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공학석사학위논문

**A Dynamic User Equilibrium Model of
Battery Electric Vehicles Considering
En-route Recharging Behaviors**

주행 중 충전 행태를 고려한
배터리 전기차의 동적 이용자 평형 모형

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Abstract

A Dynamic User Equilibrium Model of Battery Electric Vehicles Considering En-route Recharging Behaviors

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Facing with increasing demands on Battery Electric Vehicle (BEV), public interests of incorporating BEVs into existing operational and planning models of transportation systems are growing recently. Unlike gasoline vehicles, BEV users encounter range anxiety that comes from short driving range, long charging time, and insufficient charging infrastructures. BEV drivers who encounters the range anxiety problem are expected to have different route choice behaviors from the existing drivers and will resist being stuck in the middle of trips, for not having enough battery states.

Several models have been developed to reflect this aspect in static traffic assignment models. However, Dynamic Traffic Assignment (DTA) incorporating BEVs on it have hardly been researched yet. In addition to generally researched static models, the temporal approach toward BEV can broaden analytical scopes in a temporal manner and can be used for analysis related to operational planning. Therefore, this research proposes a Dynamic

User Equilibrium (DUE) model of BEVs that can reflect driver's behaviors incurred by range anxiety.

In this research, a trip-based DTA model for BEVs is developed. It is because the usable paths of a BEV are completely determined by its remaining battery state, a path-based discrete time DTA model is formulated to track the battery states of BEVs at each time interval. The model's objective function is composed of travel time and out-of-range penalty term induced from battery shortage for assigned path flows. A modified iterative flow swapping algorithm is adopted to gradually decrease the out-of-range penalty and travel time gap between shortest paths and non-shortest paths.

The suggested model is applied to an example problem of Nguyen-Dupuis network with insufficient initial battery state. As a result, given with insufficient initial battery state, traffic flows of BEV detoured to maintain their batteries to be higher than minimal comfortable amount in a dynamic transportation system. The experienced travel time for traffic flows that had the same O-D and departing time interval showed identical value and traversed their paths without running out of batteries.

Keywords: Dynamic User Equilibrium, Dynamic Traffic Assignment, Battery Electric Vehicle, Range anxiety

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Chapter 1. Introduction

1.1. Background

Over the past few decades, gasoline vehicles (GV) have significantly expanded the one-day life zones of their owners. Despite the effectiveness and efficiency GVs, the increasing and fluctuating prices of fossil fuels, and the environmental harm caused by their use have increased public interest in alternative-fuel vehicles.

There are now many different alternative-fuel vehicles, including those that run on biodiesel, hydrogen and fuel cells, and compressed natural gas. Electric vehicles (EV), which have shown rapid growth in recent years, are the most widely adopted alternative-fuel vehicles among them. During the transition period from the GV to EV types, the parallel use of fossil fuels and electric energy applied to hybrid electric vehicles (HEV) and plug-in hybrid electric vehicles (PHEV) is currently being used. Differing given its use of multiple energy sources, the battery electric vehicle (BEV), which uses electric energy as its only energy source, is considered to be the final goal of the development of the EV.

Similar to the PHEV, the BEV is charged by plugging its battery into an external electricity source. The considerably long charging time of the BEV has been a barrier to its entry into the market. The charging time to a full battery state can extend to six or even eight hours through a low-density 240 V charger. On the other hand, a rapid charger requires only 20-30 minutes for a full charge, which is still is much longer than the refueling time of a GV. On top of the recharging time problem, insufficient recharging infrastructure for EVs also presents some inconvenience to the early users of BEVs. Furthermore, given the current state of battery technology, most BEVs can be driven only about 13 km on a single full charge, which may be insufficient for one day trips

considering that not all drivers will maintain a full battery state to be ready at the beginning of each trip. Furthermore, in accordance with the long time needed to charge the battery, the insufficient charging infrastructure, and short driving range of BEVs, drivers often encounter what is termed range anxiety.

Range anxiety is a fear of running out of battery power before arriving at a final destination. Once a battery is exhausted within the short driving range of a BEV, a driver has to spend a considerable amount of time to recharge the battery, or they may be stuck in the middle of a trip. These situations induce BEV drivers to maintain their battery at a level over a critical amount, also causing them continually to make plans to recharge their battery at charging stations or at their workplace and/or home.

Along with the range anxiety problem and technical issues, BEVs of nowadays are not widely adopted yet. However, as technologies related to BEVs are being improved rapidly, an increased driving range and lower cost of BEVs are expected in the near future.

1.2. Motivation

The increasing levels of demand for BEVs and battery-charging infrastructure have increased public interest in the adoption of BEVs. Policymakers and private enterprises are expected to cope with network-level aspects stemming from the increased use of BEVs. From a macroscopic perspective on transportation planning, existing studies developed traffic assignment models to handle the traffic patterns of GVs. However, few researchers have included BEVs in traffic assignment models.

Unlike GV users, BEV users encounter range anxiety due to the long battery charging times, the insufficient recharging infrastructure and the short driving range of BEVs (Pearre et al., 2011; He et al. 2013). BEV drivers who encounters the range anxiety problem are expected to have different route choice behaviors from the existing drivers and will resist being stuck in the

middle of trips, for not having enough battery states. Therefore, modified assignment model that can include traffic flows affected by the range anxiety is required for the representation of BEVs on transportation networks.

Several static assignment models of BEVs have been developed to cope with the range anxiety of alternative fuel vehicles including BEVs (Kuby and Lim, 2005; Wang and Lin, 2009; Kim, 2010; Jiang et al., 2012; Jiang et al., 2014; He et al., 2014; Lee et al., 2014). In static traffic assignment models, the travel time of each link is defined by what is termed the volume-delay function (VDF). Generally, the VDF uses the number of vehicles on a specific link as its input variable, with the average travel cost calculated as an output variable according to the amount of vehicles on the link. Static models also use time-invariant variables in the VDF and are applied to rather long-time periods; given the time-invariant property of the models, an analysis based on such models cannot capture the time-varying traffic patterns of a transportation network.

In addition to generally researched static models, the temporal approach toward BEV drivers will also likely be required for several reasons. For BEV drivers, range anxiety not only induces them to consider where to recharge their vehicles and which route to use among spatial concerns, but also temporally affects their recharging time, which is determined by the current amount of charge in their batteries. With regard to battery-charging infrastructure, estimating the time-varying arrival and departure times of recharging BEVs can help approximate the appropriate capacities of battery chargers. For operational decision makers, the expected temporal traffic patterns of BEV drivers that come from time-varying operations can be simulated beforehand.

A temporal analysis of BEVs in a transportation network can be implemented by including BEVs in dynamic traffic assignment (DTA) models. DTA models describe time-varying traffic patterns. They can have discrete time intervals or a continuous time axis. In the dynamic models, the time time-

varying inflow and outflow of vehicles on each link determines the time-varying link travel cost. Consequently, the time-varying link cost causes different shortest paths at each time and affects drivers' temporal route choices. The DTA models are generally applied during the operational planning of a transportation system. Operational planning is aimed at reaching the best planning decisions for major operations, construction, or demand management actions that are likely to induce a temporal or spatial pattern shift of traffic among different roadway facilities at the corridor-network level (Chiu, Yi-Chang et al. 2011). Therefore, once a DTA model including BEVs is established, it is expected to be used for analyzing the temporal aspects of BEVs in network-level for operational planning purposes.

In this research, a DTA model consolidating BEVs is developed. Because the usable paths of a BEV are completely determined by its remaining battery state, a path-based discrete time DTA model is formulated to track the battery states of BEVs at each time interval. The model's objective function follows a complementarity problem where flows on unusable paths that are impossible to be traversed with given battery states necessarily incur a penalty cost. Minimizing the value of the objective function to zero guarantees that none of the assigned flows exceed their range limits while also satisfying the Wardrop's first principle. The model is expected to be used to determine the temporal traffic patterns of BEVs and as a time-varying simulation tool in relation to the increasing adoption of BEVs.

1.3. Objectives

The main objective of this research is to propose a Dynamic User Equilibrium (DUE) model considering BEVs that can be used for temporal analyses of transportation networks with BEVs. To achieve the main objective, three sub-objectives must be accomplished: modeling a DUE model with BEVs, developing a corresponding search algorithm, and applying the model to numerical example that can have practical implications.

First, a DUE model with BEVs that encompasses the recharging behaviors of BEV drivers is proposed. The recharging behaviors are assumed to include a determination of the recharging station, detouring for the purpose of battery recharging, and decisions on the recharging amount at a station. Reflecting the behavioral aspects of battery-recharging-induced flow-propagation constraints to accommodate arriving and departing BEVs at each recharging station is also considered. Furthermore, the proposed objective function is modified from the zero-extreme value minimization problem of Huang and Lam (2002) to avoid flows on unusable paths at equilibrium, where the unusable paths are defined as paths that do not have a minimal cost with or without en-route battery recharging facilities. In summary, the goal of the proposed model is simultaneously to follow both Wardrop's first principle and to realize unbroken trips with the given initial battery states.

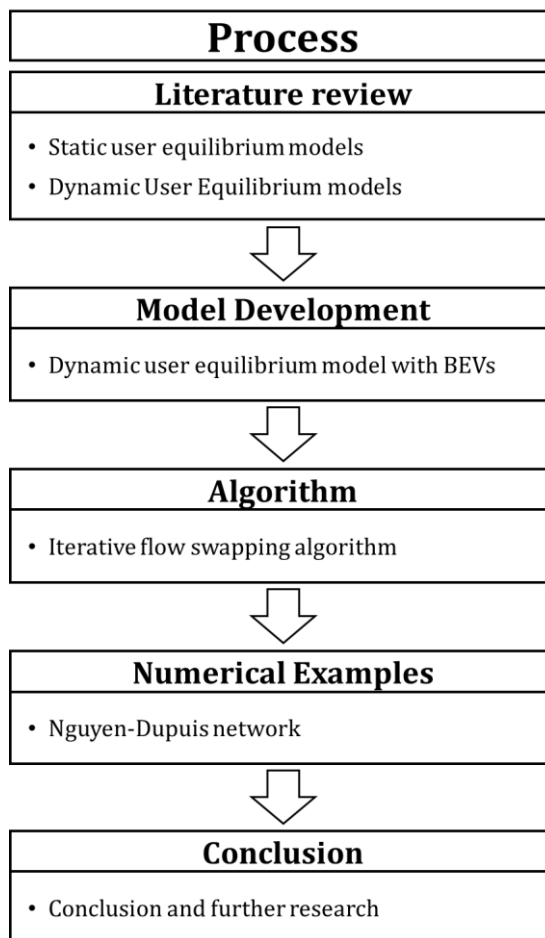
Secondly, an iterative algorithm to work in conjunction with the proposed DTA model is developed. At each iteration, the flows of paths which do not have a minimal travel cost within the same origin-destination pair are transferred to the minimal cost paths. The amount of the transferred flow is proportional to the gap between the cost of a path which is being transferred and the cost of the minimal paths. The algorithm iteratively reduces the value of the objective function toward zero until the value satisfies predefined stopping criteria.

Finally, a numerical example is analyzed to demonstrate the applicability and implications of the proposed model and algorithm. The Nguyen-Dupuis network is analyzed with predefined battery recharging stations. Computational tractability and network-level implications are then proposed through the example problem.

Achieving the objective of this study will help those who make decisions on the operational planning of BEVs do so on a more analytic basis. Examples can include a temporal operational plan for recharging stations, estimations of the proper capacities of stations, and determinations of the locations of recharging stations.

1.4. Organization of Thesis

The rest part of this research is composed as follows. Chapter 2 reviews literature on static user equilibrium models of alternative fuel vehicles and general DUE models. Chapter 3 introduces mathematical formulation of the DUE model with BEVs. Chapter 4 shows an iterative algorithm to solve the suggested DUE model with BEVs. Chapter 5 handles numerical example and shows applicability of the proposed model.



<Figure 1> Research procedure

Chapter 2. Literature Review

In this research, a DUE model encompassing the en-route recharging behaviors of BEV drivers is proposed. Related studies can be categorized into three types: static traffic assignment models with BEVs, general DTA models, and search algorithms. Precedent studies within the three categories are reviewed in this chapter.

2.1. Static User Equilibrium Models

The BEV is of course not the only mode of transportation for which drivers encounter the needs to refuel their vehicles. In a broader sense, alternative fuel vehicles which generally have range limits are also associated with this problem. There have been efforts to include range limits in traffic assignment models; however, early mathematical models did not use an equilibrium principle when addressing the range limit problem. Exogenously generated shortest paths which did not reflect variations of link travel costs via link traffic flows were considered to evaluate the appropriateness of the locations of refueling stations (Kuby and Lim, 2005; Wang and Lin, 2009; Kim, 2010). Later, the equilibrium principle was integrated into the modeling process for a better description of behavioral aspects of drivers with range limits (Jiang et al., 2012; Jiang et al., 2014; He et al., 2014; Lee et al., 2014).

Kuby and Lim (2005) proposed a model to optimize the locations of refueling facilities for alternative fuel vehicles. This model was termed the flow refueling location model (FRLM). They integrated the range limits of alternative-fuel vehicles into the FRLM, as they had not been considered before. In their study, an en-route refueling mechanism was applied while assuming that alternative fuel vehicles may have to refuel at least once to finish their trips

successfully. The proposed FRLM maximized the flows of predetermined and fixed shortest paths to finish the trips. Given that the shortest paths and flows were not based on the equilibrium principle, detouring behaviors by which drivers can lower their travel costs were not considered.

Wang and Lin (2009) proposed a refueling-station-location model based on the vehicle-routing logic of alternative fuel vehicles. Their model is a flow-based set-covering model that maximizes coverage with passing flows along the shortest paths. The vehicle-routing logic is a type of refueling criteria which determine where drivers refuel their vehicles within their range limits. Once an auxiliary solution is generated by the model, the logic is applied to evaluate the quality of the given solution. Akin to Kuby and Lim (2005), their model assumed that the flows of an O-D only traverse along predetermined shortest paths. Therefore, the model does not consider the equilibrium principle.

Kim (2010) found it necessary to improve the static representation of alternative fuel vehicles, as existing research had only considered the flows of exogenously selected shortest paths. Accordingly, a deviation-flow refueling location model (DFRLM) was proposed to consider as well flows that are not on the shortest paths. To reflect detouring flows, the DFRLM assumed that some detouring flows depend on the detouring distances of the alternative fuel vehicles. The relationship between the proportion of detouring flows and the detouring distance followed functional forms, i.e., linear, exponential, and sigmoid. Though the model was improved to include detouring flows, it also exogenously considered detouring paths. A traffic equilibrium state was devised by balancing between traffic flows, and driving and recharging times were not considered

Jiang et al. (2012) proposed a path-constrained traffic assignment model that simultaneously satisfies Wardrop's first principle and the distance constraint of paths. The model was originally developed for a representation of the equilibrium of EVs. However, en-route battery recharging by EVs was not

reflected in the model, and only route choices within the distance limit were considered. In the model, because the equilibrium is deduced based on the sum of the costs of the links existing on a path and a path-specific out-of-range penalty term, the path flows are generated endogenously regarding these cost terms. The model added a distance constraint to the general traffic assignment model according to which a path exceeding the distance limit cannot have a positive flow. The added distance constraint is as follows:

$$(D_m - l_k^{rs})f_{k,m}^{rs} \geq 0 \quad \forall k, r, s, m \quad (2.1)$$

Where,

- D_m = distance limit of m th class of vehicles
- l_k^{rs} = the length of path k from origin r to destination s
- $f_{k,m}^{rs}$ = traffic flow rate of m th class of vehicles on path k from origin r to destination s

In the model, equation (2.1) always produces a positive value. In detail, if the distance of a path exceeds the distance limit, the flows along the path must always be zero to satisfy the constraint. Otherwise, if the distance of a path is within the distance limit, path flows can have either a zero or a positive value. Therefore, even when a path satisfies the distance constraint, it may not have flows because it is not included in the set of shortest path(s).

Jiang et al. (2014) developed a network equilibrium problem which is an elaborated model for BEVs compared to the prior model of Jiang et al. (2012). Their newer model analyzes the impact of mixed gasoline and electric vehicular traffic flows on parking facilities in urban areas. The model incorporates destinations, routes and parking choices subjected to the driving range limit and the alternative travel cost composition. In the model, parking facilities are assumed to be divided into the two categories of ordinary parking garages and special parking garages. Ordinary parking garages can be utilized by both GVs and BEVs; however, special garages can only accommodate BEVs, as they are

equipped with charging stations. Based on the suggested model, several numerical example is analyzed and reasonable results were presented. It was found that as the driving range limit becomes shorter and the market penetration rate of BEVs increases, flow shifts on a transportation network become rather significant and complex. This model was specialized to analyze the impact of BEVs with the given parking facilities. The recharging of BEVs is assumed only to appear at the final destinations, at which point the special garages exist. Therefore, the model was not able to describe detouring route choices made by drivers of BEVs with low battery states when they must stop at an intermediate rapid charging station not at the final destination.

He et al. (2014) developed a user-equilibrium-based model which included the en-route recharging of BEVs. Unlike other relevant studies which mainly addressed the location problems of charging infrastructure, this research exclusively concentrated on equilibrium models based on the costs of the driving time, the recharging time and range anxiety. Several models were proposed according to whether or not the flow-dependency of battery consumption was reflected in an effort to account for the amount of recharging as a decision variable and to include the cost of range anxiety. The most advanced model in their study is takes into account the costs of range anxiety, flow-dependent battery consumption, en-route battery recharging and the various recharging performances of charging stations. The concept of a sub-path was adopted to evaluate whether a BEV can traverse a certain section of a path with a given battery state immediately before entering the section. The section of the path known as the sub-path is defined as a partial path that starts and ends at a charging station, an origin, or a destination. Evaluating the availability of a sub-path is done by comparing the required battery amount for traversing the sub-path and the battery state of the BEV just before entering it. At equilibrium, if the required amount of battery power for a sub-path is greater than the given battery state, flows are not able to be assigned to the entire path.

In the model, at equilibrium, if the path used includes a charge-depleting sub-path where the required battery consumption of the sub-path is identical to the given battery state of the BEV, the path's summation of the driving time cost and the recharging time cost may be less than or equal to the cost of the shortest path, which is composed of the costs of the driving time, the recharging time and range anxiety. Because the state of equilibrium always guarantees an identical cost of all used paths within the same O-D, the authors assumed that the cost gap between a path with a charge-depleting sub-path and the shortest path can be interpreted as the cost of range anxiety. The authors insisted that this approach does not have a behavioral basis, and comparing the magnitudes of the range anxiety costs between different charge-depleting paths is not appropriate. Although the range anxiety cost does not reflect behavioral aspects, the model was applied to analyze several numerical examples with various initial battery states which were organized in a discrete form, with multiple allowable minimum battery states of BEV drivers. It was found through this analyses that BEV drivers preferred high-power charging stations regardless of the initial battery state of their BEV. Moreover, a lower level of the allowable minimum battery state induced relatively high levels of the recharging frequency, amount and time.

Lee et al. (2013) developed a bi-level user-equilibrium-based location-problem which considered rapid charging stations for BEVs. The upper-level problem was to determine the locations of rapid charging stations, while the lower-level problem was macroscopically to simulate the traffic flows of BEV with the given candidate locations of the initial problem. The suggested model used a stochastic distribution of the initial battery states, differing from the discrete distribution in the model by He et al. (2014). This concept originated from the difficulties in capturing the initial battery state of each BEV in the real world. Because the stochastic initial battery state may not be sufficient to ensure arrival at the nearest charging station from an origin node, the concept of trip

failure was applied. In the model, each BEV driver minimized their detouring trip distance and avoided a trip failure. With various distributional forms of the initial battery states, small, medium and large networks were analyzed. Through these analyses, more realistic levels of trip failure were calculated compared to the existing charging station location problems.

2.2. Dynamic User Equilibrium Models

DTA models can be categorized into reactive assignment models and predictive assignment models. In a time-variant dynamic transportation system, reactive assignment models assume that drivers choose their shortest paths based on the present instantaneous traffic condition, meaning that drivers starting at a certain moment choose their paths according to the traffic pattern at that moment. Therefore, reactive traffic assignment models do not guarantee identically experienced travel costs for drivers who started their trips from the same origin to the same destination at the same time. On the other hand, predictive assignment models pursue identically experienced path travel costs, matching Wardrop's first principle. At a state of traffic equilibrium according to predictive models, the equilibrium state is referred to as the state of DUE. There are several analytical methods which represent DTA problem, including mathematical programming formulations, optimal control formulations and variational inequality formulations. Among them, variational inequality formulations are the most widely used approach due to their levels of analytical flexibility and convenience when used to address various DTA problems (Peeta et al., 2001). This subchapter introduces the basic concept of variational inequality in DUE and relevant studies of this subject.

A route-based DUE state with discrete time according to Huang and Lam (2002) can be represented as follows:

$$\sum_{k \in K} \sum_{r \in R} \sum_{s \in S} \sum_{p \in P_{rs}} c_p^{rs}(k, \mathbf{f}^*) [f_p^{rs}(k) - f_p^{rs*}(k)] \gg 0 \quad (2.2)$$

Where,

- \mathbf{f}^* = a vector composed of DUE path flows in accordance with time intervals
- $c_p^{rs}(k, \mathbf{f}^*)$ = the unit travel cost (on an average) incurred by commuters departing from r and selecting path p to s during time interval k in a DUE state
- $f_p^{rs}(k)$ = the flow rate of path p that enters the network from origin r to destination s during time interval k
- $f_p^{rs*}(k)$ = the flow rate of path p that enters the network from origin r to destination s during time interval k in a DUE state

Traffic flows in a DUE state are achieved if and only if the requirements of equation (2.2) are met. This equation was formulated by considering an inevitable feature of traffic flows in a DUE state. For example, when traffic flows enter the DUE state, all of the assigned paths within an O-D have identical costs, which are equal and minimal. Additionally, however, all of the unassigned paths within the O-D, where the paths have no traffic flow, have costs that are equal to or greater than the cost of the assigned paths. This relationship between the assigned/unassigned flows and the minimal/non-minimal path matches the variational inequality formulation of equation (2.2) precisely. Details of the process of its induction are covered in Huang and Lam (2002).

Friesz et al. (1993) developed an early form of a model which considered continuous time and simultaneous route choices and departure time decisions at equilibrium with an infinite dimensional variational inequality form. Because this model was intended for simultaneous choices, the path cost was composed of the link travel time as derived by VDF and the penalty cost for early or late arrivals. The penalty cost was generated using the gap between the actual arrival

time and the desired arrival time. Because the model is represented in a continuous time form, computational tractability was relatively burdensome. Furthermore, the uniqueness of the solution was not addressed in that study.

Wie et al. (1995) developed a discrete time model of simultaneous route choice and a departure-time decision equilibrium model to decrease the computational load when searching for a solution. Their model is an advanced form of a path-based variational inequality model of Friesz et al. (1993). Given the nature of path-based problems, though the discrete formulation had lightened the computational burden, considerable path generations between an O-D did not completely solve the tractability problem. In detail, a credible result of a path-based DTA problem requires a path-enumeration process prior to a traffic-assignment process. Nevertheless, a discrete-time DUE was shown to exist under certain regularity conditions.

Ran et al. (1996) suggested a link-based discrete-time variational inequality model for the simultaneous route-choice and departure-time decision problems. The proposed model avoided the burdensome path enumeration during its formulation and solution procedure. Through this simplification, the proposed model was able to consider large, dynamic transportation networks. Although the link-based model has the advantage of computational efficiency, it has a drawback because it loses the path flow information entirely. When considering the problem of incorporating BEVs into a DUE model, the path flow information is crucial because the path flow information can be used to calculate the remaining battery life at a certain time and location along a certain path.

Huang and Lam (2002) suggested an efficient path-based search algorithm and a simultaneous choice model with queues. Their model included a zero-extreme value minimization condition stemming from the variational inequality formulation. When the condition is satisfied by minimizing the objective value to zero, the DUE requirement of a given problem is also met. The zero-extreme

value minimization condition is as follows:

$$W(\mathbf{f}) = \sum_{k \in K} \sum_{r \in R} \sum_{s \in S} \sum_{p \in P_{rs}} f_p^{rs}(k) [c_p^{rs}(k, \mathbf{f}) - c_{\min}^{rs}(\mathbf{f})] \quad (2.3)$$

Where,

- \mathbf{f} = a vector composed of path flows in accordance with time intervals
- $W(\mathbf{f})$ = the function to be minimized toward zero-extreme value
- $f_p^{rs}(k)$ = the flow rate of path p that enters the network from origin r to destination s during time interval k
- $c_p^{rs}(k, \mathbf{f})$ = the unit travel cost (on an average) incurred by commuters departing from r and selecting path p to s during time interval k
- $c_{\min}^{rs}(\mathbf{f})$ = the minimal path travel cost from origin r to destination s

The zero-extreme value minimization condition transformed the existing variational inequality formulation into a conventional minimization problem, whereas existing approaches differed from the general minimization/maximization problem or were far from intuitive. To minimize the condition to zero, an iterative algorithm was adopted. During each iteration of the algorithm, within the same O-D, an evaluation of the minimal cost paths is conducted. Subsequently, the flows of other non-shortest cost paths of an O-D pair are swapped into the shortest paths in proportion to the gap between the cost of the non-shortest path and that of the shortest path. The iterative procedure is repeated until a predefined stopping criterion is met.

2.3. Review result

In subchapter 2.1, static equilibrium models and FRLM with alternative fuel vehicles were reviewed. Early static models did not use the equilibrium principle and assigned traffic flows on exogenously defined shortest paths. Later then, static equilibrium models of BEVs were developed with

endogenously assigned traffic flows on transportation networks. In subchapter 2.2, DUE models with general vehicles were reviewed. To enhance computational efficiency, it was shown that discrete time models were mainly developed. Also, based on discrete time models, the models were further divided into link-based and path-based models. Although path-based models have disadvantages on computational efficiency, their advantages on representing vehicles' trajectories have made consistent researches on it.

Based on the precedent study, to describe realistic traffic flows of BEVs, an equilibrium principle based model is developed in this research. In the developed model, traffic flows are assigned to minimize their travel time and out-of-range penalty cost simultaneously in a dynamic transportation system. The static assignment models of BEVs were previously researched from He et al. (2014). This research further transforms the static models to a dynamic assignment problem.

In the developed dynamic assignment problem with BEVs, the path-based approach is applied. It is because that link-based model is unable to represent trajectories of vehicles, and battery state which is defined by traversed trajectory cannot be calculated accordingly. Therefore, the proposed model has its objective function that is modified from the path-based problem of Huang and Lam (2002).

Chapter 3. Model Development

3.1. Problem Description

This research proposes a DUE model with BEVs that considers the en-route recharging behaviors of BEV drivers. Specifically, the model is developed as a trip-based and path-based model with discretized time. As mentioned earlier, unlike GV drivers, BEV drivers encounter what has been termed range anxiety during their trips. Range anxiety causes a BEV driver to maintain a battery state which exceeds a certain critical amount so as to avoid being trapped in the middle of a path. The proposed model reflects this route-choice behavior in that BEVs are assigned to the shortest paths that can also be used in consideration of the issue of battery consumption.

Several assumptions were established to consider the suggested route-choice behavior of BEV drivers in the DUE model. These assumptions can be classified as assumptions related to the drivers and those related to the battery of the BEV.

Regarding the assumptions of BEV drivers, the model assumes that users of an analyzed transportation network have perfect information about traffic conditions of the future. Moreover, users have full information about en-route battery conditions and the characteristics of charging. Based on these two assumptions, it was assumed that BEV drivers simultaneously determine their paths and recharging plans to minimize their travel time, which is composed of the driving time and the recharging time.

With reference to the assumptions about the battery of the BEV, it was assumed that battery power consumption by the BEV is explicitly affected by its travel distance. Additionally, because battery exhaustion during a trip rarely occurs, the initial battery state of each BEV is assumed to be sufficient for arriving at the nearest charging station.

Based on the proposed route-choice behavior of BEV drivers and the assumptions about the problem, mathematical formulation of the model is discussed in the next subchapter.

3.2. Mathematical Formulation

3.2.1 Parameters and Variables

The parameters and variables used in the proposed DUE model with BEVs can be arranged as follows:

<Table 1> Parameters and variables

Notation	Explanation
a	An index denoting a link
$A(j)$	A set of links having node j as a tail node (after j)
$b_{j,p}^{rs}(k)$	The battery charging amount at node j for BEVs departed from r to s by selecting path p
$B(j)$	A set of links having node j as a head node (before j)
$\bar{B}_p(n(p_q))$	A set of sub-paths on path p before a station $n(p_q)$
$\bar{B}_p(n(p_q))$	A set of charging stations on path p before a station $n(p_q)$ (including $n(p_q)$)
$c_p^{rs}(k)$	Experienced travel time of drivers departed from r to s by selecting path p during time interval k
$c_{\min}^{rs}(k)$	Experienced shortest travel time of drivers departed from r to s during time interval k
$\tilde{c}_p^{rs}(k)$	Experienced travel cost of drivers departed from r to s by selecting path p during time interval k
$\tilde{c}_{\min}^{rs}(k)$	Experienced travel cost of drivers departed from r to s by selecting path p during time interval k
e_{p_q}	The Required battery amount for traversing a sub-path p_q
$f^{rs}(k)$	The flow rate from r to s during time interval k

f_p^{rs}	The flow rate of a path p from r to s during time interval k
$\gamma_{p_q}^{rs}(k)$	A logical value that returns 1 if BEVs departed from r to s by selecting a path p during time interval k can traverse sub-path p_q with given battery state, otherwise 0
h	The comfortable minimal battery state
$I_{j,p}^{rs}(k)$	The number of vehicles started to charge at a node j during time interval k departed from r to s by selecting a path p
k	An index denoting a time interval
K	The total number of time intervals
$l_{j,p}^{rs}(k)$	The battery charging time at a node j for BEVs that departed from r to s by selecting path p during time interval k
$\tilde{l}_{j,p}^{rs}(k)$	The discretized time intervals of $l_{j,p}^{rs}(k)$
$L_{n(p_q),p}^{rs}(k)$	The battery state after recharging at a station $n(p_q)$ for BEVs that departed from r to s by selecting a path p during time interval k
L_{max}	The battery capacity
$n(p_q)$	An index denoting a tail node of sub-path p_q
$O_{j,p}^{rs}(k)$	The number of vehicles finished charging at a node j during time interval k departed from r to s by selecting path p
p	An index denoting a path
p_q	An index denoting a sub-path
r	An index denoting an origin
s	An index denoting a destination
$t_a(k)$	The travel time of link a at a time interval k
$\tilde{t}_a(k)$	The discretized time intervals of $t_a(k)$
$u_{a,p}^{rs}(k)$	The inflow rate of link a during time interval k departed from r to s by selecting path p
$v_{a,p}^{rs}(k)$	The exit flow rate of link a during time interval k departed from r to s by selecting path p
$x_{a,p}^{rs}(k)$	The number of vehicles on link a during time interval k departed from r to s by selecting path p
*	A superscript denoting parameters and variables of DUE

3.2.2 Objective Function

As noted above, Huang and Lam (2002) formulated a discrete time path-based DUE condition into a minimization problem with equation (2.3). For general GV's, this formulation can be used for a proper objective function. However, the DUE condition incorporating BEVs must be modified to account for the range limit problem. Therefore, in addition to the representation by Huang and Lam (2002), a similar concept stemming from the path-constrained problem of Jiang et al. (2012) was applied to the present DUE model.

Assuming that all the BEVs are in a DUE with a given network, it can be said that the BEVs always maintain retain more than the minimum required battery life before entering each sub-path. Specifically, at a DUE with BEVs, at the starting node of each sub-path along an assigned path, the remaining battery power after recharging at the starting node of the sub-path is always equal to or higher than the required amount of battery life for the upcoming sub-path. This concept can also be applied to a case without recharging. Unless this condition is satisfied for all inflows of the paths, flows that are out of range with the given battery state necessarily occur. This range limit condition can be formulated as follows:

$$\sum_{k,r,s,p,q} f_p^{rs*}(k) \gamma_{pq}^{rs*}(k) [e_{pq} - L_{n(pq),p}^{rs*}(k) + h] = 0 \quad (3.1)$$

Where,

$$\gamma_{pq}^{rs*}(k) = \begin{cases} 1 & \text{if } e_{pq} - L_{n(pq),p}^{rs*}(k) + h > 0 \\ 0 & \text{else} \end{cases}$$

The aforementioned range limit condition can further be applied to evaluate whether an auxiliary assigned traffic flow \mathbf{f} and battery charging amount \mathbf{b} allow a BEV to be driven within its range limit. If the result of the equation below exceeds zero, it can be said that for some BEVs, the range limit condition

is not met (3.1).

$$R(\mathbf{f}, \mathbf{b}) = M \sum_{k,r,s,p,q} f_p^{rs}(k) \gamma_{p_q}^{rs}(k) [e_{p_q} - L_{n(p_q),p}^{rs}(k) + h] \quad (3.2)$$

Where,

$$\gamma_{p_q}^{rs}(k) = \begin{cases} 1 & \text{if } e_{p_q} - L_{n(p_q),p}^{rs}(k) + h > 0 \\ 0 & \text{else} \end{cases}$$

Specifically, if the range limit condition is not met, equation (3.2) returns a summation of each assigned path's out-of-range penalty as calculated by multiplying a significantly large constant M , the amount of the out-of-range flows and an additionally the necessary battery life to traverse the path. Therefore, it can be said that the greater the out-of-range penalty cost is, the further the assigned flows are from the range limit condition. By applying equation (3.2) to the zero-extreme value minimization condition of Huang and Lam (2002), in the present model, the cost of each path is modified further to account for both the travel time and the out-of-range penalty. To minimize the out-of-range penalty cost to zero during the solution search process, a significantly large constant M is multiplied by the penalty term of each path. The cost composition of each path is given below.

$$\tilde{c}_p^{rs}(k) = c_p^{rs}(k) + M \sum_q f_p^{rs}(k) \gamma_{p_q}^{rs}(k) [e_{p_q} - L_{n(p_q),p}^{rs}(k) + h] \quad (3.3)$$

By adopting the above path cost concept, an objective function modified from the zero-extreme value minimization problem results, as follows:

$$\min \sum_{k,r,s,p} f_p^{rs}(k) [\tilde{c}_p^{rs}(k) - \tilde{c}_{\min}^{rs}(k)] \quad (3.4)$$

If the value of the objective function (3.4) as calculated by an assigned traffic

flow pattern equals zero, a DUE satisfying the range limit condition is established. At a DUE, all of the assigned path flows within an O-D pair have an identical and minimal travel time assuming that all of the initial battery life levels are uniform. This indicates that if the various initial battery states at the beginning of a trip are considered as discrete groups, the travel times along the paths within the same group and with the same O-D pair are also equal and minimal, as in the case with a single initial battery state.

3.2.3 Constraints

Constraints mainly referring to flow conservation, flow propagation, and the en-route battery state are proposed later. In the present model, several constraints are modified to account for the arrival and departure of BEV driver flows at battery charging stations.

First, a relationship between the number of vehicles and the inflow and outflow of vehicles on a link is presented, as follows:

$$x_{a,p}^{rs}(k+1) = x_{a,p}^{rs}(k) + \mu_{a,p}^{rs}(k) - \nu_{a,p}^{rs}(k) \quad \forall r, s, p, a; k = 1, \dots, K; \quad (3.5)$$

In the present model, the state variable $x_{a,p}^{rs}(k+1)$ indicates the number of vehicles on link a of path p which connects origin r and destination s at the time interval of $k+1$. This variable can be calculated by adding the inflow vehicles and subtracting the outflow vehicles from the number of existing vehicles on the same corresponding link at the prior time interval k .

Secondly, flow conservation constraints are constraints that maintain spatial connections between successive links and nodes. For path inflows starting at an origin node, the amount of this flow is equal to the inflows of the very first link of the assigned path. The constraint reflecting this notion is given as follows:

$$f^{rs}(k) = \sum_{a \in A(r), p} u_{a,p}^{rs}(k) \quad \forall r, s, p; r \neq s; k = 1, \dots, K; \quad (3.6)$$

In equation (3.6) above, $A(r)$ indicates a set of links having node r as a starting node. Therefore, if there is some level of demand between origin r and destination s at a time interval k , the summation of inflows of the corresponding links, within the available paths of an O-D pair rs , must be equal to the demand of the O-D pair at the same time interval.

In addition, there is an additional flow conservation constraint that guarantees spatial continuity within the intermediate links on a path. This constraint retains flows such that they move along a given path. It is formulated as follows:

$$u_{A(j),p}^{rs}(k) = v_{B(j),p}^{rs}(k) - I_{j,p}^{rs}(k) + O_{j,p}^{rs}(k) \quad \forall r, s, p, j; j \neq r, s; k = 1, \dots, K; \quad (3.7)$$

Where,

$$I_{j,p}^{rs}(k) = O_{j,p}^{rs}(k + \tilde{I}_{j,p}^{rs}(k)) \quad \text{if } j \in \text{stations}$$

$$I_{j,p}^{rs}(k) = O_{j,p}^{rs}(k) = 0 \quad \text{else}$$

The basic notion with regard to equation (3.7) is that, at an identical time interval, the outflows of the upper link within two successive links are always equal to the inflows of the lower link. This concept can always be valid for a general DUE model of GVs. However, considering BEVs and charging stations, if BEVs are charged at an intermediate charging station, the recharged BEVs will experience time-lagged flows on the basis of the charging station. In detail, if the outflows of BEVs from the upper link at time interval k charge for ten time intervals, these flows will increase the inflows of the lower link at time interval $k + 10$. Equation (3.7) reflects the aforementioned concept by distinguishing whether the intermediate node is a charging station or not.

Thirdly, the flow propagation constraint is a condition that ensures temporal continuity of traffic flows. The constraint maintains traffic flows to consistently propagate within an analyzed period according to their actual experienced travel time. Static assignment models do not cope with this constraint that the models assume traffic flows to propagate in a moment. In the presenting model, a link based and actual experienced travel time based flow propagation constraint is applied.

$$u_{a,p}^{rs}(k) = v_{a,p}^{rs}[k + \tilde{t}_a(k)] \quad \forall r, s, p, a; k = 1, \dots, K; \quad (3.8)$$

The flow propagation constraint applied in this model does not include characteristics of vehicle platoons' concentration and dispersion. Therefore, within a link a , inflow of the link at time interval k is always equal to outflow of it at time interval $k + \tilde{t}_a(k)$ where $\tilde{t}_a(k)$ indicates the link's discretized actual travel time for inflow vehicles at time interval k . Accordingly, inflow vehicles at time interval k is always identical to the outflow vehicles at time interval $k + \tilde{t}_a(k)$. This relationship can be represented by the equation (3.8). For more details of other types of the flow propagation constraint, Ran et al. (1996) elaborated several types of them.

Fourthly, to calculate a value of the out-of-range penalty, $L_{n(p_q),p}^{rs}(k)$ indicating a battery state after recharging at a station $n(p_q)$ has to be calculated. After then, a battery shortage for completing a trip on an upcoming sub-path can be calculated accordingly. Therefore, a definitional constraint representing $L_{n(p_q),p}^{rs}(k)$ can be formulated as follows:

$$L_{n(p_q),p}^{rs}(k) = L_{r,p}^{rs}(k) - \sum_{q \in \tilde{B}_p(n(p_q))} e_{p_q} + \sum_{i \in \tilde{B}_p(n(p_q))} b_{i,p}^{rs}(k) \quad \forall r, s, p, j; k = 1, \dots, K; \quad (3.9)$$

The basic concept of the definitional constraint (3.9) is rather simple. A battery state after recharging at a charging station $n(p_q)$, $L_{n(p_q),p}^{rs}(k)$, is calculated by

subtracting a used battery amount from a summation of an initial battery amount and a total recharged battery amount before departing the station j . The used battery amount is explicitly calculated by a travel distance. In the presenting model, the relationship between the used battery amount and the traveled distance is assumed to have a linear interrelation where the used battery amount can be calculated by multiplying a traveled distance and a constant indicating a required battery amount for a unit travel distance. Furthermore, recharging time at a station j within a path p is also assumed to have a linear relationship from a recharging amount as like He et al. (2014) did.

$$l_{j,p}^{rs}(k) = w_{j,1} + w_{j,2}(b_{j,p}^{rs}(k)) \quad (3.10)$$

The equation (3.10) shows the liner relationship between the recharging battery amount and the recharging time. The first term of the right-hand side, $w_{j,1}$, denotes a fixed time for recharging a BEV at a station j . A time for preparing procedure before a battery recharging can be included on it. The second term, $w_{j,2}$, refers to a pure battery recharging time which is proportional to a recharging battery amount.

Fifthly, a battery state of a BEV has to be remain equal or less than its maximum capacity. Therefore, it can be represented in the model as follows:

$$L_{j,p}^{rs}(k) \leq L_{max} \quad \forall r, s, p, j; k = 1, \dots, K; \quad (3.11)$$

Sixthly, non-negativity constraints and boundary constraints have to be satisfied.

$$x_{a,p}^{rs}(k) > 0, \quad \mu_{a,p}^{rs}(k) > 0, \quad v_{a,p}^{rs}(k) > 0 \quad \forall r, s, p, a; k = 1, \dots, K; \quad (3.12)$$

$$L_{j,p}^{rs}(k) > 0, \quad b_{j,p}^{rs}(k) > 0 \quad \forall r, s, p, j; k = 1, \dots, K; \quad (3.13)$$

Unless assuming background traffic, below constraint has to be always

guaranteed.

$$x_{a,p}^{rs}(0) = 0 \quad \forall r, s, p, a; k = 1, \dots, K; \quad (3.14)$$

Lastly, each link's VDF has to include First-In-First-Out (FIFO) condition of the below constraint:

$$\frac{t_a(t + \Delta t) - t_a(t)}{\Delta t} > -1 \quad \forall a; \quad (3.15)$$

3.2.4 Summary

The suggested model's mathematical formulation can be summarized as follows:

$$\min \sum_{k,r,s,p} f_p^{rs}(k) [\tilde{c}_p^{rs}(k) - \tilde{c}_{\min}^{rs}(k)] \quad (3.16)$$

subject to

$$x_{a,p}^{rs}(k+1) = x_{a,p}^{rs}(k) + \mu_{a,p}^{rs}(k) - \nu_{a,p}^{rs}(k) \quad \forall r, s, p, a; k = 1, \dots, K; \quad (3.17)$$

$$f^{rs}(k) = \sum_{a \in A(r), p} u_{a,p}^{rs}(k) \quad \forall r, s, p; r \neq s; k = 1, \dots, K; \quad (3.18)$$

$$u_{A(j),p}^{rs}(k) = \nu_{B(j),p}^{rs}(k) - I_{j,p}^{rs}(k) + O_{j,p}^{rs}(k) \quad \forall r, s, p, j; j \neq r, s; k = 1, \dots, K; \quad (3.19)$$

$$u_{a,p}^{rs}(k) = \nu_{a,p}^{rs} [k + \tilde{t}_a(k)] \quad \forall r, s, p, a; k = 1, \dots, K; \quad (3.20)$$

$$L_{n(p_q),p}^{rs}(k) = L_{r,p}^{rs}(k) - \sum_{q \in \tilde{B}_p(n(p_q))} e_{p_q} + \sum_{i \in \tilde{B}_p(n(p_q))} b_{i,p}^{rs}(k) \quad \forall r, s, p, j; k = 1, \dots, K; \quad (3.21)$$

$$L_{j,p}^{rs}(k) \leq L_{max} \quad \forall r, s, p, j; k = 1, \dots, K; \quad (3.22)$$

$$x_{a,p}^{rs}(k) > 0, \quad \mu_{a,p}^{rs}(k) > 0, \quad \nu_{a,p}^{rs}(k) > 0 \quad \forall r, s, p, a; k = 1, \dots, K; \quad (3.23)$$

$$L_{j,p}^{rs}(k) > 0, \quad b_{j,p}^{rs}(k) > 0 \quad \forall r, s, p, j; k = 1, \dots, K; \quad (3.24)$$

$$x_{a,p}^{rs}(0) = 0 \quad \forall r, s, p, a; k = 1, \dots, K; \quad (3.25)$$

Furthermore, to reflect BEVs' various initial battery states, the above model can further be expanded to a multi-class model that assuming each class has a different initial battery state from the others. In the below model, m denotes the classes having multiple initial battery states.

$$\min \sum_{k,m,r,s,p} f_p^{rs,m}(k) [\tilde{c}_p^{rs,m}(k) - \tilde{c}_{\min}^{rs,m}(k)] \quad (3.26)$$

subject to

$$x_{a,p}^{rs,m}(k+1) = x_{a,p}^{rs,m}(k) + \mu_{a,p}^{rs,m}(k) - \nu_{a,p}^{rs,m}(k) \quad \forall r, s, m, p, a; k = 1, \dots, K; \quad (3.27)$$

$$f^{rs,m}(k) = \sum_{a \in A(r), m, p} u_{a,p}^{rs,m}(k) \quad \forall r, s, m, p; r \neq s; k = 1, \dots, K; \quad (3.28)$$

$$u_{A(j),p}^{rs,m}(k) = v_{B(j),p}^{rs,m}(k) - I_{j,p}^{rs,m}(k) + O_{j,p}^{rs,m}(k) \quad \forall r, s, m, p, j; j \neq r, s; k = 1, \dots, K; \quad (3.29)$$

$$u_{a,p}^{rs,m}(k) = v_{a,p}^{rs,m}[k + \tilde{t}_a(k)] \quad \forall r, s, m, p, a; k = 1, \dots, K; \quad (3.30)$$

$$L_{n(p_q),p}^{rs,m}(k) = L_{r,p}^{rs,m}(k) - \sum_{q \in \tilde{B}_p(n(p_q))} e_{p_q} + \sum_{i \in \tilde{B}_p(n(p_q))} b_{i,p}^{rs,m}(k) \quad \forall r, s, m, p, j; k = 1, \dots, K; \quad (3.31)$$

$$L_{j,p}^{rs,m}(k) \leq L_{max} \quad \forall r, s, m, p, j; k = 1, \dots, K; \quad (3.32)$$

$$x_{a,p}^{rs,m}(k) > 0, \quad \mu_{a,p}^{rs,m}(k) > 0, \quad \nu_{a,p}^{rs,m}(k) > 0 \quad \forall r, s, m, p, a; k = 1, \dots, K; \quad (3.33)$$

$$L_{j,p}^{rs,m}(k) > 0, \quad b_{j,p}^{rs,m}(k) > 0 \quad \forall r, s, m, p, j; k = 1, \dots, K; \quad (3.34)$$

$$x_{a,p}^{rs,m}(0) = 0 \quad \forall r, s, m, p, a; k = 1, \dots, K; \quad (3.35)$$

Chapter 4. Algorithm

4.1. Flow Swapping algorithm

Several algorithms related to the dynamic route choice problems were developed based on iterative approaches (Smith, 1993; Tong and Wong, 2000; Huang and Lam, 2002). These algorithms have a common feature that when an auxiliary path flows are fixed, paths which are having longer travel time is partially moved to the shortest travel time paths. This flow swapping mechanism is repeatedly conducted at each iteration until a certain stopping condition is satisfied.

The iterative procedure of the algorithm of Huang and Lam (2002) was developed to solve the zero-extreme minimization problem of the equation (2.3). To make the equation (2.3) to be zero which is equivalent to the DUE, at each iteration, path flows which are having higher travel time than the shortest paths within an O-D pair are moved to the shortest paths of the same O-D pair. The moved amounts of path flows are proportional to a magnitude of the moved path's flow, and a gap between the moved path's travel time and the shortest paths' travel time. Below represents this mechanism.

$$f_p^{rs}(k)_{\tau+1} = f_p^{rs}(k)_{\tau} - \rho_{\tau} f_p^{rs}(k)_{\tau} [c_p^{rs}(k, \mathbf{f})_{\tau} - c_{\min}^{rs}(\mathbf{f})_{\tau}] \quad (4.1)$$

Where,

- \mathbf{f} = a vector composed of path flows in accordance with time intervals
- ρ_{τ} = a scaling parameter at iteration τ
- $f_p^{rs}(k)_{\tau}$ = the flow rate of path p that enters the network from origin r to destination s during time interval k at iteration τ
- $c_p^{rs}(k, \mathbf{f})_{\tau}$ = the unit travel cost (on an average) incurred by commuters departing from r and selecting path p to s during time interval k at iteration τ

$c_{\min}^{rs}(\mathbf{f})_{\tau}$ = the minimal path travel cost from origin r to destination s at iteration τ

After then, the total moved flows within an O-D pair are divided into the number of the shortest paths. The divided flows are then added to each shortest path of the same O-D pair. This step can be represented as follows:

$$f_p^{rs}(k)_{\tau+1} = f_p^{rs}(k)_{\tau} + \frac{\psi_{\tau}}{|\hat{P}_{\tau}|} \quad (4.2)$$

Where,

ψ_{τ} = a summation of moved path inflows at iteration τ
 \hat{P}_{τ} = a set of shortest paths at iteration τ

This process of swapping path flows is repeated until the below condition is satisfied where π denotes a satisfactory critical value.

$$\frac{\sum_{k,r,s,p} f_p^{rs}(k)_{\tau} [c_p^{rs}(k, \mathbf{f})_{\tau} - c_{\min}^{rs}(\mathbf{f})_{\tau}]}{\sum_{k,r,s,p} f_p^{rs}(k)_{\tau} c_{\min}^{rs}(\mathbf{f})_{\tau}} < \pi \quad (4.3)$$

4.2. Modified algorithm

Since a value of the presented model's objective function has to be decreased to zero like as the zero-extreme minimization problem from Huang and Lam (2002), a modified algorithm further including a decision process of battery charging amount is developed.

In the modified flow swapping algorithm, travel time of each path can only be calculated after deciding a battery charging amount. It is because when the charging amount is fixed, charging time can be computed according to the linear relationship of the equation (3.9). After then, for each path, a summation of a driving time and the battery charging time can be used as a travel time.

Therefore, in the modified algorithm, a process of deciding battery charging amount is newly added prior to the calculation of each path's actual travel time.

In the newly added process, since the proposed model is a trip-based model and does not include multiple en-route recharging, charging amount is equal to a battery shortage for completing each trip. Charging amount at a station $n(p_q)$ for BEVs on a path p with a comfortable battery of h is calculated as follows:

$$b_{n(p_q),p}^{rs}(k) = e_{p_q} + h - \left[L_{r,p}^{rs}(k) - \sum_{p_q \in \bar{B}(n(p_q))} e_{p_q} + \sum_{i \in \bar{B}(n(p_q))} b_{i,p}^{rs}(k) \right] \quad (4.4)$$

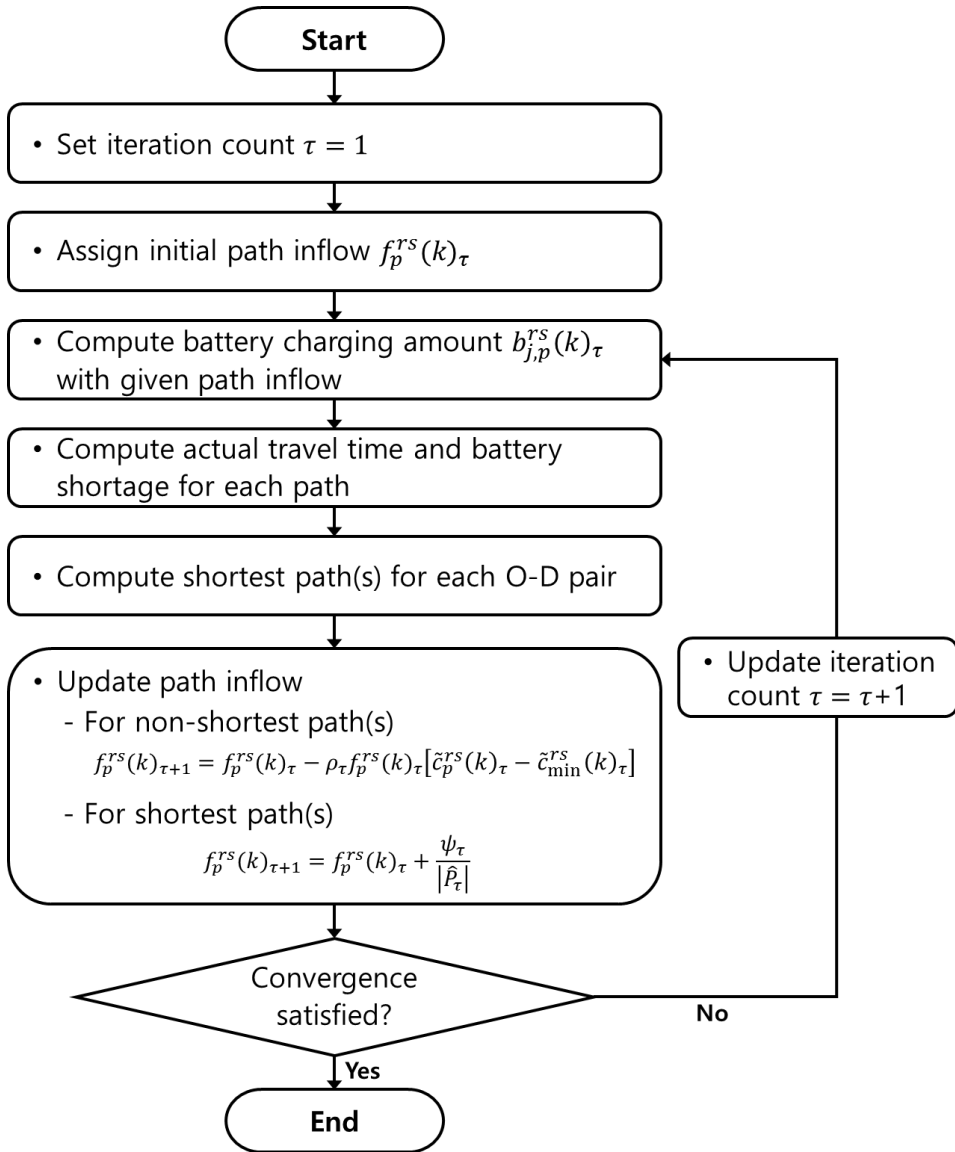
If a battery amount after charging exceeds its capacity, charging amount is limited as follows:

$$b_{n(p_q),p}^{rs}(k) = L_{max} - \left[L_{r,p}^{rs}(k) - \sum_{p_q \in \bar{B}(n(p_q))} e_{p_q} + \sum_{i \in \bar{B}(n(p_q))} b_{i,p}^{rs}(k) \right] \quad (4.5)$$

Since charging more than minimal required battery for completing a trip increases actual travel time, this phenomenon does not accord with the Wardrop's first principle. To further enhance realistic behavioral aspects, development of tour-based model is needed for the future research.

After the calculation of charging amount and travel time of an auxiliary traffic flow, a penalty cost of a path p departed at a time interval k from an origin r to destination s decides an amount of path flows to be swapped as like the equation (4.1) and (4.2). The cost gap of the equation (4.1) encompasses a degree of being apart from the shortest travel time within an O-D pair and a degree of unavailability in a view point of energy consumption. And, the modified algorithm iteratively minimizes a summation of the cost gap into zero until a termination condition of equation (4.3) is satisfied.

A flowchart of the modified algorithm having a revised path cost concept and modified condition (4.3) according to the revised cost concept is as follows:



<Figure 2> Modified flow swapping algorithm in flowchart

Besides the newly added decision process on charging amount, it is because the suggested model is a discrete time based model, an analyzed time period is divided into smaller time intervals which are having equal time unit. Accordingly, in the solution search process, a variable of continuous time form

such as link travel time $t_a(k)$ is transformed to a discrete time variable by following a condition below:

$$\tilde{t}_a(k) = i \quad \text{if} \quad i - 0.5 \leq t_a(k) \leq i + 0.5 \quad (4.6)$$

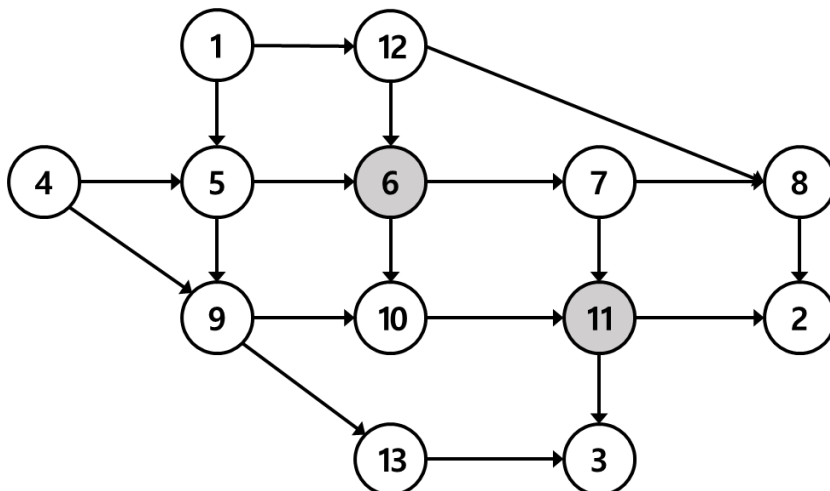
An integer i of the condition (4.6) indicates a time interval within an analyzed period and $\tilde{t}_a(k)$ denotes a discretized variable from its original continuous variable $t_a(k)$. This condition is also adopted to convert the continuous variable of charging time $l_{j,p}^{rs}(k)$ to $\tilde{l}_{j,p}^{rs}(k)$. To increase accuracy on the proposed model, making the time intervals be smaller is recommended.

Chapter 5. Numerical Example

A small size network of the Nguyen-Dupuis network is analyzed according to the proposed model and algorithm. The example network is referred from He et al. (2014). Assignment results, convergence of the proposed algorithm, and implications are discussed in this chapter.

5.1. Network properties

The Nguyen-Dupuis network with two fast charging stations are analyzed with BEVs having 24 kWh capacity of the Nissan Leaf. Each time interval is assumed to be 20 seconds on the entire analysis. In the example network, charging stations having their performance as 1.2 min/kW with fixed time of 5 minutes are located in node 6 and node 11. Demands of BEVs having 6 kWh of the initial battery state are assumed to exist on four O-D pairs. The network's geometrical figure, overall properties on the analysis, and link attributes for VDF are as follows:



<Figure 3> Nguyen-Dupuis Network

<Table 2> Overall properties of Nguyen-Dupuis network example

Division	Property
Network	
Number of nodes	13 nodes
Number of links	19 links
Average link distance	12.63 km
Total distance	240 km
Longest path distance	58 km
Charging stations	2 stations (node 6, node 11)
Charging performance	1.2 min/kW
O-D demand	
1 to 2	10,000vph for initial 400 sec
1 to 3	10,000vph for initial 400 sec
4 to 2	10,000vph for initial 400 sec
4 to 3	10,000vph for initial 400 sec
Battery	
Battery capacity	24 kWh
Initial battery	9 kWh
Fuel economy	0.18 kWh/km
Comfortable battery	2 kWh

In the analyzed network, VDF is formulated based on speed-density relationship and was used in Jayakrishanan et al. (1995) and Ran et al. (2002).

$$\begin{aligned}
 \text{if } d < d_j, \quad v &= v_{\min} + (v_{\max} - v_{\min}) \left(1 - \frac{d}{d_j}\right) \\
 \text{otherwise,} \quad v &= v_{\min}
 \end{aligned} \tag{5.1}$$

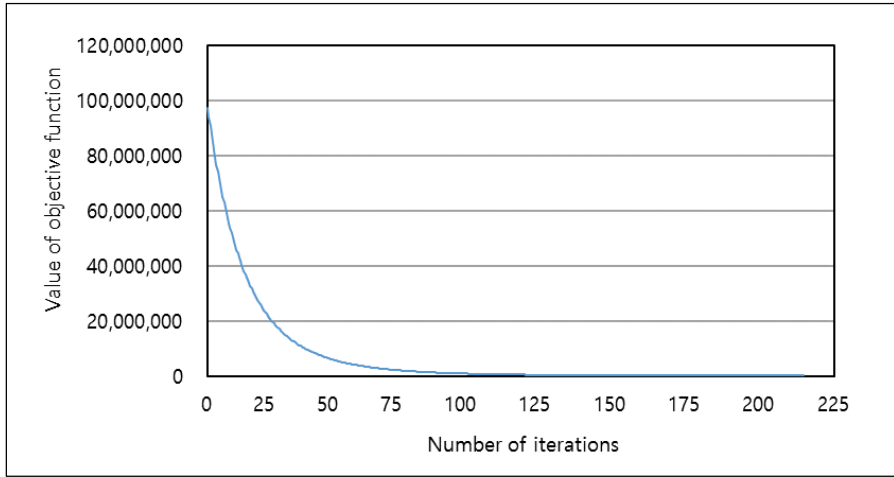
It was assumed that all the analyzed links are having single lane and the parameter d_j is 140 veh/km/lane. The variable d is calculated by transforming the variable denoting the number of vehicles on a link into the number of vehicles on a unit distance. The parameters of the analyzed links are as follows:

<Table 3> VDF parameters of Nguyen-Dupuis network example

Link	d_j (veh/km/lane)	v_{\min} (km/hr)	v_{\max} (km/hr)	Distance (km)
1→12	140	8	70	10
1→5	140	8	70	10
4→5	140	8	70	10
4→9	140	8	40	15
5→6	140	8	70	10
5→9	140	8	70	10
6→7	140	8	70	14
6→10	140	8	70	10
7→8	140	8	70	14
7→11	140	8	70	10
8→2	140	8	70	10
9→10	140	8	70	10
9→13	140	8	40	15
10→11	140	8	70	14
11→2	140	8	70	14
11→3	140	8	70	10
12→6	140	8	70	10
12→8	140	8	40	30
13→3	140	8	70	14

5.2. Assignment result

The modified flow swapping algorithm gradually approximated traffic flows to be in the DUE after 214 iterations. As decline of the scaling parameter ρ_τ in iterative process improves convergence, the scaling parameter was started as 0.0001, after then changed into 0.00005 in iteration 101 and 0.000025 in iteration 201 (Huang and Lam, 2002). It is because that the out-of-range penalty cost carries out rather high values, values of the objective function showed larger magnitude at the beginning of the algorithm. But after then, the value showed continuously decreasing slope until convergence.



<Figure 4> Convergence plot for the example network

At the DUE, all the assigned paths are as follows:

<Table 4> Assigned paths of Nguyen-Dupuis network example

Origin	Destination	Path number	Node sequence
1	2	1	1-5-6-7-8-2 (6)
		2	1-5-6-10-11-2 (6)
		3	1-5-6-7-11-2 (6)
		4	1-12-6-7-8-2 (6)
		5	1-12-6-10-11-2 (-)
		6	1-12-6-7-11-2 (-)
1	3	7	1-5-6-7-11-3 (6)
		8	1-12-6-7-11-3 (6)
		9	1-5-6-10-11-3 (6)
		10	1-12-6-10-11-3 (6)
4	2	11	4-5-6-7-8-2 (6)
		12	4-9-10-11-2 (11)
		13	4-5-6-10-11-2 (6)
		14	4-5-6-7-11-2 (6)
4	3	15	4-5-6-7-11-3 (6)
		16	4-9-10-11-3 (11)
		17	4-5-6-10-11-3 (6)

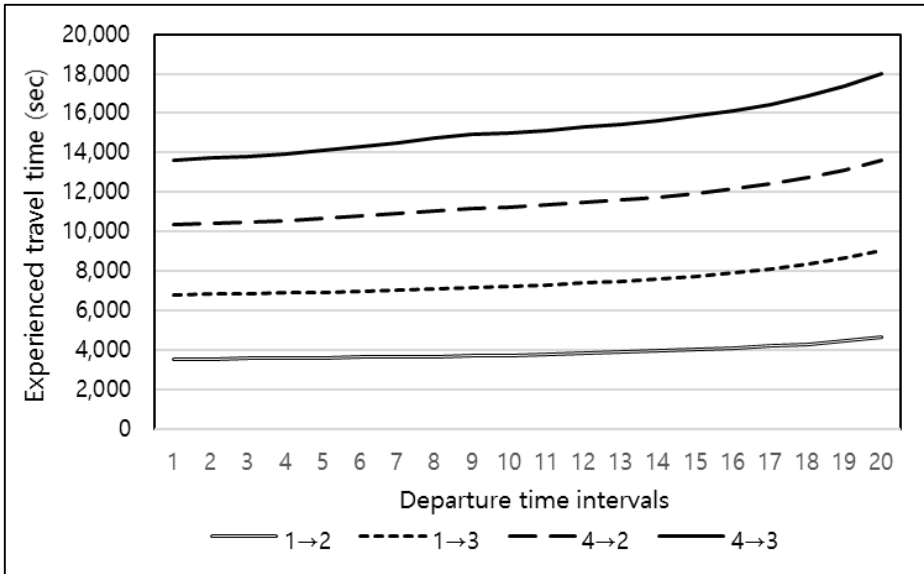
On the above table, node sequence was written with utilized station number

within parenthesis.

At the initial 20 time intervals, all the above paths had assigned traffic flows and showed almost equal travel time within the same O-D pair and the same departure time. As vehicles on the network increases and congestion occurs, it was shown that experienced travel time of each O-D pair gradually increased. The experienced travel times of each O-D pair were shown as follows:

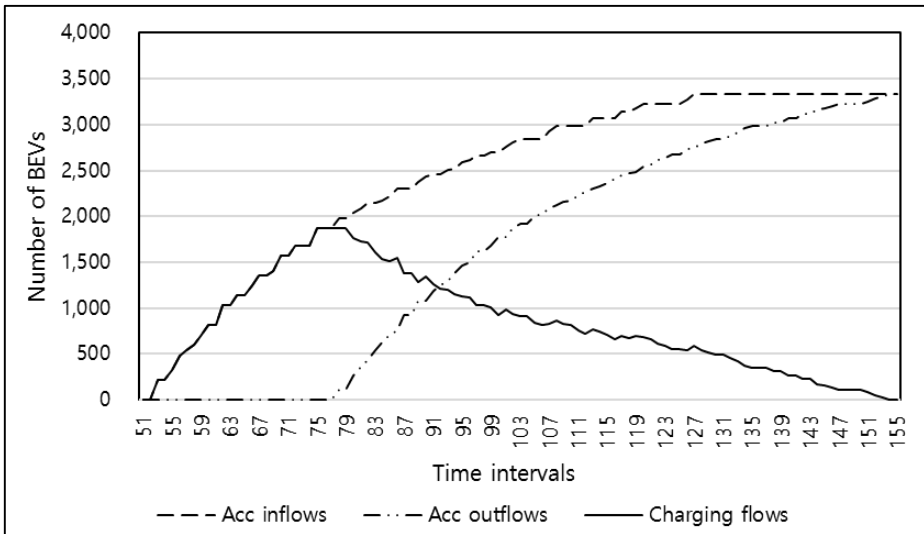
<Table 4> Travel time of Nguyen-Dupuis network example

Departure time interval	O-D pair			
	1→2	1→3	4→2	4→3
1	3,531	3,274	3,531	3,274
2	3,550	3,292	3,568	3,310
3	3,556	3,298	3,610	3,352
4	3,576	3,318	3,658	3,401
5	3,597	3,340	3,716	3,458
6	3,621	3,363	3,783	3,526
7	3,647	3,389	3,865	3,608
8	3,676	3,418	3,938	3,681
9	3,707	3,449	4,001	3,744
10	3,742	3,485	4,025	3,767
11	3,782	3,524	4,050	3,792
12	3,827	3,569	4,077	3,820
13	3,878	3,620	4,107	3,850
14	3,936	3,679	4,141	3,883
15	4,005	3,748	4,183	3,926
16	4,087	3,829	4,238	3,981
17	4,184	3,927	4,307	4,049
18	4,304	4,046	4,390	4,132
19	4,453	4,196	4,491	4,234
20	4,643	4,386	4,613	4,356

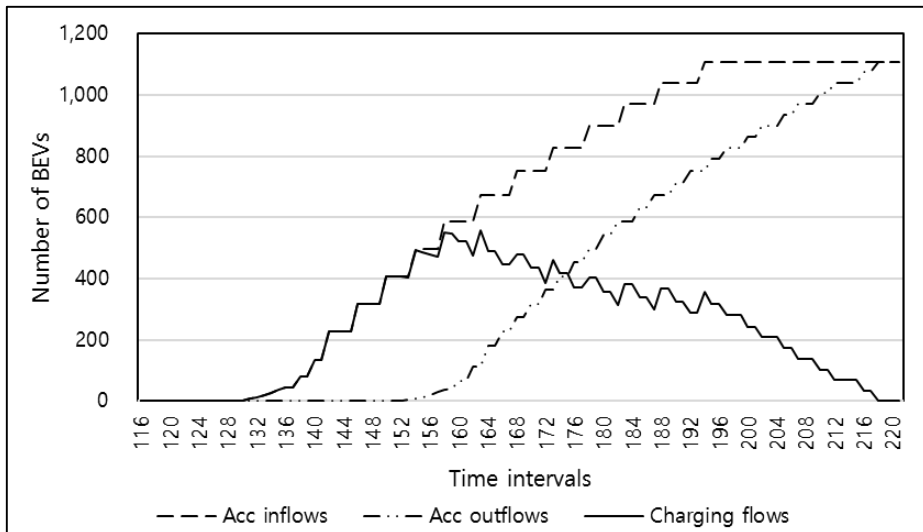


<Figure 5> Experienced travel time of four O-D pairs

As the assigned paths were concentrated on paths that are having node 6 as their en-route charging station, node 6 showed highly crowded demand over entire analyzed period. Below are plots presenting arrival, departure BEVs at stations of node 6 and 11.



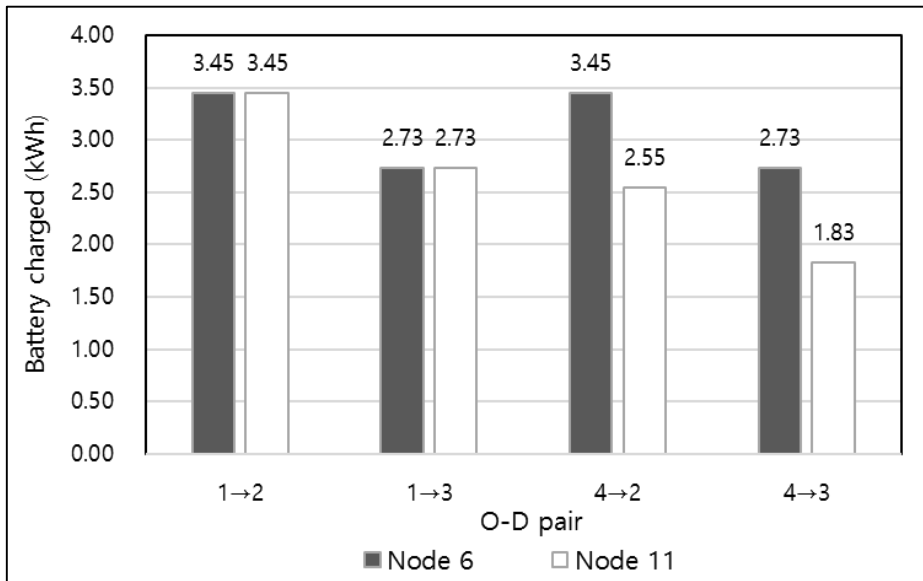
<Figure 6> Accumulative arrival and departure BEVs at node 6



<Figure 7> Accumulative arrival and departure BEVs at node 11

It is because that node 11 is rather farther than node 6 from the origin nodes, BEVs charged at node 11 started their charging later than the BEVs charged at node 6. Maximum number of charging flows also showed significant differences that node 6 had about 1,800 vehicles and node 11 had about 500 vehicles. It can be said that considerable amount of flows assigned to traverse node 6 derived the distinct difference on the maximum number of charging vehicles at a moment.

Specifically, for each O-D pair, the average amount of charged battery at each station is as follows:



<Figure 8> Average battery charged for each O-D pair

The node 6 showed higher charged amount of battery than node 11 among the four O-D pairs. The O-D pair 4→2 and 4→3 has shown rather different average charged amount between node 6 and 11. This can be interpreted that paths having node 6 as a charging station had advantage on shorter driving time than paths having node 11 as a charging station. Therefore, although charging time at node 6 was longer than that of node 11, the offset driving time made flows to be assigned on both stations.

Chapter 6. Conclusions

6.1. Summary and conclusion

Due to increasing demands on BEVs and necessity to incorporate them into operational and planning level of transportation systems, several traffic assignment models handling BEVs were researched beforehand. The existing models mainly were developed as a static assignment form, however, dynamic model with BEVs has been hardly researched. It is expected that once a dynamic model with BEVs is established, it can broaden analytical range on operational planning which requires temporal analysis. For this reason, in this research, a DUE model with BEVs reflecting rapid charging at intermediate charging stations is developed.

In the proposed model, as the usable paths of a BEV are completely determined by its remaining battery state, a path-based discrete time DTA model is formulated. The model's objective function follows a complementarity problem where flows on unusable paths that are impossible to be traversed with given battery states necessarily incur a penalty cost. In the suggested objective function, minimizing the value of it to zero guarantees that none of the assigned flows exceed their range limits and also satisfy the Wardrop's first principle.

To solve the suggested path-based model, a modified flow swapping algorithm was presented. This algorithm is an iterative algorithm that gradually swaps path flows that are assigned to non-shortest paths to that of shortest-paths. In the algorithm, the moved amount of traffic flows is proportional to the travel cost gap between the swapping path and the shortest paths. On top of this process, as the suggested model is based on BEVs, decision process of battery charging amount is included additionally.

Along with the suggested DUE model and iterative algorithm, an

example problem with the Nguyen-Dupuis network was analyzed to show an appropriateness of the suggested model and algorithm. At equilibrium, given with insufficient initial battery state, traffic flows of BEV detoured to maintain their batteries to be higher than minimal comfortable amount in a dynamic transportation system. The battery charging time and driving time offset each other, and showed identically experienced travel time for traffic flows that had the same O-D and departing time interval. Gathered from the dynamically assigned traffic flows, accumulative arrival and departure flows at each stations are suggested as plots. Furthermore, average battery charging amount is also presented for each O-D pair.

The developed model showed availability on temporal evaluations of traffic flows of BEVs. With given initial battery state of BEVs and charging stations defined by their performance parameters, traffic flows following DUE while satisfying range limit were gathered appropriately. To enhance more realistic result of this assignment problem, several aspects have to be incorporated to this model further. First, as the suggested model is a trip-based model, tour-based model is needed that many BEV users not only consider a single trip but also their entire trips within a day. This may occur significant computational burden on dynamic system, therefore, appropriate efficient algorithm has to be devised as well. Second, it is because that capacity of charging station is not unlimited in the real world, capacity constraint that can make the model in a more realistic way is required. As like the road capacity problem, non-unified travel time will occur and appropriate techniques and interpretation is required on it. Lastly, specifying the travel time is expected to be needed that driving time and charging time may not be equally weighted for BEV drivers. Furthermore, if charging station's capacity problem is covered, incorporation of waiting time on the model is needed to be reflected.

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국문 초록

배터리 전기차량(BEV)에 대한 수요가 증가함에 따라서, 기존 교통시스템의 계획 및 운영 모형에 있어 BEV를 반영하고자 하는 사회적 관심이 최근에 대두되고 있다. 일반적인 화석연료 차량과는 다르게, BEV 이용자들은 짧은 주행거리, 긴 충전시간, 불충분한 충전시설을 원인으로 하는 주행거리에 대한 불안감(range anxiety)을 가지고 있다. 이에 따라 BEV 운전자들은 기존의 운전자와는 다른 경로 선택 행태를 보일 것으로 예상되며, 불충분한 배터리 잔량으로 인하여 통행 중간에 주행이 불가능해지는 상황을 기피할 것이다.

BEV의 이러한 특성을 반영한 정적 통행배정 모형은 일부 연구가 되어왔으나, BEV를 반영한 동적통행배정 모형은 현재까지는 연구가 거의 이루어지지 않고 있다. 기존에 연구되어왔던 정적 모형들에서 더 나아가, 동적통행배정 모형은 BEV의 통행패턴에 대한 시간대별 분석을 가능하게 하여 분석범위의 확장성을 도모할 수 있다. 이에 따라, 본 연구에서는 BEV 이용자들의 통행 특성을 반영한 동적이용자평형 모형을 제시한다.

본 연구에서는 통행 기반의 동적이용자평형 모형을 제시한다. 또한 BEV가 이용할 수 있는 경로들은 배터리 잔량에 의하여 결정되며 지나온 경로의 거리에 따라서 잔량이 변화되기 때문에, 경로 기반의 모형이 제시되었다. 모형의 목적식은 최소화 문제를 따르며 최단시간경로와 그렇지 않은 경로들의 통행 시간 차이에 대한 항과, 경로를 모두 지나가기에 부족한 배터리 잔량을 기반으로 한 패널티 항으로 구성되었다. 알고리즘으로서는 변형된 flow swapping 방법이 사용되었다.

제시된 모형을 기반으로 불충분한 초기 배터리 잔량을 가정으로 하여 Nguyen-Dupuis 네트워크에서 사례분석을 시행하였다. 그 결과로서, 불충분한 초기 배터리 잔량으로 인하여, 배터리 잔량을 일정수준 이상 유지하기 위하여 충전소로 우회하는 교통량이 동적 배정방법론으로써 구현이 되었다. 또한, 동일한 O-D에서 동일한 시간대에 출발한 BEV들이 같은 통행시간을 가지게 되었으며, 통행 중 배터리 소진으로 인한 통행불가가 발생하지 않게 되어 동적이용자 평형이 구현된 것을 확인하였다.

주요어: 동적이용자평형, 동적통행배정, 배터리 전기차량,
주행거리 불안감

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