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Essays on Climate Change Computable General Equilibrium Models

기후변화 연산가능일반균형 모형에 관한 연구

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Abstract

Essays on Climate Change Computable General Equilibrium Models

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This research reviews the problems of conventional computable general equilibrium (CGE) models which are widely used for climate change policy analysis. To solve the problems, it proposes multivariate distribution approach as an alternative way of representing the production activities in model structures and assesses the possibility of its practical employment.

In the first part of this research, the basic characteristics of three well known global CGE models are reviewed and production function structures are pointed out as the main sources of the differences in carbon emission projections among models. Two experiments are introduced regarding the effects of changes in production function structures. In one experiment, the nested structure of constant elasticity substitution (CES) functions is substituting with alternative nesting structures. In another experiment, fixed input structures are partly applied for incorporating bottom-up approach with top-down mechanism of CGE models. The results show that these structural

changes cause a considerable impact on the prediction results of greenhouse gas emissions and carbon prices. Also, the experiments are extended to the comparison of GDP losses among different model structures. Simulations for the case of Korea reveal that the estimations of GDP loss differs among model structures, raising some issues on applying them into practical policy making.

In the second part, the performance of a global CGE model is analyzed in marginal abatement cost estimation when data disaggregation is applied. Extraordinary carbon prices are reported for the case of relatively large share of capital in the economies of a few developing countries. Empirical evidence indicates that the abnormal phenomenon is accounted for by the proportional relationship between capital intensity and carbon price. The analysis is extended to CES functions with a numerical analysis, concluding that the unusual phenomena may be connected to distribution parameters of CES functional forms which are most widely used in CGE models.

In the last part, multivariate distribution approach is applied for an alternative description of energy related production activities. Applying theories on the microfoundations of aggregate production functions, it is shown that a set of bottom-up microscopic information can converge to specific aggregate production functions if assumptions are imposed on the statistical distribution of local production technologies. The actual characteristics of statistical distributions were reviewed for a real dataset of energy intensive manufacturing sector of Korea. To facilitate simulations and conveniently reproduce the relationships embedded in multivariate joint distribution maps, a statistical tool called copulas is introduced in advance. After

the basic theory of copulas is briefly introduced, the performance of a copula model is investigated, revealing that a copula model is successful in describing heterogeneous microscopic information. After the introduction of copulas, a new type of CGE model is applied, in which an aggregation of local Leontief production functions takes over the role of conventional global production functions. A pilot model is composed to apply this scheme to a CGE model and it is shown that this new approach has some advantages: it eliminates the effect of the past time data and improves the precision of projection results.

Keywords : computable general equilibrium (CGE) model, structural uncertainty, constant elasticity of substitution (CES) production function, greenhouse gas emission projection, input factor distribution, copula
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Chapter 1

Overall introduction

1.1 Motivations

For decades, the growing concerns over global climate change have urged the necessity for policy schemes to curb greenhouse gas emissions. Consequently, there has been a rapid increase of the demand for quantitative analysis on the effects of those climate change policies. Usually, the analysis contains projections of future greenhouse gas emissions which are compared among various policy instruments. To examine the effects of individual policies, researchers often use complicated economic simulation models. This is because the issue of greenhouse gas is related to every part of the economy in terms of the technologies of energy use and, consequently, it is essential to understand the interactions between each economic sector. This is why many economic models are assisted by the theoretical knowledge or discoveries of economics. Among various model types, computable general equilibrium (CGE) models are one of the most preferred economic models, which are based on the well known fundamental microeconomic theory of general equilibrium.

In economic models, the activities of economic agents are often described as a collection of optimization problems. Production activities are also expressed by the frameworks of optimization of production functions under some constraints. The types of such equations are usually proposed by the scientific achievements of economics, to which some recent work began to add detailed technical information – in part or as a whole – to improve performances. However, uncertainty is inevitably immanent in almost all economic models because the employed equations are mere approximations of the real economy. This issue especially applies to the case of greenhouse gas emissions because the largest share of greenhouse gas emissions is deeply related to production activities with energy use. Therefore, it is natural to give priority to the issues on the performances of production function structure in a model for a reasonable analysis of the effectiveness of greenhouse gas emissions policies.

As a matter of fact, there exist long standing debates on the relationships between energy and the other input factors as well as the position of energy in a production function structure. As for the former case, some researches show that energy is a substitute for capital, but other studies give the contrary conclusion that energy and capital are complements with each other and the substitution elasticity is zero. Concerning the latter issue, there exists a huge difference in the function structure among models. In some models whose production functions have a nesting structure, capital is first bundled with labor and then energy is rebundled with this bundle. But other models imply that capital is directly related – whether substitute or complement – to energy, and then this bundle is rebundled with labor. These observations imply that, when energy is considered as an input factor in a production function, the uncertainty may be increased compared to the other non-energy production function cases.

For these reasons, this research focuses on the energy related production functions in CGE models which are the most widely used for climate change policy analysis. The first two chapters are for providing empirical evidences on the failures of using conventional production function frames when energy input factors are included: One is to measure how the overall projection results are sensitive to structural changes in production functions, and the other is to illustrate how a set of production functions in a CGE model can be affected by reference datasets, causing systematic errors in carbon price projections. To overcome the shortcomings of conventional function structures, the last chapter proposes joint distribution mapping as an alternative approach to ensure there is no loss of microscopic information as well as to minimize the possibility of distortions in projection results. With a brief introduction of theoretical backgrounds, this new method is applied in a simple demonstration.

1.2 Overview and outline

In Chapter 2, two experiments are performed by substituting a component of production function structure in a model with a counterpart component in other models. If a component is completely compatible with another, then the projection outcomes will be similar in both original and variant models. However, if the outcomes are not reproduced by such component switching, it means that a set of input factor bundles possibly employed by a production function component for achieving a production level cannot be overlapped with the counterpart's production possibility set. Though there are various types of production function structures in CGE models regardless of whether energy input factor is considered, this study is limited to only two cases of structural change. One is switching the energy-capital bundle structure in nested constant elasticity of substitution (CES) production functions and the other is the case of partly incorporating microscopic technical information with conventional CGE model structures by applying a number of fixed input structures.

Chapter 3 extends the structural analysis to the case of data disaggregation. Originally, the individual projection outcomes were viewed by region or sector. However, marginal abatement cost estimations by regional disaggregation in a CGE model gave unexpected results. A few developing countries exhibited a relatively high level of carbon prices, against the commonly held belief to the contrary. This is accounted for by the shortcomings of some parameters in CES production functions, which are calibrated by datasets of reference or benchmark years. Specifically, it is argued that this problem appears when the share of capital in contributions to gross production is relatively high. Chapter 3 tries to explain this issue by introducing the concept of capital intensity. After empirically deriving a connection between capital intensity and carbon price and analyzing a simple mathematical model, the analysis is extended to the CES functional form. Through it, it is proven that the unusual phenomena may have some connections to CES functions which are the most commonly used functional form in CGE models.

In Chapter 4, the concept of joint distribution mapping is proposed as an alternative approach to replace conventional function structures including CES functions. Basically, the adoption of CES functions in a CGE model requires the unrealistic assumption of a homogeneous representative agent. This is why there arises an ambiguity in establishing the relationships between economic variables as can be seen in Chapter 2. Also, the conventional calibration approaches with CES functions are vulnerable to the selection of benchmark datasets as in Chapter 3. Therefore, this research proposes a distribution approach to ensure heterogeneous stochastic properties and to minimize unfavorable vulnerability to data selection.

At first glance, joint distribution mapping seems to have no connection with production functions. However, Houthakker (1955-1956) and Jones (2005) reveal that conventional aggregate production functions can be derived under some assumptions on microscopic distributions of technical information. In this sense, if the distribution of technological information is established with assistance of the availability of abundant data, the role of production functions can be taken over by such distributions themselves. This may have great implications in composing economic models. Generally, models are classified into two categories – bottom-up and top-down. Bottom-up models only focus on microscopic technological details without consideration of interactions between economic agents. On the contrary, top-down models, including CGE models, aim to find macroeconomic equilibrium solutions with a set of economic functions and equations. In reality, there is a gap between these two modeling approaches because it has been almost impossible to understand one approach in the frame of the other. Therefore, the new distribution approach introduced in this research can help build a bridge across the divided modeling field.

Chapter 4 starts with discussions on the microfoundations of production functions, by which the succeeding replacement of conventional nested CES production structure is enabled. The replacement is done by switching the CES function structure with a joint distribution map by introducing a set of firms. Each firm has fixed input structure and the set of firms collectively converges to an 'implicit' aggregate production function. To reproduce the statistical map of the real economy for actual simulations, the concept of copulas is introduced as a convenient statistical tool to deal with the distribution map. From an estimated copula model, the sets of firms are generated to compose replicate joint distribution maps. In this way, asymptotically converged production functions can contribute to more realistic descriptions of the economy.

1.3 Contributions

The contributions of this research are summarized as follows. The first contribution is the introduction of a more systematic approach to comparative studies on the issue of structural uncertainty in CGE models in Chapter 2. The structural difference is investigated under more strictly controlled conditions compared to the previous literature of meta analysis. Also, time factor is also considered to assess the performances of a specific model structure. Most comparative studies deal with only static performances of a specific model structure. However, even if two different structures show similar level of static performances, the descriptions of the economy may change when inter-temporal dynamic process is considered during a long time period. Therefore, this research tries to view the whole dynamics of structural changes.

The second contribution of this study is providing empirical evidences of carbon price distortions on a regional basis with global CGE models in Chapter 3. Although researchers are aware of the limitations associated with parameters calibrated to a benchmark year, there are few previous studies on the degree to which the projection results can be distorted. Most literature on the robustness of economic models tacitly ignore the possibility that model structure itself may not be suitable for unusual situations. In this sense, this research tries to understand and elaborate on the structural limitations of global CGE models.

The third contribution is the efforts to find and understand the connections between bottom-up and top-down models in Chapter 4. It is shown that a bottom-up production function structure can be considered as a top-down aggregate production technology if the concept of distribution is introduced. This is an important concept to various fields related to economic models. The understandings of the theoretical connections between bottom-up and top-down can be promoted and, consequently, CGE model projection results may be utilized more broadly for practical policy making processes. The introduction of copulas as a convenient tool for preserving distribution information can be thought to be another contribution of this work. With the assistance of this statistical instrument, a new concept of CGE model is also introduced and its performance is assessed in this research.

Chapter 2

Structural differences between global climate change CGE models

2.1 Introduction

An energy-environmental economic model is a computer simulation program to describe and mimic the real world from an economic viewpoint. Recently, many global economic models have been introduced especially in the context of increasing concerns over global climate change. These simulation models are widely used by many governments, international organizations, and research institutions for long term projections of green house gas emissions and carbon prices in the emission trading market. There are various types of economic models due to the difference of opinion between research groups, and they are generally classified into the two main categories of 'top-down' or 'bottom-up', although some models contain elements of both approaches and are referred to as 'hybrid' models.

Top-down models usually view the economy as an integrated whole, reaching economic equilibrium under an environmental constraint through substituting capital, energy, labor, and so on. Top-down models tend to focus on economic processes rather than technology detail or specific market products. There are many sub-categories of top-down models and in computable general equilibrium (CGE) models, one of the sub-categories, demand and supply of each commodity or factor across a specified set of markets in the economy are balanced through a price mechanism.

Bottom-up models are suitable to answer questions about specific low-carbon technology deployment, whereas top-down models are more appropriate for answering questions about economic impacts of carbon policies. Bottom-up models typically assess distinct mitigation technologies or practices, specifically, their costs and emission reduction capabilities, as well as their substitutability with other technologies. A combination of mitigation technologies is then used to meet energy demands under environmental constraints. Bottom-up models tend to focus on the interactions within the energy system,[1] rather than its relationship with the overall economy. Hybrid models combine elements of both bottom-up technology details, which usually focus on the electricity sector, and top-down economic integration.[2]

There have been studies on whether there is a significant difference between these approaches and what the main sources of the disagreement are. However, the qualitative arguments about these issues usually draw very diverse conclusions, even conflicting with each other. For example, the conclusion of van Vuuren et al. (2009) and the evidences of OECD (2009) are contrary to each other.

van Vuuren et al. (2009) presents a comparative study by comparing the reduction potential relative to baseline, i.e., business as usual (BAU) scenario among models. It carries out comparative studies between eight

[1] For more information about the calculation method in bottom-up apporach, refer to EPA (2006) and ICF International (2008).

[2] For further information on hybrid models, refer to Hourcade et al. (2006).

different models for two cases. One is for energy-related CO_2 emissions only and the other is for total greenhouse gases. For the CO_2 only case, emission reductions differ according to the categories each model belongs to. However, for total greenhouse gas emissions case, the results from all models are comparable regardless of the adopted approach. In other words, as the paper insists, there is no systematic difference between top-down and bottom-up approaches.[3] The paper says the conclusion is consistent with the findings of IPCC (2007), comparing its outcomes with a regression result from the comparative study of the assessment report of IPCC (2007).

On the contrary, the data in OECD (2009) hint that the above argument is not valid when time is taken into account. The report reveals the projection results of mitigation potential for the US from 2020 to 2050 in 10-year intervals. In year 2020, all graphs are tangled at similar locations. However, in the subsequent years, there arises a divergence between two groups. One of them mainly consists of top-down models and the other is composed of hybrid models which can be thought to have the characteristics of bottom-up models. Among models employed, top-down models such as ADAGE, SGM, EPPA and ENV-Linkages, and one hybrid model, GTEM, commonly use the same dataset, GTAP, for the base year reference, but only GTEM shows a large shift to the right side, confirming a separation between different model types.[4]

[3]However, on a sector basis, the difference is apparent, especially in the energy sector. In the energy sector, the reduction potential of top-down models is higher than that of bottom-up models. This is mainly due to technology details which is substantial compared to other sectors.

[4]There is the possibility that the baseline is exaggerated. If then, the mitigation effect will be overestimated and the graph will tend to move to the right. Based on OECD (2009), the baseline estimates in EPPA are larger than in other CGE models, but the location of

Therefore, contrary to van Vuuren et al. (2009) or IPCC (2007), there is an apparent systematic difference between top-down and bottom-up approaches when the time factor is considered. The same argument can be easily found also in other comparative researches such as Edenhofer et al. (2006) and Amann et al. (2009). From this observation, bottom-up or hybrid models with technology details show the tendency of expanding the mitigation capacity in the long-run.

However, even though models belong to the same approach or even the same subcategory, such as CGE, the projection outcomes can be quite different between models. For this reason, credibility over models' predictability became a serious concern, often causing policy makers to become reluctant to adopt a particular simulation result. For example, in a report presented by Stanford University's Energy Modeling Forum (EMF), carbon price prediction based on CGE models for year 2030 spanned from 0.53\$/tCO_2e to 60.00\$/tCO_2e.[5] (Kim and Chang, 2008)

To compare the projection results between global models and search for some clues to this problem, many researchers, mainly in the US and European countries, have carried out cooperative studies to compare the performance of alternative global economic models dealing with climate change policy. In a series of the assessment reports published by the International Panel on Climate Change, (IPCC, 2007) the outcomes from institutions around the world are put together with a brief comprehensive analysis.

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the graph is not that far from those of other CGE models. From this observation, one can carefully infer that the difference in model approaches affects the overall results.

[5] In this sense, Hourcade and Robinson (1996) put more weight on the input dataset rather than the difference in approaches.

Similarly, the EMF of Stanford University regularly provides assessment reports after a comparison work with the participations of various research groups around the world. (Weyant and Hill, 1999; Weyant, 2004; Weyant et al., 2006) Besides, Weyant (2000), Amann et al. (2009), Klepper and Peterson (2006) have also contributed.

IPCC (2007) argues that the factors accounting for differences between cost estimates can be divided into three groups: features inherent in the economies such as high substitution possibilities at low cost, assumptions about policy such as the use of international trading in emission permits, and simplifying assumptions chosen by the model builders such as how many sector or regions are included in the model. The first two groups of factors can be controlled, but differences in model builders' approaches and assumptions persist in the treatment of substitution and technology.

Among the various factors, Fischer and Morgenstern (2006) concludes that most of the differences between models are accounted for by the modelers' assumptions by conducting a meta-analysis dealing with the carbon prices or the marginal abatement costs of achieving Kyoto targets. The strongest factor leading to lower carbon prices is the assumption of high substitutability between traded products. Other factors leading to lower prices include the greater disaggregation of product and regional markets. This suggests that any particular results about carbon prices are possibly the outcome of the particular assumptions and characterization of the problem chosen by the model builder. Consequently, these results may not be replicated by others choosing different assumptions.

Taking the conclusions of the meta analysis into account, this study tries to more strictly control the common factors between models. The previous studies have contributed to finding the main factors that cause different results among various economic models, but these works often have limitations in deriving clear-cut conclusions on how largely a factor or a component contributes to the uncertainty in carbon prices projections. This is because they cannot control the factors in each study, so the conclusions become unclear with various factors standing out. For this reason, this research came up with a method which is similar to parameter sensitivity analysis. The idea is to do a quantitative analysis on the inner structures, measuring the effect of structural change in a component of a CGE model.

This chapter aims to give a quantitative analysis on the effects of the changes in the inner structures for the well-known three global climate change CGE models – EPPA, ENV-Linkages, and GTEM. After briefly reviewing the structures of the models, a series of structural analyses were conducted in combination with rough sensitivity analyses on parametric changes. The methodology employed in this study is to substitute alternative structural components with those of other models and observe the subsequent changes in outputs. Furthermore, this study also tries to derive economic implications on the issue of applying the outcomes such as carbon prices and GDP projections.

2.2 Reviews on global CGE models

In this study, three global climate change CGE models were chosen for comparison. Each model has its own unique structure. Specifically, two

Table 1: Key features of the chosen models.

Models	EPPA	ENV-Linkages	GTEM
Developer	MIT	OECD	ABARE
Approach	TD	TD	hybrid
Sector	13	20	19
Region	16	12	13
Discount rate	4%	not determined	4%
Growth rate data	endogenous	endogenous	IMF, OECD
Energy data	?	IEA	ABARE, IEA
GHG data	EPA	IEA, EPA	ABARE, IEA
LULUCF	○	×	○
CCS	○	×	○
nuclear	○	○	○
hybrid vehicle	?	×	○
bio-fuel	○	×	○

Source: OECD (2009). Note: IEA: International Energy Agency, EPA: Environmental Protection Agency, ABARE: Australian Bureau of Agriculture and Resource Economics, LULUCF: land-use, land-use change and forestry, CCS: Carbon Capture and storage.

CGE models, EPPA, ENV-Linkages, and one hybrid CGE model, GTEM, are selected.[6] Table 1 briefly shows key features of the selected models. Also, reviews on model properties were developed in this section, based on Paltsev et al. (2005), OECD (2008), van der Mensbrugghe (2005) and ABARE (2000).

[6]These models were also introduced and became well known in Korea. Refer to Kim (2010), Kim and Chang (2008) and Lim and Kang (2000)

2.2.1 Models

Development

The EPPA (Emissions Prediction and Policy Analysis) model was invented by MIT interdisciplinary initiative, originally based on the OECD model, called GREEN. (Yang et al., 1996) The basic philosophy of the two models were similar, but, as time passed, they have evolved in different ways. The results of EPPA model projections have been widely quoted in IPCC Assessment Reports as well as the US Congressional testimony. As a matter of fact, EPPA is a part of IGSM (Integrated Global System Model) which comprises all aspects of atmosphere, ocean, land use and urban activities. This means that the greenhouse gas (GHG) abatement target set in EPPA model is originally determined based on the IGSM. However, the policy goal scenario can be easily modified according to the research purposes.

The OECD ENV-Linkages model is another successor to the OECD GREEN model, which was initially developed by OECD. (Burniaux et al., 1992) GREEN was developed into the Linkages model and became the JOBS/Polestar modeling platform. A version of that model is also currently in use at the World Bank for research in global economic development issues.

GTEM (Global Trade and Environment Model) has a different ancestor: It was developed out of the MEGABARE model of Australia (ABARE, 1996), which contained significant advancements over the GTAP model of that time. (Hertel, 1997)

Dataset

The three models are all recursive-dynamic models and all use the same dataset for the reference year. The dataset is provided by Purdue university's GTAP (Global Trade Analysis Project) database, which collects global outputs and trade data with sectoral and regional details. With the dataset, each model calibrates parameters for the reference year. Although they all start with the same information, different structures cause different trajectory paths with the passage of time.

As a default, EPPA has 16 regions and 13 sectors. The sectors are agriculture, energy intensive industry, transportation, other industry, services, electricity, crude oil, shale oil, bio-oil, refined oil, coal, natural gas, and coal-gas. ENV-Linkages has 12 regions and 20 sectors including rice, other crops, livestock, forestry, fisheries, coal, crude oil, gas, electricity, petroleum & coal products, food products, mineral products, non-ferrous metal, iron & steel, chemicals, energy intensive industries, other manufacturing, transport, services, and construction & dwellings. GTEM has 13 regions and 19 sectors which are similar to the industrial classification of ENV-Linkages.

Program

EPPA uses the MPSGE (Mathematical Programming System for General Equilibrium) as the main solver program which realizes the Arrow-Debreu equilibrium[7] in the GAMS (General Algebraic Modeling System) modeling language. EPPA is formulated and solved as an MCP (Mixed

[7]For more information, please consult Rosenthal (2010) and Rutherford (1997).

Complementarity Problem), in which the zero profit, market clearance, and income balance conditions should be satisfied.

Like EPPA, ENV-Linkages is also written in the GAMS language and, in many cases, solves the problem as an MCP through a number of available solvers.

The language program of GTEM is GEMPACK(General Equilibrium Modeling PACKage). Unlike the other CGE models whose equations have non-linear forms, GTEM transforms the set of optimization problems into a series of linearized equation system according to Johansen (1960), utilizing the percentage changes of the variables.

2.2.2 Static structure

Production

In EPPA, production technologies are described using nested CES functions. (Figure 1) When it comes to the manufacturing sector, intermediate inputs enter in a Leontief structure with the capital-labor-energy (KLE) bundle, which consists of an energy and value-added bundle. As for imported goods in the intermediate goods composites, they are first combined by region of origin and then further aggregated to create an Armington good composed of the domestic good and imports. With this technology, a representative firm chooses an output level, quantities of primary factors and intermediate inputs to maximize profits subject to the constraint of its production technology.

Like EPPA, production level in ENV-Linkages is determined by cost

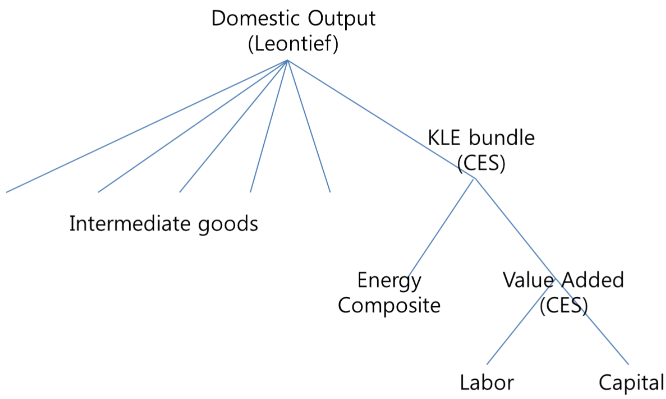

Figure 1: Production function structure of the manufacturing sector in EPPA.

minimization with an assumption of perfect markets, zero profit and constant return to scale technology. The production technology is specified as nested CES production functions in a branching hierarchy. However, some structures including value-added bundle are different from those of EPPA as shown in Figure 2. The value-added plus energy bundle is represented by a composite of labor and capital-energy composite, and the capital-energy bundle also has branches of an energy composite and a composite of capital and sector-specific factor.

GTEM also assumes that a representative firm produces goods under the assumption of perfect competition, constant returns to scale. By zero-profit assumption, the price of produced goods is equivalent to the production cost. The nesting structure of production technology in manufacturing sector is given in Figure 3. But different production structures are assumed in the electricity sector and the iron and steel sector. In these sectors, firms in these two sectors produce homogeneous outputs but they employ non-

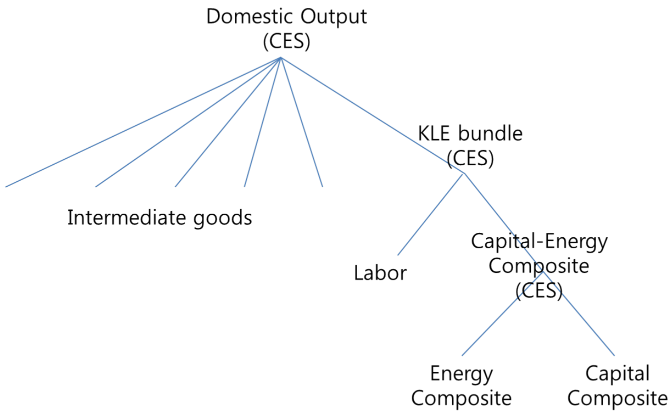

Figure 2: Production function structure of the manufacturing sector in ENV-Linkages.

homogeneous technologies.

Production in the electricity and iron and steel industries is modeled with fixed input structures (Figure 4). In detail, electricity is generated from fourteen technologies while iron and steel can be produced using either blast furnace or electric arc technologies. The imperfect substitutability between outputs of various technologies is modeled by a CRESH (Constant Ratios of Elasticities of Substitution, Homothetic) function. In both sectors, the representative producer solves the problem of minimizing the cost of producing the CRESH function of the output of technologies by choosing the output mix of possible technologies.[8]

Consumption and savings

In EPPA, a household representative agent is endowed with the factors of production, which may be sold or leased to firms. In each period, the rep-

[8]For an introduction of CRESH function, refer to Hanoch (1971).

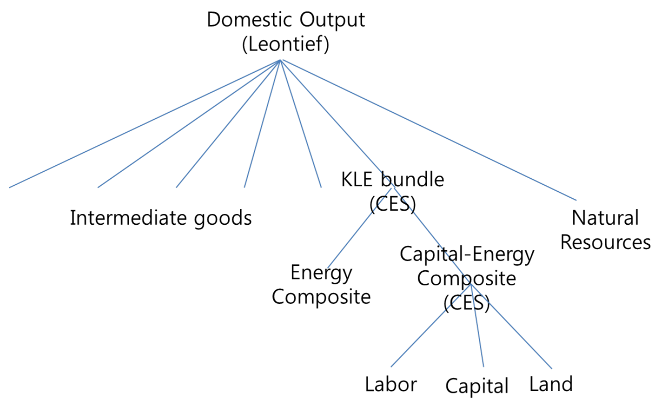

Figure 3: Production function structure of the manufacturing sector in GTEM.

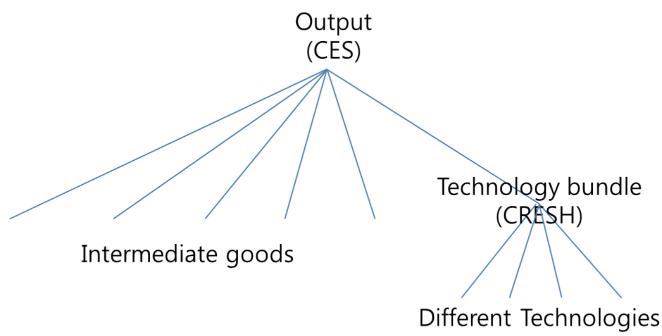

Figure 4: Production function structure of the electric power and the steel sector in GTEM.

resentative consumer chooses consumption and saving to maximize a welfare function subject to a budget constraint. Like production, preferences are represented by a CES utility function of consumption and savings. For welfare accounting, however, changes in aggregate consumption excluding savings are reported, avoiding double counting over time of changes in savings.

The structure of household includes both an energy nest and a nest that captures household transportation, separating purchases of transport fuels by households from the former. Because the consumption function is assumed to be homogeneous of degree one, the share of each good in total consumption remains unchanged regardless of a change in total consumption. To overcome the limitation, the elasticity and share parameters are a function of income between periods in EPPA.

ENV-Linkages also assumes that a representative consumer purchases goods or services. As in EPPA, the decision between consumption and savings is static instead of forward-looking, which means that saving is treated as a good and its amount is determined simultaneously with the demands for other goods. Because the price of saving is set arbitrarily equal to the average price of consumer goods, consumers are saving a constant proportion of their income. The utility function in ENV-Linkages is represented by an extended linear expenditure system (ELES), which was introduced in Lluch (1973). This function includes both the floor consumption level and the share parameters, and they are calibrated to a given set of initial consumption shares and income elasticities.

In GTEM, household demand for different commodities are determined by minimizing a constant difference in elasticity of substitution (CDE) expenditure function.[9] With a CDE expenditure function, the difference in the Allen partial elasticities of substitution between two commodities is invariant to the choice of pairs.[10] By this property, the values of income and cross-price elasticity can be derived using only income elasticities and own-price elasticities. As for savings, households in GTEM spend their nominal savings on purchasing domestic and foreign bonds. The price paid for these bonds in each region is based on domestic and global average price of investment goods

Trade

EPPA does not endogenously model international trade in factors such as capital and labor. While natural resources such as crude oil are traded as a homogeneous product, the Armington good assumption is not well applied in the case of gas. Thus the trade of natural gas can have a number of different scenarios.

In ENV-Linkages, labor factor migration is determined by a function of the expected relative wages. As in EPPA, ENV-Linkages assume the Armington specification of CES nests, in which domestic agents choose the optimal combination of the domestic good and an aggregate import good. However, ENV-Linkages adopts a nesting structure of constant elasticity of transformation (CET) functions on the export side. In ENV-Linkages, domestic suppliers optimally allocate aggregate supply across the domestic market

[9]For a detailed derivation of the CDE functional form, see Hanoch (1975).

[10]For the definition of Allen (Uzawa) partial elasticities, refer to Allen (1938) or Uzawa (1962).

and the aggregate export market.

In GTEM, international movement of both capital and labor is allowed between industries and across regions. However, these flows are not responsive to regional differences in wage rates but are determined by regional market clearing conditions and a market clearing price. GTEM also uses the CES Armington assumption to combine domestic goods with imported goods.

Carbon Policy

EPPA can incorporate a variety of emission control policies. The MPSGE solver computes the shadow price for explicit constraints on emissions and separate constraints can be represented by region, sector and greenhouse gas. An economy-wide cap can be independently set for each region and the model can be solved to find a local or global carbon price under international trade. Usually, carbon emissions are represented as an input factor in the Leontief relationships with fuel inputs.

In ENV-Linkages, taxes and permits are applied on inputs of energy from fossil-fuel producing sectors, such as refined petroleum, natural gas, coal, as well as on final demand of fossil fuel based energy. As in EPPA, regulatory policy has also been introduced in the model by imposing a shadow cost on a firm's inputs or capital. In addition, factor-income taxes at the personal level as well as factor taxes and subsidies have also been introduced.

GTEM assumes that combustion emissions of greenhouse gases are proportional to the quantity of fossil fuel combusted while non-combustion emissions are proportional to the quantity of output produced. In general, the emission intensity responds negatively to carbon tax rates and gas price, according to the respective emission response functions and their underlying parameters.

2.2.3 Dynamic process

Capital stock

EPPA specifies an investment sector where an aggregate investment good is equal to the level of savings. The accumulation of capital is calculated as investment net of depreciation according to the standard perpetual inventory assumption. There is distinction between malleable and non-malleable capital. The malleable portion of the capital stock in each sector is put into the nested CES production functions whereas the non-malleable portion of the capital stock appears in the Leontief production functions. The capital stock in each region and sector is determined by the capital vintaging procedure. In each period a fraction of the malleable capital is frozen to become part of the non-malleable portion. Some malleable capital can take advantage of intervening improvements in energy efficiency driven by the autonomous energy efficiency improvement index (AEEI) which is given exogenously.

Similarly, in ENV-Linkages, the capital accumulation is calculated by equating the current capital stock to the depreciated stock inherited from the previous period plus gross investment. If the demand for old and new capital can be less than the depreciated stock of old capital in a sector, the sector contracts over time by releasing old capital goods. In this way, the new capital vintage available to expanding industries is equal to the sum of withdrawn capital in contracting industries plus total saving in each period, which is consistent with the closure rule. In addition, the substitution elasticities among input factors are assumed to be higher with the new than with the old capital vintages.

GTEM also accumulates capital as a result of net investment. But this model does not consider the vintage of capital. Rather, it assumes one year for the installation of capital. Hence, supply of capital in the current period is determined by the last period's capital stock and investments made during the previous period.

Labor supply

In EPPA, changes in labor force size, which is computed based on the population projection, and productivity growth per worker are tracked in the model. Hence, although labor productivity is modeled as factor-augmenting, it is possible to identify the separate effects of population growth and pure productivity growth.

Likewise, the labor supply in ENV-Linkages is calibrated on exogenously given growth rates of population as well as sector-specific or aggregate labor productivity growth.

Unlike the other models, GTEM determines labor supply in each period by endogenous changes in population, given participation rates and a given unemployment rate. The unemployment rate is normally fixed in GTEM simulations through model closure.

Technical change

EPPA model introduces demand reduction factors that scale production sectors' use of energy per unit of output. The rate of growth of these factors is called the autonomous energy efficiency improvement (AEEI). This is a reduced-form parameterization of the evolution of technologically driven changes in energy demand, not induced by price changes. Also, several new advanced energy supply options have been specified. These technologies endogenously enter when they become economically competitive with existing technologies.

Likewise, ENV-Linkages also employs the AEEI in energy use. Typically it is dynamically calibrated to reproduce IEA's energy demand prospects. But the calibrations can be done by reproducing a particular exogenous fossil fuel price path. Compared to EPPA, ENV-Linkages has a lack of new energy technology options.

GTEM can simulate the impacts of various types of specific technical change without explicitly accounting for the implementation cost. In GTEM, both endogenous and exogenous technical changes can be included. The former responds to a specific model such as learning-by-doing function while the latter arises in input demand, price links and zero-profit conditions.

2.3 Model structure analysis

Two sources of uncertainties stand out in contributing to the discrepancy of the projection results among different models. One is uncertainty in parameters, and the other is uncertainty in structural forms. (Edenhofer et al., 2006) Compared to the abundant literature on the parametric uncertainty, including sensitivity analysis, there is little work on the structural uncertainty. This is because there is no unified theory or methodology as far as structural uncertainty is concerned. In fact, all models start with their own interpretations of the economy and, subsequently, this fact does not allow a direct comparison between models in terms of structural difference. In this sense, the research question should be "which structural components among different models cause the differences in the prediction results, and by which amount?" However, it is impossible to deal with all components in a model, so this study only focus on the energy-related production function structure.

This study employed a structural decomposition approach as its methodology. This study assumes that the whole computing structure in a model can be disassembled into multiple separated components. Each structural component can be switched independently from other parts, in other words, one part of the computing structure is substituted with other models' structural form, and the overall changes can be evaluated. Usually, switching of a component or a set of equations in a model is accompanied by a series of parameter calibrations to reproduce the model outcomes of the original unchanged model. However, this process tainted the self-consistency of other parts of the model since all parameters and equations should ultimately have robust theoretical foundations. Thus, this study assumes that parameters and structures in the unaltered components are determined on their own theoretical bases.

The reviews in the previous section reveal that models differ the most in both the production side and the consumption & savings side. However, it can be said that the issue of climate change or greenhouse gas emissions has a direct connections with production side. For this reason, this section quantitatively analyzes the impact of structural changes in the production side. Specifically, the EPPA model was selected as the base model: alternative structural components from the other models were applied to the corresponding parts of the base model. The reference year is 1997 according to the GTAP database version 5.

2.3.1 Change in energy-capital bundle structures

In most cases, CGE models use CES functions for the descriptions of the relations between macroeconomic variables. But there are many cases that the relations are very ambiguous and, mostly, CES functions cannot describe such situations. As for the parameters of energy-related production function, studies have led to contradictory evidences regarding substitution possibilities between energy and capital since Hudson and Jorgenson (1974) and Berndt and Wood (1975): Some researches show that energy is a substitute for capital, but other studies give contrary results, i.e., energy and capital are complements with each other and the substitution elasticity is zero. Cumulative work on this issue tentatively reveals that the two factors are substitutes cross-sectionally and complements in time-series. (Thompson and Taylor, 1995; Apostolakis, 1990) Also, in terms of model structure, there exists a huge difference in the function structure among models. For example, while the position of EPPA model or GTEM model is that capital should be first bundled with labor and then energy is rebundled with this bundle, ENV-Linkages model shows that capital is in direct relationship – whether as a substitute or complement – with energy, and then this bundle is rebundled with labor. Briefly speaking, the parameters of elasticity of substitution may differ and the same time the function structure is not invariant.

Nevertheless, some researches such as Chang (1994), Kemfert (1998) and van der Werf (2008) have tried to determine best fitted CES nesting structures and parameters for a certain sector or region. However, these works investigated only the static performance of the structures and did not check the difference in dynamic trajectories each nesting structure generates. In this case, even if two structures have similar levels of performance, the projection outcomes can significantly differ between those two structures. Thus, the comparison of function structures' performance should consider the ability to reproduce the real economy in the time domain. This is why this research carries out component switching experiments.

In this experiment, the energy-capital bundle structure in EPPA was replaced with that of ENV-Linkages. Along with the structural change, substitution parameters were also switched. It should be noted that ENV-Linkages uses two kinds of elasticity parameters – new vintage capital parameters and old vintage capital parameters. Both new and old vintage parameters can be considered as maximum and minimum values, respectively. In this research, only new capital parameters were used because the counterparts produced unacceptable results. Detailed parameter values are listed in OECD (2008).

The concept of capital vintage was originated from the belief that capital can be transformed or incarnated into physical facilities. There are different vintages between facilities deployed for production, and hence different vintages mean different level of technologies and indicate the degree of capital malleableness. Thus, new vintage capital can be substituted more easily with another production factor, while old vintage capital have been irreversibly transformed into tangible facilities.

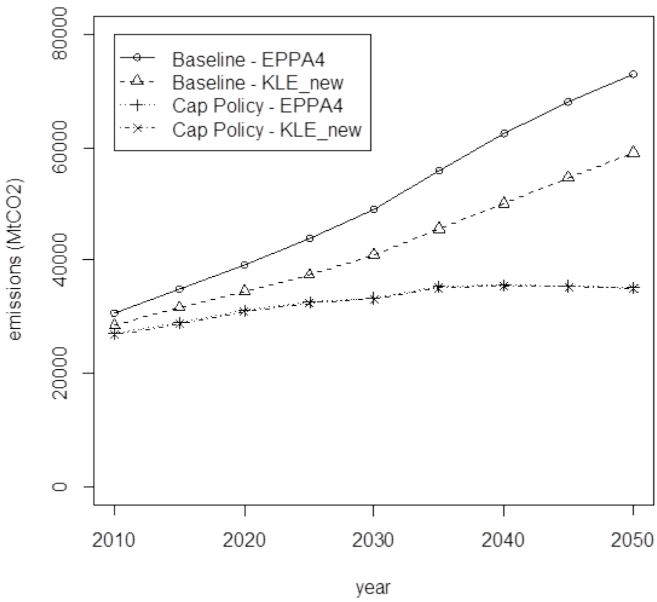

Figure 5: Emission levels for baseline and cap-policy when new capital parameters are applied.

Figure 5 and Figure 6 show the results. Parameters were changed in both capital-energy (KE) bundle and KE-labor bundle. When the structure was modified with the new vintage capital parameter, i.e., relatively high elasticity of substitution, the baseline emission level was lowered by 19.10% in the final year compared to the case of unchanged structure. The cap-policy

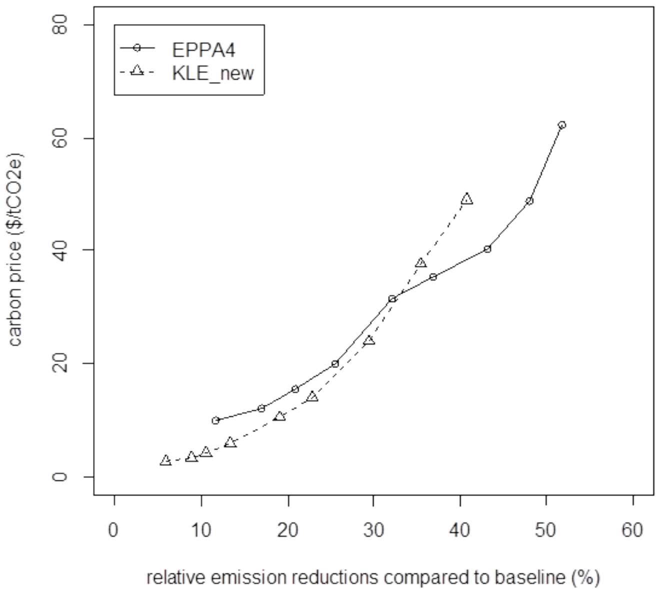

Figure 6: Moving path of carbon price and reduction potential when new capital parameters are applied.

emission level decreased by 0.45%, almost unchanged regardless of structure change. This is the situation that one can easily decrease the energy consumption by substituting capital for energy so that greenhouse gas emission level is lowered. It also affected the moving path of carbon price and reduction potential, which depicts the trajectory of a pair of carbon price and reduction potential for individual projection years. Both decreased by 21.11% and 11.11%, respectively, in the final year.

This result clearly shows that a structural change in a model can affect the overall projection outcomes even though there is no difference in the dataset. The switched part of EPPA can be thought to have the same information or interpretation for the economy with the original structural component, because both structures were built on the same dataset in both EPPA and ENV-Linkages. However, the emissions and carbon prices projections show a huge gap between the original model and its variant.

Next, Figure 7 and Figure 8 describe the results when the elasticity of substitution is lowered only between capital and energy. The parameter value was set at zero. As a result, the baseline emission level decreased by 6.02% in the last year, moving closer to the graph of the unmodified version of EPPA. The cap-policy emission level stayed almost at the same level, showing 0.54% increase. However, unlike the former case, carbon price went up by 27.41% in 2050. What can be inferred from this result is that energy cannot be easily replaced with capital. In order to meet a certain production level, producers should increase energy and eventually increase the greenhouse gas emissions through purchasing more carbon credits.

The baseline emissions projection increased as the substitution is low-

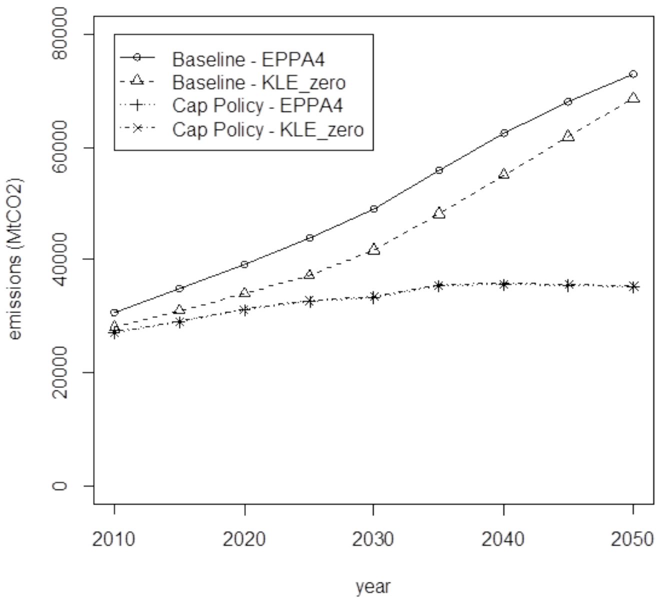

Figure 7: Emission levels for baseline and cap-policy when new capital parameters are applied except for the energy-capital substitution of zero.

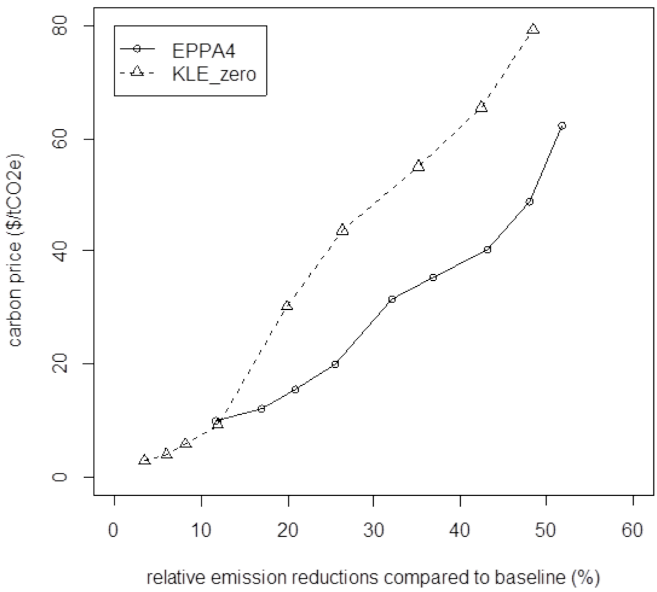

Figure 8: Moving path of carbon price and reduction potential when new capital parameters are applied except for the energy-capital substitution of zero.

ered, but it did not reach the outcome of EPPA. In other words, despite the extreme variation of the elasticity parameter, the baseline emissions projection still shows an unrecoverable difference between EPPA and its variant.[11] This implies that a structural component of a model cannot perfectly reproduce the information embedded in an alternative structure. Therefore, from the experiment of this section, it can be inferred that a change in the bundle structure can affect the overall projection result.

2.3.2 Replacement with fixed input structures

In spite of its CGE mechanism, GTEM model is generally classified as a hybrid model because it reflects bottom-up approach in the structure by applying a combination of fixed input structures, so-called ‘technology bundle.’ Technology bundle means that multiple technologies, which have different input structures, are combined to produce the same goods. In GTEM, production structures in the electricity, iron and steel industries implement this method to add the characteristics of the bottom-up models into the top-down framework.

The technology bundle approach recognizes the fact that output in some industries can only be produced by using a mix of given technologies. Each technology uses a different mix of inputs which are in fixed proportion to its output. Therefore, the feasible input space for the industry (in a top-down sense) is defined by the convex combination of the technologies with boundaries, each of which corresponds to the optimized points of individ-

[11]This is the same with a variation of substitution elasticity parameters related to labor, though it failed to give optimized carbon prices at some equilibrium points.

ual technologies. Thus, parts of isoquants that lie outside the cone-shaped boundaries are infeasible. Therefore, the technology bundle approach avoids corner solutions, in which one technology may win over all others. This contributes to a more plausible description of the actual economy, especially in handling the energy intensive sectors.[12]

This study applied the technology bundle approach to EPPA's electricity production structure. Among various production technologies, three conventional technologies were chosen – fossil fuel, nuclear, hydroelectric – as well as six advanced generation technologies proposed by EPPA scenario. The input structures of each technology are fixed by using Leontief functions, and then the simulation program is allowed to automatically choose between those technologies with different levels of substitution.[13] From Figure 9 to Figure 12, the simulation results are represented for two cases.

When the elasticity of substitution between technologies is set at 1, the baseline emissions level in the final year decreased by 7.27% compared to the case of no technology bundle approach. But in the case of cap policy, emission level was increased by 4.65%. When the elasticity value is 10, the same patterns are observed – a decrease of 5.85% and an increase of 2.80%, respectively. What is noteworthy is that, in both cases, carbon price declines in 2020.[14] In the case of substitution elasticity 1, carbon price declined more

[12]The equilibrium solutions in each sector are obtained by cost minimization in GTEM. Refer to ABARE (2000) for more information.

[13]In GTEM, the techonology substitution is realized by CRESH(Constant Ratio Elasticity of Substitution, Homothetic) function. This function was introduced to cope with the inflexibility of CES(Constant Elasticity of Substitution) function. (Hanoch, 1971, 1975) In this study, for the sake of convenience with programming, the substitution is realized in CES functions.

[14]This can be accounted for by the transition to new technologies assumed by the EPPA scenario.

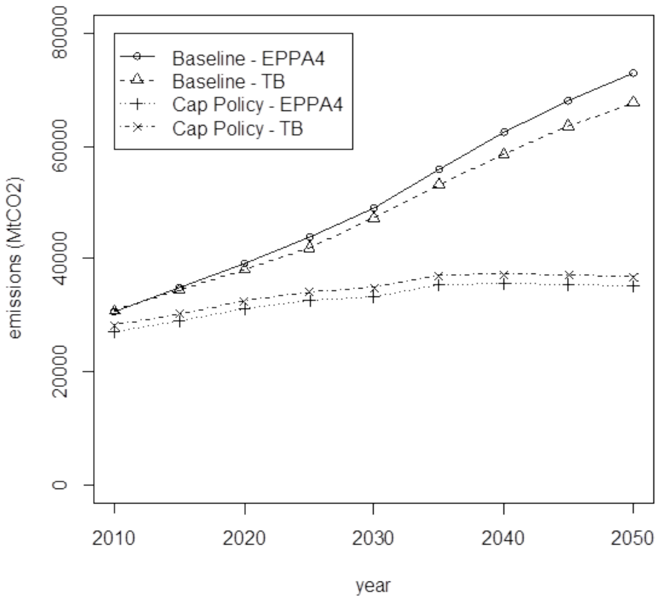

Figure 9: Emission levels for baseline and cap-policy when elasticity parameter is 1.

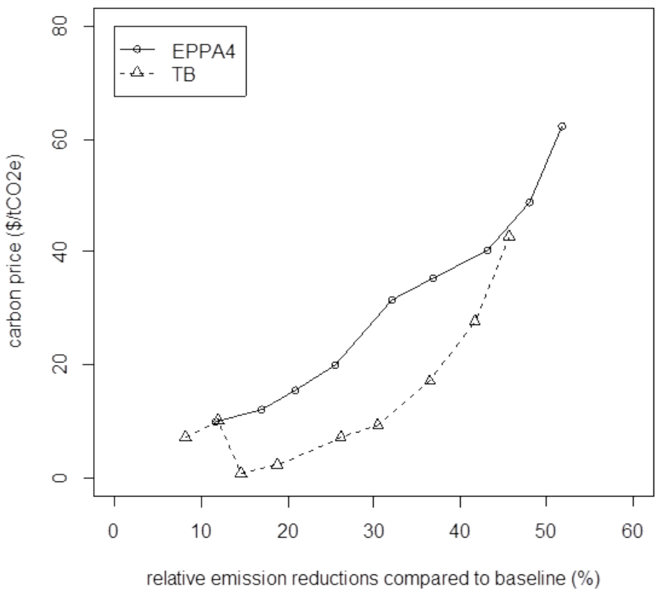

Figure 10: Moving path of carbon price and reduction potential when elasticity parameter is 1.

rapidly than in the case of 10. Consequently, in both cases, the reduction potentials were expanded compared to the case of no technology bundle model. This confirms the finding of OECD (2009) case study, mentioned in the introductory part of this chapter, that the marginal abatement cost curves of hybrid models, such as GTEM, have a tendency to move more to the right in the long run.

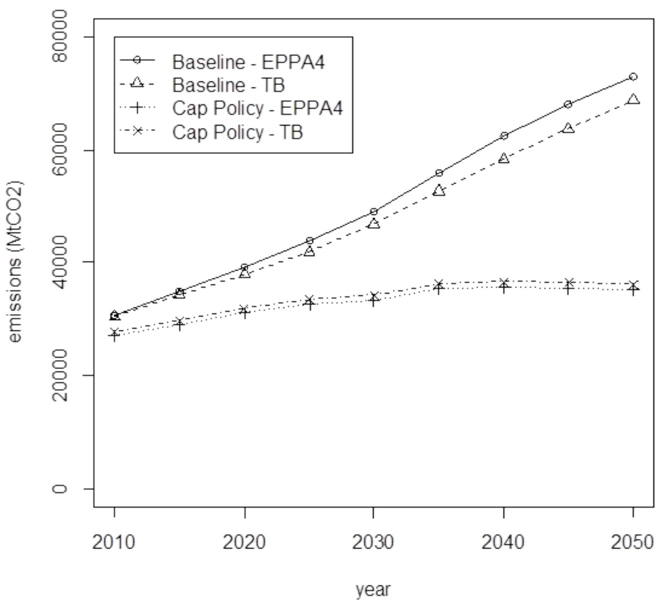

Figure 11: Emission levels for baseline and cap-policy when elasticity parameter is 10.

With the technology bundle approach, the property of fixed input structures of Leontief functions tend to narrow the gap in emissions between the two cases – baseline and cap-policy. However, in Figure 9 and Figure 11, the difference is negligible compared to the previous experiment. The main

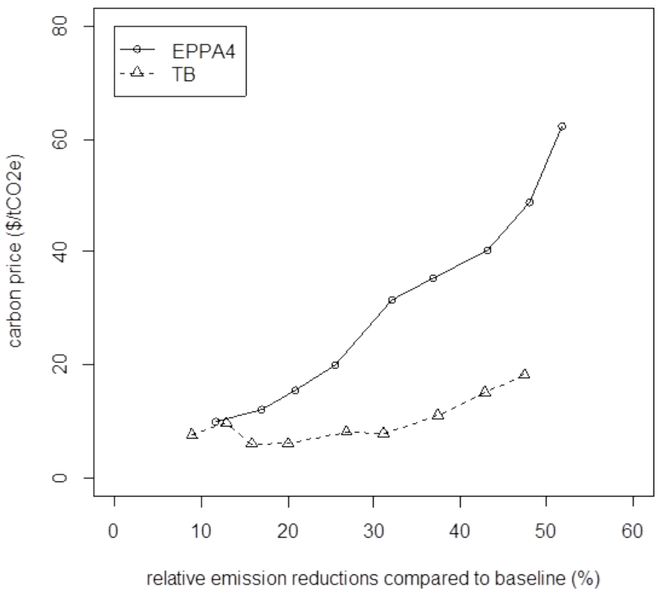

Figure 12: Moving path of carbon price and reduction potential when elasticity parameter is 10.

point of this experiment is that the equilibrium moves to the low level of carbon prices, as depicted in Figure 10 and Figure 12. The slightly increased emissions level in the cap-policy case may be accounted for by the partial modification for only electric power sector in this experiment, but the carbon price shows totally different behavior compared to the former experiment. The low carbon prices hint that technology bundle model assumes that new technologies can be implemented more freely compared to the original model. In this experiment, it is very important to determine the elasticity of substitution between technologies, which represents the availability of new technologies.

The availability of a technology can be sketched by comparing the levelized costs of electricity generation(LCOE), which are usually provided by IEA (2010). Nuclear power generation, in general, shows the lowest cost, while the technology of fossil fuel based power generation, such as coal and LNG combined cycle, shows high cost. However, fossil fuel based technologies still hold an advantage over renewable energy in terms of power generation cost. This means it is not easy to make technology transition without any help from policy intervention. The intervention can also enter a scenario related to the elasticity of substitution between technologies.

The unique property of the technology bundle approach helps to compose more realistic models from the aspect of technology transition. It excludes the unrealistic assumption that the possible decisions of economic agents are achieved only at the level of input factors with a given fixed technology structure as is usually assumed in many CGE models. Rather, it enables the choice at the level of technology, which can be considered to be the better description of reality. However, it is still unclear to what extent one can rely on the projection results of this approach. The experiment results of this section only provide a piece of evidence of the effect of a possible structural change.

2.4 Policy implications

2.4.1 Carbon price

A marginal abatement cost (MAC) measures the change in economic costs associated with a unit change in pollution abatement. An MAC curve (MACC) depicts the relationship between pollution abatement and costs and is typically positively sloped. Recently, MACC has been widely used for estimating the cost of reducing greenhouse gas emissions. It offers a simple and attractive tool for policy makers and researchers because they are straightforward to use and can directly present a cost related to certain emission reduction target.

In this section, MACC schedules of Korea for each structural change are extracted and an analysis on the impact of the emissions trading implementation on the national economy is provided. Figure 13 shows the MACC derived from EPPA and its three variants. To facilitate simulation procedure, this study only included CO_2 as the greenhouse gas emission. To compensate the expected limitations due to ignoring other greenhouse gases, it was assumed that 30% reduction of total greenhouse gas emissions by year 2020 is equivalent to 24% reduction in CO_2, taking into account the potential contributions of other gases. (Kim and Jeon, 2010) Thirty percent reduction compared to business-as-usual scenario by year 2020 is the voluntary reduction target set by the Korean government. Also, this study assumed that the global level emission trading market will be in effect. Under these assumptions, the simulation result shows that the carbon price ranges between 30–120\$/tCO_2e based on the four alternative model assumptions.

Previous studies tend to fall within this range. Kim and Jeon (2010) applied an auction allocation scenario to a variant of the ENV-Linkages model to estimate approximately 50\$/tCO_2e of carbon price, while Lim (2010) derived a minimum value of 80.6\$/tCO_2e as a weighted average cost of marginal reduction without the introduction of international emissions trading scheme.

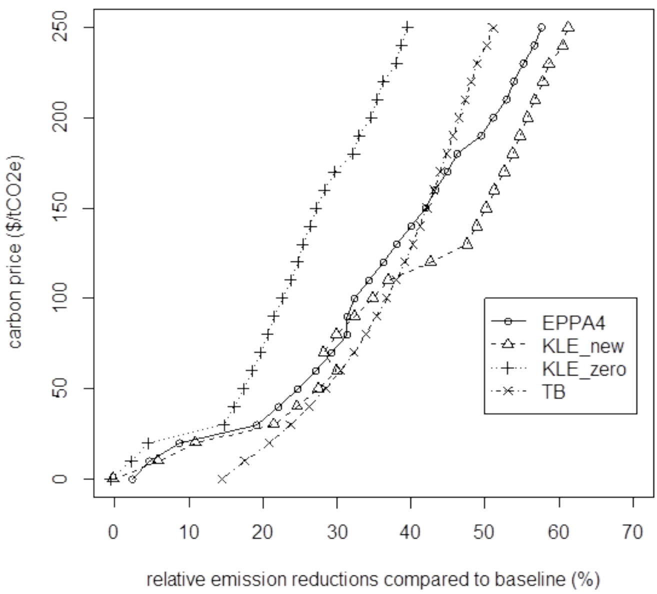

Figure 13: Marginal abatement cost curve(MACC) of South Korea in 2020.

In Figure 13, each graph differs in the overall slope according to the model types. In the range of 20–40% mitigation rate, every graph looks close to a straight line and is monotonically increasing. More specifically, KLE_zero case, in which capital and energy are first bundled with zero elasticity of substitution, represents a change of approximately \$180, showing the largest change than any other structure. KLE_new, in which capital and energy are first bundled with non-zero elasticity of substitution, shows \$90 change, while EPPA and TB, which employs technology bundle, show the variation of the width of \$110. The steep slope of the KLE_zero can be explained by the zero elasticity of substitution between capital and energy in the production function. Delarue et al. (2010) revealed that the substitution in the input structure affects the whole shape of MACC graph. With a more careful observation, it can be found that the TB type of models show the same tendency because of its Leontief structure in the technology bundles.

In all four models, carbon prices are lower than 30\$/tCO_2e in the interval of reduction level of 5-15% while the marginal abatement costs begin to increase rapidly when the reduction rate is around 20%. From this observation, it can be inferred that a gradual carbon reduction path may be more desirable than a drastic reduction alternative. Low level of carbon price at a certain mitigation potential means there is no excess demand for carbon credits and greenhouse gas reduction can be achieved comparatively easily. In other words, a large scale reduction in the short term beyond the current ability of a country may cause the government to pay a high cost, although the same reduction rate can be achieved with a relatively low level of expenditure in the long run. Thus, from this aspect, more careful consideration is needed in policy decisions on setting the mitigation target.

In TB model, at a low level of carbon price, the reduction rate is relatively high compared to other models. The individual technologies are fixed in the form of Leontief function and it is impossible to switch between input factors. Therefore, it is more effective to switch the technology function itself in order to achieve profit maximization. This property indicates that a model that includes the concept of technology switch is more suitable for policy analysis when the target level of reduction is relatively high, because the target is established under the assumption that a wide range of technology transition is available at the present time.

More specifically, in the case of a high reduction target, policies should be more drastic with a requirement of the introduction of new technologies. Under this situation, too much cost is needed to keep the existing technologies compared to the implementation of new technologies. This case is shown in the TB model simulation – high rate of reduction at a low carbon price. Thus, when high reduction targets are outlined in the policy goals, EPPA or ENV-Linkages models may not be suitable for a policy analysis method because they do not incorporate or reflect the bottom-up approach.

2.4.2 Estimation of GDP change

Finally, this study analyzed the impact of greenhouse gas mitigation policy on the gross domestic product(GDP). Figure 14 shows that the GDP loss spans from 0.48% to 4.02%. EPPA and KLE_new show relatively low levels of reduction, while TB, in which GTEM structure is partly reflected, presents about 4% decrease. Recalling that TB showed the lowest carbon

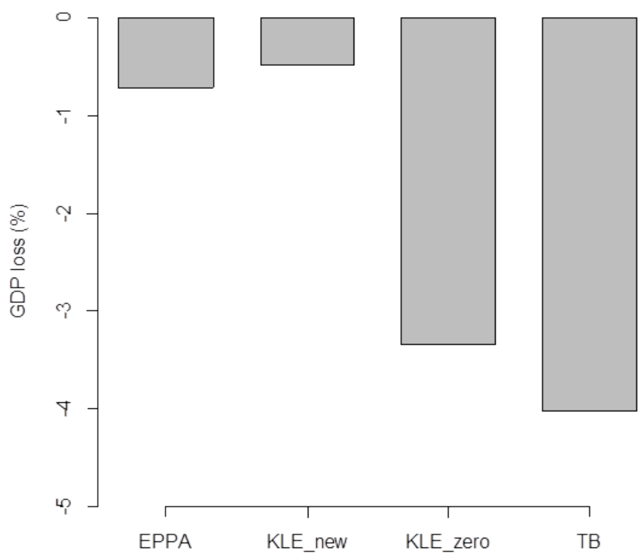

Figure 14: A Projection of GDP loss of South Korea in 2020.

price of 30$/tCO$_2$e, it is interesting that its GDP loss is the highest among the simulation models. Obviously, it is because the inflexibility of Leontief functions lowered the level of gross production, also decreasing the carbon prices.

Presidential Committee on Green Growth (2009) of Korea proposed the mid-term reduction goals by year 2020 with 0.49 percent loss of GDP, which has been reportedly calculated with the use of a variant of ENV-Linkages. According to other recent studies, however, this modest result may have been underestimated. Lim (2010), for instance, showed that GDP growth was to decline by 1.53 percent using a GTEM variant.

2.5 Conclusion

At its core, this chapter is about deepening the understanding of CGE models which are widely used for climate change policy analysis. By reviewing the basic characteristics of three well known global CGE models, this study pointed out production function structures as the main sources of the differences in carbon emissions projections among models. Then, to examine the different projection outcomes a specific structural component can give compared to other forms of model structures, two experiments were carried out for two cases: substituting with alternative nesting structures and applying fixed input structures. Also, the experiments were extended to the comparison of GDP losses among different model structures.

In the first experiment, the energy-capital bundle structure within nested production functions in EPPA model was replaced by that of ENV-Linkages model. Then the elasticity of substitution was varied between capital and energy. The result shows that this change causes a large impact on the prediction results. In the second experiment, fixed input structures defined by Leontief functional forms were applied to EPPA's electricity production sector. This approach is used in GTEM model for the purpose of combining both top-down and bottom-up approaches. Changes in the assumptions on technical information brought a significant effect especially on carbon reduction potential. This may very likely be the reason behind the hybrid models' unique behavior in OECD (2009). In addition, carbon price and GDP loss are estimated and compared for individual structurally variant models. In the process, marginal abatement cost curves (MACCs) were derived and changes in GDP were estimated for year 2020 for the case of Korea for different carbon reduction scenarios. The decrease of GDP ranged from 0.48% to 4.02% according to structural forms.

In this chapter, a more systematic approach was applied for comparative analysis on the issue of structural uncertainty in CGE models. The experiments were carried out under more strictly controlled conditions compared to the previous literature such as meta analysis. In addition, time factor was included to assess the whole dynamics of a specific structural component. Previous comparative studies only dealt with static performances of model structures and did not consider the time factor.

This chapter only focused on production functions which are thought to be directly related to greenhouse gas emissions. However, other structural components, such as household consumption models, Armington goods production functions and consumption models in transportation sectors, can also affect the projection results. Thus, a more comprehensive and in-depth researches should be conducted to encompass other main factors.

Chapter 3

Carbon prices and parameter calibration in CES function structures

3.1 Introduction

Recently, the issue of estimating precise carbon prices has been a central theme in many countries which are considering an introduction of carbon tax or emissions trading scheme. In many cases, carbon prices, i.e. greenhouse gas marginal abatement costs, are calculated by deriving the relations between carbon prices and potentials of reduction, and most of this work is carried using the concept of marginal abatement cost curve (MACC). The marginal abatement cost means the additional cost which one should incur when reducing greenhouse gas by a unit amount, and this cost is often referred to as carbon price or greenhouse gas price. The curve and shape of a MACC imply various information about the level of greenhouse gas reduction technologies of a country or an industrial sector. In this sense, the MACC method can be a good and efficient tool for decision making in carbon policies.

Broadly speaking, there are two approaches to derive a MACC, top-down and bottom-up. The top-down approach mostly means an macroeconomic model, and, among many subcategories, computable general equilibrium (CGE) models are one of the most popular methods. In this approach, the economy is described by a set of optimization problems for each representative agent, and the shadow price corresponding to the carbon emissions constraints of those simultaneous equations means the marginal abatement cost. On the contrary, the bottom-up approach is usually based on engineering information. This approach does not consider the interactions between individual agents, but only focuses on each technology's reduction potential and abatement cost. Bottom-up models do not explain the whole picture of an economy, but they can convey the technology information in detail. Recent works on deriving MACCs have shown the researchers' preferences for CGE models, which can be briefly and easily composed compared to the other approaches. On the contrary, the adoption of bottom-up models is limited by the necessity of a huge amount of information on individual technologies for each industrial sector.

There are cumulative evidences on what factors can affect the robustness of a MACC derived from CGE models. Ellerman and Decaux (1998) is an early study on the robustness of MACC derived from a CGE model, EPPA developed by MIT. It deals with opportunities for emission trading between different regions and tests the robustness of the MACCs. It analyzes the effect of a change in other regions' abatement policies on a MACC of a certain region, concluding that MACCs are robust in this case. Klepper and Peterson (2006) analyzes the influence of fossil fuel prices on MACCs using a CGE model, DART, in which fossil fuel prices are determined endogenously and depend on abatement levels all over the world. The result shows that a MACC is highly dependent on the level of fossil fuel prices while it remains unaffected by absolute or relative emission reduction levels. Morris et al. (2008) shows that the robustness of a MACC can be affected by how to construct it, i.e., how to define the baseline. Especially, it also illustrates the path dependence of a MACC through time, which means that a MACC at a certain time depends on the policies of the past. As for the effect of parametric changes, there exits a large number of literature on sensitivity analysis, including Webster et al. (2003) and Webster et al. (2009).

However, there has been little work on structural analysis of the inner components in a CGE model, except Hong and Kim (2011) and Kim and Hong (2012) which deal with the aggregated effect of a structural change. Specifically, there is little understanding of the characteristics of constant elasticity of substitution (CES) function structures in a CGE model. CES function structures usually play a great role in a description of the behavior of the representative agents in CGE models. However, people usually ignore the possibility that the CES functional form itself can affect the outcomes derived from a MACC.

Moreover, considering the recent trend of data disaggregation, the analysis of CES function structures is even more critical. With data disaggregation, an economy is divided into multiple sectors or regions and proper function structures are ascribed for each sector or region. Many CGE models adopt regional or sectoral disaggregation to incorporate the microscopic information with the macroeconomic structures of conventional CGE models. Some studies go further by combining the general equilibrium method with disaggregated partial equilibrium systems. (Narayanan et al., 2010) However, the problem is that there is no positive proof that a disaggregated data frame contributes to an actual improvement in a description of the economy.

This chapter analyzes the behavior of the CES function structures. The first part of this study starts from carbon price estimations which contradict common sense when the regional disaggregation is considered in a global CGE model. Specifically, a few developing countries exhibited a relatively high level of carbon prices, as opposed to the generally accepted belief to the contrary. An empirical analysis is added to explain the phenomenon, and a mathematical model is introduced using the concept of capital intensity. These basic pictures are extended to a CES functional form to obtain a condition for minimizing the extraordinary phenomenon in carbon price projections. This chapter illustrates that CES model structures are not suitable for encompassing all of the disaggregated situations.

3.2 Problems in regional disaggregation

3.2.1 Derivation of MACC using the EPPA model

In this section, a global MACC is derived by EPPA, which is a recursive-dynamic model developed by MIT. In the model, technological change is given exogenously and negative abatement cost is not allowed unlike usual bottom-up models. The dataset for reference year is Purdue University's GTAP.[1] EPPA uses the optimization program languages, GAMS and MPSGE, which convert the general equilibrium model into a mixed complementar-

[1] This study uses GTAP version 5 which contains the outdated data of year 1997. It should be reminded that this study does not aim to estimate a precise carbon price at the present time but focuses on the structural analysis of a CGE model. In other words, the outdatedness doesn't matter in this study.

ity problem (MCP) under the condition of zero profit, market clearing and income balance. (Rosenthal, 2010; Rutherford, 1997)

According to Paltsev et al. (2005), in EPPA, a firm of region r and sector i should solve the following profit maximization problem to achieve the production level of y when it uses k of input factor f and x of intermediate input provided from sector j.

$$\max_{y_{ri}, x_{rji}, k_{rfi}} \pi_{ri} = p_{ri} y_{ri} - C_{ri}(p_{ri}, w_{rf}, y_{ri}) \quad s.\ t. \quad y_{ri} = \varphi_{ri}(x_{rji}, k_{rfi}), \tag{3.1}$$

where π and C denote the profit and cost functions, and p and w are the prices of goods and input factors, respectively. The production technology φ is represented by a CES function which has constant returns to scale (CRTS), by which the firm's problem can be simplified into the unit cost and unit profit functions with zero profits conditions,

$$p_{ri} = c_{ri}(p_{rj}, w_{rf}) \tag{3.2}$$

where c is the unit cost function. By Shephard's Lemma, the demand of intermediate goods j and the demand for f in sector i are

$$x_{rji} = y_{ri} \frac{\partial c_{ri}}{\partial p_{rj}}, \qquad k_{rfi} = y_{ri} \frac{\partial c_{ri}}{\partial w_{rf}}. \tag{3.3}$$

The production technology is structured by multi-level nested CES functions. Although it has diversified forms for each sector, this section introduces the structure of the manufacturing sector only. The following are the variables and and their indices.

i, r, t : index for production sectors, regions, and periods,

e, l, k, ne : index for energy, labor, capital, and non-energy,

X_{irt} : sectoral gross output,

Xa_{irt} : output of Armington composite goods,

Xm_{irt} : output of imported goods,

Xd_{irt} : output of domesitic goods,

E, L, K : energy, labor, and capital,

$ELEC$: energy production in electricity sector,

$NELEC$: energy production in non-electricity sectors.

The uppermost production function for the manufacturing sector in the CES nesting structure is denoted by

$$X_i = \sum_{ne} a_{ne,i} Xa_{ne,i} + a_{elk,i} Z_{elk,i}, \tag{3.4}$$

which means that sectoral gross output X is a linear function of the Armington goods Xa and Z, composite goods of energy, labor, and capital. Z is constructed with energy and composite goods consisting of the rest input factors as the following CES function,

$$Z_{elk,i} = (a_{e,i} E_i^{\rho_{elk}} + a_{lk,i} Z_{lk,i}^{\rho_{elk}})^{1/\rho_{elk}}, \tag{3.5}$$

where ρ_{elk} is the parameter related to the elasticity of substitution, σ_{elk} between energy and labor-capital composite goods, in other words, $\sigma_{elk} = \dfrac{1}{1-\rho_{elk}}$. Likewise, if ρ_{lk} determines the substitution between labor and capital, the lower-level equation is denoted by

$$Z_{lk,i} = (a_{l,i} L_i^{\rho_{lk}} + a_{k,i} K_i^{\rho_{lk}})^{1/\rho_{lk}}. \tag{3.6}$$

In Equation 3.5, energy composite goods are described by the following equation comprised of electricity, denoted by an index, E, and non-electricity, N, sectors:

$$E_i = (a_{E,i} ELEC_i^{\rho_{EN}} + a_{N,i} NELEC_i^{\rho_{EN}})^{1/\rho_{EN}}. \tag{3.7}$$

Concerning the other branch of Equation 3.4, the Armington goods are denoted by

$$Xa_{j,i} = (a_{m,j} Xm_{j,i}^{\rho_{dm}} + a_{d,j} Xd_{j,i}^{\rho_{dm}})^{1/\rho_{dm}}, \qquad j \in \{ne, e\} \tag{3.8}$$

Imported goods, Xm, are again put by a multiple input CES function:

$$Xm_{j,i} = (\sum_r a_{r,j} Xa_{r,i,j}^{\rho_{mm}})^{1/\rho_{mm}}, \qquad j \in \{i\}. \tag{3.9}$$

On the other hand, the amount of carbon emissions, EE, for each time t is determined by

$$EE_t = \sum_e Xa_{e,t} T_e \varepsilon_e + X_{b,t} \lambda T_{refoil} \varepsilon_{refoil}, \tag{3.10}$$

where Xa_e denotes the Armington energy goods compounded of natural gas, refining and coal while $X_{b,t}$ denotes the production of carbon liquids backstop technology. T is a coefficient to convert the expenditure of energy consumption into heat units and ε is a coefficient for converting heat units into CO_2 emissions units. The release of CO_2 at the point of refined oil production, indexed by $refoil$ is expressed as a fraction, λ, of the CO_2 emitted by the corresponding refined oil at the point of consumption.

In EPPA, the representative agent of the household determines consumption and saving to maximize a welfare function subject to a budget constraint as expressed by

$$\max_{d_{ri}, s_r} W_{ri}(d_{ri}, s_r) \quad s.\ t. \quad M_r = \sum_f w_{rf} K_{rf} = p_{rs} s_r + \sum_i p_{ri} d_{ri}, \tag{3.11}$$

where s is saving, d is the final demand for commodities, K and M is the factor endowment and the income level, respectively, of the representative agent. Like the firm's problem, preferences are represented by a CES utility function. By duality and the property of linear homogeneity, a unit expenditure function corresponding to Equation 3.11 is given by

$$p_{rw} = E_r(p_{ri}, p_{rs}). \tag{3.12}$$

By Shephard's Lemma, the compensated final demand for goods and savings are respectively expressed as

$$d_{ri} = \bar{m}_r \frac{\partial E_r}{\partial p_{ri}}, \qquad s_r = \bar{m}_r \frac{\partial E_r}{\partial p_{rs}} \tag{3.13}$$

where $\bar{m}_r$ is the initial level of expenditure in each region.

The above system consisting of firm and household is closed with a set of market clearing conditions, which gives the equilibrium prices in each goods or factor market. As a simple case, ignoring the rest categories including investment, government and foreign trade, the equilibrium is determined by the following equations:

$$y_{ri} = \sum_j y_{rj} \frac{\partial C_{rj}}{\partial p_{ri}} + \bar{m}_r \frac{\partial E_r}{\partial p_{ri}}, \qquad K_{rf} = \sum_j y_{rj} \frac{\partial C_{rj}}{\partial w_{rf}} \tag{3.14}$$

To this point, a brief sketch of the whole structure of EPPA was provided according to Paltsev et al. (2005). Next, global scale MACCs were derived by the model, specifically EPPA version 4. The multi-regional and multi-sectoral model originally divide the world into 16 regions. However, in this study, the region for Korea is additionally separated and the MACCs were extracted for 17 regions. The original model can deal with an emissions trading scheme for 13 kinds of greenhouse gas, including the six kinds defined by the International Panel on Climate Change (IPCC). However, this study only deals with CO_2 due to a lack of inventory information of Korea in the reference year.

The time of extracting the MACCs was set at the year of 2020, by which the Korean government announced that it will voluntarily achieve 30% reduction of greenhouse gas compared to business-as-usual (BAU) scenario. In this experiment, a global-scale emissions trading was assumed from year 2015.[2] The details related to scenarios remained unchanged from the original version, in which the final target of greenhouse gas density is set at 550 ppmv by the year of 2100. Also, the scenarios for the employments of new technologies followed the time schedule of EPPA.

[2]This unrealistic assumption is not relevant in this study because the main goal of this study is to give an analysis of the structure of a CGE model.

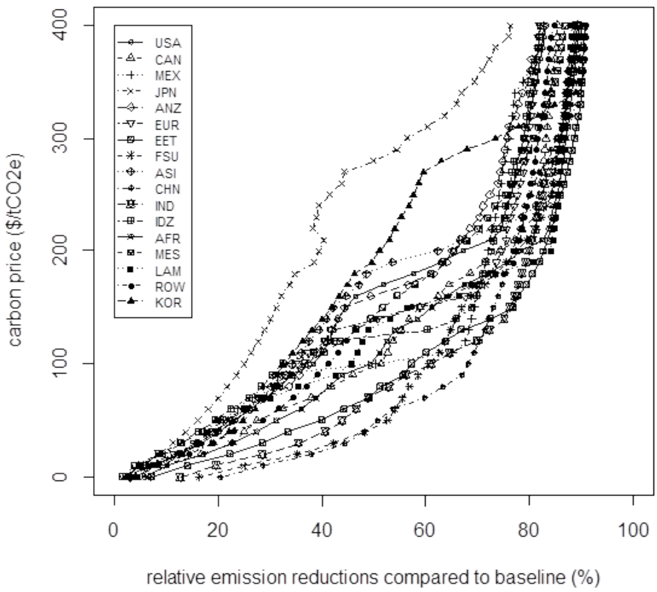

Figure 15: A marginal abatement cost curve (MACC) in 2020, produced by EPPA model simulation.

Figure 15 shows the MACCs for individual regions based on the above conditions. On the whole, Japan (JPN) shows the steepest MACC while the MACCs of the former Soviet Union (FSU) and China (CHN) are slanted the most to the bottom. The graph of Korea (KOR) shows a cusp around 250-300\$/tCO_2e of carbon price and a relatively horizontal movement up to 350\$/tCO_2e, in which there arises a critical fuel change from oil to bio-oil according to the embedded scenario of EPPA.

3.2.2 Regional deviations in carbon price

In this section, the carbon prices for individual regions are estimated and their relations with their respective GDP level are examined, under the assumption of 30% reduction of greenhouse gas by year 2020 in all regions. Generally, the level of carbon price to achieve the same reduction ratio is proportional to the degree of economic development or income level of individual countries, which differ in the efficiency of energy use. In Figure 16, Japan (JPN) shows the highest level of carbon price as expected. Also, in big developing countries, such as India (IND) and China (CHN), as well as the former communist countries, such as Eastern Europe (EET) and the former Soviet Union (FSU), the GDP per capita and carbon prices are simultaneously at low levels. However, except for those regions, there is no clear pattern consistent with the common notion: The carbon prices of the rest are concentrated in the range of 50-90\$/tCO_2e regardless of the income level. Of course, there are some limitations in this picture because, for example, Europe (EUR) region consists of countries which have diversified industrial structures. Nevertheless, it is unnatural that the carbon price of a high income country, Canada (CAN), is lower than that of low income countries such as Indonesia (IDZ) or Mexico (MEX). This contradicts the general belief that developing countries can secure more capability of carbon reduction at a low level of carbon price than developed countries, because they have relatively low level of energy efficiency and carbon reduction technology.

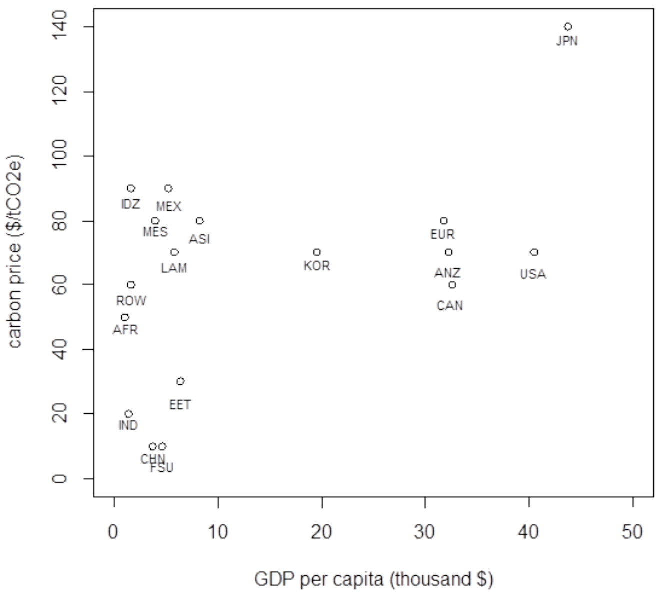

Figure 16: A simulated graph of carbon prices and GDP per capita in 2020 (EPPA).

To dive into exploring the source of the disagreement, the problem needs to be viewed from a macroscopic perspective. Table 2 summarizes the relations between CO_2 emissions and capital demand along with the estimated carbon prices for each region. In the table, (A) means the carbon prices under the assumption of 30% of CO_2 reduction, (B) is 'capital in-

Table 2: The relations between CO_2 and capital demand

region	(A) carbon price ($/tCO$_2$e)	(B) capital demand per CO_2 emissions ($/tCO$_2$e)	(C) The ratio of power sector in CO_2 emissions (%)	(D) capital demand per CO_2 emissions in power sector ($/tCO$_2$e)
USA	70	545.76	49	59.04
CAN	60	426.00	32	116.48
MEX	90	694.53	21	24.08
JPN	140	1214.59	29	333.56
EUR	80	957.18	35	103.88
EET	30	168.11	48	28.09
FSU	10	92.47	27	26.48
ASI	80	442.98	33	41.39
CHN	10	109.56	37	18.95
IND	20	192.95	40	54.48
IDZ	90	470.42	23	33.63
AFR	50	248.10	31	64.13
MES	80	262.41	18	56.80
LAM	70	901.31	12	285.47
ANZ	70	546.82	54	48.54
ROW	60	603.48	33	61.96
KOR	70	306.88	25	50.56

tensity' per CO_2 emissions, i.e., capital demands for the entire economy divided by each region's CO_2 emissions, based on GTAP database of year 1997. The concept of capital intensity is a reciprocal of carbon intensity, the widely used one.

If the entire economy can be described by a production function and CO_2 emission is regarded as one of input factors, the ratio of CO_2 emissions to the amount of capital input at the present time may indicate the substitution between these two input factors in the future. In other words, if the value of (B) is relatively high in a region, it is harder to input capital to lower carbon emissions than in other regions, which implies the region's carbon reduction activity will be relatively sluggish. For example, Japan (JPN) should pay 1,214 \$ to reduce a unit of CO_2 emissions while the former Soviet Union (FSU) spends only 92\$, which shows that it will be easier for the latter to implement policy on capital expenses to increase the reduction of carbon emissions than the former. In Figure 17 which sketches the relations between (A) and (B), there appears a clear correlation: The correlation coefficient is 0.805. From this observation, one can infer that a carbon price reflects the carbon intensity of CO_2 emissions. This is direct evidence that the difference of industrial structure, reflected in the carbon intensity, can influence carbon prices. Therefore, the values of (B) indicates the supposed average economic value of a unit of CO_2 emission in a certain region.

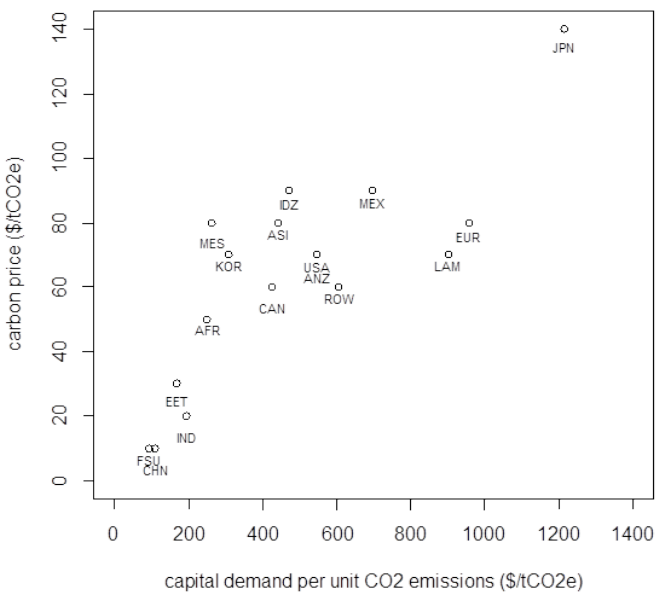

Figure 17: A simulated graph of carbon prices and capital demand per CO2 emissions in 2020 (EPPA).

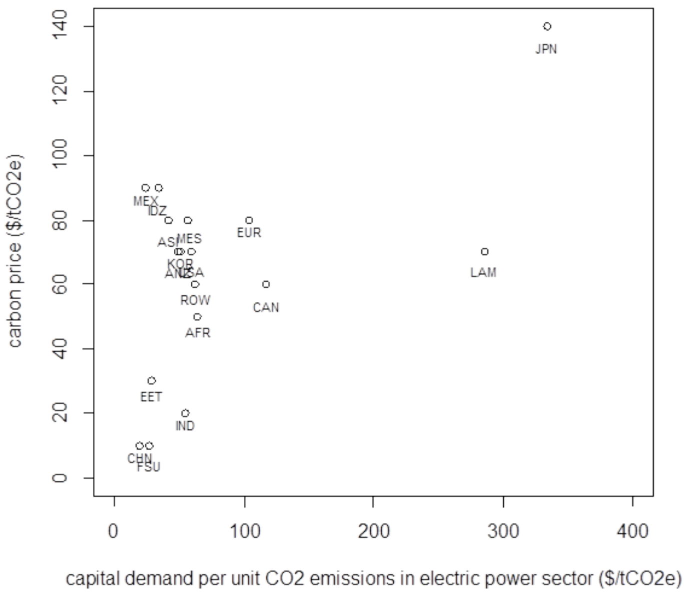

Figure 18: A simulated graph of carbon prices and capital demand per CO2 emissions in the electric power sector in 2020 (EPPA).

However, does the above observation apply also to individual sectors? To find out the answer, the electric power sector was investigated for all regions. Generally, all over the world, the electric power sector contributes the most to CO_2 emissions, followed by manufacturing, transport and construction. (IPCC, 2007) Considering that about 50% of carbon emissions allowance was allocated to the power sector in the year 2006 in the European emission trading scheme, EU-ETS, it can be easily inferred that the level of marginal abatement cost in the electric power sector will be dominant in the entire emissions trading market when the scheme is introduced. In this study, the values of (C) in Table 2 denote the shares of power sector in gross CO_2 emissions, showing around 30% of share with regional fluctuations. For the electric power generation sector, (D) of Table 2 denotes the capital input used in power sector divided by CO_2 released from the sector for each region.[3] In addition, Figure 18 depicts the relations between the capital intensity of power sector and carbon prices. In this picture, Mexico (MEX) and Indonesia (IDZ) show large deviations from the proportional pattern observed in Figure 18. In other words, in these two countries, the carbon intensity of the power sector is much lower than that of the entire economy. The carbon intensity of a sector denotes the actual technology level of that industrial sector, which means that carbon prices estimated by a CGE model fail to reflect the real economy in some regions.

Usually, a multi-regional CGE model carries out the computation only with macroscopic variables of a region or a country, excluding the detailed

[3]The data of CO_2 emissions of year 1997 was not available, then the data of year 2000 provided by Carbon Monitoring for Action (2012) was applied.

information of each industrial sector. This means an outcome which is based on the microscopic and technological information can be distorted. To put it differently, there is a chance to include capital demands of other input factor markets which are unrelated to the carbon price estimation. In fact, in the GTAP dataset, the ratios of capital demand to GDP are not over 50%, except for 64% in Mexico and 57% in Indonesia where the contribution shares of capital are relatively high. This reflects an increase of capital unrelated to the real economy or a depreciation for real property, both of which underestimate input factors other than capital. In conclusion, a CGE model is limited in its ability to filter such an imbalance among macroeconomic variables.

3.3 Mathematical analysis

3.3.1 Ratio of capital intensity

To give a more systematic analysis of the previous section, the production side of a CGE model is simplified with a mathematical model in this section.[4] First, the economy of a region is roughly divided into two sectors – energy production and general goods production – and the profit maximization problems of the representative firms of each sector are given as follows,

Energy production sector :

$$\max_{(K_1,L_1,F)} \pi_1(E) = H(K_1,L_1,F) - rK_1 - wL_1 - pF + \lambda_1(e_1 - i_1F), \quad (3.15)$$

[4] The mathematical model of this section is an modified and extended version of the analysis tools introduced in Klepper and Peterson (2006) and Copeland and Taylor (2003). The tool is combined with the concept of capital intensity newly introduced in this study.

Goods production sector :

$$\max_{(K_2,L_2,E)} \pi_2(X) = G(K_2,L_2,E) - rK_2 - wL_2 - qE + \lambda_2(e_2 - i_2E), \quad (3.16)$$

where π_1 and π_2 are the profit function for each sector, E is the output of energy production and X is the output of goods production. Also, H and G denote the production functions of energy and goods, respectively, which are increasing, homogeneous of degree one and have convex upper contour set of production possibilities. In Equation 3.15, K_1, L_1 and F are capital, labor and the amount of fossil fuel, respectively, demanded for producing energy. e_1 and i_1 are the amount of emissions from energy sector and its emission coefficient, which have the constraint of $e_1 = i_1F$. λ_1 is the Lagrangian multiplier for emissions constraint which implies the shadow price of the constraint. Likewise, from Equation 3.16, K_2, L_2 and E are capital, labor and energy, respectively, demanded for production goods under the technology H. e_2 and i_2 are the greenhouse gas emissions from the goods sector and its emission coefficient, which have the constraint of $e_2 = i_2E$ along with its shadow price of λ_2. w, r, p and q are exogenous prices of labor, capital, fossil fuel and energy, respectively. The good X is regarded as a numeraire whose price is one.

Assuming that the two sectors are not related to each other, the first order conditions for Equation 3.15 are given as follows:

$$
\begin{aligned}
H_{K_1} &= \frac{\partial H(K_1, L_1, F)}{\partial K_1} = r, && (3.17)\\
H_{L_1} &= \frac{\partial H(K_1, L_1, F)}{\partial L_1} = w, && (3.18)\\
H_F &= \frac{\partial H(K_1, L_1, F)}{\partial F} = p + i_1 \lambda_1, && (3.19)\\
e_1 &= i_1 F. && (3.20)
\end{aligned}
$$

In the same way, the conditions for Equation 3.16 are expressed as

$$
\begin{aligned}
G_{K_2} &= \frac{\partial G(K_2, L_2, E)}{\partial K_2} = r, && (3.21)\\
G_{L_2} &= \frac{\partial G(K_2, L_2, E)}{\partial L_2} = w, && (3.22)\\
G_E &= \frac{\partial G(K_2, L_2, E)}{\partial E} = q + i_2 \lambda_2, && (3.23)\\
e_2 &= i_2 E. && (3.24)
\end{aligned}
$$

These conditions determine the optimized amount of capital input, labor input, fossil fuel and energy use and carbon emissions. The carbon prices, λ_1 and λ_2, are given by rephrasing Equation 3.19 and Equation 3.23 as follows:

$$
\begin{aligned}
\lambda_1 &= \frac{1}{i_1}(H_F - p), && (3.25)\\
\lambda_2 &= \frac{1}{i_2}(G_E - q). && (3.26)
\end{aligned}
$$

These equations are obtained under the ideal condition that the two optimization problems are independently solved. However, it is unnatural that the intermediate goods E should be optimized simultaneously in both sectors. For this reason, additional assumptions are needed to improve the model.

First, let's transform the above profit maximization problems into the expenditure minimization ones by the duality property. The expenditure minimization problems are given as follows:

Energy production sector :

$$c^E(w,r,p) = \min_{(K_1,L_1,F)} rK_1 + wL_1 + pF \quad s.t. \quad H(K_1,L_1,F) = E, \qquad (3.27)$$

Goods production sector :

$$c^X(w,r,q) = \min_{(K_2,L_2,E)} rK_2 + wL_2 + qE \quad s.t. \quad G(K_2,L_2,E) = 1, \qquad (3.28)$$

where the function c^E denotes the cost function for producing E unit of energy, which is set at one by definition. Likewise, the function c^X denotes the minimized expenditure of producing one unit of goods. The above problems are interpreted as follows: If the goods production sector solves the cost minimization problem and decide to produce one unit of goods using energy E, then the energy production sector should minimize its expenditure under the energy production level E imposed by the goods production sector.

In a typical type of general equilibrium models, the sums of labor, capital and fossil fuel supplies should be constant, satisfying market clearing conditions. However, this model does not include other sectors such as household and government consumptions. The energy production E is determined in the interactions with other sectors, varying the allocations between production sectors and consumption sides. Thus, the total amount of input factors are not fixed in the above model. This enables both sectors to together achieve an equilibrium at Pareto optimal points for a given goods production level. If the sums of labor input and capital input are expressed as L and K, respectively, then the following conditions are obtained as follows:

$$c^E(w,r,p) = qE, \tag{3.29}$$

$$c^X(w,r,p) = 1, \tag{3.30}$$

$$(\frac{\partial c^E}{\partial w} + \frac{\partial c^X}{\partial w})X = L, \tag{3.31}$$

$$(\frac{\partial c^E}{\partial r} + \frac{\partial c^X}{\partial r})X = K, \tag{3.32}$$

$$\frac{\partial c^E}{\partial p} = F, \tag{3.33}$$

$$\frac{\partial c^X}{\partial q} = E, \tag{3.34}$$

$$i_1 F = e_1, \tag{3.35}$$

$$i_2 E = e_2. \tag{3.36}$$

Equation 3.29 and Equation 3.30 denote the zero profit conditions where the marginal cost of production equals the price in each market. From Equation 3.31 to Equation 3.34, the sums of individual input factors are denoted, which can vary according to the level of production. Equation 3.35 and Equation 3.36 are the carbon emissions constraints.

It is convenient to calculate a ratio for comparing capital intensities.

From the above equations, the ratio of the capital intensity of the whole economy, which puts together both goods and energy sectors, to that of the energy sector is expressed as follows:

$$\frac{\dfrac{rK}{e_1+e_2}}{\dfrac{rK_1}{e_1}} = \frac{\dfrac{rK}{i_1F+i_2E}}{\dfrac{rK_1}{i_1F}} = \frac{\dfrac{\dfrac{\partial c^E}{\partial r}+\dfrac{\partial c^X}{\partial r}}{i_1\dfrac{\partial c^E}{\partial p}+i_2\dfrac{\partial c^X}{\partial q}}}{\dfrac{\dfrac{\partial c^E}{\partial r}}{i_1\dfrac{\partial c^E}{\partial p}}} = \frac{i_1\dfrac{\partial c^E}{\partial p}\left(\dfrac{\partial c^E}{\partial r}+\dfrac{\partial c^X}{\partial r}\right)}{\dfrac{\partial c^E}{\partial r}\left(i_1\dfrac{\partial c^E}{\partial p}+i_2\dfrac{\partial c^X}{\partial q}\right)}. \quad (3.37)$$

For an application of the above equation to the two-sector case – electric and non-electric – in the previous section, the goods and energy sectors can be substituted with non-electric and electric sectors, respectively. This way, Equation 3.37 becomes

$$\frac{r\bar{K}/(e_1+e_2)}{rK_1/e_1} = \frac{i_1\left(1+\dfrac{\partial c^X}{\partial r}\Big/\dfrac{\partial c^E}{\partial r}\right)}{i_1+i_2\dfrac{\partial c^X}{\partial q}\Big/\dfrac{\partial c^E}{\partial p}}. \quad (3.38)$$

As the capital input K_1 for producing a unit of energy E decreases, in other words, $\frac{\partial c^E}{\partial r}$ decreases, the carbon intensity ratio in Equation 3.38 is raised. An increase of $\frac{\partial c^E}{\partial p}$ also amplify the ratio value. As the production possibility set of H is convex, the decrease of $\frac{\partial c^E}{\partial r}$ and the increase of $\frac{\partial c^E}{\partial p}$ are identical in meaning, contributing to the increase of carbon intensity ratio in the same direction. Likewise, if the capital input K_2 for producing a unit of goods E increases, i.e., $\frac{\partial c^X}{\partial r}$ increases, the ratio increases. This has the same direction with an decrease of $\frac{\partial c^X}{\partial q}$ by the convexity of the production possibility set of G. To put it another way, from the aspect of capital, the capital intensity ratio in Equation 3.38 increases when the share of capital in producing energy is lowered or the share of capital in the whole economy is raised. From the aspect of energy input, this is equivalent to the case of increasing consumption of fossil fuels to replace capital or less consumption of energy in the economy.

Again, in terms of energy production, the carbon price of Equation 3.25 increases as H_F increases. However, the production function H is concave and its first order derivative H_F decreases, i.e., $\frac{\partial^2 H}{\partial F^2} < 0$ when fossil fuel input F increases. Thus, if the use of fossil fuel remains at a high level, the energy production sector shows relatively low level of abatement cost. However, in this case, the ratio in Equation 3.38 is also raised. Therefore, a large capital intensity ratio does not mean a high level of carbon price. On the other hand, in the goods production sector, the production function G is also concave, like H, and $\frac{\partial^2 G}{\partial E^2} < 0$, by which the carbon price is lowered with an increase of E in Equation 3.26. However, the increase of E also lowers the ratio of Equation 3.38. Therefore, in regards to goods production, carbon price has a proportional relation with capital intensity ratio.

Returning to Table 2, the reason why carbon prices are overestimated in Mexico and Indonesia is that the contribution of capital is exaggerated over the entire economy, especially in the sectors unrelated to energy production, when using the CGE model. Considering the fact that developing countries usually show a high dependence on fossil fuels in energy production, carbon prices remain at low levels by Equation 3.25. However, in the description of the economy of Mexico and Indonesia by the CGE model, EPPA, the low level of abatement costs were dominated by the large contribution of capital in the entire economy. In terms of energy use, a low contribution of energy use in GDP reveals that capital-intensive service industry has more share than heavily energy-intensive manufacturing industries. But, some CGE models which ignore the regional differences in industrial structure may possibly fail to reflect the actual situations.

3.3.2 Extensions to the CES function

In the previous section, the shares of capital and energy in a market was denoted as the derivative terms such as $\frac{\partial c^X}{\partial r}$ or $\frac{\partial c^X}{\partial q}$. So, there is a possibility that the functional forms of such derivatives themselves amplify the over-estimation or under-estimation of carbon prices. In this sense, the CES functional form is investigated, which is employed in the EPPA model. Assume that a two-input CES production function is given by

$$f(x_1,x_2) = (\theta_1 x_1^{\rho} + \theta_2 x_2^{\rho})^{\frac{1}{\rho}}, \qquad \theta_1 + \theta_2 = 1, \tag{3.39}$$

where x_1 and x_2 denote the two individual factors, θ_1 and θ_2 are the distribution parameter between the two factors and ρ is related to the elasticity of substitution $\sigma = \frac{1}{1-\rho}$. The corresponding cost function is also a CES function with elasticity substitution $1/\sigma$ and is given by

$$c(w_1,w_2,y) = y(\theta_1^{\sigma} w_1^{1-\sigma} + \theta_2^{\sigma} w_2^{1-\sigma})^{\frac{1}{1-\sigma}}, \tag{3.40}$$

where w_1 and w_2 are prices of the input factors. With the assumption that the production y is one, the derivative of the cost function is expressed as

$$\frac{\partial c(w_1, w_2)}{\partial w_1} = \left[\theta_1 + \theta_2^{\sigma}\theta_1^{1-\sigma}\left(\frac{w_2}{w_1}\right)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}}. \tag{3.41}$$

The valuation of the derivative can be easily done by hand for the following special cases.

If $\sigma = 0$,

$$\frac{\partial c}{\partial w_1} = \left[\theta_1 + \theta_1\left(\frac{w_2}{w_1}\right)\right]^0 = 1 \tag{3.42}$$

If $\sigma = 1$,

$$\frac{\partial c}{\partial w_1} = \lim_{\sigma \to 1}(\theta_1 + \theta_2)^{\frac{\sigma}{1-\sigma}} = \lim_{\sigma \to 1}(1)^{\frac{\sigma}{1-\sigma}} = 1 \tag{3.43}$$

However, for the other values of elasticity of substitution σ, it is convenient to be assisted by a computer. First, the derivative was evaluated for three cases of $(\theta_1, \theta_2) = (0.25, 0.75), (0.5, 0.5), (0.75, 0.25)$ when $w_2/w_1 = 1$, which means the prices of the two input factors are the same. The results are shown in Figure 19. When the distribution parameters are the same, the values of the derivative are fixed at one regardless of the substitution parameter. Thus, if there are two sectors which individually have their own cost functions, the ratio between the two derivatives will remain unchanged from one in this case. However, in the other cases where the distributions between input factors are unbalanced, the ratio of the derivatives between the two sectors will change. If one sector has the distribution of (0.25, 0.75) and the other has (0.75, 0.25), like the case of Mexico and Indonesia where different distributions appeared among sectors, the ratio of the derivative of the latter to that of the former will be always larger than one, except for $\sigma = 0$ and 1.

In a CES function based CGE model, the distribution parameters are determined by the dataset of the reference year. If an input factor outweighs the others in terms of the contribution to GDP, the calibration results at that time remains in the future projections. In other words, there is virtually no chance to adjust the once-determined distribution parameters even if the actual distributions change in other time periods. This is the reason behind the carbon price distortions in the previous section.

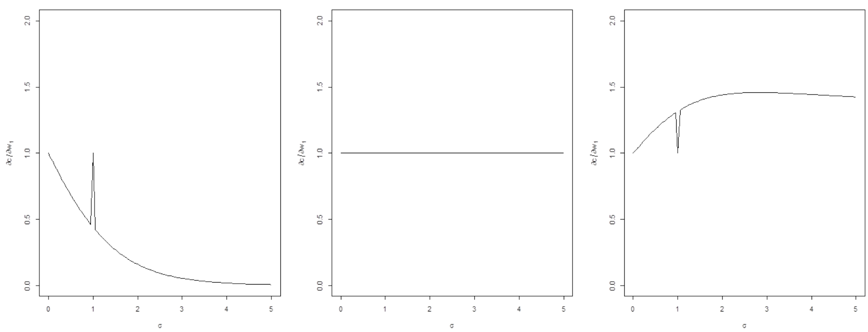

Figure 19: Plots of $\dfrac{\partial c}{\partial w_1}$ for $(\theta_1, \theta_2) = (0.25, 0.75), (0.5, 0.5), (0.75, 0.25)$, when $w_2/w_1 = 1$.

For the other cases of $w_2/w_1 = 2$ and $w_2/w_1 = 1/2$, Figure 20 and Figure 21 depict the plotting results. In these cases, the even distributions no longer guarantee an acceptable projection of carbon prices. In the former case, all graphs were raised compared to the case of same prices. On the contrary, the latter shows lowered graphs.

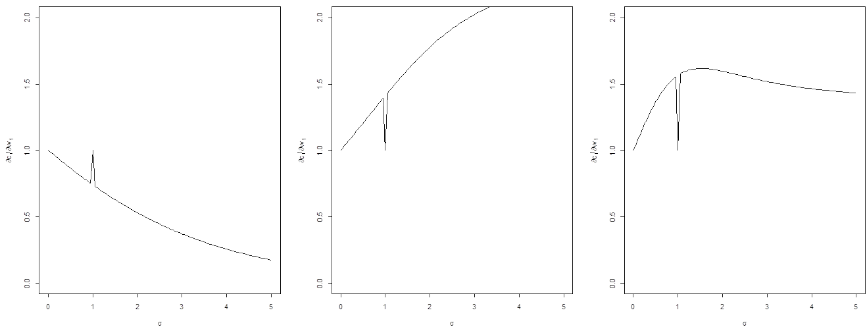

Figure 20: Plots of $\dfrac{\partial c}{\partial w_1}$ for $(\theta_1, \theta_2) = (0.25, 0.75), (0.5, 0.5), (0.75, 0.25)$, when $w_2/w_1 = 2$.

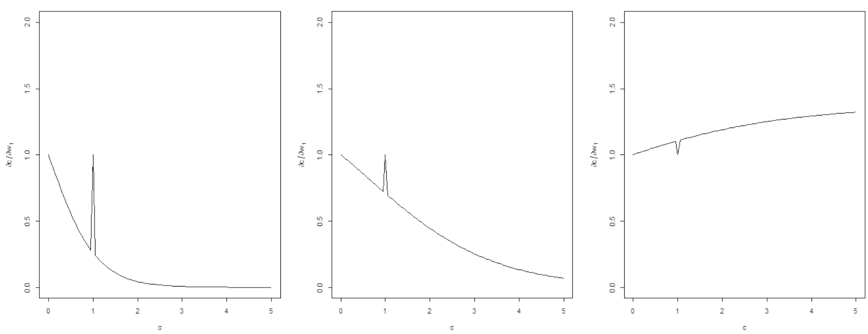

Figure 21: Plots of $\dfrac{\partial c}{\partial w_1}$ for $(\theta_1, \theta_2) = (0.25, 0.75), (0.5, 0.5), (0.75, 0.25)$, when $w_2/w_1 = 1/2$.

To suppress the influence of distribution parameters on carbon price estimation or other macroscopic variables, it is recommended to make use of the favorable cases in which the ratio of cost function derivatives is invariant to the changes in distribution or price vectors. From the observations, the exclusive cases are only when the elasticity of substitution is 0 or 1. The latter case of parameter 1 means that the CES function becomes the well known function, Cobb-Douglas function, which has also various limitations in depicting the real economy. Thus, it is necessary to find an effective way of taking advantage of the other option, Leontief functions of zero elasticity of substitution, in building a robust CGE model.

3.4 Conclusion

In this chapter, the influence of regional disaggregation, i.e., the difference in regional industrial structure on marginal abatement cost estimation in CGE models was investigated. In most researches with CGE models, it is assumed that the model structure can globally encompass almost all types of economic situations. However, the functional structures in general CGE models turned out to be unsuitable for some cases of developing countries. Specifically, the CES function structure with calibrated parameters were not able to cover a unique situation in which the share of capital in an economy is relatively high.

Marginal abatement cost curves for each region were derived using EPPA model and, unexpectedly, some developing countries exhibited high levels of carbon price. This chapter tried to explain this issue by introducing the concept of capital intensity. A connection was empirically discovered between capital intensity and carbon price and an analysis was done using a simple mathematical model. It was shown that the larger the capital share is in the entire economy, the higher the carbon price becomes. The analysis was extended to CES functions with a numerical analysis, concluding that the unusual phenomena may have some connections with distribution parameters of CES functional forms which are the most widely used in CGE models.

This chapter serves to provide empirical evidence of carbon price distortions due to the limitation embedded in a set of CES functions of global CGE models. Researchers are aware of this limitation with parameter calibrated to a benchmark year, but there has been little work about the influences of such limitation on projection results. In this sense, this research provides a good example for understanding the structural limitations of global CGE models. Also, this finding has important implications for the recent trend of data disaggregation: regional or sectoral disaggregation should be followed by investigations on the coverage of a specific model structure for each region or sector.

Chapter 4

The statistical distribution approach for a description of production activities

4.1 Introduction

Usually, a production technology is described as a combination of functions and parameters in a model. In many models, functions are considered to be merely convenient regression tools, not originating from any in-depth analysis on the microscopic mechanisms. This is why parameters should have their own statistical distributions to encompass the deviations from such microfoundations. If a mathematical function can perfectly reflect the dynamics of an economy, there should be no uncertainty with parameters employed in the function. However, most economic models do not allow any vagueness in the computational structures although they are not 'perfect prophesiers.' In this sense, it is natural to expect a series of activities to overcome such limitations by considering the probabilistic characteristics of parameters.

This issue is related to the long-standing debate over whether CGE models should be constructed using calibration or econometric methods. In the calibration method, parameters are determined by a survey of empirical literature or arbitrarily chosen for a model to replicate the data of a reference year, completely eliminating the stochastic properties. It has been criticized on the expediencies, the dependence on the quality of the reference data, and the limitation in selecting functional forms. (McKitrick, 1998) On the other hand, the estimation method can give statistical information of the parameters. However, the estimation work is often affected by data insufficiency, and consequently, there are concerns about the statistical reliability of econometric estimates.[1] (Shoven and Whalley, 1984)

However, little attention has been paid to the issue of selecting functional forms in economic models including CGE, which is astonishing because the process of parameter estimation is subject to the choice of functional forms. As a matter of fact, there have been incessant efforts to devise flexible and globally well-behaved functional forms. Such improvements were focused on the issues of consistency with theoretical restrictions of general-equilibrium theories or analytical tractability in evaluating the supply and demand responses for any price vector. As a result, preferences are limited to several frequently used functional types such as Cobb-Douglas, constant elasticity of substitution (CES), and some flexible functional forms including translog, which are usually expressed as second-order Taylor expansion. (Shoven and Whalley, 1984)

Historically, the choice of specific functions has been related to the

[1]One of the practical solutions compromising between the two sides is Bayesian approaches, in which prior information about parameters are incorporated into the estimation with observed data. Adkins et al. (2003) uses a Bayesian approach to estimate the parameters of a translog production function in a regional CGE model. However, reportedly, Bayesian approaches also have many challenges such as dimensionality of the parameter space as well as many difficulties arising from its non-linear state-space representation. (An and Schorfheide, 2007)

characteristics of elasticities.[2] In spite of their simplicity, Cobb-Douglas functions have problems with the restrictions of unitary income and uncompensated own-price elasticities and zero cross-price elasticities. CES functions solve the problem of unitary own-price elasticities, but they also have an unacceptable property: if input shares are close to zero, the compensated own-price elasticities approach the value of elasticity of substitution. Hierarchical or nested CES functions, proposed by Sato (1967), relax the problem as well as the unitary income elasticity feature of the Cobb-Douglas functions. In composing a production function structure, nested CES functions are the most widely used to allow for substitution between input factors although there is difficulty in choosing a nesting structure among various ones.

Another way to relax the restrictions on elasticities is to use flexible functional forms, such as the translog or normalized quadratic. By giving a second-order expansion approximation, they allow enough free parameters to obtain the relevant elasticities without imposing prior constraints. (McKitrick, 1998) However, a flexible functional form does not have proper curvature and can cause a failure in numerical calculation with an economic model. (Adkins et al., 2003) Although Caves and Christensen (1980) shows that the regions, in which a locally well-behaved production function is defined, can be large for some parameters, a lack of global regularity still remains problematic in most flexible functional forms.[3] (Perroni and Ruther-

[2]According to Shoven and Whalley (1984), this point is best illustrated by considering the demand side. Thus, this argument follows the convention.

[3]The regularity conditions denote monotonicity and strict quasi convexity of a production or utility function. (Caves and Christensen, 1980)

ford, 1998; Guilkey et al., 1983; Chang, 1994) In addition, modeling sophisticated technologies with flexible functional forms can aggravate the burden of estimation in terms of data availability as stated in the above argument on calibration and estimation.

Whether more flexible functional forms can be invented has no direct connection with the ability of relating a functional form to any microfoundations. More flexibility provides more mathematical platforms, but does not converge to a unique functional form derived on the basis of microfoundations. This point has been ignored in many economic modeling researches. As a positive response to this problem, there is a new set of approaches, reflecting the correlations between the activities of individual economic agents. An example is the agent based models assisted by much accumulated work of microscopic statistical distribution analysis. (Delli Gatti et al., 2008) The information on the distribution of a variable can give new perspective about the combination of heterogeneous microscopic foundations with a traditional macroscopic economic model which is usually based on the assumption of the homogeneous representative agent.

The new approaches based on heterogeneous microscopic information can also be found in interpreting conventional functional forms although there has been very little work on the microeconomic foundations of production functions. (Dupuy, 2012) Aside from relatively abundant work based on the analysis of microscopic mechanism in labor market, (Rosen, 1978; Caselli and Coleman, 2006; Lagos, 2006) there are efforts to link an aggregate production function with input factor distributions. Houthakker (1955-1956) derives Cobb-Douglas aggregate production function from the generalized Pareto type of distributions of input factors and Levhari (1968) reverses the procedure of Houthakker (1955-1956) to derive the distribution function for a CES production function. Jones (2005) shows that when the unit factor productivity of each input follows an independent Pareto distribution it leads to a Cobb-Douglas production function while Growiec (2008a) and Growiec (2008b) try to relax the restriction of independence and extend it to a CES function.

These researches can justify the use of conventional functional forms if the real economy is consistent with the assumptions on microscopic information or distributions. However, the marginal distribution and correlation behaviors of economic variables often show large deviations from such assumptions according to the characteristics of a chosen dataset. Also, when energy input is considered, additional in-depth studies should be done about various issues such as energy factor productivity. In this sense, it is necessary to consider the possibility of using a joint distribution map as a description of production activities, instead of trying to derive explicit functions which may inevitably have complex forms to satisfy the relaxations of ideal assumptions. In this chapter, it is revealed that a number of fixed input structures can converge to a multivariate distribution map and it can implicitly take over the role of conventional production functions.

The distribution map needs to be given some variations for statistically meaningful simulations. In this sense, Iyetomi et al. (2012) introduces a convenient way called copulas to simplify and parameterize the distribution map of microscopic interactions. A copula is a function that combines a multivariate distribution function with its one-dimensional marginal distribution functions. Technically, a copula is a multivariate distribution function whose one-dimensional margins are uniform on the interval $[0,1]$. (Sklar, 1959; Nelsen, 2006) Of course, copulas are merely one method among many to treat the dependence in multivariate statistics but they facilitate the bottom-up approaches in mapping and reproducing a part of the economy. (McNeil et al., 2005) Instead of cramming the marginal information of individual factors into deterministic macroscopic equations, the copula method enables us to choose between a variety of possible dependence models without a loss of the stochastic properties of each variable. This is why copulas have recently attracted interest from business practitioners. (Cherubini et al., 2004) Likewise, they can also be applied in economic modeling which should consider the preservation of the information of an economy as a priority.

This chapter introduces some theoretical models related to the micro-foundations of conventional production functions to verify the splits between ideal forms of production functions and the real economy. Then, this argument is extended to energy-related production sector in a CGE model. A pilot model is devised in which conventional nested CES production function structures are replaced with a set of firms which can be interpreted as a collection of local production technologies. The set of firms can be considered as an asymptotically converged joint distribution map. To conveniently reproduce the statistical map of the real economy for actual simulations, a copula model is composed. From an estimated copula model, multiple sets of firms are generated to compose replicate joint distribution maps. In this way, a confidence interval of projection results is derived. The results of this chapter illustrate more statistically robust outcomes can be obtained with the new pilot model compared to conventional CES function based models.

4.2 Functional forms and data distribution

4.2.1 Microfoundations of production functions

Houthakker (1955-1956), the classical literature on the linkage between production function and input distribution, considers a set of 'production cells,' which may be thought of as individual firms. Suppose that a particular cell i needs labor L_i and capital K_i to produce one unit of output Y_i by a Leontief technology. The output 'possibility' can change according to the combination of (L_i, K_i), so the possibility distribution is expressed as $\varphi(L_i, K_i)$. The prices of Y_i, L_i and K_i are p_Y, p_L and p_K. Thus, the profit function will be $p_Y - p_L L_i - p_K K_i$.

If some cells lie in an area defined by $[L_i, L_i + dL_i]$ and $[K_i, K_i + dK_i]$, the total production capacity may be written as $\varphi(L_i, K_i) dL_i dK_i$. By integrating over all the values of L_i and K_i for which production is profitable, i.e., $p_Y - p_L L_i - p_K K_i > 0$, one can obtain total output Y and total inputs L and K. Houthakker (1955-1956) shows that, when the production distribution is of the generalized Pareto type as follows,

$$\varphi(L_i, K_i) = A L_i^{\alpha-1} K_i^{\beta-1}, \quad \alpha \geq 1, \beta \geq 1, \tag{4.1}$$

the following Cobb-Douglas functional form is obtained:

$$Y \propto L^{\alpha/(\alpha+\beta+1)} K^{\beta/(\alpha+\beta+1)}. \tag{4.2}$$

Similarly, Levhari (1968) shows that a CES production function with elasticity of substituion smaller than 1 reaches a unique complex distribution function.

The model of Jones (2005) starts from pointing out a critical problem of the above model. Because of the presence of capacity constraints, if one wants to expand output, he or she has to add additional production units of lower quality technologies although the best option is to allow the firm to take its best idea and use it for every unit of production. As a result, the Cobb-Douglas style of aggregate production function is characterized by decreasing returns to scale. To avoid the constraints on production size, Jones (2005) considers a local or firm-level production function for a representative firm.

The local production technology is expressed as the following Leontief function f:

$$Y = f(a_i L, b_i K) = \min\{a_i L, b_i K\}, \tag{4.3}$$

where a_i and b_i mean the unit factor productivities (UFPs) of labor L and capital K, respectively, for a technology set i, which is called an 'idea' in Jones (2005).

Assume that the UFPs are drawn from independent Pareto distributions as follows:

$$\begin{aligned}
\Pr[a_i \leq a] &= 1 - \left(\frac{a}{\gamma_a}\right)^{-\alpha}, \quad a \geq \gamma_a > 0, \\
\Pr[b_i \leq b] &= 1 - \left(\frac{b}{\gamma_b}\right)^{-\beta}, \quad b \geq \gamma_b > 0,
\end{aligned} \tag{4.4}$$

where $\alpha > 0$, $\beta > 0$, and $\alpha + \beta > 1$.[4] Then the joint distribution of the two UFPs are given by

$$G(a,b) \equiv \Pr[a_i > a, b_i > b] = \left(\frac{a}{\gamma_a}\right)^{-\alpha} \left(\frac{b}{\gamma_b}\right)^{-\beta}. \tag{4.5}$$

Let Y_i denote output using technology i. Since the production function is Leontief, its distribution is given by the following Pareto distribution:

$$\begin{aligned} H(y) \equiv \Pr[Y_i > y] &= \Pr[a_i L > y, b_i K > y] \\ &= G(y/L, y/K) \\ &= \gamma L^{\alpha} K^{\beta} y^{-(\alpha+\beta)}, \end{aligned} \tag{4.6}$$

where $\gamma \equiv {\gamma_a}^{\alpha} {\gamma_b}^{\beta}$.

Let N denote the total number of available production technology. If N technologies are chosen independently, then the global production function $Y = F(L, K; N)$ is given as follows,

$$F(L,K;N) \equiv \max_{i=1,\dots,N} f(a_i L, b_i K), \tag{4.7}$$

which reveals that the global production function is the convex hull of local production functions. Thus, the distribution of the global production function should satisfy

$$\begin{aligned} \Pr[Y \leq y] &= (1 - H(y))^N \\ &= (1 - \gamma L^{\alpha} K^{\beta} y^{-(\alpha+\beta)})^N. \end{aligned} \tag{4.8}$$

[4] The condition that the sum of the two parameters should be greater than one is needed for the existence of the mean of the Fréchet distribution.

If a normalization is taken as $z_N \equiv (\gamma N L^{\alpha} K^{\beta})^{1/(\alpha+\beta)}$, the above equation can be rewritten as

$$\Pr[Y \leq z_N y] = \left(1 - \frac{y^{-(\alpha+\beta)}}{N}\right)^N . \tag{4.9}$$

Using $\lim_{N\to\infty}(1-x/N)^N = \exp(-x)$ for any fixed value of x, we have

$$\lim_{N\to\infty} \Pr[Y \leq z_N y] = \exp(-y^{-(\alpha+\beta)}), \tag{4.10}$$

for $y > 0$. This distribution is a Fréchet distribution. Therefore,

$$\frac{Y}{(\gamma N L^{\alpha} K^{\beta})^{1/(\alpha+\beta)}} \overset{a}{\sim} \text{Fréchet}(\alpha+\beta). \tag{4.11}$$

That is, the global production function converges asymptotically to a Fréchet distribution. If ε is a random variable drawn from the distribution, the global production function is rewritten as

$$Y \approx (\gamma N L^{\alpha} K^{\beta})^{1/(\alpha+\beta)} \varepsilon, \tag{4.12}$$

which shows that the global production function is a Cobb-Douglas function. A similar study of Growiec (2008b) shows that the CES production function is associated with Weibull distributions of UFPs. Conclusively, these researches reveal that production functions and UFP distributions can be transformed into each other, verifying that both have the same microscopic information. In other words, it is verified that the widely used functional forms, such as Cobb-Douglas and CES, implicitly have prior assumptions on the microfoundations. Therefore, if the assumptions of UFP distribution are violated in the above models, the shapes of production functions may deviate from the well known ones.

Jones (2005) introduces various examples showing Pareto distribution, including Axtell (2001) and Kortum (1997) to support the validity of its model. However, an empirical study of Cabral and Mata (2003) reveals that a firm size distribution can evolve toward a lognormal distribution as time passes while regional difference also exists across the world. Figure 22 shows the marginal distributions of UFPs for a data set, which will be used throughout this chapter.[5]. With the help of Table 3, it can be said that the UFPs derived from this data follow log-normal distributions. These observations hint that the assumptions in Jones (2005) and Growiec (2008b) is no longer valid in general cases. Thus, this can raise some questions on the employment of those conventional functional forms in economic models.

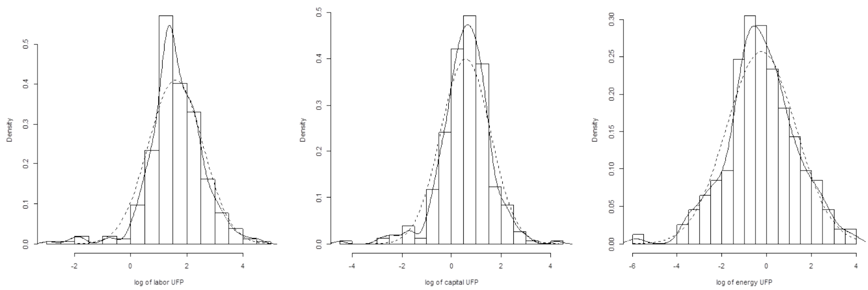

Figure 22: Histograms of unit factor productivity (UFP) of labor, capital and energy use. Solid lines depict the kernel density estimates, and dotted lines show the estimated normal distributions which have the same mean values and standard deviations with the corresponding distributions.

On the other hand, Growiec (2008a) goes one step further: It allows

[5]A detailed description of this data set is given in Section 4.2.2

Table 3: D-values of Kolmogorov-Smirnov tests for the unit factor productivities (UFPs) of the input variables. The values in parentheses are p-values for the null hypothesis of log-normal distribution.

	labor UFP	capital UFP	energy UFP
D-value	0.069	0.058	0.060
(p-value)	(0.212)	(0.397)	(0.361)

for dependence between the marginal UFP distributions. Despite a lack of any empirical evidence on the pattern of dependence, it adopted a Clayton copula to encompass such a generalization.[6] A Clayton copula is written as

$$C(u,v) = \max\{0, (u^{-\delta} + v^{-\delta} - 1)^{-1/\delta}\}, \tag{4.13}$$

where u and v are uniformly distributed over $[0,1]$, which are usually substituted with cumulative distribution functions. δ captures the degree of dependence between the marginal distributions and its support is $[-1,\infty)$. If $\delta < 0$, the variables are negatively correlated while $\delta > 0$ means they are positively correlated. $\delta = 0$ denotes independence which is the assumption of Jones (2005). With the already given Pareto distribution, the two-dimensional distribution for a technology set (a_i, b_i) is given by

$$\Pr[a_i > a, b_i > b] = \max\left\{0, \left[\left(\frac{a}{\gamma_a}\right)^{\alpha\delta} + \left(\frac{b}{\gamma_b}\right)^{\beta\delta} - 1\right]^{-1/\delta}\right\}, \tag{4.14}$$

[6]The basic theory of copulas is briefly introduced in Section 4.3.1

if $\delta \in [-1, \infty) \setminus \{0\}$, or

$$\Pr[a_i > a, b_i > b] = \left(\frac{a}{\gamma_a}\right)^{-\alpha} \left(\frac{b}{\gamma_b}\right)^{-\beta} \tag{4.15}$$

if $\delta = 0$. The latter is equivalent to Equation 4.5 in Jones (2005).

With some assumptions on the local production function from Caselli and Coleman (2006), Growiec (2008a) shows that the shape of a global production function is affected by the dependence parameter δ and some parameter values even violate the diminishing marginal utility requirement. Therefore, if there exists dependence between UFPs, the usage of Cobb-Douglas or CES functions cannot be justified. Figure 23 and Table 4 reveals that there exists dependence in the data set of this study, especially in the relationship between capital and energy technologies. Thus, at least in the modeling work for energy-related production activity in this study, those traditional functions are not the best options.

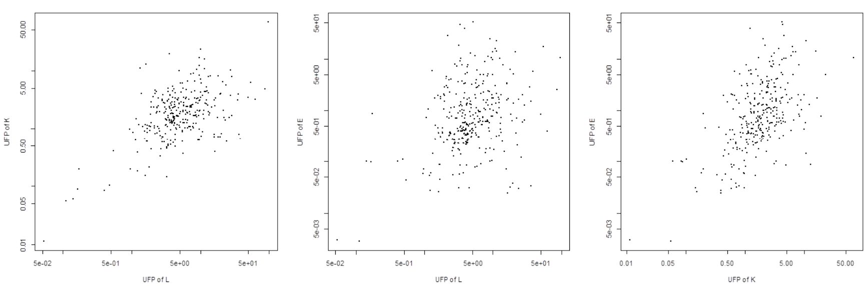

Figure 23: Scatter plots for all pairs of unit factor productivities of the input factor.

There remains another side to this argument. Even if an economy satisfies both assumptions – Pareto distribution and UFP independence, there can

Table 4: Correlation coefficients for all pairs of unit factor productivities of the input factor.

	labor UFP	capital UFP	energy UFP
labor UFP	1.000	-	-
capital UFP	0.399	1.000	-
energy UFP	0.232	0.500	1.000

still be size effects on UFPs, which is ignored in Jones (2005) and Growiec (2008b). For example, in Figure 24 and Table 5, UFPs show some correlations with the size of value added, including the relatively strong connections in the case of energy input.

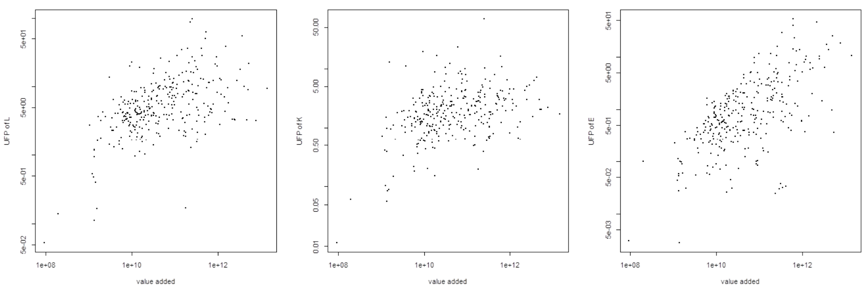

Figure 24: Scatter plots depicting the correlations of value added with each of the unit factor productivities of the three inputs.

Table 5: Correlation coefficients between value added and the three unit factor productivities.

	labor UFP	capital UFP	energy UFP
correlation with value added	0.476	0.366	0.597

Lastly, the question of whether the flexible functional forms are a possible candidate in substituting for the conventional functions still remain. Despite the issue of regularity, at a glance it seems that the flexibility with various high-order cross terms can help encompass the information of microscopic interactions. However, a flexible linear model should inevitably need more parameters to reproduce the complex non-linearity of the functions which would have concrete microfoundations. The number of parameters is restricted by the data availability as stated in the argument on the debate of calibration and estimation in Section 4.1. Anderson and Thursby (1986) shows that the estimation of the elasticities using translog models often provides no information regarding the structure of technology – the confidence intervals span both positive and negative values.

Fundamentally, parameters estimated from time-series data or cross-sectional data tend to inevitably ignore the structural change for a time period as well as the regional or sectoral difference. Also, considering the ultimate purpose of using economic models, the parameter estimation can be no more than burdensome interim processes in some cases. For these reasons, this study pays attention to the distributions of microscopic information rather than stick to conventional functional forms. This trial can be verified by the fact that a group of distributions can have information equivalent to a certain explicit functional forms as already proven in Houthakker (1955-1956) and Jones (2005). In the following series of sections, this alternative approach will be realized in the composition of a new type of CGE model after the distributional properties of a real dataset are examined and a convenient statistical tool called copulas is introduced.

4.2.2 Data analysis

Before getting into the specifics on the statistical distribution method, empirical surveys about the microscopic information with a specific data sample should be conducted beforehand. This section provides a preliminary overview of the characteristics of the data set employed in this study.

The financial information of individual firms in Korea was gathered from DART (Data Analysis, Retrieval and Transfer System) of Financial Supervisory Service of Korea (FSS) (2012). The service provides the financial statements of firms and the reported data were transformed into the fundamental quantities such as labor cost, capital and value added. While labor cost (L) clearly appeared in the reports, capital (K) was substituted by interest expense and depreciation of tangible asset, except land, plus intangible asset. The calculation of capital is based on the assumption that a company only uses the fixed asset for production, which is not movable in an annualized period. Value added (Y) was calculated by adding wages and salaries, fringe benefits, depreciation costs, public dues, paid value added tax and operating profits, according to Statistics Korea (2012). As for the energy (E) variable, Greenhouse Gas Inventory & Research Center of Korea (GIR) (2012) has accumulated the records of each firm's energy use[7] and greenhouse gas emissions for the past several years. It monitors companies which consume more than 500 TJ of energy or discharge over 125,000 tCO_2eq[8] to collect fundamental data in preparation for a carbon tax or emission trad-

[7]The energy use data exclude the cases of using energy resources as raw materials in an industrial process.

[8]These standards are valid until 2011 and the Korean government keeps lowering the entity thresholds to make its greenhouse gas policy more rigid.

ing scheme. The average price of energy was estimated from Korea Energy Economics Institute (2012b). Three hundred and eight firms were selected, which simultaneously belong to both databases in 2010.[9] The number of firms in each sector is as follows: 29 in cement & ceramic, 23 in electronic, 16 in food & beverage, 26 in machinery, 45 in steel & metal, 48 in wood & paper, 64 in petroleum & chemistry, 17 in building, 23 in power generation, and 17 in others.

First, firm size distributions are derived and investigated for each variable. Firm size distributions have been widely studied since Gibrat (1931) and they have often been described by lognormal distributions. This is known as the law of proportional effect, or as Gibrat's law, in which each firm's growth is considered a random process. After that, various types of distribution functions or regression models have been proposed to fit the empirical data of size distributions, which often have long tails. (McDonald, 1984; Axtell, 2001; Kleiber and Kotz, 2003; Yang and Tse, 2006)

Figure 25 shows the histograms of the quantities in logarithmic scale, along with estimated kernel density graphs and normal distributions which have the same mean values and standard deviations with the observed dataset. According to Gibrat (1931), any quantities related to firm sizes should have lognormal distributions, that is, their density plot in logarithmic space should converge to normal distributions. However, in Figure 25, all graphs of density estimations are skewed to the right although K seems to be the least skewed and close to a normal distribution.

The values of skewness and D-value, the statistic of two sided Kolmogorov-

[9] Fifteen firms with negative Y have been excluded from the dataset.

Smirnov goodness-of-fit test, are listed in Table 6. The skewness values show that E has the most skewed pattern and the D-value test points that E is the most aberrant from a normal distribution. Considering both statistic values, K and Y comparatively seem more close to a normal distribution than any other quantities. However, none of the quantities reject the normality hypothesis at a significance level of 0.01 in the two-sided Kolmogorov-Smirnov tests.

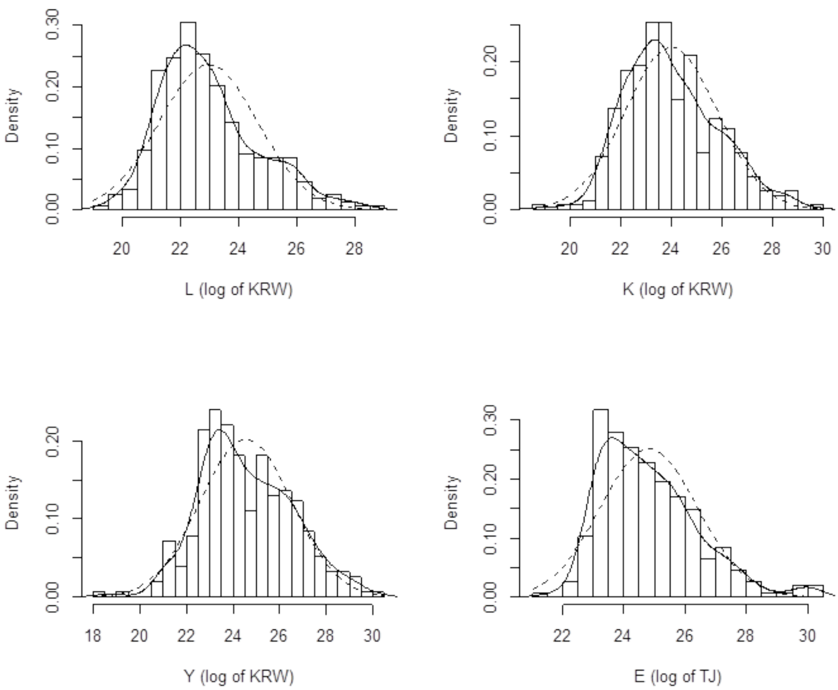

Figure 25: Histograms of labor cost(L), capital(K), value added(Y) and energy use(E). Solid lines depict the kernel density estimates, and dotted lines show the estimated normal distributions which have the same mean values and standard deviations with the corresponding quantities.

Table 6: Skewness statistics and D-values of the Kolmogorov-Smirnov test

	$\ln L$	$\ln K$	$\ln Y$	$\ln E$
skewness	0.749	0.439	0.250	0.944
D-value	0.086	0.075	0.086	0.091
(p-value)	(0.059)	(0.137)	(0.062)	(0.040)

Iyetomi et al. (2012) plotted the complementary cumulative distribution functions (CDFs) for the fundamental quantities and fitted the data with a generalized beta distribution of the second kind (GB2), considering each quantity's power-law tail property. However, K and Y in this study do not converge to any of GB2 functions. Figure 26 shows the plots of the complementary CDFs, in which K and Y is better fitted to lognormal function.[10]

The discrepancy between the Iyetomi et al. (2012) and this study can be accounted for by the difference in sample size and the problem of data truncation.[11] Segarra and Teruel (2012) points that the fitting results of a power-law firm size distribution depend on the sampling size. It also mentions that even the ages of firms can affect the result. Capasso and Cefis (2012) shows that, when endogenous or exogenous thresholds truncate the firm size distribution, a bias can arise in the estimation of the relation between firm size and variance in growth rates. Similarly, as firms in the sample data set were selected by a criterion of energy use amount in this study, the result of data analysis can be different from that of the case with the entire population of

[10]In this study, the data were fitted with a vector generalized linear model (VGLM) using iteratively reweighted least squares (IRLS) method.

[11]A firm sizes distribution can be also affected by time factor. Angelini and Generale (2008) provides a thorough and quantitative analysis of the effect of time evolution on firm sizes distributions.

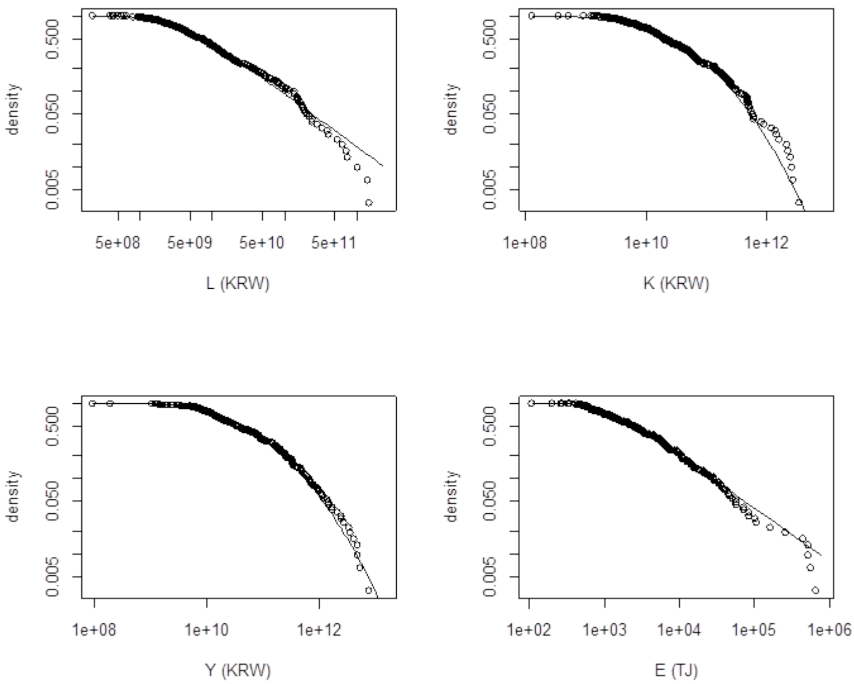

Figure 26: Graphs of the complementary CDFs of labor cost(L), capital(K), value added(Y) and energy use(E) in logarithmic plotting frame. Solid lines depict the results of fitting to real data: L and E are fitted to GB2 functions while K and Y to lognormal functions.

firms. However, the issue of data truncation can be excused in this study because the main target of it is the bottom-up analysis of *energy*-related industries. That is, this study focus on the microscopic foundations of only energy intensive sectors, not on a general picture of the entire economy.

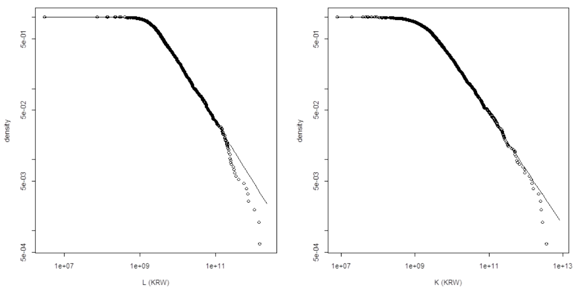

Figure 27: Logarithmic plots of the complementary CDFs of labor cost(L), capital(K) and value added(Y) for the firms listed on Korea's stock markets. Solid lines depict the results of fitting to real data: All quantities are fitted to GB2.

Returning to the issue of sample size, it will be sufficient to provide a fitted result for all listed firms of Korea's stock markets, KOSPI and KOSDAQ, although this result does not contain energy quantity.[12] In Figure 27, the complementary CDFs of L, K, Y of 1479 firms are plotted by points. A series of points are well overlapped with an estimated GB2 function plots for every quantity, except for in the tail parts of extreme values. This implies that a fitting result with a large sample can be different from a small sample case. As a more exhaustive case of literature, using data on the entire population of tax-paying firms in the United States, Axtell (2001) shows that the Zipf distribution characterizes firm size, that is, the probability that a firm is

[12]Kang et al. (2011) already evaluated the distribution and dealt with the inequality of firm sizes for the Korean firms listed on the stock markets.

larger than a certain size is inversely proportional to that size.

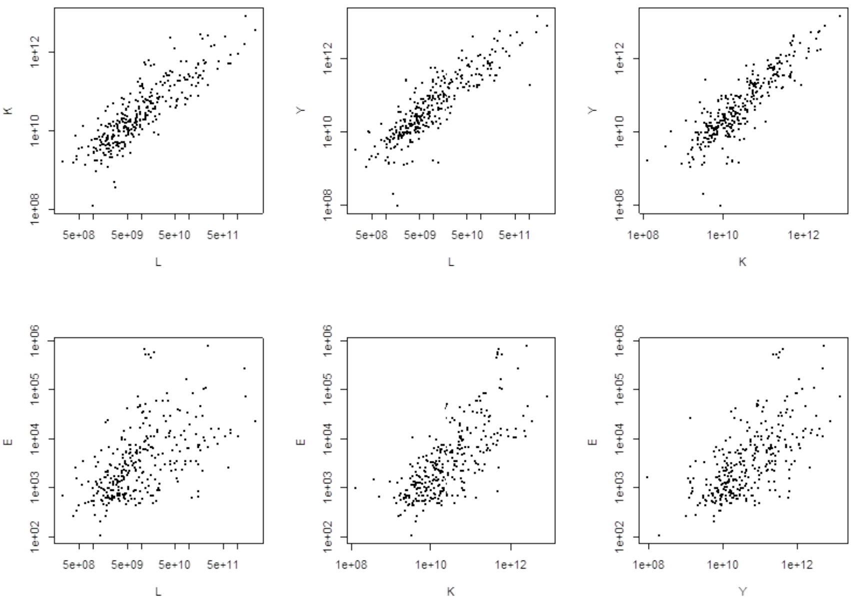

Figure 28: Scatter plots for all pairs of labor cost L, capital K, value added Y and energy E.

Table 7: Correlation coefficients for all pairs of the four variables.

	L	K	Y	E
L	1.000	-	-	-
K	0.867	1.000	-	-
Y	0.878	0.856	1.000	-
E	0.601	0.723	0.653	1.000

Figure 28 displays scatter plots for all pairs of the four quantities, labor cost L, capital K, value added Y and energy E. Also, the correlation coefficients were calculated for all pairs, which are listed in Table 7. The

Spearman's rank correlation coefficient was used, which is one of the robust non-parametric measures of the correlation between two variables. Conclusively, both graphs and coefficient values imply that the pairs containing only the conventional quantities – L, K, Y – are more mutually correlated than the other ones bundled with the non-conventional quantity E.

4.2.3 Dependence representation of the CES function

In Chapter 2, the effects of structural changes in the production functions of CGE models were sketched, showing that the discrepancy in projection results is mostly accounted for by the uncertainty of substitution elasticity parameters and function structure. However, even though it is possible to determine statistically meaningful parameters or structures, there is no guarantee of successfully reflecting the microfoundations of the economy. This is very obvious because, in such models, the microscopic information is transformed into averaged parameters in a collection of regression models. Although there have been many studies on performing a disaggregated analysis in a CGE framework for decades since Basevi (1968), they usually ignore a possible loss of the information of interactions between or within those disaggregated groups. To offer a clear illustration of the information loss with a CES function, this section presents the result of a brief experiment. A few nested CES functions were fitted to disaggregated real data and generated random datasets from the estimated models. Then, a comparison was done between the simulated outcomes and the real data in terms of correlations.

For the three input factors – L, K and E, the real dataset was fitted to the nested CES functions, proposed by Sato (1967), which allow more flexibility compared to plain non-nested versions. However, nested CES functions are not invariant to the nesting structure and there is a need for the process of selecting the most suitable function structure. Like Kemfert (1998), three possible nesting forms were tried as follows.

$$y = \gamma \left[\delta \left(\delta_1 x_1^{-\rho_1} + (1-\delta_1) x_2^{-\rho_1} \right)^{\rho/\rho_1} + (1-\delta) x_3^{-\rho} \right]^{-1/\rho}, \tag{4.16}$$

where γ determines the productivity and δ and δ_1 are the distribution coefficients of the inputs. ρ and ρ_1 determine the values of elasticity of substitution, which are $\sigma = 1/(1+\rho)$ and $\sigma_1 = 1/(1+\rho_1)$, respectively. Equation 4.16 can be diversified by the set of (x_1, x_2, x_3), which have three possible combinations, (L, K, E), (K, E, L), and (E, L, K).

It is meaningless to fit CES function models to the entire dataset because the regression models are estimated on the unrealistic assumption of homogeneity as stated previously. In order to check whether a set of CES functions can reproduce the interactions between disaggregated groups, the dataset was divided into ten sectors and the three possible CES functions were fitted to eight individual sectors.[13] The optimized combination of (x_1, x_2, x_3) was selected by comparing the R^2 statistics and randomly generated datasets were extracted from those estimated functions.

Figure 29 shows the scatter plots for one of the simulated dataset, depicting the mutual dependence between the four quantity variables. Except for the censoring phenomena in $L - Y$ and the sparsely correlated pattern in

[13]Two of the ten sectors were dropped due to occurring errors in the calculation process with the L-BFGS-B method.

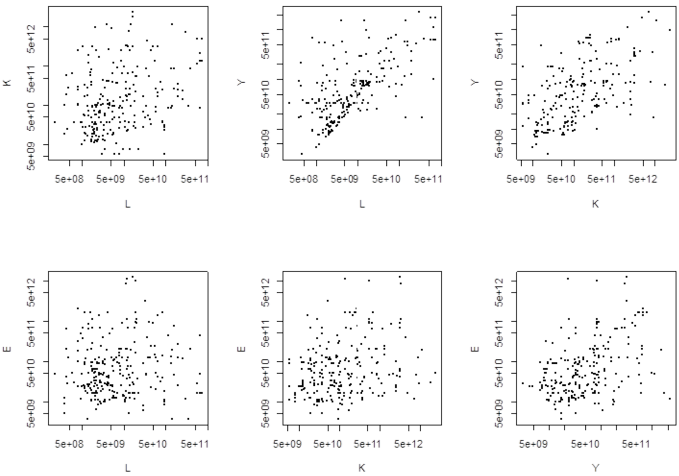

Figure 29: Scatter plots for all pairs of labor cost L, capital K, value added Y and energy E, randomly generated by the nested CES functions estimated for individual sectors.

Table 8: Comparison of correlation coefficients between real data and randomly generated data from the estimated CES function. The values in parentheses are standard deviations of the estimated statistics. In the simulation, the sample size is equal to the size of the real dataset and the iteration number is 50.

	correlation coefficients					
	$L-K$	$L-Y$	$K-Y$	$L-E$	$K-E$	$Y-E$
aggregate	0.823	0.863	0.821	0.619	0.736	0.668
disaggregate	0.226	0.584	0.584	0.026	0.208	0.357
	(0.057)	(0.046)	(0.034)	(0.054)	(0.050)	(0.053)

$K - Y$, it is hard to discern a strong tendency of any proportion among the variables. This is confirmed by the simulated values of correlation coefficient in Table 8. The comparison with the correlations of aggregate real data proves that the estimated CES function fails to reproduce the correlation information of the aggregate dataset.

A conventional economic model is merely an *ex post* sketch of the economy using regression parameters and, consequently, is inevitably accompanied by a considerable loss of information. To make up for this shortcoming, various models try to combine disaggregated data with the regression type of model frame. Nevertheless, the inherent shortcomings of such models cannot convey the entire map of interactions between sectors or individual agents. Also, obviously, a control of the level of disaggregation cannot cover the limitations: small disaggregated groups deprive the dataset of the information between individual groups while the small sample sizes due to a highly disaggregated dataset prohibit estimations of robust parameters for each group. This is why an alternative method to effectively incorporate the microscopic information is needed.

4.3 The copula model

Before getting into the construction of an alternative CGE model, the concept of copulas, a convenient multivariate distribution method, is introduced in this section. Due to their unique convenience in reproducing a statistical distribution map of the real economy, this study tries to apply the statistical tool to conventional CGE models. In fact, the copula method is only partly used later in the actual simulations of Section 4.5.2, but its importance should be emphasized because it can easily and effectively realize a distribution-based description of production activities argued in Section 4.2.2. Its basic theories and performances are dealt with in this preliminary section for this reason.

4.3.1 Copula theory

As a useful tool of measuring the dependence among stochastic variables, copulas are becoming more popular in various fields. A copula ascribes a mathematical form of expression to the correlations inherent among stochastic variables, separating the marginal CDFs.

According to Sklar (1959), a joint CDF, $F_{L,K,Y,E}(l,k,y,e)$ is a unique function of the marginal CDFs as follows:

$$F_{L,K,Y,E}(l,k,y,e) = C(F_L(l), F_K(k), F_Y(y), F_E(e)), \tag{4.17}$$

where the unique function $C(F_L(l), F_K(k), F_Y(y), F_E(e))$ is called a copula. One can simplify the expression by replacing $F_{L,K,Y,E}(l,k,y,e)$ with $F(L,K,Y,E)$ and using a new notation u_X instead of $F_X(x)$ for a certain $x = F_X^{-1}(u_X)$, where $u_X \in [0,1]$ and $X \in \{L,K,Y,E\}$:

$$F(L,K,Y,E) = C(u_L, u_K, u_Y, u_E). \tag{4.18}$$

A partial differentiation with respect to all variables gives a so called copula density $c(u_L, u_K, u_Y, u_E)$ as follows:

$$\begin{aligned} f(L,K,Y,E) &= \frac{\partial^4 F(L,K,Y,E)}{\partial L\, \partial K\, \partial Y\, \partial E} \\ &= f(L)f(K)f(Y)f(E)\frac{\partial^4 C(u_L,u_K,u_Y,u_E)}{\partial u_L \partial u_K \partial u_Y \partial u_E} \\ &= f(L)f(K)f(Y)f(E)\ c(u_L,u_K,u_Y,u_E), \end{aligned} \tag{4.19}$$

where the lower case f denotes a probability density function (PDF) corresponding to each of the CDFs. If the variables are mutually independent of each other, then the copula density $c(u_L, u_K, u_Y, u_E)$ reduces to 1.

Various forms of copulas and their mathematical properties have been explored.[14] Among them, the most well-known and widely-used family of copulas is Archimedean copulas. (Nelsen, 2006) A bivariate Archimedean copula can be constructed by a generator function ψ as follows:

$$C_A(u_1,u_2) = \psi^{-1}[\psi(u_1)+\psi(u_2)], \tag{4.20}$$

where ψ is a continuous, convex and strictly decreasing function mapping $[0,1]$ to $[0,\infty]$ with boundary conditions of $\psi(1) = 0$ and $\psi(0) = \infty$.

According to the type of the generator function ψ, the family of Archimedean copulas has branches such as Frank copula, Gumbel copula and Clayton copula. Among them, the Gumbel copula is given by

$$\psi_G(u;\theta) = (-\ln u)^{\theta}, \quad \theta \in [1,\infty]. \tag{4.21}$$

[14]For a detailed instruction of the properties of copulas, refer to McNeil et al. (2005) and Cherubini et al. (2004).

Thus, for example, the bivariate Gumbel copula can be written as

$$C_G(u_1,u_2;\theta) = \exp\{-[(-\ln u_1)^\theta + (-\ln u_2)^\theta]^{1/\theta}\}. \tag{4.22}$$

As Iyetomi et al. (2012) reports that Gumbel copulas, including one of its nesting-structured variants, give the best performances in constructing a copula model, this study will only deal with the Gumbel type of copulas. Additional comparative studies among various copula types may give answers to what kind of copulas are better fitted to the real data. However, the main target of this study is not searching for the best functional form, but is assessing the possibility of using the copula approach for catching the microscopic information of each agent's energy use. The next section provides an empirical example of constructing a copula model and judging its power of reproducing the economy.

4.3.2 Construction of a copula model

When it comes to the design of a multivariate copula with more than three variables, it is important to decide what type of tree structure the copula has. For example, in the case of the four variables L, K, Y and E, an Archimedean copula can have the following type of single-level structure:

$$C_A(u_L,u_K,u_Y,u_E;\theta) = \psi^{-1}[\psi(u_L)+\psi(u_K)+\psi(u_Y)+\psi(u_E)], \tag{4.23}$$

where all of the marginal CDFs have the same correlation structure characterized by a single parameter θ.[15] However, this is not true for the actual data, as can be roughly inferred in the scatter plots shown in Figure 28. Actually, the energy quantity shows less correlations with the other ones compared to the case of non-energy bundles. In this sense, another copula model can be composed with plural characteristic parameters as in the following example:

$$\begin{aligned}
&C_A(u_L, u_K, u_Y, u_E; \theta_1, \theta_2) \\
&= C_A(C_A(u_L, u_K, u_Y; \theta_2), u_E; \theta_1) \\
&= \psi_1^{-1}[\psi_1(\psi_2^{-1}(\psi_2(u_L) + \psi_2(u_K) + \psi_2(u_Y))) + \psi_1(u_E)],
\end{aligned} \tag{4.24}$$

where ψ_2 generates the *child* copula, $C_A(u_L, u_K, u_Y; \theta_2)$, characterized by θ_2 parameter, and the remaining margins, $C_A(u_L, u_E; \theta_1)$, $C_A(u_K, u_E; \theta_1)$ and $C_A(u_Y, u_E; \theta_1)$ are determined by ψ_1 with θ_1. This type of copulas are referred to as *non-exchangeable* Archimedean copulas. (McNeil et al., 2005) In this case, the condition of $\theta_1 \leq \theta_2$ should be satisfied for the combination of ψ_1 and ψ_2^{-1} to have a completely monotonic first order derivative, which is the *sufficient nesting condition* under which Equation 4.24 is a proper copula. (McNeil, 2008)[16]

With Equation 4.21, both Equation 4.23 and Equation 4.24 can be expressed as two Gumbel copula models as follows:

[15] In a multivariate case, note that the inverse of a generator function should be completely monotonic.

[16] More details about the properties of Archimedean copulas constructed by the nesting of generators are revealed in Joe (1997).

Model I :

$$C_G(u_L, u_K, u_Y, u_E; \theta) \\ = \exp\left\{ -[(-\ln u_L)^\theta + (-\ln u_K)^\theta + (-\ln u_Y)^\theta + (-\ln u_E)^\theta]^{1/\theta} \right\}, \tag{4.25}$$

Model II :

$$C_G(u_L, u_K, u_Y, u_E; \theta_1, \theta_2) \\ = \exp\left\{ -\left[\left(-\ln\left[\exp\left(-[(-\ln u_L)^{\theta_2} + (-\ln u_K)^{\theta_2} + (-\ln u_Y)^{\theta_2}]^{1/\theta_2} \right) \right] \right)^{\theta_1} \right.\right. \\ \left.\left. + (-\ln u_E)^{\theta_1} \right]^{1/\theta_1} \right\}. \tag{4.26}$$

Table 9: The estimated parameters and maximized log-likelihood values for Model I & II.

	Model I	Model II
parameter (st. dev.)	θ = 1.758 (0.045)	θ_1 = 1.540 (0.058)
		θ_2 = 2.385 (0.078)
log-likelihood	343.2	438.6

After setting the models, maximum likelihood estimations were carried out to obtain the optimized values of the characteristic parameters. The fitting outcomes are listed in Table 9. The comparison of log-likelihood values shows that Model II, equipped with two parameters, outperforms the single-parameter model, Model I. Hence, Model II was selected for further study in this research.

The comparison of the contour diagrams of Figure 30 with those of

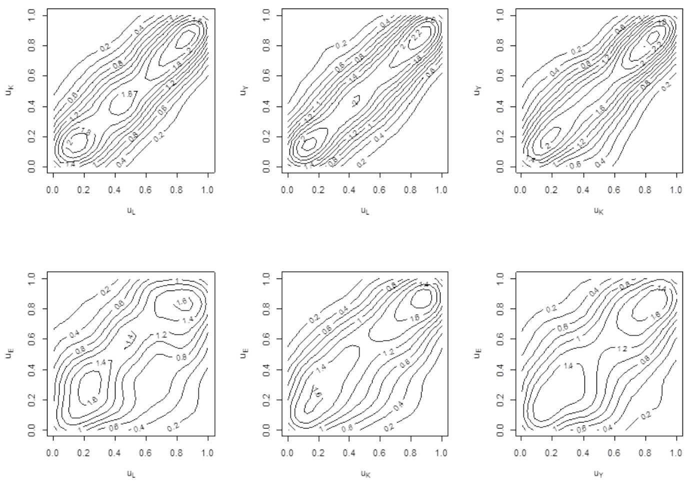

Figure 30: Contour plots of the density functions for each pair of the four variables from the real data.

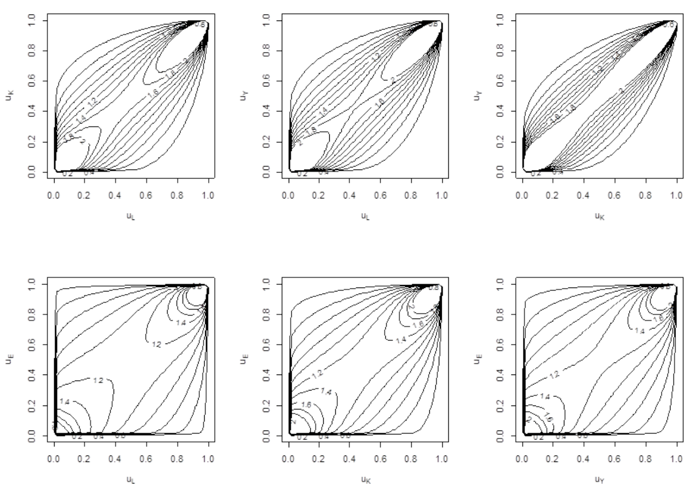

Figure 31: Contour plots of the estimated copula density functions for each pair of the four variables.

Figure 31 provides a sketch of assessing the performance level of a copula model. Figure 30 shows the contours of the copula densities of the real data in the two-dimensional spaces of all pairs, and Figure 31 depicts the contour diagrams corresponding to the copula models which are fitted to the real copula densities for each pair. Presumably, the relatively sparsely spaced contour lines in the density plots of energy-paired bundles well depict the less populated pattern in the proportional regions. In the next section, an elaboration was done on assessing the performance of the constructed copula model.

4.3.3 Performance of the copula model

This section provides some empirical results of assessing the performance of a copula model in reproducing the economy. First, a simulation was carried out with Model II and the marginal distributions were extracted for each quantity, in which no correlations among the quantities were taken into account. The results are shown in Figure 32 and Table 10. Table 10 summarizes the results of the goodness-of-fit test between the simulated marginal distributions and those of the real data. The values of Kolmogorov-Smirnov test statistics were obtained by averaging the outcomes of 1,000 tests, each of which has the sample size of 1,000. Both graphs and test statistics reveal that the copula model, Model II, is successful in deriving the marginal information of each variable. Extracting marginal distributions from a joint distribution is, however, not an exclusive speciality of copulas. The main point of using a copula model lies in reproducing mutual dependence between variables.

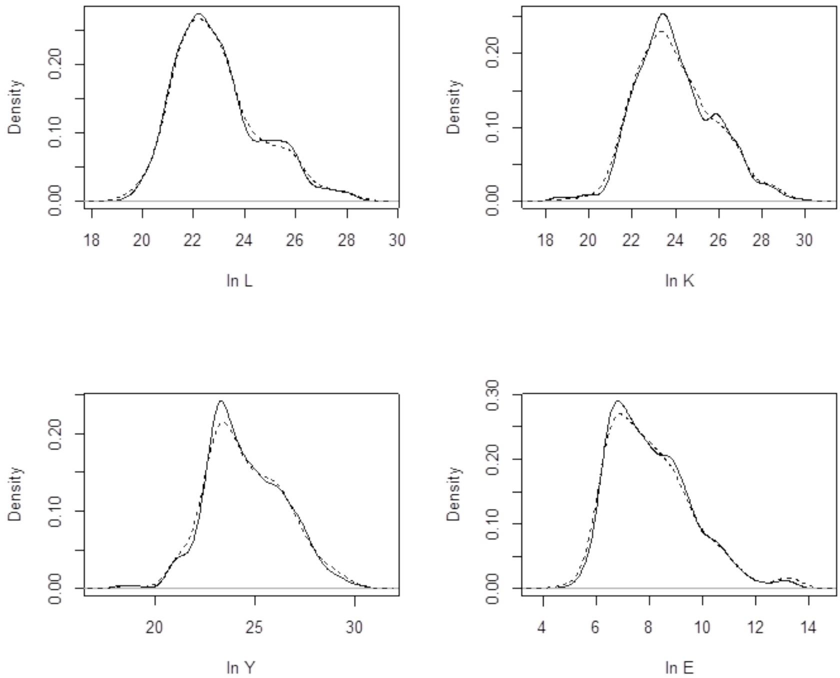

Figure 32: An example of density function plots for labor cost(L), capital(K), value added(Y) and energy use(E). Solid lines depict the kernel density estimates of the quantities generated by the estimated copula models, and dotted lines show the estimated density functions from the actual data set.

Table 10: D-values of Kolmogorov-Smirnov tests between the simulation data and the real data set. The values in parentheses are standard deviations of the test statistics. The tests were iterated 1,000 times for each quantity and the size of each generated sample is 1,000.

	$\ln L$	$\ln K$	$\ln Y$	$\ln E$
D-value	0.0271	0.0282	0.0268	0.0265
(std. dev.)	(0.0083)	(0.0081)	(0.0085)	(0.0084)

Figure 33 displays an example of pairwise scatter plots generated by a simulation with Model II. This can be compared with Figure 28 which depicts the correlations in the actual data set. Considering the location and shape of each scattered pattern, the simulation results well imitate the relations embedded in the real data. However, the quantitative results of calculating correlation coefficients in Table 11 show a disagreement with the values for the real data in Table 7: the simulated Spearman's rank correlation coefficients are less than the corresponding values for the real data.

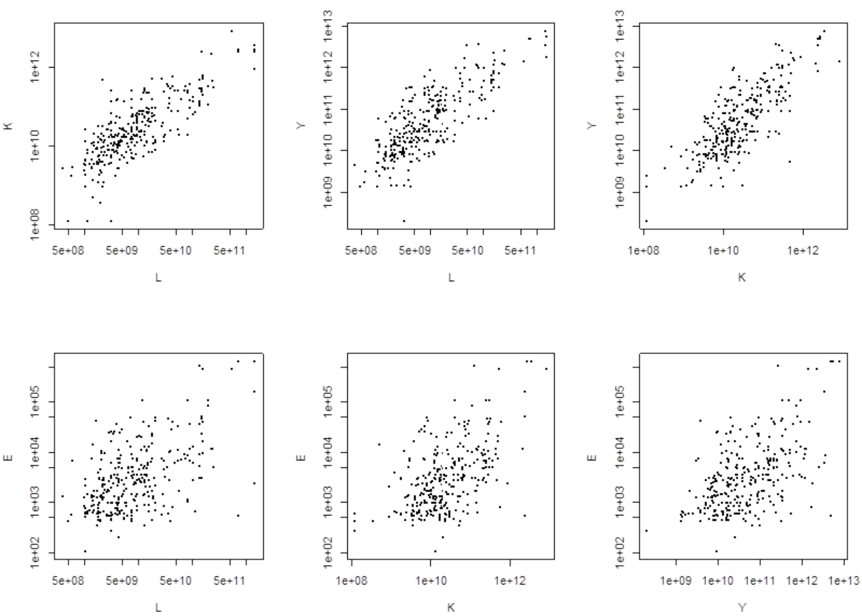

Figure 33: An example of scatter plots for all pairs of L, K, Y and E generated by a simulation with the estimated copula model. The number of data points is 300 for each graph.

The disagreement in the results of reproducing the dependence can raise some questions about the usefulness of a copula model. In fact, the

Table 11: Correlation coefficients for all pairs of the quantity variables randomly generated by the estimated copula model. The values in parentheses are standard deviations of the estimated statistics. The sample size is 1,000 and iteration number is also 1,000.

	L	*K*	*Y*	*E*
L	1.000	-	-	-
K	0.771	1.000	-	-
	(0.016)			
Y	0.769	0.769	1.000	-
	(0.016)	(0.014)		
E	0.502	0.501	0.502	1.000
	(0.026)	(0.025)	(0.029)	

best way to recover the relations in an economic model is a simulation based on the joint distribution itself. However, describing a multivariate joint distribution function often needs a number of parameters especially when the dimension increases or each marginal distribution does not have a well-known functional type. To avoid such limitations of parametric approaches, a non-parametric method can be deployed to grasp the entire map of a multivariate density function. However, the advantage of its parameter-free property is often offset by the so called *curse of dimensionality* with the computationally intensive task: the burden of computation work grows exponentially as the dimension of a non-parametric model increases. Also, a result of the kernel density estimation, which is widely chosen among various non-parametric methods, is strongly dependent on the selection of bandwidth or smoothing *parameter*. (Yatchew, 1998)

For these reasons, copulas have been studied and widely accepted in various fields to deal with the issue of correlations in an extremely easy way. Recall that there are no deep considerations about the model specification of Model II in this study: even a trial version of a Gumbel copula model is successful in imitating the correlations as can be seen in Figure 33, which is almost impossible with a traditional economic model. With more elaborations on the selection of copula families, nesting structures and characteristic parameters, a copula model can be improved to be more consistent with the actual situation in the economy.

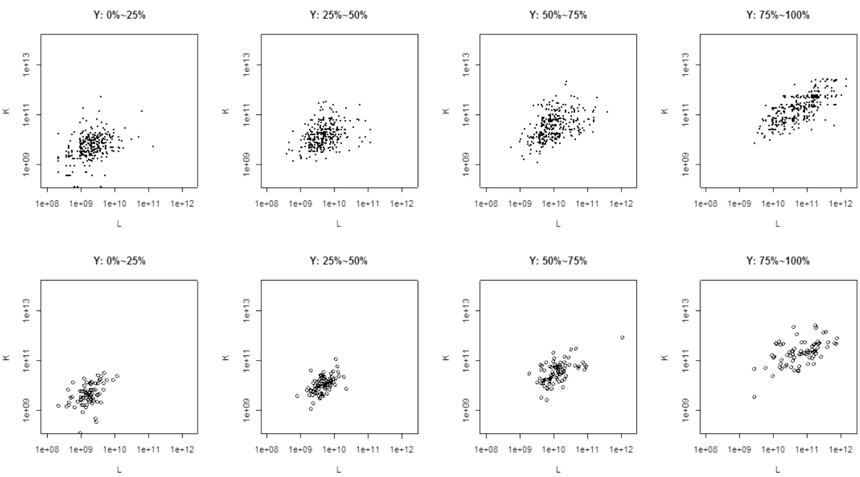

Figure 34: Scatter plots for *L*-*K* pairs from a data set generated by a simulation with the estimated copula model (upper row) and *L*-*K* pairs from the real data set (lower row). Each data set was split into four groups equally divided by the quantiles of 25%, 50% and 75% of value added.

In the next experiment, the sample was divided – for both the simulation result and the real data set – into four groups according to the level of value added of each firm: both data sets were split into four groups equally

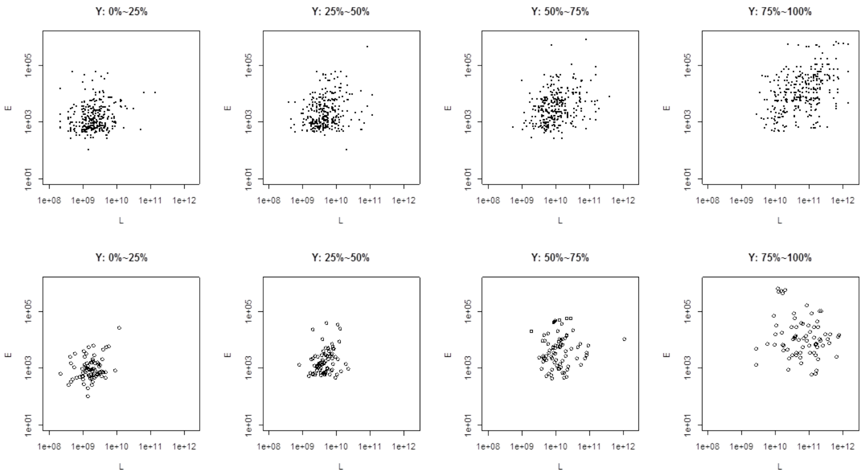

Figure 35: Scatter plots for *L*-*E* pairs from a data set generated by a simulation with the estimated copula model (upper row) and *L*-*E* pairs from the real data set (lower row). Each data set was split into four groups equally divided by the quantiles of 25%, 50% and 75% of value added.

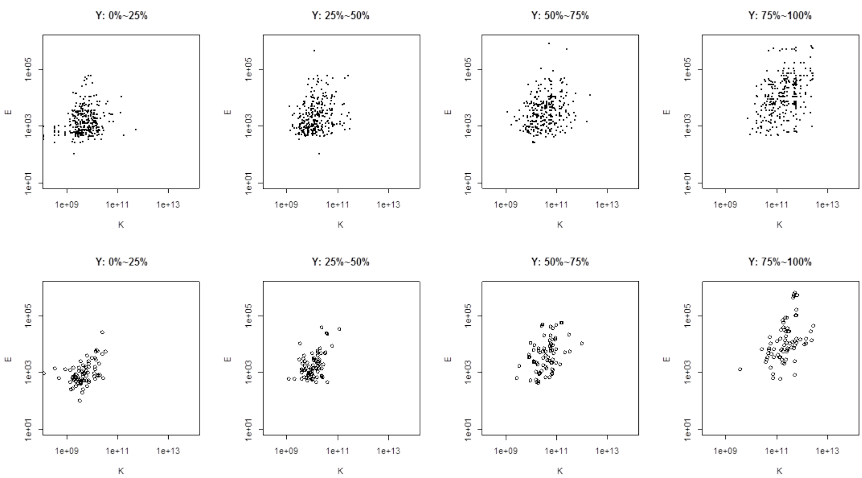

Figure 36: Scatter plots for *K*-*E* pairs from a data set generated by a simulation with the estimated copula model (upper row) and *K*-*E* pairs from the real data set (lower row). Each data set was split into four groups equally divided by the quantiles of 25%, 50% and 75% of value added.

divided by the quantiles of 25%, 50% and 75% of value added. Figure 34 compares the scatter plots for *L*-*K* pairs generated by Model II with the scatter plots for the real data. Comparing the locations and the shapes of dispersion, one can infer that the copula model well imitates the microscopic information even in a smaller sample. Also, the same argument is valid in Figure 35 and Figure 36, which compare scatter plots for *L*-*E* and *K*-*E*, respectively.

4.3.4 The copula model with data disaggregation

When a dataset is disaggregated, it is generally recommended to do a regression for individual groups because each group-wise regression model is expected to reflect the particular characteristics of the group. Thus, there arises a question over whether a similar argument is also valid for the case of a copula model. To find a reasonable answer, the effect of a disaggregation in copula analysis is examined. First, two arbitrary sectors are merged and an optimized copula model is derived for the merged dataset, generating simulated random variables. Second, on the contrary, two copula models are estimated for the individual datasets of each sector, generating another simulated dataset. Lastly, a comparison for the two cases are carried out.

Table 12 summarizes the correlation coefficients among variables for each of the ten sectors. These sectoral groups have different sample sizes. Among them, three sectors with the largest sample sizes were selected for effective model estimations. First, two of the three sectors were considered – 'steel & metal' sector and 'petroleum & chemical'. Table 13 lists the result of a comparison of correlation coefficients between real data and simulated

Table 12: Correlation coefficients of all pairs of the four variables for individual sectors.

sector	$L-K$	$L-Y$	$K-Y$	$L-E$	$K-E$	$Y-E$
cement & ceramic	0.869	0.819	0.641	0.812	0.824	0.642
electronic	0.860	0.847	0.829	0.820	0.882	0.839
food & beverage	0.950	0.903	0.891	0.791	0.779	0.841
machinery	0.913	0.902	0.901	0.795	0.877	0.839
steel & metal	0.850	0.806	0.791	0.673	0.730	0.691
wood & paper	0.778	0.803	0.685	0.732	0.855	0.617
petroleum & chemistry	0.860	0.808	0.868	0.710	0.849	0.808
building	0.890	0.949	0.887	0.725	0.706	0.718
power generation	0.779	0.843	0.941	0.431	0.663	0.676
others	0.918	0.904	0.973	0.888	0.968	0.946

data for this case. First, a simulation was performed with a unified copula function for the two sectors, and, second, a similar work was done for two individual copula models of the two individual sectors. The table reveals that the simulation result of the aggregate case is closer to the real dataset while performance of the disaggregate case is not as good in terms of reproducing correlation coefficients. Figure 38 and Figure 39 provide brief sketches of the copula density for the two cases. However, it is hard to intuitively derive a concrete conclusion from a comparison with Figure 37.

Next, the two selected sectors were substituted with ‘wood & paper’ and ‘petroleum & chemical’. Table 14 lists the result of a comparison of correlation coefficients between real data and simulated data for this case. In both cases – aggregate and disaggregate, the overall performances are less powerful than those of the previous sectoral combination. However, also in the case of ‘wood & paper’ and ‘petroleum & chemical’, the result of the aggregate case is closer to the real dataset in terms of correlation coefficients. Also, the copula density plots of Figure 41 is apparently different from that

Table 13: Comparison of correlation coefficients between real data and simulated data. The first simulation data were obtained by a unified copula function for the two sectors – ‘steel & metal’ sector and ‘petroleum & chemical’ sector, while the second dataset was from two individual copula models for the two sectors. The values in parentheses are standard deviations of the coefficients. Each simulation result is accompanied by log-likelihood value.

	correlation coefficients						log-likelihood
	$L-K$	$L-Y$	$K-Y$	$L-E$	$K-E$	$Y-E$	
real data	0.854	0.809	0.842	0.699	0.794	0.772	-
simulation aggregate case	0.801 (0.025)	0.802 (0.025)	0.800 (0.024)	0.679 (0.035)	0.678 (0.034)	0.680 (0.034)	258.4
simulation disaggregate case	0.609 (0.040)	0.610 (0.040)	0.610 (0.039)	0.547 (0.042)	0.547 (0.043)	0.546 (0.042)	199.2

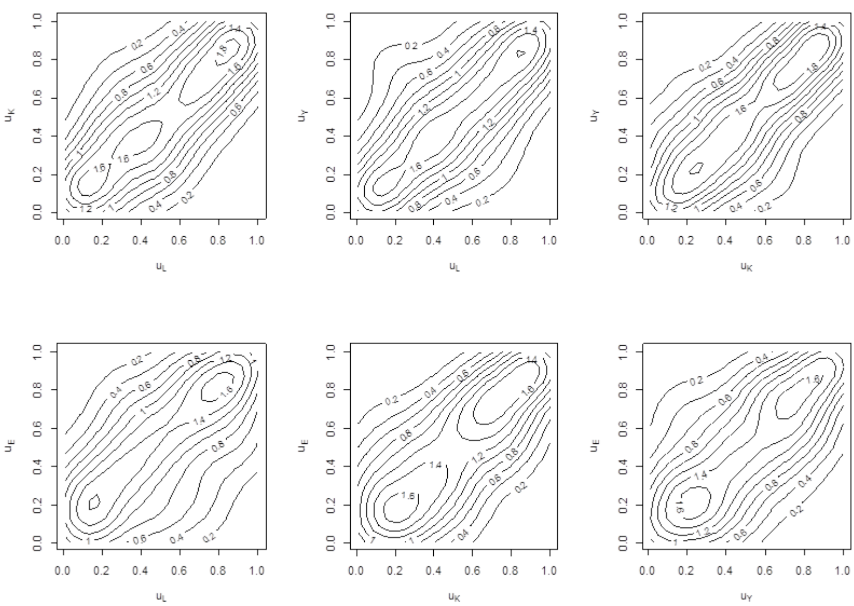

Figure 37: A contour plot of the density of real data in copula space for the merged two sectors – ‘steel & metal’ sector and ‘petroleum & chemical’ sector. The size of the dataset is 109.

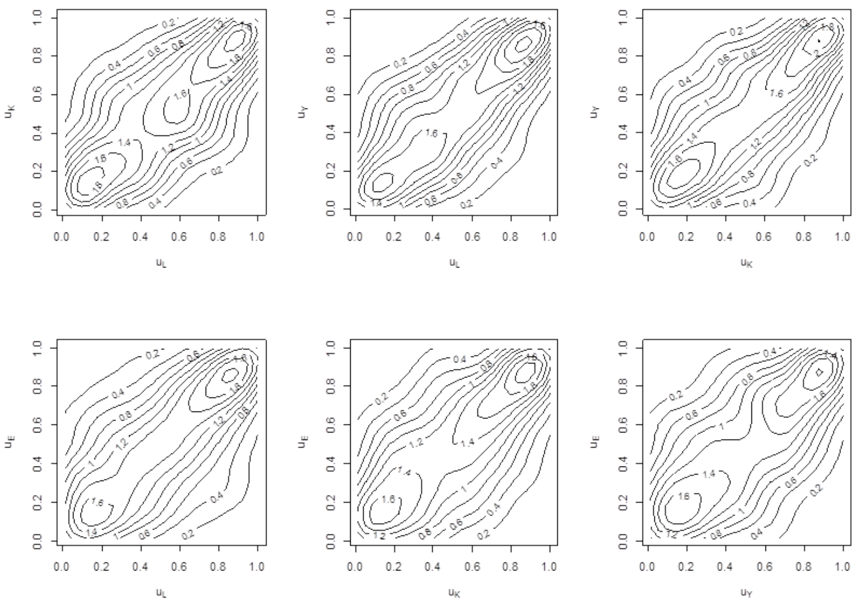

Figure 38: A contour plot of the density of simulated data in copula space. The data were generated from a unified copula model, estimated from the merged two sectors – ‘steel & metal’ sector and ‘petroleum & chemical’ sector. The sample size is 327.

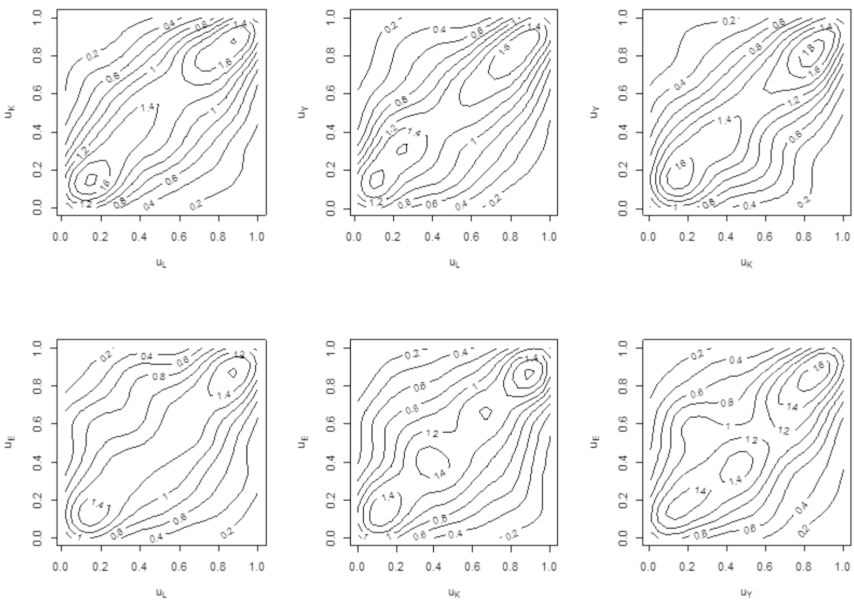

Figure 39: A contour plot of the density of simulated data in copula space. The data were generated from two copula models, individually estimated from two sectors – ‘steel & metal’ sector and ‘petroleum & chemical’ sector. The sample size is 327.

of Figure 42 which show poor performance of rebuilding the information of the real data, depicted in Figure 40.

Table 14: Comparison of correlation coefficients between real data and simulated data. The first simulation data were obtained by a unified copula function for the two sectors – ‘wood & paper’ sector and ‘petroleum & chemical’ sector, while the second dataset was from two individual copula models for the two sectors. The values in parentheses are standard deviations of the coefficients. Each simulation result is accompanied by log-likelihood value.

	correlation coefficients						log-likelihood
	$L-K$	$L-Y$	$K-Y$	$L-E$	$K-E$	$Y-E$	
real data	0.861	0.849	0.836	0.736	0.849	0.745	-
simulation aggregate case	0.616 (0.038)	0.616 (0.038)	0.615 (0.036)	0.575 (0.039)	0.574 (0.042)	0.574 (0.040)	138.8
simulation disaggregate case	0.414 (0.048)	0.416 (0.049)	0.414 (0.050)	0.414 (0.047)	0.415 (0.049)	0.417 (0.048)	87.1

The experiment results hint that there is a possibility that a disaggregation with copula modes does not help reproduce the microscopic information of mutual dependence. The argument is also verified by the comparisons of log-likelihood for each experiment. The reason of this is that, obviously, the estimation of a copula model is not about picking a general trend as in a regression model, but about mapping the entire interactions between data points. Thus, inevitably, there arises a loss of information with a disaggregation in copula analysis.

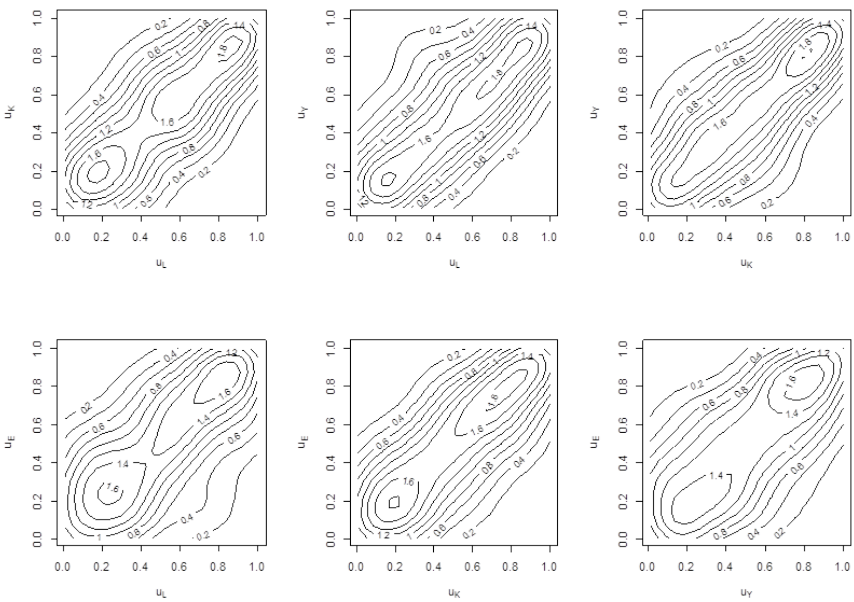

Figure 40: A contour plot of the density of real data in copula space for the merged two sectors – ‘wood & paper’ sector and ‘petroleum & chemical’ sector. The size of the dataset is 112.

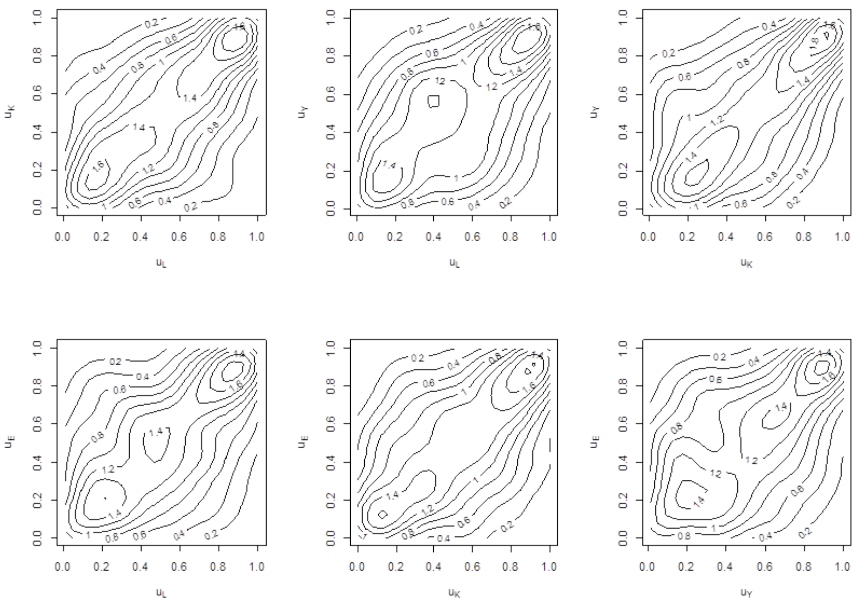

Figure 41: A contour plot of the density of simulated data in copula space. The data were generated from a unified copula model, estimated from the merged two sectors – ‘wood & paper’ sector and ‘petroleum & chemical’ sector. The sample size is 336.

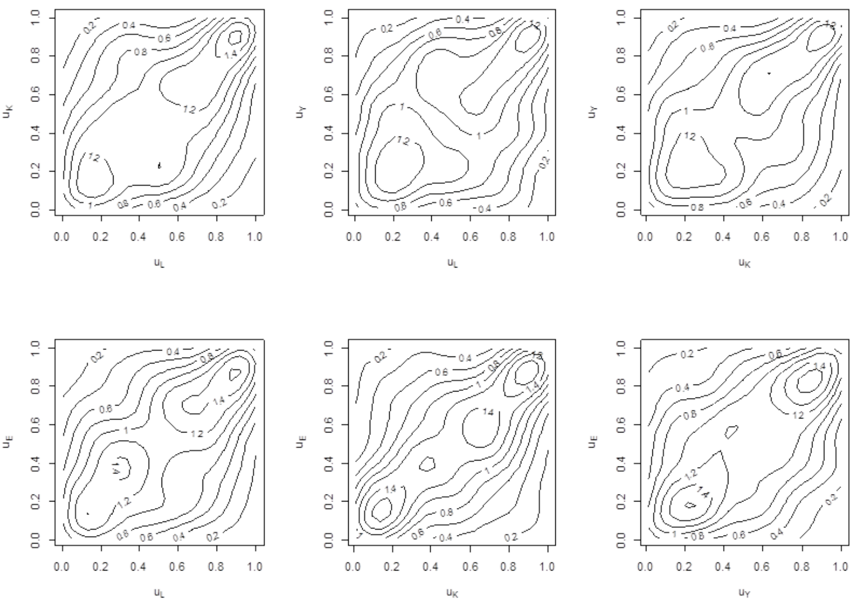

Figure 42: A contour plot of the density of simulated data in copula space. The data were generated from two copula models, individually estimated from two sectors – ‘wood & paper’ sector and ‘petroleum & chemical’ sector. The sample size is 336.

4.4 The statistical distribution approach

4.4.1 Set of firms

In this section, the details of the statistical distribution approach proposed in Section 4.2.2 are provided. Assume that the manufacturing sector of a CGE model is depicted by a set of firms. Each of the firms has a Leontief type of input structure and the production function Y_i for firm i is defined by L_i, K_i and E_i, the demand functions for input factors such as labor, capital and energy, as follows:

$$Y_i = \min\left(\frac{L_i}{a_i}, \frac{K_i}{b_i}, \frac{E_i}{c_i}\right), \tag{4.27}$$

where the coefficients, a_i, b_i and c_i, are determined from the reference year dataset and do not change, containing the information of each firm's input structure. At a glance, one can find the similarity with the Leontief local production function in Jones (2005). The coefficients in this equation can be regarded as reciprocals of the unit factor productivities, which depict the technology adopted by each firm at a chosen time. However, in this case the distribution of these coefficients are determined by an estimated joint distribution of the economic variables, not by specifically assumed distributions for the coefficients themselves as done in Jones (2005). One thing to keep in mind again is that the technology set of a firm once determined does not change in this model, which needs to be relaxed in further studies.

Next, to impose a constraint on the production capacity, the share of each firm is determined by a fixed coefficient θ_i calibrated with the reference year data:

$$Y_i \cdot p_{Y_i} = \theta_i \cdot Y \cdot p_Y, \tag{4.28}$$

where $p_{Y,i}$ means an imaginary price of the goods Y_i from a firm i, which can be regarded as an average production cost of the firm. Y is the gross output of the manufacturing sector and p_Y is the average price of goods from the sector. If the production cost increases in a firm, the output level should be lowered because the firm's input structure and the coefficient θ_i is fixed. The decision of the output level of the sector is determined under the following budget constraints,

$$Y \cdot p_Y = \sum_i Y_i \cdot p_{Y_i}. \tag{4.29}$$

The effects of the capacity restrictions may differ according to the technologies of individual firms, which is explained in Figure 43.

Figure 43 illustrates the isoquant curves of three firms in an economy. Each firm determines the optimized amount of input factors, X_1 and X_2, according to its own Leontief input structure, and the price vector revealed in the budge constraint which is depicted as the slanted solid line, corresponding to Equation 4.29. If there occurs a change in the price vector as depicted by a change from a solid line to a dashed line, each firm variates its production level. Unlike a CES function, the shares of individual input factors are unchanged. If a firm's input structure has an advantage in the situation change, the firm increases the production level.

If the three-firm example is extended to a general case, there should be more firms on the budget constraint line. If multiple firms have the same input structure, then their Leontief production functions will be overlapped

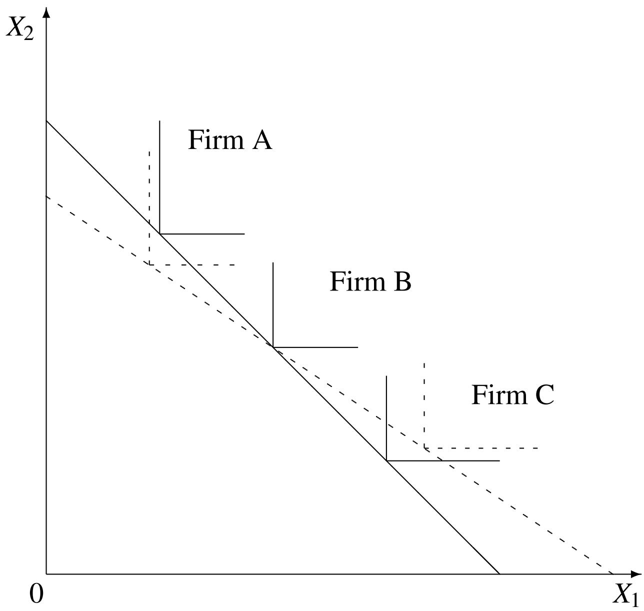

Figure 43: An illustration of isoquant curves. A change from solid line to dashed line means a change in price vector. The total budget is assumed to stay unchanged at the optimized point of firm B.

in the same location, only differing in the share of contribution to the gross production which cannot be described in an isoquant curve. Each point on the budget line, which implies its own input factors bundle, can have a different density in terms of the number of firms. Hence, one can deduce a distribution of firms along the budget line, which contains the information of correlation between input factors. The size of the set of firms should be large enough to compose a continuous aggregate isoquant curve, in which the entire information of a joint multivariate distribution can be reflected to the domain of a set of input factor bundles. This method constructs an aggregate production set by assembling the numerous Leontief 'pieces' from individual firms.

What is noteworthy is the possible time evolution of the joint distribution map, which may erode the justification of using the distribution approach especially in a long-term projection with inter-temporal economic models. Still, there is little academic work on this issue although there is some empirical studies in the side of marginal size distributions. This is why this study only considers 10 years as the time horizon for projections. This practical limitation should be tackled by in-depth further studies.

4.4.2 Properties of cost functions

In this section, the properties of the production technology described by a set of firms are investigated from the aspect of cost functions. The reason of using cost functions is that the distribution approach does not have the form of explicit functional forms in many cases. The cost functions should be nondecreasing, homogeneous of degree 1, concave and continuous in a price vector. (Varian, 1992)

First, let us derive the cost function of the model of a set of firms. The profit function of a firm i which has the production function of Equation 4.27 is

$$\pi(p_{Y_i}, p_L, p_K, p_E) = \max_{Y_i, L_i, K_i, E_i} p_{Y_i} Y_i - p_L L_i - p_K K_i - p_E E_i. \tag{4.30}$$

By the envelope theorem, the equation can be differentiated at equilibrium. Thus,

$$\frac{\partial \pi}{\partial p_L} = \frac{\partial p_{Y_i}}{\partial p_L} Y_i + p_{Y_i} \frac{\partial Y_i}{\partial p_L} - \frac{\partial L_i}{\partial p_L} p_L - L_i - \frac{\partial K_i}{\partial p_L} p_K - \frac{\partial E_i}{\partial p_L} p_E = 0, \tag{4.31}$$

where the vector (Y_i, L_i, K_i, E_i) is located at its optimal point $(Y_i^*, L_i^*, K_i^*, E_i^*)$. From Equation 4.27, $L_i = a_i Y_i$, $K_i = b_i Y_i$, $E_i = c_i Y_i$ at the optimal point, where (a_i, b_i, c_i) denotes a technology set and the values are fixed. Then, the substitution of L_i, K_i and E_i leads to

$$\left(\frac{\partial p_{Y_i}}{\partial p_L} - a_i \right) Y_i + \frac{\partial Y_i}{\partial p_L} \left(p_{Y_i} - a_i p_L - b_i p_K - c_i p_E \right) = 0. \tag{4.32}$$

By the zero-profit assumption, $p_{Y_i} Y_i = p_L L_i + p_K K_i + p_E E_i$, the terms of the second parenthesis vanishes. Then, the equation leads to

$$\frac{\partial p_{Y_i}}{\partial p_L} = a_i. \tag{4.33}$$

Likewise, the following equations can be obtained as follows:

$$\frac{\partial p_{Y_i}}{\partial p_K} = b_i \tag{4.34}$$

$$\frac{\partial p_{Y_i}}{\partial p_E} = c_i \tag{4.35}$$

On the other hand, it is assumed that the total production $p_{Y_i} Y_i$ is not affected by any change of the prices of input factors. Therefore,

$$\frac{\partial p_{Y_i}}{\partial p_L} Y_i + p_{Y_i} \frac{\partial Y_i}{\partial p_L} = 0. \tag{4.36}$$

By Equation 4.33, the above equation becomes

$$\frac{\partial Y_i}{\partial p_L} = -\frac{a_i Y_i}{p_{Y_i}}. \tag{4.37}$$

Likewise, the following equations can be derived:

$$\frac{\partial Y_i}{\partial p_K} = -\frac{b_i Y_i}{p_{Y_i}}, \tag{4.38}$$

$$\frac{\partial Y_i}{\partial p_E} = -\frac{c_i Y_i}{p_{Y_i}}. \tag{4.39}$$

Due to the relation of Equation 4.36, the total production of each firm does not vary even when there arises a change in the price system. However, the real output Y_i responds to the price changes and then the total output of an economy, $\sum_i Y_i$, varies. In this sense, it is more appropriate to use the unit cost function for the investigation of how the sector responds to price changes under budget constraints.

By the cost minimization condition for Leontief production functions, the unit cost function is written as

$$C(p_L, p_K, p_E) = \min_{\{L_i, K_i, E_i\}} \frac{\sum_i p_L L_i + p_K K_i + p_E E_i}{\sum_i Y_i} = \frac{\sum_i p_{Y_i} Y_i}{\sum_i Y_i}. \qquad (4.40)$$

The cost function of a Leontief production technology is increasing in the price of an input factor. Hence, a simple average of the costs of individual firms should be also increasing in price. Thus, the above unit cost function satisfies the properties of continuity and is nondecreasing. Also, homogeneity of degree one is satisfied: if all imaginary prices of individual firms are multiplied by a positive constant, the average price, or the unit cost, is multiplied by that constant because the outputs are all divided by that constant in Equation 4.40.

Using Equation 4.37, the first order partial differentiation with respect to labor price p_L is expressed as follows:

$$\begin{aligned} \frac{\partial C}{\partial p_L} &= \frac{\sum_i (p_{Y_i} \frac{\partial Y_i}{\partial p_L} + a_i Y_i) - \sum_i (p_{Y_i} Y_i) \sum_i \frac{\partial Y_i}{\partial p_L}}{(\sum_i Y_i)^2} \\ &= \frac{\sum_i (p_{Y_i} Y_i) \sum_i \frac{a_i Y_i}{p_{Y_i}}}{(\sum_i Y_i)^2}. \end{aligned} \qquad (4.41)$$

Likewise, the other first order partial derivatives are given as follows:

$$\frac{\partial C}{\partial p_K} = \frac{\sum_i (p_{Y_i} Y_i) \sum_i \frac{b_i Y_i}{p_{Y_i}}}{(\sum_i Y_i)^2}, \qquad (4.42)$$

$$\frac{\partial C}{\partial p_E} = \frac{\sum_i (p_{Y_i} Y_i) \sum_i \frac{c_i Y_i}{p_{Y_i}}}{(\sum_i Y_i)^2}. \qquad (4.43)$$

The constantly positive values of these equations verify the nondecreasing property of the unit cost function again. To check the remaining concavity condition, second order differentiations are carried out as follows. The following are own-price second order partial derivatives:

$$\frac{\partial^2 C}{\partial {p_L}^2} = \frac{2\sum_i p_{Y_i} Y_i}{(\sum_i Y_i)^3}\left[\left(\sum_i \frac{a_i Y_i}{p_{Y_i}}\right)^2 - \sum_i \frac{a_i^2 Y_i}{p_{Y_i}^2}\sum_i Y_i\right], \tag{4.44}$$

$$\frac{\partial^2 C}{\partial {p_K}^2} = \frac{2\sum_i p_{Y_i} Y_i}{(\sum_i Y_i)^3}\left[\left(\sum_i \frac{b_i Y_i}{p_{Y_i}}\right)^2 - \sum_i \frac{b_i^2 Y_i}{p_{Y_i}^2}\sum_i Y_i\right], \tag{4.45}$$

$$\frac{\partial^2 C}{\partial {p_E}^2} = \frac{2\sum_i p_{Y_i} Y_i}{(\sum_i Y_i)^3}\left[\left(\sum_i \frac{c_i Y_i}{p_{Y_i}}\right)^2 - \sum_i \frac{c_i^2 Y_i}{p_{Y_i}^2}\sum_i Y_i\right]. \tag{4.46}$$

Also, the cross-price second order partial derivatives can be obtained as follows:

$$\frac{\partial^2 C}{\partial p_L \partial p_K} = \frac{2\sum_i p_{Y_i} Y_i}{(\sum_i Y_i)^3}\left[\sum_i \frac{a_i Y_i}{p_{Y_i}}\sum_i \frac{b_i Y_i}{p_{Y_i}} - \sum_i \frac{a_i b_i Y_i}{p_{Y_i}^2}\sum_i Y_i\right], \tag{4.47}$$

$$\frac{\partial^2 C}{\partial p_L \partial p_E} = \frac{2\sum_i p_{Y_i} Y_i}{(\sum_i Y_i)^3}\left[\sum_i \frac{a_i Y_i}{p_{Y_i}}\sum_i \frac{c_i Y_i}{p_{Y_i}} - \sum_i \frac{a_i c_i Y_i}{p_{Y_i}^2}\sum_i Y_i\right], \tag{4.48}$$

$$\frac{\partial^2 C}{\partial p_K \partial p_E} = \frac{2\sum_i p_{Y_i} Y_i}{(\sum_i Y_i)^3}\left[\sum_i \frac{b_i Y_i}{p_{Y_i}}\sum_i \frac{c_i Y_i}{p_{Y_i}} - \sum_i \frac{b_i c_i Y_i}{p_{Y_i}^2}\sum_i Y_i\right], \tag{4.49}$$

The concavity of a cost function indicates decreasing marginal rate of technical substitution in the production side. To satisfy this condition, the Hessian matrix of the cost function should be negative semi-definite. Using the symmetry of second order cross-price partial derivatives, the Hessian matrix of the above unit cost function is expressed as follows:

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2 C}{\partial {p_L}^2} & \frac{\partial^2 C}{\partial p_L \partial p_K} & \frac{\partial^2 C}{\partial p_L \partial p_E} \\ \frac{\partial^2 C}{\partial p_L \partial p_K} & \frac{\partial^2 C}{\partial {p_K}^2} & \frac{\partial^2 C}{\partial p_K \partial p_E} \\ \frac{\partial^2 C}{\partial p_L \partial p_E} & \frac{\partial^2 C}{\partial p_K \partial p_E} & \frac{\partial^2 C}{\partial {p_E}^2} \end{pmatrix} \tag{4.50}$$

The Hessian matrix of the model of a set of firms used in this study is calculated as follows.[17]

$$\mathbf{H} = \begin{pmatrix} -3.577 & 0.090 & 3.487 \\ 0.090 & -0.599 & 0.509 \\ 3.487 & 0.509 & -3.995 \end{pmatrix}$$

The equivalent condition of the Hessian matrix $\mathbf{H}$ being negative semi-definite is that all eigenvalues are non-positive. The eigenvalues of the above matrix are 8.971×10^{-16}, -8.773×10^{-1} and -7.293, not satisfying the concavity condition. However, considering that the first eigenvalue is relatively small and near zero, the Hessian matrix $\mathbf{H}$ can be regarded as a negative semi-definite matrix and the unit cost function C can satisfy the property of concavity.

[17] The prices of labor, capital and energy are all set at one. This is the convention of general CGE models, in which the total production values are considered as real outputs multiplied by unit prices. There is no clear standard of defining real outputs, so the following calculation results can be slightly changed if outputs and prices are defined differently.

4.4.3 Elasticity of substitution

The elasticity of substitution was originally introduced by Hicks (1932) for the case of two factors, which is called Hicks elasticity of substitution (HES) by Blackorby and Russell (1989). Later, the concept was extended to multi-factor substitutions by Allen and Hicks (1934). This is called Hicks-Allen or direct partial elasticity of substitution (HAES) and defined by

$$\sigma_{ij} = \frac{\partial \ln(x_i/x_j)}{\partial \ln(p_j/p_i)}, \tag{4.51}$$

where σ_{ij} measures the change in relative inputs x_i/x_j when the relative price of the two input factors, p_i and p_j, changes. All inputs are assumed to be flexible when the cost is minimized for a fixed output. However, if all inputs except x_i and x_j are held constant, σ_{ij} is equivalent to HES.

Allen (1938) proposed another definition of the elasticity of substitution, called Allen partial elasticity of substitution (AES), as follows:

$$\sigma_{ij} = \frac{\sum_k p_k x_k}{p_j x_j} \frac{\partial \ln(x_i)}{\partial \ln(p_j)}, \tag{4.52}$$

where output and input prices except p_j are held constant. Though the definition of HAES is close to the original definition of elasticity of substitution, AES has been the most used measure of substitution in the production function literature. (Hamermesh, 1993). AES was transformed by Uzawa (1962) into the following equation:

$$\sigma_{ij} = \frac{C \dfrac{\partial^2 C}{\partial p_i \partial p_j}}{\dfrac{\partial C}{\partial p_i} \dfrac{\partial C}{\partial p_j}}, \tag{4.53}$$

where $C = C(p)$ is the unit cost function. This is often called Allen-Uzawa elasticity of substitution (AUES).

The values of elasticity of substitution derived from linear regression models are usually HES or HAES while the regression results for CES functions are often related to AES or AUES. As for the model of a set of firms introduced in the previous section, AES has been employed to directly compare the outcomes of CES functions.

There are some examples of estimating the elasticity of substitution in the CES function nesting structures. However, the estimation methods are slightly different between studies. Kemfert (1998) estimated the elasticity of substitution with a nested CES production function between capital, energy and labor for aggregated time series data for the entire German industry for the period 1960 to 1993. It investigates three different nested CES production functions and their parameters.[18]

van der Werf (2008) also estimated the parameters of nested CES production functions with capital, labor and energy as inputs after systematically comparing all nesting structures for 12 OECD countries for the period 1978 to 1996. However, it employs a different method: it applies Shephard's

[18] The direct non-linear estimations of Kemfert (1998) are accompanied by very high R^2 values and relatively low t-values, which means there is a problem of multicollinearity among the explanatory variables.

lemma to the cost function and then takes first differences to get percentage changes by the log-linearization method. (Johansen, 1960) Contrary to Kemfert (1998) in which the nesting structure where capital and labor are combined first shows the poorest performance, van der Werf (2008) found that the very nesting structure fits the data best.

Table 15 summarize the estimated elasticity of substitution of van der Werf (2008) and Kemfert (1998) only for one nesting structure where capital and labor are combined first and then energy is rebundled with this bundle. Interestingly, the two estimations for West Germany are different from each other and the result from CES function fitting of Kemfert (1998) shows higher values.

Table 15 also includes the estimation results of this study for the case of Korea. The CES nesting structure, in which capital and labor are combined first and then energy is rebundled with this bundle, was fitted to both time series data and pooled data. The time series data of labor cost, capital (depreciation of fixed asset) and value added were gathered from the national accounts data of The Bank of Korea (2012) and the energy input data were obtained from Statistics Korea (2012) and Korea Energy Economics Institute (2012a) for the period 1981 to 2010. The pooled data indicates the dataset for the year of 2010 was used in this study. Despite the wide range of standard errors, the case of time series data shows higher elasticity values.

In fact, there is little literature on non-linear CES function structure for the case of Korea. Rather, there have been many studies with translog, one of flexible functional forms. Yuhn (1991) estimated a factor-augmenting translog cost function for the Korean manufacturing sector during the period

Table 15: Estimated elasticity of substitution of van der Werf (2008), Kemfert (1998) and this study for the nesting structure where capital and labor are combined first and then energy is rebundled with this bundle. The values in parentheses are standard deviations.

		$KL-E$	$K-L$
van der Werf (2008)	Belgium	0.6053 (0.0765)	0.6154 (0.0375)
	Canada	0.1725 (0.1231)	0.5273 (0.0481)
	Denmark	0.4957 (0.0947)	0.4184 (0.0348)
	Finland	0.5415 (0.0717)	0.5525 (0.0290)
	France	0.3518 (0.0719)	0.4200 (0.0278)
	UK	0.2481 (0.0764)	0.2748 (0.0280)
	Italy	0.2417 (0.0766)	0.5216 (0.0353)
	Netherlands	0.1928 (0.0936)	0.2892 (0.0263)
	Norway	0.3255 (0.0895)	0.3800 (0.0288)
	Sweden	0.2531 (0.0756)	0.4655 (0.0254)
	USA	0.5470 (0.1100)	0.3191 (0.0278)
	West Germany	0.3311 (0.0968)	0.4271 (0.0432)
Kemfert (1998)	West Gemany	0.458*	0.822
This study (Korea)	Time series data	0.2155* (0.1603)	0.6558 (2.8268)
	Pooled data	0.1151* (0.1180)	0.2096 (0.1470)

*: Except these AES values, all values are HES.

1962 to 1981, providing AES values. It was assumed that the production technology of Korean manufacturing is related to four inputs – capital, labor, energy and materials. Lee (2001) also calculated AES with a similar translog frame for the Korean manufacturing sector from the time series data of the period 1971 to 1993. The estimation results are given in Table 16. It is hard to compare the two outcomes due to the difference in the information of standard errors. However, except for the AES values of $L-E$ which are not significant in both studies, Lee (2001) shows higher elasticity values.

Table 16: Estimated AES from Yuhn (1991) and Lee (2001). The values in parentheses are standard deviations.

	Yuhn (1991)	Lee (2001)	This study
$L-K$	0.9077 (0.1393)	1.6829	0.0507
$L-E$	0.4283 (2.2088)	-2.4121	0.7029
$K-E$	1.5728 (0.6618)	2.6301	0.3291

Table 16 also includes the AES values estimated from the dataset for the year 2010. The AES values were calculated by the unit cost function of the model of a set of firms introduced in the previous section. The AES value between labor and capital reveals that these two factors are complementary to each other and the AES of $L-E$ is higher than in the other studies.[19] Attentions should be paid to the difference in the meaning of AES between this study and the others. If the AES value of $L-E$ is zero when using time

[19]The data employed in this calculation came from the dataset of Section 4.2.2 which consists of the firms of energy intensive technologies. Hence, the implications of the elasticity parameter calculations should be followed by understanding the characteristics of this dataset. The same is valid for the composition of the pilot CGE model in the following sections, which assumes that the manufacturing sector has the same properties with the dataset of energy intensive firms of Section 4.2.2.

series data, it implies that the increase of labor cost is accompanied by the increase of energy use. However, zero AES value estimated by using micro-data of a certain time point as in the new model of this study indicates that labor and energy should be simultaneously employed to achieve a production goal. The real value of 0.7029 for $L-E$ in this study implies that the economy is divided into labor-intensive sectors and energy-intensive ones, each of which can achieve the same level of production. Interestingly, the AES values of this study do not include standard error statistics because, in the model of a set of firms introduced in previous sections, all data points directly enter the calculation process using Equation 4.53.

Aside from the above approaches, Kim et al. (2011) uses the firm level panel data of 28 industries in Korea during the period 2005 to 2008 to estimate the elasticity of substitution between labor and capital, excluding energy. It tried not only pooling OLS (ordinary least squares) but also the fixed effect model as well as the random effect model. However, the elasticity of substitution estimation results are not different between models, ranging from 0.453 to 0.479.[20]

As in Kim et al. (2011) which considers only labor and capital as input factors, the pooled data fitting was done again for labor and capital for the year of 2010 in this study. Also, the AES calculation using Equation 4.53 was carried out for the same year for comparison. To investigate the effect of sample size, the estimations were done for three datasets – the dataset of this study, the sample of all firms listed in the Korean stock markets, and the dataset of all firms listed in Financial Supervisory Service of Korea (FSS)

[20]With two inputs, HES equals AES. (Allen, 1938)

(2012), which have 308, 1519 and 15280 of sample sizes, respectively. The results are given in Table 17.

Table 17: Estimated elasticity of substitution in $L-K$ with the dataset of this study, stock market sample and the dataset of all firms listed in Financial Supervisory Service of Korea (FSS) (2012). The sample sizes are 308, 1519 and 15280, respectively. The values in parentheses are standard deviations.

	Pooled data CES fitting	AES from Equation 4.53
Dataset of this study	0.6688 (0.0927)	0.2810
Stock market sample	0.8927 (0.0937)	0.4355
Dataset of all firms	0.8478 (0.0194)	0.5731

From Table 17, the estimated values with the CES function fitted to the pooled data are higher than those of the AES calculations using Equation 4.53. Considering that the result of Kim et al. (2011) is in the range of AES calculation outcomes, the AES calculation results seem to be more acceptable than the pooled data fitting case. However, it is interesting that the AES values increase as the dataset gets bigger. It can be said that the characteristics of each dataset are different from each other and the AES calculation method is sensitive to these differences. But it is still hard to answer to whether the AES value converge to a constant value. Thus, further studies are needed on the calculations with the method of Equation 4.53.

4.5 Application of the distribution approach to CGE models

4.5.1 The pilot CGE model

In this section, the model of a set of firms dealt with in Section 4.4 is applied to the manufacturing sector of a conventional standard CGE model. Considering sectoral data availability, the manufacturing sector does not translate into a set of smaller sub-sectors. Rather, the sector is divided into a number of firms, which together compose the whole map of a joint multivariate distribution of the economic variables. The set of firms can be composed from the original dataset or generated from an estimated copula model. In the latter case, the set of 'imaginary' firms virtually contains the joint multivariate information. If the size of the set of firms increases, a more precise and continuous set of possible data points can be obtained with the information of the joint distributions intact.

The choice of the CGE model to which the model of a set of firms is applied does matter. In this study, the recursive dynamic CGE model used in Kang and Kim (2007) was selected as a reference model.[21] They originally constructed the one-nation model for an analysis on the economic impact of environmental policy interventions. However, this study removed the model's exclusive feature of covering the activities of environmental protection or resource recycling, and adopted the statistical distribution method in the manufacturing sector as stated above after simplifying the energy-related

[21]The mathematical structure of the model is outlined in Appendix A and the GAMS program code of the model of a set of firms is given in Appendix B.

sectors into a unified one.

The model of this study follows a standard form of CGE models as follows. Perfect competitive markets are assumed under market clearing conditions for each commodity or factor. Producers are price takers and their profits are assumed to be zero. These conditions are represented as a set of mixed complementarity problem (MCP) non-linear equations, which is solved by the PATH solver program, a generalization of Newton's method.[22]

The production activity was divided into seven sectors – agriculture, manufacturing, construction, service, crude oil, natural gas and energy. In the crude oil sector and natural gas sector, these 'non-competitive' commodities are only imported from foreign trade, not produced in the economy. For the convenience in composing a statistical map, competitive energy commodities such as coal, refined oil, urban gas and electricity were merged in this research, although they are different in carbon content.

The basic production function nesting structure of the unmodified original model is depicted in Figure 44. In an industrial sector, final output is from the combination of composite intermediate goods and a composite production factor bundle. The intermediate goods are produced with the intermediate commodities from each sector by Leontief technologies whereas the production factor bundle or value added bundle has a CES function technology for labor, capital and energy.

However, in this study, the manufacturing sector is modified and the model is transformed into two variants. In the first case, the original single

[22]For an introduction of the basic principles of solver programs including Newton's method, refer to Miranda and Fackler (2002).

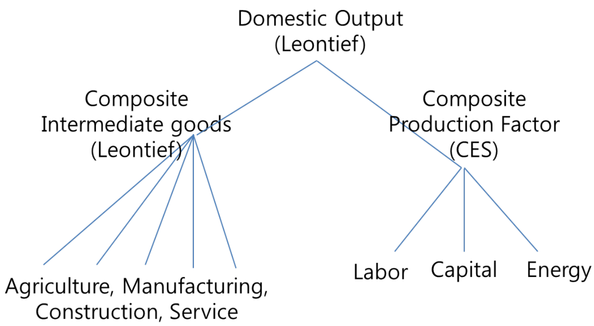

Figure 44: Production function structure in the model of Kang and Kim (2007).

level CES structure of the composite production factor bundle in the manufacturing sector turns into a nested CES function structure which has the same structure with that of EPPA in Figure 1. The second variant model switches the original single level CES structure with fixed input structures as in Figure 45. Then, in the following section, the estimation results are applied to the two variants.

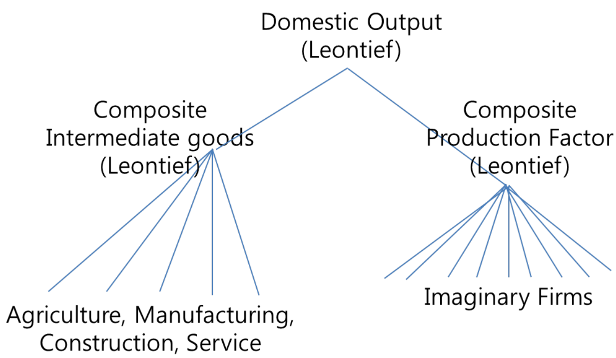

Figure 45: Production function structure in manufacturing sector in the model of a set of firms.

Final demand sectors are household, government, and investment. The representative household earns income by supplying labor and capital, and determines consumption and savings by a Cobb-Douglas utility function. The government collects direct or indirect taxes as revenue and spends them for government expenditure and savings. Savings from both sectors are added to the capital stock annually. Foreign trade also enters the final demand sectors. Imported goods are combined with domestic products as Armington goods by CES functions while foreign demand also has imperfect substitution relations with domestic demand through constant elasticity of transformation (CET) functions.

Primary factors such as labor and capital can move across sectors and their allocations are determined by factor prices. Labor is defined as total labor service from the population of the economy and is determined by the growth rate of the labor force. Capital is the total service or flow from the capital stock of the country and grows annually by adding investment net of depreciation. The depreciation is assumed to be 5% and the growth rate of labor force supply is 1% for all years in this study. However, any change of interest rate is not considered in the pilot model. Closure rules are depicted in Appendix A for the primary factors, foreign trade and individual final goods.

The default elasticity parameters, which are applied to all model structures except modified ones, are given in Table 18 in Appendix A. As for the data of the benchmark year, it is necessary to compose a social account matrix (SAM). In this study, the matrix was obtained for the reference year 2010 based on the data of national accounts and input-output tables provided by The Bank of Korea (2012). The result is given in Table 19 in Appendix A.

4.5.2 Projection results

As mentioned in Section 4.4.3, there are not many studies on CES nesting structures in terms of fitting performance. In the first experiment of this section, the estimation values of van der Werf (2008) in Table 15 were applied to a nested CES function model, in which the original single-level value added bundle structure was switched with a two-level CES nesting structure where capital and labor are combined first and then energy is rebundled with this bundle for the manufacturing sector. The results are given in Figure 46.

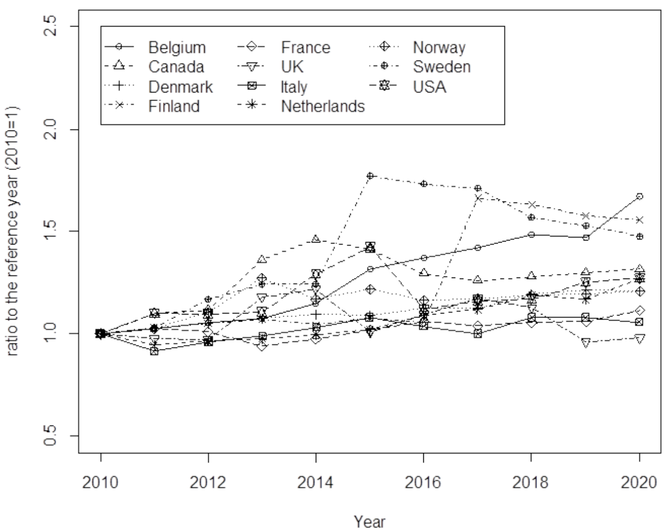

Figure 46: GDP projections of nested CES function model using the elasticity parameters of 11 countries, excluding West Germany, in van der Werf (2008) for the nesting structure where capital and labor are combined first and then energy is rebundled with this bundle.

There is a limitation in this trial because the parameters of 11 countries were applied to the CGE model composed for the case of Korea. Nevertheless it can give a glimpse of the risk of using parameters estimated from country level cross-sectional dataset. The elasticity parameters of individual countries contain their own unique memories of responses to external shocks in the past. Thus, it may be undesirable to use the similarity of two economies as a standard in borrowing the elasticity parameters.

For this reason, researchers prefer to depend on time series data in model constructions. However, there is another issue in this case because the time period of a data sample often matters. Figure 47 depicts two GDP projections of a nested CES function model for West Germany when using the two sets of elasticity parameters of van der Werf (2008) and Kemfert (1998) in Table 15. For the first few years, the two trajectories are consistent with each other but there arises a gap after a certain period. In fact, the two time series datasets are different in the period: van der Werf (2008) has the time period of 1978 to 1996 while Kemfert (1998) has the period of 1960 to 1993. Specifically, the longer time domain of Kemfert (1998) seems to lessen the ability to maintain the output level. However, aside from the time effect, the other differences in estimation methods can also affect the long-term projections.

The model of a set of firms, introduced in this study, is an alternative way to overcome such shortcomings embedded in time series data. Specifically, the newly introduced model separates the past time memories from the current economy and enables an interpretation of the economy from the viewpoint of the present time. In this way, the elasticity of substitution esti-

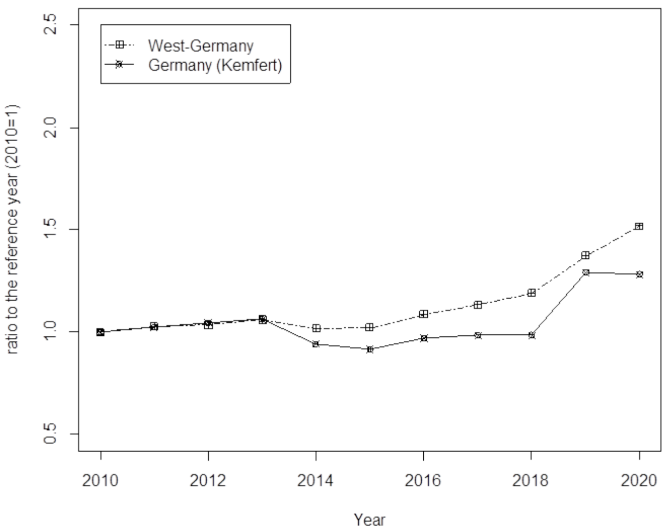

Figure 47: GDP projections of nested CES function model using two elasticity parameters for West Germany from van der Werf (2008) and Kemfert (1998) for the nesting structure where capital and labor are combined first and then energy is rebundled with this bundle.

mated by the model of a set of firms only implies how each sector determines its production level to respond to exogenous changes under the current circumstances, not including a representative agent's movement or tendency revealed in history.[23] In this sense, the new model of this study is essentially a bottom-up model because it only reflects the *status quo* economy. However, this model transforms the fixed structures of bottom-up models to a distribution based model, taking the form of a conventional macroeconomic model with the theoretical basis of Houthakker (1955-1956) and Jones (2005).

The following experiment is to check the historical information embedded in time series data for the case of Korea. First, a GDP projection with a nested CES function model was carried out for the elasticity parameter obtained from time series data of Korea. (Table 15) Then, the result was compared with projection results of using the new model with fixed input structures. This comparison is depicted in Figure 48. The two projections have trajectories different from each other and this discrepancy comes from the past time economy.

There is the third trajectory of a nested CES function model with pooled data. In other words, the parameters were estimated by the CES function nesting structure, where capital and labor are combined first and then energy is rebundled with this bundle, using the dataset for the year 2010 employed

[23]There is uncertainty in defining business as usual (BAU) or baseline scenario in climate change policy analysis. Typically, a BAU scenario considers and reflects the future technical changes by employing estimated elasticity parameters. However, in most cases, the parameters come from the past history, which is not appropriate for predicting the future economy. Therefore, it can be better to exclude the use of estimated parameters when defining the BAU scenario. Instead, the scenario of technology changes should be provided by other independent models.

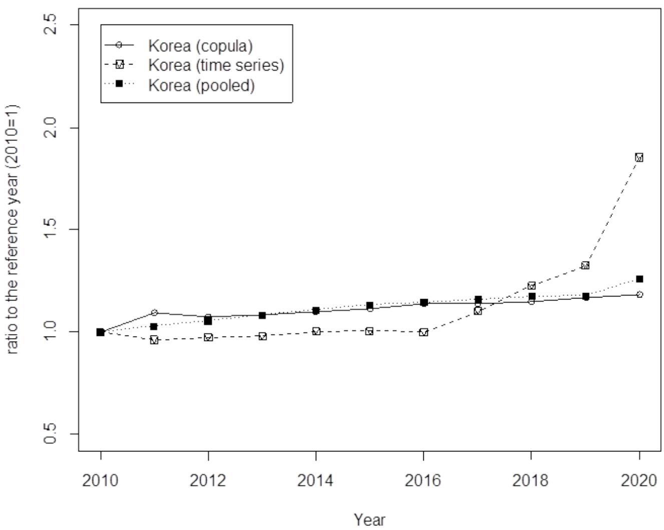

Figure 48: A comparison of three GDP projections for Korea – two projections of nested CES function model with parameters from time series data of 1981 to 2010 and pooled data of year 2010 as well as one projection of fixed input structure model with a set of firms.

in this study. Interestingly, the trajectory of this model overlaps with the fixed input structure model or the model of a set of firms. This seems to verify the usage of aggregate production functions with a certain year's dataset of microscopic technology information. However, it is said that aggregate production functions exist only under stringent conditions on local production functions. (Nataf, 1948; Fisher, 1969; Fisher et al., 1977) Moreover, the elasticity of substitution in aggregate production functions is often expressed as 'an estimate of nothing.' Nevertheless, Fisher et al. (1977) shows that the fit of aggregate production functions, including CES functions, is very good in empirical experiments with multiple 'firms.' Thus, any attempt to explain the overlapping trajectories of Figure 48 leads to nothing more than empirical and phenomenological arguments.

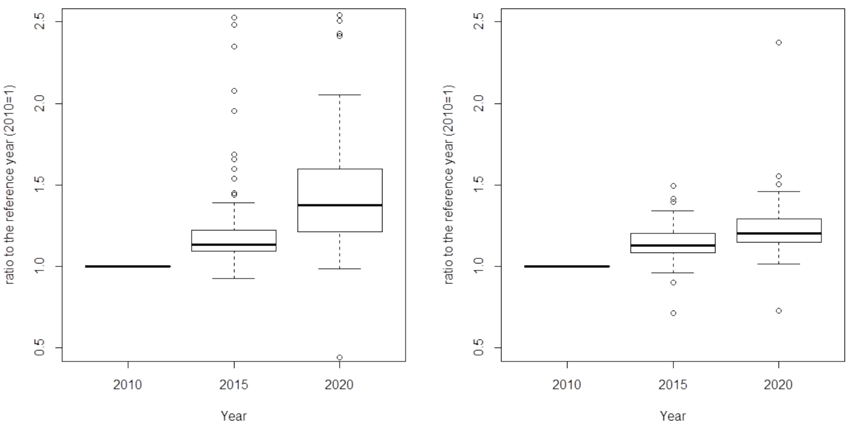

Figure 49: A comparison of GDP projections between nested CES function model (left) and fixed input structure model (right). The simulation was done 100 times. In the fixed input structure model, the number of randomly generated firms was set at 100.

Even though the two trajectories are almost consistent with each other, the projection errors may differ. Figure 49 represents a comparison of the two GDP projections between the nested CES function model and the fixed input structure model (or the model of a set of firms). The parameters of elasticity of substitution in the CES function case and the characteristic parameters of the copula model in the fixed input structure model were varied within their standard deviations.[24] In addition, in the fixed input structure model, the set of firms was altered in every iteration, which implies a change of the dependence structure. The boxplots of Figure 49 imply an improvement of the predictability of the CGE model when using the copula approach. The deviations of projection results in the fixed input structure model with the copula approach are much smaller than in the CES function case although the number of firms, 100, are smaller than the size of the real dataset, 308.

4.6 Conclusion

In this chapter, an empirical assessment of the possibility of applying the statistical distribution approach to a description of energy related production activity was given. First, the microfoundations of aggregate production functions were reviewed on a theoretical basis. It was shown that, when a concept of statistical distribution is introduced, a set of bottom-up microscopic information converges to a macroeconomic function and vice

[24]The standard errors of the parameters in the CES function case indicate that they are not statistically significant. Hence, the confidence intervals were arbitrarily lowered to secure a minimum level of statistical significance in this experiment.

versa. The characteristics of statistical distributions, including the mutual dependence between variables, were examined for a real dataset.

Before putting those theoretical findings into practice, a statistical tool called copulas was introduced to conveniently reproduce the relationships embedded in a multivariate joint distribution. After the basic theory of copulas was briefly introduced, empirical studies were carried out to investigate the performance of a two-level nested Gumbel copula model. Despite the reliance on number of parameters and nesting structures, a copula model turned out to be successful in describing the relationships embedded in an energy-included multivariate joint distribution map. To emphasize the advantage of this statistical instrument, empirical results were provided on the limitation of conventional CES functions in applying microscopic information.

After the introduction of copulas, a new type of CGE model was tried. An aggregation of local Leontief production functions was employed as an alternative global production technology description. Its properties were examined in terms of the cost function and the elasticity of substitution. Then, a pilot model was composed to apply this scheme to a CGE model. It was argued that the modified production structure gains some advantages over conventional CES function based models. First, the new model can eliminate the effects of past data, leaving the basic principle of general equilibrium intact. Second, it can improve the robustness of projection results, narrowing standard errors.

In spite of the effectiveness of the alternative method proposed in this study, there is still a strong need for more elaboration on model structure specification because a copula model can be sensitive to the inner bundle structures. This is why in-depth studies on the correlations of observed variables as well as extended explorations of a variety of copula families, including multi-parameter ones, are recommended. Additional work should be done on the properties of the size distributions of individual variables. It is natural for the properties of each distribution to change in the long-run. However, the copula method is grounded on the unverified assumption that the information embedded in a joint multivariate distribution is invariant. Thus, for an accurate long-term projection, scenarios should be developed to reflect a long-term trend of each distribution. The new concept needs to be extended to other sectors as well. In this research, the distribution approach was applied with a focus on the manufacturing sector because there is little statistical information for other sectors.

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Appendices

Chapter A

The structure of the pilot CGE model

This chapter briefly outlines the mathematical structure of the CGE model employed in Chapter 4, which has been modified from the original version of Kang and Kim (2007), almost keeping its notations.

1. Production side

   (a) Final goods production

   The aggregate final goods production xt_i consists of composite intermediate good m_i, composite input factor xb_i and indirect tax IDT_i for sector i, and it is expressed as follows:

$$xt_i = \min\left[\frac{m_i}{1-axb_i-idtr_i},\ \frac{xb_i}{axb_i},\ \frac{IDT_i}{idtr_i}\right]. \tag{A.1}$$

   When the supply of each good is determined at equilibrium by the solution of cost minimization problem, the demands, m_i, xb_i and IDT_i, and their prices, pt_i, pm_i and pxb_i, are given by

$$m_i = am_i \cdot xt_i, \tag{A.2}$$

$$xb_i = axb_i \cdot xt_i, \tag{A.3}$$

$$IDT_i = idtr_i \cdot xt_i, \tag{A.4}$$

$$pt_i = am_i \cdot pm_i + axb_i \cdot pxb_i \cdot idtr_i \cdot p_i, \tag{A.5}$$

$$am_i = 1 - axb_i - idtr_i, \tag{A.6}$$

where axb_i is the composite input factor coefficient, $idtr_i$ is the indirect tax rate, and am_i is the composite intermediate good input coefficient.

(b) Composite input factor in non-manufacturing sectors

The composite input factor production function xb_i for producing final good i is defined by the following CES function with input factors such as labor l_i, capital k_i and energy e_i:

$$xb_i(l_i, k_i, e_i) = ab_i \left[\sum_f alp(f,i) f^{\rho 1_i} \right]^{-1/\rho 1_i}, \tag{A.7}$$

$$\sum_f alp(f,i) = 1, \tag{A.8}$$

$$\sigma 1_i = 1/(1 + \rho 1_i), \tag{A.9}$$

where $f \in \{l_i, k_i, e_i\}$, ab_i is the scale parameter, $alp(f,i)$ is the distribution parameter, and $\sigma 1_i$ is the elasticity of substitution in the labor-capital-energy composite input factor.

By cost minimization with the above function, the demand for each input factor is given by

$$l_i = ab_i^{\sigma 1_i - 1} \left[alp(l,i) pxb_i / pl_i \right]^{\sigma 1_i} xb_i, \tag{A.10}$$

$$k_i = ab_i^{\sigma 1_i - 1} \left[alp(k,i) pxb_i / pk_i \right]^{\sigma 1_i} xb_i, \tag{A.11}$$

$$e_i = ab_i^{\sigma 1_i - 1} \left[alp(e,i) pxb_i / pe_i \right]^{\sigma 1_i} xb_i. \tag{A.12}$$

The price of the composite input factor is expressed as

$$pxb_i = 1/ab_i \left[\sum_f alp(f,i)^{\sigma 1_i} p(f,i)^{1-\sigma 1_i} \right]^{1/(1-\sigma 1_i)}, \tag{A.13}$$

where $p(f,i)$ is the price of input factor

(c) Composite input factor in the manufacturing sector

The composite input factor production function xbf_s for firm s is defined by lf_s, kf_s and ef_s, the demand functions for labor, capital and energy, respectively:

$$xbf_s = \min\left(\frac{lf_s}{acl_s}, \frac{kf_s}{ack_s}, \frac{ef_s}{ace_s} \right), \tag{A.14}$$

where the coefficients, acl_s, ack_s and ace_s, are calibrated to the reference year dataset. To impose a constraint on the production capacity, the share of each firm is determined by a fixed coefficient $axbf_s$:

$$xbf_s \cdot pxbf_s = axbf_s \cdot xb \cdot pxb, \tag{A.15}$$

where $pxbf_s$ means an imaginary price of the good xbf_s from a firm s, which can be regarded as an average production cost of the firm. xb is the gross output of the manufacturing sector, and pxb is the average price of goods from the sector. The production level is determined under the following budget constraint,

$$xb \cdot pxb = \sum_s xbf_s \cdot pxbf_s. \tag{A.16}$$

(d) Composite intermediate goods

Composite intermediate good m_i is given by the following Leontief function:

$$m_i = \min\left[xm_{j,i}/amm_{j,i}\right], \tag{A.17}$$

$$\sum_j amm_{j,i} = 1, \tag{A.18}$$

where j denotes non-energy sectors as well as the oil and gas sectors. $xm_{j,i}$ is the demand for commodity j in sector i, and $amm_{j,i}$ is the input coefficient of commodity j. m_i has the following relationship with $xm_{j,i}$:

$$xm_{j,i} = amm_{j,i} \cdot m_i, \tag{A.19}$$

$$pm_i = \sum_j amm_{j,i} \cdot p_j, \tag{A.20}$$

where p_j is the supply price of final good j and pm_i is the price of composite intermediate good i.

Summing $xm_{j,i}$ leads to the aggregate demand for intermediate goods as follows:

$$dm_j = \sum_i xm_{j,i}, \tag{A.21}$$

where dm_j is the sum of the demand of final good j in sector i.

2. Demand side

(a) Household

Disposable income DHI is

$$DHI = (1 - dtr)\sum_{fc}[pf(fc) \cdot qfa(fc)] + TP, \tag{A.22}$$

where $fc \in \{l,k\}$, dtr is the rate of direct tax and $qfa(fc)$ is the supply of input factors such as labor and capital. TP is the transfer income from the government.

The household utility $U(xc_i)$ is given by a Cobb-Douglas function:

$$U(xc_i) = \prod_i xc_i^{\delta_i}, \tag{A.23}$$

where xc_i is the consumption of commodity i and δ_i is the share of xc_i.

Household savings HS and the consumption demand are written as

$$HS = hsr \cdot DHI, \tag{A.24}$$

$$hc_i = \delta_i \cdot (1 - hsr)DHI/p_i, \tag{A.25}$$

where δ_i means the share of goods i in total consumption, and hsr is the share of household savings.

(b) Government demand

Tax revenue TR of the government is given by

$$TR = \sum_i idtr_i \cdot pt_i \cdot xt_i + dtr \left[\sum_{fc} pf(fc) \cdot qfa(fc) \right] + Fbor, \tag{A.26}$$

where dtr is the direct tax rate and $Fbor$ is the tariff revenue.

Government savings GS, transfer payment to household TP, and the government's consumption demand gc_i for good i are summarized as follows:

$$GS = gsr \cdot TR, \tag{A.27}$$

$$TP = tpr \cdot TR, \tag{A.28}$$

$$gc_i = \gamma_i (1 - tpr - gsr) TR / p_i, \tag{A.29}$$

where gsr is the government saving rate, tpr is the transfer expenditure rate and γ_i is the government propensity to consume.

(c) Savings and investment

Total investment TIV consists of household savings HS, government savings GS, and foreign savings $FSAV$:

$$TIV = HS + GS + FSAV. \tag{A.30}$$

Total real investment $tinv$ is calculated by dividing TIV by the price of investment good p_I:

$$tinv = TIV/p_I. \tag{A.31}$$

The investment goods corresponding to $tinv$ are produced by the following Leontief function with input $invd_i$ allocated from each industrial sector:

$$tinv = \min\left[invd_i/ainv_i\right], \tag{A.32}$$

where $ainv_i$ is the capital input coefficient for producing aggregate investment goods in sector i.

3. Foreign trade side

(a) Transformation of final goods

Final good xt_i is divided into domestic good xd_i and export good xex_i by a constant elasticity of transformation (CET) function:

$$xt_i = at_i\left[alt(i,d)xd_i^{-\rho t_i} + alt(i,ex)xex_i^{-\rho t_i}\right]^{-1/\rho t_i}, \tag{A.33}$$

where xd_i is domestic good and xex_i is export good. $alt(i,d)$ and $alt(i,ex)$ are the distribution coefficients, σt_i is the elasticity of transformation and $\rho t_i = (\sigma t_i - 1)/\sigma t_i$.

According to zero profit assumption, the producer's total revenue is expressed as

$$pt_i \cdot xt_i = pd_i \cdot xd_i + pxe_i \cdot xex_i, \tag{A.34}$$

where pt_i, pd_i and pxe_i denote the price of final goods, the price of domestic goods and the price of export goods, respectively. The price of export goods, pxe_i, is determined by the currency rate exr and the world market price of export goods, $pxew_i$, as follows:

$$pxe_i = exr \cdot pxew_i. \tag{A.35}$$

From zero profit condition and cost minimization with the CET function, the optimized supply of export goods is determined by

$$xex_i/xd_i = [pd_i/pex_i \cdot alt(i,ex)/alt(i,d)]^{\sigma t_i}. \tag{A.36}$$

(b) Armington goods

Armington good xs_i is composed with domestic goods xd_i and import goods im_i:

$$xs_i = as_i \left[als(i,d) xd_i^{-\rho s_i} + als(i,im) im_i^{-\rho s_i} \right]^{-1/\rho s_i}, \tag{A.37}$$

where $als(i,d)$ and $als(i,im)$ are the distribution coefficients, and $als(i,d) + als(i,im) = 1$. σs_i is the elasticity of substitution in the Armington goods supply function, and $\rho s_i = (1 -$

$\sigma s_i)/\sigma s_i$. The related equilibrium is determined by

$$ps_i \cdot xs_i = pd_i \cdot xd_i + pxm_i \cdot im_i, \tag{A.38}$$

where ps_i, pd_i and pxm_i are the price of Armington goods, the price of domestic goods, and the price of import goods, respectively.

The demands for xd_i and im_i are determined by

$$xd_i = as_i^{-1} \left\{ \left[als(i,d)^{\sigma s_i} pd_i^{1-\sigma s_i} + als(i,im)^{\sigma s_i} pxm_i^{1-\sigma s_i} \right]^{1/(1-\sigma s_i)} \frac{als(i,d)}{pd_i} \right\}^{\sigma s_i} xs_i, \tag{A.39}$$

$$im_i = as_i^{-1} \left\{ \left[als(i,d)^{\sigma s_i} pd_i^{1-\sigma s_i} + als(i,im)^{\sigma s_i} pxm_i^{1-\sigma s_i} \right]^{1/(1-\sigma s_i)} \frac{als(i,im)}{pxm_i} \right\}^{\sigma s_i} xs_i. \tag{A.40}$$

The price of Armington goods, ps_i, is expressed as

$$ps_i = as_i^{-1} \left[als(i,d)^{\sigma s_i} pd_i^{1-\sigma s_i} + als(i,im)^{\sigma s_i} pxm_i^{1-\sigma s_i} \right]^{1/(1-\sigma s_i)}, \tag{A.41}$$

and the price of import goods, pxm_i, is calculated by

$$pxm_i = exr \cdot pxmw_i, \tag{A.42}$$

where $pxmw_i$ is the world market price of import goods and exr is the currency rate.

4. Other conditions

   (a) Market clearing

   The clearing condition for the market of commodity i is written as

$$xs_i = \sum_j xm_{j,i} + hc_i + gc_i + invd_i, \tag{A.43}$$

   where xs_i is the supply of Armington goods, $xm_{j,i}$ is the demand for intermediate goods, hc_i is the household demand, gc_i is the government demand, and $invd_i$ is the investment demand.

   The market clearing conditions for the markets of input factors such as labor l and capital k are given by

$$\sum_i f_{fc,i} = endow_{fc}, \tag{A.44}$$

   where $endow_{fc}$ is the total stock of production input factors $fc \in \{k,l\}$. Likewise, as for the foreign trade,

$$\sum_i pxm_i \cdot im_i = \sum_i pxe_i \cdot xex_i + FSAV. \tag{A.45}$$

   The consumer price index is defined to describe the change of the price vector:

$$CPI = \sum_i alhc_i \cdot p_i, \tag{A.46}$$

where $alhc_i$ is the share of final goods consumption.

(b) Long-term scenario

The capital stock at time t is calculated by

$$KS_t = (1-\delta_t)KS_{t-1} + ati_{t-1}TIV_{t-1}, \tag{A.47}$$

where δ_t is the depreciation rate for time t. ati_{t-1} is the adjustment parameter and TIV_{t-1} is the total investment at time $t-1$. The labor supply at time t is determined by the labor growth rate n_t and the labor supply of the previous time period as follows:

$$L_t = alt_{t-1}(1+n_t)L_{t-1}, \tag{A.48}$$

where alt_{t-1} is the adjustment parameter.

Table 18 provides the default values of the elasticity of substitution for individual commodities. Table 19 is the social account matrix (SAM) of Korea for the year of 2010. The industry is divided into seven sectors: agriculture (AGRI), manufacturing (MANU), constructing (CONS), service (SERV), oil (OIL), natural gas (GAS), and energy (ENER). Also, there are other sectors such as government (G), investment (GS), and foreign trade (F). Primary input factors, labor (L) and capital (K), are also included.

Table 18: The default values of elasticity of substitution for individual commodities.

	Composite production factors	Elasticity of transformation for export goods	Elasticity of substitution in Armington goods
Agriculture	0.5	3.9	1.5
Manufacturing	0.7	2.9	2.5
Construction	0.7	0.7	2
Service	0.7	0.7	2
Energy	0.4	2.9	2

Table 19: Social account matrix of Korea in the year 2010.

	AGRI	MANU	CONS	SERV	OIL	GAS	ENER	L	K	H	G	CS	F	TOT
AGRI	3454277	34096359	345277	7279786	0	0	1849	0	0	16430639	0	2245964	790879	64645030
MANU	14343243	869200811	80251083	119758560	0	0	7756227	0	0	132778423	0	125654770	496247697	1845990814
CONS	35292	473766	29680	10489371	0	0	608597	0	0	0	0	176339167	368831	188344704
SERV	5038835	175302308	27941548	366371418	0	0	8371931	0	0	430226848	178396187	41033273	88333813	1321016161
OIL	0	19919567	0	0	0	0	60361148	0	0	0	0	32314	0	80313029
GAS	0	0	0	0	0	0	21336372	0	0	0	0	542185	0	21878557
ENER	2173760	39331721	4447067	59448896	0	0	33229684	0	0	37546726	0	1837322	32933402	210948578
L	3541130	134125699	46536518	335118374	0	0	6950967	0	0	0	0	0	7030	526279718
K	23395756	164348237	16418482	295154090	0	0	15323446	0	0	0	0	0	171261	514811272
H	0	0	0	0	0	0	0	526279718	514811272	0	0	0	0	1041090990
G	902811	30357833	12366081	42900157	0	0	24961679	0	0	90299200	0	0	-1844761	199943000
CS	0	0	0	0	0	0	0	0	0	333809154	21546813	0	-7670972	347684995
F	11759926	378834513	8968	84495509	80313029	21878557	32046678	0	0	0	0	0	0	609337180
TOT	64645030	1845990814	188344704	1321016161	80313029	21878557	210948578	526279718	514811272	1041090990	199943000	347684995	609337180	

Chapter B

Source code

```
*Source Code of the pilot CGE model,
*modified by Changhun Kim from the original code of Kang and Kim (2007).

OPTION DECIMALS=8;

*$include init_id.txt
set id /1*300/

Sets

A total sectors /AGRI,MANU,CONS,SERV,OIL,GAS,ENER,L,K,H,G,CS,F,TOT/
TA Total production output /AGRI,MANU,CONS,SERV,OIL,GAS,ENER,L,K,G/
i Industries and commodities /AGRI,MANU,CONS,SERV,OIL,GAS,ENER/
s(i) Total production sector /AGRI,MANU,CONS,SERV,ENER/
nm(s) Non-manufacturing /AGRI,CONS,SERV,ENER/
mn(s) Manufacturing /MANU/
ne(i) Non-competitive energy and non-energy sectors
/AGRI,MANU,CONS,SERV,OIL,GAS/
ie(i) Non-competitive energy /OIL,GAS/
e(i) Energy sector /ENER/
fc Composite production factor /L,K,ENER/
f(fc) Basic production factors /L,K/

*id firm id /14701*15000/
qnty quantity /lab, kap, yva, ene/
;

alias (A,AA),(i,j),(s,ss),(fc,fcc),(e,ee),(f,ff);
```

```
parameter rowSAM(*,*) Social Accounting Matrix;
$libinclude xlimport rowsam sam_10.xls sam1!a2:o16
*$include init_sam.txt
display rowsam;
parameter sam(*,*) scaled social accounting matrix;
sam(a,aa) = rowsam(a,aa)/1e6;

parameter lkye(*,*) copula generated data;
$libinclude xlimport lkye sam_10.xls lkye1!a1:e45001
display lkye;

parameters
xbf0(id), lab0(id),kap0(id),ene0(id)
tot_xbf0, tot_lab0, tot_kap0, tot_ene0
;

Parameters
xt0(i),xb0(i),m0(i),IDT0(i),xm0(i,j),l0(i),k0(i),ce0(i)
xex0(i),xd0(i),xim0(i),xs0(i)
pt0(i),pxb0(i),pm0(i),pce0,pl0,pk0,pxbf0
pex0(i),pd0(i),pim0(i),ps0(i),p0(i),pexw0(i),pimw0(i),exr0
;

xt0(i) = sum(ta,sam(ta,i));
xb0(i) = sum(fc,sam(fc,i));
m0(i) = sum(ne,sam(ne,i));
IDT0(i) = sam('g',i);
l0(i) = sam('l',i);
k0(i) = sam('k',i);
ce0(i) = sum(e,sam(e,i));

tot_xbf0=sum(id,lkye(id,'yva'));
tot_lab0=sum(id,lkye(id,'lab'));
tot_kap0=sum(id,lkye(id,'kap'));
```

```
tot_ene0=sum(id,lkye(id,'ene'));

xbf0(id) = xb0('MANU')*lkye(id,'yva')/tot_xbf0;
lab0(id) = l0('MANU')*lkye(id,'lab')/tot_lab0;
kap0(id) = k0('MANU')*lkye(id,'kap')/tot_kap0;
ene0(id) = ce0('MANU')*lkye(id,'ene')/tot_ene0;

xm0(i,j) = sam(i,j);
xex0(i)=sam(i,'f');
xim0(i)= sam('f',i);
xd0(i)=xt0(i)-xex0(i);
xs0(i)=xd0(i)+xim0(i);

pt0(i)=1;pxb0(i)=1;pm0(i)=1;pce0=1;pl0=1;pk0=1;pxbf0=1;
pex0(i)=1;pd0(i)=1;pim0(i)=1;ps0(i)=1;p0(i)=1;pexw0(i)=1;
pimw0(i)=1;exr0=1;
display xt0,xb0,m0,IDT0,xm0,ce0,l0,k0, xbf0, lab0,kap0,ene0
xex0,xd0,xim0,xs0, pt0,pxb0,pm0,pce0,pl0,pk0
pex0,pd0,pim0,ps0,pexw0,pimw0,exr0
;

parameters els(i,*) elasticity of substitution;
$libinclude xlimport els sam_10.xls els1!a2:f9
display els;

parameters
sigt(i) elasticity of transformation btw domestic and exported goods
rot(i)
altd(i) distribution parameter for domestic goods
in transformation function
altx(i) distribution parameter for exported goods
in transformation function
at(i) scale parameter of transformation ftn

sigs(i) elasticity of substitution btw domestic and imported goods
```

ros(i)
alsd(i) distribution parameter for domestic goods
in armington function
alsm(i) distribution parameter for imported goods
in armington function
as(i) scale parameter of armington ftn

am(ne,j) leontief coefficient of composite non energy
intermediate input
axb(i) leontief coefficient of composite production
factor input(fc)
idtr(i) indirect tax rate
aio(i,j) sectoral intermediate input coefficient

sig1(i) elasticity of substitution btw composite production factors
ro1(i)
al_l(i) distribution parameter for labor (l)
al_k(i) distribution parameter for capital (k)
al_e(i) distribution parameter for energy composite (e)
ab(i) scale parameter of composite production factors (fc)

axbf(id)
ac_l(id) leontief coefficient of labor
ac_k(id) leontief coefficient of capital
ac_e(id) leontief coefficient of energy

end_l endowment of labor
end_k endowment of capital
dtr direct tax rate
hsr household saving rate
tpr rate of transfer payment
alhc(i) household consumption ratio by goods
pc0(i) household consumption price
idtr(i) indirect tax rate of production sectors
gsr government saving rate

```
algc(i) government consumption ratio by goods
ainv(i) leontief coefficient of sectoral investment demand
;

sigs(i) = els(i,'elss');
ros(i)$(s(i)) = 1/sigs(i)-1 ;
alsd(i)$(xd0(i) ne 0) = 1/[ (xim0(i)/xd0(i))
**(ros(i)+1)*(pim0(i)/pd0(i)) + 1 ];
alsm(i) = 1/[ (xd0(i)/xim0(i))**(ros(i)+1)*(pd0(i)/pim0(i)) +1];
as(i)$(xd0(i) ne 0) = xs0(i)/ [alsd(i)*xd0(i)**(-ros(i))
+alsm(i)*xim0(i)**(-ros(i))]**(-1/ros(i));

sigt(i) = els(i,'elst');
rot(i)$(s(i)) = 1- 1/sigt(i);
altd(i)$(s(i)) = 1/[1+ (pex0(i)/pd0(i))*(xd0(i)/xex0(i))
**(rot(i)-1) ];
altx(i)$(s(i)) = 1/[1+ (pd0(i)/pex0(i))*(xex0(i)/xd0(i))
**(rot(i)-1) ];
at(i)$(s(i)) = xt0(i)/ [altd(i)*xd0(i)**(-rot(i))
+altx(i)*xex0(i)**(-rot(i))]**(-1/rot(i));

display sigs, ros, alsd, alsm, as, sigt, rot, altd, altx, at;

am(ne,j)$(m0(j) ne 0)= xm0(ne,j)/m0(j);
axb(i)$(xt0(i) ne 0)=xb0(i)/xt0(i);
idtr(i)$(xt0(i) ne 0)= sam('g',i)/xt0(i);
aio(i,j)$(xt0(j) ne 0) = xm0(i,j)/xt0(j);

sig1(i) = els(i,'elsb');
ro1(i)$(s(i)) = 1/sig1(i)-1 ;
al_l(i)$(s(i)) = 1/ [ {k0(i)/l0(i)}**(ro1(i)+1)*(pk0/pl0)
+ {ce0(i)/l0(i)}**(ro1(i)+1)*(pce0/pl0) +1 ];
al_k(i)$(s(i)) = 1/ [ {l0(i)/k0(i)}**(ro1(i)+1)*(pl0/pk0)
+ {ce0(i)/k0(i)}**(ro1(i)+1)*(pce0/pk0) +1 ];
al_e(i)$(s(i)) = 1/ [ {k0(i)/ce0(i)}**(ro1(i)+1)*(pk0/pce0)
```

```
+ {l0(i)/ce0(i)}**(ro1(i)+1)*(pl0/pce0)+1 ];
ab(i)$(s(i)) = xb0(i)/[ al_k(i)*k0(i)**(-ro1(i))+al_l(i)*l0(i)
**(-ro1(i)) +al_e(i)*ce0(i)**(-ro1(i)) ]**(-1/ro1(i));

axbf(id)$(xb0('MANU') ne 0)=xbf0(id)/xb0('MANU');
ac_l(id)$(xbf0(id) ne 0)=lab0(id)/xbf0(id);
ac_k(id)$(xbf0(id) ne 0)=kap0(id)/xbf0(id);
ac_e(id)$(xbf0(id) ne 0)=ene0(id)/xbf0(id);

display am,axb,idtr,aio,sig1,ro1,al_k,al_l,al_e,ab, ac_l,ac_k,ac_e;
*,rom,sigm,al_lk,al_ce,abm,romlk,sigmlk,alml,almk,abmlk;

end_l = sam('h','l');
end_k = sam('h','k');
dtr = sam('g','h')/(sam('h','l')+sam('h','k'));
hsr = sam('cs','h')/{sam('tot','h')-sam('g','h')};
tpr = sam('h','g')/sam('tot','g');
alhc(i) = sam(i,'h')/{sum(j,sam(j,'h'))};
pc0(i) = 1;
idtr(i)$(xt0(i) ne 0) = IDT0(i)/xt0(i);
gsr = sam('cs','g')/(sam('g','tot')-sam('h','g'));
algc(i) = sam(i,'g')/(sum(j,sam(j,'g')));
ainv(i) = sam(i,'cs')/sum(j,sam(j,'cs'));
display end_l,end_k,hsr,dtr,alhc,pc0, idtr, gsr,algc,ainv
;
parameters
HI0,DHI0,HS0,hc0(i),TP0,TR0,GS0,gc0(i),TIV0,pi0,IT0,INV0(i),
FTAX0,ftaxr, FSAV0,FINV0, CPI0
;
HI0 = sam('h','tot');
DHI0 = sam('h','tot')-sam('g','h');
HS0 = sam('cs','h');
hc0(i) = sam(i,'h');
TP0 = sam('h','g');
TR0 = sam('g','tot');
```

```
GS0 = sam('cs','g');
gc0(i) = sam(i,'g');
TIV0 = sam('tot','cs');

pi0 = 1;
IT0 = TIV0/pi0;
INV0(i) = sam(i,'cs');
FTAX0 = sam('g','f');
ftaxr = sam('g','f')/sum(i,xim0(i));
FSAV0 = sam('cs','f');
FINV0 = sam('f','cs');
CPI0 = 1;
display HI0,DHI0,HS0,hc0,TP0,TR0,GS0,gc0,TIV0,pi0,IT0,INV0,
FTAX0,FSAV0,FINV0
;

Variables
xs(i) armington composite goods = supply of goods
xim(i) imported goods
xd(i) domestically supplied goods
ps(i) price of supply goods
pim(i) domestic price of imported goods
pimw(i) world price of imported goods
exr exchange rate
pd(i) price of domestic supplied goods
pex(s) export price
pexw(s) world price of exported goods
xex(s) exported goods
xt(s) total production of commodity
pt(s) price of production goods

xb(s) composite production factor input
m(s) composite intermediate input
IDT(s) indirect tax
```

l(s) composite factor

k(s) capital demand by sector

ce(s) composite energy

xbf(id)

lbf(id)

kpf(id)

enf(id)

pxb(s) price of composite producton factor

pm(s) price of composite intermediate

pxbf(id)

pce price of composite energy

pl price of labor (wage level)

pk price of capital (rate of return)

xm(i,j) intermediate demand by sectors

DHI household disposible income

HS household savings

hc(i) household consumption

TR total government revenue

TP transfer payment

GS government savings

gc(i) real government consumption

TIV total investment amount

invd(i) sectoral investment by origin

FSAV foreign saving closure variable between foreign capital

in-out flow with finv

FINV foreign investment

qfa(f) stock accoumulation

CPI consumer price index

sub(s) subsidy for some sectors

p(i) armington supply price after imposing carbon tax

```
;

xs.l(i) = xs0(i);
xim.l(i) =xim0(i);
xd.l(i) = xd0(i);
ps.l(i) = ps0(i);
pim.l(i) = pim0(i);
pimw.l(i) = pimw0(i);
exr.l = exr0;
pd.l(i) = pd0(i);
xex.l(s) = xex0(s);
pex.l(s) = pex0(s);
pexw.l(s) = pexw0(s);
xt.l(s) = xt0(s);
xb.l(s) = xb0(s);
m.l(s) = m0(s);
idt.l(s) = idt0(s);
xm.l(i,j) = xm0(i,j);
ce.l(s) = ce0(s);
l.l(s) = l0(s);
k.l(s) = k0(s);

loop(id,
xbf.l(id)=xbf0(id);
lbf.l(id)=lab0(id);
kpf.l(id)=kap0(id);
enf.l(id)=ene0(id);
pxbf.l(id) = pxbf0;
);

pt.l(s) = pt0(s);
pxb.l(s) = pxb0(s);
pm.l(s) = pm0(s);

pce.l = pce0;
```

```
pl.l = pl0;
pk.l = pk0;

DHI.l = DHI0;
HS.l = HS0;
hc.l(i) = hc0(i);
TP.l = TP0;
TR.l = TR0;
GS.l = GS0;
gc.l(i) = gc0(i);
TIV.l = TIV0;
*PI.l = PI0;
*it.l = it0;
invd.l(i) = inv0(i);
*FTAX.l = FTAX0;
FSAV.l = FSAV0;
FINV.l = FINV0;

qfa.l('l') = end_l;
qfa.l('k') = end_k;
*initial value of CPI
CPI.l = CPI0;
sub.l(s) = 0;

parameters ghg(i,*) emission coefficient;
$libinclude xlimport ghg sam_10.xls ghgem!a1:c2
display ghg
;

parameters
cef(i) co2 emission ton per ton of energy
ctep(i) energy ton per million won
atc
pcb0
ct0(s)
```

```
emf0(s)
temf0
;

cef(i) = 44/12*ghg(i,'cte');
ctep(i) = ghg(i,'cetp');
atc=0;

pcb0=0;
*$include init_pcb.txt

emf0(s) = cef('ENER')*ctep('ENER')*ce0(s);
ct0(s) = (atc*pcb0)*emf0(s);

temf0 = sum(s,emf0(s));
display cef, ctep;

variables
CT(s) carbon tax
TCT
emf(s) ghg emission level by fossil fuel
temf total ghg emission level
pcb carbon price
p(i) armington supply price after imposing carbon tax
;

CT.l(s) = ct0(s);
emf.l(s)=emf0(s);
temf.l = temf0;
p.l(i) = p0(i);
pcb.l= pcb0;

Equations

E_cteq(s)
```

```
E_emfeq(s)
E_temfeq
E_TCTeq
E_peq(i)

E_qm_commodity(ne)
E_qm(e)

E_ps(i)
E_ps2(i)
E_pim(i)
E_xd(i)
E_xd2(i)
E_xim(i)
E_xim2(i)
E_pex(s)
E_xex(s)
E_pd(s)
E_xt(s)
E_pt(s)
E_xb(s)
E_IDT(s)
E_m(s)

E_pxb_nm(nm)
E_xb_nm(nm)
E_k_nm(nm)
E_l_nm(nm)
E_ce_nm(nm)

E_pxb_mn(mn)
E_xb_mn(mn)
E_l_mn(mn)
E_k_mn(mn)
E_ce_mn(mn)
```

```
E_xbf(id)
E_pxbf(id)
E_lbf(id)
E_kpf(id)
E_enf(id)

E_xm(e,s)
E_pce
E_xm(ne,s)
E_pm(s)
E_qm_l
E_qm_k
E_DHI
E_HS
E_hc(i)
E_TR
E_TP
E_GS
E_gc(i)
E_TIV
E_invd(i)
E_invd2(ie)
E_balance
E_CPI
;

E_qm_commodity(ne).. xs(ne)=e= sum(s,xm(ne,s))
+hc(ne)+gc(ne)+invd(ne);
E_qm(e).. xs(e)=e= sum(s,xm(e,s))+hc(e)+gc(e)+invd(e);

E_ps(i)$(s(i)).. ps(i)*as(i) =e= [alsd(i)**(sigs(i))*pd(i)
**(1-sigs(i)) +(alsm(i))**(sigs(i))*pim(i)**(1-sigs(i))]
**(1/(1-sigs(i)));
E_ps2(i)$(not s(i)).. ps(i) =e= pim(i);
```

```
E_pim(i).. pim(i) =e= exr*pimw(i);
E_xd(i)$(xd0(i) ne 0).. xd(i)*as(i) =e=[ {alsd(i)**(sigs(i))*pd(i)
**(1-sigs(i)) +alsm(i)**(sigs(i))*pim(i)**(1-sigs(i))}
**(1/(1-sigs(i)))*(alsd(i)/pd(i)) ]**sigs(i)*xs(i) ;
E_xd2(i)$(xd0(i) eq 0).. xd(i) =e= 0;
E_xim(i)$(s(i)).. xim(i)*as(i) =e=[ {alsd(i)**(sigs(i))*pd(i)
**(1-sigs(i)) +alsm(i)**(sigs(i))*pim(i)**(1-sigs(i))}
**(1/(1-sigs(i)))*(alsm(i)/pim(i)) ]**sigs(i)*xs(i) ;
E_xim2(i)$(not s(i)).. xim(i) =e= xs(i);
E_pex(s).. pex(s) =e= exr*pexw(s);
E_xex(s).. xex(s)/xd(s) =e= [(pd(s)/pex(s))*(altx(s)/altd(s)) ]
**sigt(s);
E_pd(s).. pt(s)*xt(s) =e= pd(s)*xd(s)+pex(s)*xex(s);
E_xt(s).. xt(s) =e= at(s)*[altd(s)*xd(s)**(-rot(s))
+(altx(s))*xex(s)**(-rot(s))]**(-1/(rot(s)));
E_pt(s).. (1+sub(s)-idtr(s))*pt(s) =e= axb(s)*pxb(s)
+(1-idtr(s)-axb(s))*pm(s);
E_xb(s).. xb(s)=e=axb(s)*xt(s);
E_IDT(s).. IDT(s)=e=idtr(s)*pt(s)*xt(s);
E_m(s).. m(s)=e=(1-axb(s)-idtr(s))*xt(s);

E_pxb_nm(nm).. pxb(nm)*ab(nm)=e={ al_k(nm)**(sig1(nm))*pk
**(1-sig1(nm)) + al_l(nm)**(sig1(nm))*pl**(1-sig1(nm))
+al_e(nm)**(sig1(nm))*pce**(1-sig1(nm)) }**(1/(1-sig1(nm)));
E_xb_nm(nm).. xb(nm) =e= ab(nm)*[al_l(nm)*l(nm)**(-ro1(nm))
+al_k(nm)*k(nm)**(-ro1(nm))+al_e(nm)*ce(nm)**(-ro1(nm))]
**(-1/(ro1(nm)));
E_k_nm(nm).. k(nm) =e= ab(nm)**(sig1(nm)-1)*{ al_k(nm)*pxb(nm)/pk}
**(sig1(nm))*xb(nm);
E_l_nm(nm).. l(nm) =e= ab(nm)**(sig1(nm)-1)*{ al_l(nm)*pxb(nm)/pl}
**(sig1(nm))*xb(nm);
E_ce_nm(nm).. ce(nm) =e= ab(nm)**(sig1(nm)-1)*{ al_e(nm)*pxb(nm)/pce}
**(sig1(nm))*xb(nm);

E_pxb_mn(mn).. xb(mn)*pxb(mn)=e=sum(id,xbf(id)*pxbf(id));
```

```
E_xb_mn(mn).. xb(mn)=e=sum(id,xbf(id));
E_l_mn(mn).. l(mn)=e=sum(id,lbf(id));
E_k_mn(mn).. k(mn)=e=sum(id,kpf(id));
E_ce_mn(mn).. ce(mn)=e=sum(id,enf(id));

E_xbf(id).. xbf(id)*pxbf(id)=e=axbf(id)*xb('MANU')*pxb('MANU');
E_pxbf(id).. pxbf(id)=e=ac_l(id)*pl+ac_k(id)*pk+ac_e(id)*pce;
E_lbf(id).. lbf(id)=e=ac_l(id)*xbf(id);
E_kpf(id).. kpf(id)=e=ac_k(id)*xbf(id);
E_enf(id).. enf(id)=e=ac_e(id)*xbf(id);

E_emfeq(s).. emf(s) =e= cef('ENER')*ctep('ENER')*ce(s);
E_CTeq(s).. CT(s) =e= atc*pcb*emf(s);
E_temfeq.. temf =e= sum(s,emf(s));
E_TCTeq.. TCT =e= sum(s, CT(s));
E_peq(i).. p(i) =e= ps(i);

E_xm(e,s).. xm(e,s)=e=ce(s);
E_pce.. pce=e=p('ENER');
E_xm(ne,s).. xm(ne,s)=e= am(ne,s)*m(s);
E_pm(s).. pm(s) =e= sum(ne,am(ne,s)*p(ne));
E_qm_l.. qfa('l') =e= sum(s,l(s));
E_qm_k.. qfa('k') =e= sum(s,k(s));
E_DHI.. DHI=e=(1-dtr)*sum(s,(pk*l(s)+pl*k(s)))+TP;
E_HS.. HS=e=hsr*DHI;
E_hc(i).. hc(i)=e=alhc(i)*(1-hsr)*DHI/p(i);
E_TR.. TR=e=dtr*sum(s,(pl*l(s)+pk*k(s)))+sum(s,(idtr(s)-sub(s))
*pt(s)*xt(s))+ftaxr*sum(i,pim(i)*xim(i))+TCT;
E_TP.. TP=e=tpr*TR;
E_GS.. GS=e=gsr*(1-tpr)*TR;
E_gc(i).. gc(i)=e=(algc(i)*(TR-TP-GS))/p(i);
E_TIV.. TIV =e= HS+GS+FSAV;
E_invd(s).. p(s)*invd(s)=e=ainv(s)*TIV;
E_invd2(ie).. p(ie)*invd(ie)=e=ainv(ie)*TIV;
E_balance.. sum(i,pim(i)*xim(i))+FINV=e=sum(s,pex(s)*xex(s))
```

```
+ftaxr*sum(i,pim(i)*xim(i))+FSAV;
E_CPI.. CPI=e=sum(i,alhc(i)*p(i));

Model sim_ncge1 /all/;

PARAMETER SELMOD;
SELMOD=1;
*IF SELMOD = 1, THEN STATIC MODEL
*IF SELMOD = 2, RECURSIVE DYNAMIC MODEL

parameters
gdp_1, emf_1, pcb_1, ct_1, k_t, ktmp, neg_cnt
;

IF( selmod=1,
CPI.fx=1;
qfa.fx('l')=qfa.l('l');qfa.fx('k')=qfa.l('k');
pimw.fx(i) = pimw.l(i);pexw.fx(s) = pexw.l(s);
exr.fx = exr0;
atc=1;

$include init_k_mn.txt

k_t= k.l('MANU');
k.fx('MANU')=ktmp*k_t;

*pcb.fx = pcb.l;
*temf.fx = temf0;

Solve sim_ncge1 using mcp;

neg_cnt=0;

loop(id,
if(xbf.l(id)<0,
```

```
xbf.fx(id)=0;
neg_cnt=neg_cnt+1;
);
);

if(neg_cnt>0,
solve sim_ncge1 using mcp;
);

gdp_1 = sum(s,hc.l(s)+gc.l(s)+invd.l(s))+sum(s,pex.l(s)*xex.l(s))
-sum(i,pim.l(i)*xim.l(i));
emf_1 = sum(s, emf.l(s));
pcb_1 = pcb.l*1e6;
ct_1 = tct.l;

display gdp_1, emf_1, pcb_1, ct_1
;
);

display xs.l,xd.l,xex.l,xim.l, xt.l,xb.l,m.l,xm.l,ce.l,l.l,k.l,
pt.l,pxb.l,pm.l,pce.l,pk.l,pl.l,qfa.l,DHI.l,hc.l,gc.l,
HS.l,TP.l,TR.l,GS.l,TIV.l,invd.l,emf.l,temf.l
;

SET
t /2010*2020/
tfirst(t),
ntfirst(t),
sim /bau_s, pol_s1*pol_s3/
bau(sim) /bau_s/
policy(sim) /pol_s1/
;
Tfirst(t) = yes$(ord(t) = 1);
Ntfirst(t) = yes$(ord(t) ne 1);
```

```
parameter DP(T,*) depreciation rate in a given period;
$libinclude xlimport DP sam_10.xls rate!A1:B12
parameter dpr(t) ;
dpr(t) = DP(t,'depre');

parameter Ira(T,*) interest rate in a given period;
$libinclude xlimport ira sam_10.xls rate!C1:D12
parameter intr(t) ;
intr(t) = Ira(t,'interest');

parameter GR_lab(t,*) growth rate of labor in period t
$libinclude xlimport GR_lab sam_10.xls rate!E1:F12
parameter gwr_l(t) ;
gwr_l(t) = GR_lab(t,'growthl');

*parameter sub_rate(t,*) sectoral subsity
*$libinclude xlimport sub_rate sam_10.xls rate!G1:H12
*parameter sub_r(t);
*sub_r(t) = sub_rate(t,'subsidy');
*display dpr,intr,gwr_l,sub_r;

parameter ainvest (t,*) investment efficiency
$libinclude xlimport ainvest sam_10.xls rate!I1:J12

parameter alabor (t,*) labor productivity change
$libinclude xlimport alabor sam_10.xls rate!K1:L12

parameters
ks(t) capital stock of period t
ls(t) labor stock of period t
Toinv(t) new investment in period t
ati(t) investment efficiency ratio
alb(t) labor productivity
*pa5(t) new environmental commodity price with subsity in period t
;
```

```
parameter abate(t,*) abatement of emission scenario 1
$libinclude xlimport abate sam_10.xls co2rate!A1:C12

parameter
s1emf(t) emission level under scenario1
;
s1emf(t) = temf0*abate(t,'sc1');

*parameter
*s1pcb(t) carbon tax rate under secnario1
*s1ct(t) carbon tax under secnario1
*;

parameters
s_gdp, s_pk, s_pl, s_p, s_l, s_k, s_emf, s_pcb, s_ct
cef_t, ctep_t, hsr_t, gsr_t, pce_t
;

IF( selmod=2,
CPI.fx = CPI0;
pimw.fx(i) = pimw0(i);
pexw.fx(s) = pexw0(s);
exr.fx = exr0;

pcb.fx = pcb0;

loop( (t,policy),

atc=0$Tfirst(t)+1$Ntfirst(t);

pce_t(t)=p.l('ENER')+atc*pcb.l*(cef('ENER')*ctep('ENER'));
p.l('ENER')=pce_t(t);

Toinv(t) = TIV.l;
```

```
ks(t)=qfa.l('k');
ls(t)=qfa.l('l');

*hsr_t=hsr*(1-0.1)**(ord(t)-1);
*gsr_t=gsr*(1-0.1)**(ord(t)-1);
*hsr=hsr_t;
*gsr=gsr_t;

ati(t) = ainvest(t,'at');
alb(t) = alabor(t,'lb');
qfa.l('k') = (ks(t)*(1-dpr(t))+Toinv(t)*ati(t))$Ntfirst(t)
+ end_k$Tfirst(t);
qfa.fx('k')=qfa.l('k');
qfa.l('l') = (ls(t)*(1+gwr_l(t))*alb(t))$Ntfirst(t)
+ end_l$Tfirst(t);
qfa.fx('l')=qfa.l('l');

*temf.l = s1emf(t);
*s1pcb(t) = pcb0;
*s1ct(t) = ct.l;

solve sim_ncge1 using mcp;

s_gdp(t,policy) = sum(s,hc.l(s)+gc.l(s)+invd.l(s))
+sum(s,pex.l(s)*xex.l(s))-sum(i,pim.l(i)*xim.l(i));
s_pk(t,policy) = pk.l;
s_pl(t,policy) = pl.l;
s_p(t,i) = p.l(i);
s_l(t,s) = l.l(s);
s_k(t,s) = k.l(s);
s_emf(t) = sum(s, emf.l(s));
s_pcb(t,policy) = pcb.l*1e6;
s_ct(t,policy) = tct.l;

);
```

```
);
display s_gdp, s_pk, s_pl, s_p, s_l, s_k, s_emf, s_pcb, s_ct, neg_cnt
;

file results /results_carbon_copula.txt/;
results.ap = 1;
put results;
*loop((t,policy), put t.tl, @12, put s_gdp(t,policy), @24,
put s_emf(t), @36, put s_pcb(t,policy) /);
put pcb_1, @12, put neg_cnt;
putclose;
```

초록

본 연구에서는 기후변화 정책분석의 수단으로 널리 사용되는 연산가능일반균형 (computable general equilibrium; CGE) 모형의 문제점들에 대해 고찰하고 이에 대한 해결방안 중 하나로서 다변량 분포 접근법을 대안적인 생산함수 기술 방법으로 제안하며 이 방법의 실제적 응용가능성을 평가한다.

이 연구의 첫 부분에서는, 우선 널리 쓰이는 글로벌 CGE 모형 세 가지의 특징들을 살펴보고 모형마다 다른 온실가스 배출량 산정 결과를 주는 가장 큰 요인으로서 생산함수 구조를 우선적으로 선정한다. 생산함수 구조의 변화가 모형의 예측결과에 얼마나 큰 영향을 미치는지 알아보기 위해 두 가지 실험을 실시한다. 먼저 어느 한 CGE 모형의 중첩 (nested) 고정대체탄력성 (constant elasticity of substitution; CES) 함수 구조를 다른 모형의 함수 구조로 변환하였을 때의 예측결과의 변화를 살펴본다. 또한 다른 실험에서는 상향식 모형의 특징을 반영하기 위해 고정 요소투입 구조를 부분적으로 적용할 경우 어떤 변화가 일어나는지 살펴본다. 실험 결과는 이와 같은 구조적 변화가 온실가스 배출량 및 탄소 가격의 예측 결과에 상당한 영향을 미치는 것을 보여주고 있다. 또한 한국의 경우에 대해 국내총생산 감소가 함수구조에 따라 달라지는 것을 확인할 수 있는데, 이는 실제 정책 수립에 CGE모형의 예측 결과를 반영할 경우 예측치의 신뢰성과 관련한 여러가지 문제가 발생할 수 있음을 시사한다.

두번째 부분에서는, 데이터 세분화 적용과 함께 한계저감비용이 산정되는 경우의 글로벌 CGE 모형의 설명력에 대한 분석을 실시한

다. 탄소가격을 지역별로 산정한 결과 경제 전체에서 자본의 기여도가 특히 높은 몇몇 개발도상국 지역에서 통념과 다르게 높은 수준의 탄소가격이 나타남을 보인다. 실증 자료와 간단한 수식 모형을 통해 이러한 이상현상은 단위 탄소발생량 당 자본집중도와 탄소가격 간 정비례 관계에 기인하는 것으로 설명된다. CGE 모형에서 널리 쓰이는 함수 형태인 CES 함수를 대상으로 한 수치분석 결과 탄소가격의 이상현상은 구체적으로 CES 함수의 비례모수 (distribution parameter) 와 관련이 있다.

마지막 세번째 부분에서는, 에너지 관련 생산함수 기술의 대안적 방법으로 다변량 분포 방법이 시도된다. 총생산함수의 미시적 기초에 대한 이론적 연구들에 의하면 미시적 생산기술에 대해 특정 통계 분포가 가정될 경우 미시적 정보의 집합은 특정한 형태의 총생산함수로 변환될 수 있다. 이를 실증적으로 살펴보기 위해 우선적으로 한국의 에너지 다소비 제조업 부문의 실제 데이터를 이용하여 미시적 통계 분포의 특성들을 살펴본다. 또한 실제 시뮬레이션을 할 때 다변량 결합 분포함수에 담긴 미시적 상관성 정보를 편리하게 재현할 수 있도록 코풀라 (copula) 라고 하는 통계적 수단을 도입하고 그 효용성에 대해 살펴본다. 코풀라의 기본적인 이론에 대해 간단히 소개한 후 실제 데이터에 의해 추정된 코풀라 모형의 설명력에 대해 분석한다. 그 결과 코풀라는 이질적인 (heterogeneous) 미시적 정보를 성공적으로 묘사할 수 있음을 확인한다. 아울러 분포 접근법이 적용된 새로운 형태의 CGE 모형이 소개된다. 이 모형에서는 레온티에프 (Leontief) 함수 형태의 개별 생산함수들의 총합이 기존에 사용되던 총생산함수의 역할을 대신한다. 새로운 모형은 기존 모형들에 비해 몇 가지 장점이 있는데, 과거 데이터의 영향을 제거할 수 있고 예측결과의 정확성을 향상시킬 수

있다.

주요어 : 연산가능일반균형 모형, 구조적 불확실성, 고정대체탄력성 생산함수, 온실가스 배출 전망, 투입요소 분포, 코풀라

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