산업공학석사 학위논문

Parametric Exponential Lévy Models Calibrated To Predict European and American Index Options: An Empirical Study.

유러피안 인덱스 옵션 및 아메리칸 인덱스 옵션 가격 추정을 위한 지수 레비 모델의 모수 추정에 관한 실증적 연구

2015 년 2 월

서울대학교 대학원
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이 논문을 산업공학 석사 학위논문으로 제출함
2015 년 1 월

서울대학교 대학원
산업공학과
장 희 수

장희수의 산업공학석사 학위논문을 인준함
2015 년 1 월

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Abstract

Parametric Exponential Lévy Models Calibrated To Predict European and American Index Options: An Empirical Study.

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We compare empirically with european option price and american option price estimated from the same parameter sets of exponential Lévy models. We use the S&P 100 index option data for recent three years. First, we calibrate the parameter sets for four exponential Lévy models : Merton, Variance−Gamma, CGMY, and Kou model. Second, We predict the european options by using Carr−Madan's Fourier transform method. We also estimate the american option prices by using numerical method. Lastly, we perform analysis of the difference true price and estimated price. In very short time to maturity, american option pricing reveals better performance than european option pricing. However, it's performance shrink fast by increasing time to maturity. Call option pricing is more accurately than put option pricing over all. Out of the money (OTM) options have larger error than the error in the money(ITM) and at the money (ATM) options have. In time to maturity's point of view, Longer time to maturities options have poor estimation performance.

Keywords : Option markets, Exponential Lévy models, Model calibration and selection, Constrained optimization.
Student Number : 2013−21082

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1 Introduction

Financial modeling with jump process (i.e. Lévy processes) have been widely employed in finance literature. It represents stylized facts of underlying asset returns and volatility smile effects of traded option prices [6]. Besides, Financial model with jump process can explain the frequently opserved steepness of the skews for very short expirations in contrast with stochastic volatility model [5].
Option pricing is achieved in two ways : nonparametric approach [8,7,9,12,18], and parametric approach. Nonparametric model which have no specified underlying dynamics is not available to path-dependent exotic option pricing. On the other hand, the Lévy processes make it possible to generate the paths of underlying and to estimate the price of path-dependent exotic options. Therefore, the parameter sets has significant meaning for the financial model.
In real market, investors make decision according to individual preference to risk. Under the no arbitrage assumption, Option price which is measured under the risk-averse measure(P measure) is same to Option price which is meaured under the risk-neutral measure (Q measure) by Girsanov's theorem. Because real market is incomplete, the option price is different from the option price estimated by model.
In this paper, The aim of our work is to compare with european option price and american option price estimated from the same parameter sets of exponential Lévy models. Under the no arbitrage assumption, All options from same underlying asset can be estimated using the same parameter set of the underlying asset's model. We do research empirically the European/American options of S&P 100 index. When the actual price and the estimated price are different, we're trying to analyze and investigate the cause.
Among various jump processes, exponential Lévy processes have been very popular because they belong to a tractable class of risk-neutral models with jumps and allow direct application of the fast fourier transform (FFT) method to the option pricing formula. On the other hand various numerical methods to solve the LCP for pricing the American options are developed. There is the implicit method coupled with the operator splitting method suggested in [11] to preserve the second order accuracy in the time and spatial variables.
We use the S&P 100 index option data for recent three years. S&P 100 index is representative index. Furthermore, It is only index options which have both European style and American style expiration. First, we calibrate the four exponential Lévy models and acquire the parameter sets for each models. Second, We predict the european options from these parameter sets. We also estimate the american option prices in accordance with the calibrated parameter set at the same time. Lastly, We compare the performance of each options.
The paper is structured as follows. In section 2 we describe the widely used exponential Lévy models and present briefly the FFT methods to evaluate European option prices using the characteristic functions of exponential Lévy models. In section 3 we represent a a numerical method to evaluate American options. Then we show the simulation results in section 4 and conclude this paper in section 5.

2 Calibrating Lévy models

In this section, we briefly represent exponential Lévy processes. Next we state the closed-form characteristic functions and four statistical moments of each exponential Lévy model. Lastly, we refer to Carr-Madan's Fourier transform methods for European option pricing

2.1 Parametric Lévy models

Let us consider a probability space $(\Omega, \mathcal{F}, \mathbf{P})$. A Lévy process X_t on $(\Omega, \mathcal{F}, \mathbf{P})$ is a cadlag stochastic process with $X_0 = 0$. It is stationary independent increments and continuious in probability. The characteristic function Φ_X and characteristic exponent Ψ_X are presented as follows by the Lévy-Khintchine formula.

$$\Phi_X(z) = \mathbb{E}[e^{iz\cdot X_t}] = e^{t\Psi_X(z)}$$
$$\Psi_X(z) = -\frac{1}{2}z \cdot Az + i\gamma \cdot z + \int_{\mathbb{R}^n} (e^{iz\cdot x} - 1 - iz \cdot x1_{|x|\leq 1})\nu(dx),$$

where $\gamma \in \mathbf{R}^n$, A is a nonnegative-definite symmetric matrix and ν is a Lévy measure on $\mathbf{R}^n \backslash \{0\}$ with

$$\int_{|x|\leq 1} |x|^2 \nu(dx) < \infty, \qquad \int_{|x|\geq 1} \nu(dx) < \infty. \tag{1}$$

The triplet (A, ν, γ) is called **Lévy triplet** of the process X_t.

Exponential-Lévy model suggests a manageable classs of risk neutral models with jumps generalizing the Black-Sholes model as in

$$S_t = S_0 \exp(rt + X_t), \tag{2}$$

where $(X_t)_{t\geq 0}$ is a Lévy process on $\mathbf{R}$ with Lévy triplet (σ^2, ν, γ). In order to guarantee that $e^{-rt}S_t$ is a martingale, we should impose additional restrictions on the Lévy triplet (σ^2, ν, γ) of X that $E^{\mathbf{Q}}[e^{X_t}] < \infty$, i.e. $\int_{|x|\geq 1} e^x \nu(dx) < \infty$ [17] and $E^{\mathbf{Q}}[e^{X_t}] = e^{t\Psi_X(-i)} = 1$ for all t under a risk neutral martingale measure $\mathbf{Q}$ [4], i.e.

$$\Psi_X(-i) = \gamma + \frac{\sigma^2}{2} + \int_{-\infty}^{\infty} (e^x - 1 - x1_{|x|\leq 1})\nu(dx) = 0. \tag{3}$$

In this paper, we consider the following risk-neutral stock price process given by $S_t = S_0 \exp((r - q)t + X_t(\theta) + \omega t)$ where r and q denote the constant continuously compounded interest rate and dividend yield respectively, and $X_t(\theta)_{t\geq 0}$ is a parameterized Lévy process on $\mathbf{R}$ with parameter set θ. Here ω is introduced to guarantee the martingale property for the price process too[3], i.e. $E^{\mathbf{Q}}[e^{X_t(\theta)+\omega t}] = e^{t\Psi_X(-i)}e^{\omega t} = 1$, giving $\omega = -\Psi_X(-i)$.

Four widely used homogeneous exponential Lévy processes are used in this paper and their statistical and computational properties are briefly summarized in the each subsections.

2.1.1 Lévy processes via jump-diffusion model

A Lévy process of jump-diffusion type has the following form:

$$X_t = \gamma t + \sigma W_t + \sum_{i=1}^{N_t} Y_i, \tag{4}$$

where $(N_t)_{t\geq 0}$ is the Poisson process counting the jumps of X and Y_i are jump size.

1) **Merton model[14]:** In the Merton model Y_i are Gaussian distribution with mean m_J and variance σ_J^2. There are 4 parameters $\theta = (\lambda, m_J, \sigma_J, \sigma)$: λ-jump intensity, m_J-mean jump size, σ_J-standard deviation of jump size, σ-diffusion volatility. The characteristic exponent $\Psi_s(z)$ of $s_t = \ln(S_t/S_0)$ is

$$\Psi_s(z) = (r - q + \omega_0)iz + \left\{-\frac{1}{2}\sigma^2 z^2 + \lambda\left(e^{(im_J z - \frac{1}{2}\sigma_J^2 z^2)} - 1\right)\right\} \tag{5}$$

where $\omega_0 = \omega + \gamma = -\frac{1}{2}\sigma^2 + \lambda(1 - \exp(m_J + \frac{1}{2}\sigma_J^2))$.

2) **Kou model:[10]** The distribution Y_i of jump sizes is an asymmetric exponential as follows:
$p_Y(x) = p\lambda_+ e^{-\lambda_+ x} 1_{x>0} + (1-p)\lambda_- e^{\lambda_- x} 1_{x<0}$, where $\lambda_\pm > 0$ determine the tail behavior of the distribution of positive and negative jump sizes and $p \in [0, 1]$ represent the probability of an upward jump. For this process, the probability distribution of returns shows an exponential tails. Due to the memoryless property of exponential random variables, analytical expressions for expectations involving first passage times can be obtained. There are 5 parameters $\theta = (\lambda, \lambda_+, \lambda_-, p, \sigma)$: λ-jump intensity, λ_+, λ_-, p-parameters of jump size distribution, σ-diffusion volatility. The characteristic exponent $\Psi_s(z)$ of $s_t = \ln(S_t/S_0)$ is given by

$$\Psi_s(z) = iz(r - q + w_0) + \left\{-\frac{1}{2}\sigma^2 z^2 + \lambda\left(\frac{p\lambda_+}{\lambda_+ - iz} + \frac{(1-p)\lambda_-}{\lambda_- + iz} - 1\right)\right\}, \tag{6}$$

where $\omega_0 = \omega + \gamma = -\frac{1}{2}\sigma^2 + \lambda\left(1 - \frac{\lambda_+ p}{\lambda_+ - 1} - \frac{\lambda_-(1-p)}{\lambda_- + 1}\right)$.

2.1.2 Lévy processes via infinite activity models

Let T_t be a subordinator, i.e. its trajectories are almost surely nondecreasing. Using the subordinating Brownian motion An infinitely activity Lévy process can be represented as follows:

$$X_t = \gamma T_t + \sigma W(T_t), \tag{7}$$

3) **The Variance Gamma model[13]:** The variance gamma process is a finite variation process with infinitely but relatively low activity of small jumps obtained by evaluating Brownian motion with drift γ and volatility σ at an independent gamma time, i.e. $X_t = \gamma T_t + \sigma W(T_t)$, where T_t is a gamma process with mean rate t and variance rate κt and the density function of the gamma time change g over a finite interval t is given by
$p_t^T(g) = e^{-\frac{g}{\kappa}} g^{\frac{t}{\kappa}-1} / (\Gamma(\frac{t}{\kappa})\kappa^{\frac{t}{\kappa}})$. There are 3 parameters $\theta = (\gamma, \kappa, \sigma)$: γ-diffusion drift, κ-variance of the subordinator, σ-diffusion volatility. The characteristic exponent $\Psi_s(z)$ of the log prices s_t is given by

$$\Psi_s(z) = (r - q + \omega)iz - \frac{1}{\kappa}\ln(1 - i\gamma\kappa z + \frac{1}{2}\sigma^2\kappa z^2) \tag{8}$$

where $\omega = \frac{1}{\kappa}\ln\left(1 - \gamma\kappa - \frac{1}{2}\sigma^2\kappa\right)$.

4) **CGMY model[3]:** The CGMY process (also called “truncated Lévy flights”) is an infinite activity tempered stable process with no Gaussian component given by $\nu(x) = \frac{c}{(-x)^{1+\alpha}} e^{\lambda_- x} 1_{x<0} + \frac{c}{x^{1+\alpha}} e^{-\lambda_+ x} 1_{x>0}$ It is of finite variation if $0 \leq \alpha < 1$ and of infinite variation if $\alpha \geq 1$. There are 4 parameters $\theta = (c, \lambda_-, \lambda_+, \alpha)$: c determine the overall and relative frequency of jumps, λ_-, λ_+ determine the tail behavior of the Lévy measure, α determine the local behavior of the process (how the price evolves between big jumps). The characteristic exponent $\Psi_s(z)$ of the log prices s_t is given by

$$\Psi_s(z) = (r - q + \omega)iz + c\Gamma(-\alpha)\{(\lambda_+ - iz)^\alpha - \lambda_+^\alpha + (\lambda_- + iz)^\alpha - \lambda_-^\alpha\}, \tag{9}$$

where $\omega = -c\Gamma(-\alpha)\{(\lambda_+ - 1)^\alpha - \lambda_+^\alpha + (\lambda_- + 1)^\alpha - \lambda_-^\alpha\}$.

2.2 Carr-Madan’s Fourier transform methods for option pricing

Carr and Madan [2] proposed pricing methods for European options which is applied where the characteristic function of the risk-neutral stock price process is explicitly known. In the previous section, we illustrated the closed-form characteristic functions of the risk-neutral stock price for the four Lévy process, e.g., Merton, Kou, Variance gamma (VG), and CGMY models. Let α which allows to use Fast Fourier Transforms(FFT) is a positive number. In our simulations, we’ve used $\alpha = 1.6$. The European call option price $C_T(k)$ of T maturity and log strike price k is generated by following as

$$C_T(k) = \frac{\exp(-\alpha k)}{2\pi} \int_{-\infty}^{+\infty} e^{-ivk}\psi_T(v)dv, \tag{10}$$

where

$$\psi_T(v) = \frac{e^{-rT}\Phi_s(v - (\alpha+1)i)}{\alpha^2 + \alpha - v^2 + i(2\alpha+1)v} \tag{11}$$

and Φ_s is the characteristic function of the log price s_t. Using Fast Fourier Transforms (FFT), we can calculate the European call option price within a second.

2.3 Calibration

Let an exp-Lèvy model $(\sigma(\theta), \nu(\theta), \gamma(\theta))$ parameterized by θ and C_i the observed prices of call and put option data for maturities T_i and strikes K_i, $i \in I$. We will resort to finding a parameter vector θ which solves the following nonlinear problems:

$$\min_\theta \sum_{i=1}^{N} w_i |C^\theta(T_i, K_i) - C_i|^2$$

where C^θ denotes the estimated option price by model for the exponential-Lèvy model with triplet $(\sigma(\theta), \nu(\theta), \gamma(\theta))$ and w_i is the weight of the difference of the option prices. From the calibration problem the calibration algorithm consists of following steps.

(1) Choose the exp-Lévy model.
(2) Choose the initial parameter set randomly which matches the constraint of the exp-Lévy model, damping parameter α and the weight of the option prices w_i. In this paper we settle $\alpha = 1.6$, $w_i = 1$.
(3) Using a derivative-free constrained optimization method, find the parameter set. In this paper, we use the Nelder-Mead Simplex method which is derivative-free nonlinear optimization method.

3 Numerical Methods for Pricing American Options

In this section, we describe a numerical method to evaluate American options when the underlying asset follows the Lévy processes. The linear complementarity problem (LCP) for pricing American options under the jump-diffusion models is

$$\begin{cases} u_\tau(\tau, x) - \mathcal{L}u(\tau, x) = \psi(\tau, x), \\ \psi(\tau, x) \geq 0, \quad u(\tau, x) \geq h(x), \quad \psi(\tau, x)\,(u(\tau, x) - h(x)) = 0, \end{cases} \tag{12}$$

in the region $(0, T] \times (-\infty, \infty)$, where the integro-differential operator $\mathcal{L}u$ is defined by

$$\begin{aligned} \mathcal{L}u(\tau, x) =& \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2}(\tau, x) + (r - d - \frac{\sigma^2}{2} - \lambda\zeta)\frac{\partial u}{\partial x}(\tau, x) \\ & - (r + \lambda)u(\tau, x) + \lambda \int_{-\infty}^{\infty} u(\tau, z) f(z - x) dz, \end{aligned} \tag{13}$$

and the initial condition $h(x)$ is one of the payoff functions of the put and call options given by respectively

$$h(x) = \max(0, K - S_0 e^x) \quad \text{and} \quad h(x) = \max(0, S_0 e^x - K).$$

There are various numerical methods to solve the LCP for pricing the American options. In this paper, we use the implicit method coupled with the operator splitting method suggested in [11] to preserve the second order accuracy in the time and spatial variables. The numerical method called *the implicit method with three time level* has the advantage that it avoids the iteration we need to solve the dense linear system at each time step. The operator splitting method, which are used to deal with the constraints in the LCP, consists of two steps. Fist, the intermediate approximation $\widetilde{U}^{n+1}$ of u^{n+1} at the $(n+1)$-th time level is computed by

$$\frac{\widetilde{U}_m^{n+1} - U_m^{n-1}}{2\Delta\tau} - \mathcal{L}_\Delta(U_m^{n-1}, U_m^n, \widetilde{U}_m^{n+1}) = \Psi_m^n,$$

where the discrete integro-differential operator $\mathcal{L}_\Delta(U_m^{n-1}, U_m^n, U_m^{n+1})$ with the boundary condition $g(\tau, x)$ is

$$\begin{aligned}
&\mathcal{L}_\Delta(U_m^{n-1}, U_m^n, U_m^{n+1}) \\
&\quad = \frac{\sigma^2}{2}\left(\frac{U_{m+1}^{n+1} - 2U_m^{n+1} + U_{m-1}^{n+1}}{2\Delta x^2} + \frac{U_{m+1}^{n-1} - 2U_m^{n-1} + U_{m-1}^{n-1}}{2\Delta x^2}\right) \\
&\quad + (r - d - \frac{\sigma^2}{2} - \lambda\zeta)\left(\frac{U_{m+1}^{n+1} - U_{m-1}^{n+1}}{4\Delta x} + \frac{U_{m+1}^{n-1} - U_{m-1}^{n-1}}{4\Delta x}\right) \\
&\quad - (r+\lambda)U_m^n + \frac{\lambda\Delta x}{2}\left(U_0^n f_{m,0} + 2\sum_{j=1}^{M-1} U_j^n f_{m,j} + U_M^n f_{m,M}\right) \\
&\quad + \lambda \int_{\mathbb{R}\setminus(-X,X)} g(\tau_n, z) f(z - x_m)\,dz.
\end{aligned} \tag{14}$$

In the second step, the approximate solutions U^{n+1} and Ψ^{n+1} for u^{n+1} and ψ^{n+1} is obtained by updating the intermediate value $\widetilde{U}^{n+1}$ as follows

$$U_m^{n+1} = \max\left(h(x_m), \widetilde{U}_m^{n+1} - 2\Delta\tau\Psi_m^n\right), \quad \Psi_m^{n+1} = \Psi_m^n + \frac{U_m^{n+1} - \widetilde{U}_m^{n+1}}{2\Delta\tau}.$$

Then we can evaluate the prices of the American options with the computational complexity of $O(MN\log_2 M)$ operations where M is the number of spatial steps and N is the number of time steps.

4 Empirical Results

In the current section, We calibrate the exponential Lévy model parameter set by using the Nelder-Mead Simplex Algorithm(NMSM).Based on the calibration result, we compare the empirical performance of European option pricing and American option pricing of the four exp-Lévy models.

4.1 Data

We used the market data from S&P 100 index options. The S&P 100 index is a stock market index of United States stocks maintained by Standard & Poor's. The S&P 100, a subset of the S&P 500, includes the largest and most established companies in the S&P 500. The S&P 100 index option contract has an underlying value that is equal to the full value of the S&P 100 index. There are two style options : S&P 100 options with American-style exercise(ticker symbol OEX) and S&P 100 options with European-style exercise(ticker symbol XEO). Since 1983 investors have used OEX to adjust their equity portfolio exposure. More than one billion OEX options have been traded. In July 2001, the CBOE introduced cash-settled S&P 100 options(ticker symbol XEO)with European-style exercise.
We use the every Wednesday data covering the range from September 2010 to August 2013 to calibrate the parameter set. Because Wednesday has the lowest possibility of being a holiday and of being affected by day of-the-week effects. Thus – total estimations for each model are obtained. We provide the risk-free rate as constant,the time to maturity weighted average risk-free rate. We used the annualized dividend yield used for implied volatility calculations on index options.
Summary statistics are reported for option prices, implied volatilities, and the number of observations. Options are categorized into four levels on the basis of time to maturity and four levels of simple-moneyness. There are summary statistics in table 1, 2: S&P100 index European options,S&P100 index American options.
In table 3 and 4, annual information of option prices and implied volatilities are provided.

Table 1
S&P 100 index European option data description

Call Option		Time to maturity≤ 30				30 <Time to maturity≤ 60				60 <Time to maturity≤ 90				90 <Time to maturity				All			
		mean	median	max	std	mean	median	max	std	mean	median	max	std	mean	median	max	std	mean	median	max	std
$\kappa < 0.99$	IV	0.5	0.30	2.99	0.50	0.24	0.21	0.72	0.09	0.24	0.22	0.55	0.07	0.24	0.23	0.52	0.05	0.29	0.23	2.99	0.26
	Price	51.38	35.5	334.2	47.18	47.80	42.07	183.95	25.84	56.92	52.15	180.55	27.27	91.97	87.15	217.35	39.02	71.31	62.65	334.2	43.24
	Observ.	**3230**				**2738**				**1332**				**7162**				**14462**			
$0.99 \le \kappa < 1.01$	IV	0.18	0.15	0.83	0.08	0.16	0.15	0.35	0.05	0.17	0.16	0.32	0.04	0.18	0.18	0.31	0.04	0.17	0.16	0.83	0.06
	Price	5.83	5.45	20.00	3.58	12.32	11.88	27.80	4.09	17.03	16.30	34.75	4.05	35.88	31.48	78.50	15.51	16.82	11.95	78.50	15.35
	Observ.	**925**				**520**				**197**				**654**				**2296**			
$1.01 \le \kappa < 1.5$	IV	0.27	0.22	1.33	0.18	0.16	0.14	0.47	0.05	0.15	0.14	0.32	0.04	0.16	0.16	0.30	0.04	0.19	0.16	1.33	0.11
	Price	0.55	0.08	14.00	1.29	1.84	0.38	21.95	2.98	3.31	1.11	29.30	4.52	14.79	10.35	73.00	14.63	6.99	1.30	73.00	11.66
	Observ.	**3898**				**3388**				**1454**				**6128**				**14868**			
$1.5 \le \kappa$	IV	0.00	0.00	0.00	0.00	0.41	0.38	0.50	0.08	0.32	0.32	0.34	0.01	0.30	0.30	0.31	0.02	0.32	0.31	0.50	0.05
	Price	0.00	0.00	0.00	0.00	0.05	0.05	0.05	0.00	0.05	0.05	0.05	0.00	0.05	0.05	0.05	0.00	0.05	0.05	0.05	0.00
	Observ.	**0**				**3**				**6**				**9**				**18**			
All	IV	0.35	0.23	3.00	0.36	0.19	0.17	0.72	0.08	0.19	0.17	0.55	0.07	0.21	0.20	0.53	0.06	0.24	0.19	3.00	0.20
	Price	21.55	3.55	334.20	38.65	21.59	8.55	183.95	27.74	28.09	15.25	180.55	31.96	55.39	43.65	217.35	48.14	37.09	20.10	334.20	43.90
	Observ.	**8053**				**6649**				**2989**				**13953**				**31644**			
Put Option		**Time to maturity**≤ 30				30 <**Time to maturity**≤ 60				60 <**Time to maturity**≤ 90				90 <**Time to maturity**				**All**			
		mean	median	max	std	mean	median	max	std	mean	median	max	std	mean	median	max	std	mean	median	max	std
$\kappa < 0.99$	IV	0.44	0.36	1.82	0.26	0.31	0.27	1.21	0.14	0.30	0.28	0.92	0.11	0.28	0.27	0.75	0.07	0.32	0.28	1.82	0.17
	Price	0.76	0.15	18.95	1.49	2.57	1.03	26.95	3.53	4.20	1.93	32.30	5.10	17.28	10.75	106.10	18.71	8.92	2.33	106.10	14.90
	Observ.	**7770**				**6525**				**3056**				**14265**				**31616**			
$0.99 \le \kappa < 1.01$	IV	0.18	0.15	0.87	0.09	0.16	0.14	0.36	0.05	0.17	0.16	0.33	0.05	0.18	0.18	0.32	0.04	0.17	0.16	0.87	0.06
	Price	6.44	6.05	23.45	3.77	14.01	13.50	31.40	4.25	19.89	19.30	36.60	4.13	47.70	40.33	108.35	23.91	21.08	13.48	108.35	21.78
	Observ.	**925**				**520**				**197**				**656**				**2298**			
$1.01 \le \kappa < 1.5$	IV	0.30	0.23	1.79	0.24	0.16	0.14	0.51	0.06	0.15	0.14	0.36	0.05	0.16	0.16	0.32	0.04	0.19	0.16	1.79	0.13
	Price	42.55	31.00	248.85	32.60	50.28	40.80	261.50	31.69	58.41	49.05	246.30	33.36	93.82	89.20	243.40	39.97	70.33	61.70	261.50	42.90
	Observ.	**2545**				**2454**				**1153**				**5689**				**11841**			
$1.5 \le \kappa$	IV	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.36	0.36	0.03	0.00	0.00	0.00	0.00	0.34	0.36	0.36	0.03
	Price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	276.85	277.85	286.30	9.99	0.00	0.00	0.00	0.00	276.85	277.85	286.30	9.99
	Observ.	**0**				**0**				**3**				**0**				**3**			
All	IV	0.39	0.30	1.82	0.26	0.26	0.23	1.21	0.14	0.25	0.23	0.92	0.12	0.24	0.24	0.75	0.08	0.28	0.24	1.82	0.17
	Price	10.69	0.58	248.85	23.30	15.52	3.18	261.50	26.39	19.26	5.55	286.30	30.22	39.38	22.25	243.40	43.12	25.44	7.95	286.30	36.99
	Observ.	**11240**				**9499**				**4409**				**20610**				**45758**			

Table 2
S&P 100 index American option data description

Call Option		**Time to maturity** ≤ 30				$30 <$ **Time to maturity** ≤ 60				$60 <$ **Time to maturity** ≤ 90				$90 <$ **Time to maturity**				**All**			
		mean	median	max	std	mean	median	max	std	mean	median	max	std	mean	median	max	std	mean	median	max	std
$\kappa < 0.99$	IV	0.73	0.58	2.99	0.54	0.17	0.17	0.23	0.03	0.44	0.34	2.25	0.32	0.33	0.28	2.42	0.17	0.41	0.30	2.99	0.32
	Price	83.44	65.45	450.65	62.51	42.15	41.45	73.15	19.32	65.12	54.35	276.30	46.21	132.66	115.75	493.85	84.96	120.99	102.70	493.85	83.06
	Observ.	**4539**				**14**				**955**				**19275**				**24783**			
$0.99 \leq \kappa < 1.01$	IV	0.20	0.17	0.84	0.10	0.11	0.11	0.12	0.01	0.16	0.14	0.43	0.07	0.17	0.16	0.36	0.04	0.17	0.16	0.84	0.07
	Price	5.00	4.65	20.30	3.33	7.68	7.50	10.40	2.63	6.51	6.40	19.10	3.45	22.14	15.70	82.25	17.25	15.79	10.90	82.25	16.01
	Observ.	**599**				**3**				**141**				**1229**				**1972**			
$1.01 \leq \kappa < 1.5$	IV	0.29	0.25	1.18	0.18	0.10	0.10	0.10	0.00	0.19	0.16	0.52	0.08	0.16	0.15	0.55	0.05	0.19	0.16	1.18	0.10
	Price	0.36	0.05	14.30	0.98	2.15	2.00	3.28	1.06	0.72	0.10	12.30	1.39	8.81	2.18	75.55	13.73	6.85	0.80	75.55	12.54
	Observ.	**2449**				**3**				**430**				**9434**				**14868**			
$1.5 \leq \kappa$	IV	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.31	0.50	0.05	0.32	0.31	0.50	0.05
	Price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.05	0.10	0.01	0.05	0.05	0.10	0.01
	Observ.	**0**				**0**				**0**				**18**				**18**			
All	IV	0.55	0.37	2.99	0.49	0.15	0.15	0.23	0.04	0.34	0.25	2.25	0.28	0.27	0.23	2.42	0.16	0.33	0.24	2.99	0.29
	Price	50.43	24.35	450.65	62.95	30.98	27.73	73.15	23.77	41.56	26.68	276.30	47.63	89.04	62.45	493.85	90.31	79.67	51.45	493.85	86.02
	Observ.	**7587**				**20**				**1526**				**29956**				**39089**			
Put Option		**Time to maturity** ≤ 30				$30 <$ **Time to maturity** ≤ 60				$60 <$ **Time to maturity** ≤ 90				$90 <$ **Time to maturity**				**All**			
		mean	median	max	std	mean	median	max	std	mean	median	max	std	mean	median	max	std	mean	median	max	std
$\kappa < 0.99$	IV	0.44	0.39	1.92	0.23	0.18	0.17	0.21	0.03	0.36	0.28	2.12	0.24	0.29	0.27	1.13	0.10	0.32	0.28	2.12	0.15
	Price	0.55	0.10	18.60	1.23	2.91	2.11	7.90	2.26	0.89	0.35	10.90	1.33	11.46	3.85	106.70	17.36	9.11	1.95	106.70	16.01
	Observ.	**4324**				**14**				**1017**				**19321**				**24676**			
$0.99 \leq \kappa < 1.01$	IV	0.20	0.18	0.90	0.10	0.13	0.13	0.13	0.00	0.17	0.14	0.59	0.07	0.17	0.16	0.37	0.05	0.18	0.16	0.90	0.07
	Price	5.48	5.05	23.80	3.55	12.12	11.80	14.80	2.54	7.13	6.95	15.35	3.26	27.89	18.05	109.30	24.82	19.56	12.15	109.30	22.42
	Observ.	**601**				**3**				**141**				**1229**				**1974**			
$1.01 \leq \kappa < 1.5$	IV	0.45	0.36	1.95	0.30	0.12	0.12	0.12	0.00	0.25	0.21	0.79	0.13	0.19	0.17	0.75	0.07	0.24	0.19	1.95	0.18
	Price	49.83	40.30	259.95	35.75	21.67	21.55	25.70	3.98	44.69	32.65	155.05	32.81	79.08	72.90	263.55	43.04	72.15	65.05	263.55	43.28
	Observ.	**2413**				**3**				**403**				**9379**				**12198**			
$1.5 \leq \kappa$	IV	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.45	0.54	0.05	0.47	0.45	0.54	0.05
	Price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	280.13	274.70	314.40	15.43	280.13	274.70	314.40	15.43
	Observ.	**0**				**0**				**0**				**18**				**18**			
All	IV	0.42	0.36	1.95	0.26	0.16	0.16	0.21	0.03	0.31	0.25	2.12	0.22	0.25	0.23	1.13	0.10	0.29	0.25	2.12	0.16
	Price	17.16	0.88	259.95	30.77	7.11	3.81	25.70	7.50	12.76	1.15	155.05	25.25	33.47	14.05	314.40	42.45	29.55	9.85	314.40	40.56
	Observ.	**7338**				**20**				**1561**				**29947**				**38866**			

Table 3
Annual information of S&P 100 index European option data

CallOption	Price					Implied Volatility					**Observ.**
	mean	median	max	min	std	mean	median	max	min	std	
2010.09 2011.08	38.458	22.700	306.400	0.025	43.552	0.248	0.206	2.998	0.091	0.213	9745
2011.09 2012.08	44.043	25.900	288.150	0.025	48.804	0.274	0.232	2.963	0.098	0.195	10662
2012.09 2013.08	29.319	14.050	334.200	0.025	37.610	0.196	0.155	2.959	0.086	0.198	11237
PutOption	**Price**					**Implied Volatility**					**Observ.**
	mean	median	max	min	std	mean	median	max	min	std	
2010.09 2011.08	24.5613	7.6	286.3	0.025	36.5839	0.3025	0.2648	1.7948	0.0635	0.1653	14192
2011.09 2012.08	30.7316	13.95	223.75	0.025	38.3938	0.3068	0.2723	1.5317	0.0546	0.1616	13388
2012.09 2013.08	22.2291	4.8	243.4	0.025	35.8138	0.2502	0.2079	1.8225	0.0488	0.1717	18178

Table 4
Annual information of S&P 100 index American option data

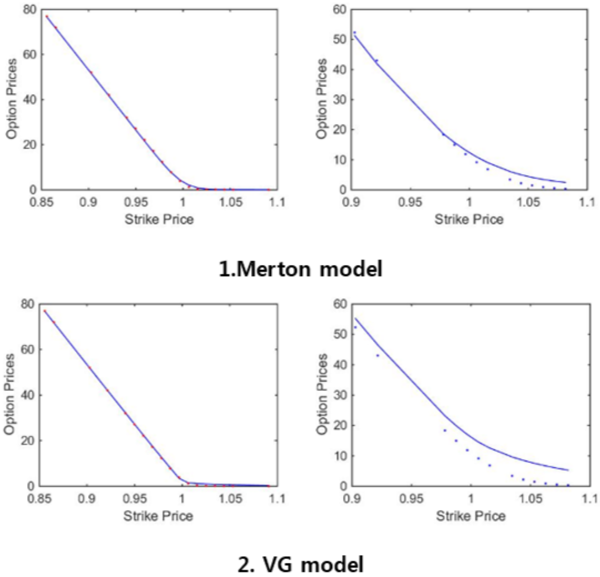
CallOption	Price					Implied Volatility					**Observ.**
	mean	median	max	min	std	mean	median	max	min	std	
2010.09 2011.08	24.561	7.600	286.300	0.025	36.584	0.303	0.265	1.795	0.064	0.165	14192
2011.09 2012.08	30.732	13.950	223.750	0.025	38.394	0.307	0.272	1.532	0.055	0.162	13388
2012.09 2013.08	22.229	4.800	243.400	0.025	35.814	0.250	0.208	1.823	0.049	0.172	18178
PutOption	**Price**					**Implied Volatility**					**Observ.**
	mean	median	max	min	std	mean	median	max	min	std	
2010.09 2011.08	31.3487	11.05	314.4	0.025	43.4484	0.3083	0.2648	1.9521	0.0968	0.1695	11885
2011.09 2012.08	35.3286	17.075	224.55	0.025	41.6205	0.3184	0.2796	1.613	0.099	0.1643	11222
2012.09 2013.08	24.0726	5.4	215.3	0.025	36.6678	0.249	0.21	2.1175	0.0743	0.1531	15759

4.2 Calibration Result

Using the S&P 100 index European options data described above, we estimate the parameter set for the four models: Merton model, The Variance Gamma model, CGMY model, and Kou model. We employ the S&P 100 European option data the trading volume of which is more than eight. The calibration result are analyzed based on four measurements. Let $\epsilon_n = C_n^{model} - C_n^{mkt}$ denote the errors. Calibration performance is evaluated by :

(1) The root mean squared error (RMSE), $\sqrt{(\sum_{n=1}^{N}(\epsilon_n^2))/N}$, represents the volatility of the difference between the model prices and the market prices.
(2) The mean percentage error (MPE), $(\sum_{n=1}^{N} \epsilon_n / C_n^{mkt})/N$, measures the directions of the percentage errors.
(3) The mean absolute error (MAE), $(\sum_{n=1}^{N} |\epsilon_n|)/N$, measures the magnitude of the errors.
(4) The mean absolute percentage error(MAPE), $(\sum_{n=1}^{N} |\epsilon_n| / C_n^{mkt})/N$, represents the magnitude of the percentage errors. where N is the total number of observations.

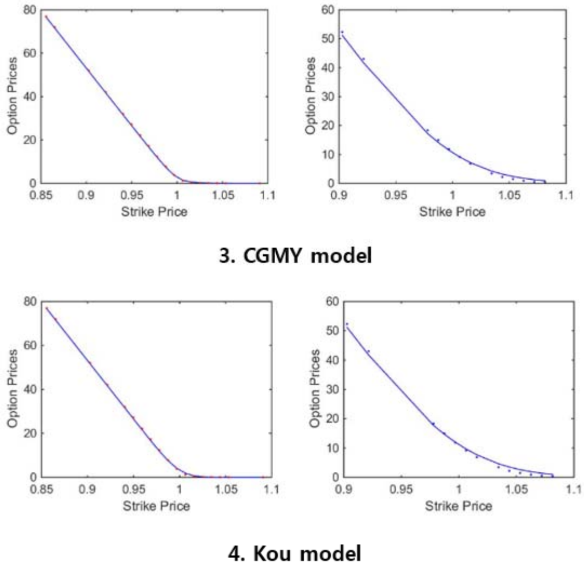
Fig. 1. The European call option prices according to the moneyness Merton and Variance-Gamma model : blue line is call option prices estimated with calibrated parameter sets and dots are true option prices. : Oct.13. 2010



1.Merton model

2. VG model

Fig. 2. The European call option prices according to the moneyness CGMY and Kou model : blue line is call option prices estimated with calibrated parameter sets and dots are true option prices. : Oct.13. 2010

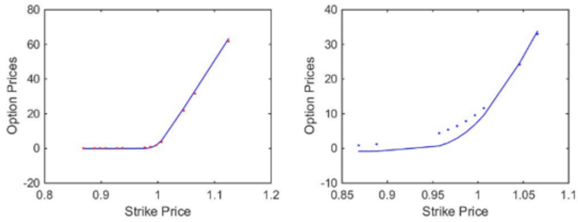


3. CGMY model

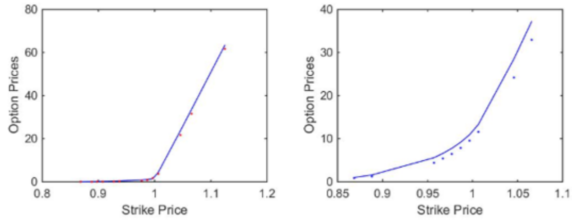
4. Kou model

We calibrate the parameter set of the four exp-Lévy models. First time, we start from the randomly chosen initial parameter sets. For the next Wednesday calibration, we select the initial parameter sets that have the top three best performance in the before Wednesday. There are the European call/put option prices according to the moneyness for Merton, VG, CGMY, and Kou model in figure 1 - figure 4. Following table 5 and 6 shows the Calibration result of call option and put option.

Fig. 3. The European put option prices according to the moneyness Merton and VG model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices. : Sep.15. 2010

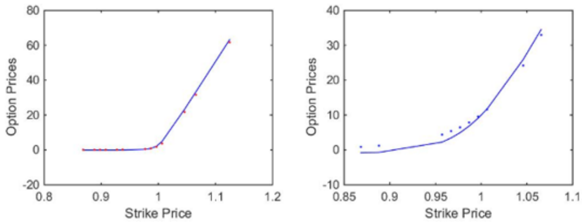


1.Merton Model

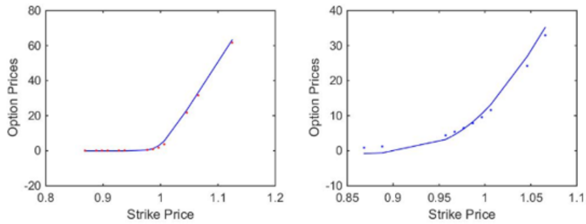


2. VG model

Fig. 4. The European put option prices according to the moneyness CGMY and Kou model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices. : Sep.15. 2010



3. CGMY model



4. Kou model

From figure 1 to figure 4 displays the true european call/put option prices and the estimated european call/put option prices which is calculated by the FFT method with an calibrated parameter sets for Merton, VG, CGMY and Kou model. According the above figures, we may assert that the calibration of the European option prices estimated using the four Lévy models is reasonable. We observe that the Merton, CGMY and Kou model describes the market prices better than VG model.

Table 5
Calibration Result of S&P index European Call options

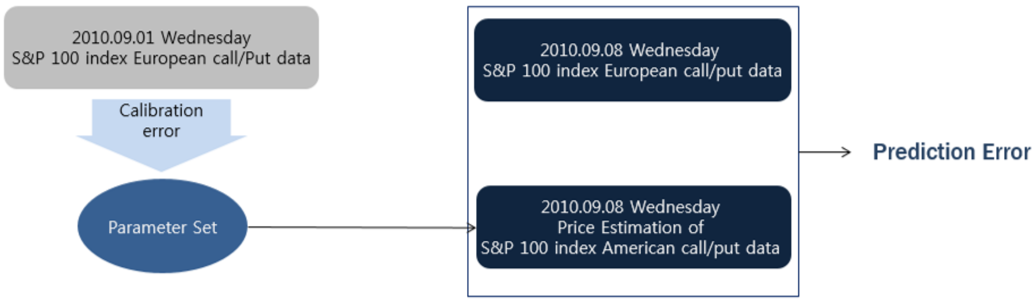
Call Option		Time to maturity ≤ 30				$30 <$ Time to maturity ≤ 60				$60 <$ Time to maturity ≤ 90				$90 <$ Time to maturity				All			
		RMSE	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**
$\kappa < 0.99$	Merton	0.4692	-0.0003	0.0830	0.0043	11.8624	-0.0027	2.5042	0.0875	2.1209	-0.1005	2.1209	0.1005	1.2500	-0.0751	1.2500	0.0751	1.8137	-0.0004	0.1362	0.0061
	VG	88.7925	0.0517	3.3522	0.1471	15.0445	0.0624	4.2752	0.1322	2.1228	0.0645	1.3323	0.0650	3.1896	0.1070	2.9589	0.1070	11.0698	0.0555	3.6158	0.1414
	CGMY	2.9642	-0.0239	2.0416	0.1040	2.2198	-0.0337	1.6715	0.0925	1.4574	-0.0093	1.1523	0.0787	1.8956	-0.0447	1.8298	0.0447	2.7379	-0.0270	1.9186	0.0998
	Kou	1.8708	0.0043	1.3971	0.0765	9.7793	-0.0113	2.1033	0.0804	1.3959	0.4614	1.3959	0.4614	1.5855	-0.2100	1.5855	0.2100	5.5905	-0.0001	1.6100	0.0782
$0.99 \leq \kappa < 1.01$	Merton	1.2484	0.0976	0.8878	0.1885	1.0897	0.0797	0.7472	0.1795	0.8363	0.1277	0.8074	0.2176	0.9761	-0.0478	0.8038	0.1341	1.1974	0.0909	0.8430	0.1854
	VG	3.1744	0.0811	2.2404	0.4069	3.6620	0.2602	2.5692	0.4919	3.4775	0.2878	2.6319	0.3351	0.8784	-0.2585	0.8703	0.2585	3.3249	0.1344	2.3367	0.4315
	CGMY	1.5290	-0.0413	0.9419	0.1743	1.2988	-0.0417	0.8351	0.1609	0.6408	-0.0988	0.5347	0.0988	0.2529	0.0191	0.2431	0.0276	1.4590	-0.0414	0.9058	0.1694
	Kou	1.0566	0.0942	0.7591	0.1693	0.9838	0.0818	0.6811	0.1682	0.7663	0.2108	0.6122	0.2422	1.6011	0.1649	1.5062	0.1649	1.0365	0.0912	0.7377	0.1693
$1.01 \leq \kappa$	Merton	2.1841	2.1163	1.3041	2.2007	1.8201	1.8587	1.1169	1.9291	0.6513	2.5397	0.5593	2.5397	2.0027	4.6460	1.4022	3.8115	2.0802	2.0709	1.2476	2.1424
	VG	3.0322	1.3608	2.0649	1.6814	3.4451	1.6682	2.3925	1.8802	0.8488	2.4251	0.8488	2.4251	0.4593	2.8989	0.3990	2.8989	3.1321	1.4516	2.1362	1.7417
	CGMY	0.9275	0.7344	0.5862	0.9042	0.7812	0.6392	0.5148	0.7876	0.7316	0.4942	0.4884	0.4942	0.9263	3.0369	0.8987	3.0369	0.8906	0.7221	0.5689	0.8848
	Kou	1.6589	0.5747	0.9235	0.8601	1.0711	0.5314	0.6533	0.9057	0.2557	0.5320	0.2045	0.7808	2.3383	0.3389	2.3383	0.3389	1.5148	0.5624	0.8449	0.8716
All	Merton	0.8479	0.2087	0.2364	0.2248	6.0702	0.9344	1.3649	1.0173	0.8269	1.8397	0.6927	1.8707	1.7118	2.9231	1.2125	2.4592	1.8192	0.2677	0.3196	0.2877
	VG	5.5726	0.5594	2.5260	0.8102	9.4576	0.6479	3.1249	0.8150	2.4833	0.2678	1.6270	0.2800	2.0042	1.4573	1.4178	1.5219	6.8955	0.5879	2.6876	0.8125
	CGMY	1.7539	0.3677	1.0137	0.5388	1.4338	0.2836	0.9102	0.4398	1.0450	0.1201	0.7518	0.2148	1.2796	1.5067	1.0998	1.5379	1.6644	0.3497	0.9838	0.5154
	Kou	1.5945	0.3147	1.0026	0.4954	5.0896	0.2655	1.0416	0.4867	0.5285	0.4522	0.3583	0.6374	1.7237	0.1362	1.6364	0.1962	3.0525	0.3008	1.0115	0.4931

Table 6

Calibration Result of S&P index European Put options

Put Option		Time to maturity≤ 30				$30 <$Time to maturity≤ 60				$60 <$Time to maturity≤ 90				$90 <$Time to maturity				All			
		RMSE	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**
$\kappa < 0.99$	Merton	6.2046	0.2293	2.6971	1.4365	5.3230	0.0298	2.2904	1.3510	0.8126	-0.3822	0.7428	0.6074	5.1071	0.8651	3.6454	1.5040	5.9471	0.1748	2.5795	1.4085
	VG	3.0327	0.3907	1.7480	1.4816	2.8108	0.2076	1.6349	1.3765	2.8603	1.4916	2.1787	1.6522	4.0912	0.8671	1.9637	1.8223	2.9860	0.3520	1.7219	1.4575
	CGMY	1.9759	-1.0763	1.2651	1.1522	1.8801	-0.9538	1.0592	1.2544	0.3583	-7.1662	0.3583	7.1662	0.1166	-1.9884	0.1138	1.9884	1.9547	-1.0559	1.2209	1.1774
	Kou	2.1323	-0.4229	1.1585	0.8169	2.1382	-0.1714	1.1718	0.9162	13.3730	1.6070	12.2438	1.6070	8.8032	1.5160	6.9749	1.5160	2.2920	-0.3368	1.2120	0.8506
$0.99 \leq \kappa < 1.01$	Merton	10.7683	0.8350	6.4386	0.8480	10.0290	0.8610	5.8864	0.8825	1.1074	0.1089	1.0910	0.1089	11.3338	1.5725	11.2188	1.5725	10.5501	0.8471	6.3078	0.8623
	VG	4.0103	0.1088	2.9101	0.4269	4.2363	0.1113	2.9821	0.4316	5.5617	0.5948	5.5617	0.5948	2.8451	0.2230	2.3608	0.3070	4.0780	0.1107	2.9339	0.4282
	CGMY	2.8991	-0.0131	1.9068	0.2542	3.4453	0.2002	2.4756	0.5005	2.8122	-0.1472	2.0308	0.1472	1.0821	-0.6096	1.0821	0.6096	2.9989	0.0224	2.0048	0.2974
	Kou	4.5575	0.3909	2.8507	0.4147	4.4827	0.6192	3.2254	0.6244	9.9161	0.5865	7.1578	0.5865	8.2188	0.4497	5.7064	0.4497	4.6101	0.4508	2.9846	0.4696
$1.01 \leq \kappa$	Merton	13.2485	0.4135	8.1551	0.4239	16.2481	0.4704	9.3946	0.4817	0.4496	-0.0034	0.4357	0.0244	13.0196	0.8951	12.9414	0.8951	14.0529	0.4293	8.4706	0.4399
	VG	5.6068	0.2029	4.4785	0.2529	6.1567	0.275	4.8936	0.316	5.7037	0.1609	4.2761	0.1609	7.7762	0.3878	7.7762	0.3878	5.778	0.2239	4.6043	0.2708
	CGMY	3.3343	0.0546	2.4172	0.1399	3.967	0.0558	3.3393	0.2807	4.1905	-0.0552	3.5562	0.0687	1.5607	0.0252	1.5607	0.0252	3.423	0.0542	2.5383	0.1573
	Kou	5.3244	0.1911	3.4801	0.2086	5.3284	0.3113	4.1825	0.3545	8.2049	0.0947	6.0568	0.22	7.1112	0.0733	5.6324	0.1743	5.4362	0.2099	3.7125	0.2363
All	Merton	8.6974	0.376	4.3034	1.1588	8.7906	0.2628	4.0539	1.1293	0.8208	-0.2735	0.7479	0.4722	8.3428	1.0456	6.7007	1.445	8.7	0.3464	4.2346	1.1494
	VG	3.7325	0.3082	2.3895	1.0918	3.8548	0.1989	2.443	1.0101	3.9611	1.1028	2.9144	1.2133	4.2294	0.7902	2.2509	1.6281	3.772	0.2855	2.406	1.0737
	CGMY	2.4214	-0.7034	1.5638	0.8283	2.4828	-0.6577	1.5267	1.0295	3.1958	-1.5142	2.3065	1.5195	0.8541	-1.3099	0.5968	1.32	2.4327	-0.6974	1.5564	0.868
	Kou	3.472	-0.1497	1.9129	0.6275	3.2249	0.0366	1.9258	0.7927	10.1616	0.5963	7.9269	0.6631	7.8835	0.6242	6.109	0.6769	3.5334	-0.0897	1.981	0.6723

Fig. 5. European and American Options Pricing Procedure by Using Calibrated Parameter Sets



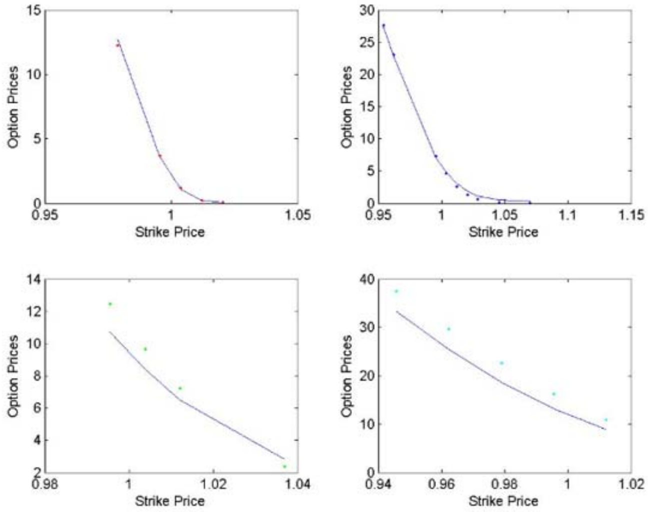
Through the table 5-6 we can check that the european call is explained better than european put by calibrated parameter sets. From the moneyness point of view,the stably good performance were obtained at ATM and ITM. The longer time to maturity, the lower performance is obtained. On the whole, RMSE has higher value than other measures. It is caused by the range of option prices which is from 10 to 100. Because the calibrated parameter sets stay in small range, we can assert that the calibrated parameter sets are in stable. We used the calibrated parameter sets to pricing next Wednesday European and American options.

4.3 Pricing European options

We next estimated S&P 100 index European option prices to show the performance of calibrated parameter sets for real market data. We use the same range of the days with the data for calibrating. Like Figure 5, We use the calibrated parameter sets in the present week for pricing the next week's options. In figure 6 - figure 13, There are estimation results for the European call/put price for the one day and different time to maturity.

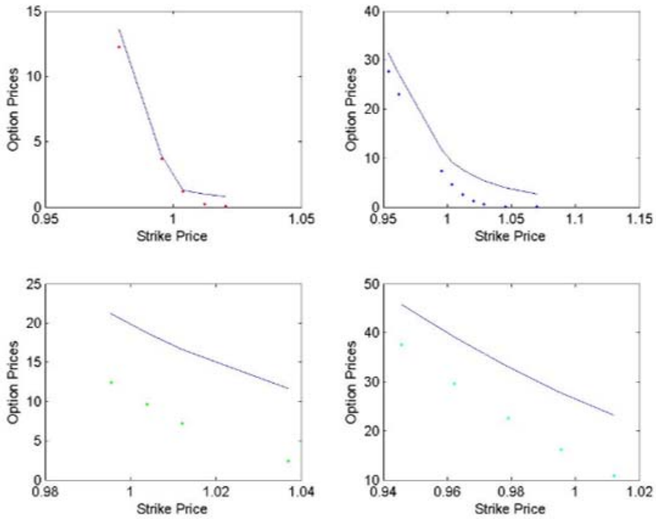
Figures shows that the European call prices have a better performance than the European put prices. Additionally Put price is sensitive for increasing of the time to

Fig. 6. The European call option prediction according to the moneyness Merton model : blue line is call option prices estimated with calibrated parameter sets and dots are true option prices.: Apr. 27. 2011



1.Merton Model

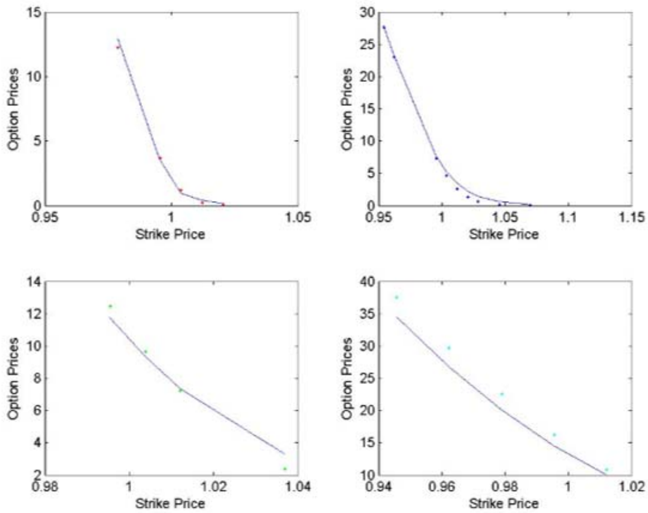
Fig. 7. The European call option prediction according to the moneyness V-G model : blue line is call option prices estimated with calibrated parameter sets and dots are true option prices.: Apr. 27. 2011



2. VG model

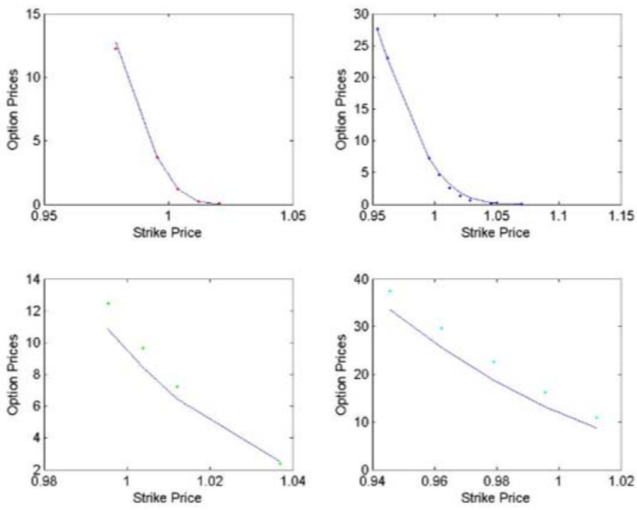
maturities.The Variance-gamma model has the worst prediction performance. We observe that CGMY and Kou model have better performance than Merton and VG model. We can infer that the different jump size for jump direction describe the market better than same jump size. We can make sure of these observations by the table 7 and 8.

Fig. 8. The European call option prediction according to the moneyness CGMY model : blue line is call option prices estimated with calibrated parameter sets and dots are true option prices.: Apr. 27. 2011



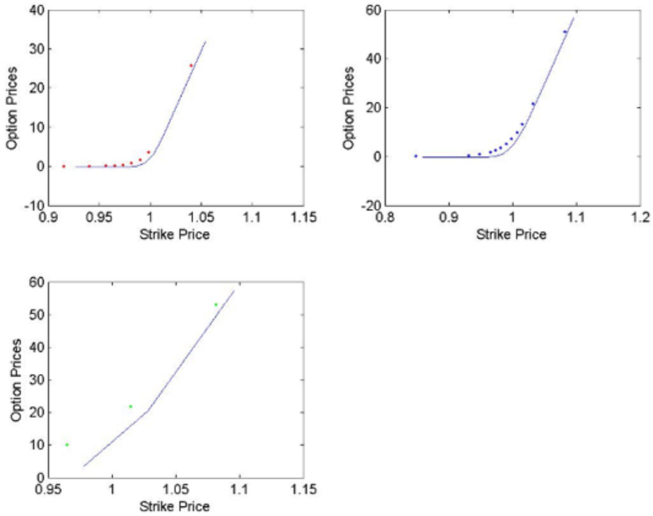
3. CGMY model

Fig. 9. The European call option prediction according to the moneyness Kou model : blue line is call option prices estimated with calibrated parameter sets and dots are true option prices.: Apr. 27. 2011



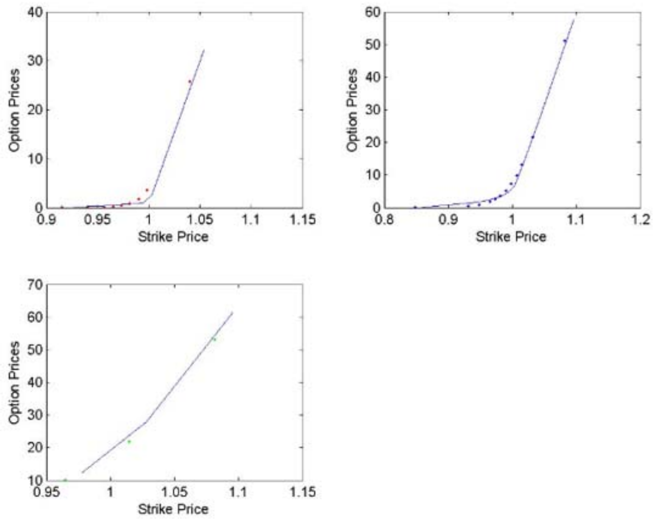
4. Kou model

Fig. 10. The European put option prediction according to the moneyness Merton model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices.: May. 30. 2012



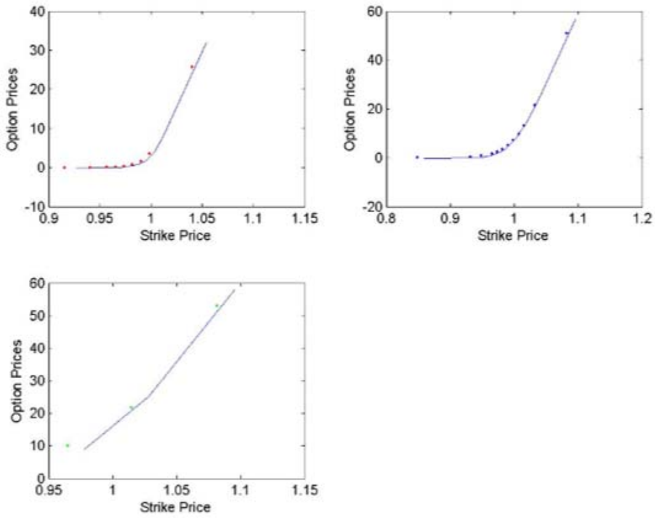
1.Merton Model

Fig. 11. The European put option prediction according to the moneyness V-G model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices.: May. 30. 2012



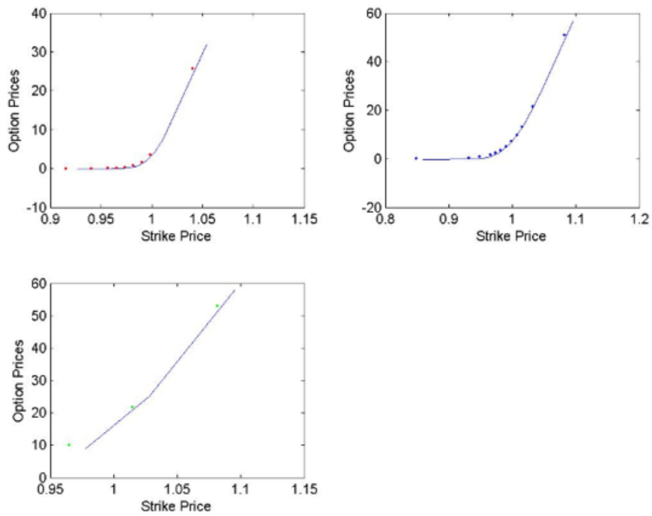
2. VG model

Fig. 12. The European put option prediction according to the moneyness CGMY model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices.: May. 30. 2012



3. CGMY model

Fig. 13. The European put option prediction according to the moneyness Kou model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices.: May. 30. 2012



4. Kou model

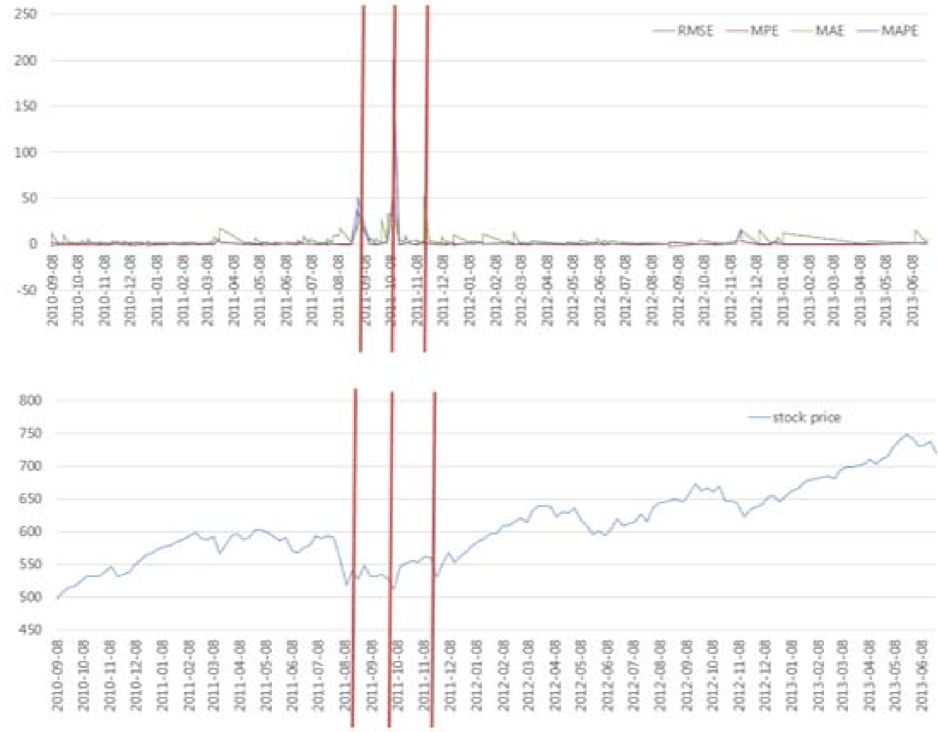
Table 7
Prediction Result of S&P index European Call options ver1.

Call Option		Time to maturity≤ 30				$30 <$Time to maturity≤ 60				$60 <$Time to maturity≤ 90				$90 <$Time to maturity				All			
		RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE
Merton	$\kappa < 0.99$	11.4854	0.0699	3.6099	0.4544	4.9481	0.0094	3.886	0.2126	11.052	-0.1077	9.2653	0.3329	29.6638	-0.2386	24.9096	0.3108	12.5251	0.0122	5.8452	0.3635
	$0.99 \leq \kappa < 1.01$	3.1927	0.5202	1.966	0.8626	6.4673	-0.0203	4.4613	0.2815	7.9083	-0.2366	5.4541	0.2366	22.3597	-0.4046	16.423	0.4046	5.1927	0.3667	2.8512	0.7123
	$1.01 \leq \kappa$	2.4904	1.9023	1.3644	2.244	6.3092	3.0521	4.2887	3.3251	12.8704	19.7966	9.9505	20.0437	5.0955	0.3127	3.9238	0.5472	4.8108	2.9458	2.6389	3.261
	All	5.5304	1.2731	1.9096	1.623	6.0572	1.9589	4.2231	2.2205	11.7075	9.4334	9.2131	9.7844	22.6406	-0.0412	15.9648	0.4114	7.2911	1.8428	3.3801	2.1713
VG	$\kappa < 0.99$	13.529	0.3275	5.1307	0.558	10.1275	0.2695	6.514	0.2865	34.9353	0.353	15.8122	0.4208	20.2416	0.1916	14.2998	0.2167	16.7236	0.3035	7.2622	0.4424
	$0.99 \leq \kappa < 1.01$	5.4195	1.2125	3.077	1.4489	17.9439	0.7379	11.2644	0.7655	11.763	0.5087	9.4046	0.5622	11.3713	0.3405	9.2238	0.3421	9.8405	1.0756	5.1175	1.2579
	$1.01 \leq \kappa$	5.1552	7.033	2.6926	7.2427	19.1984	9.3279	12.785	9.3334	36.0442	64.0124	25.0449	64.0205	19.503	4.7862	15.609	4.7926	13.7028	10.0071	6.8764	10.1436
	All	7.5149	4.5728	3.229	4.7919	17.4161	6.1495	11.189	6.1606	33.9123	30.9251	19.6031	30.9628	19.2671	1.9846	14.3154	2.0001	13.8466	6.2872	6.6477	6.4324
CGMY	$\kappa < 0.99$	11.4009	0.0174	3.4259	0.3879	4.6312	-0.0453	3.7611	0.2003	8.7146	-0.1588	7.4616	0.2933	24.7232	-0.2512	21.018	0.2729	11.412	-0.0383	5.2569	0.3164
	$0.99 \leq \kappa < 1.01$	2.5368	0.1957	1.7117	0.5885	4.9218	-0.0989	3.5433	0.2315	8.0858	-0.2158	5.6802	0.2575	18.3156	-0.4012	14.7	0.4012	4.2167	0.1094	2.4519	0.4983
	$1.01 \leq \kappa$	2.231	1.1012	1.1795	1.5656	4.0538	1.074	3.0326	1.5062	6.669	4.2453	5.2193	4.4907	6.9138	-0.203	5.2185	0.3114	3.2908	1.2033	1.9754	1.6497
	All	5.3772	0.7071	1.7154	1.1388	4.3184	0.6583	3.268	1.0343	7.7319	1.9494	6.2072	2.3045	19.7084	-0.2501	14.9163	0.2995	6.2177	0.7296	2.7918	1.1453
Kou	$\kappa < 0.99$	3.2432	-0.0548	2.0942	0.2904	3.6036	-0.051	2.866	0.1252	7.1933	-0.1718	5.0273	0.1894	23.295	-0.2474	19.2484	0.2474	7.4472	-0.0799	3.8978	0.2304
	$0.99 \leq \kappa < 1.01$	1.9612	0.0352	1.2642	0.3757	4.3026	-0.0442	3.158	0.2189	7.1675	-0.1641	4.182	0.2045	19.4295	-0.4627	16.2694	0.4627	3.8518	0.0038	2.0138	0.3387
	$1.01 \leq \kappa$	1.3802	0.5272	0.8958	1.1173	3.8186	1.1228	2.7747	1.4978	3.9354	0.7641	2.8494	1.014	6.9625	-0.0014	4.5665	0.2656	2.681	0.7032	1.6101	1.2088
	All	1.9773	0.3169	1.1951	0.8098	3.8425	0.7036	2.8473	1.0193	5.8569	0.2781	3.8974	0.5867	18.3134	-0.173	13.2768	0.2753	4.3706	0.4073	2.1821	0.8398

Table 8
Prediction Result of S&P index European Put options ver1.

Put Option		Time to maturity ≤ 30				$30 <$ Time to maturity ≤ 60				$60 <$ Time to maturity ≤ 90				$90 <$ Time to maturity				All			
		RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE
Merton	$\kappa < 0.99$	16.5442	7.7911	5.8603	8.0282	16.5395	6.9531	7.7436	7.0941	41.4598	89.2646	18.147	89.4	54.8187	28.1312	36.7419	28.4425	21.0142	10.1804	8.2709	10.389
	$0.99 \leq \kappa < 1.01$	3.8006	0.4061	2.2424	0.9893	10.4365	0.1543	6.2037	0.4122	4.9056	-0.3046	4.3571	0.3046	20.2806	-0.3642	17.6943	0.4107	6.0869	0.3427	3.1916	0.87
	$1.01 \leq \kappa$	3.6412	-0.6068	1.8774	1.476	8.1339	-0.6484	5.3479	1.2087	16.9209	-0.1558	9.7547	0.8712	25.6819	-0.033	18.1895	1.1526	7.4992	-0.5844	3.5102	1.3827
	All	7.8093	1.1228	2.6751	2.5755	11.216	1.4419	6.0821	2.6499	22.7454	15.4815	10.8185	16.3212	36.8553	8.5369	23.816	9.428	11.5051	1.7925	4.4367	3.1509
VG	$\kappa < 0.99$	17.1417	7.9784	6.1257	8.1606	18.4441	7.8934	9.2687	7.9086	47.7632	100.8973	25.6737	100.8973	50.5575	26.8319	33.6768	26.9786	21.5216	10.7556	8.9065	10.8799
	$0.99 \leq \kappa < 1.01$	3.2275	0.5866	2.0405	0.891	7.7648	0.4559	6.1305	0.5016	10.0066	0.5567	8.6435	0.6125	45.603	-0.3828	28.0797	0.6455	6.9718	0.5494	3.2034	0.8144
	$1.01 \leq \kappa$	2.7342	0.7471	1.5229	1.8556	6.9922	-0.2257	4.4059	1.1506	6.3103	0.0315	4.7895	0.5922	40.3789	-0.6653	22.3687	1.1702	8.6462	0.4409	3.0402	1.6245
	All	7.7634	2.0344	2.4636	2.8165	11.2505	1.9849	5.8945	2.8386	20.8951	17.7224	8.7333	18.1471	44.0828	7.7428	26.2058	9.0084	12.2607	2.5851	4.277	3.3932
CGMY	$\kappa < 0.99$	15.8468	7.5931	4.897	7.8402	14.6423	6.6395	5.3858	6.804	39.8776	85.811	17.6051	85.9211	46.5127	25.2589	29.2108	25.611	19.0693	9.7451	6.5425	9.9689
	$0.99 \leq \kappa < 1.01$	2.4684	0.2375	1.5266	0.6551	4.9385	0.0035	3.7442	0.2702	4.3343	-0.0019	3.8155	0.2055	18.3023	-0.4134	18.1202	0.4134	3.7446	0.1838	2.1777	0.5776
	$1.01 \leq \kappa$	3.0064	-0.8334	1.5309	1.1935	6.0361	-0.8773	4.3594	1.0851	6.7683	-0.4289	5.4051	0.6007	18.8689	-0.478	14.2494	0.7473	5.3647	-0.8204	2.7701	1.1349
	All	7.253	0.9128	2.1439	2.3001	9.0287	1.2006	4.5512	2.4804	17.7216	14.695	7.4209	15.5021	30.1001	7.3783	19.0763	8.3102	9.7602	1.5286	3.4462	2.86
Kou	$\kappa < 0.99$	16.0891	7.7822	5.3816	7.9441	14.8778	6.725	5.4648	6.8171	39.9277	85.9803	17.5366	86.0431	51.205	27.7647	30.5266	27.8074	19.8505	10.0194	6.931	10.1513
	$0.99 \leq \kappa < 1.01$	2.7408	0.5257	1.6951	0.8606	6.8655	0.184	4.0893	0.2804	2.9246	0.1439	2.3084	0.1725	9.3203	-0.1313	9.0618	0.2045	3.9715	0.451	2.2351	0.7396
	$1.01 \leq \kappa$	3.1343	-0.6194	1.5376	1.2962	6.1369	-0.7254	4.0932	1.0623	10.9682	-0.229	7.3924	0.7548	12.8338	-0.2773	10.646	0.6669	5.0084	-0.6224	2.6408	1.2021
	All	7.4016	1.1368	2.2694	2.4232	9.3238	1.3387	4.4518	2.4711	19.2316	14.8855	8.7864	15.6366	30.1512	8.2877	16.6039	8.9157	9.9761	1.7541	3.4555	2.9672

Fig. 14. Prediction measure plot and S&P 100 index time series



Like Calibration results, We identify that the performance go from bad to worse according to increasing of the time to maturities. We expected the bad performance from the mispricing. Because the calibrated parameter sets using for prediction are gained under the risk-neutral measure, the parameter sets couldn't reflect the risk premium for long time to maturity and other risks. we observe that the middle time to maturity (from 30 days to 60 days) has a better performance than the short time to maturity (Under 30days).Estimation result is widely available at two points of view: Moneyness and Time to maturity. When compared to in the money(ITM) and at the money(ATM) options, an out of the money(OTM) option isn't accurately estimated. We can infer that the three cause of the observation. First, the OTM options are usually used for speculation. It exhibits the needs for pricing according to investor's utilities. Second, there is a limit to be small due to tick trade. Lastly, relative error is large due to OTM option price is small. In the second perspective time to maturity, The longer the time to maturity, The greater estimation error is acquired. The arbitrage trades are still seems to be present for the imbalance of information.

In figure 14, When the S&P 100 index drops sharply or increase sharply, the prediction measures have a extreme bad performance. Therefore, We reestimated the price of options after revising the days which have a index's sharp move.
After revising, the total estimation scopes are reduced slightly, but hold the observations before the revision.

Table 9
Prediction Result of S&P index European Call options ver2.

Call Option		**Time to maturity** ≤ 30				$30 <$ **Time to maturity** ≤ 60				$60 <$ **Time to maturity** ≤ 90				$90 <$ **Time to maturity**				**All**			
		RMSE	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**
Merton	$\kappa < 0.99$	12.3541	-0.104	3.7865	0.2716	4.1947	-0.1124	3.3126	0.1265	10.9027	-0.2075	8.9539	0.2795	21.8962	-0.2026	16.9501	0.2099	11.3502	-0.1229	4.9354	0.2276
	$0.99 \leq \kappa < 1.01$	3.2862	0.4887	2.0385	0.8253	6.5077	-0.0537	4.4045	0.271	7.9083	-0.2366	5.4541	0.2366	6.3066	-0.2537	5.7909	0.2537	4.4078	0.3424	2.6908	0.6829
	$1.01 \leq \kappa$	2.4911	1.706	1.3492	2.0672	5.8122	1.7947	3.7535	2.1134	7.8003	0.6087	5.7875	0.9574	5.0955	0.3127	3.9238	0.5472	4.0363	1.6636	2.2376	2.0094
	All	5.7344	1.1293	1.9237	1.4878	5.6124	1.1105	3.752	1.4127	9.3777	0.1273	7.2142	0.5554	14.6991	0.0507	9.507	0.3822	6.3418	1.0466	2.8817	1.3924
VG	$\kappa < 0.99$	11.7931	0.2433	3.9128	0.4887	7.232	0.2181	4.8862	0.237	6.5852	0.1781	5.7546	0.2531	20.2416	0.1916	14.2998	0.2167	11.318	0.226	5.1944	0.3755
	$0.99 \leq \kappa < 1.01$	3.7586	0.5998	2.2489	0.8723	13.1184	0.5901	8.4257	0.6211	11.763	0.5087	9.4046	0.5622	11.3713	0.3405	9.2238	0.3421	7.4015	0.5896	3.963	0.7971
	$1.01 \leq \kappa$	3.4859	6.8814	2.2524	7.107	13.4594	5.3989	9.2229	5.4052	18.6494	3.7682	13.2007	3.7784	19.503	4.7862	15.609	4.7926	9.0244	6.2933	4.9646	6.4428
	All	6.0849	4.3667	2.572	4.6053	12.2848	3.572	8.1375	3.5845	13.7451	1.8029	9.4692	1.8467	19.2671	1.9846	14.3154	2.0001	9.3456	3.9403	4.8432	4.0998
CGMY	$\kappa < 0.99$	3.6002	0.0209	2.4517	0.3994	4.6312	-0.0453	3.7611	0.2003	9.016	-0.1852	7.7292	0.3145	12.5255	-0.1521	11.6598	0.1869	5.4735	-0.0294	3.8701	0.3173
	$0.99 \leq \kappa < 1.01$	2.1946	-0.0143	1.4436	0.413	4.9218	-0.0989	3.5433	0.2315	8.0858	-0.2158	5.6802	0.2575	11.2707	-0.3447	8.3357	0.3447	3.6101	-0.0455	2.1752	0.3634
	$1.01 \leq \kappa$	1.7157	0.8056	1.0233	1.2729	3.9273	1.0064	2.9488	1.4438	6.8367	0.6447	5.2951	0.9124	6.9138	-0.203	5.2185	0.3114	3.0535	0.8419	1.8582	1.293
	All	2.2654	0.5021	1.3654	0.9455	4.2457	0.6125	3.2167	0.9913	7.9395	0.2083	6.3424	0.5943	10.2268	-0.1927	8.4298	0.2578	3.7917	0.5025	2.3432	0.924
Kou	$\kappa < 0.99$	3.3101	-0.0419	2.143	0.2753	3.6036	-0.051	2.866	0.1252	5.9286	-0.1462	4.3287	0.1679	18.0634	-0.1803	14.1786	0.1803	5.743	-0.0622	3.2908	0.2142
	$0.99 \leq \kappa < 1.01$	1.9562	0.0014	1.2264	0.3351	4.3026	-0.0442	3.158	0.2189	7.1675	-0.1641	4.182	0.2045	12.3955	-0.4248	10.0407	0.4248	3.2213	-0.0189	1.8559	0.3062
	$1.01 \leq \kappa$	1.3875	0.5249	0.9004	1.1043	3.8186	1.1228	2.7747	1.4978	4.1057	0.268	3.0519	0.5406	6.9625	-0.0014	4.5665	0.2656	2.6963	0.6842	1.6227	1.1839
	All	1.9814	0.3175	1.1887	0.8003	3.8425	0.7036	2.8473	1.0193	5.277	0.0588	3.6736	0.3584	13.5936	-0.1165	9.424	0.2385	3.6308	0.4048	2.0119	0.8273

Table 10
Prediction Result of S&P index European Put options ver2.

Put Option		Time to maturity≤ 30				$30 <$ Time to maturity≤ 60				$60 <$ Time to maturity≤ 90				$90 <$ Time to maturity				All			
		RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE
Merton	$\kappa < 0.99$	2.7109	-0.7579	1.4416	1.5009	6.7599	-0.734	4.4489	1.2363	15.6846	-0.3436	8.4398	0.7793	27.5109	-0.0163	19.7911	1.3415	7.0071	-0.7149	3.0153	1.4082
	$0.99 \leq \kappa < 1.01$	3.8493	0.4389	2.2709	1.0075	9.2859	0.1115	5.4024	0.3839	4.9056	-0.3046	4.3571	0.3046	23.2993	-0.5166	22.233	0.5166	5.7621	0.3628	3.0608	0.8847
	$1.01 \leq \kappa$	5.711	0.4008	3.6916	0.6357	8.2844	0.1394	5.8534	0.2991	2.8909	-0.0621	2.4725	0.1273	22.6575	-0.4361	19.3077	0.4496	7.3479	0.2893	4.7058	0.5121
	All	3.6626	-0.2967	2.017	1.2423	7.5388	-0.4008	4.9318	0.8859	13.8936	-0.3001	7.2374	0.6455	26.5382	-0.1168	19.9017	1.1459	6.8624	-0.3182	3.3535	1.1359
VG	$\kappa < 0.99$	2.2819	0.8615	1.3329	2.0499	6.9459	-0.2281	4.2095	1.2414	6.3503	0.0848	4.7264	0.6082	42.936	-0.9129	22.6302	1.2779	8.8739	0.5065	2.8674	1.7795
	$0.99 \leq \kappa < 1.01$	3.2513	0.585	2.036	0.885	7.6702	0.4727	6.141	0.5221	10.0066	0.5567	8.6435	0.6125	7.8563	0.1751	7.3778	0.1751	4.5383	0.5605	2.8885	0.8105
	$1.01 \leq \kappa$	5.7226	0.3386	3.8599	0.5054	9.5252	0.3534	7.1581	0.3701	10.9739	0.415	8.486	0.415	21.9797	-0.0769	15.6446	0.3631	7.9163	0.3362	5.2705	0.4563
	All	3.3502	0.7099	1.9302	1.5274	7.7764	0.0117	5.214	0.9252	7.5429	0.1724	5.5993	0.581	38.8048	-0.7059	20.4384	1.0579	8.0483	0.4833	3.34	1.3408
CGMY	$\kappa < 0.99$	1.6881	-0.9638	1.0783	1.2041	4.4641	-0.9382	3.6024	1.1319	7.006	-0.4838	5.574	0.6328	15.3017	-0.4961	12.5274	0.8167	4.0305	-0.9276	2.2261	1.1563
	$0.99 \leq \kappa < 1.01$	2.4844	0.2693	1.5344	0.6624	4.7943	-0.0101	3.5545	0.2678	4.3343	-0.0019	3.8155	0.2055	19.3239	-0.4386	19.2199	0.4386	3.6379	0.2095	2.1059	0.5855
	$1.01 \leq \kappa$	3.3904	0.2351	2.5727	0.4626	5.3211	0.014	3.7307	0.1911	3.9866	-0.0021	3.2138	0.152	21.2057	-0.4186	17.4958	0.4186	5.1141	0.1446	3.2734	0.3668
	All	2.256	-0.4877	1.4401	0.957	4.74	-0.5741	3.6289	0.7791	6.4611	-0.3737	5.0861	0.5275	16.6409	-0.4801	13.7746	0.7286	4.1972	-0.5066	2.4079	0.896
Kou	$\kappa < 0.99$	1.8819	-0.7545	1.0596	1.3085	4.1466	-0.8208	3.1339	1.0787	8.7938	-0.4005	6.0535	0.6683	12.8487	-0.2991	10.8081	0.762	3.8135	-0.7455	2.0523	1.2139
	$0.99 \leq \kappa < 1.01$	2.7515	0.56	1.6959	0.8745	4.8189	0.1483	3.1832	0.2438	2.9246	0.1439	2.3084	0.1725	10.1029	-0.2239	9.9412	0.2239	3.3704	0.4745	2.0536	0.7487
	$1.01 \leq \kappa$	4.3324	0.4357	3.1338	0.5852	4.7115	0.0779	3.4096	0.1798	3.8415	0.0636	2.9667	0.1515	6.9776	0.0845	6.0206	0.161	4.522	0.304	3.2815	0.4353
	All	2.6639	-0.2625	1.5626	1.0877	4.3883	-0.465	3.2107	0.7404	7.9054	-0.2875	5.2915	0.552	11.9541	-0.2359	10.025	0.6315	3.8861	-0.3134	2.2923	0.9754

4.4 *Pricing American put options*

We next use the calibrated parameter sets from the S&P 100 index european options to price the S&P 100 index american put options at the same date with calibration. Numerical method in section 3 is used for american put option pricing method. There are the American put option prices according to the moneyness for Merton, VG, CGMY, and Kou model in figure 15 - figure 18.

Through table 12 American option pricing is better performance than European option pricing for the options which have the short time to maturity(T¡30days). Every model is better performance than european options pricing except the Variance-Gamma model.

We can guess that the observations from the difference with european options and american options. American options can be exercised at any time before the expiration date in contrast with european options. Therefore american option holder rarely bears holding risk in the shortest maturity. However, American option pricing performance more decreases rapidly according to the increasing the time to maturity than the European pricing. It is based on the liquidity of the expiration date which makes holding risk lower. Besides,Trading dexision of longer time to maturity options goes left or right largely by investor's utilities.

Other observations of result are same for european options. Estimation result is widely available at two points of view: Moneyness and Time to maturity. When compared to in the money(ITM) and at the money(ATM) options, an out of the money(OTM) option isn't accurately estimated. We can infer that the three cause of the observation. First, the OTM options are usually used for speculation. It exhibits the needs for pricing according to investor's utilities. Second, there is a limit to be small due to tick trade. Lastly, relative error is large due to OTM option price is small. In the second perspective time to maturity, The longer the time to maturity, The greater estimation error is acquired. The arbitrage trades are still seems to be present for the imbalance of information.

Following table 11,there is american option pricing performance after revision as above european pricing section.

Fig. 15. The American put option prediction according to the moneyness Merton model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices.: May. 02. 2012

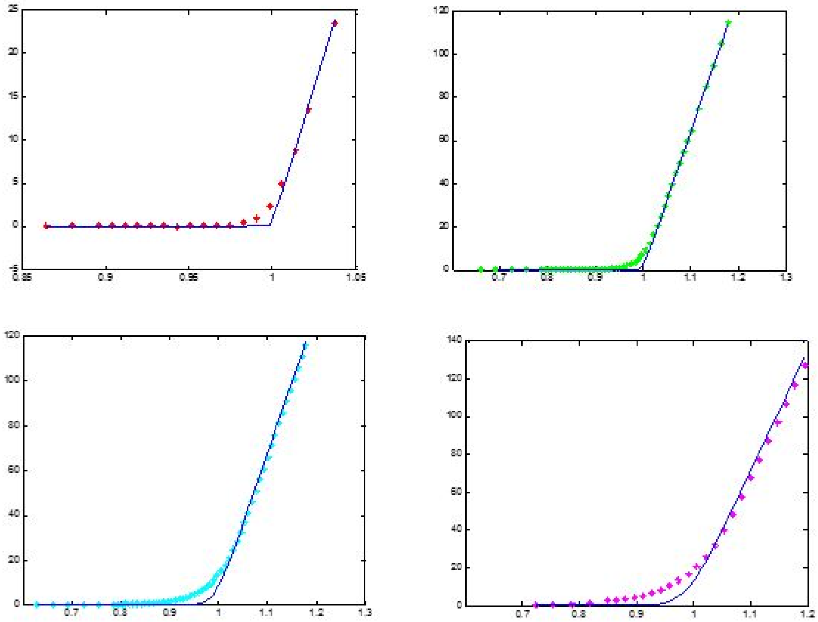


Fig. 16. The American put option prediction according to the moneyness V-G model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices.: May. 02. 2012

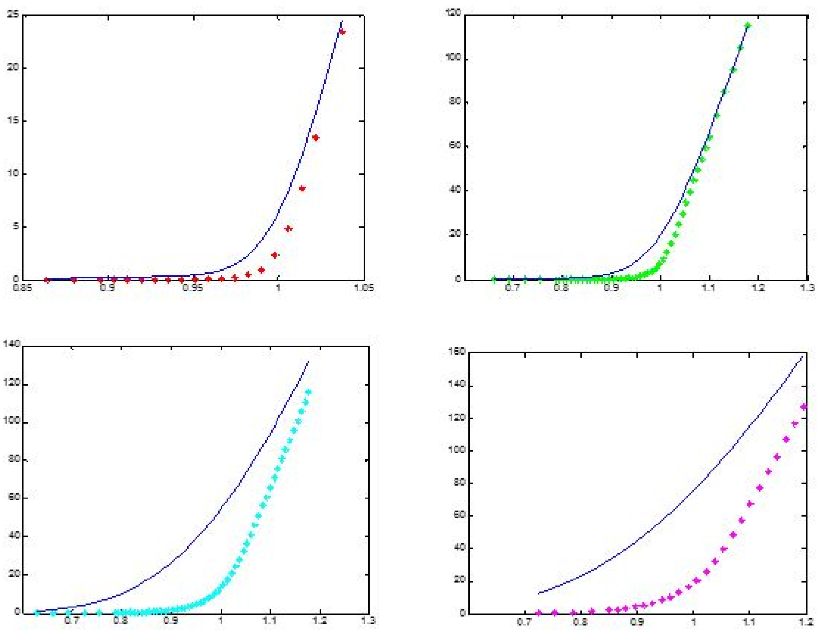


Fig. 17. The American put option prediction according to the moneyness CGMY model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices.: May. 02. 2012

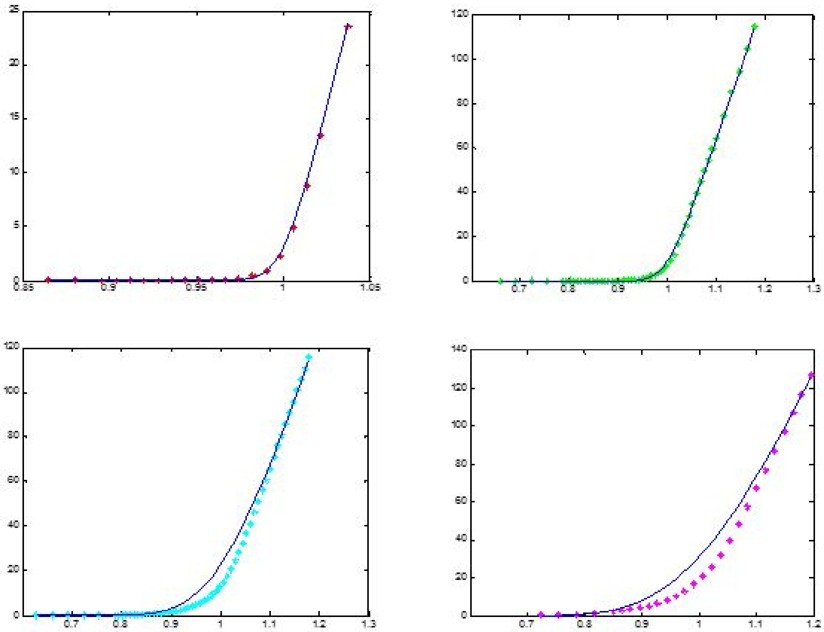


Fig. 18. The American put option prediction according to the moneyness Kou model : blue line is put option prices estimated with calibrated parameter sets and dots are true option prices.: May. 02. 2012

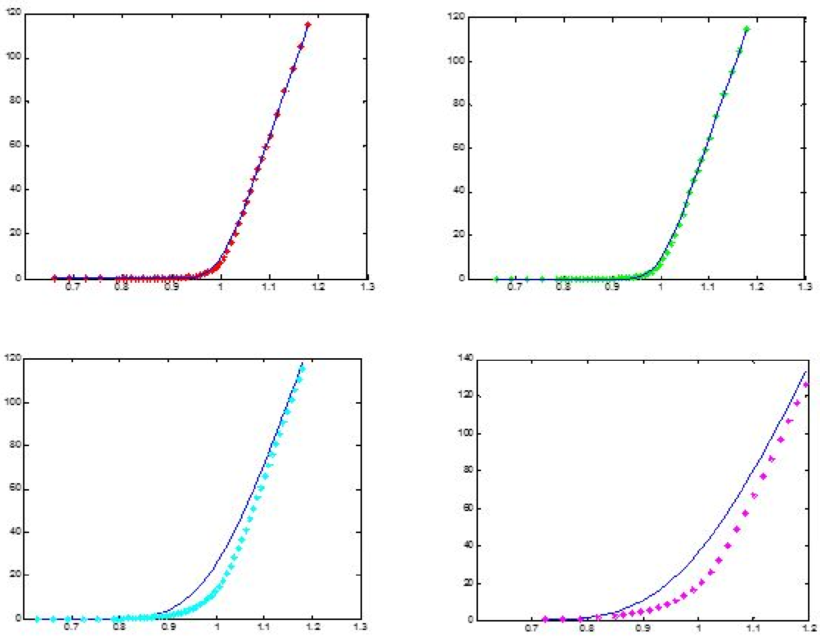


Table 11
Prediction Result of S&P index American Put options ver1.

Put Option		Time to maturity≤ 30				$30 <$Time to maturity≤ 90				$90 <$Time to maturity$\leq 1year$				$1year <$Time to maturity				All			
		RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE	RMSE	MPE	MAE	MAPE
Merton	$\kappa < 0.99$	1.4738	-0.3816	0.4283	1.1399	15.8953	48.7711	11.1051	48.9905	23.6666	5.5174	11.2359	6.0832	220.2661	131.2426	180.7542	131.344	145.2188	59.6041	82.1217	60.1453
	$0.99 \leq \kappa < 1.01$	4.204	0.4563	2.6108	0.7499	29.9904	5.5007	27.3557	5.5007	39.8816	1.784	27.726	1.828	274.3867	8.9063	231.2448	8.9223	150.8827	3.6522	80.1369	3.7597
	$1.01 \leq \kappa$	4.0213	0.0485	2.9557	0.109	25.1128	0.8852	21.34	0.8852	39.7086	0.5521	25.4111	0.5555	292.1426	3.5216	256.9203	3.5257	186.1298	1.6538	113.4559	1.6678
	All	2.7441	-0.1921	1.3534	0.7969	20.8	28.7598	15.8874	28.8859	31.0173	3.6129	16.9363	3.9585	246.9191	87.1188	206.3494	87.1868	159.9908	38.1361	92.2796	38.4853
VG	$\kappa < 0.99$	13.1693	27.2253	4.8208	27.4241	47.1442	186.4956	33.999	186.4956	79.7436	66.6346	51.1885	66.6427	289.079	193.3432	262.6536	193.3432	196.8522	117.1634	134.023	117.2049
	$0.99 \leq \kappa < 1.01$	19.6741	3.6432	11.7417	3.6479	68.8486	11.3943	59.0335	11.3943	111.6754	6.1356	88.6833	6.1356	370.4042	14.3678	346.3208	14.3678	218.0297	8.2004	145.1728	8.2018
	$1.01 \leq \kappa$	20.4371	0.3738	9.8066	0.3912	62.2522	1.9353	49.308	1.9353	104.8061	1.7125	75.2547	1.7125	386.4844	5.1637	363.3768	5.1637	257.2205	2.906	181.9964	2.9091
	All	16.1113	17.7484	6.7488	17.8809	54.432	111.4939	40.9798	111.4939	90.4968	42.0245	60.9577	42.0295	325.2121	129.0441	296.6793	129.0441	218.7844	76.0075	149.7921	76.035
CGMY	$\kappa < 0.99$	0.9303	-0.625	0.3297	0.9824	9.9906	23.7005	6.6356	23.947	10.4679	1.8488	5.5369	2.4888	116.7113	41.7135	87.1011	41.8815	76.8426	19.0897	39.6721	19.7029
	$0.99 \leq \kappa < 1.01$	3.0829	0.393	2.1161	0.6338	19.1578	3.3324	17.597	3.3324	20.0509	1.0323	15.971	1.0896	154.9324	5.1574	127.4984	5.2139	84.9977	2.1592	44.7465	2.2679
	$1.01 \leq \kappa$	3.4606	0.0205	2.5794	0.0937	13.1395	0.4542	9.7285	0.4607	15.7246	0.2431	10.5111	0.2626	167.0523	1.8496	135.0264	1.8739	105.9408	0.8491	58.661	0.8799
	All	2.2061	-0.3571	1.1425	0.6858	12.0324	14.032	8.5173	14.176	13.0736	1.2505	7.7471	1.6473	135.8348	27.9937	103.4818	28.1129	87.7696	12.363	46.1102	12.7631
Kou	$\kappa < 0.99$	0.9472	-0.4914	0.3239	1.0378	12.4387	36.1089	8.9759	36.2912	15.199	3.6501	8.8022	4.0302	1.14E+48	8.61E+44	3.29E+46	8.61E+44	7.45E+47	3.67E+44	1.4E+46	3.67E+44
	$0.99 \leq \kappa < 1.01$	3.2317	0.572	2.2209	0.7021	23.4321	4.2168	22.2039	4.2168	28.2599	1.6178	24.2064	1.6178	1.29E+46	8.66E+42	8.49E+44	8.66E+42	6.96E+45	2.5E+42	2.46E+44	2.5E+42
	$1.01 \leq \kappa$	3.4546	0.036	2.6393	0.0983	16.8586	0.6183	13.7763	0.6187	24.6965	0.4398	18.6868	0.4401	1.91E+48	7.27E+44	9.64E+46	7.27E+44	1.2E+48	2.87E+44	3.8E+46	2.87E+44
	All	2.2246	-0.256	1.1666	0.7257	15.1275	21.2903	11.6234	21.3954	19.7259	2.436	12.9536	2.6661	1.41E+48	7.92E+44	5.19E+46	7.92E+44	9.06E+47	3.24E+44	2.13E+46	3.24E+44

Table 12

Prediction Result of S&P index American Put options ver2.

Put Option		Time to maturity ≤ 30				$30 <$ Time to maturity ≤ 90				$90 <$ Time to maturity $\leq 1year$				$1year <$ Time to maturity				All			
		RMSE	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**	**RMSE**	**MPE**	**MAE**	**MAPE**
Merton	$\kappa < 0.99$	1.9001	-0.2383	0.5707	1.2213	15.2104	44.8256	10.6486	45.0099	24.0386	5.4714	11.7499	6.0061	222.4368	123.147	182.8096	123.2441	145.4736	55.275	81.9515	55.7971
	$0.99 \leq \kappa < 1.01$	4.7864	0.4409	2.9364	0.7389	29.5078	5.2797	27.0102	5.2797	40.0386	1.7577	27.9653	1.7987	278.2225	8.7936	234.7889	8.8089	150.8366	3.5288	79.46	3.6381
	$1.01 \leq \kappa$	5.3505	0.0478	3.6827	0.1191	24.0349	0.823	20.2936	0.823	40.0438	0.5476	25.938	0.5517	295.9494	3.4837	260.2217	3.4874	185.7315	1.5942	112.0095	1.611
	All	3.5991	-0.1011	1.7306	0.834	19.8455	26.9646	15.0773	27.0728	31.4692	3.5447	17.5453	3.8676	250.0441	81.3628	209.1597	81.4274	160.3079	35.0233	91.9106	35.3581
VG	$\kappa < 0.99$	12.7337	25.9437	4.7363	26.1516	48.727	195.0371	36.6436	195.0371	78.9029	63.9604	51.5595	63.9679	290.3505	184.1077	263.6872	184.1077	196.3002	111.5803	133.1905	111.6242
	$0.99 \leq \kappa < 1.01$	19.223	3.4596	11.6462	3.4769	70.1614	11.2724	60.8453	11.2724	110.1519	5.9718	87.9485	5.9718	371.3829	14.1306	347.1689	14.1306	215.8261	7.9326	142.7461	7.9378
	$1.01 \leq \kappa$	19.3807	0.3437	9.7365	0.3715	64.0979	1.924	52.3903	1.924	102.6371	1.6649	74.7342	1.6649	387.7532	5.0784	364.3425	5.0784	254.3523	2.7954	178.3231	2.8007
	All	15.5369	16.5934	6.7249	16.7334	55.9073	118.1403	43.5374	118.1403	89.2741	39.9022	61.1357	39.9067	326.7927	122.3742	298.0162	122.3742	217.709	71.7147	148.2958	71.7444
CGMY	$\kappa < 0.99$	1.4996	-0.571	0.4605	1.0081	9.4019	21.1546	6.1354	21.4241	11.2572	1.8523	6.0194	2.4788	121.7728	39.4395	90.5165	39.6038	79.5416	17.8298	40.7075	18.4386
	$0.99 \leq \kappa < 1.01$	3.9778	0.3746	2.4671	0.6295	18.7362	3.1917	17.1957	3.1917	20.8115	1.0158	16.5066	1.0858	159.4769	5.1097	130.4481	5.1673	86.2917	2.0897	44.8546	2.2088
	$1.01 \leq \kappa$	4.7323	0.0194	3.2942	0.1049	12.4785	0.4175	9.1083	0.4247	17.4222	0.2455	11.5596	0.2726	175.1329	1.8721	140.4795	1.8955	109.4553	0.8357	59.7107	0.872
	All	3.1052	-0.3181	1.5069	0.6918	11.3675	12.7823	7.9023	12.943	14.2673	1.2402	8.466	1.6275	142.1981	26.3702	107.7208	26.4862	90.9382	11.4468	47.2549	11.8423
Kou	$\kappa < 0.99$	1.548	-0.4307	0.4699	1.067	12.5513	36.169	9.2279	36.3216	16.2237	3.7088	9.3915	4.0726	1.11E+48	8.18E+44	3.13E+46	8.18E+44	7.16E+47	3.39E+44	1.29E+46	3.39E+44
	$0.99 \leq \kappa < 1.01$	4.0214	0.5481	2.5762	0.6958	23.5141	4.1078	22.3733	4.1078	29.0614	1.5951	24.5086	1.5951	1.27E+46	8.44E+42	8.28E+44	8.44E+42	6.72E+45	2.34E+42	2.29E+44	2.34E+42
	$1.01 \leq \kappa$	4.9621	0.0369	3.4105	0.1102	16.5564	0.5896	13.5389	0.59	26.3164	0.4407	19.5108	0.4423	1.85E+48	6.84E+44	9.06E+46	6.84E+44	1.13E+48	2.57E+44	3.4E+46	2.57E+44
	All	3.2392	-0.2147	1.5599	0.7332	14.9754	21.7213	11.5969	21.8111	21.0378	2.4455	13.6819	2.6636	1.37E+48	7.5E+44	4.92E+46	7.5E+44	8.66E+47	2.96E+44	1.94E+46	2.96E+44

5 Conclusion

We compare the prediction performance of european and american option prices from the parameter sets acquired by calibrating european options. Estimated parameter are stable for each day. We may assure the parameter that is locally optimal. For comparing option pricing performance, we used four error measure : RMSE,MAE,MPE, and MAPE. In very short time to maturity, american option pricing reveals better performance than european option pricing. However, It's performance shrink fast by increasing time to maturity. Call option pricing is more accurately than put option pricing over all.

Estimation result is widely available at two points of view: Moneyness and Time to maturity. When compared to in the money(ITM) and at the money(ATM) options, an out of the money(OTM) option isn't accurately estimated. We can infer that the three cause of the observation. First, the OTM options are usually used for speculation. It exhibits the needs for pricing according to investor's utilities. Second, there is a limit to be small due to tick trade. Lastly, relative error is large due to OTM option price is small. In the second perspective time to maturity, The longer the time to maturity, The greater estimation error is acquired. The arbitrage trades are still seems to be present for the imbalance of information.

the gap of performance may be based on the time to maturity, moneyness and the type of expirations. Reasoning of the pricing performance differences and the pricing method considering investor's utilities will be undertaken in future research.

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초　　록

본 논문은 3년간의 S&P100 인덱스 옵션 데이터를 사용하여 지수 레비 모형의 모수를 추정하고 동 기간의 일주일 후의 유러피안 옵션 및 아메리칸 옵션 데이터를 예측하여 실증적으로 비교하였다. 머튼, VG, CGMY, Kou 네 가지의 지수 레비 모델의 파라메터를 캘리브레이션을 통하여 추정하였다. 이를 바탕으로 하여 Carr-Madan 의 푸리에 변형을 통한 유러피안 옵션 가격 추정, 및 아메리칸 옵션 가격을 추정하였다. 마지막으로 같은 모수로부터 추정된 아메리칸 옵션과 유러피안 옵션의 가격 예측 결과를 비교해보았다. 그 결과 매우 짧은 만기에서는 아메리칸 옵션의 가격추정이 유러피안 옵션의 가격 추정보다 더 좋은 결과를 나타냈으나, 만기가 증가함에 따라 빠른 속도로 오차가 증가하는 결괄르 보였다. 콜 옵션 가격의 경우 풋 옵션보다 전체적으로 좋은 추정결과를 가져왔다. 일반적으로 알려져 있듯이 OTM 옵션의 가격예측이 ITM이나 ATM 옵션 가격 예측보다 어려운 것을 확인할 수 있었다.

주요어 : 옵션 시장, 지수 레비 모델, 모델 캘리브레이션 및 선택, 지역 최적화.

학　　번 : 2013-21082