Force Identification for Nonlinear Structures using the Dynamic Response Sensitivity

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Abstract

Force identification is an inverse procedure that estimates the applied forces based on the measured dynamic responses. This paper introduces a novel force identification method for nonlinear finite element models via sensitivity of dynamic response with respect to the load parameters in the time domain. Unlike conventional state space method, the formulation introduced in this study is based on the implicit Newmark method and Newton-Raphson algorithm to solve nonlinear flexible dynamic problems. Since the sensitivities at respective discretized time step are evaluated in geometrically nonlinear framework, robust and accurate prediction of the exciting force in nonlinear problems can be made. In order to demonstrate the effectiveness and validity of the presented method, nonlinear floating beam and 3D shell problems with various loading conditions are provided in this paper. The results show that the proposed method can accurately determine the magnitude of the input forces even with the presence of the noise effect.

Keyword : Force identification, Sensitivity analysis, Nonlinear, Finite element analysis, Newmark method

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Chapter 1. Introduction

Determination of the excitation input based on the measured dynamic responses becomes significantly important in the study of finite element analysis especially when the external force cannot be directly measured. There are mainly two categories for the force identification methods; the frequency domain approach and the time domain approach. The frequency domain method uses the frequency response functions and identifies the forces in the frequency domain. If the force needs to be determined in time domain, then inverse Fourier transform must be used [1]. In contrast, the time domain method uses the dynamic responses and directly identifies the force history in time domain. The time domain approach can avoid any transform error and be suitable for any types of forces [2].

There are three major types of the force identification methods in time domain; the state−space method, the sensitivity method and the genetics algorithm. The state−space method converts the finite element into the state space model. Ma and Chang [3] used state−space based method with Kalman filter and a recursive least−squares algorithm. The sensitivity method directly uses the finite element model to obtain the dynamic responses and their sensitivity with respect to the force parameters. Lu and Law [4] used the response sensitivity and the penalty functions for linear problems. The genetics algorithm is widely used as a global optimization technique [5]. Yan and Zhou [6] introduced the genetic algorithm−based method to adaptively identify the location and the history of the external force with its global search capability.

This paper introduces a novel method to reconstruct the history of the unknown force using the dynamic responses and their sensitivity. The sensitivity−based force identification method proposed by Lu and Law [7] is applied only in linear truss problems. This study further develops the sensitivity model constructed based on the nonlinear governing equations of motion using direct derivative method (DDM). The force identification technique proposed in this study can be applied to any structural analysis, such as static and dynamic conditions, linear and nonlinear problems, and single- and multi-degrees of freedoms. It is also possible to accurately predict the total force when the structure is pre-stressed.

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Chapter 2. Method

A flow chart is sketched in Fig.1 to show the overview of the force identification process that approximates the magnitude of the input force using the dynamic response and their sensitivities with respect to the force parameters. The procedure of the force identification consists of two main steps: 1) the governing equation of motion and 2) the recursive estimation. The governing equations of motion solve for the dynamic responses of the displacement, velocity and acceleration and their sensitivities with respect to the given input force parameters. The recursive estimation iteratively updates the force parameters until the error between the actual dynamic responses and the identified dynamic response comes within the tolerance range.

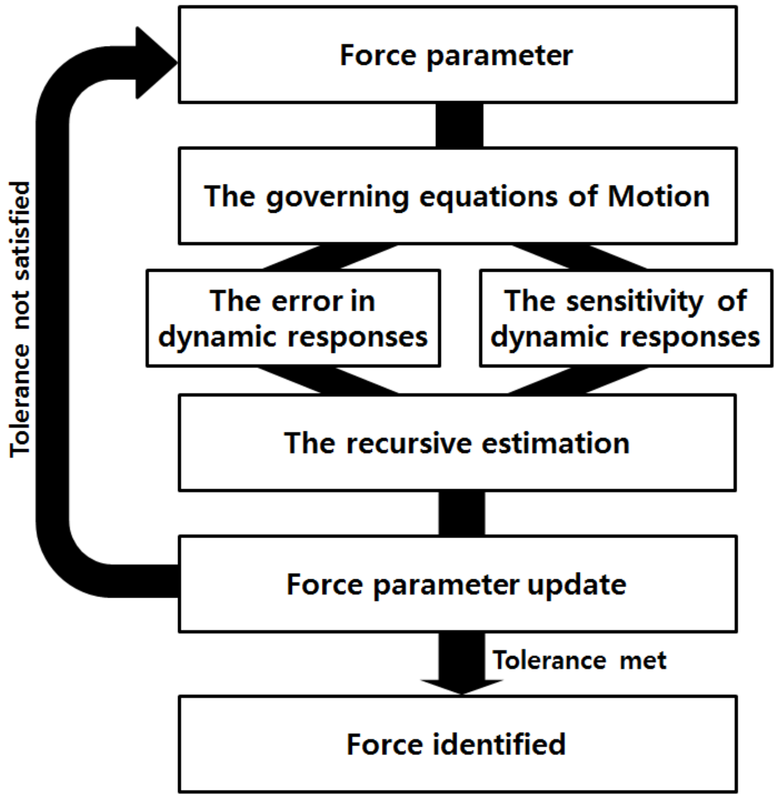


Figure 1. Flow chart of force identification via the dynamic response sensitivity with respect to the force parameters

2.1. The governing equation of motion

The first step of force identification is obtaining the dynamic responses of the structure by performing finite element analysis. For nonlinear dynamic problems, the governing equation can be written as

$$\mathbf{M}(\mathrm{p})\ddot{\mathbf{u}}(\mathrm{t},\mathrm{p}) + \mathbf{C}(\mathrm{p})\dot{\mathbf{u}}(\mathrm{t},\mathrm{p}) + \mathbf{r}(\mathbf{u}(\mathrm{t},\mathrm{p}),\mathrm{p}) = \mathbf{f}(\mathrm{t},\mathrm{p}) \tag{1}$$

where t is the time and p is the load parameter. $\mathbf{M}$ and $\mathbf{C}$ denote the mass matrix and the damping matrix. $\mathbf{r}$ is the internal force vector and $\mathbf{f}$ is the external force vector. $\mathbf{u},\dot{\mathbf{u}},\ddot{\mathbf{u}}$ are the vectors of displacement, velocity, acceleration , respectively. All the terms in the nonlinear equation of motion are dependent on the external load parameter.

The j^{th} input force can be formulated with a parameter set of a constant, sine series and cosine series as follows

$$\mathbf{f}^{\mathrm{j}}(\mathrm{t},\mathrm{p}) = \mathrm{P}_0^{\mathrm{j}} + \sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\mathrm{P}_{\mathrm{s,i}}^{\mathrm{j}}\sin(\omega_{\mathrm{s,i}}^{\mathrm{j}}\mathrm{t}) + \mathrm{P}_{\mathrm{c,i}}^{\mathrm{j}}\cos(\omega_{\mathrm{c,i}}^{\mathrm{j}}\mathrm{t})\right) \tag{2}$$

Here, $\mathbf{f}^{\mathrm{j}}$ is the j^{th} component of the external force vector. $\mathrm{P}_0^{\mathrm{j}}$ is the constant parameter. $\mathrm{P}_{\mathrm{s,i}}^{\mathrm{j}}$ is the amplitude parameter and $\omega_{\mathrm{s,i}}^{\mathrm{j}}$ is the frequency parameter of the i^{th} sine term in the j^{th} force component. Similarly, $\mathrm{P}_{\mathrm{c,i}}^{\mathrm{j}}$ and $\omega_{\mathrm{c,i}}^{\mathrm{j}}$ are amplitude and frequency parameters of the i^{th} cosine series in the j^{th} force component. By formulating the excitation force using a combination of multiple sine and cosine series, it is possible to express any force history curve in the time domain.

The nonlinear governing equation can be solved by using an incremental iterative algorithm, such as Newton-Raphson method and can be integrated numerically by using the constant-average acceleration method of Newmark-β method with $\alpha = 1/4$ and

$\beta = 1/2$. Here, α and β are the parameters that controls the accuracy and stability of the direct integration method.

2.2. The sensitivity model

The sensitivity of dynamic response can be obtained by directly differentiating the nonlinear governing equation in Eq. (1) with respect to the external load parameter $\mathbf{P}$ as follows

$$\mathbf{M}\frac{\partial \ddot{\mathbf{u}}}{\partial \mathrm{P}_n^j} + \mathbf{C}\frac{\partial \dot{\mathbf{u}}}{\partial \mathrm{P}_n^j} + \mathbf{K_T}\frac{\partial \mathbf{u}}{\partial \mathrm{P}_n^j} = \frac{\partial \mathbf{f}}{\partial \mathrm{P}_n^j} - \frac{\partial \mathbf{M}}{\partial \mathrm{P}_n^j}\ddot{\mathbf{u}} - \frac{\partial \mathbf{C}}{\partial \mathrm{P}_n^j}\dot{\mathbf{u}} - \left.\frac{\partial \mathbf{r}}{\partial \mathrm{P}_n^j}\right|_{\mathbf{u}} \tag{3}$$

where $\frac{\partial \ddot{\mathbf{u}}}{\partial \mathrm{P}_n^j}, \frac{\partial \dot{\mathbf{u}}}{\partial \mathrm{P}_n^j}, \frac{\partial \mathbf{u}}{\partial \mathrm{P}_n^j}$ represent the sensitivities of acceleration, velocity, and displacement, respectively, with respect to the n^{th} parameter $\mathbf{P}_n$ of the j^{th} force component. $\frac{\partial \mathbf{f}}{\partial \mathrm{P}_n^j}$ is the sensitivity of the external load. The derivative of internal force vector can be expressed as $\frac{\partial \mathbf{r}}{\partial \mathrm{P}_n^j} = \mathbf{K_T}\frac{\partial \mathbf{u}}{\partial \mathrm{P}_n^j} + \left.\frac{\partial \mathbf{r}}{\partial \mathrm{P}_n^j}\right|_{\mathbf{u}}$ where $\mathbf{K_T}$ is the tangent stiffness matrix and $\left.\frac{\partial \mathbf{r}}{\partial \mathrm{P}_n^j}\right|_{\mathbf{u}}$ is the sensitivity of internal force with respect to parameter with fixed $\mathbf{u}$ (Conte at el 2003). To gather all the dynamic response sensitivity terms on one side, the $\mathbf{K_T}\frac{\partial \mathbf{u}}{\partial \mathrm{P}_n^j}$ term is placed on the left side and $\left.\frac{\partial \mathbf{r}}{\partial \mathrm{P}_n^j}\right|_{\mathbf{u}}$ is moved on the right side of Eq. (3). The analytical process for the mass gradient $\frac{\partial \mathbf{M}}{\partial \mathrm{P}_n^j}$ and the damping gradient $\frac{\partial \mathbf{C}}{\partial \mathrm{P}_n^j}$ has been derived by Haukass and

Kiureghian (2005). However, since only the geometric nonlinear effect is considered in this study, the assumption that only the stiffness matrix is dependent on the parameters and the gradient can be applied to the sensitivity equation and eq. (3) can redefined as

$$ {}^{t}\mathbf{M_T}\frac{\partial \ddot{\mathbf{u}}}{\partial P_n^j} + {}^{t}\mathbf{C_T}\frac{\partial \dot{\mathbf{u}}}{\partial P_n^j} + {}^{t}\mathbf{K_T}\frac{\partial \mathbf{u}}{\partial P_n^j} = \frac{\partial \mathbf{f}}{\partial P_n^j} \tag{4} $$

$\mathbf{M_T}, \mathbf{C_T}$ and $\mathbf{K_T}$ are the tangent matrices of the mass, damping and the stiffness, which can be computed at each time step using Newton−Raphson iterations. The dynamic response sensitivities $\frac{\partial \ddot{\mathbf{u}}}{\partial P_n^j}$, $\frac{\partial \dot{\mathbf{u}}}{\partial P_n^j}$, and $\frac{\partial \mathbf{u}}{\partial P_n^j}$ can be obtained by using Newmark−β method ($\alpha = 1/4$ and $\beta = 1/2$).

2.3. The recursive estimation

The second step for force identification is to approximate the unknown input force by iteratively updating the parameters. The dynamic response and their sensitivities are now computed at every time step, the error between the calculated acceleration and the actual acceleration can be found using the following equation.

$$ \partial \ddot{\mathbf{u}} = \ddot{\mathbf{u}}_\mathbf{m} - \ddot{\mathbf{u}}_\mathbf{i} \tag{5} $$

$\ddot{\mathbf{u}}_\mathbf{m}$ is the vector of acceleration data extracted from the reference simulator and $\ddot{\mathbf{u}}_\mathbf{i}$ is the vector of the identified acceleration computed by using Eq.(1). $\partial \ddot{\mathbf{u}}$ is the difference of measured and identified outputs.

After obtaining the dynamic responses, their sensitivity with respect to the force parameter and the error compared to the reference response, the input force parameters can now be updated by using the damped least−squares method, also known as the

Levenberg–Marquardt method.

$$\mathbf{P}^{k+1} = \mathbf{P}^{k} + \left[\mathbf{S}^{T}\mathbf{S} + \lambda\mathbf{I}\right]^{-1}\mathbf{S}^{T}\left(\partial\ddot{\mathbf{u}}\right) \tag{6}$$

Here, $\mathbf{S}$ is the two–dimensional acceleration sensitivity matrix with respect to the force parameter in the time domain. For instance, if a structure is subjected to an input force consisting of N numbers of parameters during T time steps and the reference acceleration is available in M numbers of degree of freedom, then the sensitivity matrix $\mathbf{S}$ has M×T number of rows and N number of columns. $\mathbf{P}^{k}$ denotes the force parameter at k^{th} iteration and λ represents the damping coefficient. This damping coefficient is initialized by using the following equation

$$\lambda_0 = \tau \times \max(\mathrm{diag}(\mathbf{S}^{\mathbf{T}}\mathbf{S})) \tag{7}$$

τ is a user–specified value between 10^{-3} and 1. The damping coefficient is updated at each iteration based on the gain ratio G whose equation is as

$$\mathrm{G} = \frac{\partial\ddot{\mathbf{u}}^{k} - \partial\ddot{\mathbf{u}}^{k+1}}{\frac{1}{2}\mathrm{d}\mathbf{P}^{T}(\lambda\mathrm{d}\mathbf{P} - \mathbf{S}^{T}\partial\ddot{\mathbf{u}})} \tag{8}$$

$$\mathrm{d}\mathbf{P} = \left[\mathbf{S}^{T}\mathbf{S} + \lambda\mathbf{I}\right]^{-1}\mathbf{S}^{T}\left(\partial\ddot{\mathbf{u}}\right) \tag{9}$$

Here, $\mathrm{d}\mathbf{P}$ is the difference between $\mathbf{P}^{k+1}$ and $\mathbf{P}^{k}$. It can be computed using Eq.(9). When the gain ratio is greater than zero, it means that the iteration made a worse approximation and λ is increased. If the gain ratio is negative, λ becomes smaller.

2.4. Time discretization for extreme conditions

While the modified sensitivity formula can successfully determine the exciting force without going through a heavy computation in most nonlinear problems, it may result in a wrong approximation in some extreme cases. The modified nonlinear sensitivity formula assumes that the mass, damping and the internal force does not highly depend on the parameter and treats their derivatives negligible. However, in case when a highly flexible structure experiencing a large rotation or deflection, neglecting those terms accumulate errors in the dynamic sensitivity and cause the solution to fall into a wrong local minimum or disconverge. To improve the results by avoiding the error accumulation, the time discretization method needs to be applied. The basic concept of the time discretization is to divide the entire simulation time into several divisions and make parameter estimations for each division through iterations.

The force identification process for each time discretization is as follows

1. Collect the acceleration data from the reference simulation model for the n^{th} discretization.
2. Set the initial force parameter $\mathbf{P}^k$ and obtain the dynamic response of the displacement $\mathbf{u}$, velocity $\dot{\mathbf{u}}$ and acceleration $\ddot{\mathbf{u}}$ and the tangent matrices of the mass $\mathbf{M_T}$, the damping $\mathbf{C_T}$ and the stiffness $\mathbf{K_T}$ at each time step from Eq. (1).
3. Apply $\mathbf{M_T}, \mathbf{C_T}$ and $\mathbf{K_T}$ into Eq. (4) to obtain the sensitivity of displacement, velocity and acceleration with respect to the input force parameter; $\frac{\partial \ddot{\mathbf{u}}}{\partial \mathrm{P}}, \frac{\partial \dot{\mathbf{u}}}{\partial \mathrm{P}}$ and $\frac{\partial \mathbf{u}}{\partial \mathrm{P}}$ from Eq. (4).
4. Compute the acceleration difference vector $\boldsymbol{\partial}\ddot{\mathbf{u}}$ between the reference simulator and the force identification numerical model by using Eq. (5)

5. Update the force parameter vector $\mathbf{P}^{k+1}$ using Eq. (6)−(9).
6. Repeat step 2−6 until at least one of the following three conditions is satisfied.
    1) The acceleration difference between the reference model and the force identification model is smaller than the user specified tolerance.
    2) The change of the force parameter from the previous iteration to the current iteration is smaller than the tolerance.
    3) The number of iteration has reached the maximum number.
7. After the iteration has been completed in the n^{th} discretization and the solution has converged, repeat step 1−6 until the final discretization.
8. Combine the force data of each discretization to get one complete force history.

Discretizing the dynamic motion in the time domain saves the total computational time because for each iteration, the dynamic response calculation needs to be performed only within a short time period. Once the satisfactory parameter estimation is made in the previous division, then only a few iteration need to be run afterwards.

Chapter 3. Numerical examples

The reliability of the force identification proposed in this study is examined with a simple beam, a multi−body system and a 3D shell finite element model under different force conditions in each case. Comparator simulators are also constructed using the same finite element models as the identification models for all cases. Dynamic responses of the comparator simulator subjected to the user specified input force are used as reference to compute the error of the identification model at each iteration to update the load parameters. The comparator is also used as reference to verify the accuracy of the results determined from the identification simulator. In this study, the comparator and the identified simulator used the finite element and numerical algorithm, which is based on Newton−Raphson iterations and Newmark−β method with $\alpha = 1/4$ and $\beta = 1/2$. To demonstrate geometric nonlinear dynamic simulations, co−rotational beams with 6 degrees−of−freedom (DOF) for each node are used. In 3D shell analysis, MITC4 shell consisting of 5DOF for each node is used.

3.1. A nonlinear beam

A flexible nonlinear beam consisting of ten elements is subjected to a concentrated force in the direction of y−axis at the second node from the left. The beam dimension is 0.1X0.05X10 as shown in Fig. 2. The length of the beam is relatively very long to this cross sectional area to emphasize the flexible motion of the structure. The elastic modulus is $E=1/2 \times 10^{11}$Pa and the density is $\rho=7860 kg/m^3$. The effect of damping of the beam is assumed to be negligible for this example.

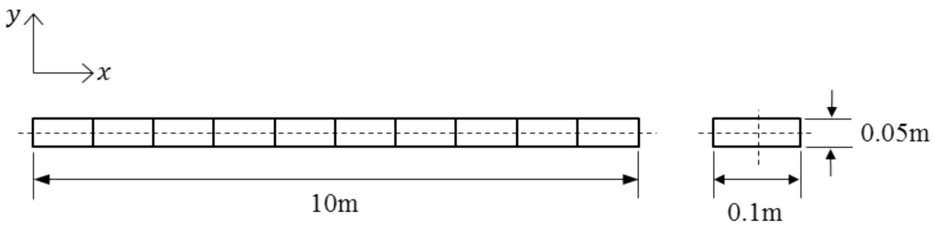
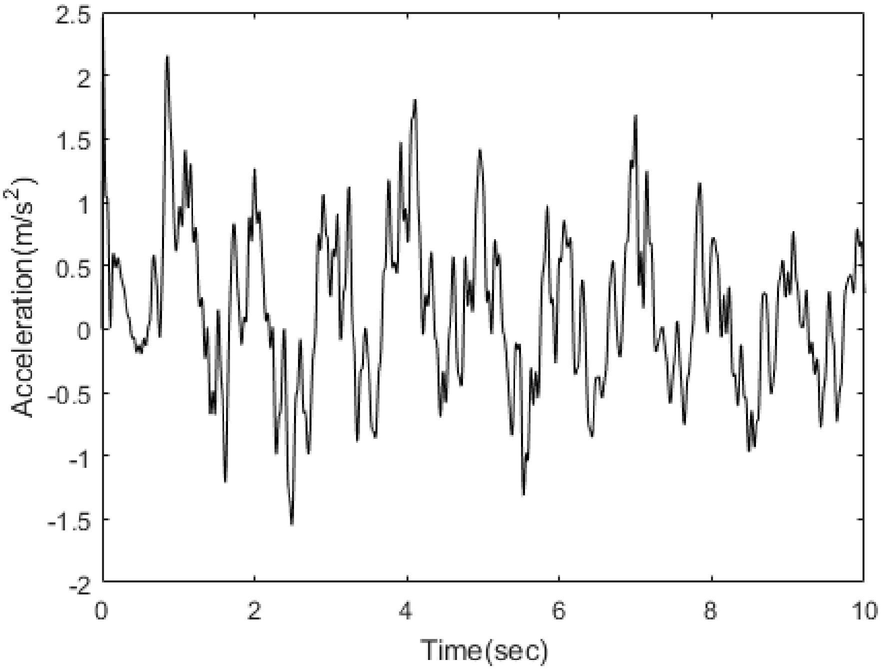
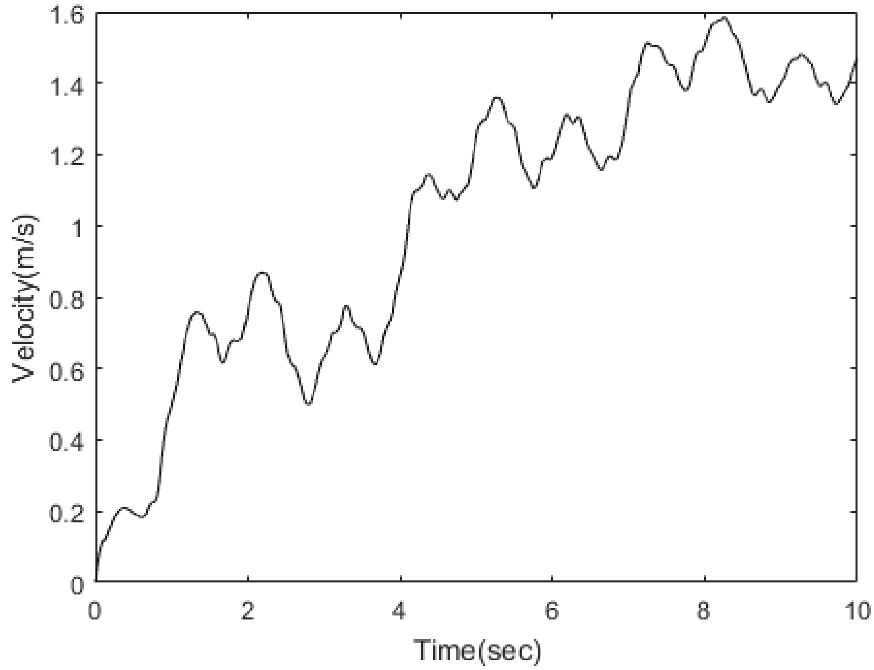


Figure 2. A simple co-rotational beam consisting of ten elements with 6 DOF for each node.

With time step size of 0.01 seconds, a random force of $F = 50 + 100\sin(0.5\pi t) + 150\cos(2\pi t)$ is applied to the reference simulator for 10 seconds. Fig. 3 is the dynamic responses of displacement, velocity and acceleration of the comparator. The acceleration from the reference simulator is needed for two purposes. First, the reference acceleration data in the time domain can be used as the initial force frequency parameters after converting into the frequency domain by using Fast Fourier Transform(FFT). FFT converts the acceleration history in the time domain into the frequency domain and finds its natural frequencies, which can be set as the initial frequency parameters of the sine and cosine series of the input force in Eq. (2). Second, it can be used as reference to find the difference in dynamic responses between the identified system and the comparator. In this study, the error in acceleration between two simulators is used when updating the load parameters in Eq. (6). The natural frequencies extracted from FFT algorithm is illustrated in Fig. 4.


Acceleration(m/s2)
Time(sec)
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
2
4
6
8
10


Velocity(m/s)
Time(sec)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
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2
4
6
8
10

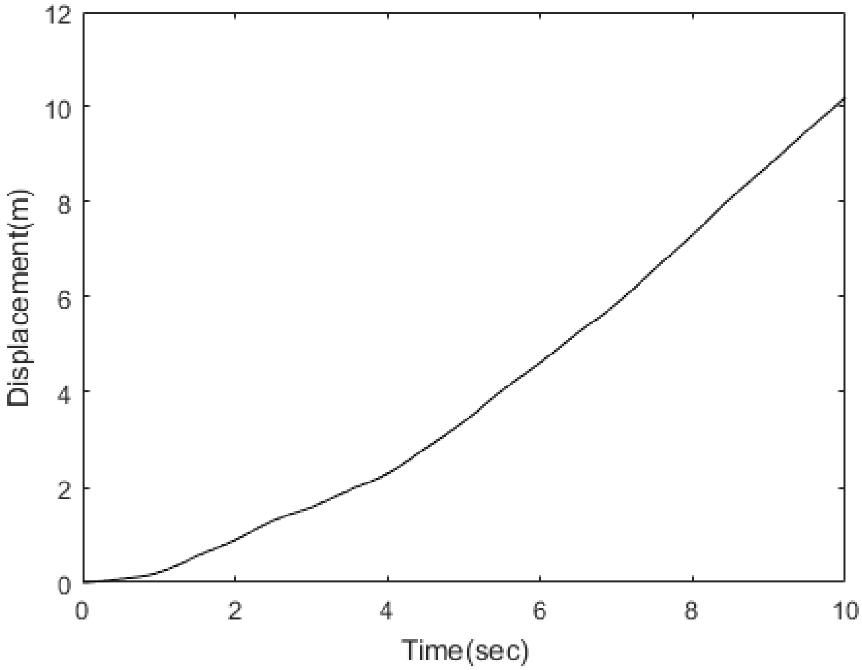


Figure 3. The dynamic responses of displacement, velocity and acceleration of the reference simulator of simple nonlinear beam

The major six natural frequencies extracted from FFT are 0.58π, 1.95π, 4.88π, 6.83π, 9.17π and 11.7π, and they are set as the initial frequency parameters of the input force The initial force parameter set becomes a 25X1 vector, which includes one constant parameter, six amplitudes of sine, six amplitudes of cosine, six frequencies of sine and six frequencies of cosine series. The constant parameter is initially set 60 and all amplitudes are set 110 for sine and cosines. $\mathbf{p} = [p_0, p_{s,1} \cdots p_{s,6}, p_{c,1} \cdots p_{c,6}, \omega_{s,1} \cdots, \omega_{s,6}, \omega_{c,1}, \cdots \omega_{c,6}]^T = [60,\ 110 \cdots 110,\ 110 \cdots 110,\ 0.58\pi,\ 1.95\pi,\ 4.88\pi,\ 6.83\pi,\ 9.17\pi,\ 11.7\pi,\ 0.58\pi,\ 1.95\pi,\ 4.88\pi,\ 6.83\pi,\ 9.17\pi,\ 11.7\pi]^T$ is the parameter vector used.

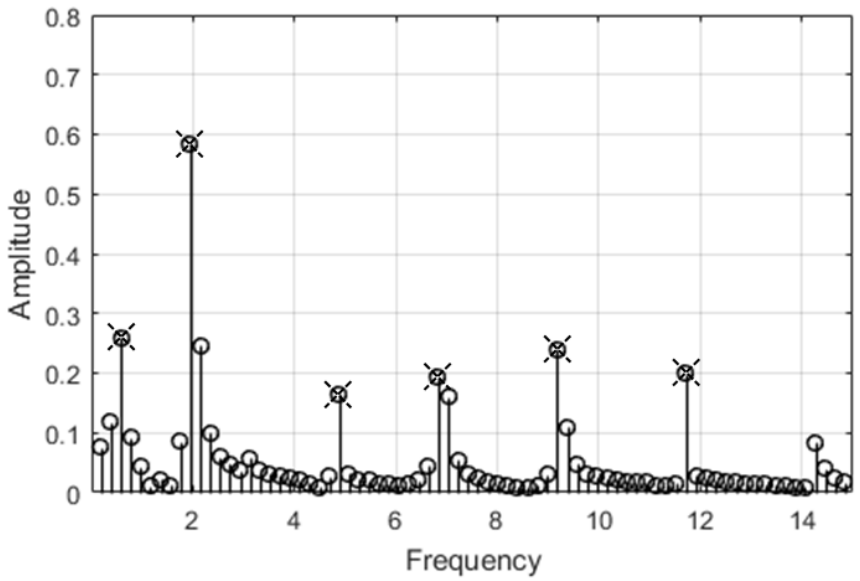


Figure 4. Six major natural frequencies found from Fast Fourier Transform

Fig. 5 shows the comparison of the identified force to the actual input force to the reference simulator. The dotted line is the actual force and the bold line is the force determined using the identification method. The solution converges after 11 iterations and the root mean square error is $8X10^{-3}$.

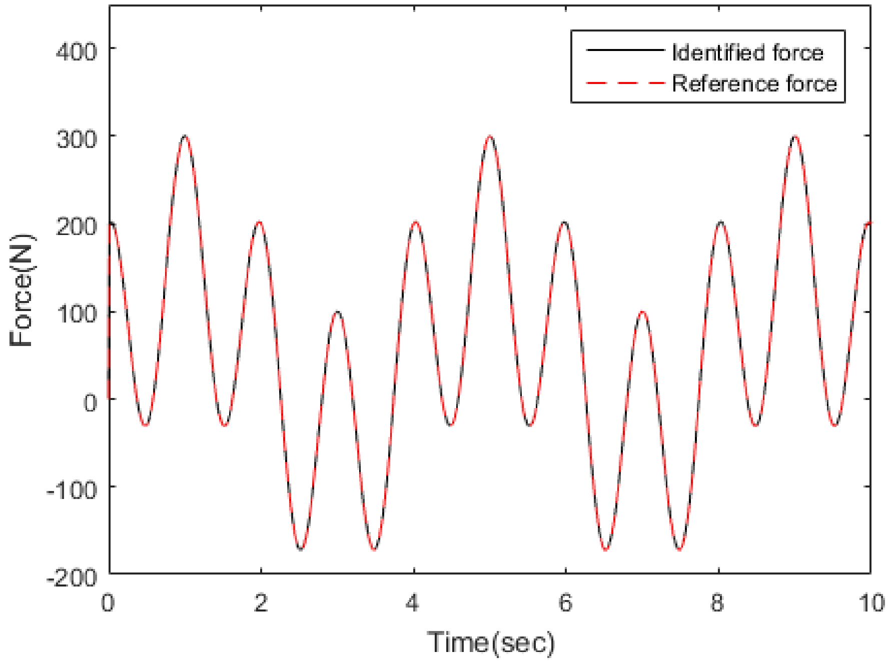


Figure 5. The force result comparison between the identification system and the comparator

3.2. Multi−body system

To illustrate a multi−body system that translates and rotates freely in all x, y and z direction, a plane shaped model is constructed with rigid and flexible beams as in Fig. 6. Each of two wings includes 10 elements with 6DOF for each node. The wing beam of 10m length is tapered with the largest cross section of 1.4X0.2m at the wing root and the smallest cross section of 0.1X0.05m. The wing has elastic modulus of E=2X10^{11}Pa, the density of ρ=7860kg/m^3 and poison ration of 0.3. The plane body and two tail wings are designed as rigid beams. The body has a dimension of 1.2X1.5X12 m (widthX heightXlength), the elastic modulus of E=2X10^{14}Pa, the density of ρ=7860kg/m^3 and poison ration of 0.3. Two tails each has a dimension of 0.8X0.12X3.6m. Other material properties of the tails are the same as the body part.

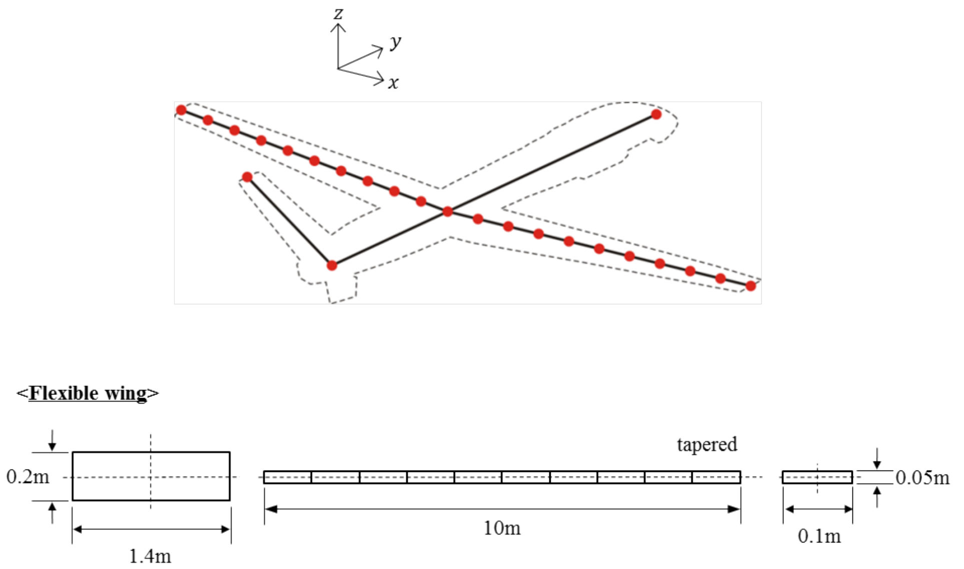
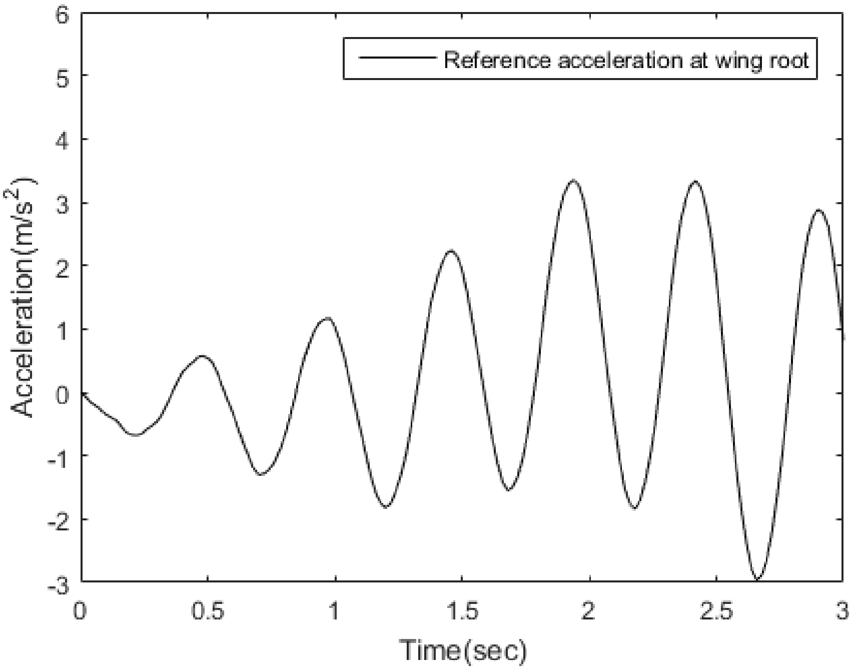


Figure 6. A plane shaped multi-body system with a rigid body, rigid tails and flexible wings

3.2.1. Pre-stressed

The plane shaped model that is moving forward at a constant velocity of 100m/s is pre-stressed with a known load of $F = -2E3 - 3E3\sin(4\pi t)$. After 1 second, it experiences a '1-cosine' discrete gust with magnitude of $F = 5E3(1-\cos(\pi t))$. The wind load is proportionally distributed to the element width over the entire bottom surface of the plane model. Only the reference acceleration data at the root and the tip of one wing of the comparator is used to inversely approximate the input load as shown in Fig. 7.

To run the force identification simulator, the initial force parameters are set $\mathbf{p} = [p_0, p_s, p_c, \omega_s, \omega_c] = [7E3,\ 1E3,\ 7E3,\ 1.2\pi,\ 1.2\pi]$. The force results comparison between the comparator and the force identification simulator is shown in Fig. 8. Because the wing is designed with highly flexible material property, the magnitude of acceleration is much higher at the tip, experiencing large fluctuation.



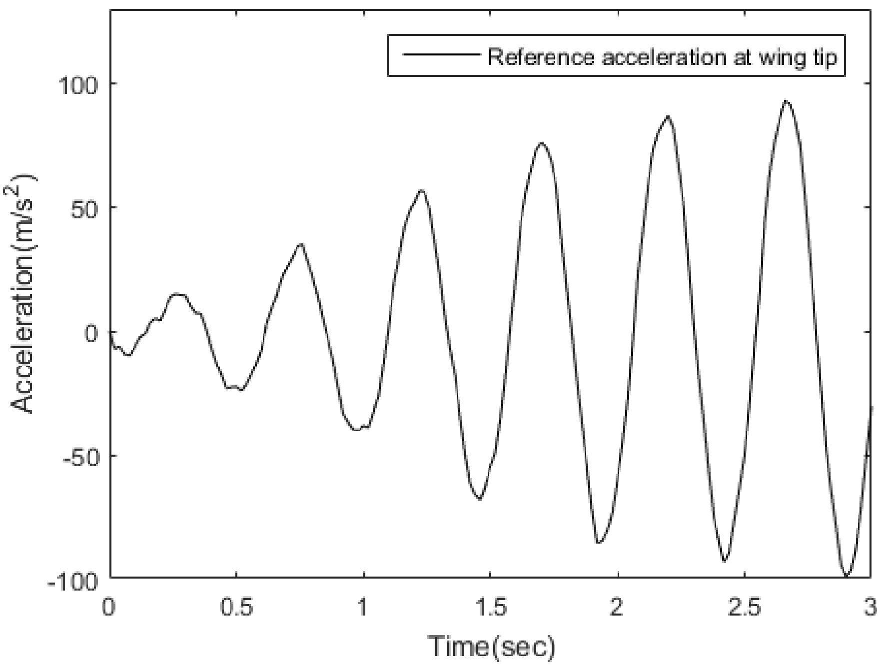
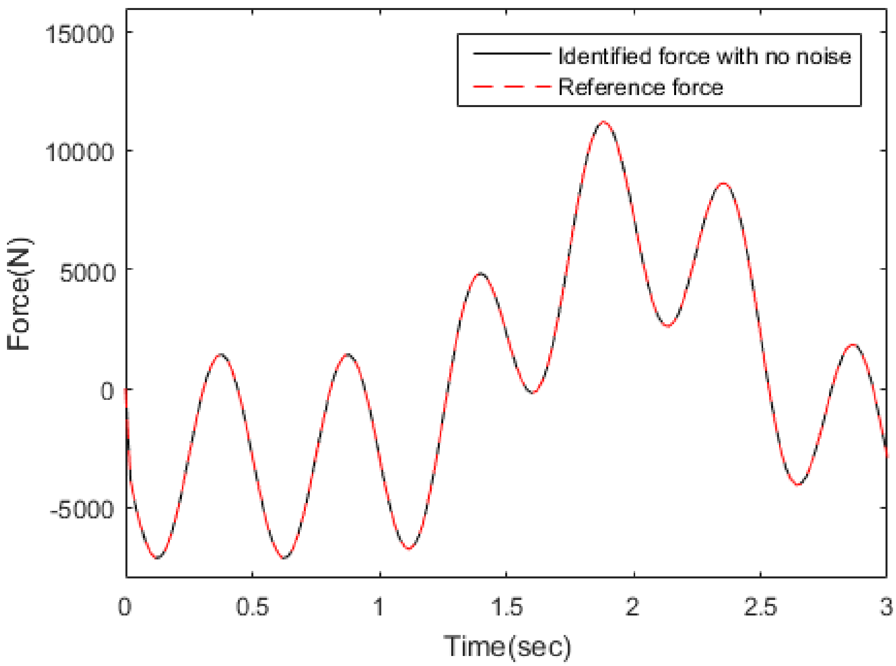


Figure 7. The acceleration history of the reference simulator at the wing root(top) and the wing tip(bottom)

Force identification is usually needed when the direct measurement of the input force applied to an experimental structure is unavailable in the lab and the measured data from gauges usually contains measurement error or noise. To demonstrate this noise in the dynamic response measurement, Gaussian white noise is added to the reference data of the comparator. Fig. 8 shows the result comparison between the actual force applied to the reference model and the identified force from the load identification system when there is 0%, 10% or 20% random noise mixed in the measured data. When there is no noise, the solution converges after 12 iterations and the resulting curve is very close to the actual force curve. In this case, the root mean square error is $1X10^{-2}$. When 10% or 20% noise is added, the identified force still correctly follows the trend of the reference data with the error of $2.6X10^{-1}$ or $2.5X10^{-1}$.



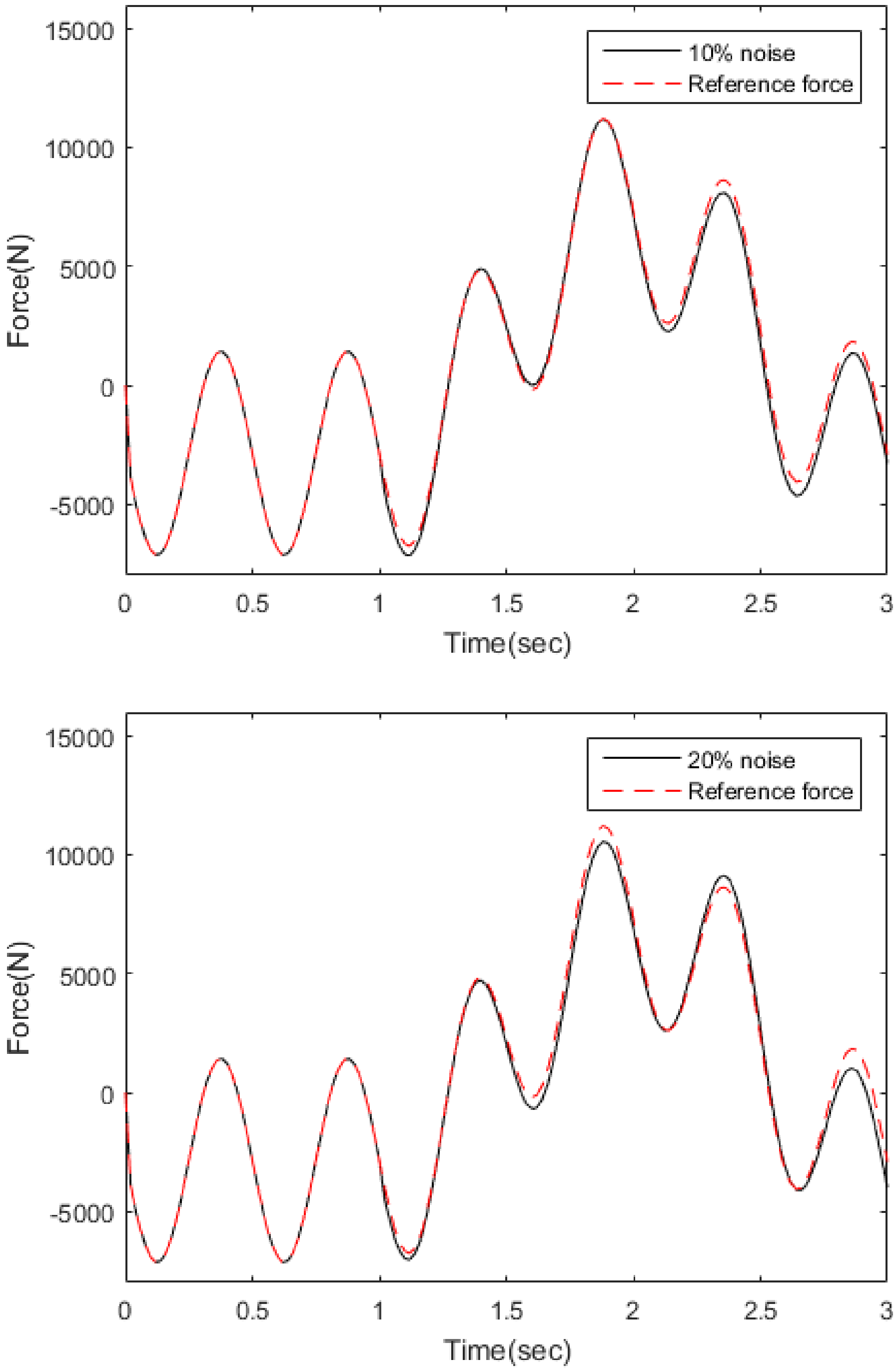


Figure 8. The identified force compared to the reference simulator when 10%(top) and 20%(bottom) noise is added to the measured acceleration data

3.2.2 Large rotation

The same plane shaped model used in the previous case is now subjected to a point force of $F = 2E5(1 - \cos(1\pi t))$ in z−direction at the tip node of one wing. This force causes the plane to rotate more than 90 degrees for 2 seconds while it is moving forward at the velocity of 50m/s. The time step △t is 0.02 seconds. No pre−load is applied in this case and the initial force parameters are $\mathbf{p} = [2.1E5,\ 1E2,\ -2.1E5,\ 1.1\pi,\ 1.1\pi]$.

The first graph of Fig. 9 is the comparison of the identified force to the actual input force. The identified force does not correctly reconstruct the actual force history. When the structure is experiencing a large rotation and deflection, the stiffness gradient with respect to the force parameter becomes non−negligible, resulting in an error in the dynamic response sensitivity.

To improve the accuracy of the results, the time discretization technique is applied to the same model. The total simulation time of 2 seconds is equally discretized into four sections, 0.5 seconds in each time division. In the first time section from 0 to 0.5 seconds, the initial force parameters set by the user are updated at each iteration. After the solution is converged, the force parameters found in this section is now used as the initial parameter input for the next time period from 0.5 to 1 second. The second graph of Fig.9 is the force results of the time discretized force identification. The force determined when the simulation time is discretized into multiple sections is significantly more accurate than when the identification analysis is performed without the time discretization technique. When the highly flexible structure is experiencing a large rotation, application of time discretization is required to obtain valid force results.

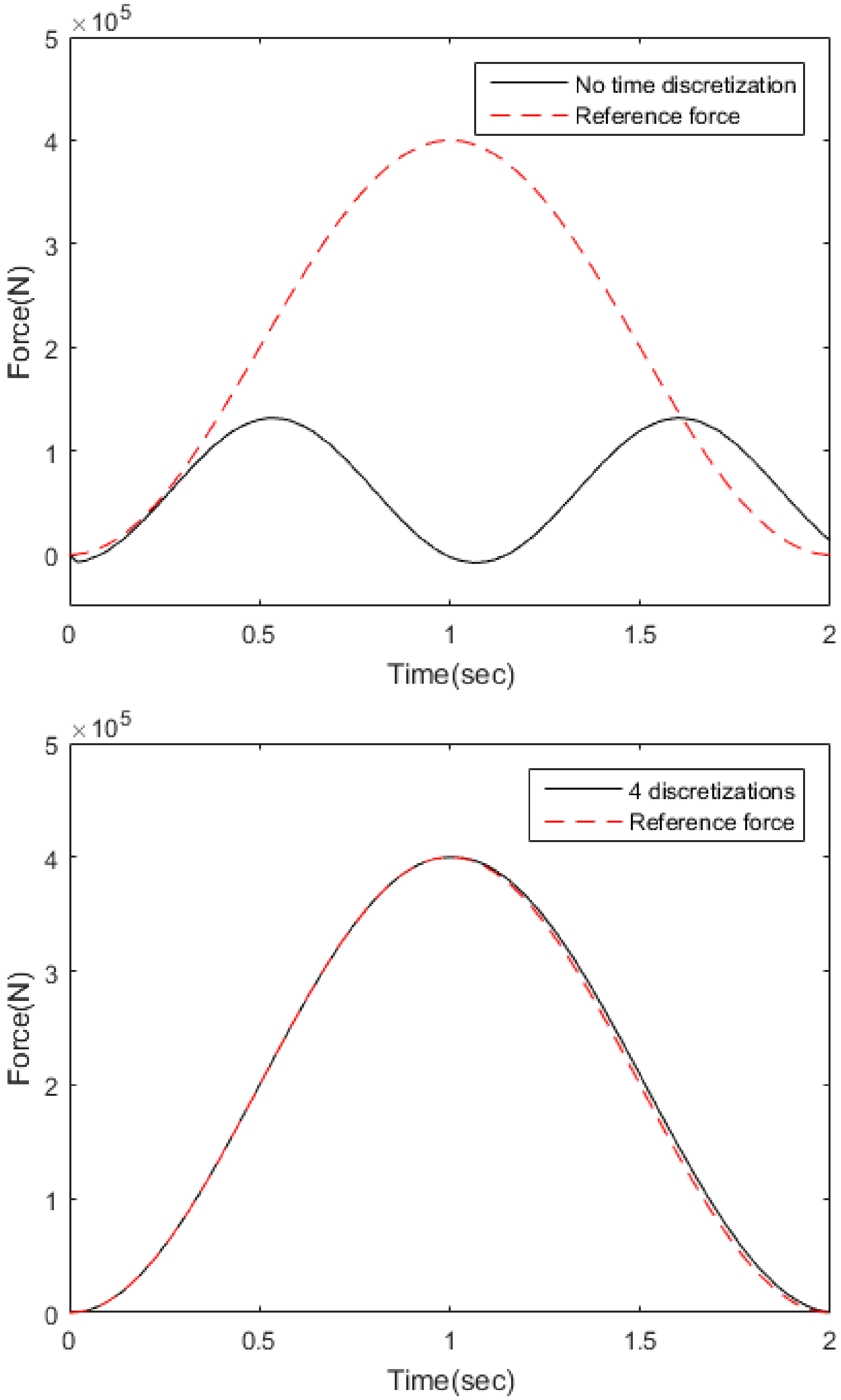


Figure 9. Force results of the identification model when the simulation time is not discretized (top) or discretized into four sections(bottom)

3.3. 3D shell

The proposed force identification method is further verified with a 3D MIT4 shell model. The elastic modulus is E=2x10^{9}Pa and the density is ρ=9800kg/m^3. The total dimension of this finite element model is 0.2X0.1X1 (widthXheightXlength). It is discretized into 20 elements with 5DOF at each node. A distribute force of $F = 200 - 200\cos(5\pi t)$ is applied at one end for 2 seconds. The initial parameters are $\mathbf{p} = [250,\ 10\ ,\ \text{-}250,\ 5.5\pi,\ 5.5\pi]^T$ and the motion is discretized into 4 divisions, making 0.5 seconds per each time division.

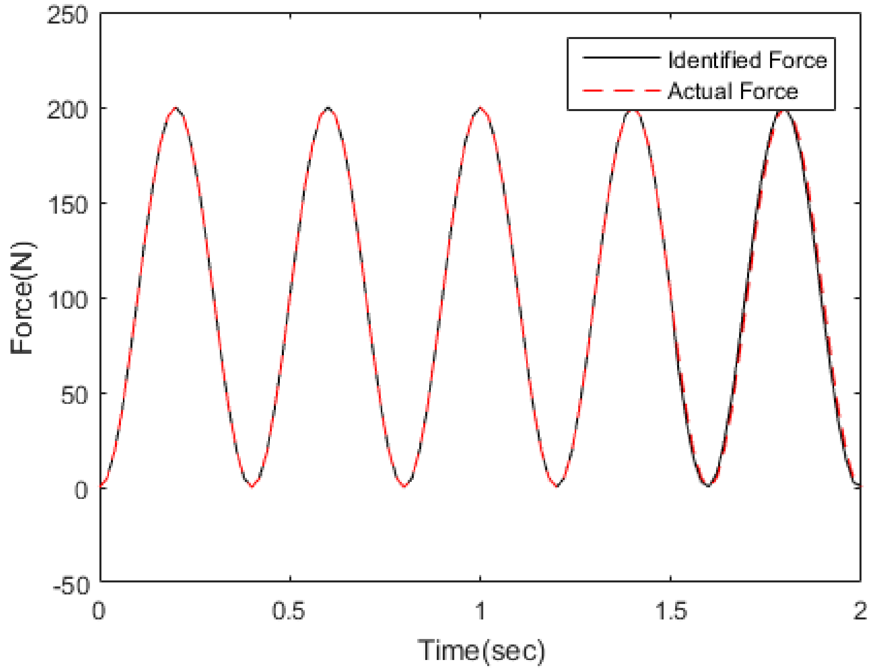


Figure 10. Force identified using the 3D shell model

The results comparison between the identified force and the actual input to the reference simulator is in Fig.10. It is proved that the very accurate approximation can be made for 3D shell finite element model using the force identification proposed in this study.

Chapter 4. Conclusion

This study introduces a novel force identification technique for nonlinear dynamic problems by using the dynamic response sensitivity. In this study, a reference simulator is used as a comparator to find the error in the dynamic responses and the force results. The identical finite element models are used for reference simulator and the force identification model. Also, they both use the identical nonlinear dynamic algorithms like Newmark−β method and Newton−Raphson iteration. To prove the validity of this force identification method, several numerical examples are tested with various nonlinear finite element models, from a simple beam to a complex multi−body system consisting of both rigid and flexible bodies. Here, co−rotational beam is used to demonstrate nonlinear beam model and 3D MIT4 is used to construct 3D nonlinear shell models.

As it is shown in the examples, the dynamic response history computed from a reference simulation model can be transferred into the frequency domain via FFT to obtain major natural frequencies. These frequencies are needed to predict the force frequency and used as initial force frequency parameters. The proposed force identification method is valid even when an additional unknown force is applied to a structure that is already pre−stressed with known force. It is proved that the unknown force history can be reconstructed regardless of the presence of any known forces. Further validation test is performed with Gaussian white noise mixed in the measured dynamic responses. The results show that the proposed identification method is not very sensitive to the white noised and therefore reasonable results can be expected from this identification method.

In some extreme cases, such as a motion with a large rotation or a large deflection, the modified sensitivity can be used with time discretization technique to improve the accuracy of the results.

Applying the time discretization method actually saves the computation time because for each iteration, the dynamic response calculation needs to be performed only within a short time period. Once the solution has been converged in the previous division, only a few iteration need to be run afterwards. The advantage of applying the time discretization may become more significant especially when handling heavy finite elements model with a large number of degrees−of−freedoms. Further verification test can be performed in the future study with more extreme motions or with more complex finite element models.

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Abstract

하중 역추적이란 구조물의 변위 또는 가속도와 같은 동적 응답 데이터를 이용하여 구조물에 작용한 미지의 외부 하중을 역으로 추적하는 역해석 기법이다. 본 논문에서는 하중 파라미터에 대한 가속도 민감도를 이용하여 동적 운동을 하는 비선형 유연 다중보에 가해진 하중을 역으로 추적할 수 있는 민감도 하중 역해석 기법을 소개한다. 시간영역에서 측정된 가속도 데이터를 고속 푸리에 변환(Fast Fourier Transform, FFT)기법을 통해 주파수영역에서 주요 고유진동수를 추정하고, 추정된 주요 파라미터를 토대로 역해석 수치 모델에서 추출된 가속도 값과 측정 가속도 값 사이의 오차가 최소화되도록 미지의 하중 파라미터를 찾아가는 최적화 기법을 사용하였다. 여기서 역해석 수치 모델은 비선형 뉴마크 시간적분법(Newmark time integration)과 뉴턴-랩슨 알고리즘(Newton-Raphson algorithm)을 응용한 유한요소 모델을 사용하며, 최적화 기법으로는 감쇠최소자승법(Damped least-squares method)를 사용한다. 본 연구에서는 민감도 하중 역추적 기법을 이용하여 다양한 종류의 연속하중 또는 급작하중을 받는 유연한 비선형 단순보 및 다중보에 대한 하중 역해석을 수행하고, 동적 응답 데이터에 백색잡음이 섞여있는 경우에 대해서도 본 기법의 신뢰성을 검증하였다.