 모델 자체에 영향을 끼칠 수 있다. 통계 이론 가운데 빈도 정리와 캘리브레이션 정리에 근거해서 우리는 경제학자가 그러한 인식적 부족을 겪는다고 주장한다: 참인 분포와 같은 것이 존재한다는 전제 하에 경제학자는 그러한 참인 분포를 알 수 없다. 따라서 이러한 통계 이론들은 경제 행위자의 인식적 결함으로 이어지며, 이는 다시 입증의 평가 대상인 경제 모형에 영향을 끼칠 수 있게 된다.

마지막으로, 이러한 세 가지 점을 결합해서 우리는 다음과 같은 이유로 시장 선택 이론이 경험적으로 입증 가능하지 않다고 결론 내린다. 세 번째 주장으로부터 우리는 경제학자는 참인

분포를 알 수 없다고 주장한다. 그러면 첫 번째와 두 번째 주장으로부터, 우리는 만일 경제학자가 참인 분포를 알지 못한다면 이러한 인식적 결여는 적어도 베이즈주의 최선의 입증 모형 하에서는 입증되는 모형에 즉각 반영된다고 주장한다. 여기서 최선의 입증 모형이란 최선의 증거에 의한 모형을 말하며, 시장 선택 모형에서 최선의 증거는 참인 분포와 가장 가까운 믿음으로부터 얻어진다. 본 논문에서 우리는 참인 분포가 존재한다면 그러한 참인 분포에서 가장 가까운 믿음이 존재한다는 것을 보이고 이로부터 최선의 입증 모형이 잘 정의된 모형임을 보인다. 이러한 최선의 입증 모형 하에서 경제학자의 인식적 결함이 모형에 즉각 반영되는 이유는, 경제학자와 경제 행위자가 같은 세계 안에 공존하기 때문이다. 즉, 최선의 모형 하에서 경제학자는 참에 가장 가까운 믿음을 증거로 삼는데, 이 때 경제학자에게 가장 정확한 믿음은 경제 행위자에게도 가장 정확한 믿음이 된다. 따라서 최선의 입증 모형 하에서, 입증 대상이 되는 경제 모형 안의 합리적 행위자는 기껏해야 이러한 가장 정확한 믿음을 가진 행위자일 수밖에 없는 것이다. 물론 이러한 가장 정확한 믿음이 참인 분포 그 자체일 수도 있다. 하지만 통계적 정리가 함축하는 바, 그러한 가장 정확한 믿음이 참인 분포인지 여부는 알려지지 않기 때문에, 가장 정확한 믿음을 합리적 믿음으로 간주하는 최선의 입증 모형은 인식적 제한이 부과된 모형이 된다.

그런데 입증을 수행하면서 데이터로부터 얻은 증거를 업데이트하는 과정에서 경제학자는 일반적으로 데이터로부터는 참인 분포를 알지 못한다는 것을 자신이 안다는 점도 증거 안에 포함시킨다. 그리고 경제학자는 가장 정확한 믿음을 갖는 행위자 역시 이러한 인식적 결함에 직면한다는 것을 알고 있기 때문에 최선의 입증 모형은 시장 선택 이론에 관한 참인 입증 모형하고 다르다고 믿는다. 그리고 경제학자는 이렇게 다른 두 모형이 그럼에도 불구하고 항상 같은 입증 결과를 가져올 수 있는지 확신할 수 없기 때문에 결과적으로 최선의 입증 모형을 가지고 참인 입증을 수행할 수 없게 된다.

이상에서 우리는 최선의 입증 모형 하에서 시장 선택 이론을 입증할 수 없음을 보였다. 그런데 어떤 가설이 최선의 증거로도 입증이 가능하지 않다는 것을 보였기 때문에 이 가설은 그 밖의 어떤 증거로도 입증이 가능하지 않다고 결론을 내린다. 이러한 논증에서 최선의 모형이 가지는 인식적 구조는 논증의 핵심 역할을 하기 때문에, 우리는 입증 불가능성 문제가 경제학자 개인의 문제가 아니라 경제 이론의 입증이 갖는 구조적인 문제라는 점을 강조한다.

주요어 (keywords) : 입증, 경제 모형, 인식적 구조, 베이즈주의, 합리성, 시장선택
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