이학박사학위논문

Studies on flavor dependent symmetry

입자의 종류마다 다른 대칭성에 관한 연구

2012년 8월

서울대학교 대학원
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김형도

이 논문을 이학박사 학위논문으로 제출함

2012년 8월

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서민석

서민석의 박사 학위논문을 인준함

2012년 8월

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   ① 본인의 논문을 보존이나 인터넷 등을 통한 온라인 서비스 목적으로 복제할 경우 저작물의 내용을 변경하지 않는 범위 내에서의 복제를 허용합니다.

   ② 본인의 논문을 디지털화하여 인터넷 등 정보통신망을 통한 논문의 일부 또는 전부의 복제배포 및 전송 시 무료로 제공하는 것에 동의합니다.

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논문 제목 : Studies on flavor dependent symmetry

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초 록

In this thesis, we study extension of the Standard Model through various symmetries, mainly flavor dependent ones, motivated by problems in particle physics which cannot be resolved within the Standard Model framework. First, as a solution to the hierarchy problem, we observe supersymmetry. The origin of the electroweak symmetry breaking scale can be understood in the context of next-to-minimal supersymmetric Standard Model with the Peccei-Quinn symmetry. As a minimal setup for hierarchy problem, effective supersymmetry, a model with the light third generation squarks, can be considered. The spectrum in the effective supersymmetry can be realized by introducing flavor dependent $U(1)'$ gauge symmetry, under which the third generation quarks and squarks are uncharged. Such kind of flavor dependent symmetry plays a crucial role in investigating the origin of fermion mass hierarchies and mixing patterns. Moreover, mixing pattern can be understood from appropriate parameterization showing intrinsic properties, such as maximal CP violation. Mixing angles are predicted by flavor dependent discrete symmetry. Structure based on D_{12} group gives Cabibbo angle 15°, solar angle 30°, and atmospheric angle 45°. These values should be modified in accordance with the up-to-date neutrino observations, reporting sizable θ_{13}. In this way, flavor dependent symmetries are expected to be good candidates for new physics beyond the Standard Model.

주요어 : Standard Model, Symmetry, Flavor dependent symmetry, Supersymmetry, Higgs, Discrete symmetry, Mixing matrix
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목 차

그 림 목 차

표 목 차

제 1 장

Introduction

Particle physics is a field studying the fundamental working mechanism of Nature: looking for the fundamental ingredients of matter and their interactions. Over the last 60 years, such investigations have been based on quantum mechanics. In quantum mechanics, the physical objects we observe are described in terms of groups and their representations, the mathematical languages for symmetry. As a result, particle physicists tend to ask questions like “Why only specific interactions are allowed, or why some nontrivial patterns appear” and explain them with models in which an appropriate symmetry is imposed. The Standard Model of particle physics is the most fruitful result of such investigations. It describes the fundamental particles and their interactions we know today in terms of gauge symmetries and their spontaneous breaking. Passing numerous experimental tests, the Standard Model is believed to be the most successful description of Nature so far. However, there are a number of problems that particle physics cannot answer yet with the Standard Model. These problems are partly due to the lack of experimental evidences, and partly of theoretical unclearness. Presently, Large Hadron Collider (LHC) experiments are expected to reveal particle physics of very high energy with unprecedentedly high accuracy. Moreover, rapid developments in observational cosmology give a good motivation for explaining the past and present of the Universe in the language of particle physics. As clues for mysteries in particle physics are discovered in experiments, the importance of model building based on new symmetry principle is increasing.

1.1 Electroweak symmetry breaking

Even though the Standard Model explains known Nature successfully, the Higgs boson, the key particle of this model, has not been found yet. As fermions are put in terms of chiral representations under the electroweak gauge group, a scalar particle transforming nontrivially under the group is needed to break this gauge symmetry. It is called the Higgs doublet scalar. After the Higgs doublet breaks gauge symmetry, there remains a scalar boson which couples to all the Standard Model particles with the strength of their masses. But the existence of a fundamental scalar gives rise to another theoretically unsatisfactory problem. Electroweak symmetry is broken around 100GeV. The highest scales we know are the Planck scale(10^{18}GeV) where gravity effects become important or the grand unified theory(GUT) scale(10^{16}GeV) where electroweak and strong gauge symmetries are unified. These scales are far above the electroweak scale. If there is no new physics between them, one has to make an enormous fine-tuning to obtain the fundamental scalar mass at the electroweak scale when the quantum corrections are taken into account. Many solutions to this hierarchy problem have been suggested. Among them, supersymmetry(SUSY) is a very plausible candidate because it is the only way to extend the Poincaré group, the spacetime symmetry at the base of special relativity. Furthermore, it may come from superstring theory, one of the prime candidates for quantum gravity. If supersymmetry is broken below 1TeV, the Higgs mass can be stabilized at the electroweak symmetry breaking scale. One of the prime motivations of the LHC accelerator has been to search for the Higgs boson and superpartners of quarks or leptons, predicted by SUSY. Recently, a signal which may be interpreted as the Higgs boson was observed around 125GeV but the superpartners based on minimal setup of SUSY, the constrained Minimal Supersymmetric Standard Model (CMSSM), are excluded to slightly above 1TeV.

On the other hand, the exclusion bounds of the superpartners of third generation quarks are not so stringent as those of the first two generations, about 400GeV. Moreover, these particles give the main contribution in stabilizing Higgs mass due to their large Yukawa couplings with the Higgs. Then, if SUSY is broken in such a way that only superpartners of third generation quarks are below 1TeV, it can still solve hierarchy problem but is not excluded by the recent LHC results. This idea is called effective SUSY, or natural SUSY. One way to realize this idea is assuming an extra U(1) gauge group, U(1)$'$ under which quarks, leptons and their superpartners except those in the third generation are charged. SUSY is broken in hidden sector which is also charged under U(1)$'$. Then SUSY breaking is transferred to the Standard Model particles and their superpartners though the U(1)$'$ interaction. The point is that only the third generation particles are not charged. For this reason, SUSY in the third generation is broken only though highly suppressed quantum effects. From this flavor dependent U(1)$'$ mediation for SUSY breaking, the mass spectrum of superpartners in effective SUSY is naturally obtained. On the other hand, if the recent 125GeV bump is the Higgs boson, the next to the MSSM (NMSSM) may be favored as it can have a heavier Higgs mass than in the MSSM. Moreover, NMSSM Higgs has a strong relation with the Peccei-Quinn symmetry, solving the strong CP problem.

1.2 Flavor structures of quarks and leptons

The flavor dependent symmetry plays an important role in explaining another problem of the Standard Model. The quark and lepton masses are generated by the Yukawa couplings multiplied by the vacuum expectation value of the Higgs doublet. So the mass hierarchies in the quark and lepton sectors come from the different magnitudes of the Yukawa couplings. However, the Standard Model does not give a reason why the couplings should be as such determined experimentally. On the other hand, the flavor violating processes are described by the mixing matrices, the CKM matrix in the quark sector and the PMNS matrix in the lepton sector. These two have very different patterns: the PMNS matrix shows a very strong mixing whereas the CKM matrix shows a very small mixing. The origin of such different patterns is not explained in the Standard Model. Furthermore, if all three real mixing angles are nonzero, one complex phase cannot be removed and then weak interaction violates CP symmetry. The experimental discovery of the weak CP violation in quark sector proves that all three real mixing angles are nonzero and the CP phase has been determined to $\simeq 90^{\rm o}$.

Such mixing matrices are related to the original patterns of Yukawa couplings, which form 3×3 matrices before diagonalization to the mass eigenstates. Therefore, to see the unexplained aspects of the flavor structure of the Standard Model particles, the structure of the Yukawa couplings should be investigated. The best approach might be considering the flavor dependent symmetry extended also to the Standard Model singlet scalars. The role of the singlets is to implement this symmetry structure in the full theory realized at high energy, for example, at the GUT scale ot Planck scale. Then, the Yukawa couplings of the Standard model can be explained by powers of these vacuum expectation values suppressed by a higher scale such as the Planck mass, as suggested by Froggatt and Nielsen. Unfortunately, we do not have sufficient information at present to know these completely. For example, the chiral nature of the Standard Model allows only left-handed fermions to participate in the weak interaction so that the mixing matrices contain the information on left handed fermions only. Thus, the flavor structure cannot be understood within the Standard Model framework. So even though the basic strategy is apparent, each model faces uniqueness problem, *i.e.* we cannot select a unique model among the various possibilities.

Therefore, at this stage, a serious consideration of hints from the various flavor structures is welcome. The fact that the mixing angle between the second and the third families in the PMNS matrix is almost 45^{o} enables the model builders to consider several discrete symmetries. The most cited example is the so-called tri-bi maximal mixing, which can be easily explained by various discrete symmetries such as the permutation groups S_3, S_4 or their subgroups like A_4. On the other hand, another consideration can be made. For example, if the lepton sector has some type of a discrete symmetry, the quark sector may be described by such a symmetry. Moreover, there is a numerical relation, the quark-lepton complementarity, which states that the sum of the corresponding mixing angles in the CKM and the PMNS matrices is about 45^o. Simplifying the mixing angles between the first two generations in the quark and lepton sectors as 15^o and 30^o, respectively, a model based on the D_{12} dihedral group can be constructed. It is less close to the experimental values than the values obtained from tri-bi maximal mixing. But the (13) element of the PMNS matrix, which is predicted to be zero in almost all discrete symmetry models including the tri-bi maximal mixing and ours, seems to be nonzero . So, corrections to these symmetry patterns are essential, and our model may have the advantage of having room for corrections compared to the tri-bi maximal mixing. Taking the quark-lepton complementarity into account, such corrections may be governed by $\lambda = \sin\theta_C$, the expansion parameter of the CKM matrix.

The fact that λ may be a good expansion parameter for both the quark and lepton mixing angles provides a possibility of model building with the Froggatt-Nielsen mechanism. In this case, λ is given by singlet vacuum expectation values suppressed by the Planck mass scale. Moreover, the phase in the mixing matrix can be explained by the vacuum expectation value of singlet(s) containing the phase. So, the CP violation in the weak interaction can be determined by this spontaneous symmetry breaking mechanism of the flavor symmetry in a complete theory. But in the Standard Model framework only, the phase can be moved here and there or even be separated. So one may ask a question of which parametrization of the CKM matrix is best.

Interestingly, if the phase of the (31) element is moved to be combined with that of the (13) element, this phase is 90°, the angle α which can be seen in the Jarlskog's triangle; in this sense the CP violation in the weak interaction is maximal.

The current issues in particle physics cannot be separated from each other. If these issues are connected under the symmetry principle, one of the most important wisdom of quantum mechanics, particle physics can have more unified picture describing Nature. For a model in particle physics to be the correct description of Nature, it should explain the microscopic world that collider experiments can prove. But this is not enough. Also it should explain the macroscopic world which is the subject of cosmology and astroparticle physics. For example, dark matter requires some special type of discrete symmetry forbidding the decay of dark matter. Many new physics models like supersymmetric extensions or extra dimensions contain such symmetries. Therefore, any new physics model should not contradict to cosmological facts, for example, the dark matter relic density. Flavor structure of the quarks and leptons are related to the baryon asymmetry in the Universe. High energy physics beyond the Standard Model could be used to explain inflation, exponential expansion of the early Universe. Even though cosmological issues are not treated in this thesis, it has great importance in a view of new physics. At present, ground based experiment has not been finding direct evidence of new physics, just confirming the Standard Model even though indirect evidences such as muon $g-2$ can be controversial. On the other hand, dark matter and baryon asymmetry provide direct and strong evidences for new physics beyond the Standard Model.

The thesis is organized as follows. First, we briefly review various aspects of the Standard Model in light of symmetries and their breaking. Then we discuss unsolved problems in the Standard Model, mainly in a theoretical point of view. Among these problems, we consider two issues: understanding electroweak symmetry breaking and flavor structure. Elctroweak symmetry breaking will be considered based on supersymmetry. Especially, for superpartner spectrums consistent with experiments, we introduce flavor dependent gauge symmetry through which supersymmetry breaking is transferred. For flavor problem, we investigate the structure of mixing matrices and construct the model based on non-Abelian discrete symmetry. Then we conclude.

제 2 장

The Standard Model of particle physics

2.1 Spontaneous breaking of electroweak gauge symmetry

Particle physicists have made models to describe Nature in light of symmetry principle[1]. Many models aim to show physics behind phenomena with a simple setup. However, such simplifications are often consistent with experimental results in very high accuracies. The Standard Model (SM) of particle physics is one of such examples. It describes particles and their interactions in terms of the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group. The theory of the strong interaction, Quantum Chromodynamics(QCD) is described by the $\mathrm{SU}(3)_c$ gauge group[2]. Fermions charged under this so participate in the strong interaction are called quarks and others are called leptons. Since SU(3) gauge theory with quarks has asymptotic freedom[3], the farther the quarks are separated, the stronger the strong interaction. On the other hand, electromagnetic and weak interactions are described by spontaneous breaking of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group[4]. One essential feature of this electroweak gauge theory is chirality of fermions, *i.e.* left- and right- handed parts of fermionic matter are not equally charged under the

$SU(2)_L \times U(1)_Y$ gauge group. Since Dirac mass of fermion can be interpreted as a coupling between left- and right- handed components, it is impossible to give the Dirac mass of chiral fermion in the theory of fermionic matters only. Therefore, gauge charged scalar should be introduced to make (scalar)-(left handed fermion)-(right handed fermion coupling). In the SM, the left handed fermions are doublets(fundamental representation) and the right handed fermions are singlet under the $SU(2)_L$, as tabulated in Table 1. Therefore, as a simplest choice, $SU(2)_L$ doublet scalar with appropriate $U(1)_Y$ charge can be chosen. This scalar is called the Higgs doublet[6]. When the Higgs doublet has vacuum expectation value(VEV), electroweak gauge symmetry is spontaneously broken, and only electromagnetic interaction $U(1)_{\rm em}$ remains as a long range force. At the same time, fermions obtain Dirac masses. Table 1 lists how quarks and leptons are charged under the SM gauge group. Each fermion in the table has three copies: three $u-$type quarks u, c, t, three $d-$type quarks d, s, b, three charged leptons e, μ, τ and three neutrinos ν_e, ν_μ, ν_τ. In this way, the SM fermions form three generations.

Higgs scalar $H = (H^+, H^0)$ is $SU(2)_L$ doublet and has $U(1)_Y$ hypercharge $1/2$, to make the gauge invariant Yukawa coupling,

$$-\mathcal{Y}^u_{ij} \bar{q}^i u^j_R \tilde{H} + \mathcal{Y}^d_{ij} \bar{q}^i d^j_R H + \mathcal{Y}^u_{ij} \bar{l}^i e^j_R H \tag{2.1}$$

where $\tilde{H} = i\sigma_2 H^\dagger$. The quarks and leptons obtain masses with Higgs VEV $(0, v/\sqrt{2})$.

Spontaneous breaking of gauge symmetry makes the gauge boson, spin-

Matters	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q=(u_L,d_L)$	**3**	**2**	$\frac{1}{6}$
u_R^c	**3**	**1**	$-\frac{2}{3}$
d_R^c	**3**	**1**	$\frac{1}{3}$
$l=(\nu_L,e_L)$	**1**	**2**	$-\frac{1}{2}$
e_R^c	**1**	**1**	1
H	**1**	**2**	$\frac{1}{2}$

표 1: The $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group charges of the SM fermions and Higgs doublet.

1 particle in the adjoint representation under the corresponding gauge group, massive. Since the gauge boson carries the fundamental force described by corresponding gauge group by being interchanged between matter currents, the range of the force is getting shorter when the gauge boson get massive. Kinetic term and gauge interaction of matter are simply written by gauge covariant derivative. When Higgs has VEV $(0, \frac{v}{\sqrt{2}})$, covariant derivative term of the Higgs

$$\begin{aligned} &\left| \begin{pmatrix} \frac{g}{2}A_\mu^3 + \frac{g'}{2}B_\mu & \frac{g}{2}(A_\mu^1 - iA_\mu^2) \\ \frac{g}{2}(A_\mu^1 + iA_\mu^2) & -\frac{g}{2}A_\mu^3 + \frac{g'}{2}B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \right|^2 \\ &= \frac{g^2v^2}{4} W_\mu^- W^{+\mu} + \frac{g^2+g'^2}{8} v^2 Z_\mu Z^\mu \end{aligned} \tag{2.2}$$

gives gauge boson mass, $M_W = \frac{1}{2}gv$ and $M_Z = \frac{1}{2}\sqrt{g^2+g'^2}v$.

For leptons gauge covariant derivative is given by

$$\bar{l}iD_\mu\gamma^\mu l + \bar{e}_R iD_\mu\gamma^\mu e_R$$
$$= \begin{pmatrix} \bar{\nu}_L & \bar{e}_L \end{pmatrix} i\big(\partial_\mu - ig\frac{\tau^i}{2}A^i_\mu - ig'(-\frac{1}{2})B_\mu\big)\gamma^\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \bar{e}_R i\big(\partial_\mu - ig'(-1)B_\mu\big)\gamma^\mu e_R \tag{2.3}$$

so gauge boson coupling to the lepton doublet l is written in a matrix form

$$\begin{pmatrix} \frac{g}{2}A^3_\mu - \frac{g'}{2}B_\mu & \frac{g}{2}(A^1_\mu - iA^2_\mu) \\ \frac{g}{2}(A^1_\mu + iA^2_\mu) & -\frac{g}{2}A^3_\mu - \frac{g'}{2}B_\mu \end{pmatrix}. \tag{2.4}$$

Off diagonal terms represent the W bosons in the weak interaction, $W^\pm = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu)$, propagating between the charged currents with the coupling $\frac{g}{2}$. Since neutrino is electromagnetically neutral, (11) element does not contain photon therefore it should be the Z boson. Defining weak mixing angle by

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}, \quad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \tag{2.5}$$

two neutral gauge bosons, photon A and weak Z bosons

$$\begin{aligned} A_\mu &= \cos\theta_W B_\mu + \sin\theta_W A^3_\mu \\ Z_\mu &= -\cos\theta_W A^3_\mu + \sin\theta_W B_\mu \end{aligned} \tag{2.6}$$

have couplings $e = \frac{gg'}{g^2+g'^2}$ and $\frac{g}{\cos\theta_W}$, respectively. Here, charges are given by $Q_{em} = T_3 + Y$ for electromagnetic interaction and $Q_Z = T_3 - Q_{\text{em}}\sin^2\theta_W$ for Z boson interaction. They give right charges to the quarks. The electromagnetic charges are $+2/3$ for the $u-$type quark and $-1/3$ for the $d-$type quark.

2.2 Gauge anomaly and cancelation in the Standard Model

The chiral nature of the SM fermions may give rise to serious problem. If the gauge symmetry is explicitly broken, unphysical states with negative norm cannot be eliminated. This contradicts to the quantum description based on unitarity. By unitarity in quantum mechanics, we mean the norm of physical state, interpreted as total probability, can be normalized to 1 before and after time evolution generated by S-matrix S satisfying $S^\dagger S = I$. More generally this condition can be written as

$$\langle A|I|B\rangle = \sum_C \langle A|S^\dagger|C\rangle\langle C|S|B\rangle \tag{2.7}$$

When $A = B$, the LHS is the norm before evolution, and the RHS is that after evolution. Since RHS is always positive, negative norm state evolves into positive norm state. So, the norm before and after the evolution are normalized to -1 and 1, respectively, and it is impossible to normalize both sates equal to 1 simultaneously. Then unitarity is violated.

In the gauge theory associated with the algebra $[\delta_\alpha, \delta_\beta] = f^\gamma_{\alpha\beta}\delta_\gamma$, decoupling negative norm states from physical process was shown by Faddeev and Popov[5]. They interpreted the gauge symmetry as redundancy. Ordinary symmetry transformation just moves one state to another in the same Hilbert space. On the other hand, gauge transformation moves a set of states in a Hilbert space into another set of states in another Hilbert space which is equivalent to the previous one. To treat one set of Hilbert space only, we fix the gauge by imposing appropriate conditions $F^A(\phi_i) = 0$, where ϕ_i is gauge charged field and A is index for gauge conditions.

Then the path integral for the fixed Hilbert space is given by

$$
\begin{aligned}
\int \frac{\mathcal{D}\phi_i}{V_{\text{gauge}}} e^{-S_0} &\sim \int \mathcal{D}\phi\delta(F^A(\phi))\mathcal{D}b_A\mathcal{D}c^\alpha e^{-[S_0+\int b_A(\delta_\alpha F^A)c^\alpha]} \\
&\sim \int \mathcal{D}\phi\mathcal{D}B_A\mathcal{D}b_A\mathcal{D}c^\alpha e^{-[S_0+i\int B_A F^A(\phi)+\int b_A(\delta_\alpha F^A)c^\alpha]},
\end{aligned}
\tag{2.8}
$$

where c^α are the fermionic and B_A are the bosonic ghosts. The new action $S_0 + i\int B_A F^A(\phi) + \int b_A(\delta_\alpha F^A)c^\alpha \equiv S_0 + S'$ is no longer gauge invariant but has a remnant of gauge symmetry which was found by Becchi, Rouet, Stora, and Tyutin[7]:

$$
\begin{aligned}
&\delta\phi_i = -i\varepsilon c^\alpha \delta_\alpha \phi, \quad \delta b_A = -\varepsilon B_A, \\
&\delta c^\alpha = -i\frac{1}{2}\varepsilon c^\beta c^\alpha f^\alpha_{\beta\gamma}, \quad \delta B_A = 0
\end{aligned}
\tag{2.9}
$$

where ε is a Grassmann variable.

This BRST symmetry has an important property, $\delta^2 = 0$, called nilpotency. The fact $\delta(b_A F^A) = i\varepsilon S'$ implies that for BRST being the symmetry of the theory, the change of physical amplitude $\langle\psi|\psi'\rangle$ under the change of the gauge fixing condition δF^A,

$$
\langle\psi|i\varepsilon\delta S'|\psi'\rangle = \langle\psi|[Q, b_A\delta F^A]_+|\psi'\rangle \tag{2.10}
$$

(Q is the BRST generator) should vanish. Therefore, physical state $|\text{phys}\rangle$ satisfies $Q|\text{phys}\rangle = 0$. However, the state annihilated by Q ($Q|\psi\rangle = 0$: such state is called BRST closed.) in general has the form of $|\psi\rangle = |\psi'\rangle + Q|\chi\rangle$, where $|\chi\rangle$ is arbitrary state and $|\psi'\rangle$ is the state annihilated by Q but not of the form of $Q|\chi\rangle$. Then, the BRST closed states modulo $Q|\chi\rangle$ are indistinguishable as they have the same norm (by nilpotency) and behave in the same way (by nilpotency and symmetry condition $[Q,H] = 0$ where H is the Hamiltonian). Therefore, the physical state should be

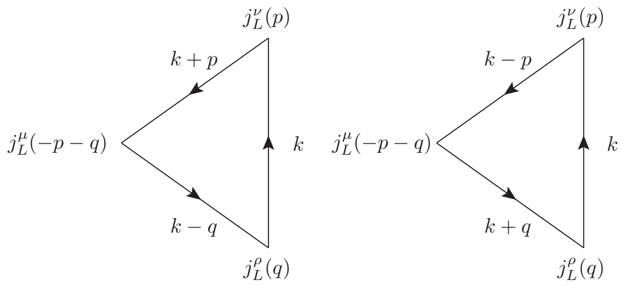

그림 1: The triangle diagram for gauge anomaly.

only one of such states, say,

$$Q|\text{phys}\rangle = 0, \quad \text{but} \quad |\text{phys}\rangle \neq Q|\chi\rangle \tag{2.11}$$

(closed but not exact). Note also that $[Q,H] = 0$ or, equivalently, $[Q,S] = 0$ implies that $Q(S|\text{phys}\rangle) = 0$. Then $S|\text{phys}\rangle$ also can be written in the form of $|\psi'\rangle + Q|\chi\rangle$, but $Q|\chi\rangle$ part, which has zero norm, eliminates unphysical intermediate states in the scattering amplitude:

$$\begin{aligned}\langle A;\text{phys}|S^\dagger S|B;\text{phys}\rangle &= \sum_C \langle A;\text{phys}|S^\dagger|C\rangle\langle C|S|B;\text{phys}\rangle \\ &= \sum_C \langle A;\text{phys}|S^\dagger|C;\text{phys}\rangle\langle C;\text{phys}|S|B;\text{phys}\rangle\end{aligned} \tag{2.12}$$

Then with the subspace of Hilbert space containing physical states only, the theory is unitary.

However, in the chiral gauge theory, gauge symmetry is broken in the quantum level[8]. Consider the massless left handed fermion described by the Lagrangian $\bar{\psi}D_\mu\gamma^\mu P_L\psi$. Then the triangle diagram shown in Fig. 1 has the term

$$\text{tr}(i\gamma^\mu t_\alpha)P_L P_L \frac{i(k+p)_\mu\gamma^\mu}{(k+p)^2 - i\varepsilon}\cdots = \text{tr}(i\gamma^\mu t_\alpha)P_L \frac{i(k+p)_\mu\gamma^\mu}{(k+p)^2 - i\varepsilon}\cdots \tag{2.13}$$

where t_α be the gauge group generator. Note that one P_L comes from the vertex and another P_L comes from the propagator. They combine to make one P_L. So, the same diagram comes from anomalous theory with the Lagrangian $\bar{\psi}D_\mu\gamma^\mu P_L\psi + \bar{\psi}\partial_\mu\gamma^\mu P_R\psi$ and it breaks the gauge symmetry as $\partial_\mu\langle \mathrm{T} j^\mu j^\nu j^\rho\rangle$ (j^μ is the current couples to the gauge boson) has extra term absent in the Ward identity, the relation between $n-$point functions representing gauge symmetry.

To obtain gauge anomaly, we may calculate the triangle diagram. On the other hand, anomaly term can be defined by $\mathcal{A} = Q\Gamma$ where Γ is quantum action. if $\mathcal{A} = 0$, quantum action is gauge invariant, so it is not anomalous. From nilpotency, we have $Q\mathcal{A} = 0$. In the language of differential form,

$$QA = -dw - [A,w]_+, \quad QF = [F,w], \quad Qw = -w^2, \quad dQ = -Qd \tag{2.14}$$

where A is gauge field one-form, $F = dA + A^2$ is field strength two form and w is the ghost, as defined in [8]. Defining characteristic class $P_n = \mathrm{tr}F^n$, we have $dP_n = n\mathrm{tr}(dF)F^{n-1} = n\mathrm{tr}(FA - AF)F^{n-1} = 0$ (since $DF = (d+A)F = dA + AF - FA = 0$) so $P_n = dQ_{2n-1}$ locally where

$$Q_{2n-1}(A,F) = n\int_0^1 dt\mathrm{tr}[AF(tA)^{n-1}] = n\int_0^1 dt t^{n-1}\mathrm{tr}[A(F + (t-1)A^2)^{n-1}]. \tag{2.15}$$

Moreover, since $QP_n = Q(dQ_{2n-1}) = -dQ(Q_{2n-1}) = 0$, $QQ_{2n-1} = dQ^1_{2n-2}$ locally where

$$Q^1_{2n-2}(v,A,F) = n(n-1)\int_0^1 dt(1-t)\mathrm{tr}[vd(AF(tA)^{n-2})] \tag{2.16}$$

for arbitrary parameter v (originated from the gauge transformation). Then, anomaly $\mathcal{A}$ in $2r$ dimension is proportional to $\int Q^1_{2r}$ as $Q\int Q^1_{2r} = 0$. This can be checked from $0 = Q(QQ_{2r+1}) = Q(dQ^1_{2r}) = -d(QQ^1_{2r})$ so $QQ^1_{2r} = d\alpha^1_{2r-1}$ locally for arbitrary α^1_{2r-1}. (The superscript 1 implies that it is linear in v.) Explicitly, when we express $Q\Gamma[A] \equiv \mathcal{A}[w,A] = \int w^\alpha(x)\mathcal{A}_\alpha(x)$, the anomaly with the left-handed field only is given by

$$\mathcal{A}^L_\alpha = -\frac{1}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma}\partial_\mu\left(A^\beta_\nu\partial_\rho A^\gamma_\sigma - \frac{i}{4}A^\beta_\nu[A_\rho,A_\sigma]^\gamma\right)D^R_{\alpha\beta\gamma} \tag{2.17}$$

where $D^R_{\alpha\beta\gamma} = \mathrm{tr}t_\alpha t_{(\beta}t_{\gamma)}$. For the right handed field, anomaly has the opposite sign. So, if the theory is vector-like, *i.e.* left- and right- handed fermions are equally charged under the gauge group, they are canceled with each other, gauge symmetry is not anomalous. However, since the SM fermions are chiral under the SM gauge group, we should check whether the SM is anomalous.

Consider anomaly of the SM within one generation. When the generators of the group for some representation R, t^R_α are equivalent to those in the complex conjugate representation, in the sense that $(it_\alpha)^* = S(it_\alpha)S^{-1}$, $D^R_{\alpha\beta\gamma} = 0$. SU(2) is the example of it, so triangle diagram with SU(2)-SU(2)-SU(2) vertices vanishes. SU(3) is not such a case, but since the SM quarks are vector-like under the SU(3)$_c$, SU(3)-SU(3)-SU(3) triangle diagram also vanishes. Other cases are given as follows:

$$\begin{aligned}
&SU(3)-SU(3)-U(1): \ \sum_{3,\bar{3}} Y = -\frac{1}{6}-\frac{1}{6}+\frac{2}{3}-\frac{1}{3} = 0\\
&SU(2)-SU(2)-U(1): \ \sum_{2} Y = 3(-\frac{1}{6})+\frac{1}{2} = 0\\
&U(1)-U(1)-U(1): \ \sum Y^3 = 6(-\frac{1}{6})^3+3(\frac{2}{3})^3+3(-\frac{1}{3})^3+2(\frac{1}{2})^3+(-1)^3 = 0\\
&\mathrm{graviton}-\mathrm{graviton}-U(1): \ \sum Y = 6(-\frac{1}{6})+3(\frac{2}{3})+3(-\frac{1}{3})+2(\frac{1}{2})+(-1) = 0.
\end{aligned} \tag{2.18}$$

Therefore the SM is anomaly free with the help of choice of U(1)$_Y$ charges of the quarks and the leptons. To look for other anomaly-free U(1) assignments [9], suppose we assign U(1) charges a, b, c, d, and e to q, u_R^c, d_R^c, l, and e_R^c, respectively. The conditions

$$
\begin{aligned}
&SU(3)-SU(3)-U(1): \quad \sum_{3,\bar{3}} Y = 2a+b+c=0\\
&SU(2)-SU(2)-U(1): \quad \sum_{2} Y = 3a+d=0\\
&U(1)-U(1)-U(1): \quad \sum Y^3 = 6a^3+3b^3+3c^3+2d^3+e^3=0\\
&\text{graviton}-\text{graviton}-U(1): \quad \sum Y = 6a+3b+3c+2d+e=0
\end{aligned}
\tag{2.19}
$$

have two solutions $b/a=-4, c/a=-2, d/a=-4, e/a=-6$ and $b=-c, a=d=e=0$. The former corresponds to the U(1)$_Y$ in SM. Two U(1)s are not compatible since they have U(1)-U(1)-U(1)$'$ and U(1)$'$-U(1)$'$-U(1) anomalies. Anomaly-free extra U(1)$'$ compatible with the U(1)$_Y$ requires more chiral fermions which are not present in the SM. Suppose such fermions are not charged under the SM gauge group. Then, for the SM particles, anomaly-free conditions are given by

$$
\begin{aligned}
&SU(3)-SU(3)-U(1)': \quad \sum_{3,\bar{3}} Y' = 2a+b+c=0\\
&SU(2)-SU(2)-U(1)': \quad \sum_{2} Y' = 3a+d=0\\
&U(1)-U(1)-U(1)': \quad \sum Y^2Y' = 6a+3(-4)^2b+3(2)^2c+2(-3)^2d+(6)^2e=0\\
&U(1)-U(1)-U(1)': \quad \sum YY'^2 = 6a^2+3(-4)b^2+3(2)c^2+2(-3)d^2+(6)e^2=0.
\end{aligned}
\tag{2.20}
$$

The solution to these conditions is $B-L$, $a=-b=-c=1/3, d=-e=-1$. Of course, we may assign U(1)$'$ to each generation differently. Then there can be more possible anomaly free U(1)$'$s.

2.3 Three-flavor model with mixing

The quarks and the leptons discovered so far form three generations[10]:

$$
\begin{aligned}
&\text{1st generation :} \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \; u_R, \; d_R, \; \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \; e_R \\
&\text{2nd generation :} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \; c_R, \; s_R, \; \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \; \mu_R \\
&\text{3rd generation :} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \; t_R, \; b_R, \; \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \; \tau_R
\end{aligned} \tag{2.21}
$$

Each of the quarks and the leptons obtains mass from the electroweak symmetry breaking, and each mass is proportional to the Yukawa coupling. The lepton masses are easily defined by pole of the propagator:

$$
m_e = 0.511\text{MeV}, \;\; m_\mu = 105.6\text{MeV}, \;\; m_\tau = 1777\text{MeV} \tag{2.22}
$$

On the other hand, the quarks do not propagate as isolated particles. They are confined inside hadrons(mesons or baryons) due to the asymptotic freedom of the strong interaction. One way to estimate the quark masses is dynamical breaking of chiral symmetry. Suppose we have three massless quarks, u, d, and s only[11]. In this case, left- and right- handed quarks are decoupled since they couple only through mass term. Gauge interaction is (left handed)-(left handed)-(gauge boson) and (right handed)-(right handed)-(gauge boson) couplings. Therefore, the theory is described by the Lagrangian $\bar{q}_L iD_\mu \gamma^\mu q_L + \bar{q}_R iD_\mu \gamma^\mu q_R$ where $q = (u, d, s)$ and it is invariant under chiral rotation,

$$
q_L \to \exp(i \sum_a \frac{\lambda^a}{2} \theta_L^a) q_L, \quad q_R \to \exp(i \sum_a \frac{\lambda^a}{2} \theta_R^a) q_R, \tag{2.23}
$$

where λ^a are SU(3) generator for fundamental representation, Gell-Mann matrices. This is equivalent to the invariance under

$$q \to \exp\Big(i\sum_a \frac{\lambda^a}{2}(\theta_V^a + \theta_A^a\gamma_5)\Big)q \tag{2.24}$$

where $\theta_{V/A} = \theta_L \pm \theta_R$ with the convention $P_{L/R} = \frac{1}{2}(1\pm\gamma_5)$. By quark confinement, $SU(3)_A$ generated by $(\lambda^a/2)\gamma_5$ is the broken and only $SU(3)_V$ generated by $(\lambda^a/2)$ remains [12]. To describe this in detail, we can parameterize the quark triplet q as

$$q(x) = \exp\Big(-i\gamma_5\sum_a \xi^a(x)\frac{\lambda^a}{2}\Big)\tilde{q}. \tag{2.25}$$

With $\langle\bar{\tilde{q}}\tilde{q}\rangle = v$ and $\langle\bar{\tilde{q}}\gamma_5\tilde{q}\rangle = 0$, spontaneous symmetry breaking $SU(3)_L\times SU(3)_R \to SU(3)_V$ occurs and $\xi(x)^a$ accompanied with the eight broken symmetry generator are interpreted as the Goldstone bosons. For effective Lagrangian, we can use the fact that it is always possible to find out ξ' such that [13]

$$\begin{aligned}&\exp\Big(i\sum_a \frac{\lambda^a}{2}(\theta_V^a + \theta_A^a\gamma_5)\Big)\exp\Big(-i\gamma_5\sum_a \frac{\lambda^a}{2}\xi(x)^a\Big)\\ &= \exp\Big(-i\gamma_5\sum_a \frac{\lambda^a}{2}\xi'^a(x)\Big)\exp\Big(i\sum_a \frac{\lambda^a}{2}\theta^a\Big).\end{aligned} \tag{2.26}$$

Then, from

$$\begin{aligned}&\exp\Big(i\sum_a \frac{\lambda^a}{2}\theta_L^a\Big)\exp\Big(-i\sum_a \frac{\lambda^a}{2}\xi(x)^a\Big) = \exp\Big(-i\sum_a \frac{\lambda^a}{2}\xi'^a(x)\Big)\exp\Big(i\sum_a \frac{\lambda^a}{2}\theta^a\Big)\\ &\exp\Big(i\sum_a \frac{\lambda^a}{2}\theta_R^a\Big)\exp\Big(i\sum_a \frac{\lambda^a}{2}\xi(x)^a\Big) = \exp\Big(i\sum_a \frac{\lambda^a}{2}\xi'^a(x)\Big)\exp\Big(i\sum_a \frac{\lambda^a}{2}\theta^a\Big)\end{aligned} \tag{2.27}$$

we obtain unitary, unimodular matrix

$$U(x) = \exp\Big(i\sum_a \xi^a(x)\lambda^a\Big). \tag{2.28}$$

which transformations like

$$U'(x) = \exp\Big(i\sum_a \frac{\lambda^a}{2}\theta_R^a\Big)U(x)\exp\Big(-i\sum_a \frac{\lambda^a}{2}\theta_L^a\Big) \tag{2.29}$$

when the Goldstone bosons transform $\xi(x) \to \xi'(x)$. Unitarity of $U(x)$ implies that the Goldstone bosons cannot have the mass term and only described by the derivative terms like $-(1/4)F^2\mathrm{Tr}\partial_\mu U\partial^\mu U^\dagger$ where F is decay constant of the meson. Therefore, Goldstone bosons are interpreted as light mesons,

$$\sum_a \frac{\lambda^a}{2}\xi^a(x) = \frac{\sqrt{2}}{F}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & K^0 \\ \bar{K}^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta^0 \end{pmatrix} \equiv \sqrt{2}\frac{B}{F}. \tag{2.30}$$

However, the quark masses break $\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R$ explicitly and give the masses to the light mesons as

$$\bar{q}M_q q = \bar{\tilde{q}}\mathrm{e}^{-i\sqrt{2}\gamma_5 B/F}\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}\mathrm{e}^{-i\sqrt{2}\gamma_5 B/F}q \tag{2.31}$$

then meson masses satisfy following relations,

$$\begin{aligned} m_{\pi^+}^2 &= m_{\pi^0}^2 = \frac{4v}{F^2}(m_u + m_d) \\ m_{K^+}^2 &= \frac{4v}{F^2}(m_u + m_s) \\ m_{K^0}^2 &= \frac{4v}{F^2}(m_d + m_s) \\ m_{\eta^0}^2 &= \frac{4v}{F^2}\frac{1}{3}(m_u + m_d + 4m_s). \end{aligned} \tag{2.32}$$

Comparing with the measured values of meson masses, $F = 184$MeV, and $v = 255$MeV, light quark masses can be obtained. The measured values are

$$\begin{aligned} &m_u = 2.5^{+0.6}_{-0.8}(1.7-3.1)\text{MeV} \;\; m_d = 5.0^{+0.7}_{-0.9}(4.1-5.7)\text{MeV} \\ &m_s = 3.8^{+1.0}_{-0.8}(3.0-4.8)\text{MeV} \end{aligned} \tag{2.33}$$

Such quark masses are called current quark masses. They are distinguished from constituent quark masses, in which the energy of 'cloud' from the gluons and the virtual quarks are taken into account. On the other hand, masses for heavy quarks, quarks heavier than confinement scale v, should be calculated taking perturbation effects into account in addition to non-perturbation effects described above. Heavy quark effective theory(HQET)[14] is representative theoretical tool. Measured values of the heavy quark masses are given by

$$\begin{aligned} &m_c = 1.29^{+0.05}_{-0.11}(1.18-1.34)\text{GeV}, \;\; m_b = 4.19^{+0.18}_{-0.06}\text{MeV} \\ &m_t = 172.9 \pm 0.6 \pm 0.9\text{GeV}. \end{aligned} \tag{2.34}$$

Such quark masses come from diagonalization of complex 3×3 Yukawa couplings. Let $\mathcal{Y}^{u,d}$ be Yukawa couplings for $U = (u,c,t)$ and $D = (d,s,b)$. Then they are diagonalized as

$$L_u \mathcal{Y}^u R_u^\dagger = \tilde{\mathcal{Y}}^u, \;\; L_d \mathcal{Y}^d R_d^\dagger = \tilde{\mathcal{Y}}^d \tag{2.35}$$

where $L_{u,d}, R_{u,d}$ are 3×3 unitary matrices and $\tilde{\mathcal{Y}}^{u,d}$ are diagonalized matrices. Equivalently, $\mathcal{Y}^2$ is diagonalized as $\tilde{\mathcal{Y}}^2 = R\mathcal{Y}^\dagger \mathcal{Y} R^\dagger = L\mathcal{Y}\mathcal{Y}^\dagger L^\dagger$. In the case of neutral currents, $\bar{U}_L\gamma^\mu U_L, \bar{U}_R\gamma^\mu U_R, \bar{D}_L\gamma^\mu D_L$, and $\bar{D}_R\gamma^\mu D_R$, unitary matrices $L_{u,d}, R_{u,d}$ does not appear. However, for charged current in the weak interaction mediated by W

boson, one combination of these matrices

$$\bar{U}\gamma^\mu D = \bar{U}\gamma^\mu L_u L_d^\dagger D \tag{2.36}$$

appears. Chiral nature of the weak interaction prevents $R_{u,d}$ from observables. This combination of unitary matrices rotating left handed fermions is called the Cabibbo-Kobayashi-Maskawa(CKM) matrix[15]: $V_{\text{CKM}} = L_u L_d^\dagger$.

One important feature of the CKM matrix is that it has one unremovable phase. This is the source of CP violation[16] in the weak interaction. Under parity and charge conjugate operations,

$$\begin{aligned} P &: \psi(t,\vec{x}) \to \gamma^0\psi(t,-\vec{x}) \\ C &: \psi(t,\vec{x}) \to C\bar{\psi}^T(t,\vec{x}), \quad C = i\gamma^2\gamma^0, \end{aligned} \tag{2.37}$$

then

$$CP : \psi(t,\vec{x}) \xrightarrow{P} \gamma^0\psi(t,-\vec{x}) \xrightarrow{C} \gamma^0 C\bar{\psi}^T(t,-\vec{x}) = -i\gamma^2\gamma^0\psi^*(t,-\vec{x}) \tag{2.38}$$

and

$$\begin{aligned} CP : & V^{+\mu}(t,\vec{x}) = \bar{\psi}_a(t,\vec{x})\gamma^\mu\psi_b(t,\vec{x}) \to -V_\mu^-(t,-\vec{x}) = -\bar{\psi}_b(t,-\vec{x})\gamma_\mu\psi_a(t,-\vec{x}) \\ & A^{+\mu}(t,\vec{x}) = \bar{\psi}_a(t,\vec{x})\gamma^\mu\gamma_5\psi_b(t,\vec{x}) \to -A_\mu^-(t,-\vec{x}) = -\bar{\psi}_b(t,-\vec{x})\gamma_\mu\gamma_5\psi_a(t,-\vec{x}). \end{aligned} \tag{2.39}$$

Therefore the interaction

$$aV_\mu^+(t,\vec{x})V^{-\mu}(t,\vec{x}) + bA_\mu^+(t,\vec{x})A^{-\mu}(t,\vec{x}) + cV_\mu^+(t,\vec{x})A^{-\mu}(t,\vec{x}) + c^*A_\mu^+(t,\vec{x})V^{-\mu}(t,\vec{x})$$

transforms to

$$aV^{-\mu}(t,-\vec{x})V^{+}_{\mu}(t,-\vec{x})+bA^{-\mu}(t,-\vec{x})A^{+}_{\mu}(t,-\vec{x})$$
$$+cV^{-\mu}(t,-\vec{x})A^{+}_{\mu}(t,-\vec{x})+c^{*}A^{-\mu}(t,-\vec{x})V^{+}_{\mu}(t,-\vec{x})$$

under CP. Here, CP is violated unless c is complex. In fact, CP violating phase cannot be removed when the number of generation is more than three. CKM matrix in n generations is complex n×n matrix so it has $2n^2$ real parameters. But unitarity conditions reduces n^2 parameters. $2n$ phases of $u-$ and $d-$type quarks can be absorbed by field redefinitions, but overall phase is irrelevant so $2n-1$ parameters are reduced. Then the total number of real parameters in the CKM matrix is $(n-1)^2$. Among them, $(1/2)n(n-1)$ parameters are mixing angles. Remaining $(1/2)(n-1)(n-2)$ parameters are the unremovable phases. For three generations ($n=3$) only one CP phase remains. Three mixing angles are parameterized by Euler angles. The conventional parametrization comes from Chau-Keung-Maiani[17],

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \qquad (2.40)$$

제 3 장

Problems in the Standard Model

3.1 Massive Neutrinos

In the SM, neutrinos[18] are massless. However, the neutrino oscillation, generation changing effect of the neutrinos, implies that they are massive, and mixings in the lepton sector appear. The charged leptons are diagonalized as $\tilde{m}_l = L_l m_l R_l$ whereas neutrinos are diagonalized as $\tilde{m}_\nu = L_\nu m_\nu R_\nu$ for Dirac mass and $\tilde{m}_\nu = L_\nu m_\nu L_\nu^T$ for Majorana mass. Then the mixing matrix called Pontecorvo-Maki-Nakagawa-Sakata(PMNS) matrix[19] is defined by $V_{\text{PMNS}} = L_l L_\nu^\dagger$.

For simplicity, suppose $L_l = I$ and consider the propagation of the massive neutrinos in the vacuum, neglecting the medium effect. The flavor basis $|\nu_\alpha\rangle$ is the superposition of the mass eigenstates $|\nu_j\rangle$ with the coefficients provided by the PMNS matrix elements,

$$|\nu_\alpha\rangle = \sum_j V_{\alpha j}^* |\nu_j\rangle \tag{3.1}$$

where the subscript α indicates e, μ, τ, and j runs from 1 to 3. The probability amplitude of observing $|\nu_{\alpha'}\rangle$ after the propagation in spacetime interval (T, L) of $|\nu_\alpha\rangle$ is given by

$$A(\nu_\alpha \to \nu_{\alpha'}) = \sum_j V_{\alpha' j} V_{j\alpha}^\dagger e^{-i(E_j T - p_j L)} \tag{3.2}$$

and the probability $P(\nu_\alpha \to \nu_{\alpha'})$ is just $|A(\nu_\alpha \to \nu_{\alpha'})|^2$,

$$\begin{aligned} P(\nu_\alpha \to \nu_{\alpha'}) &\cong P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) \\ &\cong \delta_{\alpha\alpha'} - 2|V_{\alpha 3}|^2(\delta_{\alpha\alpha'} - |V_{\alpha' 3}|^2)\left(1 - \cos\frac{\Delta m_{31}^2}{2p}L\right) \end{aligned} \tag{3.3}$$

which holds in the limit Δm_{13}^2 dominates the neutrino oscillation. Therefore, the survival of the electron type neutrino is given by

$$P(\nu_e \to \nu_e) \cong 1 - 2|V_{e3}|^2(1 - |V_{e3}|^2)\left(1 - \cos\frac{\Delta m_{31}^2}{2p}L\right) \tag{3.4}$$

which is used in the Double CHOOZ, Daya Bay and RENO experiments.A similar expression can be written for $P(\nu_\mu \to \nu_\mu)$, used in K2K and MINOS experiments. On the other hand, the appearance is given by

$$\begin{aligned} P(\nu_{\mu(e)} \to \nu_{e(\mu)}) &\cong 2|V_{\mu 3}|^2|V_{e3}|^2(1 - \cos\frac{\Delta m_{31}^2}{2p}L) \\ &= \frac{|V_{\mu 3}|^2}{1 - |V_{e3}|^2}P^{2\nu}(|V_{e3}|^2, \Delta m_{31}^2) \end{aligned} \tag{3.5}$$

where $P^{2\nu}(|V_{e3}|^2, \Delta m_{31}^2)$ indicates the probability of 2-neutrino transition, $\nu_e \to (s_{\rm atm}\nu_\mu + c_{\rm atm}\nu_\tau)$, used in MINOS experiment. Similar expression for $P(\nu_\mu \to \nu_\tau)$ is used in OPERA.

When the neutrino source has a sizable dimension ΔL and the energy resolution of detector is ΔE, we integrate over the region of neutrino source and energy resolution function. Then, a large phase $\frac{\Delta m_{31}^2}{2p}L$ in the argument of cos is averaged over and the average probability is given by

$$\overline{P}(\nu_\alpha \to \nu_{\alpha'}) = \overline{P}(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) \cong \sum_j |V_{\alpha' j}|^2|V_{\alpha j}|^2. \tag{3.6}$$

Especially, for the case of $\alpha = \alpha' = e$, the averaged probability is

$$\begin{aligned}\overline{P}(\nu_e \to \nu_e) &= \overline{P}(\bar{\nu}_e \to \bar{\nu}_e) \\ &\cong |V_{e3}|^4 + (1 - |V_{e3}|^2)^2 P^{2\nu}(\nu_e \to \nu_e)\end{aligned} \tag{3.7}$$

where

$$\begin{aligned}P^{2\nu}(\nu_e \to \nu_e) &= P^{2\nu}(\bar{\nu}_e \to \bar{\nu}_e) \\ &= 1 - \frac{1}{2}\sin^2 2\theta_{\text{sol}}(1 - \cos\frac{\Delta m_{21}^2}{2p}L),\end{aligned} \tag{3.8}$$

which has been used in the KamLand experiment.

The solar neutrino angle $\theta_{\text{sol}} = \theta_{12}$ can be determined from the solar neutrino flux observation, for example, in the SNO and the Super-Kamiokande experiments or from the detection of $\bar{\nu}_e$ neutrinos emitted from the nuclear power reactors in the KamLand. The atmospheric neutrino angle $\theta_{\text{atm}} = \theta_{23}$ measurement can be made by observing the atmospheric neutrino, the product of cosmic ray interaction in the atmosphere, in the Super-Kamiokande, or product from accelerator experiment, for example, in the K2K and MINOS experiments. Finally, the deviation from zero of V_{13} is determined by observing $P(\bar{\nu}_e \to \bar{\nu}_e)$ in the CHOOZ experiment, and $P(\nu_\mu \to \nu_e)$ in the K2K experiment.

As shown above, neutrino oscillation in vacuum shows mixing angles and absolute values of mass square differences, $|m_{ij}^2| = |m_j^2 - m_i^2|$. On the other hand, when neutrinos propagate through matter, ν_e interacts with electrons in matters via charged(W boson exchange) and neutral(Z boson exchange) currents, whereas $\nu_{\mu,\tau}$ interact only via neutral current. Such matter effect[20] enables one to know sign of $m_2^2 - m_1^2 > 0$. Therefore, what we know about the neutrino masses are $\Delta m_{\text{sol}} = m_2^2 - m_1^2$ and $\Delta m_{\text{atm}} = |m_3^2 - m_1^2|$. $\Delta m_{\text{atm}} \gg \Delta m_{\text{sol}}$ but we do not know whether m_3 is heavier(normal hierarchy) or lighter(inverted hierarchy) than m_1.

The neutrino masses are known to be smaller than 2eV[21]. One may wonder why the neutrino masses are very small compare to the quark and lepton masses. Simple modification of the SM just by adding neutrino Yukawa coupling $\mathcal{Y}^{\nu}\bar{l}\nu\tilde{H}$ does not explain this satisfactory. On the other hand, we can obtain very small mass using two preexisting scales. We already have the electroweak scale, typically represented by Higgs VEV $v = 246$GeV. Another higher scale would be Planck scale, where the gravity becomes important. However, lower scale can be chosen by introducing heavy neutrinos N and imposing Dirac mass of vector-like fermion $M\bar{N}_L N_R + h.c$ or Majorana mass, $M\bar{N^c}N$. They can be much larger than electroweak scale. As the simplest case, suppose we consider the minimal number of degrees of freedom and do not think of the gravity effect. Then SM singlet fermion with the Majorana mass can be taken. With these two scales, say, high scale (Majorana mass here) and intermediate scale(electroweak scale), much smaller scale can be obtained from (intermediate scale)^2/(high scale). This can be realized through see-saw mechanism model[22],

$$(\mathcal{Y}^{\nu})_{iJ}\bar{l}_i N_J \tilde{H} + \frac{1}{2}M_{IJ}\bar{N^c}_I N_J. \tag{3.9}$$

Note that the number of heavy neutrinos need not be the same as the number of the lepton generations, 3. In this model, the mass term in the (ν, N) basis is given by

$$\begin{pmatrix} 0_{3\times 3} & m_D \\ m_D^T & M \end{pmatrix} \tag{3.10}$$

where $m_D = v\mathcal{Y}^{\nu}$. This mass matrix is diagonalized by $(3+N)\times(3+N)$ (N is the number of heavy neutrinos N) unitary matrix

$$\begin{pmatrix} I_{3\times 3} - \frac{1}{2}m_D^* M^{-2} m_D^T & m_D^* M^{-1} \\ -M^{-1}m_D^T & I_{N\times N} - \frac{1}{2}M^{-1}m_D^T m_D^* M^{-1} \end{pmatrix} \tag{3.11}$$

and the mass eigenvalues are given by

$$\begin{pmatrix} -m_D M^{-1} m_D^T & 0 \\ 0 & M + \frac{1}{2}(M^{-1} m_D^\dagger m_D + m_D^T m_D^* M^{-1}) \end{pmatrix}. \tag{3.12}$$

The submatrix $m_\nu = -m_D M^{-1} m_D^T$ is of the form discussed above, so naturally explains tiny neutrino masses. Then, how two different scales can exist? Usually, one of them is what already known: in the case of seesaw mechanism, electroweak scale v corresponds to it. Another scale comes from symmetry breaking scale. The mass in the nonrenormalizable form (intermediate scale)^2/(high scale) implies that tree level mass is forbidden by symmetry principle. The basic idea of the seesaw mechanism is that, abnormally small neutrino mass is tiny breaking effect of 'accidental' symmetry which holds in the renormalizable interactions. By making neutrino mass Majorana, we can break lepton number conservation. Therefore, Majorana mass of heavy neutrino N is the scale where global symmetry for lepton number conservation is broken.

If neutrinos are massless or degenerate, PMNS matrix is just the identity by redefinition of the fields. However, the measured values are given by [23]

$$\begin{aligned}
&7.05\times 10^{-5}\mathrm{eV}^2 \le \Delta m_{12}^2 \le 8.34\times 10^{-5}\mathrm{eV}^2 \\
&0.25 \le \sin^2\theta_{12} \le 0.37 \\
&2.70\times 10^{-3}\mathrm{eV}^2 \le |\Delta m_{31}^2| \le 2.75\times 10^{-3}\mathrm{eV}^2 \\
&0.36 \le \sin^2\theta_{23} \le 0.67 \\
&\sin^2\theta_{13} < 0.035(0.056) \quad \text{at } 90\% \ (99.73\%) \text{ C.L.}
\end{aligned} \tag{3.13}$$

with the following BF values

$$
\begin{aligned}
(\Delta m^2_{12})_{BF} &= 7.65 \times 10^{-5}\mathrm{eV}^2, \\
(\sin^2\theta_{12})_{BF} &= 0.304, \\
(|\Delta m^2_{31}|)_{BF} &= 2.40 \times 10^{-3}\mathrm{eV}^2, \\
(\sin^2\theta_{23})_{BF} &= 0.5.
\end{aligned}
\tag{3.14}
$$

Note that the neutrino mass in the seesaw mechanism is Majorana mass, $(m_\nu)_{ij}\bar{\nu}^c_i\nu_j$. By integrating out heavy neutrinos N, it is effectively $(1/M)(LH)(LH)$ where M is typical mass scale of heavy neutrino Majorana mass, not $(1/M)(LH)^\dagger(LH)$. It is different from the Dirac mass which describes the quark and the charged lepton masses, in the form of $H\psi^\dagger\psi$. If the field is defined with the phase, $e^{i\delta}\psi$, it does not appear in the Dirac mass so physically irrelevant. On the other hand, such phase cannot be removed in the Majorana mass term. In the case of the neutrino mass, neutrinos can have the phase as $(\nu_e, e^{-i\alpha}\nu_\mu, e^{-i\beta}\nu_\tau) = P\cdot(\nu_e,\nu_\mu,\nu_\tau)$ where $P = \mathrm{diag.}(1, e^{-i\alpha}, e^{-i\beta})$. Then the phases P can be contained in the PMNS matrix making the neutrino masses real as

$$
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\alpha} & 0 \\
0 & 0 & e^{i\beta}
\end{pmatrix}.
\tag{3.15}
$$

3.2 Gauge Hierarchy Problem

Originally, gauge hierarchy problem[24] came out of the GUTconsiderations. If the SM gauge group is obtained from breaking of larger gauge group[25], electroweak and strong interactions are unified above this breaking scale. Such unifica-

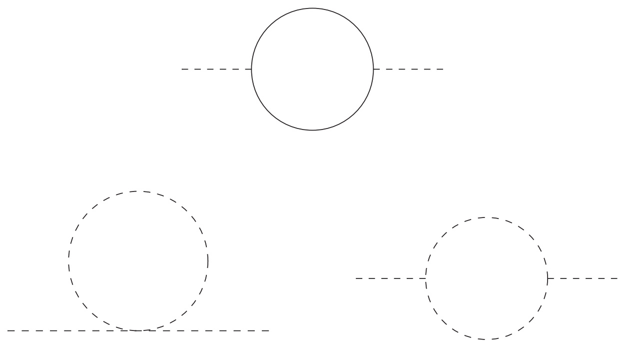

그림 2: Fermion(above) and boson(below) loop corrections to the fundamental scalar mass.

tion breaks baryon number conservation, accidental symmetry in the SM. so proton can decay. Current proton lifetime bound, 2×10^{29} years[26] imposes that GUT scale shoud be of order 10^{16}GeV. Then, one may ask why this scale is much larger than electroweak scale. This is original version of gauge hierarchy problem.

Another version of gauge hierarchy problem visits the issue of stability of fundamental scalar mass in the electroweak scale under quantum correction. In the SM, the Higgs scalar is introduced from chiral nature of the SM fermions in the weak interaction. One $\mathrm{SU}(2)_L$ doublet Higgs as a fundamental scalar in the SM is just a minimal setup: we can think of multi-Higgs doublets and even composite Higgs. In any case, gauge symmetry should be broken spontaneously, not explicitly. Otherwise the model is not unitary at least in the perturbative scheme. If we just begin with massive gauge boson, scattering amplitudes of massive gauge bosons, such as $W^+W^- \to W^+W^-$, violate unitarity at high energy[27], so such theory is just the low energy effective theory. On the other hand, if the Higgs is fundamental scalar, fine tuning problem arises. Since this fine tuning is originated from large hierarchy between electroweak scale and cutoff scale, usually taken as GUT or Planck scale, this is also hierarchy problem. To see this consider the quantum correction to the

Higgs mass from top Yukawa coupling $(y_t/\sqrt{2})H^0\bar{t}_L t_R$ in the regularization with cutoff Λ,

$$\begin{aligned}\delta m_h^2 &= i(-1)N_c \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\Big[\frac{-iy_t}{\sqrt{2}}\frac{i}{k_\mu\gamma^\mu - m_t}\frac{-iy_t^*}{\sqrt{2}}\frac{i}{k_\mu\gamma^\mu - m_t}\Big] \\ &= -\frac{N_c|y_t|^2}{8\pi^2}\Big[\Lambda^2 - 3m_t^2\ln\Big(\frac{\Lambda^2+m_t^2}{m_t^2}\Big)\Big].\end{aligned} \tag{3.16}$$

The quadratic divergence Λ^2 makes the Higgs mass in the electroweak scale fine-tuned. If the cutoff Λ is the Planck scale so that the SM is valid up to this scale, it is interpreted as very large bare Higgs mass of order $M_P^2/(8\pi^2)$ get quantum correction $-M_P^2/(8\pi^2)$ to make 10^{-16} times smaller mass m_h at the electroweak scale. If there is a new physics between the electroweak scale and M_P, smaller cutoff characterizing this new physics can be introduced and the fine tuning is lightened. Supersymmetry[28], symmetry between the boson and fermion, is one of such examples. This is easily understood by noting that boson and fermion loop have opposite sign contributions. Introducing the scalar couples to the Higgs,

$$-\frac{\lambda}{2}(H^0)^2(|\phi_L|^2+|\phi_R|^2) - H^0(\mu_L|\phi_L|^2+\mu_R|\phi_R|^2) - m_L^2\phi_L^2 - m_R^2\phi_R^2, \tag{3.17}$$

the quantum correction from this scalar interaction is given by

$$\begin{aligned}\delta m_h^2 &= \lambda N\int\frac{d^4k}{(2\pi)^4}\Big[\frac{i}{k^2-m_L^2}+\frac{i}{k^2-m_R^2}\Big] \\ &\quad + iN\int\frac{d^4k}{(2\pi)^4}\Big[\Big(-i\mu_L\frac{i}{k^2-m_L^2}\Big)^2+\Big(-i\mu_R\frac{i}{k^2-m_R^2}\Big)^2\Big] \\ &= \frac{\lambda N}{16\pi^2}\Big[2\Lambda^2 - m_L^2\ln\Big(\frac{\Lambda^2+m_L^2}{m_L^2}\Big) - m_R^2\ln\Big(\frac{\Lambda^2+m_R^2}{m_R^2}\Big)\Big] \\ &\quad - \frac{N}{16\pi^2}\Big[\mu_L^2\ln\Big(\frac{\Lambda^2+m_L^2}{m_L^2}\Big)+\mu_R^2\ln\Big(\frac{\Lambda^2+m_R^2}{m_R^2}\Big)\Big].\end{aligned} \tag{3.18}$$

If $N = N_c$, $\lambda = |y_t^2|$, $m_L = m_R = m_t$, and $\mu_L^2 = \mu_R^2 = 2\lambda m_t^2$, then the logarithmic as well as quadratic divergences vanish. That means, if the scalar has the same mass and the same number of degrees of freedom as the quark, and its coupling to the Higgs is related to the coupling between the quark and the Higgs, fine tuning from the quadratic divergence vanishes. This is supersymmetry and the scalar 'partner' of the quark is called the squark. Such new type of symmetry avoiding fine tuning is related to the naturalness. If the symmetry is enhanced when specific interaction such as mass term is absent, it is natural that this term is small: Originally forbidden by symmetry so proportional to the small symmetry breaking effects[29]. For example, gauge boson mass is naturally small since gauge symmetry is enhanced in the massless limit. In the same way, the fact that massless fermion has chiral rotation symmetry implies small fermion mass is natural. Since the scalar with the same mass as the lepton or the quark does not exist, supersymmetry is broken, then the breaking scale sets the cutoff scale Λ for new physics in which scalar partners of SM fermions appear.

The interpretation of quantum correction in this way is ambiguous in dimensional regularization since mass scale is not used. In dimensional regularization, divergence is regulated in the $4+\varepsilon$ dimension, and it is given by M^2/ε for scalar mass square correction where M is mass of particle in the loop. When the Higgs interacts with heavy particles whose masses are GUT or Planck scale, it behaves as quadratic divergence. So, dimensional regularization also has fine tuning problem provided that the Higgs is regulated by high energy physics as well as electroweak scale physics.

To see fine tuning more explicitly, it is more appropriate to investigate it in the Wilsonian picture, in which the degrees of freedom in the scale higher than the observation scale are integrated out[30]. The following arguments come from [31].

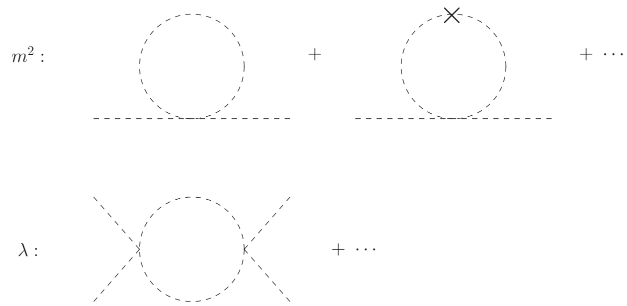

그림 3: Quantum corrections for m^2 and λ from loop diagram.

Consider the scalar field described by

$$S = \int^{\Lambda} \frac{d^4 p}{(2\pi)^d} \frac{1}{2} \phi(-p)(p^2 + m^2)\phi(p) \\ + \frac{\lambda}{4!} \int^{\Lambda} \prod_{a=1}^{4} \frac{d^d p_a}{(2\pi)^d} \delta^d (\sum_{a=1}^{4} p_a) \phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4). \tag{3.19}$$

This is the simplification of the Higgs and the squark system showing quadratic divergence in the absence of the quark. The Wilsonian renormalization undergoes the following steps:

- Divide the scale by inner region $[0, \frac{\Lambda}{N}]$ and outer region $[\frac{\Lambda}{N}, \Lambda]$ where $N = 1 + \varepsilon$.

- Take the functional integral over the outer region.

- Rescale momentum $p' = Np$ and field $\phi'(p') = N^{-D}\phi(p)$ where $-D = -(1/2)(d+2)$ is the dimension of $\phi(p)$ so that the theory which is integrated out transforms to the equivalent theory with the cutoff Λ. Then effective Lagrangian of low energy degrees of freedom is obtained and couplings reflect the quantum corrections where high scale degrees of freedoms are integrated out.

Resulting parameters are given by

$$
\begin{aligned}
m'^2 &= N^{2D-d}(m^2 + c_1\lambda - c_2 m^2\lambda), \\
\lambda' &= N^{4D-3d}(\lambda - 3c_2\lambda^2)
\end{aligned}
\tag{3.20}
$$

N dependences come from rescaling. c_i are calculated from diagrams shown in Fig. 3 and given by

$$
\begin{aligned}
c_1 &= \frac{1}{2}\int_{\text{outer region}} \frac{d^4p}{(2\pi)^d}\frac{1}{q^2} \sim \Lambda^{d-2}(1 - N^{-(d-2)}) \\
c_2 &= \frac{1}{2}\int_{\text{outer region}} \frac{d^4p}{(2\pi)^d}\left(\frac{1}{q^2}\right)^2 \sim \Lambda^{d-4}(1 - N^{-(d-4)}).
\end{aligned}
\tag{3.21}
$$

Then c_1 corresponds to the quadratic divergence and c_2 corresponds to the logarithmic divergence. The renormalized parameters are given by

$$
\begin{aligned}
&\frac{1}{\lambda'} - \frac{1}{\lambda^*} = N^{-(4D-3d)}\left(\frac{1}{\lambda} - \frac{1}{\lambda^*}\right) \\
&m'^2 - m_c^2(\lambda') = N^{2D-d}(1 - c_2\lambda)m^2 - m_c^2(\lambda')
\end{aligned}
\tag{3.22}
$$

and after repeating integrate-out n times, we have

$$
\begin{aligned}
&\frac{1}{\lambda_n} - \frac{1}{\lambda^*} = N^{-(4D-3d)n}\left(\frac{1}{\lambda_0} - \frac{1}{\lambda^*}\right) \\
&m_n^2 - m_c^2(\lambda_n) = N^{(2D-d)n} e^{-c_2\sum_{i=1}^{n-1}\lambda_i}(m_0^2 - m_c^2(\lambda_0)).
\end{aligned}
\tag{3.23}
$$

Here $\lambda^* = (N^{4D-3d} - 1)/3c_2$ and $m_c^2(\lambda) = -\lambda c_1/(1 - N^{2(D-d)})$ so quadratic divergence appears in the fixed point of the mass only, and running to the low energy is governed by logarithmic divergence only. The renormalization group flow for fundamental scalar is depicted in Fig.4. Note that $\lambda < 0$ region is not valid because it destabilizes the vacuum. If the bare mass is on the $m_c(\lambda)$, it runs down along the critical line, $m = m_c(\lambda)$, to the zero mass with $\lambda = 0$, the fixed point.(For $d = 4$, $\lambda^* = -1/3c_2 \to 0$ as $\Lambda \to \infty$.) In this case, very small scalar mass is natural. How-

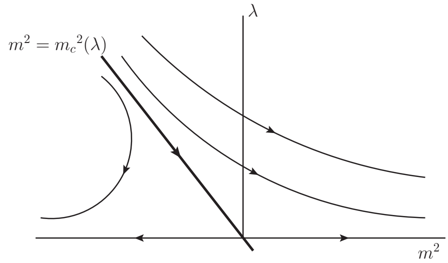

그림 4: Renomalization group running of m^2 and λ.

ever, if this mass is slightly deviated from the critical line, it flows down to the large mass. Therefore, the fine tuning problem in the Wilsonian renormalization can be interpreted as the question, why the mass at the high scale is very close to the critical line, $m = m_c(\lambda)$. Since m_c^2 determining the critical line is quadratic to the cutoff, it is the same as the interpretation in the regularization with cutoff: Bare mass should be of order $\Lambda^2/(8\pi^2)$, the same order as m_c^2 whose quadratic divergence comes from the quantum correction c_1.

If quantum correction to the field renormalization is large enough, it has a large anomalous dimension so running dimension of ϕ is large. $-D$ is no longer the same as the dimension of ϕ and anomalous dimension $\delta = -D + (d+2)/2$ affects N dependent scaling of m^2: $N^{-2\delta}$ factor is additionally multiplied to $(m_0^2 - m_c^2)$ to get $m'^2 - m_c'^2$. So mass is getting closer to the critical line as running down to the lower scale. This can be seen easily by redefinitions $\tilde{p}_n = N^{-n}p$, $\tilde{\phi}(\tilde{p}_n) = (N^{-n})^{-\frac{d+2}{2}}\phi(p)$, $\tilde{m}_n^2 = (N^{-n})^2 m^2$ and $\tilde{\lambda}_n = (N^{-n})^{4-d}\lambda$. The bare values($n = 0$) does not scale at all and

$$\begin{aligned}\tilde{m}_n^2 - \tilde{m}_c^2(\lambda_n) &= N^{(2D-d-2)n} e^{-c_2 \sum_{i=1}^{n-1} \lambda_i}(m_0^2 - m_c^2(\lambda_0)) \\ &= N^{-2\delta n} e^{-c_2 \sum_{i=1}^{n-1} \lambda_i}(m_0^2 - m_c^2(\lambda_0))\end{aligned} \tag{3.24}$$

This would be another solution to the fine-tuning problem. On the other hand, supersymmetry eliminates both quadratic and logarithmic divergences. Since large deviation from the critical line at low energy in renormalization group running is regulated by logarithmic divergence, its vanishing implies that fine-tuning problem vanishes. Choosing initial condition near the critical line has no unnaturalness.

3.3 Flavor Problem

By flavor, we mean quantum number determined by existence of the specific quark. For example, strangeness $+1$ means $\bar{s}$ quark is contained in the physical observable. If the observable does not have strange quark, strangeness is assigned to be zero. However, frequently, this term is used in the meaning of generation. So, when we say the SM gauge group is flavor universal, it means each generation is equally charged under the SM gauge group. In a view of the SM gauge group, generations are just copies of a set of the fields. The SM does not explain different properties depending on the flavor. For example, even though the u quark and the t quark, corresponding particles in the first and the third generations, have the same properties under the electroweak interaction, their masses are quite different. This makes many different phenomena such as life time. In the SM, different masses come from different magnitudes of the Yukawa couplings. But they are just free parameters determined by observations, and the SM does not explain how they have values as measured.

Such problem also arises in the mixing matrix. Yukawa couplings in general form 3×3 complex matrix(so we call it Yukawa matrix) and masses are eigenvalues of this matrix. When it is not diagonal from the beginning, unitary matrices diagonalizing it should appear, and mixing matrix is one combination of such unitary matrices. The CKM and PMNS matrices are such mixing matrices in the quark and the lepton sector, respectively. The form of mixing matrices is important because unremovable phase is source of the CP violation in the weak interaction. If any of the mixing angles vanishes, this phase can be removed by field redefinitions. Therefore, the smallest mixing angle is used to parameterize the CP violation of the weak interaction. The CKM matrix is very close to identity. The mixing between the first two generations, Cabibbo angle is the largest one, but it is just 13^{o}. Other two mixing angles are much smaller, but do not vanish, so the quark sector has weak CP violation. On the other hand, the PMNS matrix has very large mixings. The mixing between the second and the third generations,atmospheric mixing, are maximal: the mixing angle is almost 45^{o}. Mixing between the first and the second generations, solar mixing is also sizable. However, the mixing between the first and the third generations are small. If it is nonzero, weak CP phase appears when we neglect the Majorana phases. It can be measured from the neutrino oscillation.

$$\begin{aligned} A_{CP}^{\alpha'\alpha} &\equiv P(\nu_\alpha \to \nu_{\alpha'}) - P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) \\ &= 4\sum_{j>k} \mathrm{Im}(V_{\alpha' j} V_{\alpha j}^* V_{\alpha k} V_{\alpha' k}^*) \sin\frac{\Delta m_{jk}^2}{2p} L \end{aligned} \tag{3.25}$$

Especially,

$$\begin{aligned} A_{CP}^{(\mu e)} &= -A_{CP}^{(\tau e)} = A_{CP}^{(\tau\mu)} \\ &= 4J(\sin\frac{\Delta m_{32}^2}{2p} L + \sin\frac{\Delta m_{21}^2}{2p} L + \sin\frac{\Delta m_{13}^2}{2p} L) \end{aligned} \tag{3.26}$$

where $J = \mathrm{Im}(V_{\alpha' j} V_{\alpha j}^* V_{\alpha k} V_{\alpha' k}^*)$ is the Jarlskog determinant of the PMNS matrix which will be studied in detail later. Therefore, understanding the patterns of mixing matrices is important but the SM does not explain the origin of these patterns.

If we know all unitary matrices diagonalizing the Yukawa matrices, it is possible to reconstruct the Yukawa couplings before diagonalization, and also possible to study the origin of patterns of these original Yukawa matrices. If some elements of the Yukawa matrix are very small compared to other elements, we can guess that there may be some symmetry which suppresses them. In this sense, structures of the Yukawa matrices imply flavor dependent symmetry beyond the SM. However, we do not know all these unitary matrices. First, the mixing matrix is combination of unitary matrices rotating left handed fermions. We do not know how to separate it into two unitary matrices, for example L_u and L_d. Second, the mixing matrix appears in the charged current in the weak interaction, and by chiral nature of the weak interaction, right handed fermions do not make the charged current. So, there is no way to know unitary matrices rotating right handed fermions within the context of the SM. Therefore, to know them, the hint from new physics beyond the SM should be considered. At this stage, information from new physics is not enough, so even though it is possible to construct the models consistent with the measured values, there is no way to select the unique description of real world. In this sense, for plausible model construction, more experimental evidences as well as consistency with the other model outside the flavor physics itself are required.

3.4 Strong CP Problem

Strong CP problem is another fine-tuning problem in the SM[32]. In the path integral language, transition amplitude is the sum of all paths weighted by $\exp(iS/\hbar)$[33]. In the limit of $\hbar \to 0$,(classical limit) strong interference between $\exp(iS/\hbar)$s from different paths takes place, and only small portions around the extremum remain. This is why classical equation of motion makes action extremum. Among such classical solutions, special types of solutions for vacuum of the system called instanton solutions exist. They make action (for Minkowski spacetime) or energy (for Euclidean spacetime) finite. These solutions are sorted out by topological number, for example winding number. To make energy finite, vacuum configurations are assigned at infinite spacetime coordinates. When we have degenerate vacuums, there is a correspondence between vacuum configurations and coordinates at infinity, and this can be interpreted as vacuums 'wrap' the spacetime. The winding number counts how many times do the vacuums wrap spacetime. In light of topology, objects characterized by different winding numbers cannot be deformed from one to another smoothly, so the vacuum with the specific winding number is stable with respect to that with different winding number. This story arises in the non-Abelian gauge theory in the Euclidean spacetime. Consider the Euclidean Lagrangian for the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$. From the identity $\int d^4x \mathrm{Tr}(F_{\mu\nu} - \tilde{F}_{\mu\nu})^2 \geq 0$ where $\tilde{F}_{\mu\nu} = (1/2)\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ is dual field of $F_{\mu\nu}$, we know

$$E = \frac{1}{2g^2}\int d^4x \mathrm{tr} F_{\mu\nu}F_{\mu\nu} \geq |\frac{1}{2g^2}\int d^4x \mathrm{tr} F_{\mu\nu}\tilde{F}_{\mu\nu}| \tag{3.27}$$

so energy is minimum for $F_{\mu\nu} = \pm\tilde{F}_{\mu\nu}$. Since the term $\mathrm{tr}F_{\mu\nu}\tilde{F}_{\mu\nu} = 8\varepsilon_{\mu\nu\rho\sigma}\mathrm{tr}\partial_\mu[A_\nu\partial_\lambda A_\rho + (2/3)A_\nu A_\lambda A_\rho]$ is total derivative, it does not affect the perturbative Feynman rules. But it is known that[34] instanton solutions make

$$\frac{1}{2g^2}\int d^4x \mathrm{tr} F_{\mu\nu}\tilde{F}_{\mu\nu} = \frac{8\pi^2}{g^2}n \tag{3.28}$$

with n integer, interpreted as winding number. The instanton solution for $n = 1$ is given by

$$\begin{aligned} A_\mu &= \frac{r^2}{r^2+\rho^2}\omega\partial_\mu\omega^{-1} = -i\eta_{\mu\nu\rho}x_\nu\sigma_\rho\frac{1}{r^2+\rho^2}, \\ F_{\mu\nu} &= 2i\eta_{\mu\nu\rho}\sigma_\rho\frac{\rho^2}{r^2+\rho^2}, \end{aligned} \tag{3.29}$$

where $\omega = \frac{x_\mu\sigma_\mu}{r}$, $\sigma_\mu = (I, -i\vec{\sigma})$ and $\eta_{0i\mu} = -\eta_{i0j} = \delta_{ij}$, $\eta_{ijk} = \varepsilon_{ijk}$. Therefore, each instanton solutions corresponding to winding number n form equivalent, stable vacuum solutions. However, this is not the vacuum for the whole system. As shown in 5, the system with equivalent $|n\rangle$ vacuums has discrete translation symmetry *i.e.* invariance under $|n\rangle \to |n+1\rangle$ so vacuum wave function should respect this symmetry either. This is so-called θ-vacuum, $|\theta\rangle = \sum_n e^{in\theta}|n\rangle$. The new quantity θ is

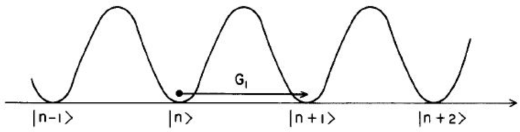

그림 5: Schematic figure for θ vacuum. y axis corresponds to the energy wall. Figure is adopted from J. E. Kim, Phys. Rep. **150** (1987), 1 in [32].

observable. Since $\langle n|\exp(-Ht)|m\rangle = \int [dA_\mu]_{n-m}\exp(-S)$,

$$\langle\theta'|e^{-Ht}|\theta\rangle = \sum_{n'}\sum_{n} e^{-i(n'\theta-n\theta)}\langle n'|e^{-Ht}|n\rangle = \sum_{n'} e^{-in'(\theta'-\theta)}\sum_{q}\int [dA_\mu]_q e^{-S-iq\theta} \tag{3.30}$$

and $\sum_{n'}\exp(-in'(\theta'-\theta)) = \delta(\theta'-\theta)$. So the action has $-iq$ in addition, and it is equivalent to the additional Lagrangian

$$\frac{\theta}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}. \tag{3.31}$$

This term has two properties. First, it breaks CP. Since $P : \vec{E} \to -\vec{E}, \vec{B} \to \vec{B}$ and $C : \vec{E} \to -\vec{E}, \vec{B} \to -\vec{B}$, $F\tilde{F}$ which is equivalent to $\vec{E}\cdot\vec{B}$ is CP odd. Second, it is related to the axial anomaly: quantum breaking of symmetry under chiral rotation, $\psi \to \exp(i\alpha\gamma_5/2)\psi$[35]. Such anomaly is given by $-\frac{\alpha}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$. If the quark mass matrix, or equivalently, Yukawa matrix has the phase, such phase can be moved to $F\tilde{F}$ term through anomaly. Then θ is redefined by $\bar{\theta} = \theta - \mathrm{Arg.Det}.M_q$. It is not accidental. Consider the Dirac operator $i\gamma_\mu D_\mu$ in Euclidean space. Let ϕ_k be the eigenvector of this Hermitian operator with eigenvalue λ_k. Since $\gamma_\mu D_\mu\gamma_5 = -\gamma_5\gamma_\mu D_\mu$,

$$i(\gamma_\mu D_\mu)(\gamma_5\phi_k) = -\gamma_5 i(\gamma_\mu D_\mu)\phi_k = (-\lambda_k)(\gamma_5\phi_k), \tag{3.32}$$

so for nonzero λ_k, we can make chirality pair $\phi_{k,\pm} = (1/2)(1 \pm \gamma_5)\phi_k$. On the other hand, for $\lambda_k = 0$, pairing is not essential so the number of $\phi_{k,+}$ need not be the same as that of $\phi_{k,-}$. Under the chiral rotation $\mathcal{U} = \psi \to \exp(i\alpha\gamma_5/2)\psi$, path integral measure $\mathcal{D}\psi\mathcal{D}\bar{\psi}$ is not invariant, but transforms to $(\mathrm{Det}\,\mathcal{U})^{-2}\mathcal{D}\psi\mathcal{D}\bar{\psi}$[36]. Then,

$$(\mathrm{Det}\,\mathcal{U})^{-2} = e^{-2\mathrm{tr}\ln U} = e^{i\int d^4x \frac{1}{2}\alpha A(x)} \tag{3.33}$$

where $A(x)$ represents the anomaly of the chiral rotation. Here,

$$\mathrm{tr}\ln\mathcal{U} = \int d^4x \langle x|\mathrm{tr}\ln\mathcal{U}|x\rangle = \int d^4x \delta^{(4)}(x-x)\ln\mathcal{U}(x) = \int d^4x \delta^{(4)}(0) i\frac{\alpha(x)}{2}\mathrm{tr}\gamma_5. \tag{3.34}$$

Note that rotation parameter α is regarded as local even though the original chiral rotation is global. It comes from deriving Noether current, the current associated with the symmetry. Pretending global symmetry local, the variation of action with respect to local parameter $\alpha(x)$ is of the form $\delta S = \int d^4x \alpha(x) J_0 + \partial_\mu\alpha(x) J^\mu = -\int d^4x \alpha(x)\partial_\mu J^\mu$. Here, the variation of action under the global transformation J_0 vanishes. When equation of motion holds, δS should be zero so $\partial_\mu J^\mu = 0$. This is Noether's theorem: symmetry is associated with the conserved current. When the symmetry is anomalous, quantum effects modifies $\langle\partial_\mu J^\mu\rangle = 0$ into $\langle\partial_\mu J^\mu - A\rangle = 0$. A is what we want to obtain.

Note that Eq. (3.34) is a product of infinity($\delta^{(4)}(0)$) and zero($\mathrm{tr}\gamma_5$). To regulate this, we introduce gauge invariant regulator, $\mathrm{tr}\gamma_5 f((D_\mu\gamma^\mu)^2/\Lambda^2)$, where $f((D_\mu\gamma^\mu)^2/\Lambda^2)$ is some function of $(D_\mu\gamma^\mu)^2/\Lambda^2$ with $f(0) = 1$ and $f(\infty) \to 0$. Then

$$\begin{aligned}\mathrm{tr}\gamma_5 f\left(\frac{(D_\mu\gamma^\mu)^2}{\Lambda^2}\right) &= \sum_x \langle\phi_k|\gamma_5 f\left(\frac{(D_\mu\gamma^\mu)^2}{\Lambda^2}\right)|\phi_k\rangle = \sum_x f\left(\frac{\lambda_k^2}{\Lambda^2}\right)\langle\phi_k|\gamma_5|\phi_k\rangle \\ &= \sum^{n_+} f(0)\langle\phi_+|\phi_+\rangle - \sum^{n_-} f(0)\langle\phi_-|\phi_-\rangle = n_+ - n_-.\end{aligned} \tag{3.35}$$

Therefore, anomaly counts difference between zero modes in $+$ and $-$ chiralities. So it affects the topological number. This is known as Atiya-Singer theorem[37].

If we take appropriate α, chiral rotation of quarks, $\bar{\theta}$ can be made zero but moved to the fermion mass term as $\bar{q}\exp(i(\alpha(\bar{\theta})/2)\gamma_5)M_q\exp(i(\alpha/2(\bar{\theta}))\gamma_5)q$, so it can be measured through CP violating process in the strong interaction such as neutron electric dipole moment[38]. The measured value is very close to zero, $|\bar{\theta}| < 0.7\times 10^{-11}$[39]. Then one may ask why CP violation in the strong interaction is so small. This is the strong CP problem. If one of the quark is massless, for example, $m_u = 0$, it is possible to assign arbitrary phase to this, so specific value of $\bar{\theta}$ is meaningless. However, all quarks seem to be massive[40]. One attractive explanation is dynamical one, suggested by Peccei and Quinn[41]. Suppose we have the 'axion' field a with the symmetry under shift, $a \to a+\varphi$, and couples to gluon $F\tilde{F}$ term so we have

$$\frac{1}{32\pi^2}\Big(\bar{\theta}+\frac{a}{F_a}\Big)F^a_{\mu\nu}F^{a\mu\nu}. \tag{3.36}$$

By shifting $a \to a - F_a\bar{\theta}$, it is just $(1/32\pi^2)(a/F_a)F^a_{\mu\nu}F^{a\mu\nu}$ and it makes action minimum at $a=0$ because

$$\begin{aligned} e^{-\int d^4xV[a]} &= \Big|\int \mathcal{D}A_\mu \prod_i \mathrm{Det}(D_\mu\gamma^\mu + m_i) e^{-\int d^4x \frac{1}{4g^2}FF - i\frac{a}{F_a}F\tilde{F}}\Big| \\ &\leq \int \mathcal{D}A_\mu \Big|\prod_i \mathrm{Det}(D_\mu\gamma^\mu + m_i) e^{-\int d^4x \frac{1}{4g^2}FF - i\frac{a}{F_a}F\tilde{F}}\Big| = e^{-\int d^4xV[0]} \end{aligned} \tag{3.37}$$

so $\int d^4xV[0] < \int d^4xV[a]$. Note that $\mathrm{Det}(D_\mu\gamma^\mu + m_i) = \prod_{\lambda_k}(-i\lambda_k + m_i)$ is always positive because chiral pairing guarantees that we have the same number of positive ($\lambda_k > 0$) and negative($-\lambda_k$) eigenvalues, so $\prod_{\lambda_k}(-i\lambda_k + m_i) = m_i^n \prod_{\lambda_k>0}(\lambda_k^2 + m_i^2)$ where $n = n_+ + n_-$ is the number of zero modes. The field a with the shift symmetry can come from Goldstone boson in the global symmetry breaking, *i.e.* $\sigma = [(F_a +$

$\rho)/\sqrt{2}]\exp(ia/F_a)$. Since a accompanies with i, it is CP odd. So, CP is broken when a is stabilized with the nonzero value, but since it favors zero, CP violation is very small. The general interactions of CP odd Goldstone boson are of the form

$$c_1\partial_\mu a\frac{1}{F_a}\bar{q}\gamma^\mu\gamma_5 q+(\bar{q}_L m q_R e^{ic_2\frac{a}{F_a}}+h.c.)+c_3\frac{a}{32\pi^2}F^a_{\mu\nu}F^{a\mu\nu}. \tag{3.38}$$

Such global symmetry is called U(1) Peccei-Quinn(PQ) symmetry. When it was firstly suggested, its breaking scale was thought of as the electroweak scale: $F_a = v$. Suppose we have two Higgs doublets, H_u and H_d responsible for the masses of the up- and down- components of the SU(2) doublet, respectively. These two Higgses have the same charge under the PQ symmetry, *i.e.* U(1) PQ transforms the Higgses as $H_u \to \exp(i\alpha)H_u$ and $H_d \to \exp(i\alpha)H_d$. The quarks are also PQ charged to make Yukawa coupling PQ singlet. When electroweak symmetry is broken, $H_{u,d} = [(v_{u,d}+\rho_{u,d})/\sqrt{2}]\exp(ia_{u,d}/v)$ and one combination of a_u and a_d, say, $-\cos\beta a_u + \sin\beta a_d$ is absorbed by Z boson to make it massive. Another combination $\sin\beta a_u + \cos\beta a_d$ is a physical field, but since global PQ symmetry is spontaneously broken, it is massless (Goldstone boson). Such type of axion, Peccei-Quinn-Weinberg-Wilczek(PQWW) axion[42] is rule out experimentally since it predicts the processes such as $K^+ \to \pi^+ + a$, which is not observed. To suppress the probability of finding out axion, we need to raise F_a to much higher scale. In this 'invisible' axion, σ is no longer the CP odd Higgs and F_a is much higher than electroweak scale.

There are two models for invisible axion. First, σ does not couple to the SM particles. instead, vectorlike heavy quark exists, and have the mass from PQ symmetry breaking via the coupling $\sigma\bar{Q}_L Q_R + h.c.$. Such model is Kim-Shifman-Vainstein-Zakharov(KSVZ) axion[43]. On the other hand, σ can couple to the Higgs but does not couples to the SM quarks to avoid the experimental bound. This is Dine-Fischler-Srednicki-Zhitniskii(DFSZ)[44] axion.

3.5 Cosmological Problem

Even though this topic is important in exploring the new physics, it is beyond the scope of discussion in this thesis. We briefly discuss some issues which can be studied by extending materials treated here.

One of mysteries in cosmology is the baryon asymmetry, a strong imbalance in baryon and antibaryon[45]. To explain this, CP violation as well as baryon number violating interactions and out-of-equilibrium condition are required[46]. If CP violation responsible for the baryon asymmetry takes place in the decay of some heavy particle into Standard Model particles, understanding flavor structure of quarks and leptons could give a good model for baryon asymmetry. Especially, leptogenesis[47] is interesting because the decaying heavy particle is the right handed heavy neutrino. Introducing heavy neutrinos is a plausible extension of the SM as it explains the very small masses of neutrinos through the seesaw mechanism. Therefore, studies on leptogenesis essentially include construction of a model for the flavor structure in the lepton sector. Then flavor dependent symmetry plays very important role as it restricts possible form of flavor structure. In this regards, leptogenesis is a good topic to extend studies on flavor dependent symmetries.

On the other hand, the total matter density of the Universe, $\Omega \equiv \rho_M/\rho_c \sim 0.3$ where $\rho_c = 3H_0^2/(8\pi G_N) = 1.9\times 10^{-26} h_0^2 \mathrm{kgm}^{-3}$ is the critical density for the flat Universe, but known baryonic matter is just $\Omega \equiv \rho_B/\rho_c \sim 0.02$. The existence of the Dark matter[48], matter explaining such missing density, is confirmed in the observations, but identity and its microscopic properties are not known yet. Moreover, the expansion of the Universe is being accelerated by dark energy, which occupies $\Omega_\Lambda \sim 0.7$, but we do not know how to explain this.

제 4 장

Supersymmetry as a solution of the gauge hierarchy problem

4.1 Current Status of the study on the electroweak symmetry breaking

There are two issues in physics of the electroweak symmetry breaking. One is identification of the Higgs: is the Higgs fundamental scalar or composite of fermions? how many Higgs does the Nature have? if there is a new physics beyond the SM, does the Higgs couple to it? if so, how does the new physics affect the electroweak symmetry breaking? The other is stabilization of the Higgs mass at the electroweak scale. Assuming the Higgs to be the elementary scalar, we need to find new physics to solve the gauge hierarchy problem. Supersymmetry is the prime example. The LHC experiments are expected to unveil the physics of electroweak symmetry breaking by finding out Higgs or evidence of new physics. However, at present, there is no solid evidence of them.

4.1.1 Higgs search at the LHC

The SM Higgs decay rates to various decay channels and their branching ratios depend on the Higgs mass[49]. For the Higgs mass $m_H < 140$GeV, $H \to b\bar{b}$ is dominant. If the Higgs is heavier than 140GeV, it will mainly decay into W^+W^- or ZZ. If such Higgs decays are not observed, we expect much smaller cross section for Higgs production. In this way, we can exclude the Higgs production

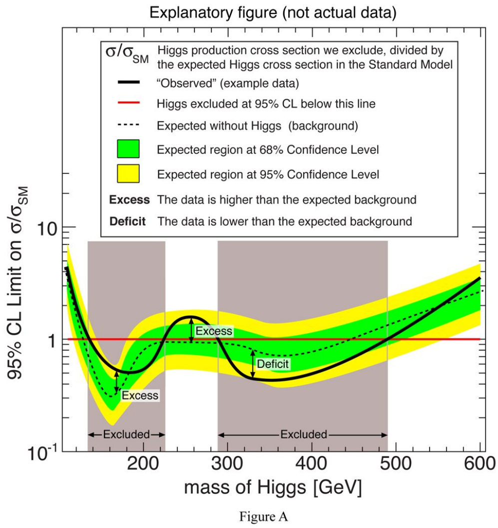

그림 6: The example of the Higgs exclusion plot. Adopted from ALTAS homepage, [50]

cross section and if the SM cross section is not excluded at some value of m_H, it can be interpreted as discovery of the SM-like Higgs at this mass.

Fig. 6[50] shows the example of exclusion plot. The dotted line shows the average expected exclusion in the absence of the Higgs. That means, with the SM without the Higgs, cross section above the dotted line is expected to be excluded. The solid line is the observed exclusion line with 95% Confidence level(C. L.). The Higgs cross section above this line is excluded from observation. If the line dips below 1.0($\sigma = \sigma_{SM}$), the Higgs is not produced with the expected cross section σ_{SM} in the corresponding Higgs mass region so this Higgs mass is excluded in the 95% certainty. In Fig. 6, the Higgs mass in the range $(135, 225)$GeV and $(290, 490)$GeV is excluded. As the luminosity is integrated, dotted line would keep going down. If the solid line does not dip below no longer but stops around $\sigma = \sigma_{SM}$, corresponding mass is the SM Higgs mass.

When new physics is taken into account, other possibilities may be consid-

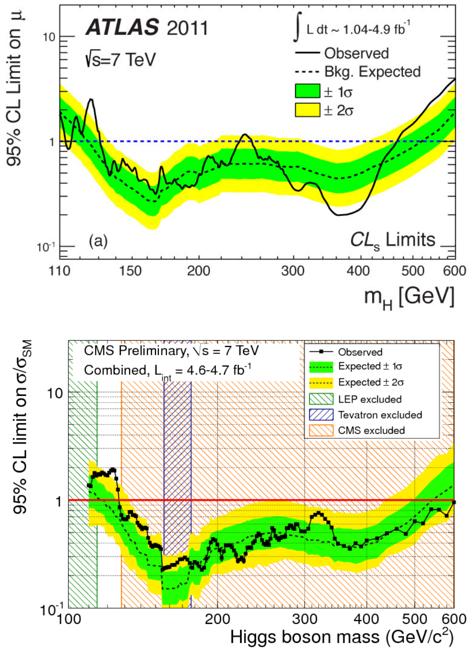

그림 7: ATLAS and CMS reports on exclusion of the Higgs production cross section.

ered. For example, Higgs production cross section can be different from that in the SM. When the Higgs decays into non-SM particles, branching ratio of observing channel can be much smaller and this can be the reason why the Higgs has not been found yet.

The LHC at CERN has two detectors searching for the Higgs and new physics beyond the SM: ATLAS and CMS. ATLAS searches for the Higgs decay channels $H \to \gamma\gamma$, $H \to ZZ^{(*)} \to l^+l^-l^+l^-$, and $H \to WW^{(*)} \to l^+\nu l^-\bar{\nu}$ to exclude (112.7,115.5)GeV, (131,237)GeV, and (251, 468)GeV at 95% confidence level(C.L.). On the other hand, CMS excluded (127,600)GeV at 95% C. L. from five decay modes, $\gamma\gamma$, $b\bar{b}$, $\tau\bar{\tau}$, W^+W^-, and ZZ. These are shown in Fig. 7. Interestingly, both experiments show apparently unexcluded the SM Higgs cross section around the similar mass region: 126GeV for ATLAS and 124GeV for CMS[51]. If these signals are not mere fluctuations, we may already discovered the Higgs.

4.1.2 Supersymmetry searches in the LHC

Even though supersymmetry(SUSY) has a good theoretical motivation as a solution to the gauge hierarchy problem, it should be broken because there is no experimental evidence of superpartners of the quarks and the leptons, called squarks and sleptons, respectively. SUSY breaking scale can be parameterized by typical mass scale of the superpartners. When SUSY is broken, the Higgs mass correction from M_P to the scale μ, say, electroweak scale is approximately given by

$$\delta m_h^2 = -\frac{3y_t^2}{8\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{\mu}{M_P}\right). \tag{4.1}$$

Then $\delta m_h^2/m_h^2$ can represent the degree of fine tuning. Let $m_h^2 \simeq 100 \sim 150$GeV. Allowing the fine tuning of the Higgs mass within factor 10, *i.e.* $|\delta m_h| \simeq 1$TeV in running from M_P to M_Z, we require $m_{\tilde{t}}^2 \simeq 1.2$TeV so SUSY breaking scale is better to be around or below 1TeV. This is why low energy SUSY characterized by sub TeV squark mass is preferred as a solution of the hierarchy problem.

Supersymmetric extension of the SM simply adds SUSY to the flavor blind SM gauge group representations. SUSY breaking effects in squark and slepton masses are free parameters as long as we do not specify SUSY breaking mechanism. Moreover, gauge bosons have their own supersymmetric partners, gauginos. Their masses are also splitted from gauge boson masses as SUSY is broken. Therefore, broken SUSY needs much more free parameters unless the exact SUSY breaking mechanism is verified. To work with the least number of free parameters, we assume that squark masses, slepton masses, gaugino masses, and A-terms (the coefficient of three scalar interaction with mass dimension one) are unified at GUT scale respectively, and are splitted by renormalization group running effect. This scenario is called Constrained Minimal Supersymmetric Standard Model(CMSSM). Experimental studies are made based on this simple model, but they can cover general cases with two features:

- SUSY model with R-parity. As can be seen later, we can impose discrete R parity under which the SM particles are even and their superpartners (sparticles) are odd. It can prevent fast decay of proton and decay of the sparticle into SM particles only. So, the Lightest Supersymmetric Particle (LSP) cannot decay into the SM particles even it is heavier than the SM particles. In cosmology, LSP can be a good candidate of Dark Matter. In collider, LSP no longer decays and escapes out of detector so observing defects of energy, called missing energy is the evidence of such particle. SUSY searches mainly focus on searching for missing energy and CMSSM is a good benchmark.

- SUSY breaking effect has a typical scale. Hadron collider is mainly sensitive to the first two generation squarks since production of the third generation squarks is suppressed. When two protons are collide with each other, squarks are produced via processes, such as $q_i q_j \to \tilde{q}_i \tilde{q}_j$ and $gg \to \tilde{q}_i \bar{\tilde{q}}_j$. To generate the third generation squark, the third generation quarks are required but they are much less contained in the proton compared to the first two generation quarks. The first two generation quarks can produce the third generation squarks through CKM mixing and squark mixing but in many models mixing with the third generation is very small. So, the third generation sqaurks are mainly produced from gluon. *e.g.* $gg \to \tilde{q}_i \bar{\tilde{q}}_j$ or $gq \to \tilde{g}\tilde{q}$ but they have velocity suppression. For example, consider the $s-$channel process $gg \to g^{(*)} \to \tilde{q}_i \bar{\tilde{q}}_j$. Since the virtual gluon can have helicity $+1$ or -1, for angular momentum conservation, both $gg \to g^{(*)}$ annihilation and $g^{(*)} \to \tilde{q}_i \bar{\tilde{q}}_j$ creation should have orbital angular momentum contribution. That's why (scalar)-(scalar)-(scalar) Feynman rules in scalar QED and gluon

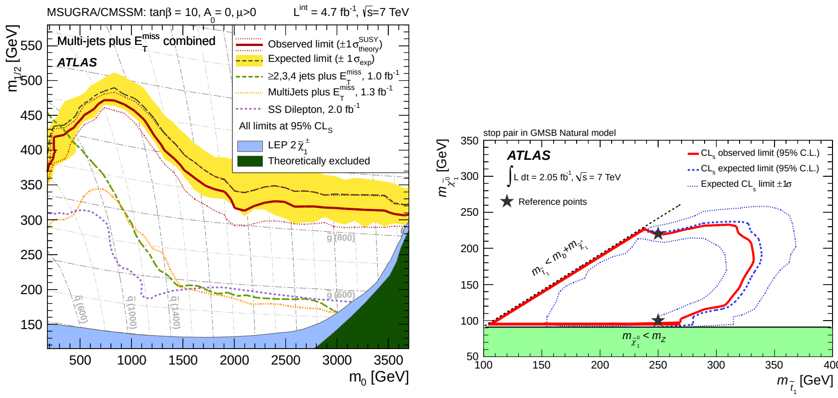

그림 8: (Left) Exclusion bound for squark and gluino based on mSUGRA/CMSSM ($\tan\beta = 10$, $A_0 = 0$, $\mu > 0$). (Right) Exclusion bound for stop $\tilde{t}$ in Gauge mediated SUSY with $\tilde{\chi}^0$ the next-to-lightest supersymmetric particle.

self interaction have a velocity dependence. Since squark velocity is less than 1, such processes have velocity suppression. $t-$channel process would be suppressed unless t is very close to the virtual squark or gluino masses. In this way, the third generation squarks have very small chance to be produced in the LHC, so even they are not found, we cannot say they are too massive. In principle, $\tilde{t}$ and $\tilde{b}$ are less bounded than the first two generation squarks. In CMSSM, squarks have almost the similar scales. If the first two generation squark masses are excluded, the third generation squarks around the similar scale are also excluded.

The recent LHC experiments report squarks and gluino based on CMSSM are excluded to about 1.4TeV as shown in the left of Fig. 8[52]. That means, low energy SUSY models with two features above are being excluded. Then there are four alternative possibilities:

1. Low energy SUSY is excluded. Then SUSY does not solve the hierarchy

problem. In this case, we have to seek for another solution consistent with experiments or ask whether the hierarchy problem is well-defined problem.

2. R-parity is violated[53]. Then LSP can decay into the SM particles and missing energy may not be detected. In this case, we should select R-parity violating operator to make the proton live long enough. SUSY may not provide the Dark matter candidate.

3. The first two generation squarks are above 1TeV but the third generation squarks are sub-TeV[54]. In fact, dominant contribution to the Higgs mass correction comes from the stop $\tilde{t}$, superpartner of top quark since only it has a large Yukawa coupling of order 1. The sbottom $\tilde{b}$ may be in the sub-TeV since superpartner of the left handed bottom quark constitutes SU(2)$_L$ doublet together with the stop. So if the third generation quarks are still in sub-TeV, hierarchy problem is still solved by SUSY. Actually, $\tilde{b}$ and $\tilde{t}$ mass bounds are lower than 400GeV as shown in the right of Fig. 8 and Ref. [55].

4. Sparticle spectrums are degenerated compared to the quark masses. For example, $\tilde{b}$ mass can be measured by detecting missing energy of neutralino $\tilde{\chi}_0$ through decaying process $\tilde{b} \to b\tilde{\chi}_0$. If mass difference between $\tilde{b}$ and $\tilde{\chi}_0$ is so small that b quark does not have enough energy to be detected over background or energy cut, $\tilde{b}$ cannot be discovered. In this case, we do not have sqaurk mass bound.

4.2 Minimal supersymmetric Standard Model

4.2.1 Model description

The Minimal Supersymmetric Standard Model (MSSM) is, literally, minimal supersymmetric extension of the SM. It just adds superpartners to all the SM particles. One difference from the SM in matter contents is that the MSSM is two Higgs doublet model. There should be two Higgs doublets, H_u and H_d, responsible for the masses of upper and lower components of the SU(2)$_L$ doublets, respectively. It has two reasons, First, supersymmetric Lagrangian comes from superpotential W by $\mathcal{L} = \int d^2\theta W$ but W is holomorphic. That means, the term $H^\dagger \bar{Q} U$ where H, Q, and U are left chiral superfields of the Higgs, SU(2)$_L$ doublet and singlet quarks respectively, is not allowed as $H^\dagger$ is a right chiral superfield. Superpotential should be the combination of either left chiral superfields only or right chiral superfields only. So we have to introduce another left chiral superfield for H_u. Second, as the Higgs scalar has fermionic superpartner(higgsino), it gives rise to anomaly. To cancel it, we should have higgsino‘s’ in vector-like pair under the SM gauge group. SU(2) doublets $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$ have U(1)$_Y$ quantum number $1/2$ and $-1/2$, respectively so their superpartners(higgsinos) cancel anomalies of SU(2)$_L$ and U(1)$_Y$.

Supersymmetric extension of the SM breaks some ‘accidental’ symmetries of the SM. In the SM, taking renormalizable terms into account only, baryon and lepton numbers are conserved. That means, we can assign quantum numbers for global U(1) symmetries in such a way that

1. quarks have $1/3$ and leptons have 0 (baryon number) and

2. quarks have 0 and leptons have 1 (lepton number).

More precisely, the SM Lagrangian is written by hand in this way to describe the Nature. Such symmetries are broken when nonrenormalizable terms are included. For example, seesaw mechanism introduces $(1/M)(lH_u)^T(lH_u)$ and it breaks the lepton number. On the other hand, MSSM breaks these two symmetries even in the holomorphic renormalizable superpotential:

$$W \supset \alpha^{ijk} Q_i L_j d_k^c + \beta^{ijk} L_i L_j e_k^c + \gamma^i L^i H_u + \delta^{ijk} d_i^c d_j^c u_k^c. \tag{4.2}$$

This can be dangerous as it predicts the proton decay, which has not been observed yet. For example, from the $s-$channel process $d_R u_R \to \tilde{b}_R^{(*)} \to l q_L$, the proton can decay into $e\bar{\pi}$, $\mu\bar{K}$, or $\nu_e \pi^+$, $\nu_\mu K^+$, etc. One way to forbid this is imposing R-parity, giving the SM particle even and sparticles odd under it.

4.2.2 Higgs sector

The superpotential for the MSSM is given by

$$W = -\mu H_u \cdot H_d - \mathcal{Y}^l_{ij} H_d \cdot L_i E^c_j - \mathcal{Y}^d_{ij} H_d \cdot Q_i D^c_j - \mathcal{Y}^u_{ij} H_u \cdot Q_i U^c_j \tag{4.3}$$

where $A \cdot B = \varepsilon_{ab} A^a B^b$, a, b are indices for SU(2)$_L$ doublet. From this, F-term contribution to the potential is given by $V_F = \sum_i |-\partial W/\partial \Phi_i^\dagger|^2$ where Φ_i represents chiral superfields in W. On the other hand, integrating out auxiliary D-terms for SU(2)$_L$ and U(1)$_Y$ gauge superfields, we obtain $\vec{D}_H = -g_2 H_i^\dagger \frac{\vec{\sigma}}{2} H_i$ and $D_H^Y = -g_Y H_i^\dagger Y H_i$ so the supersymmetric Higgs potential is given by

$$V_F + V_D = |\mu|^2 (|H_u|^2 + |H_d|^2) + \frac{1}{8}(g_Y^2 + g_2^2)(|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2}|H_u^\dagger H_d|^2 \tag{4.4}$$

Finally, the soft SUSY breaking effects comes in as

$$V_{\text{soft}} = m_u^2 |H_u|^2 + m_d^2 |H_d|^2 - (B\mu H_u \cdot H_d + h.c.) \tag{4.5}$$

The Higgs VEVs,

$$\langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} \tag{4.6}$$

where $v_{u,d}$ are taken to be real numbers, breaks electroweak symmetry. To parameterize these Higgs VEVs, we use $v = (v_u^2 + v_d^2)^{1/2}$ and $\tan\beta = v_u/v_d$ as independent parameters. Note that v can replace the Higgs VEV in the SM with one Higgs doublet. At the minimum of the Higgs potential,

$$V_{\min} = \frac{1}{32}(g_Y^2 + g_2^2)(v_u^2 - v_d^2)^2 + \frac{1}{2}m_u^2 v_u^2 + \frac{1}{2}m_d^2 v_d^2 - B\mu v_u v_d \tag{4.7}$$

and conditions $\partial V_{\min}/\partial v_u = \partial V_{\min}/\partial v_d = 0$ give conditions for the electroweak symmetry breaking at the Higgs VEVs:

$$\begin{aligned} -2B\mu &= -(m_u^2 - m_d^2)\tan 2\beta + M_Z^2 \sin 2\beta \\ |\mu|^2 &= \frac{1}{\cos 2\beta}(m_u^2 \sin^2\beta - m_d^2 \cos^2\beta) - \frac{1}{2}M_Z^2. \end{aligned} \tag{4.8}$$

The charged Higgses, H_u^+ and H_d^- form 2×2 matrix,

$$\begin{aligned} &\begin{pmatrix} m_d^2 - \frac{1}{8}(g_Y^2+g_2^2)(v_u^2 - v_d^2) + \frac{1}{4}g_2^2 v_u^2 & B\mu + \frac{1}{4}g_2^2 v_u v_d \\ B\mu + \frac{1}{4}g_2^2 v_u v_d & m_u^2 + \frac{1}{8}(g_Y^2+g_2^2)(v_u^2 - v_d^2) + \frac{1}{4}g_2^2 v_d^2 \end{pmatrix} \\ &= \Big(\frac{B\mu}{v_u v_d} + \frac{1}{4}g_2^2\Big)\begin{pmatrix} v_u^2 & v_u v_d \\ v_u v_d & v_d^2 \end{pmatrix} \end{aligned} \tag{4.9}$$

where the conditions (4.8) are used. One eigenvalue is zero, corresponding to the Goldstone mode $\sin\beta H_u^\pm - \cos\beta H_d^\pm$ and it is absorbed by W boson. Another eigenvalue is given by $(\frac{B\mu}{v_u v_d} + \frac{1}{4}g_2^2)v$, corresponding to the charged Higgs mode $H^\pm = \cos\beta H_u^\pm + \sin\beta H_d^\pm$

The neutral Higgses have real(CP even) and imaginary(CP odd) parts, $H_{u,d}^0 = \frac{1}{\sqrt{2}}(\mathrm{Re}h_{u,d}^0 + i\mathrm{Im}h_{u,d}^0)$. CP even scalar mass matrix is given by

$$\begin{pmatrix} m_d^2 - \frac{1}{8}(g_Y^2+g_2^2)(v_u^2 - v_d^2) & B\mu \\ B\mu & m_u^2 + \frac{1}{8}(g_Y^2+g_2^2)(v_u^2 - v_d^2) \end{pmatrix} = B\mu \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \tag{4.10}$$

One eigenvalue is zero for the Goldstone mode $\sin\beta \mathrm{Im} H_u^0 - \cos\beta \mathrm{Im} H_d^0$ which is absorbed by Z boson. Another mode $A = \cos\beta \mathrm{Im} H_u^0 + \sin\beta \mathrm{Im} H_d^0$ has eigenvalue $m_A^2 = 2B\mu/\sin 2\beta$. Note that m_A becomes zero for vanishing $B\mu$. In fact, when $\mu = 0$ and $B\mu = 0$, global U(1) symmetry under which H_u and H_d have the same charge is recovered. As Higgses have VEVs, this global symmetry is spontaneously broken so massless zero mode should appear. This is nothing more than PQWW axion but it cannot be the QCD axion as it is ruled out experimentally. Its quantum correction has very simplified form. Since the Goldstone mode should be massless even in the presence of quantum correction, the basic structure of the matrix proportional to

$$\begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \tag{4.11}$$

is maintained. So, quantum correction just changes overall factor $B\mu$ to $B\mu + \Delta$ and Δ is proportional to μ and A^t. Proportionality in μ can be easily understood because when $\mu = B\mu = 0$ PQWW type symmetry is enhanced so there should be massless mode. Moreover, it does not have a top loop correction since the amplitude is proportional to

$$\begin{aligned} &\mathrm{Tr}\Big[\gamma_5(\gamma\cdot p + m)\gamma_5(\gamma\cdot p + m)\Big] \\ &= \mathrm{Tr}\Big[(-\gamma\cdot p + m)(\gamma\cdot p + m)\Big] = 0. \end{aligned}$$

Stop correction comes from the (left squark)-(right squark) mixing and it is proportional to $-m_t(A^t + \mu\cot\beta)$. So in the absence of such mixing, *i.e.* $A^t = \mu = 0$, there is no quantum correction for CP odd Higgs mass.

Finally, CP even Higgs mass matrix is given by

$$\begin{pmatrix} m_d^2 - \frac{1}{8}(g_Y^2+g_2^2)(v_u^2-3v_d^2) & -B\mu - \frac{1}{4}(g_Y^2+g_2^2)v_u v_d \\ -B\mu - \frac{1}{4}(g_Y^2+g_2^2)v_u v_d & m_u^2 + \frac{1}{8}(g_Y^2+g_2^2)(3v_u^2-v_d^2) \end{pmatrix}$$
$$= \begin{pmatrix} m_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(m_A^2+M_Z^2)\sin\beta\cos\beta \\ -(m_A^2+M_Z^2)\sin\beta\cos\beta & m_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix} \quad (4.12)$$

The eigenvalues are

$$m_{H,h}^2 = \frac{1}{2}[m_A^2 + M_Z^2 \pm [(m_A^2+M_Z^2)^2 - 4M_Z^2 m_A^2 \cos^2 2\beta]^{1/2}] \quad (4.13)$$

where

$$\begin{aligned} H &= (\mathrm{Re}H_u^0 - v_u)\sin\alpha + (\mathrm{Re}H_d^0 - v_d)\cos\alpha \\ h &= (\mathrm{Re}H_u^0 - v_u)\cos\alpha - (\mathrm{Re}H_d^0 - v_d)\sin\alpha, \end{aligned} \quad (4.14)$$

$\tan 2\alpha = \tan 2\beta (m_A^2 + M_Z^2)/(m_A^2 - M_Z^2)$.

Consider the lightest CP even Higgs mass, m_h. It is smaller than $\min(m_A, M_Z)|\cos 2\beta| < \min(m_A, M_Z)$. In the decoupling limit, *i.e.* $m_A \to \infty$, heavy CP even Higgs H is also decoupled from $m_H > \max(m_A, M_Z) > m_A$ and light CP even Higgs mass satisfies $m_h < M_Z$ at the tree level. To raise m_h beyond the M_Z, large quantum correction should be required. It is known that

$$m_h^2 \leq M_Z^2 + \frac{3G_F}{\sqrt{2}\pi^2 \sin^2\beta}\left[m_t^4(\sqrt{m_t M_s})\ln\left(\frac{M_s^2}{m_t^2}\right) + \left(\frac{A^t}{M_s}\right)^2 m_t^4(M_s)(1 - \frac{1}{12}\left(\frac{A^t}{M_s}\right)^2)\right] \quad (4.15)$$

where $M_s = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ is a typical stop mass scale. Taking $M_t = 1\mathrm{TeV}$, $m_h < 132\mathrm{GeV}$.

4.2.3 sparticle masses

For later discussions, we list here various sparticle masses. Squrak and slepton mass matrices come from F- and D- term superpotential and the soft mass $M_{\tilde{q}}, M_{\tilde{u}}, M_{\tilde{d}}, M_{\tilde{l}}, M_{\tilde{e}}$ and $M_{\tilde{\nu}}$:

$$m_{\tilde{u}}^2 = \begin{pmatrix} M_u M_u^\dagger + M_{\tilde{q}}^2 + M_Z^2(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\cos 2\beta I & -M_u(A^{u*} + \mu\cot\beta) \\ -(A^{uT} + \mu^* \cot\beta)M_u^\dagger & M_u^\dagger M_u + M_{\tilde{u}}^2 + M_Z^2 \frac{2}{3}\sin^2\theta_W \cos 2\beta I \end{pmatrix}, \tag{4.16}$$

$$m_{\tilde{d}}^2 = \begin{pmatrix} M_d M_d^\dagger + M_{\tilde{q}}^2 + M_Z^2(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W)\cos 2\beta I & -M_d(A^{d*} + \mu\tan\beta) \\ -(A^{dT} + \mu^* \tan\beta)M_d^\dagger & M_d^\dagger M_d + M_{\tilde{d}}^2 - M_Z^2 \frac{1}{3}\sin^2\theta_W \cos 2\beta I \end{pmatrix}, \tag{4.17}$$

$$m_{\tilde{l}}^2 = \begin{pmatrix} M_l M_l^\dagger + M_{\tilde{l}}^2 + M_Z^2(-\frac{1}{2} + \sin^2\theta_W)\cos 2\beta I & -M_l(A^{l*} + \mu\tan\beta) \\ -(A^{dT} + \mu^* \tan\beta)M_l^\dagger & M_l^\dagger M_l + M_{\tilde{l}}^2 - M_Z^2 \sin^2\theta_W \cos 2\beta I \end{pmatrix}, \tag{4.18}$$

$$m_{\tilde{\nu}}^2 = \begin{pmatrix} M_{\tilde{l}}^2 + M_Z^2 \frac{1}{2}\cos 2\beta I & 0 \\ 0 & 0 \end{pmatrix}. \tag{4.19}$$

Here triple scalar coupling (A term) is defined by

$$H_d \cdot \tilde{q}_{Li}(\mathcal{Y}^d A^d)_{ij}\tilde{d}_{Rj}^\dagger + \tilde{q}_{Li} \cdot H_u(\mathcal{Y}^u A^u)_{ij}\tilde{u}_{Rj}^\dagger + H_d \cdot \tilde{l}_{Li}(\mathcal{Y}^e A^e)_{ij}\tilde{e}_{Rj}^\dagger + h.c. \tag{4.20}$$

Neutralino mass matrix, mass matrix for neutral gauginos(Photino and Zino) and neutral Higgsinos in the basis of $(\lambda_Y, \lambda_3, \tilde{h}_d, \tilde{h}_u)$ is given by

$$\begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}. \tag{4.21}$$

Finally, chargino mass matrix in the basis of $\frac{1}{\sqrt{2}}(\lambda_1 - i\lambda_2, \tilde{h}^+)$ (Wino and charged

Higgsino basis),

$$\begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \sin\beta & \mu \end{pmatrix}. \tag{4.22}$$

This is diagonalized as $U^* M V^\dagger$ where

$$U = O_u,$$

$$V = O_v \text{ (when det}M > 0), \;\; \sigma_3 O_v \text{ (when det}M < 0)$$

$$O_{v,u} = \begin{pmatrix} \cos\theta_{v,u} & \sin\theta_{v,u} \\ -\sin\theta v,u & \cos\theta_{v,u} \end{pmatrix}$$

$$\tan 2\theta_{u,v} = \frac{2\sqrt{2}M_W(\mu\sin\beta + M_2\cos\beta)}{M_2^2 - \mu^2 \mp 2M_W^2\cos 2\beta}. \tag{4.23}$$

4.3 Higgs sector in the next minimal supersymmetric Standard Model and Peccei-Quinn symmetry

4.3.1 μ term from Peccei-Quinn symmetry

In the MSSM summarized in the previous section, μ in the Higgs sector superpotential can be problematic. It is the only mass scale in the superpotential. All other dimensional parameters(soft masses, A term and $B\mu$) come from SUSY breaking so related to the SUSY breaking scale. μ should be in the electroweak scale for electroweak symmetry breaking but MSSM itself does not give natural explanation why μ is at this scale. This has been called the μ problem[56].

In fact, this is early version of the gauge hierarchy problem. The natural Higgs mass without fine tuning would be at the very high energy scale such as GUT or

	H_u	H_d	S_1	S_2	Z_1	Z_2	X	X'	$\overline{X}$
$Q_{\rm PQ}$	$+1$	$+1$	-1	$+1$	0	0	-2	-2	$+2$
R	$+1$	$+1$	0	0	2	2	0	0	2

표 2: The PQ and R charges of $H_{u,d}, S_{1,2}Z_{1,2}, X$ and $\overline{X}$.

Planck scale. The Higgs mass is proportional to the Higgs VEV so with order one quartic self coupling, the natural scale for electroweak symmetry breaking would be such high scale, either[57]. Once typical scale of the Higgs characterized by μ is fixed at the electroweak scale, SUSY can explain how it is stabilized at this scale. Quantum correction can be small enough, and SUSY breaking parameters adjusted to be around electroweak scale would make the Higgs to break electroweak gauge group at this scale. But SUSY does not explain why μ should be at the electroweak scale.

It may be wise to relate the SUSY breaking scale and μ term since they are the similar scales[58]. Another way might be generate this scale dynamically. It is very similar to the seesaw mechanism. If we have two scales, say, intermediate and high scale, low scale can be generated through (intermediate scale)^2/(high scale), which indicates symmetry breaking effect whose scale would be one of two scales. High scale can be GUT or Planck scale. On the other hand, Peccei-Quinn symmetry breaking can be taken as the intermediate scale, $10^9\text{GeV} < F_a < 10^{12}\text{GeV}$. Then electroweak scale about 100GeV can be easily obtained from, for example, μ term can be of order F_a^2/M_P where $F_a = 10^{10}\text{GeV}$ and $M_P \sim 10^{18}\text{GeV}$ and interpreted as breaking effect of the Peccei-Quinn symmetry. To see it from model[59], consider the superpotential with the PQ symmetry and the $U(1)_R$ symmetry shown in Table

2,

$$
\begin{aligned}
W =& - H_u H_d X + -\xi H_u H_d X' + mX\overline{X} + m' X' \overline{X} \\
& - \eta \overline{X} S_1^2 + Z_1(S_1 S_2 - F_1^2) + Z_2(S_1 S_2 - F_2^2).
\end{aligned} \tag{4.24}
$$

Here, $F_{1,2}$ are of order of the PQ scale. PQ symmetry and SUSY may be broken by such a number of terms. One linear combination of S_1 and S_2 is a Goldstone mode, the axion superfield A. In the nonlinear representation, axion superfield A is defined by $S_1 = \varphi e^{-A}$ and $S_2 = \varphi e^{A}$. $m, m' = O(M_P) \sim O(\mathrm{M_{GUT}})$. These two scales combine to make electroweak scale. The potential is

$$
V = V_F + V_D + V_{\mathrm{soft}}. \tag{4.25}
$$

The F-term potential is given by

$$
\begin{aligned}
V_F =& \left| X + \xi X' \right|^2 (|H_u|^2 + |H_d|^2) + | - H_u H_d + m\overline{X}|^2 + | - \xi H_u H_d + m'\overline{X}|^2 \\
& + |\tilde{m}\tilde{X} - \eta S_1^2|^2 + |Z_1 + Z_2|^2 |S_1|^2 + \left| -2\eta \overline{X} S_1 + (Z_1 + Z_2) S_2 \right|^2 \\
& + |S_1 S_2 - F_1^2|^2 + |S_1 S_2 - F_2^2|^2,
\end{aligned} \tag{4.26}
$$

where

$$
\begin{aligned}
& \tilde{X} = \cos\alpha X + \sin\alpha X', \\
& X_e = -\sin\alpha X + \cos\alpha X', \\
& \cos\alpha = \frac{m}{\tilde{m}}, \quad \sin\alpha = \frac{m'}{\tilde{m}}, \quad \tilde{m} = \sqrt{m^2 + m'^2}.
\end{aligned} \tag{4.27}
$$

Note here that $\tilde{X}$ and $\overline{X}$ have the mass of order $m,\ m'$ but X_e can have the mass of the order of electroweak scale. Therefore, X_e can survive at the electroweak scale and participate in the electroweak symmetry breaking.

The D-term potential is given by

$$V_D = \frac{1}{8}(g_Y^2 + g_2^2)(|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2}|H_u^\dagger H_d|^2 + \cdots, \tag{4.28}$$

and the soft term is

$$\begin{aligned} V_{\text{soft}} = &-m_u^2|H_u|^2 + m_d^2|H_d|^2 + M_1^2|Z_1|^2 + M_2^2|Z_2|^2 \\ &+ m_1^2|X|^2 + m_2^2|X'|^2 + m_3^2|\overline{X}|^2 + \mu_1^2|S_1|^2 + \mu_2^2|S_2|^2. \end{aligned} \tag{4.29}$$

The important terms determining the vacuum expectation values of S_1, S_2 and $\tilde{X}$ are

$$V' = |S_1S_2 - F_1^2|^2 + |S_1S_2 - F_2^2|^2 + |\tilde{m}\tilde{X} - \eta S_1^2|^2. \tag{4.30}$$

They are stabilized at $s_{1,2} = |S_{1,2}|$ and $\tilde{x} = |\tilde{X}|$ satisfying

$$s_1 = \sqrt{\frac{m\tilde{x}}{\eta}}, \quad \frac{s_2}{s_1} = \frac{\eta F^2}{2m\tilde{x}} \tag{4.31}$$

where $F^2 = F_1^2 + F_2^2$. Requiring $s_1 \sim s_2 \sim F = F_a$, $\tilde{x} = O(\text{TeV})$.

Below the scale m, m',heavy fields $\tilde{X}$, $\overline{X}$ are integrated out, and the superpotential becomes

$$\begin{aligned} W = &-\frac{S_1^2}{M_P}H_uH_d - f_hH_uH_dX_e \\ &+ Z_1(S_1S_2 - F_1^2) + Z_2(S_1S_2 - F_2^2) \end{aligned} \tag{4.32}$$

where

$$f_h = -\sin\alpha + \xi\cos\alpha. \tag{4.33}$$

As the PQ symmetry is broken we have the Higgs superpotential

$$W_{ew} = -\mu H_u H_d - f_h H_u H_d X_e \tag{4.34}$$

where $\mu = s_1^2/m$. Therefore, there are two ways to generate μ term. First, in the similar way to the seesaw mechanism, PQ symmetry breaking scale F_a can make electroweak scale as F_a^2/M_P. Second, surviving field X_e has a VEV around the electroweak scale. The latter case is what frequently used in the Next to the MSSM(NMSSM) model[60]. The field X_e can have the VEV around the electroweak scale with the help of soft term $m_e^2|X_e|^2$ and mixing with the Higgs. X_e potential can be stabilized when coefficient of quadratic term ($|X_e|^2$) from the soft term and supersymmetric potential is positive. One may consider quartic term $|X_e|^4$ for stabilization of negative quadratic term. To achieve this, NMSSM models usually impose S_3 discrete symmetry such that the superpotential is given by

$$-X_e H_u \cdot H_d + \frac{1}{3}\kappa X_e^3. \tag{4.35}$$

Then $V_F \supset |\partial W/\partial X_e|^2$ has the quartic self coupling $|\kappa|^2|X_e|^4$. On the other hand, in the presence of gauge U(1)$'$ symmetry where X_e is charged under it, D-term $(\tilde{g}^2/2)|X_e^\dagger X_e|^2$ can be made. This quartic term should be treated carefully. When the U(1)$'$ gauge symmetry is broken at high energy scale, *e.g.* GUT scale or PQ scale, whole D term potential is of the form

$$\begin{aligned}
\frac{1}{2}D^2 &= \frac{g'^2}{2}\left(Y_e'|X_e|^2 + Y_x'|X|^2 + \cdots\right)^2 \\
&= \frac{g'^2}{2}\left(Y_e'|X_e|^2 + Y_x'|V_{GUT} + \rho_x|^2 + \cdots\right)^2 \\
&= \frac{g'^2}{2}\left(Y_e'^2|X_e|^4 + Y_e'Y_x'V_{GUT}^2|X_e|^2 + Y_x'^2|V_{GUT}|^4 \cdots\right)
\end{aligned} \tag{4.36}$$

so it gives rise to large quadratic term for $|X_e|$ at the tree level, and fine tuning problem arises again. Therefore, such term should not appear at the electroweak scale[61]. If U(1)$'$ for quartic term exists, it should be broken at the electroweak scale[62] but it is not our case.

Therefore, we can consider the following potential for Higgs and X_e:

$$
\begin{aligned}
V = & |\mu + f_h X_e|^2(|H_u|^2 + |H_d|^2) + f_h^2|H_u H_d|^2 \\
& - m_u^2|H_u|^2 + m_d^2|H_d|^2 - (B\mu H_u H_d + \text{h.c.}) \\
& + m_e^2|X_e|^2 - (AX_e H_u H_d + \text{h.c.}) \\
& + \frac{1}{8}(g_Y^2 + g_2^2)(|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2}|H_u^\dagger H_d|^2.
\end{aligned}
\tag{4.37}
$$

In the same way as MSSM, we can decompose neutral fields into real and complex components, $\phi = \frac{1}{\sqrt{2}}(\phi^r + i\phi^i)$ where $\phi = H_u^0, H_d^0, X_e$. At the vacuum, they take VEVs v_u, v_d, x, respectively, and

$$
\begin{aligned}
V^{\min} = & \frac{1}{2}[(\mu + \frac{f_h}{\sqrt{2}}x)^2 - m_u^2]v_u^2 + \frac{1}{2}[(\mu + \frac{f_h}{\sqrt{2}}x)^2 + m_d^2]v_d^2 \\
& + \frac{f_h^2}{4}v_u^2 v_d^2 + \frac{1}{32}(g_Y^2 + g_2^2)(v_u^2 - v_d^2)^2 \\
& - B\mu v_u v_d - \frac{A}{\sqrt{2}}x v_u v_d + \frac{1}{2}m_e^2 x^2.
\end{aligned}
\tag{4.38}
$$

From $\partial V^{\min}/\partial h_u = \partial V^{\min}/\partial h_d = \partial V^{\min}/\partial x = 0$, we have three conditions:

$$
\begin{aligned}
& (\mu + \frac{f_h}{\sqrt{2}}x)^2 - m_u^2 = (\frac{A}{\sqrt{2}}x + B\mu)\frac{v_d}{v_u} - \frac{1}{8}(g_Y^2 + g_2^2)(v_u^2 - v_d^2) - \frac{f_h^2}{2}v_d^2 \\
& (\mu + \frac{f_h}{\sqrt{2}}x)^2 + m_u^2 = (\frac{A}{\sqrt{2}}x + B\mu)\frac{v_u}{v_d} + \frac{1}{8}(g_Y^2 + g_2^2)(v_u^2 - v_d^2) - \frac{f_h^2}{2}v_u^2 \\
& x(\frac{f_h^2}{2}(v_u^2 + v_d^2) + m_e^2) = \frac{A}{\sqrt{2}}v_u v_d - \frac{f_h}{\sqrt{2}}\mu(v_u^2 + v_d^2)
\end{aligned}
\tag{4.39}
$$

4.3.2 CP even Higgs mass

From $m_{ij}^2 = \langle \partial V/\partial\phi_i\partial\phi_j \rangle$, we can obtain mass matrices. For CP even scalars, in the basis of (h_u, h_d, X_e), mass matrix is given by

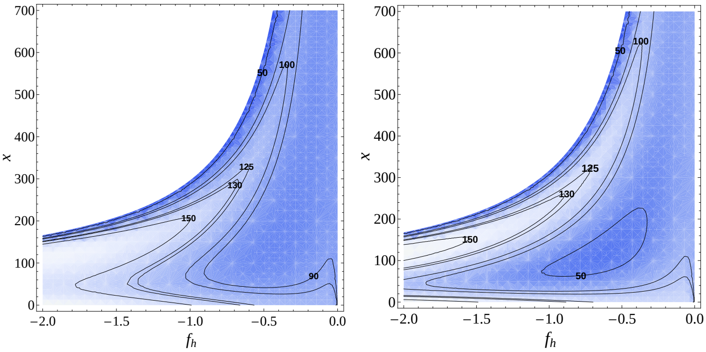

그림 9: CP even Higgs masses.

$$M_H^2 = \begin{pmatrix} \begin{matrix} m_0^2\cos^2\beta \\ +M_Z^2\sin^2\beta \end{matrix}, & \begin{matrix} \frac{1}{2}\sin 2\beta(f_h^2 v^2 \\ -m_0^2 - M_Z^2) \end{matrix}, & \begin{matrix} m_c^2\sin\beta \\ -m_c'^2\cos\beta \end{matrix} \\ \begin{matrix} \frac{1}{2}\sin 2\beta(f_h^2 v^2 \\ -m_0^2 - M_Z^2) \end{matrix}, & \begin{matrix} m_0^2\sin^2\beta \\ +M_Z^2\cos^2\beta \end{matrix}, & \begin{matrix} m_c^2\cos\beta \\ -m_c'^2\sin\beta \end{matrix} \\ \begin{matrix} m_c^2\sin\beta \\ -m_c'^2\cos\beta \end{matrix}, & \begin{matrix} m_c^2\cos\beta \\ -m_c'^2\sin\beta \end{matrix}, & M_E^2 \end{pmatrix} \tag{4.40}$$

where $M_O^2 = \frac{1}{\sqrt{2}x}(Av_u v_d - \mu f_h(v_u^2+v_d^2))$, $M_E^2 = M_O^2$, $m_c^2 = f_h(\sqrt{2}\mu + f_h x)v$, $m_c'^2 = Av/\sqrt{2}$, and $m_0^2 = (\sqrt{2}Ax + 2B\mu)/\sin 2\beta$. Note that originally (11) element is $(\mu + (f_h/\sqrt{2})x)^2 - m_u^2 + (1/8)(g_Y^2+g_2^2)(3v_u^2 - v_d^2) + (f_h^2/2)v_d^2$, (22) element is $(\mu + (f_h/\sqrt{2})x)^2 + m_d^2 + (1/8)(g_Y^2+g_2^2)(-v_u^2+3v_d^2) + (f_h^2/2)v_u^2$ and (33) element is $(1/2)f_h^2(v_u^2+v_d^2) - m_e^2$ but equivalent to those shown in the matrix with the help of Eq. (4.39).

The smallest eigenvalue, which we will identify with the Higgs, is smaller than the smallest eigenvalue of the top 2×2 submatrix[63]. Therefore,

$$2m_h^{02} \le (m_0^2 + M_Z^2) - [(m_0^2+M_Z^2)^2 - 4m_0^2 M_Z^2\cos^2 2\beta + f_h^2 v^2(f_h^2 v^2 - 2m_0^2 - 2M_Z^2)\sin^2 2\beta]^{1/2} \tag{4.41}$$

Taking quantum correction from top quark and A^t into account, the mass eigenvalues become larger. Fig. 9 shows the Higgs boson in the GeV units. We set $M_s = 1$TeV, $A^t = 800$GeV, $B = 500$GeV, $\mu = A/f_h = 200$GeV. Left panel is for $\tan\beta = 3$, right panel is for $\tan\beta = 5$.

Typically, for the large Higgs mass, we need large coupling $|f_h|$. However, too large $|f_h|$ makes perturbativity be broken down. The relevant renormalization group equations of couplings are given by

$$\begin{aligned} 16\pi^2 \frac{dg_i^2}{dt} &= b_i g_i^2 \\ 16\pi^2 \frac{dy_t^2}{dt} &= y_t^2 [f_h^2 + 6y_t^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2] \\ 16\pi^2 \frac{df_h^2}{dt} &= f_h^2 [4f_h^2 + 3y_t^2 - 3g_2^2 - \frac{3}{5} g_1^2] \end{aligned} \tag{4.42}$$

where g_i are gauge couplings($g_1 = \sqrt{5/3}g'$) so that $b_1 = 33/5, b_2 = 1, b_3 = -3$ and $t = \ln(\mu^2/M_{\rm GUT}^2)$. We see that for perturbativity up to GUT scale, say, $f_h(\mu = M_{\rm GUT}) < 2\pi$, low energy f_h should satisfy $f_h(\mu = 100{\rm GeV}) < 0.7$. When $f_h \simeq 2$, perturbativity bound is kept only below 10TeV[64]. In our case, low energy fields and couplings come from physics of PQ symmetry breaking. Then we may require that perturbativity holds up to PQ scale, $10^9 \sim 10^{12}$GeV, and $f_h(\mu = 100{\rm GeV}) <$

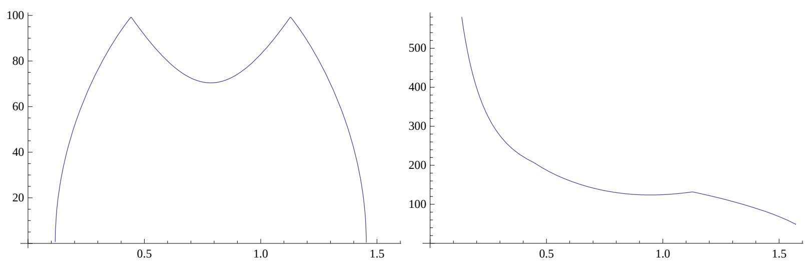

그림 10: The lightest eigenvalue versus β. x axis extends from $\theta = 0$ to $\pi/2$. The unit of y axis is GeV. f_h is fixed by -0.6, $x = 320$GeV, $M_s = 1$TeV, $A^t =$GeV, $B = 500$GeV, $\mu = A/f_h = 200$GeV.

$0.8 \sim 0.9$. Fig. 9 shows that the for given parameters, especially at $\tan\beta = 3 \sim 5$, the 125$\sim$130GeV Higgs is almostly on the perturbativity bound.

The 3×3 CP even scalar mass matrix has complicated dependence on various parameters. $\tan\beta$ dependence on the tree level mass is shown in the left of Fig. 10. However, quantum correction

$$\frac{3G_F}{\sqrt{2}\pi^2 \sin^2\beta}\left[m_t^4 \ln\left(\frac{M_s^2}{m_t^2}\right) + \left(\frac{A^t}{M_s}\right)^2 m_t^2 (1 - \frac{1}{12}\left(\frac{A^t}{M_s}\right)^2)\right], \tag{4.43}$$

where $M_s = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, gives strong dependence on $\tan\beta$ at its small values. As shown in Fig. 10, the quantum corrected Higgs mass grows as $\tan\beta$ decreases.

Now, consider f_h and x dependence of the mass matrix. To begin with, consider 2×2 matrix,

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix}. \tag{4.44}$$

The smallest eigenvalue is given by

$$\frac{1}{2}\left[(a+c) - \sqrt{(a+c)^2 - 4(ac - b^2)}\right]. \tag{4.45}$$

Note that off diagonal element b reduces the eigenvalue. In the limit $a, b \ll c$, eigenvalue is approximately $a - (b^2/c)$. This simple fact is useful to understand the Higgs mass in the model.

The lightest mass of CP even mass matrix is, mainly the Higgs-like, not X_e-like. As an illustration, consider eigenvectors of the lightest CP even mass matrix in the basis of (H_u, H_d, X_e) in the case of $\tan\beta = 3$ in Fig. 9 (Other parameters

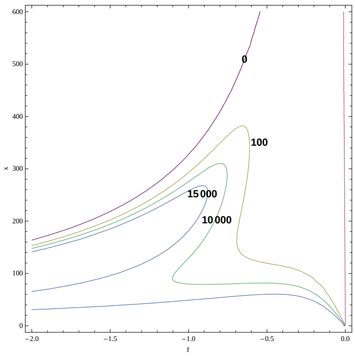

그림 11: lightest squared mass eigenvaue(GeV^2 unit) of 2×2 submatrix composed of (11), (13), (31), (33) elements.

are the same as those for Fig. 9). For $f_h = -0.6$,

$$
\begin{aligned}
x = 300\text{GeV} &: \ (0.773, 0.209, 0.599) \\
x = 250\text{GeV} &: \ (0.770, 0.211, 0.602) \\
x = 200\text{GeV} &: \ (0.781, 0.219, 0.584) \\
x = 150\text{GeV} &: \ (0.803, 0.231, 0.549) \\
x = 100\text{GeV} &: \ (0.840, 0.248, 0.482) \\
x = \ \ 50\text{GeV} &: \ (0.901, 0.278, 0.332) \\
x = \ \ 10\text{GeV} &: \ (0.948, 0.307, 0.008)
\end{aligned}
\tag{4.46}
$$

From this, we notice that the lightest mass in the CP even mass matrix is mostly H_u-like. Moreover, since (33) element is proportional to $1/x$, for large x (33) element is smaller than (22) element. So, in this region, X_e is more mixed than H_d in the lightest scalar, and (13) element plays crucial role in reducing values of the lightest scalar.

Fig. 11 shows lightest squared mass eigenvaue(GeV^2 unit) of 2×2 submatrix composed of (11), (13), (31), (33) elements. Such pattern can be explained as fol-

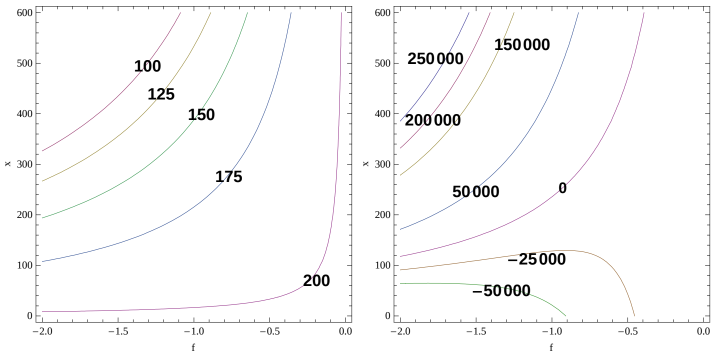

그림 12: Reduction of the Higgs mass from mixing with X_e.

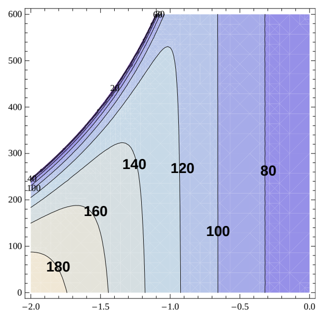

그림 13: lightest squared mass eigenvaue(GeV^2 unit) of 2×2 submatrix composed of (11), (12), (21), (22) elements.

lows. The left of Fig. 12 shows (11) element mass, $[(\sqrt{2}Ax+2B\mu)\cos^2\beta/\sin 2\beta + M_Z^2\sin^2\beta]^{1/2}$ in GeV unit. A is proportional to f_h. The right of Fig. 12 shows (13) element in GeV^2 unit. Nonzero value of it reduces the lightest eigenvalue. In particular, increasing magnitude of (13) element for small x pulls isomass line to the right where large (11) element is reduced by large (13) element.

On the other hand, for very small $x < 80$GeV, (33) element becomes very large so decoupled from the Higgs. The lightest scalar mass is mainly determined by mixing between H_u and H_d, which is shown in Fig. 13. Therefore, main contribution to the lightest scalar mass eigenvalue comes from 2×2 submatrix composed of (11), (12), (21), (22) elements, and the eigenvalue of this submatrix is given by

$$\frac{1}{2}\Big((m_0^2+M_Z^2)-(m_0^2+M_Z^2)\Big[1-4\frac{m_0^2M_Z^2\cos^2 2\beta}{(m_0^2+M_Z^2)^2}+\frac{f_h^2v^2(f_h^2v^2-2m_0^2-2M_Z^2)\sin^2 2\beta}{(m_0^2+M_Z^2)^2}\Big]^{1/2}\Big) \tag{4.47}$$

For example, consider the case of $x=0.1$GeV and $f_h=-0.6$. The second term in [ ] is about 0.06 and the third term in [ ] is about 0.04. So, the lightest mass squared value is approximately given by $(1/40)(m_0^2+M_Z^2)$. In our parameter choice, $m_0\simeq$ 577GeV. Then the lightest scalar mass is estimated by 92GeV. Including subleading effects, this can be changed, and numerical value of whole 3×3 mass matrix gives 96GeV. Quantum correction from $m_{\tilde{t}}$ and A^t can increase the Higgs mass to 125GeV.

In the region where x very close to zero, X_e does not provide VEV, but provides the Higgs quartic term $f_h|H_uH_d|^2$. Effecitve μ term, $\mu_{\rm eff}=\mu+(f_h/\sqrt{2})x$ comes out of $\mu=S_1^2/M_P$. This region allows sizable $\mu_{\rm eff}$. (Fig. 14)

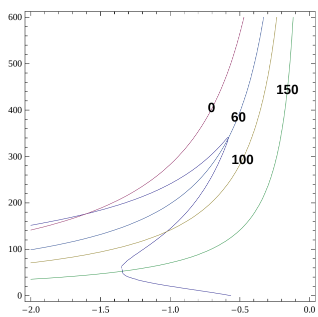

그림 14: 125GeV Higgs line compared to contours for $\mu_{\rm eff}$.

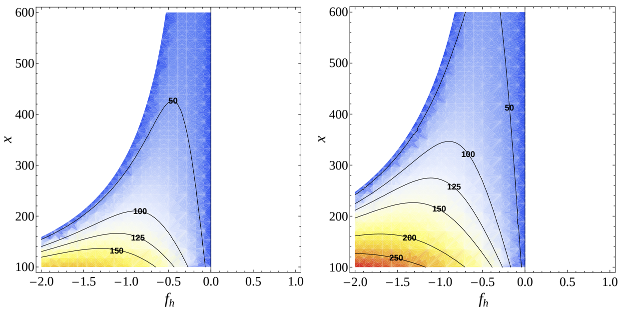

그림 15: Lightest CP odd scalar a_X mass.

4.3.3 CP odd Higgs mass

The CP odd scalar mass matrix is given by

$$M_P^2 = \begin{pmatrix} (\frac{A}{\sqrt{2}}x + B\mu)\frac{v_d}{v_u}, & (\frac{A}{\sqrt{2}}x + B\mu), & \frac{A}{\sqrt{2}}v_d \\ (\frac{A}{\sqrt{2}}x + B\mu), & (\frac{A}{\sqrt{2}}x + B\mu)\frac{v_u}{v_d}, & \frac{A}{\sqrt{2}}v_u \\ \frac{A}{\sqrt{2}}v_d, & \frac{A}{\sqrt{2}}v_u, & M_O^2 \end{pmatrix} \tag{4.48}$$

Originally, (11) element is given by $(\mu + (f_h/\sqrt{2})x)^2 - m_u^2 + (1/8)(g_Y^2 + g_2^2)(v_u^2 - v_d^2) + (f_h^2/2)v_d^2$, (22) element is $(\mu + (f_h/\sqrt{2})x)^2 + m_d^2 + (1/8)(g_Y^2 + g_2^2)(-v_u^2 + v_d^2) + (f_h^2/2)v_u^2$ and (33) element is $(1/2)f_h^2(v_u^2 + v_d^2) - m_e^2$ but equivalent to what is shown in the matrix with the help of Eq. (4.39). One eigenvalue in the direction of $(-\sin\beta, \cos\beta, 0)$ is zero, longitudinal component of Z boson. Among two remaining eigenvalues, the smaller one is

$$2m_{a_X}^2 = (m_0^2 + M_O^2) - \left[(m_0^2 + M_O^2)^2 - \frac{4\mu\tilde{M}^3}{\sin 2\beta}\right]^{1/2} \tag{4.49}$$

where $\tilde{M}^3 = 2BM_O^2 - f_h A(v_u^2 + v_d^2)$. Consider the lightest eigenvalue direction in

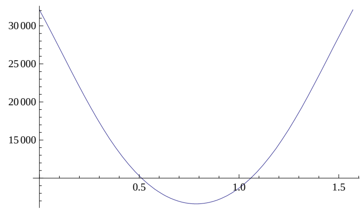

그림 16: The CP odd scalar a_X squared mass versus β. x axis extends from $\beta = 0$ to $\pi/2$. The unit of y axis is GeV^2.

large $\tan\beta$ limit. The mass matrix is of the form

$$\begin{pmatrix} xa & x & ya \\ x & \frac{x}{a} & y \\ ya & y & z \end{pmatrix} \tag{4.50}$$

where $a = \cot\beta$, and we can expand eigenvalue in terms of a. For large $\tan\beta$, $v_u > v_d$ so electroweak symmetry is broken mainly in the direction of H_u so Z boson longitudinal component is mainly CP odd part of H_u ($(1,0,0)$ direction). Then, the lightest eigenvalue direction is $(0, e_2, e_3)$ where

$$\begin{aligned} \tan\gamma &\equiv \frac{e_2}{e_3} \\ &= \frac{(1+a^2)x^2 + 2a^2y^2 - x[az + \sqrt{(1+a^2)x^2 - 2a(1+a^2)xz + a^2(4(1+a^2)y^2 + z^2)}]}{y\left[(1+a^2)x + az - \sqrt{(1+a^2)x^2 - 2a(1+a^2)xz + a^2(4(1+a^2)y^2+z^2)}\right]} \\ &\simeq -a^2\frac{y}{x} \end{aligned} \tag{4.51}$$

for small a. In our case,

$$\tan\gamma = -\frac{\frac{A}{\sqrt{2}}v}{\frac{A}{\sqrt{2}}x + B\mu}\frac{\cos^2\beta}{\sin\beta} \tag{4.52}$$

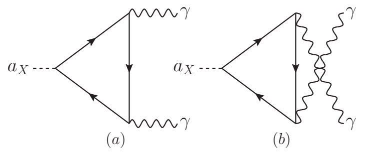

그림 17: $a_X \to \gamma\gamma$ decay through Higgsino loop.

which is very small for large $\tan\beta$. Therefore, the lightest eigenvalue is X_e-like, not the Higgs like. Actually, if $\mu = 0$, the Lagrangian recovers PQWW-type PQ symmetry (the remnant of the PQ symmetry for the QCD axion) so Goldstone mode should appear when X_e and the Higgses develop VEVs.

The lightest mass, say, mass of a_X is shown in Fig. 15 This is for $\tan\beta = 3$, $A = f_h \times 200$GeV, $B = 500$GeV, and $\mu = 150, 200$GeV, respectively. The mass m_{a_X} has β dependence as shown in Fig. 16 which is drawn for $x = 160$GeV, $f_h = -0.6$ and other parameters except $\tan\beta$ is the same as before.

In the case of extreme X_e-like a_X, it decays into $\gamma\gamma$ through the triangle diagram shown in Fig. 17. Fermion in internal loop is the Higgsinos. As low energy PQWW-type PQ symmetry, which will be called 'Higgsino symmetry' is mainly broken by X_e VEV, we can simply estimate such coupling as

$$\mathcal{L}_{a_X\gamma\gamma} = \frac{\alpha_{\rm em}}{4\pi}\frac{a_X}{x}F_{{\rm em}\mu\nu}\tilde{F}^{\mu\nu}_{\rm em} \tag{4.53}$$

and similar decays into ZZ or W^+W^- can be considered either. In the LEP, in may be produced through, for example, the $s-$channel process like $e^+(p')e^-(p) \to \gamma^{(*)}(q) \to \gamma(k)a_X(k')$ whose amplitude is given by

$$\mathcal{M} = \bar{v}(p')(-ie\gamma^\mu)u(p)\frac{-i}{q^2}\left(\frac{\alpha_{\rm em}}{2\pi}\right)\frac{1}{x}\varepsilon_{\mu\nu\rho\sigma}q^\nu k^\sigma \varepsilon(k)^\rho \tag{4.54}$$

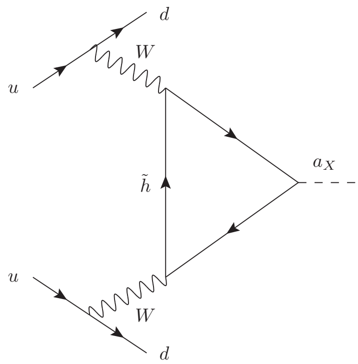

그림 18: a_X production from WW fusion.

and then the cross section is given by

$$\sigma(s) = \frac{\alpha_{\rm em}^3}{768\pi}\frac{1}{x^2}\left(1-\frac{m_{a_X}^2}{2s}\right)\left(1-\frac{m_{a_X}^2}{3s}+\frac{m_{a_X}^4}{12s^2}\right). \tag{4.55}$$

To count how many this a_X production events can take place, we can sum over (integrated luminosity at s)$\times$(cross section at s) with respect to $s > m_{a_X}$. We assume $m_{a_X} = 125$GeV and integrated luminosity in LEP II experiment is provided by [65]. Then the number of events are estimated as $\sim 5\times 10^{-3}$. In hadron collider, it may be produced from electroweak gauge boson fusion with the Higgsino triangle(Fig. 18), which has much smaller probability than the CP even Higgs produced from the gluon-gluon fusion, since electroweak coupling is smaller than the strong coupling, so production rate is suppressed about $\sim \alpha_2^4/\alpha_c^2 \sim 0.0336^4/0.118^2 \simeq 10^{-4}$. Actually, scalar production by the vector boson fusion accompanies dijets in forward direction, so it can be distinguished from scalar production from gluon-gluon fusion. As observed events at the LHC do not have forward dijets, 125GeV signal mainly come from gluon-gluon fusion. Therefore, pseudoscalar cannot explain 125GeV signal[61].

Of course, a_X also contains the Higgs components, mainly from H_d for large

$\tan\beta$, then it may decay into $b\bar{b}$ pair. The ratio of BRs to $a_X \to \gamma\gamma$ and to $a_x \to b\bar{b}$ in our example is given by

$$R = \frac{\alpha_{\rm em}^2 m_{a_X}^3}{64\pi^2 x^2} \frac{8\pi M_W^2}{3C^2 m_b^2 m_{a_X}} = \frac{\alpha_{\rm em}^2 m_{a_X}^2}{24\pi x^2} \frac{M_W^2}{C^2 m_b^2}, \tag{4.56}$$

where $C = g_2 \tan\beta \sin\gamma$ in a large $\tan\beta$ limit, and $\frac{1}{3}$ is multiplied for three colors of b. R of Eq. (4.56) is about $(m_{a_X}/67Cx)^2$. In a large $\tan\beta$ and a small μ limits, $C \approx -g_2 v \cos\beta/x$ which is very small. Therefore, a_X almost decays into $b\bar{b}$ is suppressed.

Moreover, the absence of quartic term, $|X_e|^4$ makes the X_e-like neutral fermion, say, $\tilde{X}_e$ very light. Then, tree level decay $a_X \to \tilde{X}_e\tilde{X}_e$ would be a dominant decay mode of a_X. Since a_X decay is mainly invisible, we do not have much chance to detect it.

4.4 Effective supersymmetry from flavor non-universal U(1)′ mediation

If the Higgs is a fundamental scalar, we encounter the gauge hierarchy problem. As described in Sec. 3.2, it arises from quadratic divergence in the Higgs mass correction and such quadratic divergence comes from fermion loop. However, since Yukawa couplings are very small except top quark Yukawa coupling, contributions from all quarks and leptons but top quark arenegligibly small. For this reason, if we want to construct minimal model for solving hierarchy problem, it would be sufficient to search for top-like contribution whose coupling to the Higgs is comparable with that of top quark so that it can reduce quadratic divergence against the top quark contribution. Then one may ask what makes only top-like contribution resides in sub TeV region whereas other new physics are at very heavy scale. To answer to this question, we have to introduce flavor dependent new physics.

In this section, we consider SUSY as a candidate of new physics as a solution of hierarchy problem. As an application of flavor dependent symmetry to SUSY, we consider the effective SUSY, in which all squarks but stop mass are heavy enough so that they are out of reach of the LHC searches.

4.4.1 Supersymmetry breaking mediation mechanism

SUSY should be broken since sparticles with the same mass as quarks, leptons, or gauge bosons have not been found yet. It can be broken spontaneously when potential has nonzero vacuum value. SUSY algebra $[Q_A, \bar{Q}_{\dot{B}}]_+ = 2\sigma^{\mu}_{A\dot{B}} P_\mu$ or, equivalently, $P_\mu = (1/4)\bar{\sigma}^{\dot{B}A}_\mu [Q_A, \bar{Q}_{\dot{B}}]_+$ implies that $H = P_0 = (1/4)[Q_1, \bar{Q}_{\dot{1}}]_+ + (1/4)[Q_2, \bar{Q}_{\dot{2}}]_+$. Then, if $\langle \mathrm{VAC}|H|\mathrm{VAC}\rangle \neq 0$, $Q_A|\mathrm{VAC}\rangle$ is in general nonzero so vacuum is not invariant under the SUSY transformation: SUSY is spontaneously broken. However, SUSY is not likely to be broken within the MSSM sector. To see this consider the structure of the mass matrices of fermion and boson in the broken SUSY. Gauge boson obtain the mass via the Higgs mechanism,

$$(m_V^2)^i_j = D^a_j D^{bi} + D^{ai} D^b_j \tag{4.57}$$

For scalars, the potential is given by

$$\begin{aligned} V &= F_i F_i^\dagger + \frac{1}{2} D^a D^a \\ F_i &= -\frac{\partial W^\dagger}{\partial \phi_i^\dagger}, \quad D^a = -\eta^a - g_a \phi_i^\dagger (T^a \phi)_i. \end{aligned} \tag{4.58}$$

The scalar mass is given by $\langle \partial V / \partial \Phi_i \partial \Phi_j \rangle$ where $\Phi = \phi, \phi^\dagger$, so in the basis of $(\phi, \phi^\dagger)$,

$$\begin{pmatrix} W^{\dagger ik} W_{kj} + D^{ai} D^a_j + D^{ai}_j D^a & W^{\dagger ijk} W_k + D^{ai} D^{aj} \\ W_{ijk} W^{\dagger k} + D^a_i D^a_j & W_{ik} W^{\dagger kj} + D^a_i D^{aj} + D^{aj}_i D^a \end{pmatrix}. \tag{4.59}$$

On the other hand, fermion and gaugino masses come from

$$-\sqrt{2}g_a(T\lambda)^a\psi_i\langle\phi_i\rangle - \frac{1}{2}\psi_i\psi_j W_{ij} + h.c. \tag{4.60}$$

Then, the mass matrix in the (λ,ψ) basis is given by

$$\begin{pmatrix} W_{ij} & -\sqrt{2}D_i^a \\ \sqrt{D}_j^a & 0 \end{pmatrix} \tag{4.61}$$

and the supertrace is given by

$$\begin{aligned} \mathrm{sTr}m^2 &= \sum_J (-1)^{2J}(2J+1)m_J^2 \\ &= \mathrm{Tr}m_S^2 - \mathrm{Tr}(m_F^\dagger m_F + m_F m_F^\dagger) + 3m_V^2 = -2\mathrm{Tr}(T^a D^a). \end{aligned} \tag{4.62}$$

Let us consider the mass squared matrices for each electromagnetic charge ($2/3$, $-1/3$, -1 and 0) separately. Since $SU(3)_c$ and $U(1)_{\rm em}$ should not be broken, only color and electromagnetic neutral D terms may be contribute to the SUSY breaking: D_0, D term for $U(1)_Y$ and D_3, that for T_3 component of $SU(2)_L$. Then we have

$$\begin{aligned} \mathrm{sTr}M_l^2 &= g_2 D_3 - g_Y D_0 \\ \mathrm{sTr}M_\nu^2 &= -g_2 D_3 + g_Y D_0 \\ \mathrm{sTr}M_u^2 &= -g_2 D_3 + g_Y D_0 \\ \mathrm{sTr}M_d^2 &= g_2 D_3 - g_Y D_0 \end{aligned} \tag{4.63}$$

which implies that $\mathrm{sTr}M_e^2 + \mathrm{sTr}M_\nu^2 = \mathrm{sTr}M_u^2 + \mathrm{sTr}M_d^2 = 0$ and there should be some sfermions lighter than the fermion[66]. This is ruled out by experiment.

In this regard, SUSY breaking sector need to be secluded from the MSSM sector. By secluded, we mean it should not affect the phenomenology of the SM (or

MSSM) without SUSY breaking. So we call it ‘hidden sector’. Even hidden and the MSSM (‘visible’) sector are completely separated, both are regulated by the gravitational interaction, so SUSY breaking can be transferred to the visible sector. This is gravity mediation. On the other hand, hidden sector can couple to the some unknown fields charged under the SM gauge group. In this case, these fields play the role of ‘messenger’ transferring the SUSY breaking in the hidden sector to the visible sector. This scenario is called gauge mediation. Under the messenger scale, at which messengers are integrated out, sparticle masses and their interaction obtain soft SUSY breaking term. Soft terms come into the quantum correction to the Higgs mass as $\tilde{M}^2 \ln(\Lambda/\tilde{M})$, instead of Λ^2. It replaces the quadratic divergence with the more ‘soft’ logarithmic divergence, and it is why such terms are called ‘soft’ breaking term. The form of soft term therefore depends on the SUSY breaking mechanism.

4.4.2 Effective Supersymmetry

Now, consider the current experimental status. The low energy SUSY models with two features, 1. R-parity is conserved and 2. SUSY breaking is described by the common scale, are almostly ruled out. To maintain the motivation of low energy SUSY as a solution of the gauge hierarchy problem, we should consider either R-parity violation constrained by proton stability or two different soft mass scales in the SUSY breaking. Let us focus on the latter case[67].

As mentioned previously, the third generation squarks are less constrained and those in sub-TeV are not ruled out in experiments. Moreover, the large Yukawa coupling of the third generation gives decisive contribution to the Higgs mass correction. So even though other squark masses are beyond the reach of the current LHC search, the third generation can be (sub-)TeV and it still solves the hierarchy problem. Such idea is called effective SUSY or natural SUSY[68]. In many models, stops are lighter than other squarks. As can be seen in the squark mass matrix, left- and right- squark mixing is proportional to the quark mass, For example, $-M_u(A^u+\mu\cot\beta)$ for $u-$type squarks($\tilde{u},\tilde{c},\tilde{t}$). For the first two generations, these terms are very small due to the small Yukawa couplings. On the other hand, the third generation has sizable Yukawa couplings then such large off-diagonal term makes the lightest mass eigenvalue lighter. Moreover, the squark soft masses run in the form of $d\tilde{m}/d\ln\mu = (1/8\pi^2)(\mathcal{Y}^2(m_h^2+m_{qL}^2+m_{q^c}^2)-g_a^2M_a^2)$ where m_h is the Higgs soft term, m_u or m_d. Then rough estimation gives $\delta\tilde{m}^2 \sim (\mathcal{Y}^2/8\pi^2)\ln(\mu/\Lambda) < 0$ for large Yukawa coupling. Therefore, the third generation squark can run to the lighter mass scale. In this regard, light stop is favored.

More progressively, we can consider the model making the third generation squark masses and other squark mass scales drastically different. In construction of a model for it, flavor dependent U(1) gauge symmetry is useful. Suppose new U(1) gauge symmetry, say, U(1)$'$, under which the quark superfields in the first two generation are charged, but those in the third generation are uncharged. SUSY is broken in the hidden sector and messenger is charged under U(1)$'$. Then SUSY breaking can be transmitted to the visible sector through U(1)$'$ gauge interaction. What about lepton sector? If we prefer the simplest model, we can make sleptons heavy enough likely to the first two generation squarks. Then we don't need to concern the lepton sector any more. However, there are some motivations of considering light((sub-)TeV) slepton(s). First, one may require anomaly-free U(1)$'$ from appropriate assignments of U(1)$'$ charges to the quark and lepton superfields. Then, some lepton superfields can be uncharged under it and corresponding sleptons can have (sub-)TeV mass. Second, as the quarks and the leptons are charged under flavor dependent U(1)$'$, the U(1)$'$ charges can affect the flavor structures. For example, we can find the origin of mixing patterns appearing in the CKM or PMNS matrices from such U(1)$'$. Then there may be (sub-)TeV sleptons determined by U(1)$'$ charge assignments. Third, there are some phenomena which may require new physics at (sub-)TeV. For example, muon $g-2=2a_\mu$ still has deviation from the SM prediction[23],

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = (255 \pm 63 \pm 49) \times 10^{-11}. \tag{4.64}$$

Such deviation may be due to lack of understanding on the hadron physics then it would be explained within the SM framework. However, it can also be interpreted as a smoking gun of new physics, such as SUSY. MSSM contribution to muon $g-2$ is given by[69]

$$\frac{a_\mu^{\rm SUSY}}{1\times 10^{-9}} \simeq 1.5\left(\frac{\tan\beta}{10}\right)\left(\frac{300{\rm GeV}}{m_{\tilde{\nu}}}\right)^2\left(\frac{\mu M_2}{m_{\tilde{\nu}}^2}\right). \tag{4.65}$$

If we assume that new physics contribution entirely comes from SUSY, Δa_μ can be used to estimate the scale of sneutrinos, and even can constrain upper bound of slepton mass as sub-TeV[70]. Of course, even though they can be motivations of considering light sleptons, it does not mean the sleptons have to be light. One can assign non-anomalous U(1)$'$ charges consistent with mixing patterns, but also make all the sleptons heavy enough. Muon $g-2$ deviation can be attributed to the QCD effects which may have not been noticed. Light sleptons are entirely optional. However, in this thesis, we consider light sleptons(Fig. 19):

1. Many U(1)s may contribute in the mediation. Here we choose the simplest possibility that only one U(1)$'$ with the superpartner Zprimino (Z'-ino) is effective in the mediation.

2. The SUSY breaking source does not carry the weak hypercharge Y, or the low energy SM does not result. The messenger sector at $M_{\rm mess}$ carries the Z' charge Y'.

3. The superpartners of the third family fermions, (t, b, τ, ν_τ) do not carry the U(1)$'$ charge Y'. This item realizes the effSUSY.

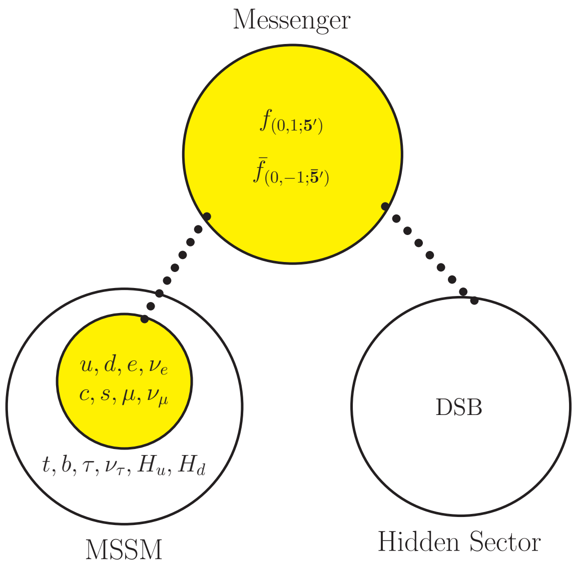

그림 19: SUSY breaking mediation through U(1)$'$ under which the third generation matter and the Higgses are uncharged

4. The Higgs doublets do not carry the U(1)$'$ charge Y'. The SU(2)$_W\times$U(1)$_Y$ breaking is naturally achieved by a running of Higgs boson masses.

Suppose SUSY is broken in the hidden sector with confining gauge group, for example, SU(5)$'$. The messenger fields, carrying the hidden sector color such as the SU(5)$'$ charge have the following $(Y, Y'; \mathrm{SU}(5))$

$$f(0,1;\mathbf{5}'),\ \ \bar{f}(0,-1;\bar{\mathbf{5}}'). \tag{4.66}$$

and the third family members do not carry the Y' charges. In addition, Higgs doublets also do not carry the Y' charges. Then, a light Higgs boson and the light 3rd family members are obtained naturally.

We may take some variations. For the lepton sector, one of the first two generations, instead of the third generation may be uncharged under this U(1)$'$. We will see this example later to explain mixing in the lepton sector. To explain heavy, nearly degenerate first two generation squarks, we may introduce SU(2)$'$ for them and the third generation squark is singlet under it[71]. We do not consider this case here for the following reason. Suppose we look for the origin of gauge symmetries from more high energy physics, for example, orbifold compactification of superstring[72]. To eliminate tachyonic state, string theory has both bosonic and fermionic degrees of freedom related by SUSY which is called superstring. For unitarity, negative norm state is not allowed and this condition predicts the 10 dimensional spacetime. For anomaly cancelation, gauge group on superstring should be SO(32) or $E_8 \times E_8$[73]. In order to obtain realistic model, we need to compactify extra 6 dimensions, and the SM gauge group and its chiral representation should be obtained from such compactification. However, we usually obtain more than one SUSY ($\mathcal{N} > 1$) in such compactifications. With $\mathcal{N} > 1$ SUSY, fermions in chiral or complex representation cannot be obtained. Therefore for chiral representation, we introduce discrete symmetry on extra dimensions. By identifying points related by such discrete group transformation, we can mode out multiple SUSY. This is called orbifold compactification. In this process, many U(1)s come out. For example, $E_8 \times E_8$ can be broken down to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)^3 \times SU(5)' \times U(1)'$ where primed groups come from E_8'[74]. Such U(1)s are in general flavor dependent, and we may find combinations of U(1)s under which the third generation quarks and the Higgses are not charged. For this reason, we prefer U(1)s for mediator rather than other gauge groups. On the other hand, mediation of SUSY breaking may take place not only through U(1)$'$ but also through the flavor universal SM gauge group. In this case, the third generation squark masses can be heavy enough but still lighter than other squarks. To make third generation squark masses low enough, SUSY breaking scale can be made lower. Consider the example where the

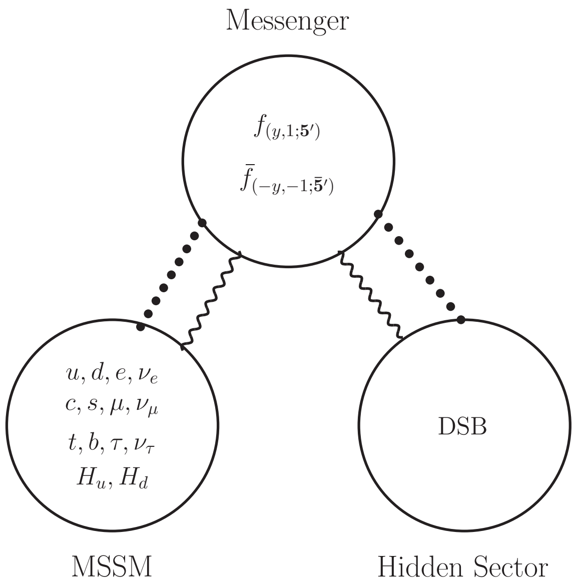

그림 20: SUSY breaking mediation through U(1)$'$ under which the third generation matter and the Higgses are uncharged and U(1)$_Y$.

messenger fields carry the Y charge,

$$f(1,1;\mathbf{5}'),\ \ \bar{f}(-1,-1;\bar{\mathbf{5}}'). \tag{4.67}$$

Then SUSY breaking is transferred to the visible sector as follows:

1. Many U(1)s may contribute in the mediation. In addition, U(1)$_Y$ of the SM also can be effective as a SUSY breaking mediator. These gauge bosons are Z' and B, and their superpartners are called Zprimino $\tilde{Z}'$ and Bino.

2. The SUSY breaking source does not carry the weak hypercharge Y, or the low energy SM does not result. The messenger sector carries both the weak hypercharge Y and the Z' charge Y'.

3. The superpartners of the third family fermions do not carry the Z' charge Y'.

light families	Y	Y'	3rd family and $H_{d,u}$	Y	Y'
$q_{1,2}$	$\frac{1}{6}$	$\frac{1}{3}$	(t,b)	$\frac{1}{6}$	0
$u^c_{1,2}$	$\frac{-2}{3}$	$\frac{-1}{3}$	t^c	$\frac{-2}{3}$	0
$d^c_{1,2}$	$\frac{1}{3}$	$\frac{-1}{3}$	b^c	$\frac{1}{3}$	0
$l_{1,2}$	$\frac{-1}{2}$	-1	(ν_τ,τ)	$\frac{-1}{2}$	0
$e^c_{1,2}$	1	1	τ^c	1	0
$N^c_{1,2}$	0	1	N^c_3	0	0
			H_d	$\frac{-1}{2}$	0
			H_u	$\frac{1}{2}$	0

표 3: The $Y' = B-L$ charges of the SM fermions, Higgs doublets and heavy neutrinos.

This item realizes the effective SUSY .

4. Higgs doublets do not carry the Z' charge Y'.

5. The $SU(2)_W \times U(1)_Y$ breaking is done by a fine-tuning between parameters of the Higgs boson mass matrix.

4.4.3 Soft mass terms and sparticle spectrum from flavor non-universal U(1)′ mediation

In this section, we obtain soft terms from U(1)′ mediation[75]. To be specific, we present minimal case here. For matter contents, MSSM matter fields(the quarks, the leptons, the Higgs) and the heavy neutrinos in the seesaw mechanism are considered, and we do not introduce more SM charged matters under the messenger scale. With these matters only, we can consider the anomaly-free U(1)′, for example, $Y' = B-L$ for the first two generations and $Y' = 0$ for the third generation as listed in Table 3. Messengers form vector-like U(1)′ charged pair.

At the messenger scale M_{mess}, the messengers obtain mass from the effective

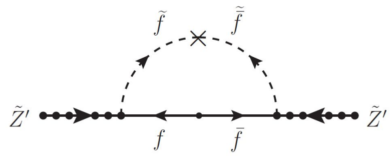

그림 21: The mass diagram of Zprimino. The SUSY breaking insertion from SUSY breaking in the hidden sector is $\times$. The bulleted line is $\tilde{Z}'$. This soft mass is added to the SUSY mass.

superpotential $Xf\bar{f}$ with spurion $X = M_{\rm mess} + \theta^2 F_{\rm mess}$ as

$$\begin{pmatrix} M^2_{\rm mess} & F_{\rm mess} \\ F_{\rm mess} & M^2_{\rm mess} \end{pmatrix} \tag{4.68}$$

Nonzero $F_{\rm mess}$ makes scalar masses split and different from the fermionic superpartners of f and $\bar{f}$ which have common mass $M_{\rm mess}$. In this way, SUSY breaking is transferred to the messenger sector. Then, $Z'-$ino and the MSSM gauginos acquire soft masses as(Fig. 21)

$$\begin{aligned} M_{\tilde{Z}'}(M_{\rm mess}) &= -\frac{N'_{\rm mess} g^2_{\tilde{Z}'}(M_{\rm mess})}{16\pi^2}\frac{F_{\rm mess}}{M_{\rm mess}} \\ M_a(M_{\rm mess}) &= 0 \end{aligned} \tag{4.69}$$

where $N'_{\rm mess} = \sum_i Y_i'^2$ is the number of messengers. Since we consider $f,\ \bar{f}$ pair, $N'_{\rm mess} = 2$.

Suppose U(1)$'$ is broken at scale $M_{Z'}$ lower than the messenger scale. At $M_{Z'} < \mu < M_{\rm mess}$, Z$'$-ino obtains mass $M_{Z'} + M_{\tilde{Z}'}$. In one-loop, gaugino soft term runs in the same way as the gauge coupling, *i.e.*

$$\frac{dM_{\tilde{Z}'}}{d\ln\mu} = \frac{b_{Z'} g^2_{Z'}}{8\pi^2} M_{Z'}, \quad \frac{d}{d\ln\mu}\frac{1}{g^2_{Z'}} = -\frac{b_{Z'}}{8\pi^2} \tag{4.70}$$

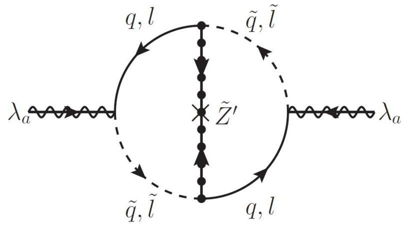

그림 22: The mass diagram of the SM gauginos. The SUSY breaking from Zprimino sector is shown as $\times$. The $\tilde{Z}'$ line is a bulleted line.

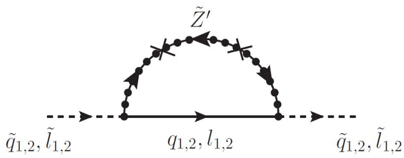

그림 23: The first two family sfermion($\tilde{q}_{1,2}, \tilde{l}_{1,2}$) mass diagrams. The SUSY breaking from Zprimino sector is shown as $\times$.

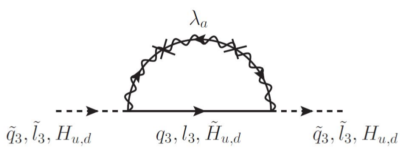

그림 24: The mass diagrams for the third family sfermion($\tilde{q}_3, \tilde{l}_3$) and Higgs bosons. The SUSY breaking from the SM gauginos are shown as $\times$.

so,

$$\frac{d}{d\ln\mu}\Big(\frac{M_{\tilde{Z}'}}{g_{Z'}^2}\Big) = 0. \tag{4.71}$$

Under $M_{Z'}$, both Z' gauge boson and Z'-ino are integrated out. (We assume here that $M_{Z'} \gg M_{\tilde{Z}'}$.) Then Z'-ino running is summarized as

$$\frac{M_{\tilde{Z}'}(\mu)}{g_{Y'}^2(\mu)} = -\frac{1}{8\pi^2}\frac{F_{\text{mess}}}{M_{\text{mess}}}, \tag{4.72}$$

This also holds for the SM gauginos. But at one-loop level, SUSY breaking effect does not appear. Including leading SUSY breaking effect(Fig. 22),

$$\frac{dM_a}{d\ln\mu} = \frac{b_a g_a^2}{8\pi^2}M_a + \frac{c_a g_a^2}{(8\pi^2)^2}g_{Z'}^2 M_{\tilde{Z}'}, \quad \frac{d}{d\ln\mu}\frac{1}{g_a} = -\frac{b_a}{8\pi^2} \tag{4.73}$$

so

$$\frac{d}{d\ln\mu}\left(\frac{M_a}{g_a^2}\right) = \frac{c_a}{8\pi^2 b_{Z'}}\frac{dM_{\tilde{Z}'}}{d\ln\mu}. \tag{4.74}$$

where c_a are given by

$$\begin{aligned} c_Y &= \sum\left[6\left(\frac{1}{6}\right)^2 Y_Q'^2 + 3\left(\frac{1}{3}\right)^2 Y_{U^c}'^2 + 3\left(\frac{1}{3}\right)^2 Y_{D^c}'^2 + 2\left(\frac{1}{2}\right)^2 Y_L'^2 + Y_{E^c}'^2\right], \\ c_2 &= \sum\left[3Y_Q'^2 + Y_L'^2\right], \\ c_3 &= \sum\left[2Y_Q'^2 + Y_{U^c}'^2 + Y_{D^c}'^2\right]. \end{aligned} \tag{4.75}$$

In our case($Y' = B - L$), $c_Y = 92/27$, $c_2 = 8/3$, and $c_3 = 8/9$. For more accurate and systematic analysis, refer to [76] in which extra gauge boson interaction broken by Yukawa couplings mediates SUSY breaking.

At $\mu < M_{Z'}$, the U(1)$'$ vector multiplet is decoupled, so the MSSM gaugino masses are determined by

$$\frac{d}{d\ln\mu}\left(\frac{M_a}{g_a^2}\right) = 0 \tag{4.76}$$

so

$$\frac{M_a(\mu)}{g_a^2(\mu)} = -\frac{c_a g_{Y'}^2(M_{Z'})}{(8\pi^2)^2} M_{\tilde{Z}'}(M_{Z'}) \ln\left(\frac{M_{\text{mess}}}{M_{Z'}}\right). \tag{4.77}$$

Note that

$$M_1 : M_2 : M_3 = c_1 g_1^2 : c_2 g_2^2 : c_3 g_3^3 \simeq c_1 : 2c_2 : 6c_3 \tag{4.78}$$

at low energy scale. The MSSM gauginos obtain soft masses as $M_a \sim 10^{-4} M_{\tilde{Z}'}$.

On the other hand, the first two generation sfermions directly couple to U(1)$'$ as (Fig. 23):

$$m^2_{\tilde{q}_{1,2},\tilde{l}_{1,2}} = Y'^2_{q_{1,2},l_{1,2}} M^2_{\tilde{Z}'}, \tag{4.79}$$

at the messenger scale. The low energy soft scalar masses are determined by

$$\frac{dm^2_{\tilde{q}_{1,2},\tilde{l}_{1,2}}}{d\ln\mu} \simeq -\frac{Y'^2_{q_{1,2},l_{1,2}}}{2\pi^2} g^2_{Z'} M^2_{\tilde{Z}'} \tag{4.80}$$

because $M_a \sim 10^{-4} M_{\tilde{Z}'} \ll M_{\tilde{Z}'}$ and the Yukawa couplings for the first two families are negligibly small.

Finally, the third generation sfermions and the Higgs doublets are not charged under U(1)$'$ so $m^2_{\tilde{q}_3,\tilde{l}_3,H_{u,d}}(M_{\text{mess}}) = 0$ but obtain soft masses through renormalization group running(Fig. 24):

$$8\pi^2 \frac{dm^2_{\tilde{q}_3}}{d\ln\mu} = y_t^2 P_t + y_b^2 P_b - \left(\frac{16}{3} g_3^2 M_3^2 + 3 g_2^2 M_2^2 + \frac{1}{15} g_1^2 M_1^2\right)$$
$$8\pi^2 \frac{dm^2_{\tilde{t}^c}}{d\ln\mu} = 2 y_t^2 P_t - \left(\frac{16}{3} g_3^2 M_3^2 + \frac{16}{15} g_1^2 M_1^2\right)$$
$$8\pi^2 \frac{dm^2_{\tilde{b}^c}}{d\ln\mu} = 2 y_b^2 P_b - \left(\frac{16}{3} g_3^2 M_3^2 + \frac{4}{15} g_1^2 M_1^2\right)$$

$$
\begin{aligned}
8\pi^2 \frac{dm^2_{\tilde{l}_3}}{d\ln\mu} &= y_\tau^2 P_\tau - \left(3g_2^2 M_2^2 + \frac{3}{5} g_1^2 M_1^2\right) \\
8\pi^2 \frac{dm^2_{\tilde{\tau}^c}}{d\ln\mu} &= 2y_\tau^2 P_\tau - \frac{12}{5} g_1^2 M_1^2 \\
8\pi^2 \frac{dm^2_{H_u}}{d\ln\mu} &= 3y_\tau^2 P_t - \left(3g_2^2 M_2^2 + \frac{3}{5} g_1^2 M_1^2\right) \\
8\pi^2 \frac{dm^2_{H_d}}{d\ln\mu} &= 3y_b^2 P_b + y_\tau^2 P_\tau - \left(3g_2^2 M_2^2 + \frac{3}{5} g_1^2 M_1^2\right)
\end{aligned} \tag{4.81}
$$

where

$$
\begin{aligned}
P_t &= m^2_{\tilde{q}_3} + m^2_{\tilde{t}^c} + m^2_{H_u} + A^{t\,2} \\
P_b &= m^2_{\tilde{q}_3} + m^2_{\tilde{b}^c} + m^2_{H_d} + A^{b\,2} \\
P_\tau &= m^2_{\tilde{l}_3} + m^2_{\tilde{\tau}^c} + m^2_{H_d} + A^{\tau 2}.
\end{aligned} \tag{4.82}
$$

For A-term, $A^{t,b,\tau} = 0$ at the messenger scale, but can be induced through renormalization group running:

$$
\begin{aligned}
8\pi^2 \frac{dA^t}{d\ln\mu} &= 6y_t^2 A^t + y_b^2 A^b - \left(\frac{16}{3} g_3^2 M_3 + 3g_2^2 M_2 + \frac{13}{15} g_1^2 M_1\right) \\
8\pi^2 \frac{dA^b}{d\ln\mu} &= y_t^2 A^t + 6y_b^2 A^b + y_\tau^2 A^\tau - \left(\frac{16}{3} g_3^2 M_3 + 3g_2^2 M_2 + \frac{7}{15} g_1^2 M_1\right) \\
8\pi^2 \frac{dA^\tau}{d\ln\mu} &= 3y_b^2 A^b + 4y_\tau^2 A^\tau - \left(3g_2^2 M_2 + \frac{9}{15} g_1^2 M_1\right)
\end{aligned} \tag{4.83}
$$

In summary, the third generation and the Higgses do not couple to $U(1)'$ gauge superfields directly, so it acquire soft masses indirectly. As the SM gauginos also obtain SUSY breaking in a indirect way and Yukawa interaction gives correction to the sparticle masses through fermion loop where soft masses are not come in, the SUSY breaking effect in the third generation is two more loop suppressed compared to those in the first two generations. More explicitly, taking gaugino-quark loop into account, the first two generation squark soft term is approximately $g^2_{Y'} M^2_{\tilde{Z}'}$ whereas the third generation squark soft term is approximately $g_a^4 g^2_{Y'} M^2_{\tilde{Z}'}$. So loop suppression from gauge couplings makes mass hierarchy between squarks.

Moreover, too heavy the first two generation squarks can make stop mass tachyonic through two loop contribution as dominant terms in renormalization group (RG) equation of stop is given in the form

$$\frac{d}{dt}m_{\tilde{t}}^2 = -8\sum_i \tilde{\alpha}_i C_i^f M_i^2 + 8\Big[\sum_i c_i \tilde{\alpha}^2 C_i^f + \cdots\Big]\tilde{m}_{1,2}^2 \tag{4.84}$$

where C_i^f is the Casimir for stop and $\tilde{\alpha}_i = g_i^2/16\pi^2$. $K-\bar{K}$ mixing determines minimal value of $\tilde{m}_{1,2}^2$ and non-tachyonic condition determines the ratio of stop to heavy squark mass less than 0.2. Then stop should be heavier than 4TeV[77].

Now, let us discuss the spectrum of our model. Since we considered the effective SUSY from broken U(1)$'$ gauge group, we need two scales: SUSY breaking scale and gauge symmetry breaking scale. In the model, messenger scale where SUSY breaking is transferred to flavor dependent U(1)$'$ gauge boson and its superpartner is 10^{14}GeV, U(1)$'$ gauge boson mass is 10^8GeV,and soft mass for gaugino is 10^6GeV. U(1)$'$ gaugino mass would be $10^8+10^6 \simeq 10^8$GeV. For scale between $10^8 \sim 10^{14}$GeV, we have to consider RG running of MSSM and U(1)$'$ gauge boson. Stop is massless at 10^{14}GeV but heavy squark and gaugino masses make stop run to obtain mass. At 10^8GeV where U(1)$'$ gauge boson and gaugino are integrated out. stop is massive at this scale. For scale between $10^8 \sim 10^5$GeV, where 10^5GeV is mass of the heavy squarks, MSSM renormalization group running is applied. Stop mass at 10^8GeV is initial condition. Heavy squarks would affect two loop RG running of stop mass within $10^5 \sim 10^{14}$GeV. Large two loop runnning within this wide range makes stop tachyonic. Threshold effect may alleviate tachyonic catastrophy[78] but it is not enough to make stop nontachyonic. Finally, below heavy squark mass (10^5GeV), only stop RG running is taken into account.

4.4.4 U(1)$'$ charge assignments reflecting flavor structure

We may consider another type of U(1)$'$ charge assignments. Previous model considers U(1)$_{B-L}$ which cancels anomalies within each generation. But such a type of anomaly cancelations is not a dogma, just simplification. We may cancel anomalies within two or more generations.

In this section, we try to relate flavor structure with U(1)$'$ gauge group. From this, it may be possible to construct the model which solves gauge hierarchy problem and flavor problem simultaneously and consistent with experiments. To solve the flavor problem completely, we have to explain both mass hierarchies and mixing matrices, but in this thesis, we mainly concentrate on the structure of mixing matrices only. Actually, the flavor dependent symmetry in supersymmetric model may provide interesting flavor structure. Flavor dependent symmetry can restrict the form of Yukawa matrices before diagonalization. As both the SM matters and their superpartners are U(1)$'$ charged in a flavor dependent way, mixings in the quarks(leptons) have the similar pattern to that of squarks(sleptons). Such restriction from symmetry is the basic reasoning for Minimal Flavor Violation(MFV) hypothesis: "any flavor violation originates from Yukawa structure of the SM"[79].

Consider first the u quark sector. It has global SU(3)$_q\times$SU(3)$_u$ flavor symmetry in the absence of quark masses and flavor dependent U(1)$'$ symmetry. As the third generation is not charged under U(1)$'$, the global family symmetry is broken and only SU(2)$_q\times$SU(2)$_u$ for the first two generations remains. Then the mixing between the first two generation is natural. The same holds in the d quark sector. Then it could explain why mixing of the third generation with the first two generations is much smaller than mixing between the first two generations.

The mixing in the lepton sector is more complicated, since PMNS matrix has large mixing. We can throw this problem away by raising all the slepton masses heavy enough by assigning nonzero charges, but let us suppose that the deviation of muon $g-2$ from the SM value implies sub-TeV slepton mass, say, $\tilde{\nu}_\mu$. Then, the second generation lepton doublet is uncharged under $\mathrm{U}(1)'$. Anomalies are not canceled in each generation, but canceled in the whole matter contents. Also assume that the second generation μ^c superfield is uncharged either. Then, naturally, $O(1)$ coupling can be attached as

$$l_1 \cdot H_d e^c + l_1 \cdot H_d \tau^c + l_2 \cdot H_d \mu^c + l_3 \cdot H_d e^c + l_3 \cdot H_d \tau^c \tag{4.85}$$

From this, the leading term of charged lepton mass matrix is given by

$$\begin{pmatrix} a & 0 & a' \\ 0 & 1 & 0 \\ a' & 0 & a \end{pmatrix} \tag{4.86}$$

which is diagonalized by unitary matrix,

$$U_l = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4.87}$$

If PMNS matrix has the form of

$$V_{\rm PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{4.88}$$

(Solar mixing angle is about $\pi/4$ and no θ_{13}), unitary matrix diagonalizing neutrino mass is given by

$$U_\nu = \begin{pmatrix} \frac{1}{2}+\frac{1}{2\sqrt{2}} & -\frac{1}{2}+\frac{1}{\sqrt{2}} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2}-\frac{1}{2\sqrt{2}} & -\frac{1}{2}-\frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix} \tag{4.89}$$

If neutrino masses have normal hierarchy, so that diagonalized to the form of diag.$(0,0,1)$, the neutrino mass matrix in flavor basis is given by

$$m_\nu = v_u^2 \mathcal{Y} M^{-1} \mathcal{Y}^T \propto \begin{pmatrix} \frac{1}{4} & -\frac{1}{2\sqrt{2}} & -\frac{1}{4} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{4} & \frac{1}{\sqrt{2}} & \frac{1}{4} \end{pmatrix}. \tag{4.90}$$

Suppose heavy neutrinos have U(1)$'$ charges for the second and the third generations, and zero charge for the first generation. Then Yukawa matrix $\mathcal{Y}$ in the seesaw mechanism is of the form

$$\mathcal{Y} = \begin{pmatrix} 0 & a & a \\ 1 & 0 & 0 \\ 0 & a & a \end{pmatrix}. \tag{4.91}$$

Now, U(1)$_{B-L}$ is broken in the Majorana mass M. Introducing superfields Φ_1 with U(1)$_{B-L}$ charge -2, Φ_2 with charge -1. To cancel the anomaly, there should be Φ_1^c and Φ_2^c with charges opposite to those of Φ_1 and Φ_2, respectively. the VEVs of $\Phi_{1,2}$ breaks U(1)$_{B-L}$ and

$$\begin{aligned} & M_1 N_1^c N_1^c + \Phi_1 (N_1^c N_2^c + N_1^c N_3^c + N_2^c N_1^c + N_3^c N_1^c) \\ & + \Phi_2 (N_2^c N_2^c + N_2^c N_3^c + N_3^c N_2^c + N_3^c N_3^c) \end{aligned} \tag{4.92}$$

give the Majorana mass matrix M,

$$M = M_1 \begin{pmatrix} 1 & c & c \\ c & b & d \\ c & d & b \end{pmatrix}. \tag{4.93}$$

with $b \simeq d$. Redefining neutrinos (N_1, N_2, N_3) to$(N_1, N_2, -N_3)$The neutrino mass matrix is proportional to

$$m_\nu = \mathcal{Y} M^{-1} \mathcal{Y}^T = \frac{1}{b+d-2c^2} \begin{pmatrix} 2a^2 & -2ac & -2a^2 \\ -2ac & b+d & 2ac \\ -2a^2 & 2ac & 2a^2 \end{pmatrix}. \tag{4.94}$$

which is very similar to (4.90).

In this way, basic patterns of mixing matrices can be understood in the presence of flavor dependent U(1)$'$ symmetry. Subleading breaking effects would explain deviation of such basic patterns from observed values. Especially, when this breaking is made of VEV with the phase, it will be the source of CP violation in the weak interaction.

4.4.5 Flavor problem in the supersymmetry

In general, new physics can enhance some phenomena which should be suppressed. Flavor changing neutral current(FCNC) and CP violation are such examples. If observations report sizable values of these effects, they may provide hints for new physics. To understand this, calculating both new physics contribution and the SM contribution in exact values are important. Especially, when taking hadron process into account, more exact calculation of the QCD effect is important. Many of such effects are loop suppressed. For CP violation from the phase, single diagram does not tend to show it since overall phase can be absorbed by redefinition

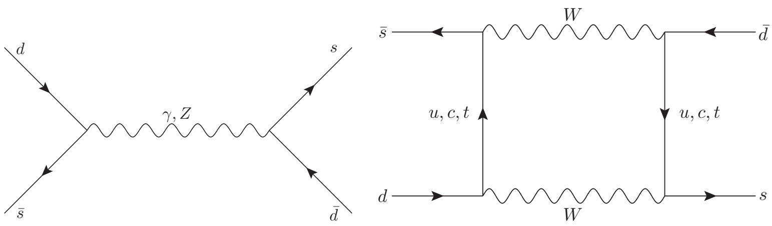

그림 25: FCNC in the SM.

of external fields. Phase effect can be seen from interference with other diagrams and in many cases, loop diagrams should be taken into account. For FCNC effects, leading contribution comes from loop effect. One of the famous example is $K-\bar{K}$ mixing. Tree level diagram like left of Fig. 25 is forbidden because mixing in the neutral current does not appear even we move from the flavor basis to the mass basis $\bar{s}\gamma^\mu d \to \sum_i \bar{s}V_{is}^*\gamma^\mu V_{id}d = \bar{s}\gamma^\mu d$ by unitarity of $V = P_L L_d + P_R R_d$. So, the leading contribution is one-loop box diagram (right of Fig. 25). Aside from loop suppression, it has additional suppression come from unitarity of mixing matrix, known as Glashow-Iliopoulos-Maiani(GIM) mechanism[80]. This diagram contains the factor

$$\Big(\sum_{i=u,c,t} V_{id}^* \frac{1}{\gamma\cdot k - m_i} V_{is}\Big)^2 \tag{4.95}$$

where V is the CKM matrix. Let m_0 be the common mass scale of the virtual quarks. We can express the mass of each quark as $m_i = m_0 + \Delta m_i$ then the factor above is

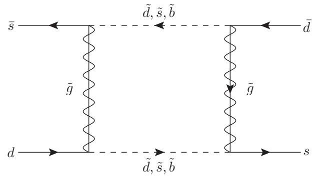

그림 26: FCNC in the SUSY.

written as

$$\begin{aligned}
&\Big(\sum_{i=u,c,t} V_{id}^{*}\frac{1}{\gamma\cdot k-m_i}V_{is}\Big)^2=\Big(\sum_{i=u,c,t} V_{id}^{*}V_{is}\frac{1}{\gamma\cdot k-(m_0+\Delta m_i)}\Big)^2\\
&=\Big[\sum_{i=u,c,t} V_{id}^{*}V_{is}\Big(\frac{1}{\gamma\cdot k-m_0}+\frac{1}{\gamma\cdot k-m_0}\Delta m_i\frac{1}{\gamma\cdot k-m_0}\Big)\Big]^2\\
&=\Big(\sum_{i=u,c,t} V_{id}^{*}V_{is}\frac{1}{\gamma\cdot k-m_0}\Delta m_i\frac{1}{\gamma\cdot k-m_0}\Big)^2.
\end{aligned} \tag{4.96}$$

The leading contribution of order $O((1/m_0)^2)$vanishes by unitarity of the CKM matrix, and subleading order $O((\Delta m_i/m_0^2)^2)$ remains.

For scale lower than M_W, $K-\bar{K}$ mixing is described by the four-Fermi effective operator,

$$\mathcal{L}_{\text{eff}}^{|\Delta S|=2}=C^{|\Delta S|=2}\bar{d}\gamma^{\mu}(1+\gamma_5)s\bar{d}\gamma^{\mu}(1+\gamma_5)s+h.c. \tag{4.97}$$

and $C^{|\Delta S|=2}$ from the SM box diagram is given by

$$C^{|\Delta S|=2}=\frac{g_2^2}{M_W^2}\sum_{i=u,c}\lambda_i^{*}\lambda_j^{*}F^{ij}\Big(\frac{m_{i,j}^2}{M_W^2}\Big), \tag{4.98}$$

where $\lambda_i\equiv V_{id}V_{is}^{*}$. Note that t quark is integrated out as $m_t>M_W$ so we do not consider it. Mixing parameter for $K-\bar{K}$ mixing is $\Delta m_K=(3.483\pm0.006)\times10^{-12}$MeV and the SM estimation explains roughly 80% of it [81]. New physics may introduce sizable mixing effect. For example, extra U(1)$'$ may lead to the tree-level FCNC, with the diagram of the same type as left of Fig. 25. To suppress this, we should impose either heavy Z' gauge boson mass or very small coupling. SUSY also contribute to FCNC process through one loop diagram shown in Fig. 26. We can consider the super-GIM mechanism[82] where virtual squarks contribute,

$$\Big(\sum_{i=\bar{d},\bar{s},\bar{b}} U_{id}^{*} \frac{1}{k^2 - \tilde{m}_i^2} U_{is} \Big)^2 = \frac{1}{k^2 - \tilde{m}_0^2} \Big[\sum_i U_{id}^{*} U_{is} \Delta \tilde{m}_i^2 \Big]^2 . \tag{4.99}$$

Then FCNC from SUSY, $C_{\rm SUSY}^{|\Delta S=2|}$ is given by

$$\frac{g_s^4}{\tilde{m}^6} \Big[\sum_i U_{id}^{*} U_{is} \Delta \tilde{m}_i^2 \Big]^2 \tag{4.100}$$

where $\tilde{m}$ is the typical scale of the squarks and gluino. In the case of gauge mediation though the SM gauge group[83], soft mass is flavor universal. Then in the quark mass basis, squark mass is of the form

$$\begin{pmatrix} m_d^2 + M_0^2 & 0 & 0 & -m_d A' & 0 & 0 \\ 0 & m_s^2 + M_0^2 & 0 & 0 & -m_s A' & 0 \\ 0 & 0 & m_b^2 + M_0^2 & 0 & 0 & -m_b A' \\ -m_d A' & 0 & 0 & m_d^2 + M_0'^2 & 0 & 0 \\ 0 & -m_s A' & 0 & 0 & m_s^2 + M_0'^2 & 0 \\ 0 & 0 & -m_b A' & 0 & 0 & m_b^2 + M_0'^2 \end{pmatrix} \tag{4.101}$$

and diagonalized by

$$U = \begin{pmatrix} c_1 & 0 & 0 & -s_1 & 0 & 0 \\ 0 & c_2 & 0 & 0 & -s_2 & 0 \\ 0 & 0 & c_3 & 0 & 0 & -s_3 \\ s_1 & 0 & 0 & c_1 & 0 & 0 \\ 0 & s_2 & 0 & 0 & c_2 & 0 \\ 0 & 0 & s_3 & 0 & 0 & c_3 \end{pmatrix} \tag{4.102}$$

which mixes left and right handed squarks but does not violate the flavor.

In the case of the effective SUSY, soft term from U(1)$'$ mediation is not flavor universal, so it can have flavor violating effect. However, the first two generation squarks are heavy enough, we can consider the third generation effect only[84]. Moreover, as mixing is similar to that of quark, the first two generation quarks($d\bar{s}$ in K meson) do not mix with the third generation squark too much. So in this case, we can be safe from FCNC problem. The same can hold for $D-\bar{D}$ mixing where $D = \bar{u}c$, composed of the first two generation quarks[85]. So, stringent bound may come from $B-\bar{B}$ mixing.

제 5 장

Flavor Problem in a view of flavor dependent symmetry

In the SM, mass hierarchies and mixing pattern come from the structure of the Yukawa couplings in a flavor basis, *i.e.* undiagonalized basis. But what we know from observations are not sufficient to guess the original forms of the Yukawa couplings in a matrix form. Moreover, the SM does not fix the form of Yukawa couplings due to the flavor universal nature of the SM gauge group. To add new type of flavor dependent symmetry determining flavor structure, we may get motivation outside the flavor physics. One of such example could be flavor dependent $U(1)'$, mediator of SUSY breaking for effective SUSY spectrum. In this chapter, we investigate flavor problem within the realm of flavor physics. For this, we first study the structure of mixing matrix in the quark sector, CKM matrix, focusing on the CP violation in the weak sector and $\lambda = \sin\theta_C$ expansion. They can be interpreted as violation effects from basic pattern provided by some kinds of flavor dependent symmetry. We also consider the PMNS matrix in a parallel way. As an example of such symmetry, we present the structure of the CKM and PMNS matrices from non-Abelian discrete symmetry, D_{12}.

5.1 Structure of the CKM matrix

5.1.1 Parameterizations of the CKM matrix

If we do not assume the fourth or more generations, CKM matrix is 3×3 unitary matrix. Its moduli is measured as

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix} \tag{5.1}$$

It can be parameterized by three mixing angles, Euler angles and one unremovable phase. When firstly suggested by Kobayashi and Maskawa[15], they parameterized CKM matrix as

$$\begin{aligned} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & e^{i\delta'} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & s_3 & -c_3 \end{pmatrix} \\ = & \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta'} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta'} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta'} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta'} \end{pmatrix} \end{aligned} \tag{5.2}$$

On the other hand, the widely used parametrization comes from Chau-Keung, and similarly, by Maiani[17],

$$\begin{aligned} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\phi} \\ 0 & 1 & 0 \\ -s_{13} e^{i\phi} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ = & \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\phi} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\phi} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\phi} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\phi} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\phi} & c_{23} c_{13} \end{pmatrix} \end{aligned} \tag{5.3}$$

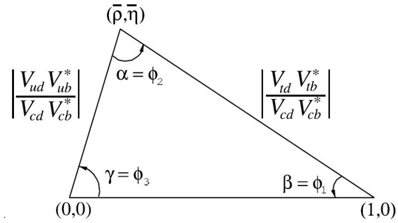

그림 27: Unitary triangle for the CKM matrix [23].

Both are just different parameterizations of the same matrix. Moreover, Such parameterizations do not concern which angle is very large or which angle is negligible. In 1983, Wolfenstein noticed that $|V_{cb}| \sim |V_{us}|^2$. Among three mixing angles, Cabibbo angle θ_C, mixing between the first two generations, is the largest. From them, he expanded CKM matrix in terms of $\lambda = \sin\theta_C = |V_{us}|$[86]. Then,

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \tag{5.4}$$

and these parameterizations correspond to

$$s_1 : s_2 : s_3 = \lambda : 0.75\lambda^2 : 0.31\lambda^2,\ \ s_{12} : s_{23} : s_{13} = \lambda : 0.81\lambda^2 : 0.31\lambda^3. \tag{5.5}$$

Further discussion on parametrization of the CKM matrix can be found in [87].

Measured values with the global fit is given by

$$\begin{aligned} \lambda = 0.2253 \pm 0.0007,\ \ A = 0.808^{+0.022}_{-0.015}, \\ \bar\rho = 0.132^{+0.022}_{-0.014},\ \ \bar\eta = 0.341 \pm 0.013. \end{aligned} \tag{5.6}$$

where $\bar\rho = \rho(1-\lambda^2/2+\cdots)$, $\bar\eta = \eta(1-\lambda^2/2+\cdots)$ come from $\bar\rho + i\bar\eta = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$. The barred parameters $\bar\rho, \bar\eta$ show one special property of the CKM matrix. From unitarity of the CKM matrix, we have

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0. \tag{5.7}$$

Dividing both sides by $V_{cd}V_{cb}^{*}$, we obtain the closed triangle shown in Fig. 27 with $(\bar{\rho},\bar{\eta})$ being the complex vertex. Then the angle $\alpha=(89.0^{+4.4}_{4.2})^{\rm o}$ is very close to $90^{\rm o}$[23]. This is why we call the CP violation in the weak interaction maximal. In commonly used Chau-Keung-Maiani parametrization, $\phi=(67.19^{+2.40}_{-1.76})^{\rm o}$ so maximal CP violation is not apparent. Within the SM framework, this does not matter because the phase can be moved anywhere by phase redefinition of the quarks without affecting phenomena. However, if there is a flavor dependent symmetry so that the Yukawa matrix in the flavor basis is fixed, maximal CP violation can be regarded as an important feature of the Yukawa matrices. In this sense, adopting the parametrization of the CKM matrix with 90^{o} phase can be a good parametrization[88].

Since

$$\alpha\equiv {\rm Arg.}\Big(-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\Big), \tag{5.8}$$

parametrization

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&e^{i\delta}\end{pmatrix}\begin{pmatrix}1&0&0\\0&c_2&s_2\\0&-s_2&c_2\end{pmatrix}\begin{pmatrix}c_1&s_1&0\\-s_1&c_1&0\\0&0&e^{i\delta'}\end{pmatrix}\begin{pmatrix}1&0&0\\0&c_3&-e^{i\delta}s_3\\0&e^{-i\delta}s_3&c_3\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&0&e^{-i\delta}\end{pmatrix}$$
$$=\begin{pmatrix}c_1&s_1c_3&-s_1s_3\\-s_1c_2&c_1c_2c_3+s_2s_3e^{-i\delta}&-c_1c_2s_3+s_2c_3e^{-i\delta}\\e^{i\delta}s_1s_2&-e^{i\delta}c_1s_2c_3+c_2s_3&c_2c_3+c_1s_2s_3e^{i\delta}\end{pmatrix} \tag{5.9}$$

has $\delta = \alpha = 89.0^{\circ}$. With this parametrization, three Euler angles are given by

$$\begin{aligned} \theta_1 &= 13.0^{\circ} = 0.227 \\ \theta_2 &= 2.42^{\circ} = 0.0423 \\ \theta_3 &= 1.54^{\circ} = 0.0276 \end{aligned} \tag{5.10}$$

so angles have hierarchy $\theta_1 = O(\lambda)$, and $\theta_{2,3} = O(\lambda^2)$. Note that unphased part of $V_{td}V_{tb}^*$ given by $-c_1 s_1 s_2^2 s_3 = O(\lambda^7)$ is very small compared to the phased part, $-s_1 c_2 s_2 c_3 = O(\lambda^3)$. Therefore, to a good approximation, the separated phases of V_{ub} and V_{td} are moved and merged to make maximal mixing. This can be seen in the expansion in terms of $\lambda = |V_{us}|$:

$$\begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8}-\frac{\lambda^6}{16}(1+8\kappa_b^2), & \lambda, & \lambda^3\kappa_b\left(1+\frac{\lambda^2}{3}\right) \\ -\lambda+\frac{\lambda^5}{2}(\kappa_t^2-\kappa_b^2), & \begin{array}{l} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8}-\frac{\lambda^6}{16} \\ -\frac{\lambda^4}{2}(\kappa_t^2+\kappa_b^2-2\kappa_b\kappa_t e^{-i\delta}) \\ -\frac{\lambda^6}{12}\left(7\kappa_b^2+\kappa_t^2-8\kappa_t\kappa_b e^{-i\delta}\right) \end{array}, & \begin{array}{l} \lambda^2\left(\kappa_b-\kappa_t e^{-i\delta}\right) \\ -\frac{\lambda^4}{6}(2\kappa_t e^{-i\delta}+\kappa_b) \end{array} \\ -\lambda^3\kappa_t e^{i\delta}\left(1+\frac{\lambda^2}{3}\right), & \begin{array}{l} -\lambda^2\left(\kappa_b-\kappa_t e^{i\delta}\right) \\ -\frac{\lambda^4}{6}(2\kappa_b+\kappa_t e^{i\delta}) \end{array}, & \begin{array}{l} 1-\frac{\lambda^4}{2}(\kappa_t^2+\kappa_b^2-2\kappa_b\kappa_t e^{i\delta}) \\ -\frac{\lambda^6}{6}\left(2[\kappa_b^2+\kappa_t^2]-\kappa_t\kappa_b e^{i\delta}\right) \end{array} \end{pmatrix}. \tag{5.11}$$

Similar expansion making V_{ub} and V_{td} simple was originally suggested by [89]. If $\delta = 0$, there is no CP violation. Moreover, if either κ_b or κ_t vanish, one or more mixing angles vanish. Then, phase can be eliminated by phase redefinitions of the quarks, so CP is not violated.

If we do not consider the phase, Euler angles are just what was used in Kobayashi-Maskawa parametrization. But phase was put differently. In fact, in Kobayashi-Maskawa parametrization, determinant is not unity but $e^{i\delta'}$. However, the phase in determinant can be related to the phase of the quark mass matrix, Arg.Det.M_q. This can be rotated away with the help of the Peccei-Quinn symmetry redefining θ term in $G\tilde{G}$ as $\bar{\theta} = \theta + \text{Arg.Det.}M_q$. So one can start with Arg.Det.$M_q = 0$ and det$V_{CKM} = 1$. In this case, one can see CP violation of the CKM matrix easily. This is discussed in the next section.

5.1.2 Jarlskog determinant

To parameterize CP violation of the mixing matrix, Jalskog suggested the following quantity, Jarlskog determinant[90].

$$J = \left| -\frac{\text{Det.}C}{2(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)} \right| \quad (5.12)$$

where

$$C = -i[M_u M_u^\dagger, M_d M_d^\dagger]. \quad (5.13)$$

So, Jarlskog determinant can be one way of parameterizing CP violation in the quark mass matrices independent of mass eigenvalues[91]. The result is given by

$$J = |\text{Im}V_{km}^* V_{lm} V_{kn} V_{ln}^*| = |\text{Im}V_{mk}^* V_{ml} V_{nk} V_{nl}^*| \quad (5.14)$$

The same quantity also comes from unitarity of the CKM matrix. For example, the unitarity condition implies

$$\begin{aligned} & V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \\ & \to V_{ud}V_{ub}^*V_{td}^*V_{tb} + V_{cd}V_{cb}^*V_{td}^*V_{tb} = -|V_{td}^*V_{tb}|^2 \\ & \to \text{Im}V_{ud}V_{ub}^*V_{td}^*V_{tb} = -\text{Im}V_{cd}V_{cb}^*V_{td}^*V_{tb} \\ & \to |\text{Im}V_{ud}V_{ub}^*V_{td}^*V_{tb}| = |\text{Im}V_{cd}V_{cb}^*V_{td}^*V_{tb}| \end{aligned} \quad (5.15)$$

and this is nothing more than Jarlskog determinant J. The measured value is given by $J = (2.91^{+0.19}_{-0.11}) \times 10^{-5}$ and it is parameterized in Chau-Keung-Maiani parametrization as $J = c_{12}s_{12}c_{13}^2 s_{13}c_{23}s_{23}\sin\delta$. Since $s_{12} = O(\lambda)$, $s_{23} = O(\lambda^2)$, and $s_{13} = O(\lambda^3)$, $J = O(\lambda^6)$.

The Jarlskog determinant is the product of four matrix elements, But when Det$V$ is real *i.e.* equal to one, more simplification can be made. To see this, consider the parametrization (5.9). The Jarlskog determinant is given by $c_1c_2c_3s_1^2s_2s_3\sin\delta$, and it is of order λ^6 as $s_{12} = O(\lambda)$, $s_{23} = O(\lambda^2)$, and $s_{13} = O(\lambda^2)$. This is expected because Jarlskog determinant is unique property of the CKM matrix, and it is independent of parameterizations. One important feature of our parameterizations is that since δ is almost 90^{o} it does not have more suppression. In this sense, Jarlskog determinant is maximal for a given λ^6 order. Also, this parametrization has a unit determinant. To make the determinant real, imaginary parts of the six elements of determinant, product of three matrix elements, cancel with each other. Moreover, each element has the same imaginary number, Jalskog determinant:

$$\begin{aligned}
V_{11}V_{22}V_{33} &= c_1^2c_2^2c_3^2 + c_1^2s_2^2s_3^2 + 2c_1c_2c_3s_2s_3\cos\delta \\
&\quad - c_1c_2c_3s_1^2s_2s_3e^{i\delta} \\
-V_{11}V_{23}V_{32} &= c_1^2c_2^2s_3^2 + c_1^2s_2^2c_3^2 - 2c_1c_2c_3s_2s_3\cos\delta \\
&\quad + c_1c_2c_3s_1^2s_2s_3e^{i\delta} \\
V_{12}V_{23}V_{31} &= s_1^2s_2^2c_3^2 - c_1c_2c_3s_1^2s_2s_3e^{i\delta} \\
-V_{12}V_{21}V_{33} &= s_1^2c_2^2c_3^2 + c_1c_2c_3s_1^2s_2s_3e^{i\delta} \\
V_{13}V_{21}V_{32} &= s_1^2c_2^2s_3^2 - c_1c_2c_3s_1^2s_2s_3e^{i\delta} \\
-V_{13}V_{22}V_{31} &= s_1^2s_2^2s_3^2 + c_1c_2c_3s_1^2s_2s_3e^{i\delta}.
\end{aligned} \tag{5.16}$$

or

$$\begin{aligned}
V_{11}V_{22}V_{33} &= c_{12}^2c_{23}^2c_{13}^2 - c_{12}s_{12}c_{23}s_{23}c_{13}^2s_{13}e^{i\delta}, \\
-V_{11}V_{23}V_{32} &= c_{12}^2s_{23}^2c_{13}^2 + c_{12}s_{12}c_{23}s_{23}c_{13}^2s_{13}e^{i\delta}, \\
V_{12}V_{23}V_{31} &= s_{12}^2s_{23}^2c_{13}^2 - c_{12}s_{12}c_{23}s_{23}c_{13}^2s_{13}e^{i\delta},
\end{aligned}$$

and

$$
\begin{aligned}
-V_{12}V_{21}V_{33} &= s_{12}^2 c_{23}^2 c_{13}^2 + c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13}e^{i\delta}, \\
V_{13}V_{21}V_{32} &= c_{12}s_{12}c_{23}s_{23}s_{13}\cos\delta - c_{12}s_{12}c_{23}s_{23}c_{13}c_{13}^2 s_{13}e^{i\delta} \\
&\quad + s_{12}^2 c_{23}^2 s_{13}^2 + c_{12}^2 s_{23}^2 s_{13}^2, \\
-V_{13}V_{22}V_{31} &- c_{12}s_{12}c_{23}s_{23}s_{13}\cos\delta + c_{12}s_{12}c_{23}s_{23}c_{13}c_{13}^2 s_{13}e^{i\delta} \\
&\quad + c_{12}^2 c_{23}^2 s_{13}^2 + s_{12}^2 s_{23}^2 s_{13}^2.
\end{aligned} \tag{5.17}
$$

Therefore, Jarlskog determinant can be expressed as a product of three matrix elements, and it can be just read off from the imaginary part of one of elements in determinant, for example, $\mathrm{Im}V_{ub}V_{cs}V_{td}$. To see this more explicitly[92], we denote indices for matrix elements as numbers, not quark names, *e.g.* $V_{uc} \equiv V_{12}$. The unit, real determinant condition is written as

$$
\begin{aligned}
1 =& V_{11}V_{22}V_{33} - V_{11}V_{23}V_{32} + V_{12}V_{23}V_{31} \\
& - V_{12}V_{21}V_{33} + V_{13}V_{21}V_{32} - V_{13}V_{22}V_{31}.
\end{aligned} \tag{5.18}
$$

Multiplying $V_{13}^* V_{22}^* V_{31}^*$ on both sides,

$$
\begin{aligned}
V_{13}^* V_{22}^* V_{31}^* =& |V_{22}|^2 V_{11}V_{33}V_{13}^* V_{31}^* - V_{11}V_{23}V_{32}V_{13}^* V_{31}^* V_{22}^* \\
& + |V_{31}|^2 V_{12}V_{23}V_{13}^* V_{22}^* - V_{12}V_{21}V_{33}V_{13}^* V_{31}^* V_{22}^* \\
& + |V_{13}|^2 V_{21}V_{32}V_{31}^* V_{22}^* - |V_{13}V_{22}V_{31}|^2.
\end{aligned} \tag{5.19}
$$

Consider the second term on the RHS, $-V_{11}V_{23}V_{32}V_{13}^* V_{31}^* V_{22}^*$. It contains a factor $V_{32}V_{22}^*$, which is equal to $-V_{31}V_{21}^* - V_{33}V_{23}^*$ by the unitarity of V. Then, $-V_{11}V_{23}V_{32}V_{13}^* V_{31}^* V_{22}^* = V_{11}V_{23}V_{13}^* V_{21}^* |V_{31}|^2 + V_{11}V_{33}V_{13}^* V_{31}^* |V_{23}|^2$. Especially, the second term $V_{11}V_{33}V_{13}^* V_{31}^* |V_{23}|^2$ combines with the first term of Eq. (5.19), $|V_{22}^2| V_{11}V_{33}V_{13}^* V_{31}^*$ to make $(1 - |V_{21}|^2) V_{11}V_{33}V_{13}^* V_{31}^*$. In the same way, for the fourth term on the RHS of Eq. (5.19), $-V_{12}V_{21}V_{33}V_{13}^* V_{31}^* V_{22}^*$ containing the factor $V_{33}V_{31}^* = -V_{23}V_{21}^* - V_{13}V_{11}^*$, can be rewritten as $-V_{12}V_{21}V_{33}V_{13}^* V_{31}^* V_{22}^* =$

$V_{12}V_{23}V_{13}^*V_{22}^*|V_{21}|^2 + V_{12}V_{21}V_{11}^*V_{22}^*|V_{13}|^2$. Here, the first term $-V_{12}V_{23}V_{13}^*V_{22}^*|V_{21}|^2$ combines with the third term on the RHS of Eq. (5.19), $|V_{31}|^2V_{12}V_{23}V_{13}^*V_{22}^*$ to make $(1-|V_{11}|^2)V_{12}V_{23}V_{13}^*V_{22}^*$.

In summary, Eq. (2) can be rewritten as

$$
\begin{aligned}
V_{13}^*V_{22}^*V_{31}^* &= (1-|V_{21}|^2)V_{11}V_{33}V_{13}^*V_{31}^* \\
&+ V_{11}V_{23}V_{13}^*V_{21}^*|V_{31}|^2 + (1-|V_{11}|^2)V_{12}V_{23}V_{13}^*V_{22}^* \\
&+ |V_{13}|^2(V_{12}V_{21}V_{11}^*V_{22}^* + V_{21}V_{32}V_{31}^*V_{22}^*) \\
&- |V_{13}V_{22}V_{31}|^2.
\end{aligned}
\tag{5.20}
$$

Now, the unitarity plays an important role in simplifying this expression. Let the imaginary part of $V_{11}V_{33}V_{13}^*V_{31}^*$ be J. From $V_{11}^*V_{13} + V_{21}^*V_{23} + V_{31}^*V_{33} = 0$, we know $|V_{11}|^2|V_{13}|^2 + V_{11}V_{23}V_{13}^*V_{21}^* + V_{11}V_{33}V_{13}^*V_{31}^* = 0$; so the imaginary part of $V_{11}V_{23}V_{13}^*V_{21}^*$ is $-J$. From $V_{11}V_{31}^* + V_{12}V_{32}^* + V_{13}V_{33}^* = 0$, we have $V_{11}V_{33}V_{13}^*V_{31}^* + V_{12}V_{33}V_{32}^*V_{13}^* + |V_{13}^*V_{33}|^2 = 0$. And, from $V_{12}^*V_{13} + V_{22}^*V_{23} + V_{32}^*V_{33} = 0$, we have $V_{12}V_{33}V_{32}^*V_{13}^* + V_{12}V_{23}V_{22}^*V_{13}^* + |V_{12}^*V_{13}|^2 = 0$. These two combine to show that the imaginary part of $V_{12}V_{23}V_{22}^*V_{13}^*$ is J. On the other hand, from $V_{11}^*V_{12} + V_{21}^*V_{22} + V_{31}^*V_{32} = 0$, we know $V_{21}V_{32}V_{22}^*V_{31}^* + V_{12}V_{21}V_{11}^*V_{22}^* + |V_{21}^*V_{22}| = 0$; so the imaginary part of $(V_{21}V_{32}V_{22}^*V_{31}^* + V_{12}V_{21}V_{11}^*V_{22}^*)$ is zero. Then, the imaginary part of the RHS of Eq. (5.20) is $[(1-|V_{21}|^2) - |V_{31}|^2 + (1-|V_{11}|^2)]J = J$. Therefore, the imaginary part of $V_{13}^*V_{22}^*V_{31}^*$ (the LHS of Eq. (5.20)) is J. Maximality of CP violation in the weak interaction characterized by $\delta = 90^{\mathrm{o}}$ can be visualized in the unitarity triangle. Original definition of Jarlskog determinant is product of four matrix elements, more precisely, two matrix elements and two complex conjugates of matrix elements. In unitarity condition $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, if we do not divide both sides by $V_{cd}V_{cb}^*$, each side of unitaity triangle is composed of one matrix element and one complex conjugate of matrix element. Therefore, the twice of area of the unitarity triangle, or area of the parallelogram is just the Jarlskog determinant. If

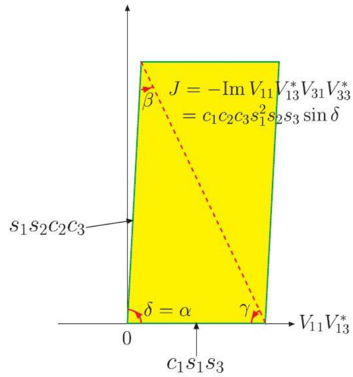

그림 28: Rotated Jarlskog triangle showing maximal CP violation.

the magnitudes of two sides are fixed, the area is maximal when the angle between them is 90°. This is shown in Fig. 28.

5.1.3 Interpretation of the Wolfenstein parametrization

The Wolfenstein's idea that $\lambda = \sin\theta_C$ can be an expansion parameter for the CKM matrix may have a physical interpretation. If we have flavor dependent symmetry, it can restrict the form of the Yukawa matrix. U(1)$'$ in the effective SUSY may be one of examples. Suppose that the basic pattern of the CKM matrix determined from certain symmetry principle is identity. Then expansion of the CKM matrix in terms of λ implies that Yukawa coupling constructed from the symmetry principle has breaking effects parameterized by λ. Suppose we have the scalar ϕ which is the SM singlet but charged under the flavor dependent symmetry. When it has VEV, symmetry is broken and Yukawa matrix has powers of $\langle\phi\rangle/M$ which can be the λ. For realization, one may assign flavor dependent symmetry charges such that couplings $\bar{q}Hu$ is not a singlet of such symmetry so that it has to be coupled to ϕ. So Yukawa coupling has the form of nonrenormalizable term $(\langle\phi\rangle/M)^n\bar{q}Hu$ to make singlet of flavor dependent symmetry. In this case, basic pattern is symmetry breaking effect in the leading order, and it can have λ expansion form. The subleading effects or small explicit breaking effects may be responsible for deviation from the measured values. Moreover, when one of such ϕs has the VEV with the phase, CP phase can be interpreted as a spontaneous breaking effect of flavor dependent symmetry.

The flavor problem of the SM asks two questions: mixing pattern and mass hierarchy. They all come from the Yukawa matrices. Then how can we relate these two aspects of the flavor problem? Writing Yukawa couplings in the form of $(\langle\phi\rangle/M)^n\bar{q}Hu$ is originally come from Froggatt and Nielsen[93], to explain the mass hierarchy in the quark sector by introducing flavor dependent U(1) symmetry. Expressing mass ratios as some powers of λ, mass hierarchies and mixing pattern can be related. Weinberg pointed out the numerical similarity, $\lambda \simeq \sqrt{m_d/m_s}$[94]. With this point of view many efforts have been made to construct the original form of the Yukawa couplings at the GUT scale, texture[95], which produces measured mixing angles and mass hierarchies at the electroweak scale. Especially, it is favored that some elements of the Yukawa coupling at high energy vanish, as forbidden by symmetry principle.

On the other hand, recent observations show that the PMNS matrix, mixing in the lepton sector has large mixing. This is different from mixing pattern of the CKM matrix, very close to identity. To explain this, non-Abelian discrete symmetries have been used[96]. One representative example is tri-bi maximal mixing, suggested by Harrison, Perkins, and Scott[97]:

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \tag{5.21}$$

In fact, similar type of mixing was studied by Pakvasa and Sugawara[98] to explain Cabibbo angle in the CKM matrix and widely used thereafter to explain large mass hierarchy. Suppose the Yukawa matrix in 'democratic form',

$$\frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{5.22}$$

This matrix is not democratic at all in the mass eigenbasis, as it is diagonalized to diag.$(0,0,1)$. The unitary matrix diagonalizing it is

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix} \tag{5.23}$$

where $\omega = \exp(i2\pi/3)$, the solution to the equation $\omega^2+\omega+1=0$. Since eigenvalues in the first two generations are degenerated, we can rotate them freely. When it is combined with maximal mixing, 45^o rotation, it becomes

$$V_0 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{\frac{2}{3}} & 0 & \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{6}} & -\frac{i}{\sqrt{2}} & \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{6}} & -\frac{i}{\sqrt{2}} & \sqrt{\frac{1}{3}} \end{pmatrix}. \tag{5.24}$$

Suppose $u-$, $d-$ quark sector and charged lepton sector have a such structure. For CKM matrix, since $L_u = L_d = V_0$, the CKM matrix is identity in the leading order. Deviations from identity in the quark sector are parameterized by λ, explaining the first two generation mass hierarchy and λ expansion in the Wolfenstein parametrization. On the other hand, if neutrino sector is diagonalized with basis changing matrix, PMNS matrix has a tri-bi maximal mixing.

However, tri-bi maximal mixing may be irrelevant for mass hierarchy. The unitary matrix (5.23) diagonalizes the mass matrix with the permutation structure

$$\begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix} \tag{5.25}$$

to diag.$(a+\omega b+\omega^* b^*, a+\omega b^*+\omega^* b, a+b+c)$. As this mass matrix has three independent real numbers, mass hierarchy may not be considered. Only permutation pattern matters. Maximal mixing also irrelevant for the mass hierarchy. 45^{o} mixing diagonalizes the mass matrix of the form

$$\begin{pmatrix} a & b & 0 \\ b & a & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5.26}$$

Then the sameness of diagonal and that of off-diagonal do matter but mass hierarchy may not be imposed. In fact, U(1)$'$ symmetry in effective SUSY does not consider the mass hierarchy too much either. Many discrete symmetry model buildings on the PMNS matrix mainly focuses on the permutation pattern of the Yukawa matrices and mass hierarchy is not an important issue. We will see an example in the next section.

5.2 Quark and Lepton Mixings from discrete D_{12} symmetry

Whereas tri-bi maximal pattern mainly concerns the permutation structure of the Yukawa matrix, our example here considers breaking of the discrete symmetry with phase. It predicts the basic structure of the PMNS matrix as

$$V_{\text{PMNS}} = \begin{pmatrix} \cos\frac{\pi}{6} & \sin\frac{\pi}{6} & 0 \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{6} & \frac{1}{\sqrt{2}}\cos\frac{\pi}{6} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{6} & \frac{1}{\sqrt{2}}\cos\frac{\pi}{6} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{5.27}$$

On the other hand, we try to explain one sizable angle in the CKM matrix, Cabibbo angle. So, basic structure of the CKM matrix is given by

$$V_{\text{CKM}} = \begin{pmatrix} \cos\frac{\pi}{12} & \sin\frac{\pi}{12} & 0 \\ \sin\frac{\pi}{12} & \cos\frac{\pi}{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.28}$$

We set the Cabibbo angle by 15^{o}, slightly different from measured value 13^{o}. This comes from hypothetical relation, so called quark-lepton complementarity

$$\theta_{\text{sol}} + \theta_C \simeq 45^{\text{o}}. \tag{5.29}$$

which states that the sum of the corresponding angles (the mixing between the first and the second generations here) in the CKM and the PMNS matrix is 45^{o}[99]. To obtain these patterns, we employ dihedral group D_{12} as a flavor dependent symmetry[100].

5.2.1 Properties of dihedral group D_{12} and breaking pattern

The dihedral group D_{2N} represents the symmetry of a regular polygon of $2N$ sides. Its properties are:

1. It is isomorphic to $Z_{2N} \rtimes Z_2$ (cyclic rotation + reflection).

2. It is generated by two generators a and b,

$$\begin{aligned} a &: (x_1, x_2, \cdots, x_{2N}) \to (x_{2N}, x_1, \cdots, x_{2N-1}) \\ b &: (x_1, x_2, \cdots, x_{2N}) \to (x_1, x_{2N}, \cdots, x_2) \end{aligned} \tag{5.30}$$

which satisfies

$$a^{2N} = e, \;\; b^2 = e, \;\; bab = a^{-1}. \tag{5.31}$$

3. Its irreducible representations are

$$\begin{aligned} &\text{Four singlets} : 1_{++}, 1_{--}, 1_{+-}, 1_{-+} \\ &(N-1) - \text{doublets} : 2_k (k = 1, \cdots, N-1) \end{aligned} \tag{5.32}$$

For a (complex) 2_k doublet basis, a and b are represented by

$$a = \begin{pmatrix} e^{2\pi ik/2N} & 0 \\ 0 & e^{-2\pi ik/2N} \end{pmatrix}, \;\; b = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{5.33}$$

For a (complex) 1_{ij} singlet basis, i is the eigenvalue of b and j is the eigenvalue of ab.

4. Tensor products satisfy the following.

- Singlet times singlet multiplication,

$$1_{s_1 s_2} \times 1_{s'_1 s'_2} = 1_{s''_1 s''_2} \tag{5.34}$$

where $s''_1 = s_1 s'_1$ and $s''_2 = s_2 s'_2$.

- Singlet times doublet multiplication,

$$(w)(\mathbf{1_{++}}) \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}(\mathbf{2_k}) = \begin{pmatrix} wx_1 \\ wx_2 \end{pmatrix}(\mathbf{2_k}),\ (w)(\mathbf{1_{--}}) \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}(\mathbf{2_k}) = \begin{pmatrix} wx_1 \\ -wx_2 \end{pmatrix}(\mathbf{2_k}), \tag{5.35}$$

$$(w)(\mathbf{1_{+-}}) \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}(\mathbf{2_k}) = \begin{pmatrix} wx_2 \\ wx_1 \end{pmatrix}(\mathbf{2_k}), (w)(\mathbf{1_{-+}}) \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}(\mathbf{2_k}) = \begin{pmatrix} wx_2 \\ -wx_1 \end{pmatrix}(\mathbf{2_k}). \tag{5.36}$$

where the boldface symbols inside the brackets show the D_{2N} representations.

- Doublet times doublet multiplication,

(a) For $k+k' \neq N$ and $k-k' \neq 0$,

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}(\mathbf{2_k}) \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(\mathbf{2_{k'}}) = \begin{pmatrix} x_1y_1 \\ x_2y_2 \end{pmatrix}(\mathbf{2_{k+k'}}) + \begin{pmatrix} x_1y_2 \\ x_2y_1 \end{pmatrix}(\mathbf{2_{k-k'}}). \tag{5.37}$$

(b) For $k+k' = N$ and $k-k' \neq 0$,

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}(\mathbf{2_k}) \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(\mathbf{2_{k'}}) = (x_1y_1+x_2y_2)(\mathbf{1_{+-}}) + (x_1y_1-x_2y_2)(\mathbf{1_{-+}}) + \begin{pmatrix} x_1y_2 \\ x_2y_1 \end{pmatrix}(\mathbf{2_{k-k'}}) \tag{5.38}$$

(c) For $k+k' \neq N$ and $k-k' = 0$, (which will be frequently used)

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}(\mathbf{2_k}) \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(\mathbf{2_{k'}}) = (x_1y_2+x_2y_1)(\mathbf{1_{++}}) + (x_1y_2-x_2y_1)(\mathbf{1_{--}}) + \begin{pmatrix} x_1y_1 \\ x_2y_2 \end{pmatrix}(\mathbf{2_{k+k'}}). \tag{5.39}$$

(d) For $k+k' = N$ and $k-k' = 0$,

$$\begin{aligned} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}(\mathbf{2_k}) \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(\mathbf{2_{k'}}) =& (x_1y_2+x_2y_1)(\mathbf{1_{++}}) + (x_1y_2-x_2y_1)(\mathbf{1_{--}}) \\ &+ (x_1y_1+x_2y_2)(\mathbf{1_{+-}}) + (x_1y_1-x_2y_2)(\mathbf{1_{-+}}). \end{aligned} \tag{5.40}$$

When D_{2N} charged field has VEV, it is spontaneously broken. For a D_{2N} doublet, suppose that the VEV is chosen as

$$\langle H(2_k)\rangle \sim \begin{pmatrix} e^{\frac{-2\pi i}{2N}km} \\ 1 \end{pmatrix}. \tag{5.41}$$

Note that $\langle H(2_k)\rangle$ is the eigenvector of ba^m with eigenvalue 1, and hence it is still invariant under the action of ba^m. Therefore, by the VEV of Eq. (5.41) D_{2N} is broken down to the smaller group generated by ba^m. Since $(ba^m)^2 = 1$, the remaining group should have a subgroup Z_2 generated by ba^m. The symmetry breaking pattern for this vacuum choice is as follows:

- When j divides $2N$ ($m = 0, 1, \cdots \frac{2N}{j} - 1$), D_{2N} is broken down to

$$D_{2N} \xrightarrow{\mathbf{2}_j} D_j = \langle a^{2N/j}, ba^m\rangle. \tag{5.42}$$

  Note that $a^{2N/j}$ generates Z_j since $(a^{2N/j})^j = 1$. Therefore, the group generated by $a^{2N/j}, ba^m$ is $Z_j \rtimes Z_2 = D_j$.

- When j does not divide $2N$ ($m = 0, 1, \cdots, 2N-1$), D_{2N} is broken down to

$$D_{2N} \xrightarrow{\mathbf{2}_j} Z_2 = \langle ba^m\rangle \tag{5.43}$$

- A successive application of doublet VEVs lead to (a) When k divides j with $m_j = m_k$,

$$D_{2N} \xrightarrow{\mathbf{2}_j} D_j \xrightarrow{\mathbf{2}_k} D_k. \tag{5.44}$$

(b) When k does not divide j with $m_j = m_k$,

$$D_{2N} \xrightarrow{\mathbf{2}_j} D_j \xrightarrow{\mathbf{2}_k} Z_2. \tag{5.45}$$

Of course, one can choose an arbitrary value for the VEV, and [101] lists all the possible symmetry breaking patterns and the resulting subgroups.

5.2.2 Model for the CKM matrix

To obtain appropriate structure of the Yukawa matrices for observed the CKM and the PMNS matrices, we have to assign D_{12} charges to the quarks and the leptons. Moreover, the Higgs may be charged, but in this case, too many Higgses in the different representation of the D_{12} group are required. Many neutral Higgses give rise to FCNC problem[102], but FCNC from Yukawa coupling is very small as long as the top quark is not taken into account. Instead, we may assume that the Higgs doublet is not charged and introduce scalars in the Froggatt-Nielsen scheme. They have VEVs suppressed by their mass scale explaining the Yukawa couplings. Such scalars are called 'flavons'.

In our case, we consider the multi-Higgs case. Higgses are charged under D_{12} as

$$H_0^u : \mathbf{1}_{++}, \begin{pmatrix} H_1'^u \\ H_2'^u \end{pmatrix} : \mathbf{2}_1, \begin{pmatrix} H_1''^u \\ H_2''^u \end{pmatrix} : \mathbf{2}_3 \tag{5.46}$$

$$H_0^d : \mathbf{1}_{++}, H_0'^d : \mathbf{1}_{++}, \begin{pmatrix} H_1'^d \\ H_2'^d \end{pmatrix} : \mathbf{2}_2 \tag{5.47}$$

$$H_0^l : \mathbf{1}_{++}, H_0'^l : \mathbf{1}_{++}, \begin{pmatrix} H_1'^l \\ H_2'^l \end{pmatrix} : \mathbf{2}_2. \tag{5.48}$$

For H^ls to couple to leptons but not to quarks and for H_0^d to couple to quarks but not to leptons, we can introduce a leptonic Z_3 discrete symmetry such that charged singlet leptons, lepton doublets and H^ls carry Z_3 quantum number 1 and all the other fields, except the singlet neutrinos, carry Z_3 quantum number 0. Moreover, H_u and H_d are distinguished by their different $U(1)_Y$ quantum numbers. To avoid unwanted H_u and H_d mixing, we can assign U(1) PQ symmetry, as will be seen later. Note that we have not introduces following Higgses which mix the D_{12} doublet and singlet fermions:

$$\begin{pmatrix} H_1^u \\ H_2^u \end{pmatrix} : \mathbf{2}_1, \ \begin{pmatrix} H_1^d \\ H_2^d \end{pmatrix} : \mathbf{2}_1, \ \begin{pmatrix} H_1^l \\ H_2^l \end{pmatrix} : \mathbf{2}_1. \tag{5.49}$$

Even though we write some couplings with the fields of (5.49) below, we will eventually set those entries zero, either by not introducing the lowest order D_{12} representations as above or by assuming their vanishing VEVs.

For quark sector, we assign D_{12} charges as follows:

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} : \mathbf{2}_1, \ \ Q_3 : \mathbf{1}_{++}$$

$$\begin{pmatrix} u^c \\ c^c \end{pmatrix} : \mathbf{2}_2, \ \ t^c : \mathbf{1}_{++}, \ \ \begin{pmatrix} d^c \\ s^c \end{pmatrix} : \mathbf{2}_1, \ \ b^c : \mathbf{1}_{++} \tag{5.50}$$

The tensor product of $Q_3(\mathbf{1}_{++}) \times t^c(\mathbf{1}_{++})$ implies that it can couple to $H_0^u(\mathbf{1}_{++})$, leading to the coupling, viz. Eq. (5.39),

$$y_1^u H_0^u \bar{t}_L t_R \tag{5.51}$$

where y_1^u is the Yukawa coupling constant.

On the other hand, since $\mathbf{2}_2$ Higgs does not exist,

$$Q_3(\mathbf{1}_{++}) \times \begin{pmatrix} u^c \\ c^c \end{pmatrix} (\mathbf{2}_2)$$

cannot make D_{12} singlet, but

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} (\mathbf{2}_1) \times t^c(\mathbf{1}_{++})$$

can couple to

$$\begin{pmatrix} H_1^u \\ H_2^u \end{pmatrix} (\mathbf{2}_1).$$

So, we consider the coupling

$$y_3^u(H_2^u \bar{u}_L t_R + H_1^u \bar{c}_L t_R) \tag{5.52}$$

where we used Eq. (5.39). Consideration of

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} (\mathbf{2}_1) \times \begin{pmatrix} u^c \\ c^c \end{pmatrix} (\mathbf{2}_2)$$

allows its coupling, via Eq. (5.39), to

$$\begin{pmatrix} H_1^{\prime u} \\ H_2^{\prime u} \end{pmatrix} (\mathbf{2}_1) \text{ and } \begin{pmatrix} H_1^{\prime\prime u} \\ H_2^{\prime\prime u} \end{pmatrix} (\mathbf{2}_3),$$

i.e. the following Yukawa coupling

$$y_4^u(H_1^{\prime u} \bar{u}_L c_R + H_2^{\prime u} \bar{c}_L u_R) + y_5^u(H_2^{\prime\prime u} \bar{u}_L u_R + H_1^{\prime\prime u} \bar{c}_L c_R). \tag{5.53}$$

These couplings are summarized by the following up mass matrix

$$M^{(u)} = \begin{pmatrix} y_5^u H_2^{\prime\prime u} & y_4^u H_1^{\prime u} & y_3^u H_2^u \\ y_4^u H_2^{\prime u} & y_5^u H_1^{\prime\prime u} & y_3^u H_1^u \\ 0 & 0 & y_1^u H_0^u \end{pmatrix} \tag{5.54}$$

One can construct a desirable mixing matrix by taking the zero VEV of $(H_1^u, H_2^u)^T$, which represents $(\mathbf{2}_1 - \mathbf{1}_{++})$ quark mixing if not vanished. One may also think of it as $(H_1^u, H_2^u)^T$ Higgs is forbidden by some kinds of symmetry. That means, $\mathbf{1}_{++}$ and $\mathbf{2}_1$ quarks are completely separated.

The D_{12} symmetry is broken down to a smaller symmetry generated by b, by assigning the VEVs as

$$\begin{aligned} \begin{pmatrix} H_1^u \\ H_2^u \end{pmatrix} (\mathbf{2}_1) = v_u \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} H_1^u \\ H_2^u \end{pmatrix} (\mathbf{2}_2) = v_u' \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \\ y_4^u \begin{pmatrix} H_1^{\prime u} \\ H_2^{\prime u} \end{pmatrix} (\mathbf{2}_1) = w_u \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \\ y_5^u \begin{pmatrix} H_1^{\prime\prime u} \\ H_2^{\prime\prime u} \end{pmatrix} (\mathbf{2}_3) = z_u \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \\ y_1^u H_0^u = x_u. \end{aligned} \tag{5.55}$$

Not introducing Eq. (5.49) is equivalent to setting $v_u = 0$ and $v_u' = 0$ in the mass matrix, and we consider only $\mathbf{2}_2$ vacuum and D_{12} is then broken down to D_2 generated by a^6 and ba^6, where a and b are generators of D_{12} defined in Appendix. Thus, the mass matrix becomes

$$M^{(u)} = \begin{pmatrix} w_u & z_u & 0 \\ z_u & w_u & 0 \\ 0 & 0 & x_u \end{pmatrix} \tag{5.56}$$

which is diagonalized by the following unitary matrix,

$$U_u = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{-\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.57}$$

Then, the mass eigenvalues appear as

$$\begin{aligned} \tilde{M}^{(u)2} &= U_u(M^{(u)}M_u^{(u)\dagger})U_u^\dagger \\ &= \begin{pmatrix} (w_u - z_u)^2 & 0 & 0 \\ 0 & (w_u + z_u)^2 & 0 \\ 0 & 0 & x_u^2 \end{pmatrix} \end{aligned} \tag{5.58}$$

which allow three independent mass values for the u, c, and t quarks. Calculating the down type quark Yukawa couplings in the same way, we obtain

$$M^{(d)} = \begin{pmatrix} y_5^d H_2'^d & y_4^d H_0'^d & y_3^d H_2^d \\ y_4^d H_0'^d & y_5^d H_1'^d & y_3^d H_1^d \\ y_2^d H_2^d & y_2^d H_1^d & y_1^d H_0^d \end{pmatrix} \tag{5.59}$$

The D_{12} symmetry is broken down to a D_2 generated by ba and a^6, by assigning VEVs (for $v_d = 0$) as

$$\begin{aligned} \begin{pmatrix} H_1^d \\ H_2^d \end{pmatrix} (\mathbf{2}_1) &= v_d \begin{pmatrix} \mathrm{e}^{-i\phi} \\ 1 \end{pmatrix}, \\ y_5^d \begin{pmatrix} H_1'^d \\ H_2'^d \end{pmatrix} (\mathbf{2}_2) &= w_d \begin{pmatrix} \mathrm{e}^{-2i\phi} \\ 1 \end{pmatrix}, \\ y_1^d H_0^d = x_d, &\ \ y_4^d H_0'^d = z_d \end{aligned} \tag{5.60}$$

where we choose $\phi = \frac{2\pi}{12}$, the smallest angle with the dodeca-symmetry. Not introducing Eq. (5.49) is equivalent to setting $v = 0$ in the mass matrix, and we obtain the following d quark mass matrix,

$$M^{(d)} = \begin{pmatrix} w_d & z_d & 0 \\ z_d & w_d e^{-2i\phi} & 0 \\ 0 & 0 & x_d \end{pmatrix} \tag{5.61}$$

which is diagonalized by the unitary matrix

$$U_d = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}e^{i\phi} & 0 \\ -\frac{1}{\sqrt{2}}e^{-i\phi} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5.62}$$

Then, the diagonalized mass matrix squared becomes

$$\begin{aligned} \tilde{M}^{(d)2} &= U_d(M^{(d)}M^{(d)\dagger})U_d^\dagger \\ &= \begin{pmatrix} w_d^2 + z_d^2 - 2w_d z_d \cos\phi & 0 & 0 \\ 0 & w_d^2 + z_d^2 + 2w_d z_d \cos\phi & 0 \\ 0 & 0 & x_d^2 \end{pmatrix} \end{aligned} \tag{5.63}$$

Then, The CKM mixing matrix becomes

$$V_{\mathrm{CKM}} = U_u U_d^\dagger = \begin{pmatrix} e^{-i\phi/2}\cos\frac{\phi}{2} & ie^{i\phi/2}\sin\frac{\phi}{2} & 0 \\ ie^{-i\phi/2}\sin\frac{\phi}{2} & e^{i\phi/2}\cos\frac{\phi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.64}$$

Note that the (11) element of V_{CKM} gives the Cabibbo angle $\theta_C = \frac{\phi}{2} = 15^\circ$.

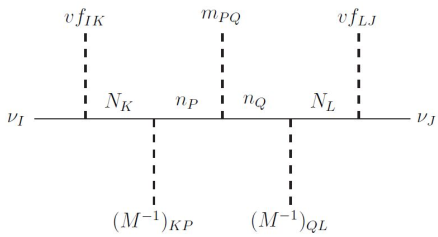

그림 29: Double seesaw mechanism. Adopted from J. E. Kim and J. -C. Park in [103].

5.2.3 Double seesaw mechanism model for the PMNS matrix

To obtain the PMNS matrix, the seesaw mechanism should be used. Here, we employ the special type, so called double seesaw mechanism. For this, we introduce two kinds of heavy neutrinos, (n_1, n_2, n_3) and (N_1, N_2, N_3). In this double seesaw mechanism, the Dirac flavor structure is screened in the neutrino mass matrix, and hence the light-neutrino mass matrix becomes directly proportional to a heavy-neutrino (n) mass matrix.

In the following renormalizable Yukawa couplings

$$f_{IJ}^{(lN)} N^I H^{\nu N} L^J + f_{IJ}^{(Nn)} N^I n^J S^{nN} + f_{IJ}^{(nn)} n^I n^J S^n, \tag{5.65}$$

we require the condition $f_{IJ}^{(lN)} \propto f_{IJ}^{(Nn)}$. Such an (almost) exact proportionality could arise in the context of GUT[103]. Suppose L_i and n_i belong to the same multiplet of a larger gauge group, say, F_1, and $H^{\nu N}$ and S^{nN} belong to the same multiplet, say S. Let F_2 be the multiplet to which N neutrinos belong. Then both $f_{IJ}^{(lN)} N^I H^{\nu N} L^J$ and $f_{IJ}^{(Nn)} N^I n^J S^{nN}$ come from the same interaction, SF_1F_2, with a common coupling constant. If the see-saw scale is at the high energy scale so that the splitting of couplings are not so large, then $f_{IJ}^{(lN)}$ is almost the same as $f_{IJ}^{(Nn)}$. For example, in the SU(6) GUT model[104], one of right handed neutrino (n in this case) and lepton doublet belong to the same representation, say $\overline{\mathbf{6}}^{M}$, another right handed neutrino N is an SU(6) singlet and S belongs to $\mathbf{6}^S$ representation. Then, the first two terms in Eq. (5.65) have the same origin, $f(\bar{\mathbf{6}}^M N \mathbf{6}^S)$. When SU(6) is broken down to SU(5)$\times$U(1), splitting of the coupling f into f^{lN} and f^{Nn} occurs, at the order of $\frac{f^2}{16\pi^2}\ln(\frac{M_{see\ saw}}{M_{GUT}})$. Supposing $M_{see\ saw} \sim 10^{14}$ GeV , $M_{GUT} \sim 10^{16}$ GeV, and $f \sim O(1)$ then the splitting effect is about 0.03, *i.e.* only 3 per cent. On the other hand, we can also construct a term $\mathbf{15}^M \bar{\mathbf{6}}^M \bar{\mathbf{6}}^H$ to form the Yukawa coupling. As splitting $\bar{\mathbf{6}}^M \to \bar{\mathbf{5}}^M + n$ occurs, we obtain various terms where n couples to the SM matter as well as to the as-yet-unobserved massive particles. Since the Yukawa coupling of the SM particles (in the SU(5) language, $y(\mathbf{10}^M \bar{\mathbf{5}}^M \bar{\mathbf{5}}^H)$) should be present, it might be hard to prevent all these terms toward the screening in the double see-saw mechanism. But even in this case, the coupling y could be much smaller than f since $y < O(10^{-2})$, and the screening effects in double see-saw mechanism is a very good approximation. For example, the τ lepton mass is about 1.8 GeV at electroweak scale and therefore its Yukawa coupling is about 10^{-2}. Since the RG equation of each Yukawa coupling is proportional to the Yukawa coupling itself, we expect that the correction from unified Yukawa coupling is small, $\frac{y^2}{16\pi^2}\ln(\frac{M_{EW}}{M_{GUT}}) \sim O(10^{-2} - 10^{-3})$, which means that y is still much smaller than the $O(1)$ coupling f even at the GUT scale.

We give the following D_{12} assignments for the SM leptons,

$$
\begin{aligned}
L_1 : \mathbf{1}_{++}, \quad & \begin{pmatrix} L_2 \\ L_3 \end{pmatrix} : \mathbf{2}_1 \\
e^c : \mathbf{1}_{++}, \quad & \begin{pmatrix} \mu^c \\ \tau^c \end{pmatrix} : \mathbf{2}_1
\end{aligned}
\tag{5.66}
$$

For the heavy-neutrinos whose mass matrix is proportional to the light-neutrino mass matrix, we assign

$$\begin{pmatrix} n_1 + in_2 \\ n_1 - in_2 \end{pmatrix} : \mathbf{2}_2 \;\; n_3 : \mathbf{1}_{++} \; . \tag{5.67}$$

Note that we combined two Majorana neutrinos to make a complex field required for a doublet representation of D_{12}. We need not specify the representation content of N_i if it applies to the double see-saw mechanism.

For charged lepton masses, we use the Higgs doublets presented in Eq. (5.47). Then, the mass matrix of charged leptons is given by

$$M^{(l)} = \begin{pmatrix} y_1^l H_0^l & y_2^l H_2^l & y_2^l H_1^l \\ y_3^l H_2^l & y_5^l H_2^{\prime l} & y_4^l H_0^{\prime l} \\ y_3^l H_1^l & y_4^l H_0^{\prime l} & y_5^l H_1^{\prime l} \end{pmatrix} \tag{5.68}$$

The D_{12} symmetry is broken down to D_2, generated by a^6 and ba^6, by assigning the VEVs as

$$\begin{pmatrix} H_1^l \\ H_2^l \end{pmatrix} (\mathbf{2}_1) = v_l \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \;\; y_5^l \begin{pmatrix} H_1^{\prime l} \\ H_2^{\prime l} \end{pmatrix} (\mathbf{2}_2) = w_l \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{5.69}$$
$$y_1^l H_0^l = x_l, \;\; y_4^l H_0^{\prime l} = z_l.$$

Note that we introduced H^l's which are different from H^d's. Not introducing Eq. (5.49) is equivalent to setting $v = 0$ in the mass matrix, and the $\mathbf{1}_{++}$ lepton and the $\mathbf{2}_1$ leptons are not mixed,

$$M^{(l)} = \begin{pmatrix} x_l & 0 & 0 \\ 0 & w_l & z_l \\ 0 & z_l & w_l \end{pmatrix} . \tag{5.70}$$

The charged lepton mass squared, $M_l M_l^\dagger$, is diagonalized by

$$U_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{5.71}$$

In models with the screening of the Dirac flavor structure in the neutrino mass matrix, the light neutrino mass matrix is assumed to be proportional to the heavy n neutrino mass matrix, $M^{(\nu)} \propto M^{(n)}$. So the number of heavy Majorana neutrinos n is the same as that of the SM doublet neutrinos ν. The SM singlet neutrinos n are required to obtain masses by the VEVs of SM singlet Higgs fields S. So, the needed SM singlet Higgs fields S is

$$\begin{gathered} S_0^n : \mathbf{1}_{++}, \;\; S_0^{\prime n} : \mathbf{1}_{++}, \\ \begin{pmatrix} S_1^n \\ S_2^n \end{pmatrix} : \mathbf{2}_1, \begin{pmatrix} S_1^{\prime n} \\ S_2^{\prime n} \end{pmatrix} : \mathbf{2}_4 \end{gathered} \tag{5.72}$$

To forbid S to couple to charged leptons or quarks, we need to assign Z_3 quantum number as stated. Therefore, S and n neutrinos have Z_3 quantum number -1.

Now, the neutrino mass matrix can be written as

$$M^{(\nu)} = \begin{pmatrix} y_4^n 2S_0^{\prime n} + y_5^n(S_1^{\prime n} + S_2^{\prime n}) & iy_5^n(S_2^{\prime n} - S_1^{\prime n}) & y_3^n(S_1^n + S_2^n) \\ iy_5^n(S_2^{\prime n} - S_1^{\prime n}) & y_4^n 2S_0^{\prime n} - y_5^n(S_1^{\prime n} + S_2^{\prime n}) & iy_3^n(S_2^n - S_1^n) \\ y_2^n(S_2^n + S_1^n) & iy_2^n(S_2^n - S_1^n) & y_1^n S_0^n \end{pmatrix} \tag{5.73}$$

We require that the D_{12} symmetry is broken down to D_2 generated by a^3 and *ba* (for $v_n = 0$)

$$\begin{pmatrix} S_1^n \\ S_2^n \end{pmatrix} (\mathbf{2}_1) = v_n \begin{pmatrix} \mathrm{e}^{-i\phi/2} \\ \mathrm{e}^{i\phi/2} \end{pmatrix},$$

$$y_5^n \begin{pmatrix} S_1'^n \\ S_2'^n \end{pmatrix} (\mathbf{2}_4) = w_n \begin{pmatrix} \mathrm{e}^{-i\phi} \\ \mathrm{e}^{i\phi} \end{pmatrix}, \tag{5.74}$$

$$y_1^n S_0^n = x_n, \;\; y_4^n S_0'^n = z_n$$

where $\phi = \frac{2\pi}{12} \times 2$. Also, taking $v = 0$, we obtain

$$M^{(\nu)} = \begin{pmatrix} 2(z_n + w_n \cos\phi) & -2w_n \sin\phi & 0 \\ -2w_n \sin\phi & 2(z_n - w_n \cos\phi) & 0 \\ 0 & 0 & x_n \end{pmatrix} \tag{5.75}$$

which is diagonalized by

$$U_\nu = \begin{pmatrix} \cos\frac{\phi}{2} & -\sin\frac{\phi}{2} & 0 \\ \sin\frac{\phi}{2} & \cos\frac{\phi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5.76}$$

$$\tilde{M}^{(\nu)} = U_\nu M^{(\nu)} U_\nu^\dagger = \begin{pmatrix} 2(z_n + w_n) & 0 & 0 \\ 0 & 2(z_n - w_n) & 0 \\ 0 & 0 & x_n \end{pmatrix}. \tag{5.77}$$

The three independent neutrino masses can be fitted to the observed neutrino mass ratios from the neutrino oscillation data.

Therefore, the PMNS matrix is calculated as

$$V_{\text{PMNS}} = U_l U_\nu^\dagger = \begin{pmatrix} \cos\frac{\pi}{6} & \sin\frac{\pi}{6} & 0 \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{6} & \frac{1}{\sqrt{2}}\cos\frac{\pi}{6} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{6} & \frac{1}{\sqrt{2}}\cos\frac{\pi}{6} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{5.78}$$

5.2.4 Vacuum stability in D_{12} breaking

The vacuum choices for desired quark and lepton mixing angles must be consistent with the Higgs potential. Couplings between Higgs and their complex conjugates are restricted by $SU(2)_L \times U(1)_Y \times U(1)_\Gamma \times Z_3 \times D_{12}$ where $U(1)_\Gamma$ is the PQ symmetry and Z_3 is the leptonic one discussed below Eq. (5.48). For example, by the $U(1)_Y$ symmetry, $H_u H_d$ and $(H_u H_d^\dagger)(H_u^\dagger H_d)$ are allowed, whereas $(H_u H_d^\dagger)^2$ is forbidden.

In Higgs potential, the most problematic terms are those containing D_{12} doublets H'^d, S^n, and S'^n, which have non-trivial phases so that we have to verify whether our phase choice is not spoiled. By imposing another symmetry such as the PQ symmetry or a Z_2 symmetry, we can forbid the unwanted terms. We show how this possibility is realized for D_{12} doublets. The potential containing D_{12} singlets can be treated in the same way.

Consider the tree level Higgs potential made of D_{12} doublets. For the quartic tensor products, the following terms are allowed,

$$
\begin{aligned}
&(H'^{u}H'^{u})(H'^{d}H'^{d}),\ \ (H'^{u}H'^{u})(H'^{u\dagger}H'^{u\dagger})\\
&(H'^{u}H'^{u})(H'^{u\dagger}H'^{d}),\ \ (H'^{d}H'^{d})(H'^{d\dagger}H'^{d\dagger})\\
&(H'^{d}H'^{d})(H'^{d\dagger}H'^{u}),\ \ (H'^{u}H'^{d})(H'^{u\dagger}H'^{d\dagger})\\
&(H'^{u}H'^{d})(H'^{u\dagger}H'^{u}),\ \ (H'^{u}H'^{d})(H'^{d\dagger}H'^{d})\\
&(H'^{d\dagger}H'^{u})(H'^{u\dagger}H'^{d}),\ \ (H'^{u}H'^{d})(H'^{u}H'^{d})\\
&(H'^{u\dagger}H'^{u})(H'^{u\dagger}H'^{u}),\ \ (H'^{d\dagger}H'^{d})(H'^{d\dagger}H'^{d})\\
&(H'^{u\dagger}H'^{u})(H'^{d\dagger}H'^{d})
\end{aligned}
\tag{5.79}
$$

and their Hermitian conjugates. Suppose we introduce the PQ charge +1 to both H'^{u} and H'^{d}. H'^{u} might be replaced by H''^{u}, but in this case the term such as $(H_u'^{\dagger}H_u'')(H_d'^{\dagger}H_d') + h.c.$ do not minimize our vacuum phase choice. For both H'^{u} and H''^{u} not to appear in the same tree level quartic terms, we assign different PQ charges to H'^{u} and H''^{u}. Then, the following terms survive,

$$
\begin{aligned}
&(H'^{u}H'^{u})(H'^{u\dagger}H'^{u\dagger}),\ \ (H'^{d}H'^{d})(H'^{d\dagger}H'^{d\dagger})\\
&(H'^{u}H'^{d})(H'^{u\dagger}H'^{d\dagger}),\ \ (H'^{d\dagger}H'^{u})(H'^{u\dagger}H'^{d})\\
&(H'^{u\dagger}H'^{u})(H'^{u\dagger}H'^{u}),\ \ (H'^{d\dagger}H'^{d})(H'^{d\dagger}H'^{d})\\
&(H'^{u\dagger}H'^{u})(H'^{d\dagger}H'^{d})
\end{aligned}
\tag{5.80}
$$

and terms with H'^{u} replaced by H''^{u}. The Lagrangian contains the following terms,

$$
\begin{aligned}
|H_1'^u|^2|H_2'^u|^2,\ \ |H_1'^u|^4+|H_2'^u|^4 \\
|H_1''^u|^2|H_2''^u|^2,\ \ |H_1''^u|^4+|H_2''^u|^4 \\
|H_1'^d|^2|H_2'^d|^2,\ \ |H_1'^d|^4+|H_2'^d|^4 \\
(|H_1'^u|^2+|H_2'^u|^2)^2,\ \ (|H_1'^u|^2-|H_2'^u|^2)^2 \\
(|H_1'^d|^2+|H_2'^d|^2)^2,\ \ (|H_1'^d|^2-|H_2'^d|^2)^2 \\
(H_1'^u H_1'^d)(H_1'^{u\dagger} H_1'^{d\dagger})+(H_2'^u H_2'^d)(H_2'^{u\dagger} H_2'^{d\dagger}) \\
(H_2'^u H_1'^d)(H_2'^{u\dagger} H_1'^{d\dagger})+(H_1'^u H_2'^d)(H_1'^{u\dagger} H_2'^{d\dagger}) \\
(H_1''^u H_1'^d)(H_1''^{u\dagger} H_1'^{d\dagger})+(H_2''^u H_2'^d)(H_2''^{u\dagger} H_2'^{d\dagger}) \\
(H_2''^u H_1'^d)(H_2''^{u\dagger} H_1'^{d\dagger})+(H_1''^u H_2'^d)(H_1''^{u\dagger} H_2'^{d\dagger}) \\
(H_2'^{d\dagger} H_1'^u)(H_1'^{u\dagger} H_2'^d)+(H_1'^{d\dagger} H_2'^u)(H_2'^{u\dagger} H_1'^d) \\
(H_2'^{d\dagger} H_2'^u)(H_2'^{u\dagger} H_2'^d)+(H_1'^{d\dagger} H_1'^u)(H_1'^{u\dagger} H_1'^d) \\
(H_2'^{d\dagger} H_1''^u)(H_1''^{u\dagger} H_2'^d)+(H_1''^{d\dagger} H_2''^u)(H_2''^{u\dagger} H_1'^d) \\
(H_2'^{d\dagger} H_2''^u)(H_2''^{u\dagger} H_2'^d)+(H_1'^{d\dagger} H_1''^u)(H_1''^{u\dagger} H_1'^d) \\
(|H_1'^u|^2+|H_2'^u|^2)(|H_1'^d|^2+|H_2'^d|^2) \\
(|H_1''^u|^2+|H_2''^u|^2)(|H_1'^d|^2+|H_2'^d|^2) \\
(H_2''^{u\dagger} H_1''^u)^2+(H_1''^{u\dagger} H_2''^u)^2
\end{aligned}
\tag{5.81}
$$

Our phase choice of VEVs must be consistent with the above potential. To investigate it in more detail, we pay attention to the last term. The other terms are not introducing phases. Let δ_1 and δ_2 be phases of $H_1''^u$ and $H_2''^u$, respectively. For Hermiticity and D_{12} invariance, the coupling constant should be real. The last term depends on phases through

$$
\cos(2(\delta_1-\delta_2)) \tag{5.82}
$$

and our vacuum choice $\delta_1 = \delta_2 = 0$ minimize it provided the coupling constant is negative. It is worth to note here that, if at least one of two D_{12} Higgs doublets were in the same representation, it is very hard to minimize the potential toward the desired vacuum property. For example, suppose that both H'^u and H'^d are in the same representation. In this case, the following terms are allowed.

$$(H_1'^u H_2'^d)(H_2'^{u\dagger} H_1'^{d\dagger}) + h.c. \tag{5.83}$$

For the invariance under the generator b of D_{12}, the overall coefficient must be real. Let α_1^u, α_2^u, α_1^d, α_2^d be the phases of Higgs VEV of $H_1'^u, H_2'^u, H_1'^d$, and $H_2'^d$, respectively. So, this quartic term has the phase dependence $\cos(\alpha_1^u - \alpha_2^u - \alpha_1^d + \alpha_2^d)$ and our vacuum choice does not minimize it.

The quadratic terms allowed by gauge and PQ symmetries are, viz. Eq. (5.46),

$$H'^{u\dagger} H'^u, \quad H''^{u\dagger} H''^u, \quad H'^{d\dagger} H'^d \tag{5.84}$$

and their Hermitian conjugates. D_{12} singlets are

$$\begin{aligned} &|H_1'^u|^2 + |H_2'^u|^2, \\ &|H_1''^u|^2 + |H_2''^u|^2 \\ &|H_1'^d|^2 + |H_2'^d|^2 \,. \end{aligned} \tag{5.85}$$

These quadratic terms may introduce negative mass squared toward achieving the VEVs of neutral members of the Higgs doublets.

The forbidden terms at tree level can appear integrating out heavy fields whose VEVs possibly break the assumed symmetries. These could be used to explain the vacuum choice of H'^d and therefore explains how D_{12} can be the flavor symmetry. For example, consider the quartic terms made of D_{12} doublet Higgs without conjugate (or starred) fields. Then, we have

$$\mathbf{1}_{++}: \ (H_2^{\prime u}H_1^{\prime d})(H_1^{\prime u}H_2^{\prime d}), \ (H_1^{\prime u}H_1^{\prime d})(H_2^{\prime u}H_2^{\prime d}) \tag{5.86}$$

$$\mathbf{1}_{+-}: \ (H_1^{\prime u}H_1^{\prime d})^2 + (H_2^{\prime u}H_2^{\prime d})^2 \tag{5.87}$$

$$\mathbf{1}_{-+}: \ (H_1^{\prime u}H_1^{\prime d})^2 - (H_2^{\prime u}H_2^{\prime d})^2 \tag{5.88}$$

$$\begin{aligned} \mathbf{2}_2: & \begin{pmatrix} (H_2^{\prime u}H_1^{\prime d})^2 \\ (H_1^{\prime u}H_2^{\prime d})^2 \end{pmatrix} \\ & \begin{pmatrix} (H_1^{\prime u}H_2^{\prime d})(H_1^{\prime u}H_1^{\prime d}) \\ (H_2^{\prime u}H_1^{\prime d})(H_2^{\prime u}H_2^{\prime d}) \end{pmatrix} \end{aligned} \tag{5.89}$$

$$\mathbf{2}_4: \begin{pmatrix} (H_2^{\prime u}H_1^{\prime d})(H_1^{\prime u}H_1^{\prime d}) \\ (H_1^{\prime u}H_2^{\prime d})(H_2^{\prime u}H_2^{\prime d}) \end{pmatrix} \tag{5.90}$$

Note that the term given in Eq. (5.86) is forbidden by the PQ symmetry assigning $+1$ to $H^{u\prime}, H^{d\prime}$ and $+2$ to $H^{u\prime\prime}$.

Let us introduce a D_{12} doublet $\mathbf{2}_4$ which is denoted as a SM singlet scalar Φ,

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} : \mathbf{2}_4 \tag{5.91}$$

Using Φ, the allowed quartic couplings are obtained. In addition, we note

- The dimension-5 D_{12} allowed couplings are

$$\lambda[\Phi_1^\dagger(H_2^{\prime u}H_1^{\prime d})(H_1^{\prime u}H_1^{\prime d}) + \Phi_2^\dagger(H_1^{\prime u}H_2^{\prime d})(H_2^{\prime u}H_2^{\prime d})] \tag{5.92}$$

- The dimension-6 D_{12} allowed couplings are

$$\begin{aligned} &\zeta_1 \Phi_1^\dagger \Phi_2^\dagger (H_2'^u H_1'^d)(H_1'^u H_2'^d) + \zeta_2 \Phi_1^\dagger \Phi_2^\dagger (H_1'^u H_1'^d)(H_2'^u H_2'^d) \\ &+ \zeta_3 [\Phi_2^{\dagger 2} (H_2'^u H_1'^d)(H_1'^u H_1'^d) + \Phi_1^{\dagger 2} (H_1'^u H_2'^d)(H_2'^u H_2'^d)]. \end{aligned} \tag{5.93}$$

Here, $\mathbf{2}_8$ is shown to be equivalent to $\mathbf{2}_4$ by applying a D_{12} transformation b of Eq. (5.33)

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} (\mathbf{2}_8) \ : \ \mathbf{2}_4. \tag{5.94}$$

Operators with dimension more than 7 are highly suppressed and hence they can be ignored. All effective quartic terms coupling to Φ do not give the vacuum we want to obtain. So, the unwanted terms must be forbidden by some symmetry or at least highly suppressed. For example, if we choose the VEV of Φ as $\frac{1}{\sqrt{2}}(\exp(-\mathrm{i}2\pi/3), 1)^{\mathrm{T}}$, only the dimension-5 operator is independent of the phase choices given in Eqs. (5.55) and (5.60). However, this vacuum choice is dangerous. With our discrete symmetry, a dimension-6 operator of the form

$$(\Phi_1^3 + \Phi_2^3)(\Phi_1^{\dagger 3} + \Phi_2^{\dagger 3}) \tag{5.95}$$

is not forbidden. Moreover, this term favors the direction which makes $\langle \Phi_1^3 \rangle + \langle \Phi_2^3 \rangle = 0$. With this dimension 6 potential, our vacuum choice is not the minimum. To forbid Eq. (5.92), we introduce a Z_2 symmetry: $\Phi \to -\Phi$.

Since dimension-5 operators are forbidden, we may choose an alternate direction $\Phi \propto \frac{1}{\sqrt{2}}(1, \exp(-\mathrm{i}\pi/3))^{\mathrm{T}}$. Then, our vacuum choice corresponds to the minimum.

5.3 Realistic parameterizations for the PMNS matrix

As shown in the previous section, the basic pattern of the PMNS matrix can be explained in terms of non-Abelian discrete symmetry and seesaw model. However, observed values show deviation from the discrete symmetric pattern. One important issue is nonzero (13) element of the PMNS matrix. Vanishing this element implies that one of the mixing angle is zero. In this case, if we neglect the Majorana phase, lepton sector does not have CP violation in the weak interaction. Only when three mixing angles do not vanish, the unremovable phase in the PMNS matrix, the Dirac phase appear. Most model based on the non-Abelian discrete symmetry predicts that θ_{13}, or θ_3 vanishes. However, subleading breaking effects of such symmetry can introduce nonzero value. Nonzero Dirac phase can appear as the breaking effect of the discrete symmetry with the phase.

Recently, the T2K collaboration reported a large θ_{13}[105]. At the 90% confidence limit, they report $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ for $\sin^2 2\theta_{23} = 1.0$, $|\Delta m^2_{23}| = 2.4\times 10^{-3}\mathrm{eV}^2$, $\delta = 0$ and normal(inverted) hierarchy. The BF points are 0.11(0.14). The MINOS group also reported that a vanishing θ_{13} is disfavored[105]. Based on the global neutrino data analysis shows a sizable θ_{13}, as well as a deviation of θ_{23} from $\pi/4$. The best fit values in their analysis, which will be used in the estimation here, are as follows[106]:

$$\sin^2\theta_{12} = 0.306(0.312), \quad \sin^2\theta_{13} = 0.021(0.025), \quad \sin^2\theta_{23} = 0.42. \tag{5.96}$$

Nonzero θ_{13} is confirming in RENO, Daya Bay, and Double Choose[107]. Double Chooz reports $\sin^2 2\theta_{13} = 0.086 \pm 0.041(\mathrm{stat}) \pm 0.030(\mathrm{syst})$, or, at 90% CL, $0.017 < \sin^2 2\theta_{13} < 0.16$, Daya Bay reports $\sin^2 2\theta_{13} = 0.092 \pm 0.016(\mathrm{stat}) \pm 0.005(\mathrm{syst})$. and RENO reports $\sin^2 2\theta_{13} = 0.103 \pm 0.013(stat) \pm 0.011(sys)$

In this regard, we should put the correction to the discrete symmetry patterns. Suppose the basic pattern provided by D_{12} symmetry[108]. With the traditional Chau-Keung-Maiani parametrization, we obtain a parametrization where the mixing of V_{23} and V_{33} is maximal. Kept to $O(\beta)$, with the (13) element being of order β, we have

$$\begin{pmatrix} \frac{1}{2}(\sqrt{3}-\beta-\frac{\sqrt{3}}{2}(1+B^2)\beta^2) & \frac{1}{2}(1+\sqrt{3}\beta-\frac{1}{2}(1+B^2)\beta^2) & B\beta \\ \begin{array}{c} -\frac{1}{2\sqrt{2}}(1+\sqrt{3}(1+Be^{-i\delta})\beta \\ -(A+\frac{1}{2}+Be^{-i\delta})\beta^2) \end{array} & \begin{array}{c} \frac{1}{2\sqrt{2}}\Big(\sqrt{3}-(1+Be^{-i\delta})\beta \\ -\sqrt{3}(A+\frac{1}{2}+Be^{-i\delta})\beta^2\Big) \end{array} & \frac{1}{\sqrt{2}}(1+(A-\frac{B^2}{2})\beta^2)e^{-i\delta} \\ \begin{array}{c} \frac{1}{2\sqrt{2}}\Big(e^{i\delta}+\sqrt{3}(e^{i\delta}-B)\beta \\ +([A-\frac{1}{2}]e^{i\delta}+B)\beta^2\Big) \end{array} & \begin{array}{c} -\frac{1}{2\sqrt{2}}\Big(\sqrt{3}e^{i\delta}-[e^{i\delta}-B]\beta \\ +\sqrt{3}([A-\frac{1}{2}]e^{i\delta}+B)\beta^2\Big) \end{array} & \frac{1}{\sqrt{2}}\Big(1-(A+\frac{B^2}{2})\beta^2\Big) \end{pmatrix} \tag{5.97}$$

With the BF values above (Eq. 5.96),

$$\beta = 0.062, \;\; B = 2.32, \;\; A = 1.28 \tag{5.98}$$

and the CP phase $\delta = 0$ as assumed in the measurement.

For the modified Kobayashi-Maskawa parametrization, giving $O(\beta)$ correction to $\theta_{2,3}$ gives

$$\begin{pmatrix} \frac{1}{2}(\sqrt{3}-\beta-\frac{\sqrt{3}}{2}\beta^2) & \frac{1}{2}\Big(1+\sqrt{3}\beta-\frac{1}{2}(1+B^2)\beta^2\Big) & \frac{B}{2}(1+\sqrt{3}\beta)\beta \\ \begin{array}{c} -\frac{1}{2\sqrt{2}}\Big(1+(\sqrt{3}-A)\beta \\ -[\frac{1}{2}+\frac{A^2}{2}+\sqrt{3}A]\beta^2\Big) \end{array} & \begin{array}{c} \frac{1}{2\sqrt{2}}\Big(\sqrt{3}-[1+\sqrt{3}A-2Be^{-i\delta}]\beta \\ -(\frac{\sqrt{3}}{2}(1+A^2+B^2) \\ -A(1+2Be^{-i\delta}))\beta^2\Big) \end{array} & \begin{array}{c} -\frac{e^{-i\delta}}{\sqrt{2}}\Big(1-\frac{1}{2}(e^{i\delta}\sqrt{3}B-2A)\beta \\ -\frac{1}{2}(A^2+B^2-e^{i\delta}(\sqrt{3}A+1)B)\beta^2\Big) \end{array} \\ \begin{array}{c} -\frac{e^{i\delta}}{2\sqrt{2}}\Big(1+(\sqrt{3}+A)\beta \\ +[\sqrt{3}A-\frac{A^2}{2}-\frac{1}{2}]\beta^2\Big) \end{array} & \begin{array}{c} \frac{e^{i\delta}}{2\sqrt{2}}\Big(\sqrt{3}-[1-\sqrt{3}A+2Be^{-i\delta}]\beta \\ -\frac{1}{2}[\sqrt{3}(1+A^2+B^2) \\ +2A(1-2Be^{-i\delta})]\beta^2\Big) \end{array} & \begin{array}{c} \frac{1}{\sqrt{2}}\Big(1+(\frac{\sqrt{3}}{2}Be^{i\delta}-A)\beta \\ -\frac{1}{2}[A^2+B^2+e^{i\delta}B(1-\sqrt{3}A)]\beta^2\Big) \end{array} \end{pmatrix} \tag{5.99}$$

and

$$\beta = 0.078, \;\; B = 3.3, \;\; A = 3.8. \tag{5.100}$$

Among the mixing angles obtained from the D_{12} model, $\theta_C = 15^o$, $\theta_{sol} = 30^o$ are deviated from measured values, $\theta_C \simeq 13^o$, $\theta_{sol} \simeq 33^o$. Such deviations, as well as nonzero mixing angles which were zero in the model can be expressed in some powers of $\beta \equiv \theta_{sol} - \pi/6$. Especially, θ_{13} of the PMNS matrix can be parameterized by $\theta_{13} = B\beta$, where $B \simeq 2$. It might be a modest modification from $\theta_{13} = 0$.

However, in terms of $\lambda = \sin\theta_C$, $\theta_{13} \simeq \lambda/\sqrt{2}$ *i.e.* of order of λ. Since D_{12} model explains λ in the CKM matrix, it would be a good challenge to obtain θ_{13} in the context of D_{12}. In this case, either $M^{(l)}$ or $M^{(\nu)}$ have rather complicated form. More than two Higgs would be responsible for one of the Yukawa matrix elements and their VEV with phase might be fine-tuned. For example, to explain a certain Yukawa matrix element proportional to λ, we have to express it in the form of $\langle H_x + H_y \rangle$ where x, y are representations of D_{12} to which $H_{x,y}$ belong, and take VEVs as $\langle H_x \rangle = v_x \exp(i\pi/12)$ and $\langle H_y \rangle = -v_x \exp(-i\pi/12)$.

On the other hand, we may use the quark-lepton complementarity in other way. We may set three mixing angles in the PMNS matrix by $\theta_{12} = \theta_{23} = \pi/4$, $\theta_{13} = 0$ and CKM matrix by identity at leading order. It would be implemented by another discrete symmetry. Then, λ parameterizes the subleading effect of discrete symmetry breaking. We can make λ expansion for PMNS matrix by setting solar angle by $\theta_{12} = \pi/4 - \lambda$ and expressing deviation of atmospheric angle θ_{23} from $\pi/4$ and that of θ_{13} from zero by some powers of λ.

When the nonzero θ_{13} is confirmed, the next issue would be measuring the Dirac phase δ, weak CP violation in the lepton sector. The Jalrskog determinant can be measured from neutrino oscillation, as $P(\nu_\alpha \to \nu_{\alpha'}) - P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'})$ is proportional to the Jarlskog determinant. At leading order,

$$J = \frac{\sqrt{3}}{8}|V_{13}|\sin\delta. \tag{5.101}$$

Before closing this section, we visit two more issues. We explain very small neutrino mass naturally using the seesaw mechanism. Seesaw mechanism predicts the presence of Majorana mass term and in general, Majorana phase should appear. Then the PMNS matrix should be modified by multiplying

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \tag{5.102}$$

on the right side. Such phases can be measured through neutrinoless double beta ($0\nu\beta\beta$)decay [109], $(Z,A) \to (Z\pm2,A)+2e^{\mp}$. The $0\nu\beta\beta$ decay rate is proportional to the squared effective neutrino mass,

$$\langle m_{\beta\beta}\rangle^2 = |\sum_i V_{ei}^2 m_{\nu_i}|^2 \tag{5.103}$$

and in terms of exact form of PMNS matrix element, it is given by

$$|c_1^2 m_1^2 + s_1^2 c_3^2 e^{i\alpha} m_2^2 + s_1^2 s_3^2 e^{i\beta} m_3^2|^2 \tag{5.104}$$

in the modified Kobayashi-Maskawa parametrization and

$$|c_{12}^2 c_{13}^2 m_1^2 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2^2 + s_{13}^2 e^{i(\beta-\delta)} m_3^2|^2 \tag{5.105}$$

in the Chau-Keung-Maiani parametrization.

On the other hand, in the early Universe, heavy neutrinos decay into the leptons[47], and CP violation effect in decay can give rise to the lepton number asymmetry. This can transferred to the baryon number asymmetry through sphaleron process. The SM extended to the seesaw mechanism preserves $B-L$. At the quantum level, the SM fermions are chiral under the weak interaction so B

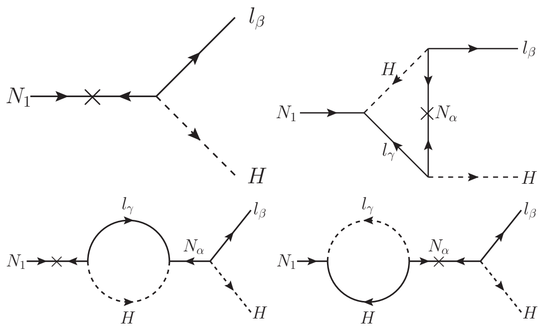

그림 30: Diagrams responsible for CP violating interference in the heavy neutrino decay.

and L global symmetry is anomalous proportional to $W_{\mu\nu}\tilde{W}^{\mu\nu}$. Then there are many vacuums with different $B+L$ winding numbers but even in this case, $B-L$ is not anomalous. If vacuum to vacuum transition takes place, The excess of lepton over antilepton can transferred to that of baryon over antibaryon. In ordinary case, such transition is made by tunneling so very suppressed. On the other hand, In the high temperature, transition 'over'(not tunneling) the potential barrier is possible, and this is the sphaleron process[110]. Once baryon asymmetry is produced this should be fixed as the Universe becomes the state of out-of-equilibrium. Such scenario is leptogenesis. The CP violation in the lightest heavy neutrino decay from the interference between tree and loop effects depicted in Fig. 30 is given by [111]

$$\varepsilon \equiv \sum_l \frac{\Gamma(N_{a_1} \to Hl) - \Gamma(N_{a_1} \to \bar{H}\bar{l})}{\Gamma(N_{a_1} \to Hl) + \Gamma(N_{a_1} \to \bar{H}\bar{l})} = -\frac{3}{16\pi}\frac{1}{(\mathcal{Y}^\dagger\mathcal{Y})_{a_1a_1}}\sum_{b\neq a_1}\mathrm{Im}\Big[\frac{(\mathcal{Y}^\dagger\mathcal{Y})_{ba_1}}{M_b}\Big] \tag{5.106}$$

where a_1 is the index for the lightest heavy neutrinos. When neutrino is diagonalized from the seesaw mechanism,

$$\tilde{m}_\nu = L_\nu(-\mathcal{Y}M_N^{-1}\mathcal{Y}^T)L_\nu^T \tag{5.107}$$

where L_ν is what appears in the PMNS matrix $V = L_l L_\nu^\dagger$. Then, Yukawa matrix $\mathcal{Y}$ is diagonalized in the form of $\tilde{\mathcal{Y}} = L_\nu \mathcal{Y} U_N^{-1}$ where U_N is unknown unitary matrix. Then the combination $\mathcal{Y}^\dagger \mathcal{Y}$ in ε does not depend on L_ν. That means, what we know from ground observation can be irrelevant for the leptogenesis. Of course, ε is just the total decay rate, and if we consider decay to lepton in each flavor separately, PMNS matrix parameter can appear in the leptogenesis. Even in this case, by arbitrariness of the U_N, leptogenesis is very insensitive to the PMNS matrix parameters[112]. To see this explicitly[113], note that unitary matrix can be written in the form, $U = \exp(i\phi) P U_{KM} Q$, where P and Q are diagonal matrices in the form of $\mathrm{diag.}(1, \exp(i\phi'), \exp(i\phi''), \cdots)$ and U_{KM} is the CKM matrix type unitary matrix. With n generations, each of P and Q has $(n-1)$ independent phases and U_{KM} has $(1/2)(n-1)(n-2)$ independent phases. Hence, $n \times n$ unitary matrix has $(1/2)n(n+1)$ phases in total, and Yukawa matrix

$$\mathcal{Y} = L_\nu \tilde{\mathcal{Y}} U_N \equiv (e^{i\phi_\nu} P_\nu L_{KM} Q_\nu) \tilde{\mathcal{Y}} (e^{i\phi_N} P_N U_{KM} Q_N) \tag{5.108}$$

seem to have $n(n+1)$ degrees of freedom. However, as $P_{\nu,N}$, $\mathcal{Y}$, and $Q_{\nu,N}$ are diagonal, so commute with each other. Then $Q_\nu \tilde{Y} P_N$ can be written in the form of $P' \tilde{Y}$. On the other hand, overall phases $\exp(i\phi_{\nu,N})$ and P_ν are absorbed by field redefinitions. Therefore, $2 + (n-1) + (n-1) = 2n$ phases are eliminated, $\tilde{Y}$ has $n(n-1)$ phases in total. Among them, $(1/2)(n-1)(n-2)$ phases in L_ν is the Dirac phase and $n-1$ phases in P' is the Majorana phases in the PMNS matrix. We have more phases which affect the CP violation in the heavy neutrino decay.

Other ways to parameterizing extra degrees of freedom are possible. For example, in the basis where M_N is diagonalized, $\tilde{m}_\nu = -L_\nu \mathcal{Y} \tilde{M}_N^{-1} \mathcal{Y}^T L_\nu^T$, we separate $\tilde{M}_N^{-1}$ into $\tilde{M}_N^{-1/2} \tilde{M}_N^{-1/2}$ and $\tilde{m}_\nu$ into $\tilde{m}_\nu^{1/2} \tilde{m}_\nu^{1/2}$. Then we can rewrite the diagonalized neutrino mass as $1 = OO^T$ where $O = \tilde{m}_\nu^{1/2} L_\nu \tilde{\mathcal{Y}} \tilde{M}_N^{-1/2}$ is the complex unitary matrix[114].

제 6 장

Conclusion

In this thesis, we considered the two problems in the SM mainly in light of flavor dependent symmetry. To make the electroweak gauge symmetry $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ spontaneously broken, we need the scalar charged under this gauge symmetry, the Higgs. But such fundamental scalar mass at electroweak scale requires large fine tuning between electroweak scale and Planck scale. As a solution to this hierarchy problem, we employ the supersymmetry(SUSY), symmetry between the bosons and fermions. We firstly observed the SUSY model explaining the LHC results which may be interpreted as evidence of the Higgs scalar. We investigate the CP even and odd Higgs mass by combining SUSY with the Peccei-Quinn symmetry, which is introduced to explain very small CP violation in the strong interaction. However, direct evidence for SUSY has not been found yet. Exclusion of the squarks in the sub TeV scale threatened the motivation of low energy SUSY as a solution of the hierarchy problem. As a viable possibility of low energy SUSY, we considered the effective SUSY, only the third generation squarks, responsible for the stable the Higgs mass at electroweak scale, have the mass of the sub TeV. As a model for it, we introduce extra $\mathrm{U}(1)'$ gauge group under which the third generation quark and the Higgs superfields are not charged. As SUSY breaking in the hidden sector come to the visible sector through $\mathrm{U}(1)'$ interaction, effective SUSY can be easily obtained. Moreover, it may be related to the flavor structure, the second problem of the SM we considered. The mass hierarchy and structure of mixing matrices, the CKM and the PMNS matrices cannot be understood in the context of the SM only, as the SM gauge group is flavor universal. We observed that Cabibbo angle expansion or $\lambda = \sin\theta_C$ expansion of the CKM matrix can be a hint for flavor dependent symmetry based on the Froggatt-Nielsen mechanism. Flavor structure has many ambiguities, so we try to find the important feature of the CKM matrix. Especially, focusing on the maximal CP violation, we suggest the parametrization of the CKM matrix which shows it apparently. We interpret λ expansion and CP violating phase as breaking effects of basic pattern provided by flavor dependent symmetry and consider the structure of the CKM and the PMNS matrix based on the non-Abelian discrete symmetry D_{12}. We more discuss nonzero θ_{13}, which may be observed in the present experiments.

Such studies require more analysis in detail, and the model can be changed for consistency with future experiments. On the other hand, low energy models we considered here give some questions. First, in many models concerning the physics at high energy scale, intermediate scale between $10^8 \sim 10^{12}$GeV appears. Seesaw scale, Peccei-Quinn scale, and messenger scale in SUSY breaking are their examples. Some of them are used to explain very small scale in terms of intermediate and high energy scale, so in this case, we put the problem at the electroweak scale or below to the unobserved scales. But the fact that these scales are concentrated in such ranges may imply that many problems in the SM are not separated with each other but related. Building model for each phenomena, we have to introduce many symmetries and exotic particles and in many cases, but it looks rather complicated. As we consider the GUT for simpler gauge group than the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$, we may ask of the existence of simpler, and unified feature of physics beyond the SM. If flavor dependent symmetry is what Nature really has, we should ask why such symmetries are broken at such intermediated scale and whether it could be understood in a unified way with the symmetries in the SM. In the regard, thinking of origin of such symmetries may have important meaning.

We hope future experiments can give hints for such questions.

참고 문헌

[1] S. Weinberg, *The Quantum Theory of Fields*, Vol 1 (Foundations), 2 (Modern Applications), 3 (Supersymmetry), Cambridge University Press (1995, 1996, 2000).

[2] M. Y. Han and Y. Nambu, "Three-Triplet Model with Double SU(3) Symmetry", Phys. Rev. **139**, B1006 (1965)

[3] D. J. Gross and F. Wilczek, "Ultraviolet Behavior of Non-Abelian Gauge Theories", Phys. Rev. Lett. **30**, 1343 (1973); H. D. Politzer, "Reliable Perturbative Results for Strong Interactions?", Phys. Rev. Lett. **30**, 1346 (1973).

[4] S. L. Glashow, "Partial Symmetries of Weak Interactions", Nucl. Phys. **22**, 579 (1961); S. Weinberg, "A Model of Leptons", Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in *Elementary Particle Theory* (N. Svartholm, Ed.), Almqvist and Wiksell, 1969, p.367.

[5] L. Faddeev and V. N. Popov, "Feynman Diagrams for the Yang-Mills Field", Phys. Lett. **25B**, 29 (1967); Review on the Faddeev-Popov scheme in this thesis follows E. Kiritsis, *String Theory in a Nutshell*, Princeton University Press(2007).

[6] P. Anderson, "Plasmons, gauge invariance and mass". Phys. Rev. **130**, 439 (1963); F. Englert and R. Brout "Broken Symmetry and the Mass of Gauge Vector Mesons". Phys. Rev. Lett. **13** 321 (1964); P. Higgs, Peter (1964)"Broken Symmetries and the Masses of Gauge Bosons". Phys. Rev. Lett. **13** 508; G. Guralnik, C. R. Hagen,and T. W. B. Kibble, "Global Conservation Laws and Massless Particles", Phys. Rev. Lett. **13** 585(1964); G.S. Guralnik "Gauge Invariance and the Goldstone Theorem" (1965 Feldafing talk), Mod. Phys. Lett. A **26** 1381 (2011) [arXiv:1107.4592]; P. Higgs, "Spontaneous Symmetry Breakdown without Massless Bosons". Phys. Rev. **145** 1156 (1966); For a review on the history of Higgs studies, see J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, "A Historical Profile of the Higgs Boson", [arXiv:1201.6045]; For a review, see J. F. Gunion, H. E. Haber, G. Kane, and S. Dawson, "The Higgs Hunter's Guide" (Frontiers on Physics Vol 80) Addison-Wesley, (1990); A. Djouadi, "The Anatomy of Electro-Weak Symmetry Breaking. I: The Higgs boson in the Standard Model", Phys.Rept.**457**, 1

(2008)[arXiv:hep-ph/0503172]; A. Djouadi, "The Anatomy of Electro-Weak Symmetry Breaking. II: The Higgs bosons in the Minimal Supersymmetric Model", Phys.Rept.**459**, 1 (2008)[arXiv:hep-ph/0503173].

[7] C. Becchi, A. Rouet and R. Stora, "Renormalization of the abelian Higgs-Kibble model", Comm. Math. Phys. **42**, 127 (1975); C. Becchi, A. Rouet and R. Stora, "Renormalization of gauge theories", Ann. Phys. **98**, 287 (1976); I.V. Tyutin, "Gauge Invariance in Field Theory and Statistical Physics in Operator Formalism", Lebedev Physics Institute preprint 39 (1975), [arXiv:0812.0580].

[8] For a pedagogical introduction, A. Bilal, "Lectures on Anomalies", [arXiv:0802.0634]; R. A. Bertlmann, *Anomalies in Quantum Field Theory*, 2nd Ed. International Series of Monographs on Physics Vol 91 , Oxford University Press (2001).

[9] S. M. Barr, B. Bednarz, and C. Benesh, "Anomaly constraints and new U(1) gauge bosons ", Phys. Rev. **D34** 235 (1986).

[10] For a review, see H Fritzsch and Z.-Z Xing, "Mass and flavor mixing schemes of quarks and leptons", Prog. Part. Nucl. Phys. **45**, 1 (2000) [arXiv:hep-ph/9912358].

[11] M. Gell-Mann "The Eightfold Way: A theory of strong interaction symmetry". Cal. Tech. Synchroton Laboratory Report CTSL-20 (1961); M. Gell-Mann. "The Eightfold Way: A theory of strong interaction symmetry". in M. Gell-Manm and Y. Ne'emann. *The Eightfold Way*. Benjamin, New York (1964) and Westview Press, (2000); Y. Ne'eman, "Derivation of strong interactions from gauge invariance". Nucl. Phys. **26**, 222 (1961).

[12] M. Gell-Mann and M Lévy, "The axial vector current in beta decay", Nuov. Cim. **16**, 705 (1960).

[13] S. Coleman, J. Wess and B. Zumino, "Structure of Phenomenological Lagrangians. I", Phys. Rev. **177**, 2239(1969); C. G. Callan, S. Coleman, J. Wess, and B. Zumino, "Structure of Phenomenological Lagrangians. II", Phys. Rev. **177**, 2247 (1969).

[14] N. Isgur and M. B. Wise, Phys. Lett. **232B** (1989) 113; **208B** (1988) 504; E. Eichten and B. Hill, Phys. Lett. **234B** (1990) 511; For a review, see, for example, H. Georgi, "Heavy quark effective field theory" HUTP-91-A039, http://www.people.fas.harvard.edu/ hgeorgi/tasi.pdf.

[15] Footnote in M. Gell-Mann and M Lévy, "The axial vector current in beta decay", Nuov. Cim. **16**, 705 (1960); N. Cabibbo, "Unitary Symmetry and Leptonic Decays", Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi, T. Maskawa, "CP-Violation in the Renormalizable Theory of Weak Interaction" Prog. Theor. Phys. **49**, 652(1973).

[16] For a review, see, for example, I. I. Bigi and A. I. Sanda,*CP Violation*(Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology), Cambridge University Press (2009); G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation*(International Series of Monographs on Physics), Oxford University Press, (1999).

[17] L. Maiani, "CP violation in purely lefthanded weak interactions ",Phys. Lett. 62 B, 183 (1976); L. Maiani, in *Preceedings of the 1977 International Symposium on Lepton and Photon Interaction at High Energies* (DESY, Hamburg, 1977), 0.867; L.L. Chau and W.-Y. Keung, "Comments on the Parametrization of the Kobayashi-Maskawa Matrix", Phys. Rev. Lett. **53**, 1802 (1984).

[18] M. Fukugita and T. Yanagida, "Physics of Neutrinos and Applications to Astrophysics", Springer,(2003).

[19] B. Pontecorvo, "Mesonium and anti-mesonium", Sov.Phys.JETP **6** 429 (1957) and Zh.Eksp.Teor.Fiz.**33** 549,1957; Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles", Prog.Theor.Phys.**28** 870,(1962).

[20] L. Wolfenstein (1978). "Neutrino oscillations in matter", Physical Review **D17** 2369; S. P. Mikheyev and A. Yu. Smirnov, "Resonance Amplification of Oscillations in Matter and Spectroscopy of Solar Neutrinos", Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913(1985)]; P. Mikheyev and A. Yu. Smirnov, "Resonant amplification of neutrino oscillations in matter and solar neutrino spectroscopy", Nuov. Cim. **9C**, 17 (1986); H. A. Bethe, "Possible Explanation of the Solar-Neutrino Puzzle", Phys. Rev. Lett. **56**, 1305 (1986).

[21] Ch. Krauss et al, "Final results from phase II of the Mainz neutrino mass search in tritium beta decay", Eur.Phys.J. **C40** (2005) 447, [arXiv:hep-ex/0412056]; For cosmological bound from supernova SN1987A, G. Pagliaroli, F. Rossi-Torres, and F. Vissani, "Neutrino mass bound in the standard scenario for supernova electronic antineutrino emission", Astropart.Phys. **33** (2010) 287, [arXiv:1002.3349 [hep-ph]].

[22] P. Minkowski,"$\mu \to e\gamma$ at a rate of one out of 10^9 muon decays?", Phys. Lett. **67B**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, Tsukuba, Japan, 1979, ed. by O. Sawada and A. Sugamoto (KEK Report No. KEK-79-18, 1979) p. 95; T. Yanagida, "Various Schemes of Neutrino Mixing"Prog. Theor. Phys. **64**, 1103 (1980); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, ed. by D.Z. Freedom and P. van Nieuwenhuizen (North-Holland, Amsterdam, 1979); R.N. Mohapatra and G. Senjanovic,"Neutrino Mass and Spontaneous Parity Nonconservation", Phys. Rev. Lett. **44**, 912 (1980).

[23] Particle Data Group (K. Nakamura et al.), J. Phys. G**37**, 075021 (2010).

[24] S. Weinberg, in "*Proceedings of the Conference on Gauge Theories and Modern Field Theory held at Northwestern University, 1975*", MIT Press(1976); E. Gildener, "Gauge Symmetry Hierarchies", Phys. Rev. **D14**, 1667 (1976).

[25] J. C. Pati and A. Salam, "Unified Lepton-Hadron Symmetry and a Gauge Theory of the Basic Interactions", Phys. Rev. **D8** (1973) 1240; H. Georgi and S. L. Glashow, "Unity of All Elementary Particle Forces", Phys. Rev. Lett **32**(1974) 438; J. C. Pati and A. Salam, "Lepton Number as the Fourth Color", Phys. Rev. **D10** (1974) 275;H. Fritzsch and P. Minkowski, "Universality of the Basic Interactions"Phys. Lett. **53B** (1974) 373; H. Georgi, H. Quinn, and S. Weinberg, "Hierarchy of Interactions in Unified Gauge Theories", Phys. Rev. Lett. **33** (1974) 451; H. Georgi, "The State of the Art-Gauge Theories", in *Proceedings if the American Institute of Physics* (1974); ;H. Fritzsch and P. Minkowski, "Unified Interactions of Leptons and Hadrons", Ann. Phys. **93**(1975) 93; For reviews of group theory for GUT, see R. Slansky, "Group Theory for Unified Model Building",Phys. Rep. **79**(1981) 1; H. Georgi, "*Lie Algebras in Particle Physics: From Isospin to Unified Theory*" 2nd Ed. ,Westview Press (1999).

[26] SNO collaboration, "Constraints on Nucleon Decay via "Invisible" Modes from the Sudbury Neutrino Observatory", Phys. Rev.Lett. **92** (2004) 102004[arXiv:hep-ex/0310030].

[27] For a pedagogical review, see R. Kleiss, "Derivation of the Minimal Standard Model Lagrangian" in *Physics up to 200 TeV* (28th International School of Subnuclear Physics "Ettore Majorana", Erice, Italy, 16 - 24 Jul ,1990), New York, NY : Plenum, (1991); D. A. Dicus and V. S. Mathur,"Upper Bounds on the Values of Masses in Unified Gauge Theories", Phys Rev. **D 7**, 3111

(1973); B. W. Lee, C. Quigg, and H. B. Thacker, “Weak interactions at very high energies: The role of the Higgs-boson mass” Phys. Rev. **D 16**, 1519 (1977).

[28] For a review, H. E. Haber and G. L. kane, “The Search for Supersy,,etry: Probing Physics Beyond the Standard Model”, Phys. Rep. **117**,75 (1985); H. P. Nilles,“Supersymmetry, Supergravity, and Particle Physics”, Phys. Rep. **110**, 1 (1984);J. Wess and J. Bagger, *Supersymmetry and Supergravity”*, Princeton University Press,(1992);M. Drees, R. M. Godbole, and P. Roy, *Theory and Phenomenology of Sparticles: An aAccount of Four-dimensional* $N = 1$ *Supersymmetry in High Energy Physics*, World Scientific (2004).

[29] D. t’Hooft, “Naturalness, Chiral Symmetry, and Spontaneous Chiral Symmetry Breaking” in *Recent Developments in Gauge Theories”* (NATO ASI series B: Physics Vol. 59), Plenum press, (1979).

[30] For a review, K.G. Wilson, “The renormalization group: critical phenomena and the Kondo problem”, Rev. Mod. Phys. **47** 773(1975); K. G. Wilson and J. Kogut, “The renormalization group and the ε expansion”, Phys. Rep.**12**, 75 (1974).

[31] H. Aoki and S. Iso, “Hierarchy problem revisited from the Wilsonian Renormalization Group”, [arXiv:1201.0857]

[32] J. E. Kim, “Light Pseudoscalars, Particle Physics and Cosmology”, Phys.Rept. **150**, 1 (1987); J. E. Kim and G. Carosi, “Axions and the Strong CP Problem”, Rev. Mod. Phys. **82**, 557 (2010), [arXiv:0807.3125 [hep-ph]].

[33] R. P. Feynman, “The Space-Time Formulation of Nonrelativistic Quantum Mechanics”, Rev. Mod. Phys. **20**, 367(1948).

[34] A. A. Belavin, A. M. Polyakov, A. S. Schwarz, and Yu. S. Tyipkin, “Pseudoparticle Solutions of the Yang-Mills Equations”, Phys.Lett.**B59**,85 (1975); G. ’t Hooft, “Computation of the quantum effects due to a four-dimensional pseudoparticle”, Phys. Rev. **D 14**, 3432 (1976).

[35] S. C. Adler, “Axial-Vector Vertex in Spinor Electrodynamics”, Phys. Rev. **177**, 2426; J. S. Bell and R. Jackiw, “A PCAC Puzzle: $\pi^0 \to \gamma\gamma$ in the Sigma Model”, Nuovo Cim. **A60**,47 (1969).

[36] K. Fujikawa, “Path-Integral Measure for Gauge-Invariant Fermion Theories”, Phys. Rev. Lett. **42**, 1195 (1979)

[37] M. F. Atiyah, Michael and I. M. Singer, "The Index of Elliptic Operators on Compact Manifolds", Bull. Amer. Math. Soc. **69** 322 (1963).

[38] H. Y. Cheng, "The strong CP problem revisited", Phys. Rep. **158**, 1; R.J. Crewther, "Chiral properties of quantum chromodynamics", (lecture presented at the Intern. Summer Institute on Theoretical Physics, Kaiserslautern, Germany (August 13-24, 1979)), http://www.physics.adelaide.edu.au/theory/staff/crewther/chiral.pdf.

[39] C. A. Backer et al, "An Improved experimental limit on the electric dipole moment of the neutron", Phys.Rev.Lett.**97**,131801(2006),[arXiv:hep-ex/0602020].

[40] M. R. Buckley and H. Murayama, "Is strong CP invariance due to a massless up quark?", Phys.Rev.Lett.**90**,021601(2003), [arXiv:hep-lat.0112029].

[41] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Pseudoparticles", Phys. Rev. Lett. **38**, 1440; R. D. Peccei and H. R. Quinn, "Constraints imposed by CP conservation in the presence of pseudoparticles",Phys. Rev. D. **16**, 1791.

[42] S. Weinberg, "A New Light Boson?", Phys. Rev. Lett. **40**, 223(1978); F. Wilczek, "Problem of Strong P and T Invariance in the Presence of Instantons", Phys. Rev. Lett. **40**, 279(1978).

[43] J. E. Kim, "Weak-Interaction Singlet and Strong CP Invariance", Phys. Rev. Lett.**43**, 103; M. A. Shifman, V. I. Vainstein, and V. I. Zakharov, "Can confinement ensure natural CP invariance of strong interactions?", Nucl. Phys.**B166**, 493.

[44] A. P. Zhitnitskii,"On Possible Suppression of the Axion Hadron Interactions", Yad. Fiz. 31,. 497-504 (1980) [Sov. J. Nucl. Phys. **31**,260 (1980)]; M. Dine, W. Fischler, and M. Srednicki,"Supersymmetric technicolor", Nucl. Phys. **B189**, 575; M. Dine, W. Fischler, and M. Srednicki, "A simple solution to the strong CP problem with a harmless axion", Phys. Lett. **B104**, 199.

[45] For a pedagogical introduction, see, for example, A. Riotto, "Theories of Baryogenesis", CERN-TH/98-204, [arXiv:hep-ph/9807454]; E. W. Kolb and M. S. Turner, in *The Early universe*, Frontiers in physics Vol 69, Westview (1994).

[46] A. Sakharov, “Violation Of Cp Invariance, C Asymmetry, And Baryon Asymmetry Of The Universe”, Pisma Zh.Eksp.Teor.Fiz.**5**32(1967), JETP Lett.**5**,24 (1967), Sov.Phys.Usp.**34** 392 (1991).

[47] M. Fukugita and T. Yanagida. “Baryogenesis without grand unification”, Phys. Lett., **B174**, 45 (1986); For a review, see, for example, S. Davidson, E. Nardi, and Y. Nir, “Leptogenesis”, Phys.Rept. **466**,105 (2008) [arXiv:0802.2962].

[48] For a review, see, for example, G. Jungman, M. Kamionkowski, and K. Griest, “Supersymmetric Dark Matter”, Phys. Rep. **267**, 195 (1996) [arXiv:hep-ph/9506380]; G. Bertone, D. Hooper, and J. Silk, “Particle Dark Matter: Evidence, Candidates, and Constraints”, Phys. Rep. **405**, 279 (2005)[arXiv:hep-ph/0404175].

[49] LHC Higgs Cross Section Working Group, Handbook of LHC Higgs cross sections: 1. Inclusive Observables [arXiv:1101.0593], 2. Differential Distributions [arXiv:1201.3084];https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CrossSections .

[50] http://www.atlas.ch/news/2011/simplified-plots.html

[51] ATLAS Collaboration, “Combination of Higgs Boson Searches with up to 4.9fb^{-1} of pp Collision Data Taken at a center-of-mass energy of 7 TeV with the ATLAS Experiment at the LHC”, ATLAS-CONF-2011-163 (2011); CMS Collaboration, “Combination of CMS searches for a Standard Model Higgs boson,” CMS PAS HIG-11-032 (2011); J. Erler, “Weighing in on the Higgs.” [arXiv:1201.0695 [hep-ph]].

[52] ATLAS collaboration, “Search for squarks and gluinos using final states with jets and missing transverse momentum with the ATLAS detector in sqrt(s) = 7 TeV proton-proton collisions”, CERN-PH-EP-2011-145 (2011) [arXiv:1109.6572[hep-ex]]; CMS collaboration, “Search for Supersymmetry at the LHC in Events with Jets and Missing Transverse Energy”, Phys. Rev. Lett **107** 221804 (2011),[arXiv:1109.2352[hep-ex]]; For recent result, ATLAS collaboration, “Hunt for new phenomena using large jet multiplicities and missing transverse momentum with ATLAS in 4.7 fb^{-1} of $\sqrt{s} = 7$ TeV proton-proton collisions”, [arXiv:1206.1760[hep-ex]].

[53] For a review, see, for example, R.Barbier, C.Berat, M.Besancon, M.Chemtob, A.Deandrea, E.Dudas, P.Fayet, S.Lavignac, G.Moreau, E.Perez, and Y.Sirois.

"R-parity violating supersymmetry", Phys.Rept.**420**,1 (2005), [arXiv:hep-ph/0406039]

[54] M. Dine, A. Kagan, and S. Samuel,"Naturalness in supersymmetry, or raising the supersymmetry breaking scale", Phys. Lett. **B243** 250(1990); S. Dimopoulos, G. F. Giudice, "Naturalness Constraints in Supersymmetric Theories with Non-Universal Soft Terms", Phys. Lett. **B357** 573 (1995) [arXiv:hep-ph/9507282]; A. Pomarol and D. Tommasini, "Horizontal Symmetries for the Supersymmetric Flavor Problem" Nucl. Phys. **B466** 3 (1996)[arXiv:hep-ph/9507462]; A. G. Cohen, D. B. Kaplan and A. E. Nelson, "The More Minimal Supersymmetric Standard Model", Phys. Lett. **B388** 588(1996) [arXiv:hep-ph/9607394].

[55] ATLAS collabortion, "Search for scalar bottom pair production with the ATLAS detector in pp Collisions at $\sqrt{s} = 7$TeV", CERN-PH-EP-2011-195(2011),[arXiv:1112.3832]; ATLAS collaboration, "Search for Scalar Top Quark Pair Production in Natural Gauge Mediated Supersymmetry Models with the ATLAS Detector in pp Collisions at $\sqrt{s} = 7$TeV", [arXiv:1204.6736].

[56] J. E. Kim and H. P. Nilles, "The $\mu-$Problem and the Strong CP Problem", Phys. Lett. **B**138, 150 (1984).

[57] J. Polchinski, "Effective Field Theory and the Fermi Surface" (lectures given in TASI 1992) [arXiv:hep-th/9210046].

[58] M. Drees, private discussion in seminar.

[59] J. E. Kim , H. P. Nilles and M. -S. Seo, "Singlet superfield extension of the minimal supersymmetric standard model with Peccei-Quinn symmetry and a light pseudoscalar Higgs boson at the LHC", [arXiv:arXiv:1201.6547[hep-ph]].

[60] For a review, see, for example, U. Ellwanger, C. Hugonie and A. M. Teixeira, "The Next-to-Minimal Supersymmetric Standard Model" Phys. Rep. **496** 1 (2010) [arXiv:0910.1785 [hep-ph]].

[61] K. Choi, private discussion.

[62] M. Cvetic, D.A. Demir, J.R. Espinosa, L. Everett, and P. Langacker, "Electroweak Breaking and the mu problem in Supergravity Models with an Additional U(1)", Phys.Rev.**D56**,2861 (1997) Erratum-ibid.**D58**,119905(1998)[arXiv:hep-ph/9703317].

[63] M. Drees, "Supersymmetric models with extended Higgs sector", Int. J. Mod. Phys. **A4** (1989) 3635.

[64] D. J. Miller, R.Nevzorov, and P. M. Zerwas, "The Higgs Sector of the Next-to-Minimal Supersymmetric Standard Model", Nucl. Phys. **B 681** (2004) 3 [arXiv:0304049].

[65] ALEPH Collaboration, "Search for $\gamma\gamma$ decays of a Higgs boson produced in association with a fermion pair in e^+e^- collisions at LEP",Phys.Lett.**B487**, 241 (2000) [arXiv:hep-ex/0008004].

[66] S. Dimopoulos and H. Georgi, "Softly broken supersymmetry and SU(5)" Nucl. Phys. **B193**,150 (1981).

[67] K. S. Jeong, J. E. Kim, and M. -S. Seo, "Gauge mediation to effective SUSY through U(1)s with a dynamical SUSY breaking, and string compactification", Phys.Rev. **D84** 075008 (2011), [arXiv:1107.5613[hep-ph]]

[68] R. Kitano and Y. Nomura,"Supersymmetry, Naturalness, and Signatures at the LHC", Phys. Rev. D **73**, 095004 (2006) [arXiv:hep-ph/0602096];Masaki Asano, Hyung Do Kim, Ryuichiro Kitano, and Yasuhiro Shimizu, "Natural Supersymmetry at the LHC", JHEP **12** 019 (2010)[arXiv:1010.0692[hep-ph]]; R. Essig, E. Izaguirre, J. Kaplan, and J. G. Wacker, "Heavy Flavor Simplified Models at the LHC", JHEP **01** 074 (2012)[arXiv:1110.6443]; Y. Kats, P. Meade, M. Reece, and D.Shih, 'The Status of GMSB After 1/fb at the LHC",JHEP 02 115 (2012) [arXiv:1110.6444]; C. Brust, A. Katz, S. Lawrence, and R. Sundrum, "SUSY, the Third Generation and the LHC"[arXiv:1110.6670]; M. Papucci, J. T. Rutherman, and A. Weiler, "Natural SUSY Endures"[arXiv:1110.6926].

[69] T. Moroi, "Muon anomalous magnetic dipole moment in the minimal supersymmetric standard model", Phys. Rev. **D 53**, 6565 (1996)[arXiv:hep-ph/9512396].

[70] J. L. Feng and K. T. Matchev, "Supersymmetry and the anomalous magnetic moment of the muon", Phys. Rev. Lett. **86** 3480 (2001)[arXiv:hep-ph/0102146]; L. L. Everett, G. L. Kane, S. Rigolin, and L-T. Wang, "Implications of muon g-2 for supersymmetry and for discovering superpartners directly", Phys. Rev. Lett. **86** (2001) 3484 [arXiv:hep-ph/0102145]; E. A. Baltz and P. Gondolo, "Implications of muon anomalous magnetic moment for supersymmetric dark matter", Phys. Rev. Lett. **86** (2001) 5004 [arXiv:hep-ph/0102147]; U. Chattopadhyay and P. Nath, Upper limits on sparticle masses

from g-2 and the possibility for discovery of SUSY at colliders and in dark matter searches", Phys. Rev. Lett. **86** (2001) 5854 [arXiv:hep-ph/0102157].

[71] A. Delgado and M. Quiros, "The Least Supersymmetric Standard Model", Phys.Rev. **D85** 015001(2012) [arXiv:1111.0528 [hep-ph]].

[72] K. -S. Choi and J. E. Kim, *Quark and Leptons from Orbifolded Superstring*(Lecture notes in physics Vol. 696), Springer

[73] K. Becker, M. Becker, and J. H. Schwarz, *String Theory and M-Theory: A Modern Introduction* Cambridge University Press(2007); E. Kiritsis, *String Theory in a Nutshell*, Princeton University Press (2007); J. Polchinski, *String Theory* Vol 1 *An Introduction to the Bosonic String* and 2*Superstring Theory and Beyond* (Cambridge Monographs on Mathematical Physics), Cambridge University Press (2005); M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* Vol 1 *Introduction* and 2*Loop Amplitudes, Anomalies and Phenomenology*(Cambridge Monographs on Mathematical Physics), Cambridge University Press (1988).

[74] J. E. Kim, "Gauge mediated supersymmetry breaking without exotics in orbifold compactification", Phys. Lett. **B656** 207 (2007) [arXiv:0706.0293].

[75] P. Langacker, G. Paz, L.-T. Wang, and I. Yavin, "Z'-mediated Supersymmetry Breaking", Phys.Rev.Lett.**100**,041802(2008)[arXiv:0710.1632[hep-ph]]; P. Langacker, G. Paz, L.-T. Wang, and I. Yavin, "Aspects of Z'-mediated Supersymmetry Breaking", Phys.Rev.**D77**,085033(2008)[arXiv:0801.3693[hep-ph]].

[76] N. Craig, M. McCullough, and J. Thaler, "The New Flavor of Higgsed Gauge Mediation", [arXiv:1201.2179[hep-ph]]; N. Craig, M. McCullough, J. Thaler, "Flavor Mediation Delivers Natural SUSY", [arXiv:1203.1622[hep-ph]].

[77] N. Arkani-Hamed and H. Murayama, " Can the supersymmetric flavor problem decouple?", Phys. Rev. **D 56** R6733 [arXiv:hep-ph/9703259].

[78] C. Tamarit, "Decoupling heavy sparticles in hierarchical SUSY scenarios: Two-loop Renormalization Group equations", [arXiv:1204.2292], "Decoupling heavy sparticles in Effective SUSY scenarios: Unification, Higgs masses and tachyon bounds", [arXiv:1204.2645].

[79] G. D'Ambrosio, G.F. Giudice, G. Isidori, and A. Strumia, "Minimal Flavour Violation: an effective field theory approach", Nucl.Phys.**B645**,155-187

(2002); For a pedagogical introduction, see, for example, A. J. Buras, "Minimal Flavour Violation", Acta Phys.Polon.**B34**,5615 (2003).

[80] S. L. Glashow, J. Iliopoulos, and L. Maiani,"Weak Interactions with Lepton-Hadron Symmetry", Phys. Rev. **D2**, 1285(1970); For $B-\bar{B}$ mixing, J. S. Hagelin, "Mass mixing and CP violation in the $B-\bar{B}$ system", Nucl. Phys. **B193** (1981) 123..

[81] S. Herrlich and U. Nierste, "Enhancement of the K_L-K_S Mass Difference by Short Distance QCD Corrections Beyond Leading Logarithms", Nucl.Phys. **B419** 292 (1994)[arXiv:hep-ph/9310311].

[82] J. S. Hagelin, S. Kelly and T. Tanaka, "Supersymmetric flavor-changing neutral currents: exact amplitudes and phenomenological analysis", Nucl. Phys. **B415** 293 (1994);F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini,"A complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model", Nucl. Phys. B477 (1996) 321[arXiv:hep-ph/9604387]; A. Masiero and L. Silvestrini, "wo Lectures on FCNC and CP Violation in Supersymmetry",[arXiv:hep-ph/9711401].

[83] For a review, see, for example, G. F. Giudice and R. Rattazi, "Theories with gauge-mediated supersymmetry breaking", Phys. Rep. **322**, 419 (1999) [arXiv:hep-ph/9801271]; G. F. Giudice and R. Rattazzi,"Extracting Supersymmetry-Breaking Effects from Wave-Function Renormalization", Nucl. Phys. **B511**, 25 (1998)[arXiv:hep-ph/ 9706540]; N. Arkani-Hamed, G. F. Giudice, M. A. Luty, and R. Rattazzi, "Supersymmetry-breaking loops from analytic continuation into superspace", Phys. Rev. **D58**, 115005 (1998)[arXiv:hep-ph/9803290].

[84] G. F. Giudice, M. Nardecchia and A. Romanino, "Hierarchical Soft Terms and Flavor Physics", Nucl. Phys. **B813** (2008) 156 [0812.3610 [hep-ph]].

[85] G. F. Giudice, G. Isidori, and P. Paradisi, "Direct CP violation in charm and flavor mixing beyond the SM", [arXiv:1201.6204],

[86] L. Wolfenstein, "Parametrization of the Kobayashi-Maskawa Matrix", Phys. Rev. Lett. **51**, 1945 (1983).

[87] Y.H. Ahn, H.-Y. Cheng, S. Oh, "Remarks on the Qin-Ma Parametrization of Quark Mixing Matrix",Phys.Lett. **B701** 614 (2011) [arXiv:1105.0450 [hep-ph]].

[88] J. E. Kim and M. -S. Seo, "Parametrization of the CKM matrix", Phys. Rev. **D84** 037303 (2011) [arXiv:1105.3304 [hep-ph]]

[89] A. J. Buras and R. Fleischer, in *Heavy Flavours II* ed. by A. J. Buras and M. Lindner, World Scientific (1998); N. Qin and B. -Q. Ma, "A new simple form of quark mixing matrix", Phys.Lett.**B695**,194 (2011)[arXiv:1011.6412[hep-ph]];N. Qin and B. -Q. Ma, "Parametrization of fermion mixing matrices in Kobayashi-Maskawa form", Phys.Rev.**D83**,033006 (2011)[arXiv:1101.4729[hep-ph]].

[90] C. Jarlskog, "Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Nonconservation", Phys. Rev. Lett. **55**, 1039 (1985); C. Jarlskog, "A Basis Independent Formulation of the Connection Between Quark Mass Matrices, CP Violation and Experiment", Z. Phys. **C29**, 491(1985); C. Jarlskog, in *CP Violation* ed. by C. Jarlskog, World Scientific, (1989).

[91] T. Suzuki, "Some Formulas for Invariant Phases of Unitary Matrices by Jarlskog", J.Math.Phys.**50**,123526(2009)[arXiv:0907.2353[math-ph]].

[92] J. E. Kim and M. -S. Seo, "Comment on "Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Nonconservation"", [arXiv:1201.3005 [hep-ph]].

[93] C. D. Froggatt and H. B. Nielsen, "Hierarchy of Quark Masses, Cabbibo Angles and CO-Violation", Nucl. Phys. **B147** 277(1979).

[94] S. Weinberg, "The Problem of Mass", in *Festschrift for I. I. Rabi*, Trans. New York Aced. Sci. **38** 185 (1977); H. Fritzsch, "Quark Masses and Flavor Mixing", Nucl. Phys. **B155**, 189 (1979).

[95] P. Ramond, R. G. Roberts, and G. G. Ross, "Stitching the Yukawa Quilt", Nucl. Phys. **B406**, 19 (1993)[arXiv:hep-ph/9303320].

[96] For a review, see, for example, Hajime Ishimori, Tatsuo Kobayashi, Hiroshi Ohki, Hiroshi Okada, Yusuke Shimizu, and Morimitsu Tanimoto, "Non-Abelian Discrete Symmetries in Particle Physics", Prog. Theor. Phys. Suppl. **183** 1 (2010)[arXiv:1003.3552[hep-th]]

[97] P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal Mixing and the Neutrino Oscillation Data", Phys. Lett. **B557**, 76(2003)[arXiv:hep-ph/0202074]; For model for it, see, for example, P. F. Harrison and W. G. Scott, "Permutation Symmetry, Tri-bimaximal Neutrino Mixing and the S_3

Group Characters, Phys. Lett. **B557**, 76 (2003)[arXiv:hep-ph/0302025]; E. Ma and G. Rajasekaran, “Softly broken A_4 symmetry for nearly degenerate neutrino masses, Phys. Rev. **D64**, 113012 (2001)[arXiv:hep-ph/0106291]; A. Zee, Obtaining the neutrino mixing matrix with the tetrahedral group, Phys. Lett. **B630** 58 (2005) [arXiv:hep-ph/0508278]; G. Altarelli and F. Fergulio, Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions, Nucl. Phys. **B720** 64 (2005) [arXiv:hep-ph/0504165]; Tri-bimaximal neutrino mixing, A4 and the modular symmetry , Nucl. Phys. **B741** 215(2006) [arXiv:hep-ph/0512103]; E. Ma, Tribimaximal neutrino mixing from a supersymmetric model with A4 family symmetry , Phys. Rev. **D73**, 057304 (2006) [arXiv:hep-ph/0511133];X. -G. He, Y. -Y. Keum, and R. R. Volkas, “A_4 Flavour Symmetry Breaking Scheme fir Understanding Quark and Neutrino Mixing Angles”, JHEP **04**,039 (2006)[arXiv:hep-ph/0601001]; G. Altarelli, F. Fergulio and Y. Lin, “Tri-bimaximal neutrino Mixing from orbifolding”, Nucl. Phys. **B775** 31 (2007) [arXiv:hep-ph/0610165]; S. F. King and M.Malinsky, “A_4 family symmetry and quark-lepton unification”,Phys. Lett. **B645** 351(2007) [arXiv:hep-ph/00610250]; M. Honda and M.Tanimoto, “Deviation from tri-bimaximal neutrino mixing in A_4 flavor symmetry”, Prog. Theor. Phys. **119** (2008) 583[arXiv:0801.0181 [hep-ph]]; T. Fukuyama, “Mass matrix modelbroken from A_4 to $2 \leftrightarrow 3$ symmetry”, arXiv:0804.2107[hep-ph]; F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, “Lepton flavour violation in models with A_4 flavour symmetry”,Nucl. Phys. **B809** 218 (2009)[arXiv:0807.3160 [hep-ph]]; S. Baek and M. C. Oh, “Neutrino mass matrix in triplet Higgs models with A4 symmetry”, Phys.Lett. **B690** 29(2010) [arXiv:0812.2704 [hep-ph]]; F. Bazzocchi, L. Merlo and S. Morisi, “Fermion masses and mixings in a S4 based model” ,Nucl. Phys. **B816** 204(2009)[arXiv:0901.2086 [hep-ph]]; M. Hirsh, S. Morisi and J. W. F. Valle, “A_4-based tri-bimaximal mixing within inverse and linear seesaw schemes”, Phys. Lett.**B679**454 (2009)[arXiv:0905.3056 [hep-ph]]; W. Grimus, L. Lavoura and P. O. Ludl, “Is S_4 the horizontal symmetry of tri-bimaximal lepton mixing? ”, J. Phys. **G36** 115007 (2009)[arXiv:0906.2689 [hep-ph]]; C. Hagedorn, E. Molinaro and S. T. Petcov, “Majorana phases and leptogenesis in see-saw models with A_4 symmetry”, JHEP **09** 115 (2009) [arXiv:0908.0240[hep-ph]]; A. Hayakawa, H. Ishimori, Y. Shimizu, and M. M. Tanimoto, “Deviation from tri-bimaximal mixing and flavor symmetry breaking in a seesaw type A_4 model”, Phys. Lett. **B680** 334 (2009) [arXiv:0904.3820 [hepph]]; Y. Lin, Tri-bimaximal neutrino mixing from A_4

and $\theta_{13} \sim \theta_C$", Nucl. Phys. **B824** 95 (2010) [arXiv:0905.3534[hep-ph]].

[98] S. Pakvasa and H. Sugawara, "Discrete Symmetry and Cabibbo Angle", Phys. Lett. **B73**, 61 (1978).

[99] C. Giunti and M. Tanimoto, "CP-violation in bilarge lepton mixing", Phys. Rev. **D 66**113006 (2002) [hep-ph/0209169];P.H. Frampton, S.T. Petcov and W. Rodejohann, "On deviations from bimaximal neutrino mixing", Nucl. Phys. **B 687**31 (2004) [hep-ph/0401206]; C.H. Albright, "Bounds on the neutrino mixing angles for an SO(10) model with lopsided mass matrices", Phys. Rev. **D 72** 013001 (2005) [hep-ph/0502161]; A. Datta, L. Everett and P. Ramond, "Cabibbo haze in lepton mixing", Phys. Lett. **B 620** 42 (2005)[hep-ph/0503222]; S. Antusch and S.F. King, "Charged lepton corrections to neutrino mixing angles and CP phases revisited," Phys. Lett. **B 631** 42 (2005) [hep-ph/0508044]; F. Plentinger and W. Rodejohann, "Deviations from tribimaximal neutrino mixing", Phys. Lett. **B 625**264 (2005) [hep-ph/0507143]; J. E. Kim and J. -C. Park, Quantum numbers of heavy neutrinos, tri-bi-maximal mixing through double seesaw with permutation symmetry, and comment on $\theta_{\rm sol} + \theta_C \simeq \pi/4$, JHEP**05** 017 (2006)[arXiv:hep-ph/0512130]; A.Y. Smirnov, "Neutrinos: '$\cdots$ Annus mirabilis??", in *Venice 2003, Neutrino Oscillations* [arXiv:hep-ph/0402264]; M. Raidal, "Relation between the neutrino and quark mixing angles and grand unification", Phys. Rev. Lett. **93** 161801(2004) [hep-ph/0404046]; H. Minakata and A.Y. Smirnov, "Neutrino mixing and quark lepton complementarity", Phys. Rev. **D 70**073009 (2004) [hep-ph/0405088]; P.H. Frampton and R.N. Mohapatra, "Possible gauge theoretic origin for quark-lepton complementarity", JHEP **01** 025 (2005)[hep-ph/0407139]; S.K. Kang, C.S. Kim and J. Lee, Importance of threshold corrections in quark-lepton complementarity, Phys. Lett. **B 619** 129 (2005) [hep-ph/0501029];J. Ferrandis and S. Pakvasa, "Quark-lepton complementarity relation and neutrino mass hierarchy", Phys. Rev. **D 71**033004 (2005) [hep-ph/0412038]; S. Antusch, S.F. King and R.N. Mohapatra, "Quark lepton complementarity in unified theories", Phys. Lett. **B 618**150 (2005) [hep-ph/0504007]; S.F. King, "Predicting neutrino parameters from SO(3) family symmetry and quark-lepton unification", JHEP **08**105 (2005) [hep-ph/0506297]; M. Lindner, M.A. Schmidt and A.Y. Smirnov, "Screening of Dirac flavor structure in the seesaw and neutrino mixing", JHEP **07** 048 (2005) [hep-ph/0505067]; T. Ohlsson, "Bimaximal fermion mixing from the quark and leptonic mixing matrices", Phys. Lett. **B622** 159(2005)

[hep-ph/0506094]; H. Minakata, “Quark-lepton complementarity: a review”, [arXiv:hep-ph/0505262].

[100] J. E. Kim and M. -S. Seo, “Quark and lepton Mixing Angwith a Dodeca-Symmetry”, JHEP**02** 097 (2010) [arXiv:1005.4684[hep-ph]].

[101] A. Blum, C. Hagedorn, and M. Lindner, “Fermion Masses and Mixings from Dihedral Flavor Symmetries with Preserved Subgroups”, Phys. Rev. **D77** 076004 (2008)[arXiv:0709.3450]; A. Blum, C. Hagedorn, and Hohenegger, “θ_C from the Dihedral Flavor Symmetries D_7 and D_{14}”, JHEP **03** 070 (2008)[arXiv:0710.5061].

[102] S. L. Glashow and S. Weinberg, ‘Natural Conservation Lwas for Neutral Current”, Phys. Rev. **D15** 1958 (1976).

[103] J. E. Kim and J. -C. Park, M. Linder, M. A. Schmidt, and A. Y. Smirnov, in [99]

[104] J. E. Kim, “Reason for SU(6) Grand Unification”, Phys. Lett. **B107** 69 (1981).

[105] T2K Collaboration, “Indication of Electron Neutrino Appearance from an Accelerator-Produced Off-Axis Muon Neutrino Beam”, Phys. Rev. Lett. **107**, 041801 (2011) [arXiv:1106.2822[hep-ex]]; MINOS Collaboration, “Improved search for muon-neutrino to electron-neutrino oscillations in MINOS”, Phys.Rev.Lett.**107** 181802(2011) [arXiv:1108.0015 [hep-ex]].

[106] G.L. Fogli, E. Lisi , A. Marrone , A. Palazzo, and A.M. Rotunno, “Evidence of $\theta_{13} > 0$ from global neutrino data analysis” Phys.Rev. **D84** 053007(2011)[arXiv:1106.6028 [hep-ph]].

[107] Double Chooz Collaboration, “Indication for the disappearance of reactor electron antineutrinos in the Double Chooz experiment”, Phys.Rev.Lett. **108**, 131801 (2012) [arXiv:1112.6353]; Daya Bay Collaboration, “Observation of electron-antineutrino disappearance at Daya Bay”, Phys. Rev. Lett. **108** 171803 (2012) [arXiv:1203.1669]; RENO Collaboration, “Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment”, Phys. Rev. Lett. **108** 191802 (2012) [arXiv:1204.0626].

[108] J. E. Kim and M. -S. Seo, “Parametrization of PMNS matrix based on dodeca-symmetry”, Int.J.Mod.Phys. **A27** 1250017 (2012) [arXiv:1106.6117 [hep-ph]].

[109] H. Primakoff and S. P. Rosen,“Double Beta Decay”, Rep. Prog. Phys. **22**, 121 (1959); For a recent review, see, for example, F. T. Avignone, III, S. R. Elliott and J. Engel, “Double Beta Decay, Majorana Neutrinos, and Neutrino Mass”, Rev. Mod. Phys. 80, 481 (2008)[arXiv:0708.1033[nucl-ex]].

[110] S. Yu. Khlebnikov and M. E. Shaposhnikov,“The statistical theory of anomalous fermion number non-conservation”, Nucl. Phys. B308885(1988); P. Arnorld and L. McLerran, “Sphalerons, small fluctuations, and baryon-number violation in electroweak theory”, Phys. Rev. D36, 581 (1987); J. A. Havey and M. S. Turner, “Cosmological baryon and lepton number in the presence of electroweak fermion-number violation”, Phys. Rev. **D42**, 3344 (1990);B.A. Campbell, S. Davidson, J. Ellis, and K. A. Olive,“On the baryon, lepton-flavour and right-handed electron asymmetries of the universe”, Phys. Lett. **B297**, 118(1992);M. Shaposhnilkov, “Sphaleron induced baryon number non-conservation”, Nucl. Phys. **B**(Proc. Suppl.)**26**, 78 (1992).

[111] J. Liu and G. Segré,“Reexamination of generation of baryon and lepton number asymmetries by heavy particle decay”, Phys. Rev. **D48**, 4609 (1993)[arXiv:hep-ph/9304241]; M. Flanz, E. A. Paschos, and U. Sarkar,“ Baryogenesis from a lepton asymmetric universe”, Phys. Lett. **B345**248 (1995) [arXiv:hep-ph/9411366]; Phys. Lett. **B384**487E (1995) ; L. Covi, E. Roulet, and F. Vissanio,“CP violating decays in leptogenesis scenarios”, Phys. Lett. **B384** 169(1996) [arXiv:hep-ph/9605319]; W. BuchmAuller and M. PlAumacher,“CP asymmetry in Majorana neutrino decays”, Phys. Lett. **B431** (1998) 354 [arXiv:hep-ph/9710460]; W. BuchmAuller and M. PlAumacher, “Matter antimatter asymmetry and neutrino properties”, Phys. Rep. **320**329 (1999) [arXiv:hep-ph/9904310];A. Pilaftsis, “CP violation and baryogenesis due to heavy Majorana neutrinos”, Phys. Rev. **D56**, 5431 (1997)[arXiv:hep-ph/9707235].

[112] S. Davidson, J. Garayoa, F. Palorini, and N. Rius,“Insensitivity of flavoured leptogenesis to low energy CP violation”, Phys. Rev. Lett. 99 (2007) 161801 [arXiv:0705.1503[hep-ph]].

[113] S. Pascoli, S. T. Petkov, and W. Rodejohann, “On the connection of leptogenesis with low energy CP violation and lepton flavor violating charged lepton decays”, Phys. Rev. **D68**093007(2003)[arXiv:hep-ph/0302054].

[114] J. A. Casas and A. Ibarra,“Oscillating neutrinos and $\mu \to e\gamma$”, Nucl. Phys. **B618**, 171 (2001) [arXiv:hep-ph/0103065].

Abstract

입자의 종류마다 다른 대칭성에 관한 연구

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본 논문에서는, 표준모형으로 설명하지 못하는 입자 물리의 문제들을 다루기 위하여 다양한 대칭성, 주로 입자의 종류마다 다른 대칭성을 이용한 표준모형의 확장을 논의한다. 우선, 위계 문제에 대한 설명으로, 초대칭을 도입한다. 전약 상호작용이 깨지는 스케일의 근원을 페차이-퀸 대칭성에 의한 다음으로 최소인 표준모형의 초대칭 확장으로 이해한다. 위계 문제를 푸는 가장 간단한 경우로, 세번째 세대 짝쿼크들이 가벼운 유효초대칭을 다룬다. 유효초대칭에서의 짝쿼크 질량들은 입자의 종류다마 다른, 즉 세번째 세대에 작용하지 않는 U(1)′ 대칭성을 도입하여 설명한다. 이러한 입자마다 다른 대칭성은 입자들의 질량 위계와 섞임을 설명하는데 중요한 역할을 한다. 특히, 섞임 행렬은 최대 CP깨짐과 같은 고유한 특징을 반영한 기술로 이해할 수 있다. 섞임각들은 입자 종류마다 다른 대칭성으로 설명할 수 있다. D_{12} 군을 이용하여 카비보각 15^o, 태양섞임 30^o, 대기 섞임 45^o 를 얻는다. 이들 값은 어느 정도 크기를 가지는 θ_{13} 등 최근 중성미자 실험 결과에 의하여 수정되어야 한다. 본 논문에서 다루었듯이, 입자 종류마다 다른 대칭성은 표준 모형

너머의 새로운 물리가 될 수 있다.

Keywords : 표준모형, 대칭성, 플래버에 의존하는 대칭성, 초대칭, 힉스, 불연속대칭성, 섞임행렬
Student Number : 2006-20333

감사의 글

종종 시간의 흐름에 대해 생각하게 됩니다. 정말 당연하게 옆에 있다고 생각하는 것들도 시간이 지나면 사라지고 잊혀집니다. 그런 것들 중에 문득 생각이 나서 다시 한번 보고 싶는 것들이 있곤 합니다. 그립다 라고 하는 감정이겠지요. 10여년 동안 서울대학교에서 물리를 공부하는 것에 너무 익숙해져 버린 것이 아닌가 하는 생각이 들었습니다. 당연히 날 새면 내가 있어야 할 곳이었지만 어느 순간부터 이제 헤어져야 할 때가 아닐까 하는 생각이 듭니다. 그렇게 떠나고 한참 시간이 지나면 다시 그리워질까요.

공부라는 것이 참 익숙한 말이지만, 어떨 때는 낯설게 느껴지기도 합니다. 인간이 생각을 하게 되면서 자신에 대해, 그리고 자신의 주변에 대해 이해하려고 했고, 때로는 지극히 자기 중심적으로 가다가 때로는 완전히 반대로 가려고 하는 일이 여러 번 있었던 것 같습니다. 그런 여러 과정을 거치면서 꽤 주변에 대해 멋지게 이해하고 있다고 생각되기는 하지만 곧바로 다시 궁금한 것이 생겼지요. 제가 몸담고 있는 입자물리를 생각해 보면 정말 인간이 이런 것을 생각해 낼 수 있구나 하고 감탄하면서도 어떨 때는 내가 하고 있는 일이 정말 자연을 이해하는 '옳은' 방향일까 하는 의문이 들기도 합니다. 그래서 괴테가 이야기했듯이 인간은 노력하면 할 수록 헤매는 법일지도 모르겠습니다. 정말 물리 공부를 한다고 느끼는 것은, 제 생각에는 단순하게 뭔가를 찾아내는 것에서 즐거움을 느끼는 것이 아니라 정말 그것이 무엇을 의미하는지, 어떤 '전체'가 뒤에 있는지 궁금해 하는 것이 아닐까 싶습니다. 그래서 그것에 충실하자고 노력을 해 온 것 같지만, 그다지 만족스럽다고 생각하지는 않습니다. 때로는 앞에 있는 것에 너무 신경을 쓰다가 옆에 있는 볼만한 것들을 지나치기도 했고, 많은 것을 포기한 것 같기도 합니다. 인간 관계가 넓은 것도 아니고 그렇다고 좋은 인상을 주는 사람이 된 것도 아니고. 아주 똑똑해서 튀는 사람도 아닌데 그나마 '이해'를 하기 위해 헤매다 보니 어떨

때는 질문을 던지는 방법도 잊어버린 것 같습니다. 어떨 때는 선택한 것도 만족스럽지 않고 놓친 것도 많은데 내가 정말 가지고 있는 것이 무엇일까 하는 생각이 들기도 합니다. 정말 제대로 헤매고 있는 것 같지요. 그 길이 어디까지 갈지, 제 삶에 어떤 결과를 줄지는 저도 모르겠습니다. 항상 그런 불안함을 가지면서도 앞으로 갈 수밖에 없는 것이 인생일까요.

제가 이렇게 모자란 점이 많음에도 이 정도나마 할 수 있었던 것은 많은 분들의 도움이 있었기 때문입니다. 사람과 사람의 만남 혹은 사람과 사람의 관계에는 사람의 힘만으로는 할 수 없는 어떤 것이 있는 것이 아닐까 하는 생각이 듭니다. 앞으로도 많은 사람들과 만나고 많은 관계가 만들어지겠지만, 지금까지의 일들을 생각해 보면 감사하고 싶은 분들이 참 많은 것 같습니다.

우선 저와 30여년 동안 함께 해 주신 엄마께 가장 감사드립니다. 그동안 많은 힘드신 일이 있었음에도 저를 항상 먼저 생각해 주셨고 공부 이외의 일로 불편함이 없도록 많은 것을 도와주셨습니다. 이제 건강도 챙기시고 여유를 많이 가지셨으면 합니다. 매일 보지만 매일 걱정되고 매일 생각이 나게 됩니다. 저도 자신 있게 엄마의 좋은 아들이라고 말할 수 있다면 참 좋겠습니다. 언제가 될지 몰라서 안타깝지만 노력해 보겠습니다.

누나도 이제 직장 생활에 어느 정도 안정되어 가는 것 같아서 다행입니다. 좋은 인연 만나서 행복하게 사시길,

6년 동안 지도교수님으로써 많은 도움을 주신 김형도 교수님께도 감사드립니다. 제가 모자람이 많은 사람이라 그다지 이해가 빠른 사람이 아닌데, 이것 저것 설명해 주실 때 많은 인내심을 발휘하신 것 같습니다.

대학원 생활 동안 아마 가장 큰 영향을 미치신 분이라면 김진의 교수님이겠지요. 평소때 감사하다는 말을 많이 못 한 것 같아서 죄송합니다. 같이 연구하면서 여러 가지 많은 것을 가르쳐 주셨고, 제가 가지고 있는 입자물리에 대한 관점에 가장 많은 영향을 주셨습니다. 입자물리에 대한 여러 가지 이야기를 듣고 논문을 쓰면서 이것 저것 찾아보고 의논할 때가 대학원 생활에서 가장 재미있었던 순간이었던 것 같습니다. 생각해 보니 gmail 에 있는 편지들의 90%가 교수님과 교환한 것들이군요. 어리 버리한 저에게 많은 배려를 해

주신 것도 정말 감사합니다.

바쁘신 와중에 심사에 참여해 주신 교수님들도 계셨습니다. 최기운 교수님께서는 졸업 논문의 초대칭과 Higgs에 관한 논의에 많은 도움을 주셨습니다. 오류를 바로잡고 놓친 부분을 생각할 수 있게 되어 정말 감사합니다. 이수종 교수님과 이원종 교수님, 김수봉 교수님께도 감사드립니다.

대학원 생활을 해 오면서 연구원으로, 혹은 선배로 서울대를 거쳐가신 많은 분들이 계셨습니다. 계범석 교수님, 박성찬 교수님, 신서동 박사님, 이현민 박사님, 최기영 교수님, 최강신 교수님과는 많은 주제에 대해 이야기할 기회가 있었고, 그때마다 많은 것을 배울 수 있었습니다. 앞으로 논문 쓸 기회가 있었으면 하는 생각이 듭니다. 금용연 교수님, 김연우 박사님 모두 배울 것이 많은 분들인데 제대로 물리 이야기해 볼 기회가 많이 없어서 많이 아쉽습니다. 박종철 박사님, 허지행 박사님, 배규정 박사님, 김도윤 박사님, 김지훈 박사님 모두 연구실 선배로 많은 도움을 주셨던 것 같습니다. 모두 계실 때 연구실 분위기가 많이 떠들썩 해서 한편으로는 이렇게 말을 많이 해도 될까 하는 생각도 들고, 한편으로는 뭔가 한다는 것이 생산적인 것 같아서 좋기도 했습니다. 입자물리를 하는 분들이야 워낙 개성이 많다보니 그런 것일까요. 후배로써 헛소리도 하고 해서 좀 답답하셨을지도 모르겠는데, 저도 이제 벌써 졸업할 때가 되고 나니 제가 아직도 모르는 것이 너무 많은 것이 아닌가 하는 생각마저 듭니다.

후배인 모도영과 곽혜정, 정태현의 건승을 바랍니다. 좋은 연구 주제를 찾고 좋은 연구를 할 수 있기를. 그리고 좀 더 깊은 생각을 할 수 있기를 바라며...

입자물리를 하면서 많은 만남이 있어왔고, 저에게 영향을 주신 분 들도 참 많은 것 같습니다. 학부때부터 많은 것을 보여주신 송재원선배, 제 연구실 혹은 옆 방 등에서 열심히 일하고 있는 박재성, 배진범씨, 곽승호씨, 김재원씨, 김희연씨 등, 격자이론방에서 저는 잘 모르는 세계를 공부해 오신 김형진씨, 윤보람씨, 김장호 등등... 일일이 적기 힘든 많은 사람들과 많은 이야기를 해왔고, 인간적으로나 학문적으로나 여러 가지 감사할 점이 많은 것 같습니다.

이제 저도 졸업을 합니다. 앞으로 어떻게 될지, 궁금하기도 하고 걱정되기도 하지만, 1학년 때 천문학을 가르쳐 주시며 깊은 인상을 남기신 홍승수 교수님 말씀대로 일단 코가 가리키는 방향으로 한번 가 보려고 합니다. 인생이 불안하면서도 재미가 있는 점은 아마 예측하기 참 힘들다는 점이겠지요.