공학석사 학위논문

Prediction of Stress in High-Strength Strand at Flexural Strength of PSC Beam

PSC 보의 휨 강도에서 고강도 강연선의 응력 예측

2015년 8월

서울대학교 대학원

건설환경공학부 구조전공

박 재 현

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2015

Jae-Hyun Park

← 2cm →

← 2.5cm →

← 4cm →

← 3cm →

← 2cm →

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지도 교수 조 재 열

이 논문을 공학석사 학위논문으로 제출함

2015 년 7 월

서울대학교 대학원
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박재현의 석사 학위논문을 인준함

2015 년 7 월

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Abstract

Prediction of Stress in High-Strength Strand at Flexural Strength of PSC Beam

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The tensile strength of prestressing strands which is globally used in PSC members is 1,860 MPa. Recently, 2,160 MPa and 2,400 MPa high-strength PS strands whose tensile strengths are higher than 1,860 MPa PS strand were developed in 2008 and 2011 respectively by domestic technique. Korean industrial standard allowed the use of high-strength PS strands and then the use of high-strength PS strands increased. However, existing design codes do not consider the influence of high-strength PS strands.

Therefore, this study examined the applicability of existing approximate equations for PS strand stress at flexural strength in PSC member with high-strength PS strands in design standards. The result shows that existing design standards does not predict nonlinear stress behavior of high-strength PS strands at pre-yielding range. Thus, this study proposed an approximate equation for PS strand stress at flexural strength of a member considering nonlinear stress behavior of high-strength PS strands. With the proposed equation, PS strand stress at flexural strength and flexural strength of member can be predicted accurately even at high-strength PS strands.

Keywords: ACI 318-14, CSA A23.3-14, Flexural strength, High-strength PS strands, PS strand stress

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Chapter 1 Introduction

1.1 Research Backgrounds

Nowadays, prestressing concrete structures generally shortened to PSC structures are widely used in various fields of building and construction such as a skyscraper, a nuclear power plant structure and a bridge. To improve performance of these PSC structures, the needs of performance enhancement in materials have been increased. Strength and performance of various constructional materials have improved with technological advances. Recently, high-strength PS strands which have more increased tensile strength than existing PS strands were developed. Existing 1,860 MPa PS strand was commercialized in the early 1980s, and there has been no increase in tensile strength for about the past 30 years. In Korea, 2,160 MPa high-strength PS strand which has 16% increased tensile strength was developed in 2008 as a part of Mega Structure Steel Development Project managed by the Ministry of Knowledge Economy. Also, 2,400 MPa high-strength PS strand which has 29% increased tensile strength was developed in 2011 which was managed by Super Long Span Bridge R&D center. Use of high-strength PS strands reduces area of PS strands and the number of anchorage in PSC structures. Also, it improves economic efficiency and workability in construction. In the case of a bridge, extending span range is possible with high-strength PS strands, and it makes a bridge more slender. Due to these merits, the use of high-strength PS strands in

PSC structures has been increased.

Fig. 1.1 PS strands in construction field

To apply the newly developed 2,160 MPa and 2,400 MPa high-strength PS strands in practice, standardization of the material in industrial standards and development of new anchorage system available for high-strength PS strands are required. Also, evaluating the applicability of design standards for high-strength PS strands and revision of design standards are needed. As a result of the efforts to commercialize the high-strength PS strands, strand type C which indicates 2,160 MPa PS strand (SWPC7CL) and strand type D which indicates 2,400 MPa PS strand (SWPC7DL) were added in KS D 7002 which was revised in 2011. Also, new anchorage systems for high-strength PS strands were developed at Super Long Span Bridge R&D center and an efficiency evaluation standard for the anchorages which is called KCI-PS101 was established at concrete standard specification in 2009. However, the current local and global design standards such as KCI, ACI, and CSA do not reflect properties of the high-strength PS strands. For this reason, an evaluation of the applicability of the design standards for high-strength PS strands and revision of the design equations are needed.

Table 1.1 Type and sign of PS strands in KS D 7002

Type		Sign	Ultimate strength (MPa)
7-Wire PS Strand	A	SWPC7AN, SWPC7AL	1,720
	B	SWPC7BN, SWPC7BL	1,860
	C	**SWPC7CL**	**2,160**
	D	**SWPC7DL**	**2,400**

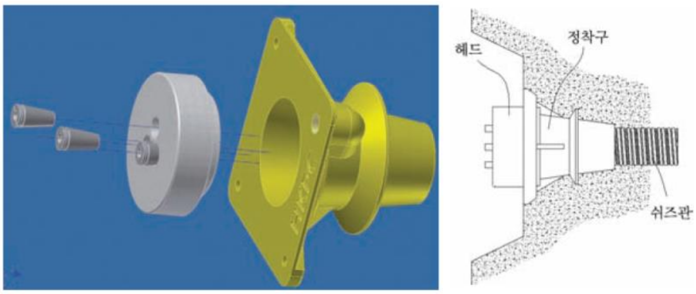


Fig. 1.2 Anchorage system of PS strand

In Structural Concrete Design Code (KCI, 2012), it is defined that PS strands used in prestressed concrete should follow KS D 7002, so there is no need to legally revise the design code in order to apply high-strength PS strand at PSC structures. However, the design code needs a conformity assessment of using high-strength PS strands and also needs revision of design equations.

1.2 Research Objectives

To apply high-strength PS strands into PSC structures, an appropriate design standard reflecting the influence of high-strength PS strands is required and proper test results should be supported to revise the design standards. Prior studies focused on the comparison of existing design standards and the results of performance evaluation tests for flexural members using high-strength PS strands. However, the performance evaluation test for a flexural member with PS strands is difficult to conduct, and it takes a lot of time and money. Thus, a theoretical evaluation is needed instead of the performance evaluation test. Through the theoretical evaluation, a wide range that the performance evaluation test does not cover can be evaluated. Eventually, it is possible to deduce the revision direction of design standards with the theoretical evaluation of the applicability of design standards.

As a part of the theoretical evaluation, the preceding research (Eu-Jeong Choi, 2015), entitled “Code Validation for Flexural Strength of PSC Members with High-Strength Strand”, was conducted. In the preceding research, material properties of high-strength PS strands were standardized using the results of uniaxial tensile strength test for 2,160 MPa and 2,400 MPa PS strands, and material models indicating stress-strain relationship for high-strength PS strands were proposed. Also, the study analyzed the applicability of an approximate equation for PS strand stress at flexural strength of a member in ACI 318-11 ,which is a design code adopted in USA and Korea, by comparing the equation and the sectional analysis results. The study argued that the existing design equation in ACI 318 is valid up to compression controlled strain limit as a result of analysis.

However, because ACI 318 does not limit the amount of PS strand, the equation should be valid from tension controlled section to compression controlled section regardless of the amount of PS strand. In addition, the design equation in ACI 318 is based on the post-yielding behavior of strand in rectangular section so it does not consider the influence of sectional shape and pre-yielding behavior of PS strands.

This study conducted an analytical study on various approximate equations for PS strand stress at flexural strength of a member used in ACI 318-14 and CSA A23.3-14. Through the analytical study on the equations, factors affecting PS strand stress at flexural strength were understood. Also, the study examined the applicability of the design standards for high-strength PS strands with the analysis of PS strand stress at flexural strength and flexural strength of a member using high-strength PS strands. Consequentially, an approximate equation for PS strand stress at flexural strength of a member considering the pre-yielding behavior in PS strand stress was proposed in the study, and the proposed equation is available for all type of PS strand including high-strength PS strands.

1.3 Construction of Research

The objective of this study is a proposal of an approximate equation for PS strand stress at flexural strength of a member with high-strength PS strands. The study covers a validation of current design standards, a proposal of an approximate equation, and a validation of the proposed equation.

In chapter 2, the approximate equations for PS strand stress at flexural strength of a member proposed by ACI 318-14 and CSA A23.3-14 were analyzed. Through the analysis, various factors affecting PS strand stress at flexural strength were understood. Also, the equations were validated by comparing the PS strand stresses obtained by strain-compatibility-based sectional analysis and the approximate equations in design standards for 4 types of PS strands which are classified by tensile strength and yield ratio.

In chapter 3, the equations were validated by comparing flexural strength obtained from the sectional analysis and the approximate equations in design standards for the four types of PS strands.

In chapter 4, based on the analysis results in chapter 2 and 3, an approximate equation for PS strand stress at flexural strength which is available for high-strength PS strands considering pre-yielding behavior in PS strand stress was proposed, and also the proposed equation was validated.

Chapter 2 PS Strand Stress at Flexural Strength

PS strand stress at flexural strength is required to get flexural strength of PSC structure. Thus, PS strand stress at flexural strength should be known exactly to predict flexural strength accurately. When getting PS strand stress at flexural strength, strain-compatibility-based sectional analysis is conducted to get the accurate PS strand stress. However, in practice, sectional analysis is hardly used because it needs an accurate material behavior and iterative calculation. As an alternative to sectional analysis, the approximate equations for PS strand stress at flexural strength are used in practice. Through the approximate equations, PS strand stress at flexural strength can be obtained using more simple calculation with less design parameters.

In Structural Concrete Design Code (KCI, 2012), an approximate equation for PS strand stress at flexural strength used in ACI 318 is adopted. In CSA A23.3, a design code in Canada, which is an approximate equation for PS strand stress at flexural strength has a different form comparing to ACI 318.

Compared to a general steel reinforcement, PS strand has a different stress-strain relationship. First, a yield point of PS strand is indistinct. Also, the stress-strain relationship of PS strand after the proportional limit of elastic range is nonlinear. The approximate equations for PS strand stress at flexural strength should consider these properties of PS strand.

Through the approximate equations for PS strand stress at flexural strength used in various design standards, factors affecting behavior of PS strand stress at flexural strength are understood. For the PSC member using high-strength PS strand, the sectional analysis results of PS strand stress at flexural strength and the approximate equations in design codes are compared in order to examine the applicability of the equations in case of high-strength PS strands.

2.1 Approximate Equations for PS Strand Stress

2.1.1 ACI 318-14

In ACI 318-14 Eq. (20.3.2.3.1), PS strand stress at flexural strength for $f_{se}>0.5f_{pu}$ is defined as following.

$$f_{ps} = f_{pu}\left(1-\frac{\gamma_p}{\beta_1}\left(\rho_p\frac{f_{pu}}{f_c'}+\frac{d}{d_p}\frac{f_y}{f_c'}\left(\rho-\rho'\right)\right)\right) \tag{2.1}$$

f_{pu} is a tensile strength of PS strand, β_1 is a coefficient related to the equivalent rectangular stress block, and ρ_p is a prestressing reinforcement ratio. γ_p is a coefficient related to yield ratio and the value is 0.55 when $f_{py} / f_{pu} \geq 0.80$, 0.40 when $f_{py} / f_{pu} \geq 0.85$, and 0.28 when $f_{py} / f_{pu} \geq 0.90$. d is an effective depth of member and d_p is a distance from extreme compression fiber to centroid of prestressing reinforcement. f_y is a yield strength of steel reinforcement and f_c' is a compressive strength of concrete. ρ is a reinforcement ratio of tensile steel and ρ' is a reinforcement ratio of compressive steel.

As factors affecting PS strand stress at flexural strength in the Eq. (2.1), the effect of yield ratio is considered in the equation as the coefficient γ_p. Also, the effect of compressive strength of concrete is considered in the equation as the coefficient β_1 and the effect of steel reinforcement is considered as ρ and ρ'.

However, the effect of sectional shape is not considered because the section which has flange such as I-type section and T-type section is treated as a rectangular section. Also, it does not consider the nonlinear stress behavior of PS strands after the proportional limit.

A design equation for PS strand stress at flexural strength in ACI 318 was first proposed at ACI 318-63 Eq. (18-3) in 1963 as shown in following Eq. (2.2).

$$f_{ps} = f_{pu}\left(1 - 0.5\rho_p \frac{f_{pu}}{f'_c}\right) \tag{2.2}$$

The Eq. (2.2) is based on the general strength concrete member using only PS strands which have 1,860 MPa stress relieved PS strand and having the rectangular section. It does not consider the effect of various factors such as strand type, concrete strength, general steel reinforcement, sectional shape, etc. Thus, the equation (2.1) considering various factors was first proposed by Mattock and adopted in ACI 318-83 in 1983. This equation is still used in ACI 318-14 which is the latest revision of ACI 318.

In ACI 318-14, the section of flexural member is classified by ductility. The section in which the net tensile strain is over 0.005 is called tension controlled section. The section in which the net tensile strain is under 0.002 is called compression controlled section. The section in which the net tensile strain is between 0.002 and 0.005 is called transition section. In RC member, the amount of reinforcement is limited by net tensile strain indirectly. However, there is no limitation in the amount of PS strands in PSC member. For that reason, the PS strand stress at flexural strength should be predicted in all section.

2.1.2 CSA A23.3-14

In CSA A23.3-14 Eq. (18.1), PS strand stress at flexural strength for f_{pe}>0.6f_{py} and $c/d_p \le 0.5$ is defined as following.

$$f_{ps} = f_{pu}\left(1-k_p\left(\frac{c}{d_p}\right)\right) \tag{2.3}$$

$$k_p = 2\left(1.04-\left(\frac{f_{py}}{f_{pu}}\right)\right) \tag{2.4}$$

A distance from extreme compression fiber to neutral axis is approximated as following.

$$\frac{c}{d_p} = \frac{c_p}{d_p} + \frac{c_s}{d_p} - \frac{c_s'}{d_p} - \frac{c_f}{d_p} = \frac{c_{pu}}{d_p}\frac{f_{ps}}{f_{pu}} + \frac{c_{sf}}{d_p} \tag{2.5}$$

Substituting Eq. (2.5) into Eq. (2.3), PS strand stress at flexural strength can be indicated as following.

$$f_{ps} = f_{pu}\left(\frac{1-k_p\left(\frac{c_{sf}}{d_p}\right)}{1+k_p\left(\frac{c_{pu}}{d_p}\right)}\right) \tag{2.6}$$

f_{pu} is a tensile strength of PS strand, c is distance from extreme compression fiber to neutral axis, and d_p is a distance from extreme compression fiber to centroid of prestressing reinforcement. k_p is a coefficient related to yield ratio, and it can be calculated with Eq. (2.4). c_p is a portion of c that indicates distance from the extreme compression fiber to the neutral axis except the flange corresponding to the tension force by prestressing reinforcement. c_s is a portion of c that indicates distance from the extreme compression fiber to the neutral axis except the flange corresponding to the tension force by tensile steel reinforcement. c_s' is a portion of c that indicates distance from the extreme compression fiber to the neutral axis except the flange corresponding to compression force by compressive steel reinforcement. c_f is a portion of c that indicates distance from the extreme compression fiber to the neutral axis corresponding to the flange.

As a factor affecting PS strand stress at flexural strength in Eq. (2.3), the effect of yield ratio is considered in the equation by the coefficient k_p. Also, a change of the neutral axis due to a steel reinforcement and a sectional shape is considered. However, the effect of compressive strength of concrete is not considered in the equation. Also nonlinear stress behavior after proportional limit which is the material property of PS strands is not reflected well in the equation.

Before 1994, a following form of approximate equation for PS strand stress at flexural strength was used.

$$f_{ps} = f_{pu}\left(1 - k_p\left(\frac{c}{d_p}\frac{f_{pu}}{f_{ps}}\right)\right) \tag{2.7}$$

$$\frac{f_{ps}}{f_{pu}} = \frac{1}{2}\left[1 - k_p\left(\frac{c_{pu}}{d_p}\right)\right] + \sqrt{\left(1 - k_p\left(\frac{c_{pu}}{d_p}\right)\right)^2 - 4k_p\frac{c_{sf}}{d_p}} \tag{2.8}$$

Because the form of Eq. (2.7) is a parabolic, designers have to solve the quadratic equation like Eq. (2.8) to get the PS strand stress at flexural strength, so the calculation is complicated. To get the PS strand stress with simple calculation, Loov proposed an approximate equation like Eq. (2.3) which is a hyperbolic in 1988. With Eq. (2.3), designers can calculate PS strand stress at flexural strength more easily. Eq. (2.3) was adopted in CSA A23.3 in 1994. This equation is still used in CSA A23.3-14 which is the latest revision of CSA A23.3.

In CSA A23.3-14, the amount of PS strands is indirectly limited by limiting the equation to the range of $c/d_p \leq 0.5$.

2.2 Validation of Current Design Standards

The approximate equations for PS strand stress at flexural strength used in design codes were proposed prior to the time when the high-strength PS strands were developed. Thus, it needs to examine the applicability of the existing approximate equations for the high-strength PS strands. To validate the approximate equations, strain-compatibility-based sectional analysis was conducted and the results of sectional analysis were compared with the approximate equations.

2.2.1 Sectional Analysis

Through the sectional analysis performed with strain compatibility condition and force equilibrium condition, PS strand stress at flexural strength can be obtained. To get the exact PS strand stress at flexural strength, accurate material properties of PS strands, concrete, and steel reinforcement are needed. For the sectional analysis to get the PS strand stress for high-strength PS strands, parameters and material models were determined. Following the analysis procedure, sectional analysis was conducted.

2.2.1.1 Parameters of Analysis

Strand types classified by tensile strength and yield ratio were set as a parameter. The sectional analysis was conducted by increasing the amount of

PS strands. To analyze the effects of using high-strength PS strands, PS strands were classified into 1,860 MPa, 2,160 MPa, and 2,400 MPa according to tensile strength and also classified into 0.85, 0.90, and 0.94 according to yield ratio. Properties of strand type are indicated in Table 2.1.

Table 2.1 Classification of PS strands

Strand type	f_{pu} (MPa)	f_{py}/f_{pu}
A	1,860	0.85
B	1,860	0.90
C	2,160	0.94
D	2,400	0.94

Increasing the amount of PS strands, total 15 groups of a cross sectional area for PS strands were used as shown in Table 2.2. To compare flexural strength, same reinforcement index for prestressing reinforcement is used in each group.

Table 2.2 Area of PS strands by strand type

	1,860 MPa strand		2,160 MPa strand		2,400 MPa strand	
	Area (mm^2)	$\rho_p \frac{f_{pu}}{f_{ck}}$	Area (mm^2)	$\rho_p \frac{f_{pu}}{f_{ck}}$	Area (mm^2)	$\rho_p \frac{f_{pu}}{f_{ck}}$
1	383	0.0212	330	0.0212	297	0.0212
2	766	0.0424	660	0.0424	594	0.0424
3	1150	0.0636	990	0.0636	891	0.0636
4	1533	0.0848	1320	0.0848	1188	0.0848
5	1916	0.1060	1650	0.1060	1485	0.1060
6	2299	0.1272	1980	0.1272	1782	0.1272
7	2683	0.1484	2310	0.1485	2079	0.1485
8	3066	0.1696	2640	0.1697	2376	0.1697
9	3449	0.1908	2970	0.1909	2673	0.1909
10	3832	0.2120	3300	0.2121	2970	0.2121
11	4215	0.2332	3630	0.2333	3267	0.2333
12	4596	0.2544	3960	0.2545	3564	0.2545
13	4979	0.2756	4290	0.2757	3861	0.2757
14	5362	0.2968	4620	0.2970	4158	0.2970
15	5745	0.3180	4950	0.3182	4455	0.3182

I-type section as shown in Figure 2.1 was used. This sectional shape is the reduced form of standard cross section of 30m-span PSC beam from Korea Expressway Corporation to be fit with 20m-span PSC beam.

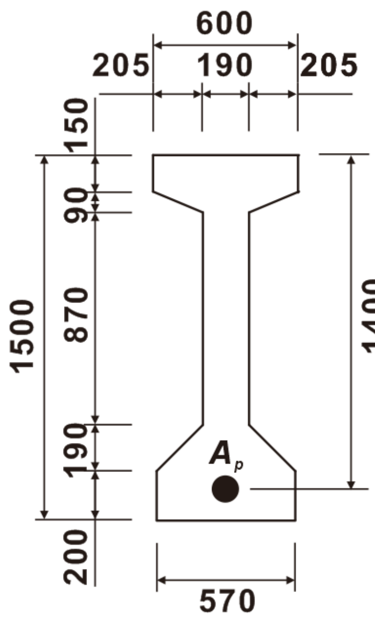


Fig. 2.1 Sectional shape of a member (unit: mm)

2.2.1.2 Material Models

For the sectional analysis, stress-strain relationships of materials are required. Material models for concrete and PS strands were selected for the sectional analysis.

The Thorenfeldt et al. (1987) was used as the material model of concrete. The stress-strain relationship of concrete is defined as Eq. (2.9). Compressive strength of concrete used in the analysis is 40 MPa and stress-strain curve for Thorenfeldt model of 40 MPa concrete is shown in Figure 2.2.

$$\frac{f_c}{f_{ck}} = \frac{n(\varepsilon_c / \varepsilon_o)}{n-1+(\varepsilon_c / \varepsilon_o)^{nk}} \tag{2.9}$$

f_c is a compressive stress of concrete at ε_c, f_{ck} is a compressive strength of concrete, ε_c is a strain of concrete, ε_o is a strain of concrete at compressive strength, n is a coefficient related to a shape of the curve, and k is a coefficient related to the gradient at descent part of the curve.

n, E_c, ε_o, and k are defined as following.

$$n = 0.8 + \frac{f_{ck}}{17.2} \tag{2.10}$$

$$E_c = 3320\left(\sqrt{f_{ck}}\right) + 6900 \tag{2.11}$$

$$\varepsilon_o = \frac{f_{ck}}{E_c}\left(\frac{n}{n-1}\right) \tag{2.12}$$

$$\begin{cases} k = 1.0 & \text{for } \dfrac{\varepsilon_c}{\varepsilon_o} \le 1.0 \\ k = 0.67 + \dfrac{f_{ck}}{62} & \text{for } \dfrac{\varepsilon_c}{\varepsilon_o} > 1.0 \end{cases} \tag{2.13}$$

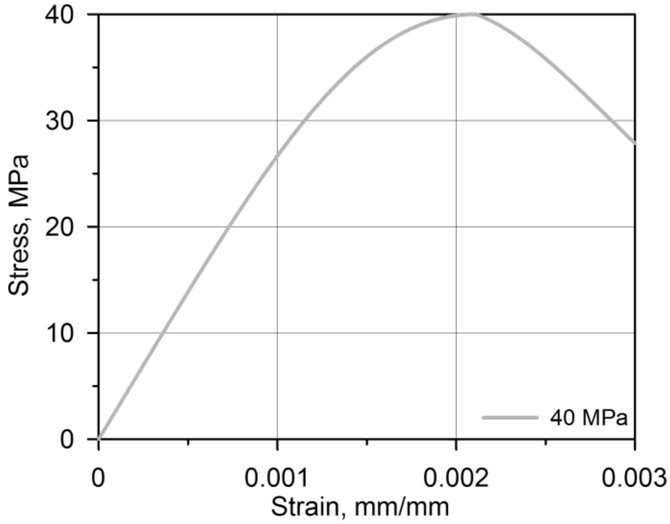


Fig. 2.2 Stress-strain curve for Thorenfeldt model of 40 MPa concrete

The modified Ramberg-Osgood models were used as the material model of PS strands.

$$f_{ps} = E\varepsilon_{pf} \left\{ A + \frac{1-A}{\left[1+\left(B\varepsilon_{pf}\right)^{C}\right]^{1/C}} \right\} \le f_{pu} \tag{2.14}$$

f_{ps} is a stress of PS strand, E is the modulus of elasticity, f_{pu} is a tensile strength of PS strand, ε_{pf} is a strain of PS strand, the constant A is a coefficient related to the gradient of curve after yield point, the constant B is a coefficient related to yield strain, the constant C is a coefficient related to curvature.

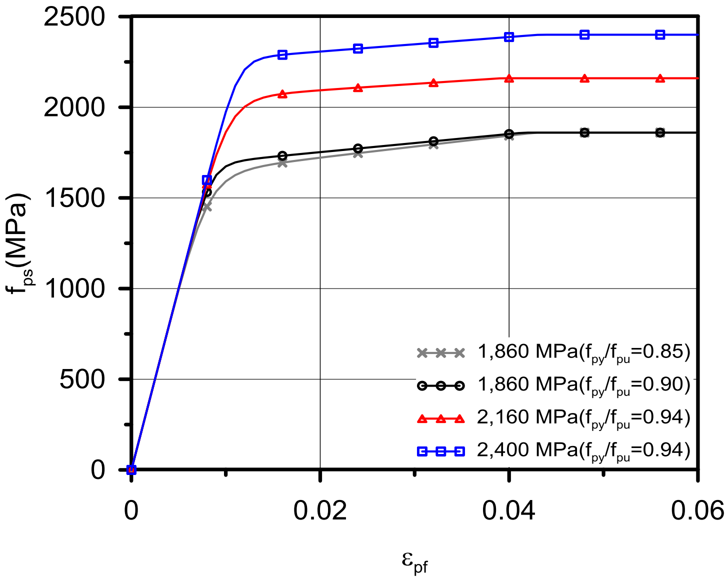


Fig. 2.3 Stress-strain curve for material model of PS strand

Table 2.3 Constants for Modified Ramberg Osgood model of PS strands

Properties	Tensile strength of PS strand (MPa)			
	1,860*	1,860**	2,160	2,400
Modulus of elasticity (MPa)	200,000	200,000	200,000	200,000
Constant *A*	0.03	0.025	0.017	0.020
Constant *B*	121	118	97	88
Constant *C*	6	10	8	13

* Stress relieved, ** Low relaxation

The coefficients of modified Ramberg-Osgood models for 1,860 MPa PS strand recommended by Michael P. Collins (1991) were used, and the coefficients for 2,160 MPa and 2,400 MPa PS strands recommended by Eu-Jeong Choi (2015) were used. The stress-strain relationships for the material models of PS strands are shown in Figure 2.3, and the coefficients for each PS strand are indicated in Table 2.3.

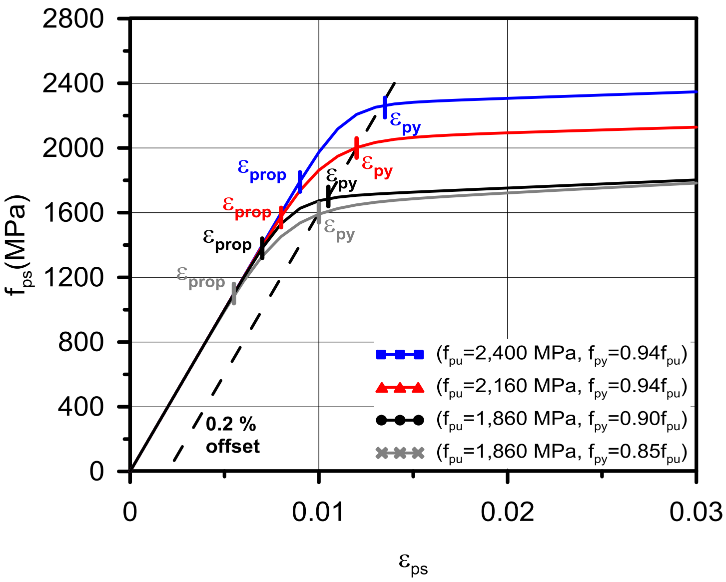


Fig. 2.4 Proportional limit and yield point of PS strand

The yield points for the material models of PS strands were defined by 0.2% offset method, and the proportional limits of PS strands recommended by Eu-Jeong Choi (2015) were used. Strains at proportional limits and yield points for PS strands are shown in Figure 2.4 and Table 2.4.

Table 2.4 Proportional limit and yield point of PS strand

Ultimate strength (MPa)	Yield ratio	ε_{prop}	ε_{py}
1,860	0.85	0.0055	0.0100
1,860	0.90	0.0070	0.0105
2,160	0.94	0.0082	0.0120
2,400	0.94	0.0092	0.0130

2.2.1.3 Results of Sectional Analysis

For the I-type sectional members with strand type A, B, C and D, PS strand stresses at flexural strength were obtained through the sectional analysis. Compressive strength of concrete is 40 MPa. An effective prestressing stress after post-tensioning is 50% of tensile strength of PS strand. The analysis was conducted with total 15 cross sectional area of PS strand. f_{ps} / f_{pu} is plotted on the y axis that indicates PS strand stress, and ρ_p (f_{pu} / f_{ck}) is plotted on the x axis that indicates reinforcement index for prestressing reinforcement. The results of sectional analysis for the type A, B, C, and D were plotted in Figure 2.5.

1,860 MPa PS strands is classified by the method of manufacture. For stress relieved PS strand, it has a yield ratio of 0.85. For low relaxation PS strand, it has a yield ratio of 0.90. As shown in Figure 2.5, for the same tensile strength, PS strand stress for strand type B which has increased yield ratio is higher than that of strand type A. Also, PS strand stresses for strand type C and D which have a yield ratio of 0.94 are higher than strand type B. In other words, as yield ratio increases, PS strand stress at flexural strength in post-yielding range increases. For the strand type C and D which have same yield ratio, the behaviors of PS strand stresses at flexural strength are similar regardless of tensile strength. That is, not tensile strength but yield ratio influences the stress behavior of PS strand at flexural strength in post-yielding range.

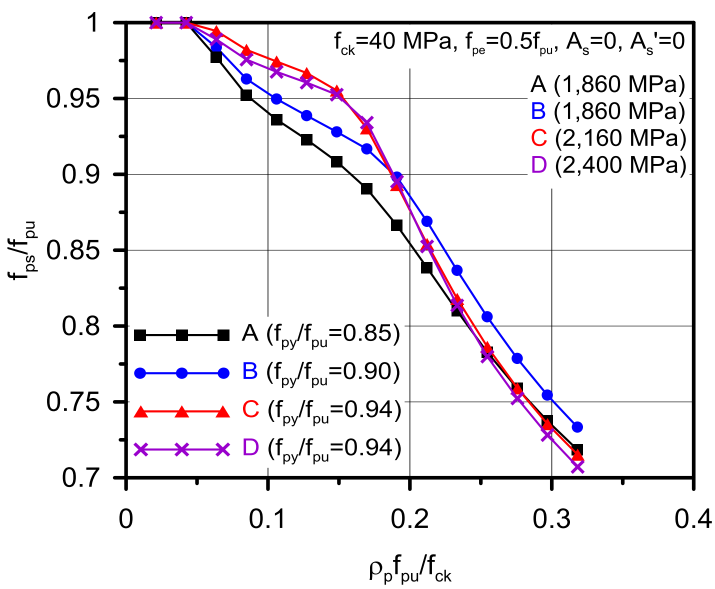


Fig. 2.5 Result of sectional analysis

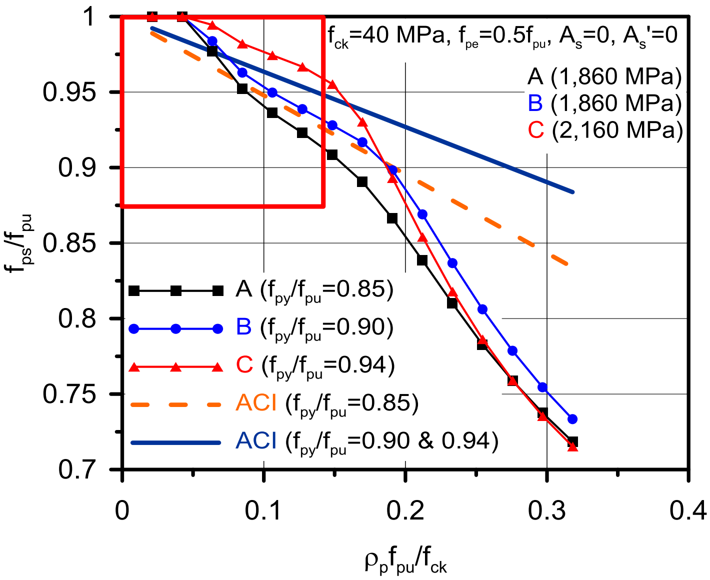
Meanwhile, as the amount of PS strands increases, PS strand stress at flexural strength decreases. At some point, the gradient of PS strand stress decreases. This is because the sectional shape of the member is I-type. As the distance from the extreme compression fiber to the neutral axis get longer than the thickness of upper flange, the width of concrete section receiving compression rapidly decreases and it causes rapid increase of the distance from extreme compression fiber to the neutral axis. Thus, strain and stress of PS strands rapidly decrease and the gradient of PS strand stress decreases.

2.2.2 Verification of Approximate Equations

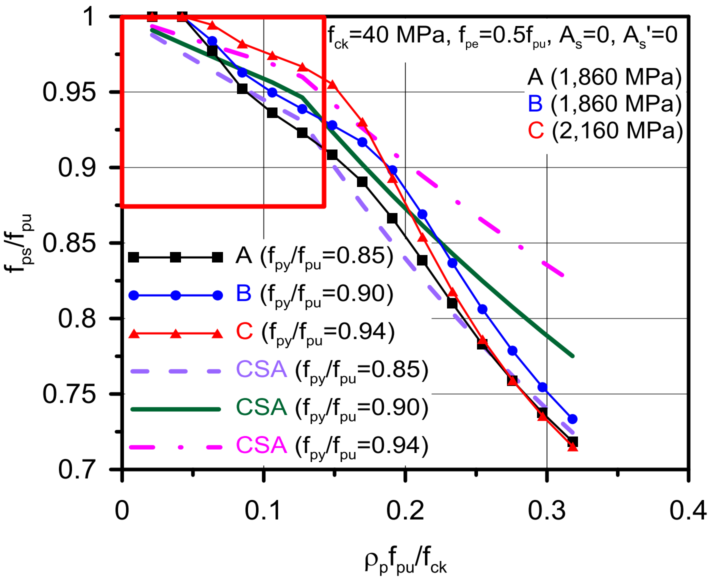
2.2.2.1 Influence of Yield Ratio

Figure 2.6 (a) shows the comparison of sectional analysis results and the approximate equation for PS strand stress in ACI 318-14. The equation considers the difference of yield ratio using the coefficient γ_p. The coefficient is all 0.28 when $f_{py} / f_{pu} \geq 0.90$, however, so the equation does not predict the increase in stress at post-yielding range shown in the left side of the upper part of graph as yield ratio increases in high-strength PS strands.

Figure 2.6 (b) shows the comparison of sectional analysis results and the approximate equation for PS strand stress in CSA A23.3-14. The equation considers the difference of yield ratio using the coefficient k_p. Because the coefficient considers all yield ratio, the equation predicts the increase in stress at post-yielding range as yield ratio increases in high-strength PS strands.



(a) ACI 318-14



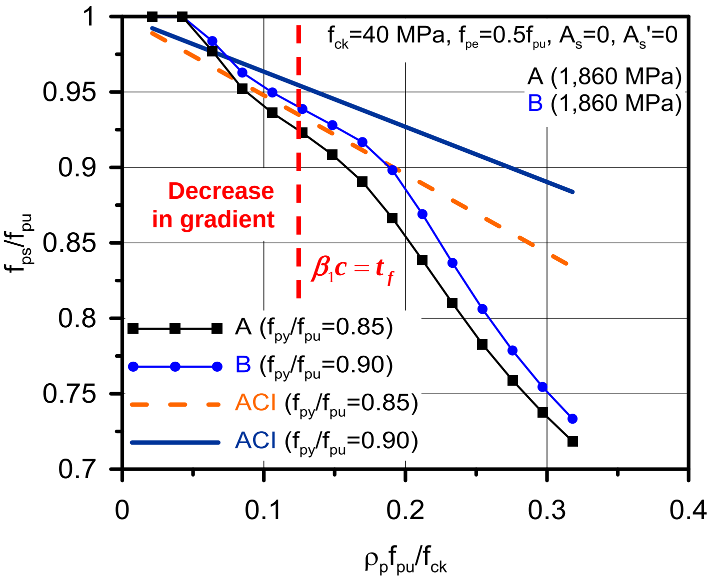
(b) CSA A23.3-14

Fig. 2.6 Consideration of yield ratio in approximate equations

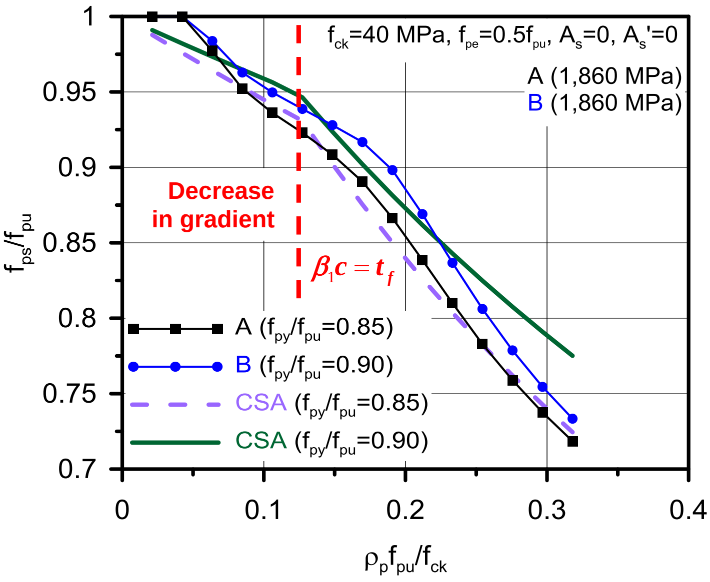
2.2.2.2 Influence of Sectional Shape

As shown in Figure 2.7 (a), the sectional width of I-type member at compression zone reduces as the distance from the extreme compression fiber to the neutral axis goes deeper than the thickness of upper flange. As the sectional width reduces, the distance from the extreme compression fiber to the neutral axis rapidly increases to maintain force equilibrium. Thus, the gradient of PS strand stress decreases as strain of PS strand reduces. The approximate equation for PS strand stress at flexural strength in ACI 318-14 is based on the rectangular sectional member so it does not consider the influence of sectional shape. For this reason, the equation does not predict the decrease in the gradient of stress and overestimates the PS stand stress.

As shown in Figure 2.7 (b), the approximate equation for PS strand stress at flexural strength in CSA A23.3-14 considers the influence of sectional shape. As the depth of equivalent rectangular stress block gets larger than the thickness of flange, CSA A23.3-14 defines the neutral axis considering the width of web. Thus, the equation predicts the decrease in the gradient of PS strand stress at flexural strength as shown in the graph.



(a) ACI 318-14



(b) CSA A23.3-14

Fig. 2.7 Consideration of sectional shape in approximate equations

2.2.2.3 Influence of Pre-yielding Behavior

Because of the material property of PS strands, stress-strain relationship of PS strand from proportional limit to yield point is nonlinear as shown in Figure 2.4. The approximate equations for PS strand stress should predict these nonlinear stress behavior. Stress-strain relationships obtained by CSA A23.3 Eq. (18.1) are compared with the stress-strain relationships from material models to identify the predictability of the equation, shown in Figure 2.8. With Eq. (2.15) proposed in Loov (1988), the strain of PS strand at specific neutral axis can be obtained. Substituting Eq. (2.15) into Eq. (2.3), stress-strain relationship of PS strand for CSA A23.3-14 can be obtained. The stress-strain relationships of PS strands for CSA A23.3-14 are expressed up to the limit point of PS strands. The lines with symbol show the case of CSA A23.3-14 and the lines without symbol indicates the case of the material models. Straight line shown in the graph is 0.2% offset line for indicating yield points of PS strands.

$$\varepsilon_{ps} = \varepsilon_{pe} - 0.0027 + 0.003\frac{d_p}{c} \tag{2.15}$$

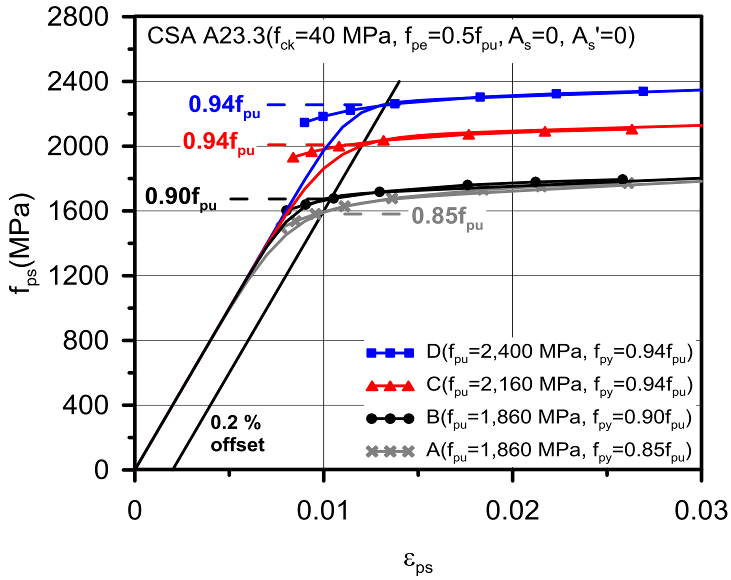


Fig. 2.8 Stress-strain relationships of PS strands at CSA A23.3

As a result of analysis, the approximate equation for PS strand stress at flexural strength in CSA A23.3-14 predicts nonlinearity in the stress of PS strand type A and B which indicate 1,860 MPa PS strands. However, in the case of strand type C and D indicating high-strength PS strands, CSA A23.3-14 overestimates the stress in pre-yielding range. This shows that it does not predict nonlinearity in the stress of high-strength PS strands. Thus, the equation overestimates stresses for the high-strength PS strands at flexural strength in the pre-yielding range as shown in Figure 2.9.

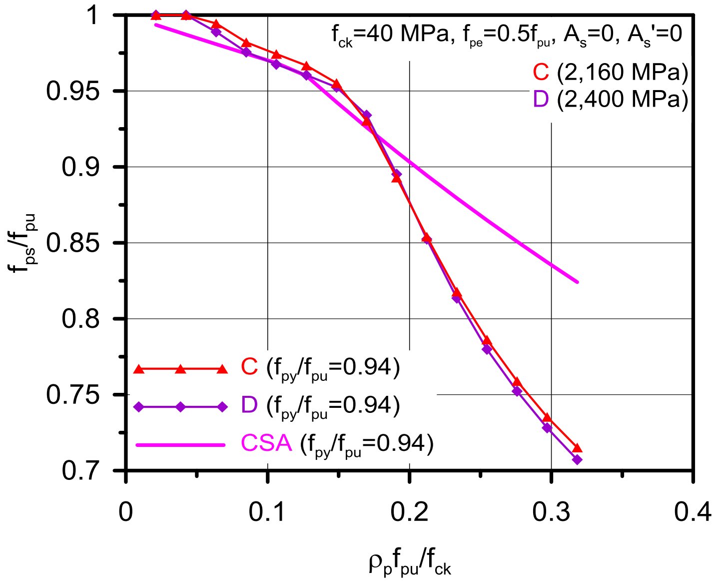
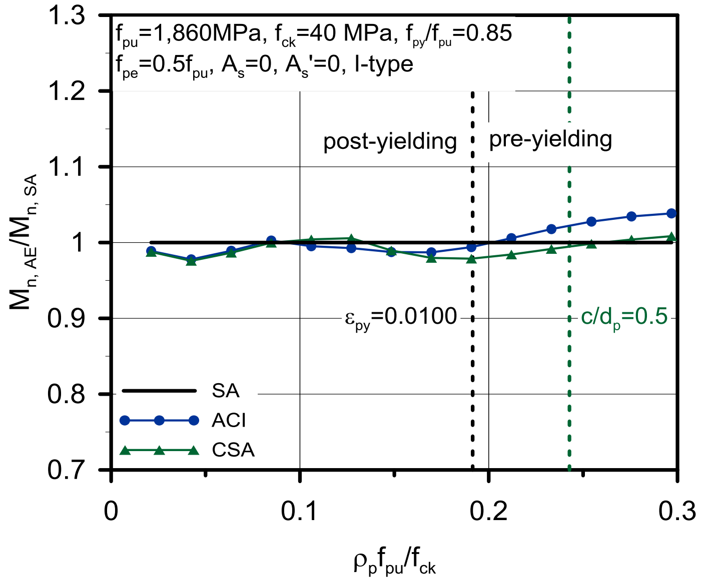


Fig. 2.9 Pre-yielding behavior of high-strength strands in CSA A23.3

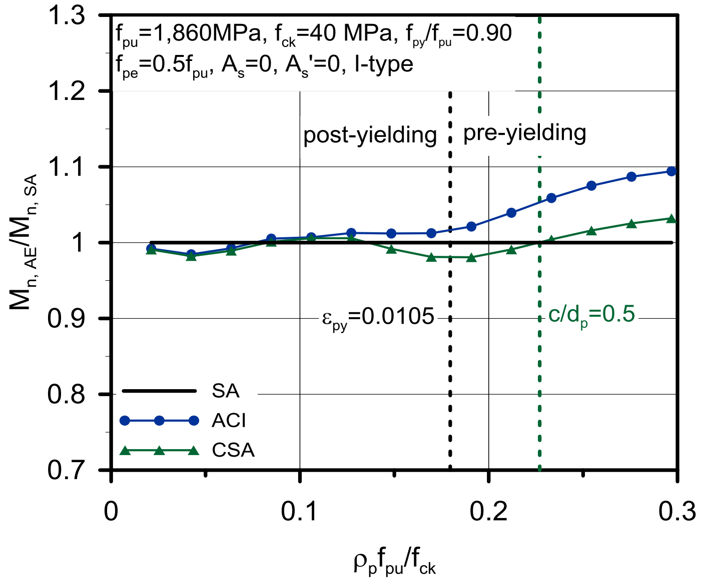
Chapter 3 Flexural Strength Analysis

3.1 Comparison of Flexural Strength

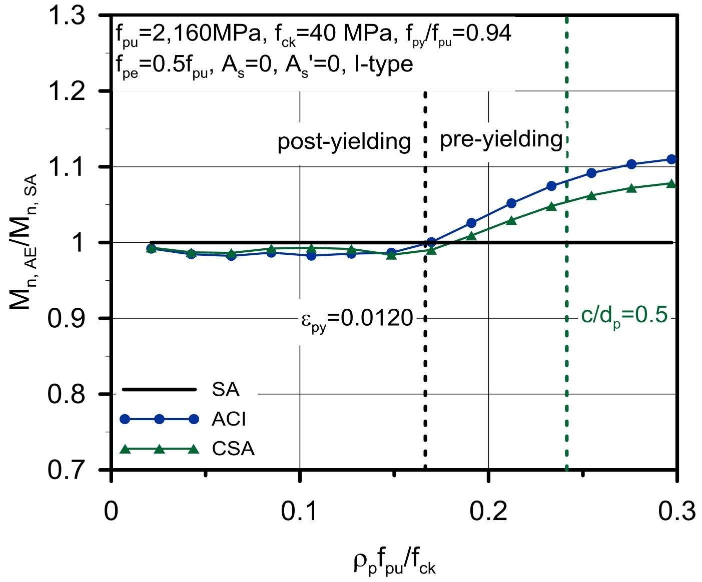
In chapter 2, PS strand stresses at flexural strength of I-type sectional members by strand type were analyzed with sectional analysis and approximate equations in the design standards. Based on the analysis results for PS strand stress, the flexural strengths of the members were analyzed. Errors of flexural strengths were analyzed by comparing flexural strengths which are obtained from ACI 318-14 and CSA A23.3-14 with flexural strength obtained from the sectional analysis. The results of flexural strength analysis are shown in Figure 3.1. In the graph, x axis indicates the reinforcement index for prestressing reinforcement, and y axis indicates the ratio of flexural strength obtained from sectional analysis to flexural strength obtained from design standards. The reinforcement index corresponding to the strain at the yield point is plotted with the dotted line. The left side of the line is the post-yielding range and the right side of the line is pre-yielding range. The point where c/d_p=0.5 is also plotted with the dotted line.



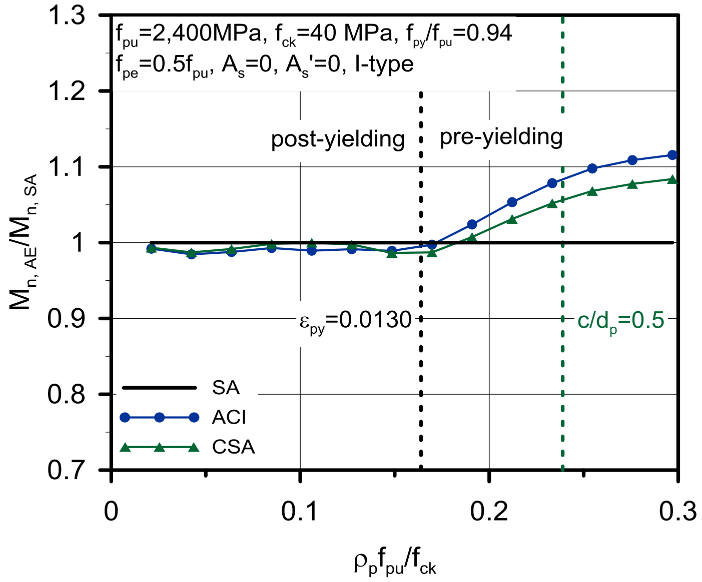
(a) Strand type A



(b) Strand type B



(c) Strand type C



(d) Strand type D

Fig. 3.1 Results of flexural strengths with design standards

Figure 3.1 (a) shows the result of flexural strength prediction for strand type A in ACI 318-14 and CSA A23.3-14. In the 1,860 MPa stress-relieved PS strand, CSA predicts flexural strength more conservative than ACI, and both design standards predicts flexural strength well. At the point where c/d_p=0.5, the ratio of flexural strengths for each design standard is $M_{n,ACI}/M_{n,SA}$=1.02 in ACI 318-14, and $M_{n,CSA}/M_{n,SA}$=0.99 in CSA A23.3-14. That is, the error of flexural strength at where c/d_p=0.5 is 2% in ACI 318-14 and 1% in CSA A23.3-14.

Figure 3.1 (b) shows the result of flexural strength prediction for strand type B in ACI 318-14 and CSA A23.3-14. In the case of ACI 318-14, the design code predicts flexural strength non-conservative because ACI over-estimates the strand stress at pre-yielding range. In the case of CSA A23.3-14, it predicts flexural strength more conservative than ACI, so it predicts flexural strength more accurate than ACI. At the point where c/d_p=0.5, the ratio of flexural strengths for each design standard is $M_{n,ACI}/M_{n,SA}$=1.06 in ACI 318-14, and $M_{n,CSA}/M_{n,SA}$=1.00 in CSA A23.3-14. Therefore, the error of flexural strength at c/d_p=0.5 is 6% in ACI 318-14 and 0% in CSA A23.3-14.

Figure 3.1 (c) shows the result of flexural strength prediction for strand type C in ACI 318-14 and CSA A23.3-14. For the high-strength PS strand, both design codes predict flexural strength non-conservative because they over-estimate the strand stress at pre-yielding range. At the point where c/d_p=0.5, the ratio of flexural strengths for each design standard is $M_{n,ACI}/M_{n,SA}$=1.08 in ACI 318-14, and $M_{n,CSA}/M_{n,SA}$=1.06 in CSA A23.3-14. As a result, the error of flexural strength at where c/d_p=0.5 is 8% in ACI 318-14 and 6% in CSA A23.3-

14.

Figure 3.1 (d) shows the result of flexural strength prediction for strand type D in ACI 318-14 and CSA A23.3-14. For the high-strength PS strand, both design codes also predict flexural strength non-conservative because they over-estimate the strand stress at pre-yielding range. At the point where c/d_p=0.5, the ratio of flexural strengths for each design standard is $M_{n,ACI}/M_{n,SA}$=1.08 in ACI 318-14, and $M_{n,CSA}/M_{n,SA}$=1.06 in CSA A23.3-14. That is, the error of flexural strength at where c/d_p=0.5 is 8% in ACI 318-14 and 6% in CSA A23.3-14.

As a result of flexural strength analysis, CSA A23.3-14 predicts flexural strength more accurate than ACI 318-14. However, CSA A23.3-14 does not consider the nonlinear stress behavior at pre-yielding range of the high-strength PS strands. Therefore, the error of flexural strength for the high-strength PS strand increases. To predict flexural strength of member with the high-strength PS strands, the design equation should approximate the PS strand stress considering nonlinear stress behavior of PS strand at pre-yielding range.

Chapter 4 Proposal of an Approximate Equation

4.1 Consideration of Pre-yielding Behavior

As a result of the analysis shown in chapter 2 and 3, CSA A23.3-14 is more appropriate for the high-strength PS strands than ACI 318-14. However, all existing design standards does not predict PS strand stress at pre-yielding range of high-strength PS strands. In ACI 318-14, PS strand stress at pre-yielding range should be predicted as the amount of PS strands increases, because there is no limitation in the amount of PS strands at the calculation of flexural strength. In CSA A23.3-14, there is a limitation in the amount of PS strands at the calculation of flexural strength. Though there is limit on the amount of PS strands, CSA over-estimates PS strand stress at the part of pre-yielding range from yield point to limit point of PS strand as above-mentioned in 2.2.2.3. Thus, CSA A23.3-14 Eq. (18.1) should be revised at the pre-yielding range of the strain from the proportional limit to yield point as shown in Figure 4.1.

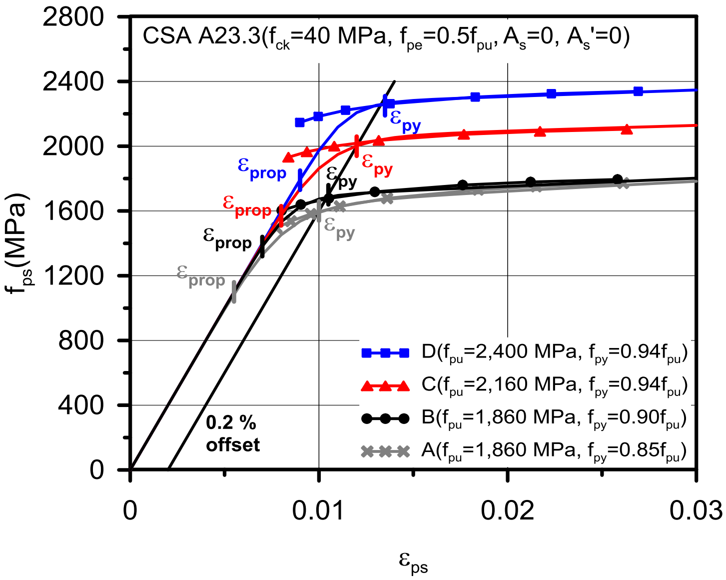


Fig. 4.1 Stress-strain relationships of PS strands at CSA A23.3

As it is shown in 2.2.1.3 Figure 2.5, stress behavior of PS strand is irrelevant to yield ratio at pre-yielding range. Considering these properties, an approximate equation for PS strand stress at flexural strength of a member is proposed by classifying the section into pre-yielding range and post-yielding range and adapting different equation in each section to predict the stress behavior at pre-yielding range.

In the proposed approximate equation for PS strand stress at flexural strength of a member, for $f_{pe}>0.5f_{pu}$, PS strand stress in the range where $\varepsilon_{ps} \geq \varepsilon_{py}$ is defined as following.

$$\frac{f_{ps}}{f_{pu}} = \left(1 - k_p\left(\frac{c}{d_p}\right)\right) \tag{4.1}$$

The coefficient is as following,

$$k_p = 2\left(1.04 - \left(\frac{f_{py}}{f_{pu}}\right)\right) \tag{4.2}$$

For the post-yielding range where PS strand strain in flexural strength is higher than yield strain at material model of PS strand, CSA A23.3-14 Eq. (18.1) is used.

For $f_{pe}>0.5f_{pu}$, PS strand stress in the range where $\varepsilon_{py}>\varepsilon_{ps}\geq\varepsilon_{prop}$ is defined as following.

$$\frac{f_{ps}}{f_{pu}} = \left(1 - k_e\left(\frac{c}{d_p}\right)\right) + \frac{2}{3}\left(k_e - k_p\right)\left(\frac{c_{py}}{d_p}\right) \tag{4.3}$$

The coefficient is as following,

$$k_e = 0.58 \tag{4.4}$$

$$\frac{c_{py}}{d_p} = \frac{0.003}{\varepsilon_{py} - (\varepsilon_{pe} - 0.0027)} \tag{4.5}$$

For the pre-yielding range where PS strand strain in flexural strength is between the strain at proportional limit and yield point of the material model of PS strand, the coefficient k_e is a constant as indicated in Eq. (4.4) in order to let the stress behavior be irrelevant to yield ratio. Also, using Eq. (4.5) which indicates the ratio of the distance from extreme compression fiber to neutral axis and the distance from extreme compression fiber to centroid of prestressing reinforcement at yield strain, a correction term related to prevention of discontinuity at yield point is added to Eq. (4.3).

The coefficient k_e=0.58 is obtained by striking an average of optimized coefficient values with the minimum sum of squared error for total four types of PS strands, and the values are obtained by non-linear regression analysis at the range from proportional limit to yield point. The results of non-linear regression for the coefficient k_e are shown in Table 4.1.

Table 4.1 Non-linear regression results for the coefficient k_e

Strand type	Tensile strength (MPa)	Yield ratio	Optimized k_e	R^2
A	1,860	0.85	0.59	1.00
B	1,860	0.90	0.59	1.00
C	2,160	0.94	0.57	0.99
D	2,400	0.94	0.58	0.99
Average			0.58	0.99

At the elastic range before proportional limit, the approximate equation for PS strand stress at flexural strength is not defined because PS strand stress at flexural strength can be analyzed using elastic analysis without iteration in calculation.

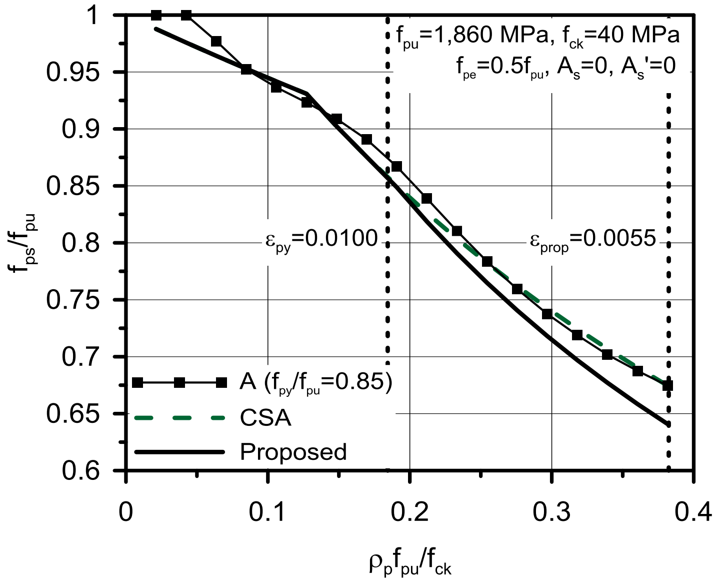
4.2 Validation of a Proposed Equation

To validate the proposed approximate equation for PS strand stress at flexural strength of a member, the results of sectional analysis and the proposed equation are compared as shown in Figure 4.2.

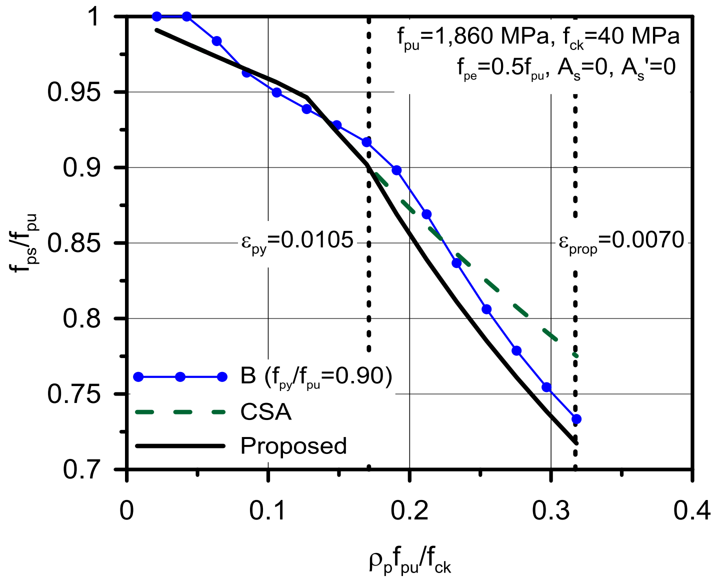
For the strand type A and B as shown in Figure 4.2 (a) and (b), the proposed equation predicts PS strand stress at flexural strength of a member same with existing CSA A23.3-14 Eq. (18.1) in the post-yielding range. In the pre-yielding range from proportional limit to yield point, the proposed equation predicts PS strand stress more conservative than CSA A23.3-14 Eq. (18.1).

For the strand type C indicating high-strength PS strand as shown in Figure 4.2 (c), the proposed equation predicts PS strand stress at flexural strength of a member same as CSA A23.3-14 Eq. (18.1) in the post-yielding range. In the pre-yielding range from proportional limit to yield point, the proposed equation predicts PS strand stress more accurate than existing CSA A23.3-14 Eq. (18.1) that over-estimates stress of high-strength PS strands. This is because the proposed equation considers the non-linear stress behavior of the high-strength PS strands.

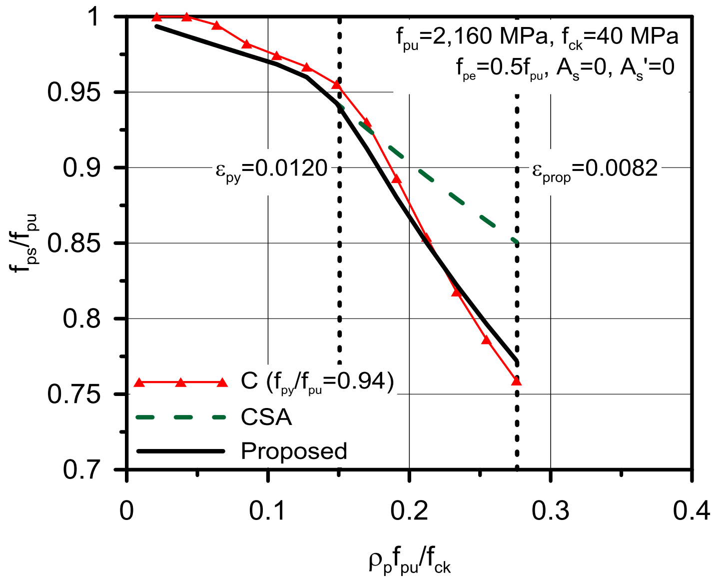
For the strand type D indicating high-strength PS strand as shown in Figure 4.2 (d), the proposed equation predicts PS strand stress at flexural strength of a member in pre-yielding range more accurate than CSA A23.3-14 Eq. (18.1) as same as the case of strand type C.



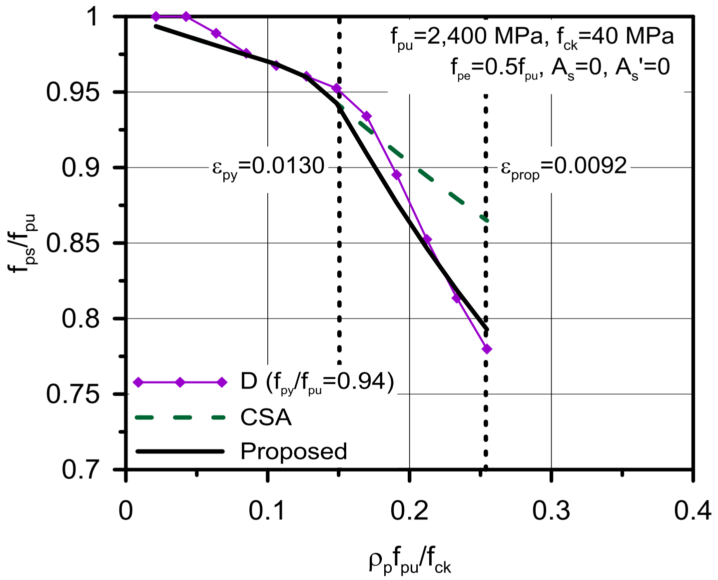
(a) Strand type A



(b) Strand type B



(c) Strand type C



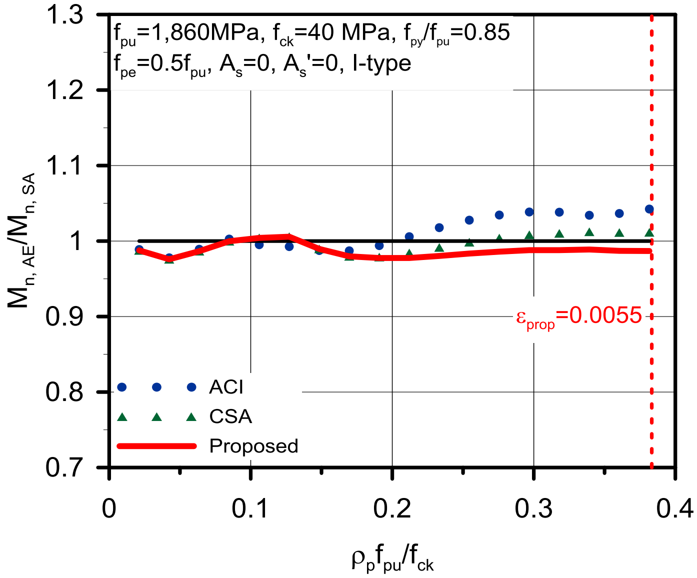
(d) Strand type D

Fig. 4.2 Strand stress at flexural strength with the proposed equation

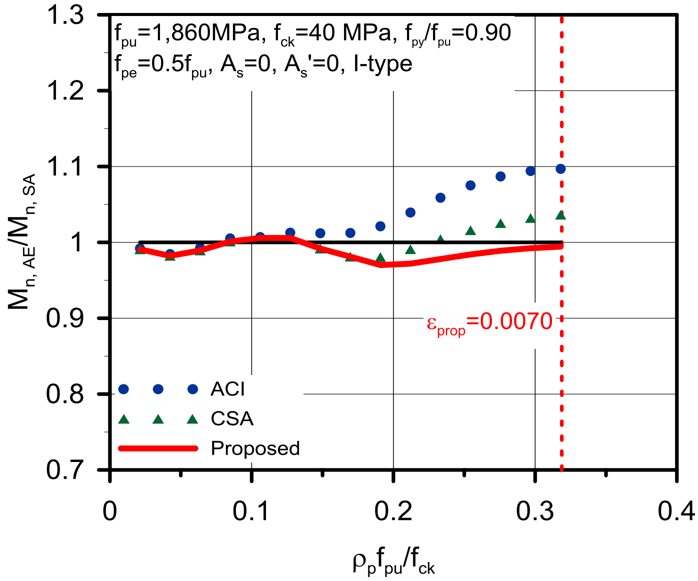
4.3 Flexural Strength Analysis

Flexural strength of a member with each strand type was obtained using the proposed approximate equation for PS strand stress. After that the errors of flexural strength obtained from ACI 318-14, CSA A23.3-14, and the proposed equation were compared and the results of flexural strength analysis are shown in Figure 4.3. In the graph, x axis indicates the reinforcement index for prestressing reinforcement, and y axis indicates the ratio of flexural strength obtained from the sectional analysis to flexural strength obtained from approximate equations. The reinforcement index corresponding to the strain at proportional limit is plotted with the dotted line. In the graph, because the proposed equation predicts PS strand stress at pre-yielding range more accurate than existing equations, the error of flexural strength obtained from the proposed equation is much lower than that from the existing equations especially in high-strength PS strands.

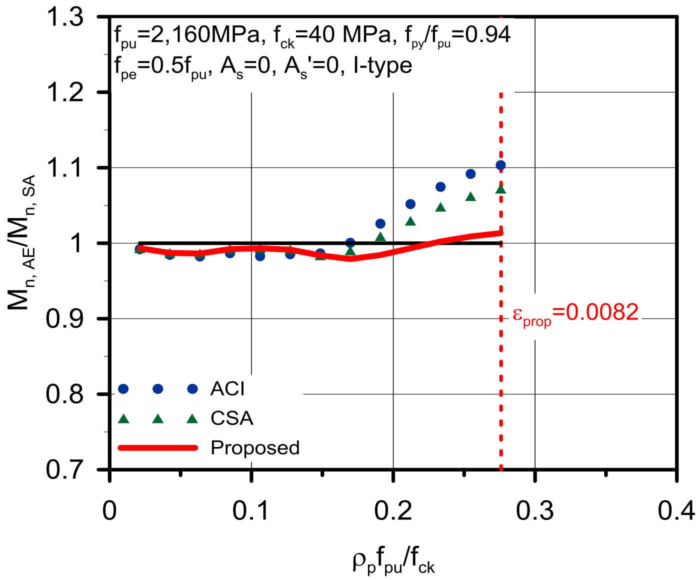
Figure 4.3 (a) and (b) show the result of flexural strength error obtained from the proposed equation for strand type A and B. For 1,860 MPa PS strands, the proposed equation predicts flexural strength similar with CSA A23.3, and it predicts flexural strength more conservative than both design codes. The ratio of flexural strengths for the proposed equation is $M_{n,proposed}/M_{n,SA}$=0.99 at ε_{prop}=0.0055 for the strand type A, $M_{n,proposed}/M_{n,SA}$=0.99 at ε_{prop}=0.0070 for the strand type B. That is, the error of flexural strengths at proportional limit strain is all 1% in the proposed equation.



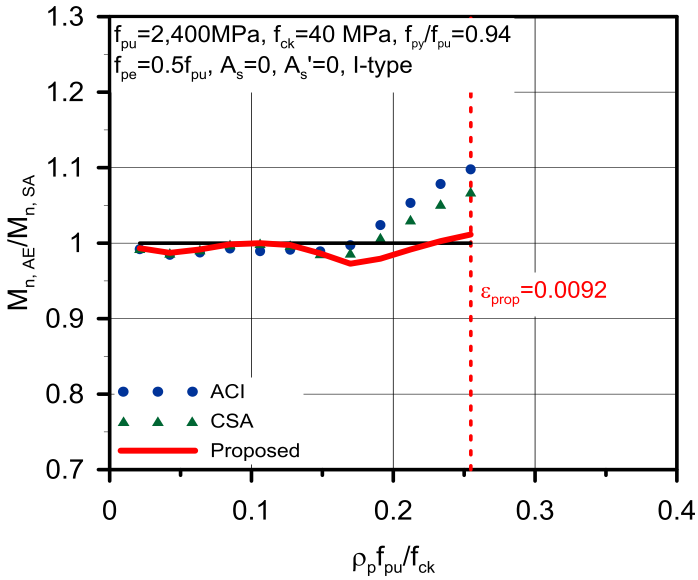
(a) Strand type A



(b) Strand type B



(c) Strand type C



(d) Strand type D

Fig. 4.3 Result of flexural strength with the proposed equation

Figure 4.3 (c) and (d) show the result of flexural strength error obtained from the proposed equation for strand type C and D indicating high-strength PS strands. For the high-strength PS strands, the proposed equation predicts flexural strength more accurate than both ACI 318-14 and CSA A23.3-14, especially at pre-yielding range. The ratio of flexural strengths for the proposed equation is $M_{n,proposed}/M_{n,SA}$=1.01 at ε_{prop}=0.0082 for the strand type C, and $M_{n,proposed}/M_{n,SA}$=1.01 at ε_{prop}=0.0092 for the strand type D. That is, the error of flexural strengths at proportional limit strain is all 1% in the proposed equation.

Chapter 5 Conclusion

This study analyzed the applicability of approximate equations used in ACI 318-14 and CSA A23.3-14 for the high-strength PS strand stress in terms of strand stress and flexural strength. ACI 318-14 does not predict the increase in PS strand stress at post-yielding range according to the increase in yield ratio. Also, ACI does not consider the effect of sectional shape so it overestimates the PS strand stress as the amount of PS strands increases. Moreover, ACI predicts PS strand stress non-conservative because it does not consider non-linear stress behavior of PS strands at pre-yielding range. CSA A23.3-14 considers the influence of yield ratio and sectional shape in the equation. However, CSA does not consider non-linear stress behavior of PS strands at pre-yielding range. Thus, an approximate equation for PS strand stress at flexural strength which considers stress behavior at pre-yielding range is proposed. The section is classified into the pre-yielding and the post-yielding range, and a different approximate equation is proposed for each range to predict stress behavior for all range. With the proposed equation, errors of flexural strength for a member using high-strength PS strands reduced.

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국문초록

현재 전 세계적으로 프리스트레스 부재에 사용되고 있는 PS 강연선의 인장강도는 1,860 MPa이다. 최근 국내에서는 기존에 사용되고 있는 PS 강연선보다 인장강도를 증가시킨 2,160 MPa 및 2,400 MPa 급 고강도 PS 강연선을 개발하였다. 이에 따라 고강도 PS 강연선의 사용을 허용하였고, 이후 PSC 구조물에 고강도 PS 강연선의 도입이 점차 증가하고 있는 추세이다. 하지만 기존의 국내 및 국외의 설계기준에서는 강연선의 고강도화에 따른 영향을 반영하지 못하고 있다.

따라서 본 연구에서는 고강도 PS 강연선에 대하여 기존 설계기준들의 강연선 응력 근사식의 적합성 평가를 실시하여 기존 설계 식의 적용 가능성을 검토하였다. 그 결과, 기존 설계기준들은 고강도 PS 강연선의 항복 이전의 비선형 응력 거동을 예측하지 못하였다. 따라서 항복 이전의 고강도 PS 강연선의 비선형 응력 거동을 고려하는 휨 강도에서의 PS 강연선 응력 근사식을 제안하였다. 제안 식을 통해 고강도 PS 강연선을 사용한 경우에 대해서도 휨 강도와 강연선 응력을 정확히 예측할 수 있게 된다.

주요어: ACI 318-14, CSA A23.3-14, 휨 강도, 고강도 PS 강연선, PS 강연선 응력

학 번: 2013-20929