경영학석사학위논문

Critical Factors for Platforms' Profit in Access Fee Charging Two-Sided Market

가입비를 부과하는 양면시장에서 플랫폼 수익에 영향을 주는 주요인들에 대한 연구

2014년 2월

서울대학교 대학원

경영학과 생산관리전공

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이 논문을 경영학석사 학위논문으로 제출함

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Abstract

In many of two-sided market studies, network externality has been the most attention receiving variable. Li et al. suggested in their work that if platform does not differentiate itself enough, attempt to increase network externality could actually harm platform's profit. In this work, a model was designed to accommodate buyer side access fee, which was not considered in Li et al.'s model.

Examining newly constructed model, several implications are derived. There are difference makers in each of optimization cases; also no-difference makers. Moreover access fee differences cause uncertainties that are possible to control with platforms' access fees for either seller or buyer. Furthermore the finding of common no-difference maker could free platforms from what these no-difference makers represent. Also application of the results to case of professional SNS shows access fee discount of certain platform could be an unnecessary discount.

This paper contribute to two-sided market study by providing hints of where to allocate its resource and where not to; also gives shows how each of variables are related and even interacts (particularly seller side access fee and buyer side access fee). Furthermore, the study verifies result of Li et al.'s study and shows their result might be lack of generality.

Keywords: Two-sided market; Cross group network externality; Within group network utility; Platform differentiation; Transportation cost; Access fee.

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1. Introduction

Profit maximization has been the first priority for the most of Profit Corporation and is still supposed to be the first priority. For this priority, called profit maximization, many of economic theories have been established and verified in numbers of industries. One of the most well-known theory would be 'price equals marginal cost in perfectly competitive market.' Because the price would be same as the marginal cost, price tends to have rather positive sign than negative sign. There is, however, a certain type of markets where price goes even below than zero by forms of subsidies. According to Evans (2003) it is the market where "Prices do not and prices cannot follow marginal costs in each side of the market." In other words, the theory of marginal cost equals the price cannot fully explain platform operators' subsidies to buyers. These subsidies often take forms as free services or discount or even free goods for buyers or sellers could enjoy. For instance, Facebook users could use its services for free of charge; mobile phone network operators give subsidies to their mobile phone network users. These free services or subsidies are possible because there is another party of service users. Interaction between these two parties generates cross group network externalities. Cross group network externality is one of the reason how platform operators are able to provide such services for free or even for providing subsidies. Conventional industrial organization theories could not be applied or would be required some of modifications in order to provide explanations on these subsidies.

One of the most effective approaches to explain the situation is two-sided markets theory. According to Rysman (2009), the market is "broadly speaking a two-sided market is one in which 1) two sets of agents interact through an intermediary or platform, and 2) the decisions of each set of agents affects the outcomes of the other set of agents, typically through an externality." Although definition of Rysman is not completely define the market, Rysman's definition clearly represent important features of the market, features are the two groups of agents and platform; the features are critical components that generate cross group network externality. Cross group network externality could be found in services of Facebook. For instance, Facebook users can use the service for free of charge because Facebook will maintain its profit from advertisers and/or from other 3rd party interlocked programs providers, advertisers and 3rd party members will only deal with Facebook if the use of Facebook provides them positive utilities. If positive utility could be provided then advertisers will access Facebook advertising service and Facebook will be able to charge access fee to advertisers; also the access fee will keep Facebook in business. If this is the case then the most important thing for Facebook would be to ensure that there are enough number of Facebook users so that advertisers' cross group network externality could be maximized, which will lead to more access fee from the advertisers that will increase revenue of Facebook.

So far most of two-sided market studies only emphasized cross group network externalities, not within group network utilities, utilities that represent interactions among each side of groups. Li et al. (2010) introduced a concept of within group network utility and showed that if competing platforms are in equilibrium and if both platforms' within group network utilities are same (not differentiated) then higher cross group network externality actually lowers profits for the platform. This result highlights a variable that has been neglected by many of studies. The variable, however, should not be disregarded if one's aim is to maximize its profit. Moreover it brings curiosity whether the result of Li et al. could be applied to two-sided markets in general. . In this study, a model was designed to accommodate buyer side access fee, which was not considered in Li et al.'s model.

Examining newly constructed model, several implications are derived. There are difference makers in each of optimization cases; also no-difference makers. Also access fee differences cause uncertainties that are possible to ignore when competing platforms' access fees for either seller or buyer are same as the other platform. Furthermore the finding of common no-difference maker could free each cases' platform from those no-difference makers. Also application to case of professional case shows, some of access fee discount could be unnecessary discount.

This paper contribute to two-sided market study by providing hints of where to allocate its resource and where not to; also gives shows how each of variables are related and even interacts (particularly seller side access fee and buyer side access fee). Furthermore, the study verifies result of Li et al.'s study and shows their result might be lack of generality.

2. Literature Review

Study of two-sided market theory began earnest in 2000s. Since articles of Rochet & Tirole (2006), Caillaud & Jullien (2003), and Armstrong (2006) there have been studies of general theories regarding optimal pricing and externalities; since Evans (2003) and Wright (2004) there have been papers on platform competition effects (Rhee 2010). For two-sided market is relatively new to academia, there have been several endeavor to define the market. Rochet & Tirole (2006) defined two-sided market as "markets in which one or several platforms enable interactions between end-users and try to get the two (or multiple) sides "on board" by appropriately charging each side. That is, platforms court each side while attempting to make, or at least not lose, money overall"; Rysman (2009) defined that broadly speaking a two-sided market is one in which 1) two sets of agents interact through an intermediary or platform, and 2) the decisions of each set of agents affects the outcomes of the other set of agents, typically through an externality.

Armstrong (2006), Caillaud & Jullien (2003), Schiff (2003), Rochet & Tirole (2003 and 2004), Parker and Van Alstyen (2005) studied concerning subsidy (which group receives subsidies from platforms). Armstrong (2006), Caillaud & Jullien (2003), Schiff (2003), Parker and Van Alstyen (2005) measured cross group network externality in their papers. Caillaud & Jullien (2003), Schiff (2003), Rochet & Tirole (2003 and 2004) looked into access charge and usage charge in their articles.

While most of two-sided market study emphasizes variables mentioned above; Li et al. (2010) attempts to measure competition between subjects within the same platform, for instance, between the different sellers competing for a consumer on an auction site. From this attempt, Li et al. successfully modelled competition between the sellers for the buyers, including the competition between the platforms themselves. Platform industry Li et al. modelled is online book stores, Alibris.com and Half.com, which provide similar functionalities however attract different types of customers. Alibris.com sells collectable or rare books whereas Half.com sells discounted college student books. There is transportation cost for customers in both platforms, the cost faced by customers when customers try to orient themselves to particular platforms. This transportation cost acts as a disutility for a customer, disutility that could be searching cost or any inconvenience in real life example. Li et al. employed Hotelling's competition model (Hotelling 1929) for competition between sellers for buyers. Employing Hotelling's competition model, Li et al. set up the proportion of the number of buyers for platform 1 as n_B^1 and buyers for platform 2 as n_B^2; Also the proportion of the number of sellers for platform1 and platform 2 as n_S^1 and n_S^2 respectively. Here Li et al. assumed that sellers and buyers single homes. In the model, Li et al. used a parameter α to represent cross group network externality, which means each groups' utility will be affected by other side groups actions. A parameter β has used in the model to show within group network utility. Suppose a user could find many of their acquaintance or friends from Facebook then, the user's within group network utility would be higher than any user who could not find that many of acquaintance or friends from Facebook. Below is Li et al.'s equations for utility of marginal buyer on each platforms.

$$u_B^1 = \alpha_1 n_S^1 + \beta_1 n_B^1 + v - (\hat{p} - a n_S^1) - n_B^1 \mathrm{t} \tag{1}$$

$$u_B^2 = \alpha_2(1 - n_S^1) + \beta_2(1 - n_B^1) + v - (\hat{p} - a(1 - n_S^1)) - (1 - n_B^1)t \tag{2}$$

Equation 1 shows that utility of buyers on platform 1 is a function of cross group network externality, the number of sellers, within group network utility, the number of buyers, gross utility the buyers can derive from the product (ν), monopoly price($\hat{p}$), coefficient of competition between sellers, and transportation cost. $(\hat{p} - an_S^1)$ shows actual price a buyer pays to sellers.

Equation 2 shows in almost same logics as equation 1 shows. The difference would be expressions of the proportions of numbers of buyers and sellers, which denoted as $(1 - n_B^1)$ and $(1 - n_S^1)$ respectively.

$$u_S^1 = \left(n_B^1 - \gamma(\hat{p} - an_S^1)\right)(\hat{p} - an_S^1) - F_1 \tag{3}$$

$$u_S^2 = \left((1 - n_B^1) - \gamma(\hat{p} - a(1 - n_S^1))\right)(\hat{p} - a(1 - n_S^1)) - F_2 \tag{4}$$

Equation 3 shows that utility of sellers on platform 1 is a function of the number of buyers, coefficient of price sensitivity among buyers, monopoly price($\hat{p}$), coefficient of competition between sellers, the number of sellers, and fixed fee charged to sellers on platform. A part of equation $\left(n_B^1 - \gamma(\hat{p} - an_S^1)\right)$ shows the proportion of the number of buyers on platform 1. Equation 4 shows the utility of sellers on platform 2. Although Li et al. argued that equation 3 and 4 are utilities of sellers on platforms, it would be less complicated if equation 3 and 4 are regarded as profits of sellers on both platforms respectively.

$$\Pi_1 = n_S^1 F_1 \tag{5}$$

$$\Pi_2 = (1 - n_S^1)F_2 \tag{6}$$

Equation 5 is revenue what platform 1 generate from sellers. Equation 6 is the revenue of platform 2.

In Li et al.'s paper, they assumed that in both platform sellers and buyers will reach equilibrium thus u_B^1=u_B^2 and u_S^1=u_S^2. Therefore $u_B^2 = 1 - u_B^1$ and, $u_S^2 = 1 - u_S^1$. They also set conditions that $0 \leq u_B^1$, u_B^2, u_S^1, u_S^2, for otherwise sellers or buyers would not have incentives to use any of platforms. With these conditions, Li et al. solved equation and derived results, equation 7 and 8.

$$F_1 = \frac{\begin{array}{c}-\hat{\mathrm{p}}(2\alpha_1+4\alpha_2-\beta_1+\beta_2)-3a^2\left(-1+\gamma(-2t+\beta_1+\beta_2)\right)\\ +a\left(3t+\alpha_1+2\alpha_2-2\beta_1-\beta_2+6\hat{\mathrm{p}}(-1-2t\gamma+\gamma\beta_1+\gamma\beta_2)\right)\end{array}}{6t-3(\beta_1+\beta_2)} \tag{7}$$

$$F_2 = \frac{\begin{array}{c}-\hat{\mathrm{p}}(4\alpha_1+2\alpha_2+\beta_1-\beta_2)-3a^2\left(-1+\gamma(-2t+\beta_1+\beta_2)\right)\\ +a(3t+2\alpha_1+\alpha_2-\beta_1-2\beta_2+6\hat{\mathrm{p}}(-1-2t\gamma+\gamma\beta_1+\gamma\beta_2))\end{array}}{6t-3(\beta_1+\beta_2)} \tag{8}$$

Solving equation 7 and 8 in two different cases that case 1: $\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2$, and case 2: $\alpha_1 = \alpha_2$, $\beta_1 \neq \beta_2$. With case 1, equation 9 and 10 were obtained.

$$F_1 - F_2 = \frac{(a-2\hat{p})(\alpha_1-\alpha_2)}{6(-t+\beta)} \tag{9}$$

$$\Pi_1 - \Pi_2 = \frac{(a-2\hat{p})(\alpha_1-\alpha_2)}{6(-t+\beta)} \tag{10}$$

With case 2, equation 11 and 12 were obtained.

$$F_1 - F_2 = -\frac{(a-2\hat{p})(\beta_1-\beta_2)}{6t-3(\beta_1+\beta_2)} \tag{11}$$

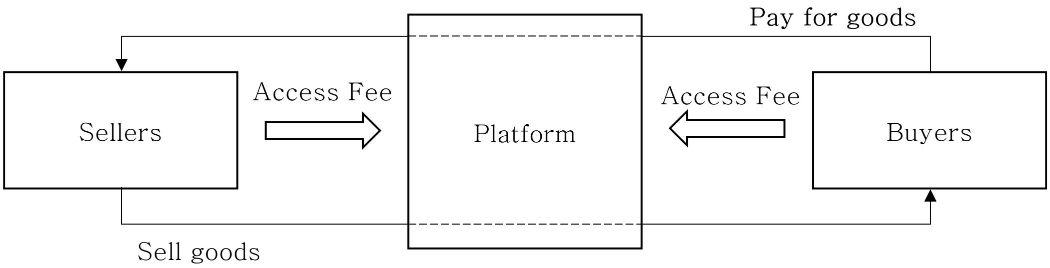
$$\Pi_1 - \Pi_2 = -\frac{(a-2\hat{p})(\beta_1-\beta_2)}{6t-3(\beta_1+\beta_2)} \tag{12}$$

Solving equation (1) ~ (6) with two case conditions of [$\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2$], and [$\alpha_1 = \alpha_2$, $\beta_1 \neq \beta_2$] respectively, Li et al. form a conclusion that when $\alpha_1 > \alpha_2$ and if platforms are not differentiated enough (such as $t < \beta$) then higher cross group network externality actually lower profits for the platform for the former condition; reverse effect is observed for the latter condition. The result what Li et al. has found clearly shows critical variable of profit organizations. There is, however, a room to extend the model. Thus later of this paper Li et al.'s model will be extended by adding another variable.

3. Model

New model is constructed based on what Li et al.'s suggested and is extended to have access fee to buyers. To avoid any confusion access fee to seller has denoted as F_S^i and access fee to buyer has denoted as F_B^i (i=1 and 2).

Figure 1. Access Fee to buyers and sellers



n_S^i and n_B^i (i=1, 2) follows H. Hotelling's spatial distribution and normalized to 1; buyers are marginal buyers, who will choose where has higher utility; access fee is fixed fee charged by each platforms. PC Operating Systems could be an example for this model (Evans 2007). Also some of Social Network Service and Real estate agencies could be examples for this type of business model.

For this market, utilities of buyers would be changed for buyers have to pay access fee, which is denoted as F_B^1.

$$u_B^1 = \alpha_1 n_S^1 + \beta_1 n_B^1 + v - (\hat{p} - a n_S^1) - n_B^1 \mathrm{t} - F_B^1 \qquad (1\text{-}1)$$

$$u_B^2 = \alpha_2(1 - n_S^1) + \beta_2(1 - n_B^1) + v - (\hat{p} - a(1 - n_S^1)) - (1 - n_B^1)t - F_B^2 \qquad (1\text{-}2)$$

Utilities of sellers would not be changed; however, the variable ‘F’ (access fee to sellers Li et al. used) will be written as F_S^i to avoid any confusion with access fee to buyers.

$$u_S^1 = \left(n_B^1 - \gamma(\hat{p} - an_S^1)\right)(\hat{p} - an_S^1) - F_S^1 \tag{1-3}$$

$$u_S^2 = \left((1 - n_B^1) - \gamma(\hat{p} - a(1 - n_S^1))\right)(\hat{p} - a(1 - n_S^1)) - F_S^2 \tag{1-4}$$

Platforms profit source will be altered for there are buyers who pay access fees, which are added as extra profit source of platforms. .

$$\Pi_1 = n_S^1 F_S^1 + n_B^1 F_B^1 \tag{1-5}$$

$$\Pi_2 = (1 - n_S^1) F_S^1 + (1 - n_B^1) F_B^2 \tag{1-6}$$

4. Analysis and Results

Purpose of this analysis is to find out what makes differences in platform's profit and also to find out what does not make differences in platforms' profit. For platforms' profit is function of access fee of sellers (F_S) and access fee of buyers (F_B), optimal value for F_S and F_B will be calculated according to two cases (Case 1 is when $\alpha_1 \neq \alpha_2$ and $\beta_1 = \beta_2$; Case 2 is $\alpha_1 = \alpha_2$ and $\beta_1 \neq \beta_2$) first; then Π_1 - Π_2 will be calculated based on each of optimal values of F_S or F_B that are already calculated; After that overall results of Π_1 - Π_2 will be examined; difference makers and no-difference makers will be suggested. At the end, common difference makers and common no-difference makers for platforms' profit will be derived. Wolfram Mathematica ver. 9.0.1.0 has used to calculate equations; script for Mathematica is attach in appendix.

This analysis contain 6 subsections, 1. Access fee to sellers − case 1; 2. Access fee to sellers − case 2; 3. Access fee to buyers − case 1; 4. Access fee to buyers − case 2; 5. Summary of Analysis; 6. Case − Professional SNS. Each of subsections from 4.1 to 4.4 will show optimal values for access fees, difference of each platforms profit, and Li et al.'s result (for comparison purpose).

4.1. Optimized to Seller Side Access Fee − case 1

Optimal value for Access Fee to Sellers – case 1 ($\alpha_1 \neq \alpha_2,\ \beta_1 = \beta_2$)

$F_S^1 : \frac{-1}{6(t-\beta)}\big(-3a^2 - 3at + 3a\beta - 6a^2t\gamma + 6a^2\beta\gamma - 2a\alpha_2 + 3aF_B^1 + 2\alpha_2F_B^1 + 2\hat{p}(3a + 6at\gamma -$

$6a\beta\gamma + \alpha_1 + 2\alpha_2 + F_B^1 - F_B^2) + 3aF_B^2 + \alpha_2F_B^2 + \alpha_1(-a + 2F_B^1 + F_B^2)\big),$

$F_S^2 : \frac{1}{6(t-\beta)}(3a^2 + 3at - 3a\beta + 6a^2t\gamma - 6a^2\beta\gamma + a\alpha_2 - 3aF_B^1 - \alpha_2F_B^1 + \alpha_1(2a - F_B^1 - 2F_B^2) -$

$3aF_B^2 - 2\alpha_2F_B^2 - 2\hat{p}(3a + 6at\gamma - 6a\beta\gamma + 2\alpha_1 + \alpha_2 - F_B^1 + F_B^2))$

$\Pi_1 - \Pi_2$ when $\alpha_1 \neq \alpha_2$ and $\beta_1 = \beta_2$ is below;

$$\frac{\alpha_1(-a+2\hat{p}+2F_B^1+F_B^2)+\alpha_2(a-2\hat{p}-F_B^1-2F_B^2)+(-4\hat{p}+3(a+t-\beta-F_B^1-F_B^2))(F_B^1-F_B^2)}{6(t-\beta)}$$

Although each of access fees are calculated in conditions of α, β equalities combinations and substituted to Π_1 - Π_2 ($n_S^1F_S^1 + n_B^1F_B^1 - (1 - n_S^1)F_S^1 + (1 - n_B^1)F_B^2$), it is still challenging to extract meaningful result. Thus extra step was taken, the step is to set $F_B^1 = \mathrm{x}F_B^2 = xF_B$. Because other side platform's access fees can be described as multiples of the other side platform's access fee. Extra step processed result follows below;

$\Pi_1 - \Pi_2$ when $\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2$ and $F_B^1 = \mathrm{x}F_B^2 = xF_B$ is

$$\frac{(-a+2\hat{p}+(x+1)F_B)(\alpha_1-\alpha_2)+(x\alpha_1-\alpha_2)F_B+\big(2(a-2\hat{p})-3(x+1)F_B+3(t-\beta)\big)(x-1)F_B+a(x-1)F_B}{6(t-\beta)}$$

This step reduces some of variables and allows re-arrange of equation; re-arranged equation contains some of partial terms that have definite signs; also the result of the extra step implies what makes differences in terms of the sign of Π_1 - Π_2. First of all, when $n_S^1 = \frac{1}{2}$, "$-a + 2\hat{p}$" is always positive. It is because $(\hat{p} - an_S^1)$ is the actual price buyer pays to sellers and thus will be positive for otherwise sellers have no incentives to sell; if $n_S^1 = \frac{1}{2}$, then the partial equation $(\hat{p} - an_S^1)$ is $(\hat{p} - \frac{a}{2})$ which has positive sign. Thus $(-a + 2\hat{p})$ is always positive. Also there is no assumptions of subsidies on access fees, thus access fees of any side would have positive values. Thus partial term $(x + 1)F_B$ has positive value. In similar logic, partial term $2(a - 2\hat{p}) - 3(x + 1)F_B$ will have negative signs. Then left terms are what decides sign of equation, $\Pi_1 - \Pi_2$. These terms what decide the equation's sign will be called 'difference makers''; the difference makers and interpretations follows below;

□ $(\alpha_1 - \alpha_2)$: difference of cross group network externalities of both platforms;

□ $(\mathrm{x}\alpha_1 - \alpha_2)$: effect of access fee difference on network externality;

□ $(t - \beta)$: degree of differentiation of platforms;

□ $(\mathrm{x} - 1)$: effect of access fee differences.

Although both "difference of cross group network externalities of both platforms" and "degree of differentiation of platforms" could be found in Li et al.'s paper, "$\mathrm{x}\alpha_1 - \alpha_2$" and "x-1" are unique findings from this study. These unique partial terms seems rooted from buyer side access fees for otherwise there is no explanation for the 'x'. Then it could be argued that buyer side access fee caused this differences.

Intuitively, since buyers are sole source of utility for sellers in this model, and what's optimized is seller side access fee, it is plausible that the cause is the buyer side access fee. This interpretation could be strengthened when compared with the result of $\Pi_1 - \Pi_2$ and when x=1. Below is a result of $\Pi_1 - \Pi_2$ when $\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2$, and $x = 1$

$$-\frac{(a - 2\hat{p} - 3F_B)(\alpha_1 - \alpha_2)}{6(t - \beta)}$$

Comparing the results of both when x is not decided and when x is 1, it can be observed that some of terms either absorbed by other terms or even disappears. The absorbed or disappeared partial term is $[(x\alpha_1 - \alpha_2)F_B + \big(2(a - 2\hat{p}) - 3(x+1)F_B + 3(t - \beta)\big)(x - 1)F_B + a(x - 1)F_B]$. This term is almost ineffective when competing platform charges same amount of buyer side access fee, yet start to make uncertainties when both platforms charge different rate of buyer side access fees. Thus this type of partial terms will be called 'uncertainty by fee differences'. Buyer side access fee still affects $\Pi_1 - \Pi_2$ even when x=1 (both platforms charge same amount of buyer side access fee) as much as $3F_B$.

Another interesting thing is when considering the difference of each platforms' profit, variables such as $\hat{p}$, a, γ, and ν does not have critical impact on the degree of profit (although $\hat{p}$ and a does appear in the equation, the signs of each parts these two variables belong are already decided; besides, γ does not appeared at all). It could mean that these variables does not make any differences in terms of platforms' profit; in this particular case it could be argued that the market platforms belong is free of γ.

4.2. Optimized to Seller Side Access Fee − case 2

Optimal values for Access Fee to Sellers – case 2 $(\alpha_1 = \alpha_2, \beta_1 \neq \beta_2)$ are

F_S^1: $-\frac{1}{6t-3\beta_1-3\beta_2}\left(-3a^2 - 3at - 3a\alpha - 6a^2t\gamma + a(2+3a\gamma)\beta_1 + a\beta_2 + 3a^2\gamma\beta_2 + 3aF_B^1 + 4\alpha F_B^1 + \hat{p}\left(-(1+6a\gamma)\beta_1 + (1-6a\gamma)\beta_2 + 2\left(3(a+\alpha+2at\gamma) + F_B^1 - F_B^2\right)\right) + 3aF_B^2 + 2\alpha F_B^2\right)$,

F_S^2: $\frac{1}{6t-3\beta_1-3\beta_2}(3a^2 + 3at + 3a\alpha + 6a^2t\gamma - a(1+3a\gamma)\beta_1 - 2a\beta_2 - 3a^2\gamma\beta_2 - 3aF_B^1 - 2\alpha F_B^1 - 3aF_B^2 - 4\alpha F_B^2 + \hat{p}((-1+6a\gamma)\beta_1 + (1+6a\gamma)\beta_2 - 2(3(a+\alpha+2at\gamma) - F_B^1 + F_B^2)))$

$\Pi_1 - \Pi_2$ when $\alpha_1 \neq \alpha_2$ and $\beta_1 = \beta_2$ is below;

$$\frac{\beta_2(a-2\hat{p}-3F_B^1)+(3(a+t)+\alpha-4\hat{p}-3F_B^1-3F_B^2)(F_B^1-F_B^2)+\beta_1(-a+2\hat{p}+3F_B^2)}{6t-3\beta_1-3\beta_2}$$

Although each of access fees are calculated in conditions of α, β equalities combinations and substituted to Π_1 - Π_2 ($n_S^1F_S^1 + n_B^1F_B^1 - (1-n_S^1)F_S^1 + (1-n_B^1)F_B^2$), it is still challenging to extract meaningful result. Thus extra step was taken in this case as well as the former case, the step is to set $F_B^1 = \mathrm{x}F_B^2 = xF_B$. Because other side platform's access fees can be described as multiples of the other side platform's access fee. Extra step processed result follows below;

$\Pi_1 - \Pi_2$ when $\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2$, and $F_B^1 = \mathrm{x}F_B^2 = xF_B$ is

$$\frac{(\beta_1-\beta_2)(-a+2\hat{p})+3(\beta_1-x\beta_2)F_B+(2(a-2\hat{p})-3(x+1)F_B+a+3t+\alpha)(x-1)F_B}{6t-3(\beta_1+\beta_2)}$$

This extra step reduces some of complexity of the equation and thus eases interpretation. Definite signs of partial terms could be found in above equation. As shown in 'Optimized to Seller Side Access Fee − case 1', $(-a+2\hat{p})$ is positive, $2(a-2\hat{p})-3(x+1)F_B$ is negative, and $a+3t+\alpha$ is positive. What's left are difference makers.

□ $(\beta_1-\beta_2)$: difference of within network utilities of both platforms;

□ $(\beta_1-\mathrm{x}\beta_2)$: effect of access fee difference on within network utilities;

□ $(\mathrm{t}-\frac{(\beta_1+\beta_2)}{2})$: degree of platforms' differences.

Although both "difference of within network utilities of both platforms" and "degree of platforms' differences" could be found from Li et al.'s paper, "$\beta_1-\mathrm{x}\beta_2$" is unique findings from this study. This unique partial terms also seem rooted from buyer side access fees for otherwise there is no explanation for the 'x'. Then the argument of 'buyer side access fee affecting seller side access fee' could be reinforced. In other words the idea of 'buyers are sole source of utility for sellers in this model, and what's optimized is seller side access fee, it is plausible that the cause is the buyer side access fee' is being received another base. This interpretation could be reinforced when compared with the result of $\Pi_1-\Pi_2$ and when x=1. Below is a result of $\Pi_1-\Pi_2$ when $\alpha_1\neq\alpha_2$, $\beta_1=\beta_2$, and $x=1$

$$\{-\frac{(a-2\hat{p}-3F_B)(\beta_1-\beta_2)}{6t-3(\beta_1+\beta_2)}\}$$

Comparing the results of both when x is not decided and when x is 1, it can be observed that some of terms either absorbed by other terms or even disappears. The absorbed or disappeared partial term is $[(3(\beta_1 - x\beta_2)F_B + (2(a - 2\hat{p}) - 3(x+1)F_B + a + 3t + \alpha)(x-1)F_B]$. This term is almost ineffective when competing platform charges same amount of buyer side access fee, yet start to make uncertainties when both platforms charge different rate of buyer side access fees. Thus the partial term '$[(3(\beta_1 - x\beta_2)F_B + (2(a - 2\hat{p}) - 3(x+1)F_B + a + 3t + \alpha)(x-1)F_B]$' is the 'uncertainty by fee differences' in this case. Buyer side access fee still affects $\Pi_1 - \Pi_2$ even when x=1 (both platforms charge same amount of buyer side access fee) as much as $3F_B$ interestingly.

Another Interesting thing is when considering the difference of each platforms' profit, variables such as $\hat{p}$, γ, and ν does not have critical impact on the degree of profit (although $\hat{p}$ does appear in the equation, the signs of each parts $\hat{p}$ belongs are already decided; besides, γ does not appeared at all as it does not in 'Optimized to Seller Side Access Fee - case 1'). It could mean that $\hat{p}$ does not make any differences in terms of platforms' profit; in this particular case the market platforms belong is free of γ.

4.3. Optimized to Buyer Side Access Fee − case 1

Optimal values for Access Fee to Buyers - case 1 ($\alpha_1 \neq \alpha_2, \beta_1 = \beta_2$) are

F_B^1: $\frac{1}{3a(1+2a\gamma-4\gamma\hat{p})}\Big(3a^2 + 3at - 3a\beta + 6a^2t\gamma - 6a^2\beta\gamma + a\alpha_2 - a^2\gamma\alpha_2 - \alpha_2F_S^1 + 3aF_S^2 + \alpha_2F_S^2 +$
$\alpha_1\left(a(2 + a\gamma) - F_S^1 + F_S^2\right) - \hat{p}\left((3 + 2a\gamma)\alpha_1 + (3 - 2a\gamma)\alpha_2 + 2\left(3a + 6at\gamma - 6a\beta\gamma + 2F_S^1 +\right.\right.$
$\left.\left.F_S^2\right)\right)\Big)$,

F_B^2: $\frac{1}{3a(1+2a\gamma-4\gamma\hat{p})}(3a^2 + 3at - 3a\beta + 6a^2t\gamma - 6a^2\beta\gamma + 2a\alpha_2 + a^2\gamma\alpha_2 + 3aF_S^1 + \alpha_2F_S^1 +$
$\alpha_1(a - a^2\gamma + F_S^1 - F_S^2) - \alpha_2F_S^2] - \hat{p}((3 - 2a\gamma)\alpha_1 + (3 + 2a\gamma)\alpha_2 + 2(3a + 6at\gamma - 6a\beta\gamma +$
$F_S^1 + 2F_S^2)))$

$\Pi_1 - \Pi_2$ when $\alpha_1 \neq \alpha_2, \beta_1 = \beta_2$ is

$$\frac{(\alpha_1(a+2a^2\gamma-4a\gamma\hat{p}-2F_S^1+2F_S^2)-\alpha_2(a+2a^2\gamma-4a\gamma\hat{p}+2F_S^1-2F_S^2)+(3a(-1+a\gamma)+\hat{p}-6a\gamma\hat{p}-3F_S^1-3F_S^2)(F_S^1-F_S^2))}{3a(1+2a\gamma-4\gamma\hat{p})}$$

With condition of equality combinations of α, β, $\Pi_1 - \Pi_2$ is flooded with variables. Thus similar approach, the extra step ($F_B^1 = \mathrm{x}F_B^2 = xF_B$) was taken to analyze the case. The result of extra step processed stated below;

$\Pi_1 - \Pi_2$ when $\alpha_1 \neq \alpha_2, \beta_1 = \beta_2, F_S^1 = \mathrm{x}F_S^2 = xF_S$ is

$$\frac{(\alpha_1-\alpha_2)a(1+2a\gamma-4\gamma\hat{p})+(\alpha_1+\alpha_2)(-2x+2)F_S+(3a\gamma(a-2\hat{p})-3(x+1)F_S+(\hat{p}-3a))(x-1)F_S}{3a(1+2a\gamma-4\gamma\hat{p})}$$

This extra step reduces some of complexity of the equation and thus eases interpretation. Although not many, definite signs of partial terms could be found in above equation. $3a\gamma(a-2\hat{p})-3(x+1)F_S$ is negative. Also condition of when $a(1+2a\gamma-4\gamma\hat{p})$ is positive could be found. If rearrange $a(1+2a\gamma-4\gamma\hat{p})$ then it is "$a(1+2\gamma(a-2\hat{p}))$"; for $(a-2\hat{p})<0$ when $n_S^1=\frac{1}{2}$, if γ is smaller than $\frac{1}{4\times(\hat{p}-\frac{a}{2})}$ then the partial term is always positive. Also if $n_S^1=\frac{1}{2}$, then $(\hat{p}-\frac{a}{2})$ is the actual price buyers pay to sellers. If read each of variables in the partial term $(1+2\gamma(a-2\hat{p}))$, it is interaction among price buyer pays, price sensitivity among buyers and competition among sellers. Thus the partial term will be called 'impact of purchasing factors'. Difference makers of this particular case are written below;

□ $(\alpha_1-\alpha_2)$: difference of cross group network externality of both platforms;

□ $(-\mathrm{x}+1)$: effect of seller side access fee differences;

□ $(\hat{p}-\frac{1}{2}a-\frac{5}{2}a)$: the actual price and competition among sellers;

□ $a(1+2a\gamma-4\gamma\hat{p})$: impact of purchasing factors.

This result shows cross group network externality still affects platforms profit as appeared in case of 'Optimized to Seller Side Access Fee − case 1'. Also $(-\mathrm{x}+1)$ shows when optimized to buyer side access fee, seller side access fee affects platforms' profit. This have a thread of connection with the idea of 'buyer side access fee affecting seller side access fee'. Meanwhile, the actual price buyer pays to sellers $(\hat{p}-\frac{1}{2}a)$, degree of competition, and price sensitivity among buyers seems have interactions through the partial terms of $(\hat{p} - \frac{1}{2}a - \frac{5}{2}a)$ and $a(1 + 2a\gamma - 4\gamma\hat{p})$. Below is $\Pi_1 - \Pi_2$ when $\alpha_1 \neq \alpha_2, \beta_1 = \beta_2, F_S^1 = \mathrm{x}F_S^2 = xF_S$ and when x=1

$$\frac{\alpha_1 - \alpha_2}{3}$$

So far this is the simplest result equation. Same amount of buyer side access fees by competing platform enables this form. In other words, the 'uncertainty by fee differences' disappears when each competing platforms charge same amount of access fees. In this case, the 'uncertainty by fee differences' is $[(\alpha_1 + \alpha_2)(-2x + 2)F_S + (3a\gamma(a - 2\hat{p}) - 3(x + 1)F_S + (\hat{p} - 3a))(x - 1)F_S]$.

It is surprising that platform differentiation ('t') does not appear in results. Although 't' does appear in optimized F_B, subtracting of Π_1 and Π_2 offsets the effect of 't'; thus the variable 't' is not a difference maker. Another interesting thing is when considering the difference of each platforms' profit, variables such as β, t, and ν does not have any impact; these do not even appear in the equations. It could mean that β, t, and ν do not make any differences in terms of platforms' profit; thus in this particular case the market platforms belong is free of β, t, and ν .

4.4. Optimized to Buyer Side Access Fee − case 2

Optimal values for Access Fee to Buyers - case 2 $(\alpha_1 = \alpha_2, \beta_1 \neq \beta_2)$ are

$F_B^1: \frac{1}{3a(1+2a\gamma-4\gamma\hat{p})}\big(3a^2 + 3at + 3a\alpha + 6a^2t\gamma - a(1+2a\gamma)\beta_1 - 2a\beta_2 - 4a^2\gamma\beta_2 - 2\alpha F_S^1 +$

$3aF_S^2 + 2\alpha F_S^2 - 2\hat{p}(3a + 3\alpha + 6at\gamma - 2a\gamma\beta_1 - 4a\gamma\beta_2 + 2F_S^1 + F_S^2)\big),$

$F_B^2: \frac{1}{3a(1+2a\gamma-4\gamma\hat{p})}(3a^2 + 3at + 3a\alpha + 6a^2t\gamma - 2a(1+2a\gamma)\beta_1 - a\beta_2 - 2a^2\gamma\beta_2 + 3aF_S^1 +$

$2\alpha F_S^1] - 2\alpha F_S^{2`} - 2\hat{p}(3a + 3\alpha + 6at\gamma - 4a\gamma\beta_1 - 2a\gamma\beta_2 + F_S^1 + 2F_S^2))$

$\Pi_1 - \Pi_2$ when $\alpha_1 = \alpha_2, \beta_1 \neq \beta_2$ is

$$\frac{(a(1+2a\gamma-4\gamma\hat{p})\beta_1 + a(-1-2a\gamma+4\gamma\hat{p})\beta_2 + (-4\alpha+3a(-1+a\gamma)+\hat{p}-6a\gamma\hat{p}-3F_S^1-3F_S^2)(F_S^1-F_S^2))}{3a(1+2a\gamma-4\gamma\hat{p})}$$

The extra step was applied to $\Pi_1 - \Pi_2$. $\Pi_1 - \Pi_2$ when $\alpha_1 = \alpha_2, \beta_1 \neq \beta_2, F_S^1 = \mathrm{x}F_S^2 = xF_S$ is stated below;

$$\frac{(\beta_1-\beta_2)a(1+2a\gamma-4\gamma\hat{p}) + (3a\gamma(a-2\hat{p}) - 3(x+1)F_S + (\hat{p}-3a-4\alpha))(x-1)F_S}{3a(1+2a\gamma-4\gamma\hat{p})}$$

After the extra step, familiar sets of variables appears, sets such as impact of purchasing factors, -2(the actual price), and (x-1). If process a similar analysis procedure, the sign of '$3a\gamma(a-2\hat{p}) - 3(x+1)F_S$' is negative; signs of left sets are undecided. The list of difference makers in this case is written below;

□ $(\beta_1 - \beta_2)$: difference of within group network utility of both platforms;

□ $(\mathrm{x} - 1)$: effect of access fee differences;

□ $(\hat{p} - \frac{1}{2}a - \frac{5}{2}a - 4\alpha)$: actual price, competition among sellers, and cross-group network externality;

□ $a(1 + 2a\gamma - 4\gamma\hat{p})$: impact of purchasing factor.

This result shows within network utility still affects platforms profit as appeared in case of 'Optimized to Seller Side Access Fee – case 2'. Also $(\mathrm{x} - 1)$ shows when optimized to buyer side access fee, seller side access fee affects platforms' profit. This also have a thread of connection with the idea of 'buyer side access fee affecting seller side access fee'. The partial term "$(\hat{p} - \frac{1}{2}a - \frac{5}{2}a - 4\alpha)$" is similar to $(\hat{p} - \frac{1}{2}a - \frac{5}{2}a)$ of 'Optimized to Seller Side Access Fee – case 1'. There is, however, an added part, '-4α', which shows stronger effect as much strong as 4α. Below is $\Pi_1 - \Pi_2$ when $\alpha_1 \neq \alpha_2, \beta_1 = \beta_2, F_S^1 = \mathrm{x}F_S^2 = xF_S$ and when x=1

$$\frac{\beta_1 - \beta_2}{3}$$

Again a very simple result was obtained. Same amount of buyer side access fees by competing platform enables this form. In other words, the 'uncertainty by fee differences' disappears when each competing platforms charge same amount of access fee. In this case, the 'uncertainty by fee differences' is $[(3a\gamma(a - 2\hat{p}) - 3(x + 1)F_S + (\hat{p} - 3a - 4\alpha))(x - 1)F_S]$.

Platform differentiation (‘t’) also does not appear in this results. Although ‘t’ does appear in optimized F_B, subtracting of Π_1-Π_2 may offsets the effect of ‘t’; thus the variable ‘t’ is not a difference maker. When x=1, the result becomes very simple. What matters is within group network utility alone. Another interesting thing is when considering the difference of each platforms’ profit, variables t, and ν does not have impact on the difference of the profits; these do not even appear in the equations. It could mean in this particular case the market platforms belong is free of t, and ν .

4.5. Summary of Analysis

4.5.1. Summary of Difference Makers

Table 1. Summary of Difference Makers

	Case	Type	With optimal F_S	With optimal F_B
$\Pi_1 - \Pi_2$	Case 1 $\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2$	Difference Maker	$(\alpha_1 - \alpha_2)$ $(x\alpha_1 - \alpha_2)$ $(t - \beta)$ $(x - 1)$	$(\alpha_1 - \alpha_2)$ $(-x + 1)$ $\left(\hat{p} - \frac{1}{2}a - \frac{5}{2}a\right)$ $a(1 + 2a\gamma - 4\gamma\hat{p})$
		No-Difference Maker	$\hat{p}$, a, γ , and ν	β , t, and ν
	Case 2 $\alpha_1 = \alpha_2$, $\beta_1 \neq \beta_2$	Difference Maker	$(\beta_1 - \beta_2)$ $(\beta_1 - x\beta_2)$ $(t - \frac{(\beta_1 + \beta_2)}{2})$	$(\beta_1 - \beta_2)$ $(x - 1)$ $\left(\hat{p} - \frac{1}{2}a - \frac{5}{2}a\right)$ $a(1 + 2a\gamma - 4\gamma\hat{p})$
		No-Difference Maker	$\hat{p}$, γ , and ν	t, and ν

In case with optimal F_S, Difference Makers are α, β, t, and F_B. These variables are not a part of sellers' utility function but that of buyers'; Possible reason why these variables affect platforms' profit because sellers' source of profit is related to buyers'. If this is the case then platform might want to provide subsidies for buyer side access fee; Also platforms' differentiation does matter in this case. $\hat{p}$, a, γ, and ν are No-difference makers.

In case with optimal F_B, Difference Makers are α, β, F_S, a, the actual price, and impact of purchasing factors. The reason why F_S and 'a' matters might be because too much of seller side access fee and competition among sellers could lower the number of sellers hence affect the cross group network externality that will lead to lower buyers' utilities. If this is case, then platform might want to provide subsidies for sellers' access fee to ease the effect; also the actual price and impact of purchasing factors influence platforms' profit. t, and ν are no-difference maker in this case.

4.5.2. Summary of Uncertainty by Fee Differences

Table 2. Summary of Uncertainty by Fee Differences

	Conditions	Cases	Uncertainties by fee differences
$\Pi_1 - \Pi_2$	With optimal F_S	Case 1	$(x\alpha_1 - \alpha_2)F_B + \big(2(a - 2\hat{p}) - 3(x+1)F_B + 3(t-\beta)\big)(x-1)F_B + a(x-1)F_B$
		Case 2	$3(\beta_1 - x\beta_2)F_B + (2(a - 2\hat{p}) - 3(x+1)F_B + a + 3t + \alpha)(x-1)F_B$
	With optimal F_B	Case 1	$(\alpha_1 + \alpha_2)(-2x+2)F_S + (3a\gamma(a - 2\hat{p}) - 3(x+1)F_S + (\hat{p} - 3a))(x-1)F_S$
		Case 2	$(3a\gamma(a - 2\hat{p}) - 3(x+1)F_S + (\hat{p} - 3a - 4\alpha))(x-1)F_S$

According to each of fee optimizations, the uncertainty by access fee (if described more precisely, it is uncertainty by access fee differences between competing platforms) tends to have similar equations. Regardless what to optimize, the source of optimized side (opposite side of optimized) seems to have concerns about paying different amount of access fee compare to competing platform.; these concerns, however, could be eliminated if competing platforms match the fees to each other.

4.5.3. Summary of Common Difference Makers

Table 3. Summary of Common Difference Makers

	Types	With optimal F_S	With optimal F_B
$\Pi_1 - \Pi_2$	Common Difference Maker	t and F_B	$a(1 + 2a\gamma - 4\gamma\hat{p})$ and F_S
	Common No-Difference Maker	$\hat{p}$ and γ	t

Common difference makers of $\Pi_1 - \Pi_2$ with optimal F_S are t and F_B. The differentiation of platforms affects numerator and denominator of equations of $\Pi_1 - \Pi_2$; The access fee to buyers affects numerator of equation $\Pi_1 - \Pi_2$. Common no-difference maker of $\Pi_1 - \Pi_2$with optimal F_S is γ . Price sensitivity among buyer would not make any differences on $\Pi_1 - \Pi_2$. When optimal value of F_S is the aim of the platform then what makes difference between competing platforms are t and F_B; also in this case, the platforms are free of γ.

Common difference makers of $\Pi_1 - \Pi_2$with optimal F_B are ($1+2a\gamma-4\gamma\hat{p}$) and F_S. The impact of purchasing factors affects numerator and denominator of equation of $\Pi_1 - \Pi_2$ when $\alpha_1 = \alpha_2, \beta_1 \neq \beta_2$ and seller side access fees are different. This is a relation between the actual price and price sensitivity; also the access fee to sellers affects numerator of equation $\Pi_1 - \Pi_2$ when $\alpha_1 = \alpha_2, \beta_1 \neq \beta_2$ and seller side access fees are different. Common no-difference maker of $\Pi_1 - \Pi_2$ with optimal F_B is t; the differentiation of platforms would not make any differences on $\Pi_1 - \Pi_2$. When optimal value of F_S is the aim of the platform then what makes difference between competing platforms are the impact of purchasing factors and F_S; In this particular market, if competing platforms charge same amount of seller side access fee then difference makers could be simplifies as simple as either α or β. Although these are not common difference makers, these will be only concern for the platforms; Also in this environment of optimal F_B, the platforms are free of ‘t’.

4.6. Case - Professional Social Network Services

Two Professional social network services were chosen; LinkedIn and Viadeo. Both firms provide professional networking service; main source of profit of both platforms is the seller (in this case job seeker). Below is a comparison table of both services (Full comparison table can be found in appendix).

Table 4. Comparison of LinkedIn and Viadeo

	LinkedIn	Viadeo
Users	259 Mil	55 Mil
Monthly Visitors	97 Mil	4.6 Mil
Available Languages	Dutch, English, French	Chinese, Dutch, English, French, Italian, Portuguese, Spanish
Minimum Age to Join	18	-

		LinkedIn	Viadeo
Profile Formatting	Custom Skins	N	Y
	Music plays on page capability	N	Y
	Partial customization of page layout	N	Y
	Standard text editing	Y	Y
Education Breakdown	**College**	**88%**	**38%**
	No College	**9%**	**43%**
	Unknown	3%	19%
Age Demographic	**35+**	**80%**	**57%**
	18-34	**19%**	**44%**
	13-17	1%	-
Subscription Fee	Buyer Side	USD 39.95	**USD 9.95**
	Seller Side	USD 19.95	USD 9.95
Revenue (2012)		USD 972 Mil	≒ USD 40 Mil (2009)
Net Income (2012)		USD 21 Mil	-

(Find The Best, 2014)

Although both services claims professional social network services, there are few differences that seems critical for their characteristics. If consider the profile formatting, Viadeo seems provide more customization than LinkedIn does; Also in Education Breakdown section, 88% of LinkedIn users have at least college degree whereas that of Viadeo is 38%. Also there are differences in age groups.

Although in reality it is not easy to compute transportation cost for both platforms, differences of the age group, educational background and customization of the service provide hints of dissimilarities. Also the difference of the number of users and monthly visitors gives hint of cross group network externality and within group network utility.

If the 'Access Fee to sellers' models applied in this case (LinkedIn as a platform 1 and Viadeo as platform 2), then buyer side access fee scheme of Viadeo might be a unnecessary discount. Although what Viadeo does on seller side access fee would be a good decision in terms of profit maximization. For buyer side access fee of LinkedIn is about 4 times higher than that of Viadeo, the case 1 of seller side access fee equation will form as written below;

$$\frac{(-a+2\hat{p}+5F_B)(\alpha_1-\alpha_2)+(4\alpha_1-\alpha_2)F_B+\big(2(\mathrm{a}-2\hat{p})-5F_B+3(t-\beta)\big)3F_B+3\mathrm{a}F_B}{6(t-\beta)}$$

If re-arrange the equation above, it will be arranged as below;

$$\frac{(-45F_B+6(\mathrm{a}-2\hat{p})F_B-(\mathrm{a}-2\hat{p})(\alpha_1-\alpha_2)+((9(t-\beta)+3(\alpha_1+a)+6(\alpha_1-\alpha_2)F_B}{6(t-\beta)}$$

Because of the differences mentioned earlier, if we assume $t > \beta$, then what matters are whether '$(-45F_B + 6(\mathrm{a} - 2\hat{p})F_B$' is bigger than $-(\mathrm{a} - 2\hat{p})(\alpha_1 - \alpha_2) + ((9(t - \beta) + 3(\alpha_1 + a) + 6(\alpha_1 - \alpha_2)F_B$; since there are insufficient information about the value of each variables, it cannot be certainly concluded which set of terms are bigger; however if judged on the basis of two platforms revenue and net income, it seems like this is the case of where F_B should be raised (Viadeo's revenue and net income is not publically announced; hence it means that the revenue and net income is not high enough). If this is the case, lower buyer side access fee scheme of Viadeo is unnecessary discount that will not contribute to platform's profit.

5. Conclusion

5.1 Conclusion

This study contributes to Two-Sided Market studies by providing model of access fee charging platform competitions and its analysis. Modelling access fee charging platforms' competition, the study found out various implications. From the analysis, the study showed that buyer side access fee affects seller side access fee; furthermore showed the degree/strength of the effect of access fee to each other; moreover this study showed how the effect could be controlled using access fee scheme. This study also suggested sets of variables, the difference makers and the no-difference makers. By comparing these difference makers and no-difference makers, the study extracted the common difference makers and common no-difference makers. The common difference makers are variables what platform should manage and research; the common no-difference makers are variables that platforms could be free from allocating resources.

From the findings, some of implications could be extracted. The implications are if platform is in too much of uncertainty managing its business then the platform could set its access fee same as its competing platforms according to the findings of this study. This will dramatically reduce the uncertainty that is named 'uncertainty by fee differences' in this study. Another implication is, as shown in case analysis, according to situation of platforms, platform could set appropriate access fee schemes and avoid unnecessary discount on access fees.

Beside this study extends the study of Li et al. by adding buyer side access fee; this study shows importance of the transportation cost is only valid when platform charges seller side access fee only and not valid when platform charges buyer side access fee; thus shows the result of Li et al.'s study might be lack of generality.

5.2 Limitations and suggestions

The study assumes marginal buyers and sellers and also assumes two competing platforms are in equilibrium. This made the study simpler, however does not fully represent real-life competition among platforms. The future study could consider non-marginal buyers and sellers. The study assumes single homing of sellers. In reality, sellers often multi-homes. (i.e. program developers in computing systems, employer who uses professionals' SNS.) Future study could be formulated to consider multi-homing of sellers. The study assumes variables such as α and β as non-manipulative for assuming manipulating these variables without affecting other variables would be of non-sense; future study could include equations that set α and β as dependent variables so that interactions between α and β and other variables could be observed.

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Appendix

1. Summary of Model sets

Models	Conditions	
	Buyers	Sellers
Li et al.	pay for goods	access fee to platform
New Model	pay for goods access fee to platform	access fee to platform

2. Notations

n_B^1	The proportion of the buyers who prefer platform 1
$1 - n_B^1$	The proportion of the buyers who prefer platform 2
t	Unit transportation cost (buyer side)
n_S^1,	The proportion of the sellers who join platform 1
$1 - n_S^1$,	The proportion of the sellers who join platform 2
α_i	Cross-group network externalities of platform i, (i=1, 2)
β_i	Within-group network utility of platform i, (i=1, 2)
u_B^1	Buyers' utilities on platform i, (i=1, 2)
u_S^1	Sellers' utilities on platform i, (i=1, 2)
v	Gross utility derived from the purchased product
$\hat{p}$	Monopoly price a seller can charge for the product he sells
a	Coefficient of competition among sellers
γ	Coefficient of price sensitivity among buyers
Π_1	Net profit of platform
F_B^{i}i	Access fee charged to buyers on platform i, (i=1, 2)
F_S^{i}i	Access fee charged to sellers on platform i, (i=1, 2)

3. Summary of equation sets

Equations	Remarks
$u_B^1 = \alpha_1 n_S^1 + \beta_1 n_B^1 + v - (\hat{p} - a n_S^1) - n_B^1 \mathrm{t}$ $u_B^2 = \alpha_2(1 - n_S^1) + \beta_2(1 - n_B^1) + v - (\hat{p} - a(1 - n_S^1)) - (1 - n_B^1)t$ $u_S^1 = \left(n_B^1 - \gamma(\hat{p} - a n_S^1)\right)(\hat{p} - a n_S^1) - F_1$ $u_S^2 = \left((1 - n_B^1) - \gamma(\hat{p} - a(1 - n_S^1))\right)(\hat{p} - a(1 - n_S^1)) - F_2$ $\Pi_1 = n_S^1 F_1$ $\Pi_2 = (1 - n_S^1) F_2$	Li et al.'s.
$u_B^1 = \alpha_1 n_S^1 + \beta_1 n_B^1 + v - (\hat{p} - a n_S^1) - n_B^1 \mathrm{t} - F_B^1$ $u_B^2 = \alpha_2(1 - n_S^1) + \beta_2(1 - n_B^1) + v - (\hat{p} - a(1 - n_S^1))$ $\quad -(1 - n_B^1)t - F_B^2$ $u_S^1 = \left(n_B^1 - \gamma(\hat{p} - a n_S^1)\right)(\hat{p} - a n_S^1) - F_S^1$ $u_S^2 = \left((1 - n_B^1) - \gamma(\hat{p} - a(1 - n_S^1))\right)(\hat{p} - a(1 - n_S^1)) - F_S^2$ $\Pi_1 = n_S^1 F_S^1 + n_B^1 F_B^1$ $\Pi_2 = (1 - n_S^1) F_S^2 + (1 - n_B^1) F_B^2$	Access Fees to Buyer side have added

4. Summary of Results.

With optimal F_S		Case 1
$\Pi_1 - \Pi_2$	$\alpha_1 \neq \alpha_2, \beta_1 = \beta_2$	$\frac{(-a+2\hat{p}+(\mathrm{x}+1)F_B)(\alpha_1-\alpha_2)+(\mathrm{x}\alpha_1-\alpha_{2)}\ F_B+ \big(2(\mathrm{a}-2\hat{p})-3(x+1)F_B+3(t-\beta)\big)(x-1)F_B+\mathrm{a}(\mathrm{x}-1)F_B}{6(t-\beta)}$
	$\alpha_1 \neq \alpha_2,\ \beta_1 = \beta_2,$ $F_B^1 = \mathrm{x}F_B^2 = xF_B$	$\frac{(-a+2\hat{p}+(\mathrm{x}+1)F_B)(\alpha_1-\alpha_2)+(\mathrm{x}\alpha_1-\alpha_2\ )F_B +\big(2(\mathrm{a}-2\hat{p})-3(x+1)F_B+3(t-\beta)\big)(x-1)F_B+\mathrm{a}(\mathrm{x}-1)F_B}{6(t-\beta)}$
	$\alpha_1 \neq \alpha_2,\ \beta_1 = \beta_2,$ $x = 1$	$-\frac{(a-2\hat{p}-3F_B)(\alpha_1-\alpha_2)}{6(t-\beta)}$
	Li et al.	$-\frac{(a-2\hat{p})(\alpha_1-\alpha_2)}{6(t-\beta)}$

With optimal F_S		Case 2
$\Pi_1 - \Pi_2$	$\alpha_1 = \alpha_2, \beta_1 \neq \beta_2$	$\frac{\beta_1\big(-a+2\hat{p}+3F_B^2\big)+\beta_2\big(a-2\hat{p}-3F_B^1\big) +(3(a+t)+\alpha-4\hat{p}-3F_B^1-3F_B^2)(F_B^1-F_B^2)}{6t-3\beta_1-3\beta_2}$
	$\alpha_1 = \alpha_2,\ \beta_1 \neq \beta_2,$ $F_B^1 = \mathrm{x}F_B^2 = xF_B$	$\frac{(\beta_1-\beta_2)(-a+2\hat{p})+3(\beta_1-x\beta_2)F_B +(2(a-2\hat{p})-3(x+1)F_B+a+3t+\alpha)(x-1)F_B}{6t-3(\beta_1+\beta_2)}$
	$\alpha_1 = \alpha_2,\ \beta_1 \neq \beta_2,$ $x = 1$	$-\frac{(a-2\hat{p}-3F_B)(\beta_1-\beta_2)}{6t-3(\beta_1+\beta_2)}$
	Li et al.	$-\frac{(a-2\hat{p})(\beta_1-\beta_2)}{6t-3(\beta_1+\beta_2)}$

With optimal F_B		Case 1
$\Pi_1 - \Pi_2$	$\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2$	$\frac{(\alpha_1(a+2a^2\gamma-4a\gamma\hat{p}-2F_S^1+2F_S^2) -\alpha_2(a+2a^2\gamma-4a\gamma\hat{p}+2F_S^1-2F_S^2) +(3a(-1+a\gamma)+\hat{p}-6a\gamma\hat{p}-3F_S^1-3F_S^2)(F_S^1-F_S^2))}{3a(1+2a\gamma-4\gamma\hat{p})}$
	$\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2$, $F_B^1 = \mathrm{x}F_B^2 = xF_B$	$\frac{(\alpha_1-\alpha_2)a(1+2a\gamma-4\gamma\hat{p})+(\alpha_1+\alpha_2)(-2x+2)F_S +(3a\gamma(a-2\hat{p})-3(x+1)F_S+(\hat{p}-3a))(x-1)F_S}{3a(1+2a\gamma-4\gamma\hat{p})}$
	$\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2$, $x = 1$	$\frac{\alpha_1 - \alpha_2}{3}$

With optimal F_B		Case 2
$\Pi_1 - \Pi_2$	$\alpha_1 = \alpha_2$, $\beta_1 \neq \beta_2$	$\frac{(a(1+2a\gamma-4\gamma\hat{p})\beta_1+a(-1-2a\gamma+4\gamma\hat{p})\beta_2 +(-4\alpha+3a(-1+a\gamma)+\hat{p}-6a\gamma\hat{p}-3F_S^1-3F_S^2)(F_S^1-F_S^2))}{3a(1+2a\gamma-4\gamma\hat{p})}$
	$\alpha_1 = \alpha_2$, $\beta_1 \neq \beta_2$, $F_B^1 = \mathrm{x}F_B^2 = xF_B$	$\frac{(\beta_1-\beta_2)a(1+2a\gamma-4\gamma\hat{p})+(3a\gamma(a-2\hat{p}) -3(x+1)F_S+(\hat{p}-3a-4\alpha))(x-1)F_S}{3a(1+2a\gamma-4\gamma\hat{p})}$
	$\alpha_1 = \alpha_2$, $\beta_1 \neq \beta_2$, $x = 1$	$\frac{\beta_1 - \beta_2}{3}$

5. Full Comparison of LinkedIn and Viadeo.

		LinkedIn	Viadeo
Users		259 Mil	55 Mil
Monthly Visitors		97 Mil	4.6 Mil
Available Languages		Dutch, English, French	Chinese, Dutch, English, French, Italian, Portuguese, Spanish
Minimum Age to Join		18	-
Profile Formatting	Custom Skins	N	Y
	Music plays on page capability	N	Y
	Partial customization of page layout	N	Y
	Standard text editing	Y	Y
Site Features	Photo Uploading	Y	Y
	Private Message Sending	Y	N
	Public Message Posting	N	Y
	Video Uploading	Y	Y
Networking Features	Event/Activity Invites	Y	Y
	File Sharing	Y	N
	Groups	Y	Y
	Multi user games	N	Y
	Private Messages	N	Y
	Real time updates from other users	Y	Y
Search Features	Filtering	Y	N
	Live search results	Y	Y
	Search suggestions	N	Y
Search Options	Age	N	Y
	Email Address	Y	Y

		LinkedIn	Viadeo
	Interests	Y	N
	Keywords	Y	Y
	Name	Y	Y
	Online Now	N	Y
Education Breakdown	College	88%	38%
	No College	9%	43%
	Unknown	3%	19%
Age Demographic	35+	80%	57%
	18-34	19%	44%
	13-17	1%	-
Subscription Fee	Buyer Side	USD 39.95	**USD 9.95**
	Seller Side	USD 19.95	USD 9.95
Revenue (2012)		USD 972 Mil	≒ USD 40 Mil (2009)
Net Income (2012)		USD 21 Mil	-

Mathematica Script and Results

Solving the F_S^1, F_S^1, F_B^1, F_B^2 in Access Fee Model using Solve[ ]

Making sub and super scripted variables in Mathematica

```
Remove["Global`@*"]

nB1 = Subsuperscript[n, B, 1]
nS1 = Subsuperscript[n, S, 1]
(* t *)
(* α *)
alpha1 = Subscript[α, 1]
alpha2 = Subscript[α, 2]
(* β *)
beta1 = Subscript[β, 1]
beta2 = Subscript[β, 2]
AFS1 = Subsuperscript[F, S, 1]
AFS2 = Subsuperscript[F, S, 2]
AFS = Subscript[F, S]
AFB1 = Subsuperscript[F, B, 1]
AFB2 = Subsuperscript[F, B, 2]
AFB = Subscript[F, B]
uB1 = Subsuperscript[u, B, 1]
uB2 = Subsuperscript[u, B, 2]
(* v *)
Phat = OverHat[P]
(* a *)
(* γ *)
Pi1 = Subscript[Π, 1]
Pi2 = Subscript[Π, 2]
(* s *)
(* d *)
```

n_B^1

n_S^1

α_1

α_2

β_1

β_2

F_S^1

F_S^2

F_S

F_B^1

F_B^2

F_B

u_B^1

u_B^2

$\hat{P}$

Π_1

Π_2

Defining six functions

```
(* defining functions and equations *)
(* function 1 *) uB1 = alpha1 * nS1 + beta1 * nB1 + v - (Phat - a * nS1) - nB1 * t - AFB1
(* function 2 *)
uB2 = alpha2 * (1 - nS1) + beta2 * (1 - nB1) + v - (Phat - a * (1 - nS1)) - (1 - nB1) * t - AFB2
(* function 3 *) uS1 = (nB1 - γ * (Phat - a * nS1)) * (Phat - a * nS1) - AFS1
(* function 4 *)
uS2 = ((1 - nB1) - γ * (Phat - a (1 - nS1))) * (Phat - a * (1 - nS1)) - AFS2
(* function 5 *) Pi1 = nS1 * AFS1 + nB1 * AFB1
(* function 6 *) Pi2 = (1 - nS1) AFS2 + (1 - nB1) * AFB2
```

$v - \hat{P} - F_B^1 - t\, n_B^1 + \beta_1\, n_B^1 + a\, n_S^1 + \alpha_1\, n_S^1$

$v - \hat{P} - F_B^2 - t\left(1 - n_B^1\right) + \beta_2\left(1 - n_B^1\right) + a\left(1 - n_S^1\right) + \alpha_2\left(1 - n_S^1\right)$

$-F_S^1 + \left(\hat{P} - a\, n_S^1\right)\left(n_B^1 - \gamma\left(\hat{P} - a\, n_S^1\right)\right)$

$-F_S^2 + \left(1 - n_B^1 - \gamma\left(\hat{P} - a\left(1 - n_S^1\right)\right)\right)\left(\hat{P} - a\left(1 - n_S^1\right)\right)$

$F_B^1\, n_B^1 + F_S^1\, n_S^1$

$F_B^2\left(1 - n_B^1\right) + F_S^2\left(1 - n_S^1\right)$

Defining six basic equations with conditions

```
eqn1 = uB1 == uB2
eqn2 = uS1 == uS2
```

$v - \hat{P} - F_B^1 - t\, n_B^1 + \beta_1\, n_B^1 + a\, n_S^1 + \alpha_1\, n_S^1 == v - \hat{P} - F_B^2 - t\left(1 - n_B^1\right) + \beta_2\left(1 - n_B^1\right) + a\left(1 - n_S^1\right) + \alpha_2\left(1 - n_S^1\right)$

$-F_S^1 + \left(\hat{P} - a\, n_S^1\right)\left(n_B^1 - \gamma\left(\hat{P} - a\, n_S^1\right)\right) == -F_S^2 + \left(1 - n_B^1 - \gamma\left(\hat{P} - a\left(1 - n_S^1\right)\right)\right)\left(\hat{P} - a\left(1 - n_S^1\right)\right)$

Solving eqn1 and eqn 2 with respect to n_B^1 and n_S^1

```
nBS1sol = Solve[{eqn1, eqn2}, {nB1, nS1}][[1]]
```

$$\Big\{n_B^1 \to -\Big(\big(a + 2\,a^2\,\gamma - 4\,a\,\gamma\,\hat{P}\big)\,\big(-a + t - \alpha_2 - \beta_2 - F_B^1 + F_B^2\big) - (2\,a + \alpha_1 + \alpha_2)\,\big(-a - a^2\,\gamma + \hat{P} + 2\,a\,\gamma\,\hat{P} + F_S^1 - F_S^2\big)\Big) \Big/ \Big(-\big(a - 2\,\hat{P}\big)\,(2\,a + \alpha_1 + \alpha_2) + \big(a + 2\,a^2\,\gamma - 4\,a\,\gamma\,\hat{P}\big)\,(-2\,t + \beta_1 + \beta_2)\Big),$$

$$n_S^1 \to \Big(\big(a - 2\,\hat{P}\big)\,\Big(\big(a + 2\,a^2\,\gamma - 4\,a\,\gamma\,\hat{P}\big)\,\big(-a + t - \alpha_2 - \beta_2 - F_B^1 + F_B^2\big) - (2\,a + \alpha_1 + \alpha_2)\,\big(-a - a^2\,\gamma + \hat{P} + 2\,a\,\gamma\,\hat{P} + F_S^1 - F_S^2\big)\Big)\Big) \Big/ \Big(\big(a + 2\,a^2\,\gamma - 4\,a\,\gamma\,\hat{P}\big)\,\Big(-\big(a - 2\,\hat{P}\big)\,(2\,a + \alpha_1 + \alpha_2) + \big(a + 2\,a^2\,\gamma - 4\,a\,\gamma\,\hat{P}\big)\,(-2\,t + \beta_1 + \beta_2)\Big)\Big) - \frac{-a - a^2\,\gamma + \hat{P} + 2\,a\,\gamma\,\hat{P} + F_S^1 - F_S^2}{a\,\big(1 + 2\,a\,\gamma - 4\,\gamma\,\hat{P}\big)}\Big\}$$

```
(* 2 Condition *)
cond1 = beta1 == beta2;
cond2 = alpha1 == alpha2;
```

```
nBS1sol1 = Simplify[nBS1sol, cond1] /. beta2 → β
nBS1sol2 = Simplify[nBS1sol, cond2] /. alpha2 → α
```

$$\Big\{n_B^1 \to \Big(-a\,\big(1 + 2\,a\,\gamma - 4\,\gamma\,\hat{P}\big)\,\big(a - t + \beta + \alpha_2 + F_B^1 - F_B^2\big) + (2\,a + \alpha_1 + \alpha_2)\,\big(a + a^2\,\gamma - (1 + 2\,a\,\gamma)\,\hat{P} - F_S^1 + F_S^2\big)\Big) \Big/ \Big(2\,a\,(t - \beta)\,\big(1 + 2\,a\,\gamma - 4\,\gamma\,\hat{P}\big) + \big(a - 2\,\hat{P}\big)\,(2\,a + \alpha_1 + \alpha_2)\Big),$$

$$n_S^1 \to \Big(a^2 + a\,t + 2\,a^2\,t\,\gamma + a\,\alpha_2 + a\,F_B^1 - 2\,\hat{P}\,\big(a + 2\,a\,t\,\gamma - 2\,a\,\beta\,\gamma + \alpha_2 + F_B^1 - F_B^2\big) - a\,F_B^2 - 2\,t\,F_S^1 + 2\,t\,F_S^2 - \beta\,\big(a + 2\,a^2\,\gamma - 2\,F_S^1 + 2\,F_S^2\big)\Big) \Big/ \Big(-2\,\hat{P}\,(2\,a + 4\,a\,t\,\gamma - 4\,a\,\beta\,\gamma + \alpha_1 + \alpha_2) + a\,(2\,a + 2\,t + 4\,a\,t\,\gamma + \beta\,(-2 - 4\,a\,\gamma) + \alpha_1 + \alpha_2)\Big)\Big\}$$

$$\Big\{n_B^1 \to -\Big(-a\,\big(1 + 2\,a\,\gamma - 4\,\gamma\,\hat{P}\big)\,\big(a - t + \alpha + \beta_2 + F_B^1 - F_B^2\big) + 2\,(a + \alpha)\,\big(a + a^2\,\gamma - (1 + 2\,a\,\gamma)\,\hat{P} - F_S^1 + F_S^2\big)\Big) \Big/ \Big(-2\,(a + \alpha)\,\big(a - 2\,\hat{P}\big) + a\,\big(1 + 2\,a\,\gamma - 4\,\gamma\,\hat{P}\big)\,(-2\,t + \beta_1 + \beta_2)\Big),$$

$$n_S^1 \to \frac{a + a^2\,\gamma - \hat{P} - 2\,a\,\gamma\,\hat{P} - F_S^1 + F_S^2}{a + 2\,a^2\,\gamma - 4\,a\,\gamma\,\hat{P}} + \Big(\big(a - 2\,\hat{P}\big)\,\Big(-a\,\big(1 + 2\,a\,\gamma - 4\,\gamma\,\hat{P}\big)\,\big(a - t + \alpha + \beta_2 + F_B^1 - F_B^2\big) + 2\,(a + \alpha)\,\big(a + a^2\,\gamma - (1 + 2\,a\,\gamma)\,\hat{P} - F_S^1 + F_S^2\big)\Big)\Big) \Big/ \Big(a\,\big(1 + 2\,a\,\gamma - 4\,\gamma\,\hat{P}\big)\,\Big(-2\,(a + \alpha)\,\big(a - 2\,\hat{P}\big) + a\,\big(1 + 2\,a\,\gamma - 4\,\gamma\,\hat{P}\big)\,(-2\,t + \beta_1 + \beta_2)\Big)\Big)\Big\}$$

Substituting n_B^1 and n_S^1 into Π_1 and Π_2 then differentiate them by F_S^1 and F_S^2 being constants

```
DPi1AFS1cond1 = D[Pi1 /. nBS1sol1, AFS1]
DPi1AFS2cond1 = D[Pi1 /. nBS1sol1, AFS2]
DPi2AFS1cond1 = D[Pi2 /. nBS1sol1, AFS1]
DPi2AFS2cond1 = D[Pi2 /. nBS1sol1, AFS2]
DPi1AFS1cond2 = D[Pi1 /. nBS1sol2, AFS1]
DPi1AFS2cond2 = D[Pi1 /. nBS1sol2, AFS2]
DPi2AFS1cond2 = D[Pi2 /. nBS1sol2, AFS1]
DPi2AFS2cond2 = D[Pi2 /. nBS1sol2, AFS2]
```

$$\frac{(-2a-\alpha_1-\alpha_2)\,F_B^1}{2a\,(t-\beta)\left(1+2a\gamma-4\gamma\hat{P}\right)+\left(a-2\hat{P}\right)(2a+\alpha_1+\alpha_2)}+\left((-2t+2\beta)\,F_S^1\right)\Big/\left(-2\hat{P}\,(2a+4at\gamma-4a\beta\gamma+\alpha_1+\alpha_2)+a\,(2a+2t+4at\gamma+\beta\,(-2-4a\gamma)+\alpha_1+\alpha_2)\right)+\left(a^2+a\,t+2a^2t\gamma+a\alpha_2+aF_B^1-2\hat{P}\left(a+2at\gamma-2a\beta\gamma+\alpha_2+F_B^1-F_B^2\right)-aF_B^2-2tF_S^1+2tF_S^2-\beta\left(a+2a^2\gamma-2F_S^1+2F_S^2\right)\right)\Big/\left(-2\hat{P}\,(2a+4at\gamma-4a\beta\gamma+\alpha_1+\alpha_2)+a\,(2a+2t+4at\gamma+\beta\,(-2-4a\gamma)+\alpha_1+\alpha_2)\right)$$

$$\frac{(2a+\alpha_1+\alpha_2)\,F_B^1}{2a\,(t-\beta)\left(1+2a\gamma-4\gamma\hat{P}\right)+\left(a-2\hat{P}\right)(2a+\alpha_1+\alpha_2)}+\left((2t-2\beta)\,F_S^1\right)\Big/\left(-2\hat{P}\,(2a+4at\gamma-4a\beta\gamma+\alpha_1+\alpha_2)+a\,(2a+2t+4at\gamma+\beta\,(-2-4a\gamma)+\alpha_1+\alpha_2)\right)$$

$$-\frac{(-2a-\alpha_1-\alpha_2)\,F_B^2}{2a\,(t-\beta)\left(1+2a\gamma-4\gamma\hat{P}\right)+\left(a-2\hat{P}\right)(2a+\alpha_1+\alpha_2)}-\left((-2t+2\beta)\,F_S^2\right)\Big/\left(-2\hat{P}\,(2a+4at\gamma-4a\beta\gamma+\alpha_1+\alpha_2)+a\,(2a+2t+4at\gamma+\beta\,(-2-4a\gamma)+\alpha_1+\alpha_2)\right)$$

$$1-\frac{(2a+\alpha_1+\alpha_2)\,F_B^2}{2a\,(t-\beta)\left(1+2a\gamma-4\gamma\hat{P}\right)+\left(a-2\hat{P}\right)(2a+\alpha_1+\alpha_2)}-\left((2t-2\beta)\,F_S^2\right)\Big/\left(-2\hat{P}\,(2a+4at\gamma-4a\beta\gamma+\alpha_1+\alpha_2)+a\,(2a+2t+4at\gamma+\beta\,(-2-4a\gamma)+\alpha_1+\alpha_2)\right)-\left(a^2+a\,t+2a^2t\gamma+a\alpha_2+aF_B^1-2\hat{P}\left(a+2at\gamma-2a\beta\gamma+\alpha_2+F_B^1-F_B^2\right)-aF_B^2-2tF_S^1+2tF_S^2-\beta\left(a+2a^2\gamma-2F_S^1+2F_S^2\right)\right)\Big/\left(-2\hat{P}\,(2a+4at\gamma-4a\beta\gamma+\alpha_1+\alpha_2)+a\,(2a+2t+4at\gamma+\beta\,(-2-4a\gamma)+\alpha_1+\alpha_2)\right)$$

$$\frac{2\,(a+\alpha)\,F_B^1}{-2\,(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)}+\left(-\frac{1}{a+2a^2\gamma-4a\gamma\hat{P}}-\left(2\,(a+\alpha)\left(a-2\hat{P}\right)\right)\Big/\left(a\left(1+2a\gamma-4\gamma\hat{P}\right)\left(-2\,(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)\right)\right)\right)F_S^1+\frac{a+a^2\gamma-\hat{P}-2a\gamma\hat{P}-F_S^1+F_S^2}{a+2a^2\gamma-4a\gamma\hat{P}}+\left(\left(a-2\hat{P}\right)\left(-a\left(1+2a\gamma-4\gamma\hat{P}\right)\left(a-t+\alpha+\beta_2+F_B^1-F_B^2\right)+2\,(a+\alpha)\left(a+a^2\gamma-(1+2a\gamma)\,\hat{P}-F_S^1+F_S^2\right)\right)\right)\Big/\left(a\left(1+2a\gamma-4\gamma\hat{P}\right)\left(-2\,(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)\right)\right)$$

$$
-\frac{2\,(a+\alpha)\,F_B^1}{-2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)+a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,(-2\,t+\beta_1+\beta_2)}+
\left(\frac{1}{a+2\,a^2\,\gamma-4\,a\,\gamma\,\hat{P}}+\left(2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)\right)\Big/
\left(a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,\left(-2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)+a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,(-2\,t+\beta_1+\beta_2)\right)\right)\right)\,F_S^1
$$

$$
-\frac{2\,(a+\alpha)\,F_B^2}{-2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)+a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,(-2\,t+\beta_1+\beta_2)}+
\left(\frac{1}{a+2\,a^2\,\gamma-4\,a\,\gamma\,\hat{P}}+\left(2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)\right)\Big/
\left(a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,\left(-2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)+a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,(-2\,t+\beta_1+\beta_2)\right)\right)\right)\,F_S^2
$$

$$
1+\frac{2\,(a+\alpha)\,F_B^2}{-2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)+a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,(-2\,t+\beta_1+\beta_2)}+
\left(-\frac{1}{a+2\,a^2\,\gamma-4\,a\,\gamma\,\hat{P}}-\left(2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)\right)\Big/
\left(a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,\left(-2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)+a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,(-2\,t+\beta_1+\beta_2)\right)\right)\right)\,F_S^2-
$$

$$
\frac{a+a^2\,\gamma-\hat{P}-2\,a\,\gamma\,\hat{P}-F_S^1+F_S^2}{a+2\,a^2\,\gamma-4\,a\,\gamma\,\hat{P}}-\left(\left(a-2\,\hat{P}\right)\,\left(-a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,\left(a-t+\alpha+\beta_2+F_B^1-F_B^2\right)+
2\,(a+\alpha)\,\left(a+a^2\,\gamma-(1+2\,a\,\gamma)\,\hat{P}-F_S^1+F_S^2\right)\right)\right)\Big/
\left(a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,\left(-2\,(a+\alpha)\,\left(a-2\,\hat{P}\right)+a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\,(-2\,t+\beta_1+\beta_2)\right)\right)
$$

Solving the simultaneous equations $\frac{\partial}{\partial F_S^1}\Pi_1 = 0$ and $\frac{\partial}{\partial F_S^2}\Pi_2 = 0$ with respect to F_S^1 and F_S^2

```
AFS1AFS2solCond1 =
 Solve[{DPi1AFS1cond1 == 0, DPi2AFS2cond1 == 0}, {AFS1, AFS2}] // Simplify
AFS1AFS2solCond2 =
 Solve[{DPi1AFS1cond2 == 0, DPi2AFS2cond2 == 0}, {AFS1, AFS2}] // Simplify
```

$$\Big\{\Big\{F_S^1 \to -\frac{1}{6\,(t-\beta)}\Big(-3a^2 - 3at + 3a\beta - 6a^2 t\gamma + 6a^2\beta\gamma - 2a\alpha_2 + 3aF_B^1 + 2\alpha_2 F_B^1 + 2\hat{P}\left(3a + 6at\gamma - 6a\beta\gamma + \alpha_1 + 2\alpha_2 + F_B^1 - F_B^2\right) + 3aF_B^2 + \alpha_2 F_B^2 + \alpha_1\left(-a + 2F_B^1 + F_B^2\right)\Big),$$

$$F_S^2 \to \frac{1}{6\,(t-\beta)}\Big(3a^2 + 3at - 3a\beta + 6a^2 t\gamma - 6a^2\beta\gamma + a\alpha_2 - 3aF_B^1 - \alpha_2 F_B^1 + \alpha_1\left(2a - F_B^1 - 2F_B^2\right) - 3aF_B^2 - 2\alpha_2 F_B^2 - 2\hat{P}\left(3a + 6at\gamma - 6a\beta\gamma + 2\alpha_1 + \alpha_2 - F_B^1 + F_B^2\right)\Big)\Big\}\Big\}$$

$$\Big\{\Big\{F_S^1 \to -\frac{1}{6t - 3\beta_1 - 3\beta_2}\Big(-3a^2 - 3at - 3a\alpha - 6a^2 t\gamma + a\,(2 + 3a\gamma)\,\beta_1 + a\beta_2 + 3a^2\gamma\beta_2 + 3aF_B^1 + 4\alpha F_B^1 + \hat{P}\left(-(1 + 6a\gamma)\,\beta_1 + (1 - 6a\gamma)\,\beta_2 + 2\left(3\,(a + \alpha + 2at\gamma) + F_B^1 - F_B^2\right)\right) + 3aF_B^2 + 2\alpha F_B^2\Big),$$

$$F_S^2 \to \frac{1}{6t - 3\beta_1 - 3\beta_2}\Big(3a^2 + 3at + 3a\alpha + 6a^2 t\gamma - a\,(1 + 3a\gamma)\,\beta_1 - 2a\beta_2 - 3a^2\gamma\beta_2 - 3aF_B^1 - 2\alpha F_B^1 - 3aF_B^2 - 4\alpha F_B^2 + \hat{P}\left((-1 + 6a\gamma)\,\beta_1 + (1 + 6a\gamma)\,\beta_2 - 2\left(3\,(a + \alpha + 2at\gamma) - F_B^1 + F_B^2\right)\right)\Big)\Big\}\Big\}$$

Substituting n_B^1 and n_S^1 into Π_1 and Π_2 then differentiate them by F_B^1 and F_B^2 being constants

```
DPi1AFB1cond1 = D[Pi1 /. nBS1sol1, AFB1]
DPi1AFB2cond1 = D[Pi1 /. nBS1sol1, AFB2]
DPi2AFB1cond1 = D[Pi2 /. nBS1sol1, AFB1]
DPi2AFB2cond1 = D[Pi2 /. nBS1sol1, AFB2]
DPi1AFB1cond2 = D[Pi1 /. nBS1sol2, AFB1]
DPi1AFB2cond2 = D[Pi1 /. nBS1sol2, AFB2]
DPi2AFB1cond2 = D[Pi2 /. nBS1sol2, AFB1]
DPi2AFB2cond2 = D[Pi2 /. nBS1sol2, AFB2]
```

$$-\frac{a\left(1 + 2a\gamma - 4\gamma\hat{P}\right)F_B^1}{2a\,(t-\beta)\left(1 + 2a\gamma - 4\gamma\hat{P}\right) + \left(a - 2\hat{P}\right)(2a + \alpha_1 + \alpha_2)} + \left(\left(a - 2\hat{P}\right)F_S^1\right) \Big/ \left(-2\hat{P}\,(2a + 4at\gamma - 4a\beta\gamma + \alpha_1 + \alpha_2) + a\,(2a + 2t + 4at\gamma + \beta\,(-2 - 4a\gamma) + \alpha_1 + \alpha_2)\right) + \left(-a\left(1 + 2a\gamma - 4\gamma\hat{P}\right)\left(a - t + \beta + \alpha_2 + F_B^1 - F_B^2\right) + (2a + \alpha_1 + \alpha_2)\left(a + a^2\gamma - (1 + 2a\gamma)\,\hat{P} - F_S^1 + F_S^2\right)\right) \Big/ \left(2a\,(t-\beta)\left(1 + 2a\gamma - 4\gamma\hat{P}\right) + \left(a - 2\hat{P}\right)(2a + \alpha_1 + \alpha_2)\right)$$

$$\frac{a\left(1+2a\gamma-4\gamma\hat{P}\right)F_B^1}{2a(t-\beta)\left(1+2a\gamma-4\gamma\hat{P}\right)+\left(a-2\hat{P}\right)(2a+\alpha_1+\alpha_2)}+\left(\left(-a+2\hat{P}\right)F_S^1\right)/$$
$$\left(-2\hat{P}(2a+4at\gamma-4a\beta\gamma+\alpha_1+\alpha_2)+a(2a+2t+4at\gamma+\beta(-2-4a\gamma)+\alpha_1+\alpha_2)\right)$$

$$\frac{a\left(1+2a\gamma-4\gamma\hat{P}\right)F_B^2}{2a(t-\beta)\left(1+2a\gamma-4\gamma\hat{P}\right)+\left(a-2\hat{P}\right)(2a+\alpha_1+\alpha_2)}-\left(\left(a-2\hat{P}\right)F_S^2\right)/$$
$$\left(-2\hat{P}(2a+4at\gamma-4a\beta\gamma+\alpha_1+\alpha_2)+a(2a+2t+4at\gamma+\beta(-2-4a\gamma)+\alpha_1+\alpha_2)\right)$$

$$1-\frac{a\left(1+2a\gamma-4\gamma\hat{P}\right)F_B^2}{2a(t-\beta)\left(1+2a\gamma-4\gamma\hat{P}\right)+\left(a-2\hat{P}\right)(2a+\alpha_1+\alpha_2)}-\left(\left(-a+2\hat{P}\right)F_S^2\right)/$$
$$\left(-2\hat{P}(2a+4at\gamma-4a\beta\gamma+\alpha_1+\alpha_2)+a(2a+2t+4at\gamma+\beta(-2-4a\gamma)+\alpha_1+\alpha_2)\right)-$$
$$\left(-a\left(1+2a\gamma-4\gamma\hat{P}\right)\left(a-t+\beta+\alpha_2+F_B^1-F_B^2\right)+\right.$$
$$\left.(2a+\alpha_1+\alpha_2)\left(a+a^2\gamma-(1+2a\gamma)\hat{P}-F_S^1+F_S^2\right)\right)/$$
$$\left(2a(t-\beta)\left(1+2a\gamma-4\gamma\hat{P}\right)+\left(a-2\hat{P}\right)(2a+\alpha_1+\alpha_2)\right)$$

$$\frac{a\left(1+2a\gamma-4\gamma\hat{P}\right)F_B^1}{-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)}-$$
$$\frac{\left(a-2\hat{P}\right)F_S^1}{-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)}-$$
$$\left(-a\left(1+2a\gamma-4\gamma\hat{P}\right)\left(a-t+\alpha+\beta_2+F_B^1-F_B^2\right)+2(a+\alpha)\left(a+a^2\gamma-(1+2a\gamma)\hat{P}-F_S^1+F_S^2\right)\right)/$$
$$\left(-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)\right)$$

$$-\frac{a\left(1+2a\gamma-4\gamma\hat{P}\right)F_B^1}{-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)}+$$
$$\frac{\left(a-2\hat{P}\right)F_S^1}{-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)}$$

$$-\frac{a\left(1+2a\gamma-4\gamma\hat{P}\right)F_B^2}{-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)}+$$
$$\frac{\left(a-2\hat{P}\right)F_S^2}{-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)}$$

$$1+\frac{a\left(1+2a\gamma-4\gamma\hat{P}\right)F_B^2}{-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)}-$$
$$\frac{\left(a-2\hat{P}\right)F_S^2}{-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)}+$$
$$\left(-a\left(1+2a\gamma-4\gamma\hat{P}\right)\left(a-t+\alpha+\beta_2+F_B^1-F_B^2\right)+2(a+\alpha)\left(a+a^2\gamma-(1+2a\gamma)\hat{P}-F_S^1+F_S^2\right)\right)/$$
$$\left(-2(a+\alpha)\left(a-2\hat{P}\right)+a\left(1+2a\gamma-4\gamma\hat{P}\right)(-2t+\beta_1+\beta_2)\right)$$

Solving the simultaneous equations $\frac{\partial}{\partial F_B^1}\Pi_1 = 0$ and $\frac{\partial}{\partial F_B^2}\Pi_2 = 0$ with respect to F_B^1 and F_B^2

```
AFB1AFB2solCond1 =
  Solve[{DPi1AFB1cond1 == 0, DPi2AFB2cond1 == 0}, {AFB1, AFB2}] // Simplify
AFB1AFB2solCond2 =
  Solve[{DPi1AFB1cond2 == 0, DPi2AFB2cond2 == 0}, {AFB1, AFB2}] // Simplify
```

$$\Big\{\Big\{F_B^1 \to \frac{1}{3\,a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)}\Big(3\,a^2+3\,a\,t-3\,a\,\beta+6\,a^2\,t\,\gamma- 6\,a^2\,\beta\,\gamma+a\,\alpha_2-a^2\,\gamma\,\alpha_2-\alpha_2\,F_S^1+3\,a\,F_S^2+\alpha_2\,F_S^2+\alpha_1\,\left(a\,(2+a\,\gamma)-F_S^1+F_S^2\right)- \hat{P}\,\left((3+2\,a\,\gamma)\,\alpha_1+(3-2\,a\,\gamma)\,\alpha_2+2\,\left(3\,a+6\,a\,t\,\gamma-6\,a\,\beta\,\gamma+2\,F_S^1+F_S^2\right)\right)\Big),$$

$$F_B^2 \to \frac{1}{3\,a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)}\Big(3\,a^2+3\,a\,t-3\,a\,\beta+6\,a^2\,t\,\gamma-6\,a^2\,\beta\,\gamma+2\,a\,\alpha_2+ a^2\,\gamma\,\alpha_2+3\,a\,F_S^1+\alpha_2\,F_S^1+\alpha_1\,\left(a-a^2\,\gamma+F_S^1-F_S^2\right)-\alpha_2\,F_S^2- \hat{P}\,\left((3-2\,a\,\gamma)\,\alpha_1+(3+2\,a\,\gamma)\,\alpha_2+2\,\left(3\,a+6\,a\,t\,\gamma-6\,a\,\beta\,\gamma+F_S^1+2\,F_S^2\right)\right)\Big)\Big\}\Big\}$$

$$\Big\{\Big\{F_B^1 \to \frac{1}{3\,a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)}\Big(3\,a^2+3\,a\,t+3\,a\,\alpha+6\,a^2\,t\,\gamma-a\,(1+2\,a\,\gamma)\,\beta_1-2\,a\,\beta_2-4\,a^2\,\gamma\,\beta_2- 2\,\alpha\,F_S^1+3\,a\,F_S^2+2\,\alpha\,F_S^2-2\,\hat{P}\,\left(3\,a+3\,\alpha+6\,a\,t\,\gamma-2\,a\,\gamma\,\beta_1-4\,a\,\gamma\,\beta_2+2\,F_S^1+F_S^2\right)\Big),$$

$$F_B^2 \to \frac{1}{3\,a\,\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)}\Big(3\,a^2+3\,a\,t+3\,a\,\alpha+6\,a^2\,t\,\gamma-2\,a\,(1+2\,a\,\gamma)\,\beta_1-a\,\beta_2-2\,a^2\,\gamma\,\beta_2+ 3\,a\,F_S^1+2\,\alpha\,F_S^1-2\,\alpha\,F_S^2-2\,\hat{P}\,\left(3\,a+3\,\alpha+6\,a\,t\,\gamma-4\,a\,\gamma\,\beta_1-2\,a\,\gamma\,\beta_2+F_S^1+2\,F_S^2\right)\Big)\Big\}\Big\}$$

Result

```
(AFS1 - AFS2) /. AFS1AFS2solCond1 // FullSimplify
(AFS1 - AFS2) /. AFS1AFS2solCond2 // FullSimplify
```

$$\left\{\frac{-\alpha_1\left(a-2\hat{P}+F_B^1-F_B^2\right)+4\hat{P}\left(-F_B^1+F_B^2\right)+\alpha_2\left(a-2\hat{P}-F_B^1+F_B^2\right)}{6\,(t-\beta)}\right\}$$

$$\left\{\frac{-\left(a-2\hat{P}\right)\beta_1+\left(a-2\hat{P}\right)\beta_2-2\left(\alpha+2\hat{P}\right)\left(F_B^1-F_B^2\right)}{6\,t-3\,\beta_1-3\,\beta_2}\right\}$$

```
(AFS1 - AFS2) /. AFS1AFS2solCond1 /. AFB2 → AFB /. AFB1 → AFB // FullSimplify
(AFS1 - AFS2) /. AFS1AFS2solCond2 /. AFB2 → AFB /. AFB1 → AFB // FullSimplify
```

$$\left\{-\frac{\left(a-2\hat{P}\right)(\alpha_1-\alpha_2)}{6\,(t-\beta)}\right\}$$

$$\left\{-\frac{\left(a-2\hat{P}\right)(\beta_1-\beta_2)}{6\,t-3\,\beta_1-3\,\beta_2}\right\}$$

```
(Pi1 - Pi2) /. nBS1sol1 /. AFS1AFS2solCond1 // FullSimplify
(Pi1 - Pi2) /. nBS1sol2 /. AFS1AFS2solCond2 // FullSimplify
```

$$\left\{\frac{1}{6\,(t-\beta)}\left(\alpha_2\left(a-2\hat{P}-F_B^1-2\,F_B^2\right)+\left(-4\hat{P}+3\left(a+t-\beta-F_B^1-F_B^2\right)\right)\left(F_B^1-F_B^2\right)+\alpha_1\left(-a+2\hat{P}+2\,F_B^1+F_B^2\right)\right)\right\}$$

$$\left\{\frac{1}{6\,t-3\,\beta_1-3\,\beta_2}\left(\beta_2\left(a-2\hat{P}-3\,F_B^1\right)+\left(3\,(a+t)+\alpha-4\hat{P}-3\,F_B^1-3\,F_B^2\right)\left(F_B^1-F_B^2\right)+\beta_1\left(-a+2\hat{P}+3\,F_B^2\right)\right)\right\}$$

```
(Pi1 - Pi2) /. nBS1sol1 /. AFS1AFS2solCond1 /. AFB2 → AFB /. AFB1 → AFB //
 FullSimplify
(Pi1 - Pi2) /. nBS1sol2 /. AFS1AFS2solCond2 /. AFB2 → AFB /. AFB1 → AFB //
 FullSimplify
```

$$\left\{-\frac{\left(a-2\hat{P}-3\,F_B\right)(\alpha_1-\alpha_2)}{6\,(t-\beta)}\right\}$$

$$\left\{-\frac{\left(a-2\hat{P}-3\,F_B\right)(\beta_1-\beta_2)}{6\,t-3\,\beta_1-3\,\beta_2}\right\}$$

```
(AFB1 - AFB2) /. AFB1AFB2solCond1 // FullSimplify
(AFB1 - AFB2) /. AFB1AFB2solCond2 // FullSimplify
```

$$\left\{-\frac{1}{3\,a\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)}\left(\alpha_2\left(a+2\,a^2\,\gamma-4\,a\,\gamma\,\hat{P}+2\,F_S^1-2\,F_S^2\right)+\left(3\,a+2\,\hat{P}\right)\left(F_S^1-F_S^2\right)-\alpha_1\left(a+2\,a^2\,\gamma-4\,a\,\gamma\,\hat{P}-2\,F_S^1+2\,F_S^2\right)\right)\right\}$$

$$\left\{-\left(a\left(-1-2\,a\,\gamma+4\,\gamma\,\hat{P}\right)\beta_1+a\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\beta_2+\left(3\,a+4\,\alpha+2\,\hat{P}\right)\left(F_S^1-F_S^2\right)\right)\Big/\left(3\,a\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\right)\right\}$$

```
(AFB1 - AFB2) /. AFB1AFB2solCond1 /. AFS2 → AFS /. AFS1 → AFS // FullSimplify
(AFB1 - AFB2) /. AFB1AFB2solCond2 /. AFS2 → AFS /. AFS1 → AFS // FullSimplify
```

$$\left\{\frac{1}{3}\left(\alpha_1-\alpha_2\right)\right\}$$

$$\left\{\frac{1}{3}\left(\beta_1-\beta_2\right)\right\}$$

```
(Pi1 - Pi2) /. nBS1sol1 /. AFB1AFB2solCond1 // FullSimplify
(Pi1 - Pi2) /. nBS1sol2 /. AFB1AFB2solCond2 // FullSimplify
```

$$\left\{\frac{1}{3\,a\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)}\left(-\alpha_2\left(a+2\,a^2\,\gamma-4\,a\,\gamma\,\hat{P}+2\,F_S^1-2\,F_S^2\right)+\left(3\,a\,(-1+a\,\gamma)+\hat{P}-6\,a\,\gamma\,\hat{P}-3\,F_S^1-3\,F_S^2\right)\left(F_S^1-F_S^2\right)+\alpha_1\left(a+2\,a^2\,\gamma-4\,a\,\gamma\,\hat{P}-2\,F_S^1+2\,F_S^2\right)\right)\right\}$$

$$\left\{\frac{1}{3\,a\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)}\left(a\left(1+2\,a\,\gamma-4\,\gamma\,\hat{P}\right)\beta_1+a\left(-1-2\,a\,\gamma+4\,\gamma\,\hat{P}\right)\beta_2+\left(-4\,\alpha+3\,a\,(-1+a\,\gamma)+\hat{P}-6\,a\,\gamma\,\hat{P}-3\,F_S^1-3\,F_S^2\right)\left(F_S^1-F_S^2\right)\right)\right\}$$

```
(Pi1 - Pi2) /. nBS1sol1 /. AFB1AFB2solCond1 /. AFS2 → AFS /. AFS1 → AFS //
 FullSimplify
(Pi1 - Pi2) /. nBS1sol2 /. AFB1AFB2solCond2 /. AFS2 → AFS /. AFS1 → AFS //
 FullSimplify
```

$$\left\{\frac{1}{3}\left(\alpha_1-\alpha_2\right)\right\}$$

$$\left\{\frac{1}{3}\left(\beta_1-\beta_2\right)\right\}$$

국문초록

가입비를 부과하는 양면시장에서 플랫폼 수익에 영향을 주는 주요인들에 대한 연구

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양면시장에 관한 다수의 연구에서 교차네트워크 외부성은 가장 주목을 받는 변수 중의 하나이다. Li et al. 의 연구에서 Li는 플랫폼이 특정 정도의 플랫폼 차별화를 하지 않은 채 교차 네트워크 외부성을 높이려는 시도는 플랫폼의 수익을 악화시킨다고 주장을 하였다. 당 연구에서는 Li et al. 의 모델에 access fee를 추가한 새 모델을 만들어 Li et al.의 주장의 일반적 유효성을 조사하였다.

새 모델을 통해 얻어진 다수의 유효한 결과들은 각 케이스 별로 최적화된 가입비 모델에 difference makers 및 no-difference makers 가 존재함을 밝혔다. 또한 가입비의 차이로 인해 발생하는 플랫폼의 수익에 차이는 각 경쟁플랫폼에서 같은 수준의 가입비를 부과하는 경우 차이가 제거됨을 밝혔다. 또한 각 가입비 부과 플랫폼 별로 공통의 no-difference makers가 존재함을 밝혔다. 더불어 당 모델의 결과들을 적용한 케이스 조사를 통해 가입비를 낮추는 것이 반드

시 플랫폼 수익에 도움이 되지는 않는다는 것도 밝혔다.

양면시장에 대한 당 연구의 기여는 당 연구가 플랫폼들이 어느 곳에 자원을 집중하고 어느 곳에 집중하지 않아도 되는지를 제시했다는 것이며, 연구에 사용된 변수들이 어떻게 관련이 되어 있고 특히 seller와 buyer의 가입비의 경우 서로 어떻게 상호작용을 하는 지를 밝혔다는 것이다. 추가적으로 Li et al.의 연구 결과가 일반성이 부족함을 보였다.