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Diffusion of Innovation in Small-world Networks with Social Interactions

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Abstract

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The advent of the Internet, mobile communications, and social network services has stimulated social interactions among consumers, allowing people to affect one another's innovation adoptions by exchanging information more frequently and more quickly. Previous diffusion models, such as the Bass model, however, face limitations in reflecting such recent phenomena in society. These models are weak in their ability to model interactions between agents; they model aggregated-level behaviors only. The agent-based model, which is an alternative to the aggregate model, is good for individual modeling, but it is still not based on an economic perspective of social interactions so far.

This study assumes the presence of social utility from other consumers in the adoption of innovation and investigates the effect of individual interactions on innovation diffusion by developing a new model called the interaction-based diffusion model. By comparing this model with previous diffusion models, the study also examines how the proposed model explains innovation diffusion from the perspective of economics. In addition, the study recommends the use of a small-world network topology instead of cellular automata to describe innovation diffusion.

This study develops a model based on individual preference and heterogeneous social interactions using utility specification, which is expandable and, thus, able to encompass various issues in diffusion research, such as reservation price. Furthermore, the study proposes a new framework to forecast aggregated-level market demand from individual-level modeling. The model also exhibits a good fit to real market data. It is expected that the study will contribute to our understanding of the innovation diffusion process through its microeconomic theoretical approach.

Keywords: Innovation Diffusion, Agent Based Model, Interaction Based Model, Small-world Network, Demand Forecasting

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Chapter 1. Introduction

1.1 Research Background

Innovations, such as new products or new services, spread throughout the world via a process of diffusion. Following the pioneering work of Rogers (1962), which established the fundamentals of innovation diffusion, the modeling and forecasting of innovation diffusion has become an interesting topic in the field of social science, especially in new product marketing.

Innovation diffusion models experienced a turning point following advances in personal communication tools, such as the World Wide Web, mobile telephones, mobile Internet, and social network services. Such technologies rapidly accelerated interpersonal communication and innovation diffusion. It has become important to analyze diffusion patterns with considering social interactions among consumers. That is, a new approach that covers interpersonal interactions is necessary for diffusion research.

Such concern has already been propounded right after the important work of Bass (1969). Russell (1980) pointed out that, though Bass model is powerful for describing market data, its theoretical background is weak. Bass model has dominated the diffusion research field, but neither it nor its extensions can provide a concrete economic explanation of a model or a consideration of individual interactions. Though numerous modifications of the Bass model have been developed to cover various aspects of innovation diffusion, few research studies have tried to encompass individual interactions or to establish a fundamental theory of diffusion (Meade and Islam, 2006).

To overcome the limitations of the Bass model, a new research flow called the agent-based model (ABM) arose. The ABM approach generated a great deal of interest among diffusion researchers, since it enabled individual modeling and provided an easy way to perform experiments in various society networks. ABM studies have uncovered several new aspects of innovation diffusion that could not have been observed using an aggregated-level approach. However, ABM studies are not able to analyze aggregated-level market data without the help of individual-level surveys. Furthermore, despite the advantages of ABMs, few attempts have been made to develop a theoretical model.

To date, there is no consensus regarding the theoretical foundation of diffusion models. What is known is that the Bass model fits very well to real market data and that the concepts proposed by Rogers are still valid for understanding aspects of innovation diffusion. Why a Bass-shaped diffusion curve is found in most innovation diffusions, even though the Bass model does not encompass any explanatory variables or individual preferences, is the core question of this study.

This study focuses on social interactions among consumers as the key to understanding diffusion patterns. It is becoming more important to specify social interactions in diffusion models. Peres et al. (2010) developed a new definition for innovation diffusion, describing it as 'The process of the market penetration of new products and services that is driven by social influences[1].' This definition emphasizes the role of social interactions in innovation diffusion. Thus, social interaction is the keystone in this study's development of a new diffusion model.

1.2 Objective of the Study

This study uses a theoretical approach to investigate the role of social interactions in innovation diffusion. The study's main research objective is to develop a new diffusion model driven by social interactions among consumers. This model should have the following desirable properties.

First, the developed model should be based on microeconomic theory. The adoption of innovation is determined by the individual preferences of each consumer. Thus, utility theory is the best candidate for explaining innovation adoption.

Second, the model should be based on individual behaviors. Since innovation is usually adopted by individuals, individual-level modeling is required to understand the nature of innovation diffusion. The ABM approach is a good candidate for this.

Third, the model should reflect the nature of social networks. As Rogers (1962) stated, one important factor in diffusion rates is the degree of network interconnectedness. Thus, the model should be based on a realistic network topology.

Fourth, the model should allow for the heterogeneity of individuals. An assumption of heterogeneity is essential to differentiating individual behaviors from aggregated

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[1] Peres et al. (2010), p. 92.

behaviors. Particularly, the model should reflect, not only heterogeneity of taste, but also heterogeneity of interactions.

Fifth, the model should be flexible enough to draw various diffusion patterns. The flexibility of the model will determine its generalizability. It is necessary to clarify that the model should work properly in any circumstance. Thus, the model's flexibility must be investigated.

Last, but not the least, the model should be applicable to aggregated-level data. A lower dependence on data means a higher practicability within the model. If the model can explain aggregated-level data without any additional assumptions, then the model can also explain why aggregate models fit well to real market data.

In sum, the purpose of this study is to develop a generalized, theoretical, individual-level model to reflect the economics of social interaction and to explain aggregated-level behaviors of innovation diffusion.

1.3 Outline of the Study

The study consists of six chapters. In Chapter 2, an overview of innovation diffusion, based largely on Rogers (1962), is presented to facilitate an understanding of the research motivation of the study. Then, three categories of diffusion models are reviewed: the Bass model and its extensions, the agent-based model, and the individual behavior-based diffusion model. The characteristics and the pros and cons of each model are discussed. After that, a discrete choice model called the interaction-based model is introduced. This model is explained in detail, since it is the key to building the diffusion model proposed in this study. Finally, the limitations of previous literatures are summarized, and a desirable direction for the study is presented.

Chapter 3 proposes a new diffusion model called the interaction-based diffusion model (IBDM), which was developed by the author. Firstly, the details of the model's specifications are discussed. Then, the social network topology used in the model is introduced. A comparison of various topologies is presented, and literatures on social network topologies are reviewed briefly. Lastly, the diffusion process described in the model is explained in detail.

Chapter 4 shows the various simulation results of the IBDM. The basic assumptions used in the simulations are introduced first, along with reasonable explanations. Then, the characteristics of the model are examined through an investigation of various scenarios. The effects of each parameter on the diffusion patterns are discussed, and a comparison with the Bass model is presented. Through these experiments, the properties and contributions of the model are discussed.

Chapter 5 presents a rough fitting of the model to real market data in order to investigate the model's empirical availability. Firstly, the model's adjustment for estimation is discussed in detail. Then, the fitting methodology used in the analysis is explained. For the fitting, the study uses data on mobile subscriptions in three countries (Korea, Germany, and USA). After the details of the data and the fitting results are presented, the limitations of the model with regard to the empirical analysis are discussed.

Finally, Chapter 6 summarizes the implications of the study and discusses its contributions and limitations. Following this discussion, potential topics for future research are suggested.

Chapter 2. Literature Review

2.1 Overview of the Diffusion of Innovation

Ever since Schumpeter (1934) highlighted innovation as a core tenet of business activity, a vast body of research has explored innovation and its role in business through the fields of economics and business management. Three decades later, Everett Rogers (1962)[2] developed the idea of the 'Diffusion of Innovation,' which focuses on the process of the spread of innovation. This concept became an important consideration for establishing marketing strategy. In this sub-chapter, the key idea of innovation diffusion is demonstrated briefly to facilitate an understanding of its properties.

It is known that Gabriel Tarde (1895) conducted the first organized research into the diffusion of innovation (Kinnunen, 1996). His was a pioneering work on people's behaviors and social interactions in relation to social and cultural ideas. Tarde also introduced the concept of a distinction between invention and imitation.

Although Tarde was the first to introduce the concept of the diffusion of innovation, it may be fate that the famous work of Rogers, *Diffusion of Innovations*, served as the foundation for the entire modern field of diffusion of innovation. Rogers developed a firm definition of diffusion of innovation, describing it as 'The process by which an innovation is communicated through certain channels over time among the members of a social system.' In this definition, Rogers pointed out that there are four key elements that

[2] This is the first edition of his work; the fifth edition was published in 2003.

contribute to diffusion: innovation, communication channels, time, and social systems. In more detail, innovation encompasses whole objects that are newly recognized by individuals or groups, including ideas, practices, or goods. Communication channels refer to the means by which messages move from one individual to another, such as mass media and interpersonal channels. Time is an important element because it distinguishes diffusion research from other behavioral research fields. The last element—social system—refers to a set of interrelated units that are engaged in joint problem solving designed to accomplish a common goal.

The importance of Rogers' work in business studies can be summarized in two points: First, he introduced the five key factors that influence the rate of diffusion, and second, he categorized the adopters who participate in the product lifecycle of innovation. These two contributions are essential to our understanding of the diffusion process—and, thus, are essential to establishing an appropriate research objective.

The five key factors suggested by Rogers are as follows: perceived attributes of innovations, types of innovation decisions, communication channels, nature of the social systems, and the extent of change agents' promotion efforts. Firstly, perceived attributes of innovations consist of relative advantage, compatibility, complexity, trialability, and observability. Although this first factor is the only factor directly related to innovation itself, few research studies have explored the relation between the perceived attributes of innovations and their rates of diffusion. Most studies attempt to determine what important attributes exist (Fliegel and Kivlin, 1966) or how attributes relate to adopters' characteristics (Ostlund, 1974). Other researchers have focused on determine which attributes are the most important to adopters (Jansson, 2011; Lin, 2011; Makse and Volden, 2011). Srivastava et al. (1985) conducted one of the few studies showing a relationship between attributes of innovation and diffusion rates. However, this study was also limited in that it counted only information costs and losses of principal as attributes of innovation—an approach that differs significantly from that of Rogers' original factors. As Gerrard and Cunningham (2003) pointed out, each attribute has a complex effect on the diffusion rate—which is why it is hard to analyze the effect of any particular attribute.

The second factor is the type of innovation decision. Rogers suggests that there are three different types of innovation decisions: optional, collective, and authority. Optional decisions refer to decisions made by individuals; collective decisions require a consensus among group members; and authority decisions are made by a small group of members, such as experts or men in power. Since collective and authority decisions include the idea of social choice, most diffusion research studies focus only on optional decision making. (Mahajan et al., 1990) Some studies also show diffusion patterns in collective innovation decisions (Granovetter and Soong, 1983); however, they still focus on collective behaviors, not the diffusion rate.

The third factor, communication channels, is most important in diffusion research. Mass media channels influence society immediately and widely. By contrast, interpersonal channels influence people with similar socioeconomic backgrounds. The effects of mass media and interpersonal channels on diffusion rates were highlighted earlier, particularly in the context of the pioneering work of Bass (1969). The details of this factor are discussed later in the study.

Within the fourth factor—the social system—societal norms and the degree of network interconnectedness are considered to be related to diffusion rates. As the word 'norm' connotes, research studies on norms are mostly qualitative, focusing on social and cultural barriers to the diffusion of innovation (Greenhalgh et al., 2004). On the other hand, the degree of network interconnectedness is becoming more important in diffusion research, as the research on social networks themselves has advanced (Valente, 1995). Since this study also focuses on the effects of social networks, further details on this factor are provided later in the study.

The last factor that Rogers suggested—extent of change agents' efforts—concerns the change agent, or the individual who influences clients' innovation decisions in a direction deemed desirable by a change agency. The studies on change agents focus mostly on the agents' roles and methods in diffusion processes (Haider and Kreps, 2004).

As stated above, among the five factors, the most important factors for understanding diffusion rates in quantitative research are communication channels and social systems. This study also focuses on those two factors: social interactions within communication channels and small-world networks within social systems.

Another importance of Rogers' work comes from that he also suggested a categorization of adopters on the basis of innovativeness, as shown in Figure 1.

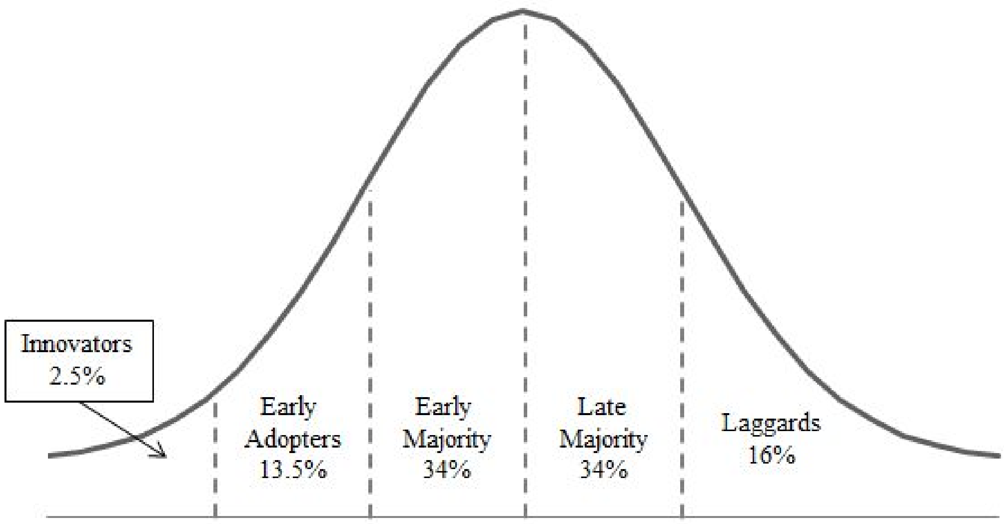

Figure 1. Adopter categorization on the basis of innovativeness[3]

As shown in Figure 1, Rogers defined five adopter categories: innovators, early adopters, early majority, later majority, and laggards. The percentages are derived from the base of mean and standard deviation of a normal distribution; for example, innovators are double standard deviation far from the mean.

This categorization is important for two reasons. First, it assumes that the product lifecycle of an innovation forms a bell shape over time. In other words, it can be said that such a lifecycle forms an S-shaped curve. This assumption motivated Bass (1969) to develop a mathematical model called the Bass model. Secondly, this categorization places the innovativeness of adopters along the time dimension, which means that more

[3] Source: Rogers (2003)

innovative people adopt innovations earlier. This is important because it implies that adopters in earlier stages and later stages are different—and, thus, that the aspect of diffusion varies over the dimension of time. Particularly, the role of innovators is very important to diffusion rate of an innovation because innovators import the innovation from outside the social system's boundaries.

Extensive studies have been done on diffusion rates based on Rogers' categorization and factors. Mathematical models for analyzing diffusion over time are called, simply, 'diffusion models.'

2.2 Diffusion Models

Diffusion models are developed to analyze diffusion rates and to forecast future aspects of diffusion. The first significant research into diffusion models was conducted by Mansfield (1961). Even before Rogers published his work, Mansfield pointed out the importance of innovation in enterprises and role of the imitation effect. Mansfield suggested a logistic function for the diffusion curve—an approach that was widely accepted by future researchers because it was is more powerful and simple than Rogers' (1962) or Bain's (1963) work, which suggested normal density as a diffusion curve. Following Mansfield's work, Chow (1967) developed the Gompertz diffusion model to overcome the symmetricity of the logistic curve.[4] Chow claimed that the diffusion rate is not symmetric, but, rather, that the rate is higher at early stages and declines in later

[4] Before Chow (1967), Gregg et al. (1964) introduced the Gompertz model, which included other functions.

stages.

These two models—the logistic and Gompertz models—are still commonly used in diffusion research. However, it would not be incorrect to say that Bass (1969) established a concrete and powerful foundation for the diffusion model, called the Bass model. In the following sub-chapter, the Bass model and its derivatives are introduced to develop our understanding of the major flow of diffusion research. Then, agent-based models (ABMs) and other models using individual choice are introduced to facilitate our understanding of the necessity of individual-based research.

2.2.1 Aggregate Models

It is very important to understand Frank M. Bass's (1969) pioneering work because, not only is it mathematically well-designed and powerful for practical use, but it also established a standard for diffusion model research. Models prior to the Bass model, such as those by Mansfield and Rogers, were limited in that their functional forms were too simple and considered only internal influences. Chow developed a more generalized form, but he still could not separate the effects of the various communication channels. The Bass model overcomes the problem of symmetricity and also provides appropriate explanations for the parameters.

The assumptions of the Bass model are as follows: First, the Bass model assumes only the first purchase of new products or services. That is, there are no repeated purchases in the market. This is because the research objective of diffusion models concerns the diffusion of innovation, which deals only with the first acceptance of newly introduced products—not with the products' demand curve. For this reason, Bass focused only on durable goods. Secondly, the Bass model assumes that there is a fixed market potential, which means that the total number of potential adopters is constant. This might seem to be strong; however, it can be explained fairly if we consider that a product's market potential can be the whole population of a society and that there is no further assumption for this in the Bass model.

The Bass model basically follows the functional form of a hazard function, which is used to explain the diffusion of contagions. Bass assumed that the likelihood of purchase at time T is as follows:

$$\frac{f(T)}{1-F(T)} = p + q \cdot F(T) \qquad \text{Eq. (1)}$$

where $f(T)$ is the likelihood of purchase at time T and $F(T)$ is its cumulative function. Bass explained p as 'Innovation factors,' which represent the external influences that promote the adoption of an innovation through mass media, such as advertising. q represents 'Imitation factors,' which are the internal influences (e.g., word-of-mouth) of previous adopters. The final diffusion curve can be derived from equation (1), as shown in equation (2).

$$\frac{dN(t)}{dt} = g(t)(\bar{N} - N(t)) = (p + \frac{q}{\bar{N}} N(t))(\bar{N} - N(t)) \quad \text{Eq. (2)}$$

The notation of this equation follows the adjustment of Mahajan and Peterson (1985). $N(t)$ is defined as the cumulative number of adopters at time t. Therefore, $dN(t)/dt$ becomes the rate of diffusion at time t—or, in the discrete time dimension, the sales at t. $\bar{N}$ is the market potential of the product and is assumed to be constant, as mentioned above. $g(t)$ is the adoption probability at time t, which determines the aspect of diffusion. If $g(t)$ follows the functional form of only $qN(t)$, equation (2) draws a logistic curve, which is a simple form of Mansfield's (1961) model.

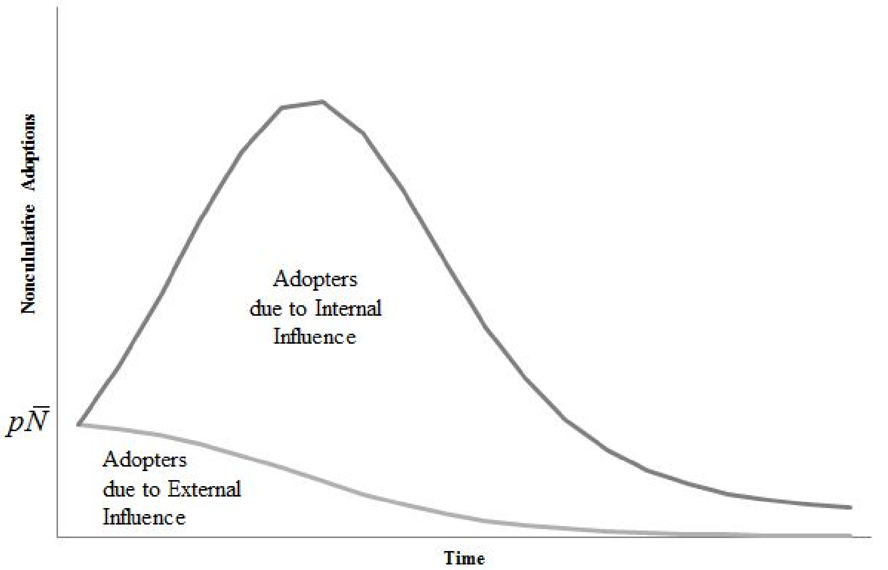

Figure 2. Adopters due to external and internal influences in the Bass model[5]

[5] Source: Mahajan et al. (1990)

As shown in Mahajan et al. (1990), the Bass model can separate the effects of external and internal influences on innovation adoption (Figure 2). As seen in the figure, the proportion of the external influence effect on diffusion diminishes over time, while the internal influence effect becomes greater in later periods. This implies that mass media is important only in the earlier periods, while the word-of-mouth effect is dominant in later periods—which is exactly what Rogers (1962) claimed in his book.

Although the Bass model is simple and works well with real market data, after a decade, Bass (1980) admitted that the model (1969) is incomplete because it does not consider typical and important economic variables (e.g. price), which certainly influence diffusion patterns. Bass (1980) attempted to construct a link between such economic variables and his model, but the altered model was still criticized by Russell (1980) for its complicatedness and weaknesses. Many attempts to include decision variables, such as price, have been made, and some have been significant (Jain and Rao, 1990; Jain 1992). One such attempt, which resulted in a powerful and convincing model, was developed by Bass and his colleagues. The model is called the generalized Bass model (GBM) (Bass et al., 1994)

The GBM simply added a function $x(T)$, which is defined as 'current marketing effort,' to the probability of adoption in equation (1), as described in the following equation.

$$\frac{f(T)}{1-F(T)} = [p + q \cdot F(T)] \cdot x(T) \cdots\cdots \text{Eq. (3)}$$

This $x(T)$ incorporates the effects of decision variables, such as price or advertising. Bass et al. (1994) proposed a basic mapping function for $x(T)$, as follows.

$$x(T) = 1 + \left[\frac{\Delta \Pr(T)}{\Pr(T-1)}\right]\beta_1 + \left[\frac{\Delta \mathrm{ADV}(T)}{\mathrm{ADV}(T-1)}\right]\beta_2 \cdots\cdots \text{Eq. (4)}$$

where $\Pr(T)$ is the price of the good at time T and $\mathrm{ADV}(T)$ is the amount of advertising expenditures at time T. Bass et al. (1994) show that the GBM fits very well with real market data and is superior to other diffusion models, including the original Bass (1969) model.

The GBM is widely used in diffusion research because it is simple, powerful, and easy to customize. As Bass et al. (1994) noted, $x(T)$ can be anything on which the researcher want to focus and can take various mathematical forms according to need. Although there is some criticism that the GBM implies an odd advertising strategy (Fruchter and Van den Bulte, 2011), the GBM is still the dominant model because it has been empirically validated many times and because it preserves the key properties of the Bass model. (Krishnan and Jain, 2006).

Other extensions have also released some of the assumptions of the Bass model (Meade and Islam, 1995, 2006). Several studies, such as those by Mahajan and Peterson

(1978) and Mahajan and Muller (1979), proposed a dynamic diffusion model, which assumes that the market potential $\overline{N}$ is not constant, but, rather, a function of exogenous variables. Lilien et al. (1981) developed a multi-adoption model, which allows for repeated purchases—something that is not allowed in the Bass model. Bayus (1987) and Bayus et al. (2000) proposed the multi-product model, which focuses on the relations among contingent goods. Norton and Bass (1987) developed the Norton-Bass (NB) model, which is also called the multi-generation diffusion model. The NB model concerns multi-generation products, such as random-access memory (RAM), assuming competition between earlier and later generations. Jiang and Jain (2012) extended the NB model to the generalized Norton-Bass model by separating consumers who have already purchased old versions of a product and those who have not. Mahajan and Peterson (1979) proposed the space and time diffusion model, which extends the Bass model by adding a space dimension to the analysis. Finally, there are also extensions of logistic and Gompertz models, such as the log-logistic model of Tanner (1978), the flexible-logistic model of Bewley and Fiebig (1988), and the local-logistic model of Meade (1985), for the logistic model, and the shifted Gompertz of Bemmaor (1994) and the gamma/shifted Gompertz of Bemmaor and Lee (2002) for the Gompertz model. These models are summarized in Table 1.

Table 1. Extensions of the Bass model and their characteristics

Model	Characteristic	References
Generalized Bass Model	Marketing effort is added to include price and advertising	Bass et al. (1994)
Dynamic Diffusion Model	Market potential is a function of exogenous variables	Mahajan and Peterson (1978) Mahajan and Muller (1979)
Multi-adoption Model	Repeated purchases are allowed	Lilien et al. (1981)
Multi-product Model	Two or more products effect one another	Bayus (1987) Bayus et al. (2000)
Multi-generation Model	There is competition between former and later generation	Norton and Bass (1987) Jiang and Jain (2012)
Space and time Model	Diffusion also spreads into the space dimension	Mahajan and Peterson (1979)

In spite of all of these extensions, the problem with the Bass model raised by Russell (1980) is still uncovered. Russell (1980) argued that the Bass model is not actually an individual-level model; thus, an economic explanation of the Bass model is difficult. Bass, himself, also admitted this limitation and suggested an individual-level diffusion model as a direction for future primary research (Bass, 2004). That is why Kiesling et al. (2012) classified such extensions of the Bass model and other growth models as aggregate models. Kiesling et al. (2012) also pointed out that aggregate models consider neither the heterogeneity of consumers nor social dynamics. To overcome this limitation, a new approach called the agent-based model (ABM) approach arose.

2.2.2 Agent-Based Models

The aggregated-level approach of Bass and its extensions generally assumes a large degree of homogeneity in the population of adopters (Goldenberg et al., 2000). It is more realistic to assume that, however, adopters have different preferences for new products/services based on their different characteristics. This is why Russell (1980) criticized the limitation of aggregated approaches, which originate from the simplified assumption of homogeneity and consider only aggregated levels of diffusion. Moreover, aggregate models have additional limitations, such as being inappropriate for answering what-if type questions (which are used to examine market responses in advance) and lacking predictive and explanatory powers (Kiesling et al., 2012).

To overcome these limitations of Bass-type diffusion models, agent-based modeling has been increasingly researched in the diffusion field in recent years (Kiesling et al., 2012). ABMs were originally developed in sociology. Schelling (1971) proposed a basic ABM to explain the segregation of social groups. Following Schelling's work, ABMs have been adopted in various fields of business-related research and applied by several large firms (Rand and Rust, 2011). ABMs are appealing as computational studies of systems of interacting autonomous entities, each with dynamic behaviors and heterogeneous characteristics (Heckbert et al., 2010). Since the 1980s, numerous ABM-related researches have been published in keeping with advances in computer technology (Macy and Willer, 2002). Moreover, ABMs have become more popular over time; since the 1990s, there has been an increasing trend in the number of related articles per year (Heath et al., 2009).

Eliashberg and Chatterjee (1985) suggested using ABMs in diffusion research to provide a better framework for understanding the diffusion process and a better tool for managerial action. They also developed the first adaptation of an ABM-like method for diffusion research (Chatterjee and Eliashberg, 1990). The model differs from aggregate diffusion studies because it is based on individual preferences and specific heterogeneity assumptions and because it establishes an appropriate link between individual decision-making and aggregate dynamics. However, it also suffers from a limitation, in that it does not include the aspect of social network. After a decade, Goldenberg et al. (2001), whose work attracted the interest of Bass (2004), led the advent of the use of ABM in the main flow of diffusion research. They proposed a simple process for diffusion, considering weak ties and strong ties in interpersonal contacts. Since the work of Goldenberg and his colleagues, there have been numerous studies with various characteristics.

ABMs can be categorized by their key criterion: the method of modeling consumer adoption behavior (Kiesling et al., 2012). There are various ways to model consumer adoption behavior, but the simple decision rules approach is dominant in the field. The simple decision rules approach assumes that an agent decides to adopt an innovation whenever his or her threshold is reached. For example, a simple decision rule can be described as follows (Alkemade and Castaldi, 2005): First, consumers who have already adopted an innovation talk about the product to their neighbors. Second, a consumer adopts the product if the word-of-mouth received from these adopters exceeds his or her 'exposure threshold' and does not exceed his or her 'over-exposure threshold.' Another way to explain a simple decision rule is given by Goldenberg et al. (2001), who used the following equation to describe the threshold for adopters.

$$p(t) = (1-(1-\alpha)(1-\beta_w)^j(1-\beta_s)^m) \quad \text{Eq. (5)}$$

where $p(t)$ is the individual threshold level, α represents the market effort (e.g., advertising), β_w represents interpersonal contacts from weak ties, and β_s is interpersonal contacts from strong ties. The numbers of weak ties and strong ties are indicated by j and m, respectively. Goldenberg et al. (2001) assumed that there is a randomly distributed threshold U for each adopter and that, if $U < p(t)$, then the individual adopts the innovation[6]. Bohlmann et al. (2010) proposed a probabilistic approach that assumes that there is a certain probability of reaching the threshold. These simple decision rules have advantages in their simplicity and ease of customization.

Another major approach is called the state transition approach. This kind of model sets multiple states of agents, such as 'potential adopter' and 'adopter.' Goldenberg and

[6] Goldenberg et al. (2001) regarded diffusion as a set of moves from a non-informed to an informed state.

Efroni (2001) conducted a pioneering work using this approach. They assumed that an agent can transform from state '0' (non-awareness) to state '1' (awareness) through spontaneous discovery or word-of-mouth. They computed a diffusion pattern from a constant probability of spontaneous discovery and a probability function based on the number of agents already in state '1.' The advantage of using the state transition approach is that it can model multiple states, such as 'awareness,' 'information seeking,' 'adoption,' and 'word-of-mouth spreading' (Thiriot and Kant, 2008). Using this approach, it is possible to overcome a limitation of the Bass (1969) model: that is, its lack of distinction between awareness and persuasion. However, the state transition model is still similar to the simple decision rules approach in the manner of diffusion process.

Even though utility is a natural candidate for modeling individual adoption decisions, there are few research studies using utilitarian approaches (Kiesling et al., 2012). In their pioneering ABM research study, Chatterjee and Eliashberg (1990) used utilities to model individual choice; however, their work does not show a diffusion pattern in a social structure. Delre et al. (2007) developed a utilitarian ABM with following equation.

$$U_{ij} = \beta_{ij} \cdot x_{ij} + (1-\beta_{ij}) \cdot y_{ij} \quad \text{Eq. (6)}$$

where U_{ij} is the utility of agent i from product j, x_{ij} represents social influence, y_{ij} is individual preference for the product, and β_{ij} is the weight of two components. It is assumed that agent i adopts the product if $U_{ij} \geq U_{ij,MIN}$ for the uniformly distributed constant $U_{ij,MIN}$. As Kiesling et al. (2012) criticized, however, Delre et al. (2007) and other utilitarian research studies do not differ from other threshold models. Kiesling et al. (2012) also commented that integrating utility into ABMs of innovation diffusion is still far off.

ABMs have several advantages over other modeling techniques commonly mentioned in review papers (Bonabeau, 2002; Garcia, 2005; Zenobia et al., 2009). First, ABMs are especially necessary in the case that the degree of heterogeneity increases in the modeled system (Bonabeau, 2002). The assumption of agents' heterogeneity enables ABMs to represent society dynamics more realistically than other models that assume an average behavior of individual agents (Garcia, 2005). From the perspective of innovation diffusion research, one of the biggest advantages of ABMs is that they describe both micro-level individual adoption and the macro-level diffusion phenomenon simultaneously.

Not only can ABMs be applied to flows, markets, organizations, and diffusion research (Bonabeau, 2002), but they can also be applied to almost every topic of social science because they represent bottom-up, fundamental microscopic modeling, which captures the emergent phenomena resulting from the interactions of individual entities. In the field of diffusion research, ABMs are particularly expected for developing new diffusion theories (Garcia and Jager, 2011; Zenobia et al., 2009). For example, Garcia

(2005) addressed the effects of network externalities, word-of-mouth networks, modeling tipping points, social networks and viral marketing as four possible research issues for ABMs. In this context, there exist various research studies that use ABMs as their tools for analysis.

Overall, previous ABM research studies can be classified into two categories: one for theoretical research studies that presented new ABM construction and/or simulation methods and another for empirical research studies that applied ABMs to specific innovations. Research studies in the former category have focused on various topics, such as the effects of supply chain policies (Amini et al., 2012), the targeting and timing of promotions (Delre et al., 2007), social influences from social networks and their structure (Laciana and Rovere, 2011; Pegoretti et al., 2012), the role of online sampling (Schlesinger and Parisi, 2001), and consumer and brand agents (Schramm et al., 2010).

Empirical researches using ABM, which are included in the latter category, have been applied to diverse sectors, such as agriculture (Berger, 2001; Rebaudo and Dangles, 2013; Schreinemachers et al., 2010); environmental policy and/or eco-innovations (Carrillo-Hermosilla, 2006; Desmarchelier et al., 2013; Schwarz and Ernst, 2009); automobile markets (Kim et al., 2011; Zhang et al., 2011); hardware/software markets (Lee et al., 2013; Zaffar et al., 2011); public health, including pharmaceutical markets (Perez and Dragicevic, 2009; Pombo-Romero et al., 2013; Rahmandad and Sterman, 2008); and new energy systems (Sopha et al., 2011).

Despite their numerous advantages and potential applicability, existing ABMs still have several limitations. For example, applying ABMs to the field of innovation diffusion, how to represent consumer adoption behaviors and social influences is a key aspect of modelling (Kiesling et al., 2012). However, in previous literatures, theoretical bases for modeling consumer adoption behaviors are obscure, and utilitarian approaches are particularly scarce. Second, despite substantial improvements, some previous research studies have adopted overly simple and strong assumptions about the interactions among agents. Furthermore, in their simulation processes, many previous research studies have failed to consider the characteristics of innovations and/or new products, resulting in insufficient implications from the perspectives of product design and pricing. Lastly, it is hard to find ABM research studies using only aggregated-level data, which increases the power of the model and enables a comparison with Bass-like models.

2.2.3 Diffusion models based on individual behavior

Diffusion patterns depend heavily on agents' preference structures. As seen in Sections 2.2.1 and 2.2.2, numerous studies have attempted to merge consumer preferences with aggregated-level sales. The discrete choice model is the most suitable model for understanding consumers' preference structures. Thus, several studies have used discrete choice models and innovation diffusion models.

To understand discrete choice models, it is important to know that discrete choice models are derived from the random utility model proposed by Marschak (1960). The random utility model assumes that a decision maker n obtains a certain level of utility U_{nj} from each alternative j and chooses an alternative i if and only if $U_{ni} > U_{nj}\ \forall j \neq i$. The utility can be decomposed into two parts: representative utility (V_{nj}) and the random part (ε_{nj}), which captures the utility that cannot be observed. This relationship is described in the following equation (Train, 2009).

$$U_{nj} = V_{nj} + \varepsilon_{nj} = \beta \cdot x_{nj} + \varepsilon_{nj} \quad \cdots\cdots \text{Eq. (7)}$$

As expressed in equation (7), representative utility is often specified as a linear function of observable attributes of alternative x_{nj}. Since there is no way to specify the random part, ε_{nj}, it is natural to set a probability distribution function of ε_{nj}. Luce and Suppes (1965) and McFadden (1973) found that, given a type-I extreme value distribution, the choice probability that consumer n chooses alternative i can be summarized very simply (Equation 8).

$$P_{ni} = \Pr(U_{ni} > U_{nj}\ \forall j \neq i) = \frac{e^{\beta x_{ni}}}{\sum_j e^{\beta x_{nj}}} \quad \cdots\cdots \text{Eq. (8)}$$

This is called the logit model. The logit model has been widely adopted in various research fields because it is simple and easy to estimate for empirical analyses. The adoption of innovation can be regarded as a binary choice situation, since an agent decides whether or not he/she is going to adopt the innovation. Thus, it seems natural to link the discrete choice model to the diffusion model. Nevertheless, only a few research studies have attempted to link them.

One of the difficulties in linking the discrete choice model and the diffusion model is that the discrete choice model is designed to analyze brand-level adoption, while the diffusion model focuses on market-level penetration. Lee et al. (2006) developed a framework using a dynamic utility function and the Bass model to forecast future product-level sales of large-screen TVs. In their first step, they estimated the coefficient β s in equation (7) using conjoint survey data. Assuming that the prices of TVs decrease exponentially, they derived a dynamic utility model that allowed for time-variant attributes. After estimating the Bass model for the whole TV market, they used the following equation to derive a product-level diffusion curve.

$$y_j(t) = Y(t)S_{jt} = Y(t) \cdot \frac{\left[\sum_{i=1}^{N} \left(\frac{e^{V_{ijt}}}{\sum_{k=1}^{J} e^{V_{ikt}}} \right) \right]}{N} \quad \text{Eq. (9)}$$

where $Y(t)$ is the estimated cumulative sales of all large-screen TVs and $y_j(t)$ is the cumulative sales of product j. S_{jt} is defined as the market share of product j at time

t. The market share is calculated from the logit formula of dynamic utility, V_{ijt} is the representative dynamic utility of agent i, N denotes the total number of survey respondents, and J denotes the total number of products. The study is significant in that it established a new framework and provided sufficient empirical implications; however, it is also limited in that it still depends heavily on the Bass model without modification.

Cho (2007) proposed new diffusion models using a utilitarian approach. From the simple utility structure described in the following equation, the study derived more than eight different models considering factors like prices, technological improvements, network externalities, and waiting costs.

$$U_i = V_i - \varepsilon_i \cdot P \quad \text{Eq. (10)}$$

where U_i denotes the utility of agent i, V_i denotes the representative utility, and P is the price of innovation. ε_i is defined as the marginal utility of income for agent i. By adding factors and assumptions, one of the proposed models, which is very similar to the Bass model, is derived as expressed in the following equation.

$$m_t = M\left(\frac{1-e^{-(\alpha+\gamma)t}}{1+\overline{B}e^{-\alpha t}}\right) \quad \text{Eq. (11)}$$

where m_t is the sales at time t; M denotes the size of the target market, which is smaller than the market potential; α represents the effect of technological innovation; and γ represents the experience effect. $\overline{B}$ is defined as the discounted difference in network externalities between two periods. The study contributed to establishing the link between the random utility model and the Bass model. The study has a limitation, however, in that it does not consider any individual interactions.

Dugundji and Gulyas (2008) introduced a nested logit model and a random utility model into ABMs. They set the utility as the following equation, derived choice probability using a nested logit model, calculated the probability by using individual-level data, and then drew diffusion series for modes of transportation, such as automobiles, motorcycles, and bicycles.

$$U_{ni} = \alpha_i + V(S_n, z_{in}) + \beta \frac{N_a}{N_g} \quad \text{Eq. (12)}$$

where U_{nj} denotes the utility of agent n from alternative i and V denotes the representative utility. α_n is an alternative specific constant. S_n represents the observable socio-demographic characteristics of agent n, and z_{in} represents observable attributes. The last term is important because it reflects the social interaction effect. N_g is defined as the total number of agents in a reference group and N_a is defined as the number of adopters in the group. β is the parameter to be estimated. The study shows the possibility of using the utilitarian approach in ABMs, while considering social interactions. However, the study does not have any relation to Bass or other diffusion models.

These individual choice-based approaches have a common limitation in that they require individual-level data and are not appropriate for analyzing aggregated market-level data. Furthermore, there is no consideration of social interaction in the random utility model. This is why it is important to explore the work of Brock and Durlauf (2001), who introduced the concept of social utility into the discrete choice model.

2.3 Interaction-Based Model

Historically, individual-level modeling was not a major concern in social science since there were doubts that aggregating individual behaviors would necessarily lead to the aggregate-level behavior. After Schelling's (1971) work, however, the idea of social interactions had begun to receive interest in the field of economics. Föllmer (1974) proposed an earlier approach to reflect interactions among agents in microeconomics. It assumes that the preference of an agent may be random and that this randomness may depend on the endowments of other agents. Two decades later, Brock (1993) developed a discrete choice model that incorporated 'social utility' by embedding the interaction effects from the neighbors of an agent. In the meantime, Durlauf (1993) proposed a model for interacting industries from the perspective of profit maximization, which supposes that the aggregate behavior of the last period affects current individual payoffs. Incorporating each of these perspectives, Brock and Durlauf (2000) suggested a discrete choice model called the 'interaction-based model,' for which they developed a finalized version in 2001.

Brock and Durlauf (2001) considers a binary choice situation of a total of I numbers of agents. The choice of each agent is i, denoted by an indicator variable ω_i. ω_i is set to have a value of 1 when an agent decides to adopt the innovation and of -1 when the agent does not. Each individual makes a decision to maximize his/her payoff function V as seen in the following equation.

$$\max_{\omega_i \in \{-1,1\}} V(\omega_i, \mathbf{Z}_i, \varepsilon_i(\omega_i)) \cdots\cdots \text{Eq. (13)}$$

where $\mathbf{Z}_i$ represents the vector of observable attributes and ε_i represents unobservable random shocks. Shocks are supposed to be relevant only for each choice; thus, $\varepsilon_i(1)$ and $\varepsilon_i(-1)$ are distinct. Following McFadden's (1973) logit model, it can be assumed that $\varepsilon_i(\omega_i)$ follows the type-I extreme value distribution. Then, the probability density of random shocks can be derived in a logistic form, as follows:

$$\mu(\varepsilon_i(-1) - \varepsilon_i(1) \le z) = \frac{1}{1+\exp(-\beta_i z)}; \quad \beta_i \ge 0 \cdots\cdots \text{Eq. (14)}$$

where $\mu(\cdot)$ denotes the probability measure commonly denoted by $\Pr(\cdot)$, z is the stochastic variable, and β_i is supposed to be dependent on $\mathbf{Z}_i$. Therefore, the formula has the functional form $\beta_i = \beta(\mathbf{Z}_i)$.

The interaction-based model is derived from these assumptions. By adding the influences of other agents' choices, equation (13) can be rewritten as follows:

$$\max_{\omega_i \in \{-1,\ 1\}} V(\omega_i, \mathbf{Z}_i, \mu_i^e(\boldsymbol{\omega}_{-i}), \varepsilon_i(\omega_i)) \cdots\cdots\cdots\cdots\cdots\cdots \text{Eq. (15)}$$

where $\mu_i^e(\boldsymbol{\omega}_{-i})$ is defined as individual i's beliefs regarding the choices of other agents (i.e., the expected behavior of others). $\boldsymbol{\omega}_{-i} = (\omega_1, \ldots, \omega_{i-1}, \omega_{i+1}, \ldots, \omega_I)$ denotes the vector of other agents' choices. In the model, it is assumed that $\varepsilon_i(\omega_i)$ is independent of other agents' choices.

To derive the specific properties of the model, two major parametric assumptions are introduced. The first assumption concerns the payoff function. Based on the common specification of the discrete choice model shown in equation (7), Section 2.2.3, the payoff function is decomposed into three terms, as follows:

$$V(\omega_i, \mathbf{Z}_i, \mu_i^e(\boldsymbol{\omega}_{-i}), \varepsilon_i(\omega_i)) = u(\omega_i, \mathbf{Z}_i) + S(\omega_i, \mathbf{Z}_i, \mu_i^e(\boldsymbol{\omega}_{-i})) + \varepsilon_i(\omega_i) \cdots\cdots \text{Eq. (16)}$$

where $u(\omega_i, \mathbf{Z}_i)$ represents the deterministic private utility, which contrasts with the following $S(\omega_i, \mathbf{Z}_i, \mu_i^e(\boldsymbol{\omega}_{-i}))$, which represents the deterministic 'social utility.' The presence of S differentiates the model from other discrete choice models.

The second assumption concerns social utility. It is assumed that social utility embodies a quadratic conformity effect, which was suggested by Bernheim (1994). Based on the theory of Bernheim (1994), social utility can be described per the following equation.

$$S(\omega_i, \mathbf{Z}_i, \mu_i^e(\boldsymbol{\omega}_{-i})) = -\mathrm{E}_i\left[\sum_{j \neq i} \frac{J_{i,j}}{2}(\omega_i - \omega_j)^2\right] \cdots\cdots \text{Eq. (17)}$$

where $\mathrm{E}_i[\cdot]$ denotes the expectation measure and $\frac{J_{i,j}}{2}$ represents the 'interaction weight,' which relates the choice of agent i to that of j. Brock and Durlauf (2000) clarified that $J_{i,j}$ is typically assumed to be nonnegative. Equation (17) can be expressed in a simpler form using the fact that $\omega_i^2 = \omega_j^2 = 1$.

$$S(\omega_i, \mathbf{Z}_i, \mu_i^e(\boldsymbol{\omega}_{-i})) = \sum_{j \neq i} J_{i,j}(\omega_i \cdot \mathrm{E}_i[\omega_j] - 1) \cdots\cdots \text{Eq. (18)}$$

The deterministic utility is expressed in a linear function following the typical discrete choice model, which is shown in equation (7), Section 2.2.3:

$$u(\omega_i, \mathbf{Z}_i) = h_i\omega_i + k_i \cdots\cdots \text{Eq. (19)}$$

where h_i and k_i are functions of $\mathbf{Z}_i$. Combining equation (18) and equation (19), the choice probability can be derived as follows:

$$\begin{aligned}&\mu(\omega_i \mid \mathbf{Z}_i, \mu_i^e(\boldsymbol{\omega}_{-i})) = \\ &\mu(h_i\omega_i + \sum_{j \neq i} J_{i,j}\omega_i \mathrm{E}_i[\omega_j] + \varepsilon_i(\omega_i) > -h_i\omega_i + \sum_{j \neq i} J_{i,j}\omega_i \mathrm{E}_i[\omega_j] + \varepsilon_i(-\omega_i))\end{aligned} \quad \cdots \text{Eq. (20)}$$

Applying equation (14), equation (20) can be arranged in a proportional form, as shown in the following equation:

$$\mu(\omega_i \mid \mathbf{Z}_i, \mu_i^e(\boldsymbol{\omega}_{-i})) \sim \exp(\beta_i h_i \omega_i + \sum_{j \neq i} \beta_i J_{i,j} \omega_i \mathrm{E}_i[\omega_j]) \cdots\cdots\cdots\cdots \text{Eq. (21)}$$

Brock and Durlauf (2001) supposed the interaction weight $J_{i,j}$ to be a constant J, which implies the homogeneity of social interactions, labeled as 'global interaction.' From this, Brock and Durlauf (2001) derived many propositions, such as the existence of self-consistent equilibria, the number of equilibria, and the local stability of these equilibria. Their study also showed the economic properties of this model and derived a function representing the likelihood of enabling econometric analysis. Based on the model, Brock and Durlauf (2002) developed a multinomial logit version of the interaction-based model.

Though it seems natural that the interaction-based model is a good candidate for explaining the diffusion model, few research studies close to this model exist. As introduced in Section 2.2.2, Dugundji and Gulyas (2008) proposed a nested logit version of the interaction-based model and applied it to the diffusion model using ABM methodology. However, the specification of social utility in this study is even simpler than that used by Brock and Durlauf (2001), as seen in equation (12), Section 2.2.2. Pombo-Romero et al. (2013) developed an ABM diffusion model based on the Ising model in physics. Pombo-Romero et al. (2013) showed that the derived model has a very similar formation to the model of Brock and Durlauf (2001). Such research studies, however, continue to assume the interaction weight to be constant; thus, they disregard the heterogeneity of individual social utility.

2.4 Research Motivation

Although Rogers (1962) proposed numerous generalizations and explanations related to innovation diffusion, many are still left unsolved. In particular, research studies about the diffusion rate, which can be called diffusion model research studies, face certain critical limitations.

Aggregate diffusion models, such as logistics, Gompertz, and Bass models and their extensions, are developed from the perspective of mathematical fitness—not that of economics or of other theoretical backgrounds. This is why Russell (1980) criticized the

Bass model for its lack of economic explanations for parameters. Such aggregate models also often disregard individual preferences and interactions among agents. They focus only on the aggregated-level data and the aspect of diffusion from the properties of certain innovation.

ABMs are well suited for building individual-level analyses, but many are based on simple rules, not economic theories. Empirical studies of ABMs are mostly dependent on individual-level data, which are difficult to gather and challenging in terms of deriving generalized implications. ABMs have another limitation in that they usually disregard the properties of innovation, which are considered important by Rogers (1962). Individual choice-based models suffer from problems similar to those faced by ABMs.

Many innovation diffusion review papers propose directions for future diffusion research (Bass, 1995, 2004; Kiesling et al. 2012; Meade and Islam, 2006; Peres et al. 2010). Common such directions can be summarized in the following three ways: First, an individual-level approach is needed. This is natural because the decision of innovation adoption is usually made by each agent, not by an aggregated-level authority. Individual-level modeling is also essential to explaining how each agent's behavior relates to the aggregated diffusion pattern. Secondly, the ways in which social networks affect diffusion patterns should be investigated. Rogers (2003) considered the structure of social networks to be one of five important factors of innovation diffusion. From these two issues, it can be seen that the reason for the prevalence of ABM research studies in the field recently is that former models only considered aggregated-level behaviors and communication channels. The third issue involves establishing a concrete theoretical background for diffusion models. Bass (1995) pointed out the lack of a T-E (theoretical-to-empirical) approach in aggregate models, and Kiesling et al. (2012) criticized the lack of a utilitarian approach in ABM models. The last issue concerns the social interaction of agents. Many ABMs assume only simple rules for social interactions, without using any theoretical backgrounds. Peres et al. (2010) expected that such research studies would become particularly prevalent due to the advent of social network services. From the latter two issues, the reason for using a utilitarian approach based on the Brock and Durlauf (2001) model can be clarified. There is a limitation in the Brock and Durlauf (2001) model, however, in that it models only global interactions, which can be interpreted as aggregated-level network externalities.

Some recent research studies have had research motivations similar to that of this study. Choi et al. (2010) examined the network effect on innovation diffusion. Their study assumed a utility structure similar to that of the interaction-based model and used a small-world network as the network topology. However, their work was based on a simple network externality for the network effect, rather than on interactions among agents. In addition, it did not assume any heterogeneity among interactions.

Cho et al. (2012) investigated the characteristics of opinion leaders in innovation diffusion. Their study allowed for social influences from distant agents and encompassed various levels of social influences using the strength of a tie, which was called ‘intimacy.’ However, there are significant differences between the work of Cho et al. (2012) and this study, in that the Cho et al. (2012) study was not based on utility theory and had different social network structure: cellular automata.

Pombo-Romero et al. (2013) modeled imitative behaviors in choosing drugs among primary care physicians in Spain. Their model was based on the Ising model of physics, which has a functional form similar to that of the Brock and Durlauf (2001) model and assumes a small-world network for its network topology. However, it does not assume the heterogeneity of interactions, nor does it assume the influence of distant agents. Furthermore, though it shows a similar functional form, it is hard to say that the model is based on utilitarian framework.

This research presents a new diffusion model that encompasses the strengths of existing models or overcomes their limitations, as seen in Figure 3. First, the concept of social utility from consumer interactions is concretely included in modeling consumer adoption behaviors, so that the theoretical basis for the construction and interpretation of the model is reinforced. Second, the model is built on the basis of the microeconomic theory of Brock and Durlauf (2001). Furthermore, the relative influence of interactions among agents is assumed to have a distribution, which is a more realistic and generalized approach than that taken in Brock and Durlauf's (2001) global interaction. For this, the interaction topology of the agents is assumed to be a small-world network, and the distribution is assumed to allow for differences in interactions between agents, according to the distance in the social network. Third, attributes of innovation, such as price and the degree of network externalities, are included in the model, which enables managers to draw more beneficial implications for marketing purposes. Lastly, unlike other ABMs, the model is not dependent on individual data. Following the traditions of diffusion research, such as the work of Bass (1969), this model has an ultimate goal of analyzing aggregated-level market data from an individual-level model. In sum, this research seeks to develop a generalized, microeconomic ABM diffusion model that considers social interactions as drivers.

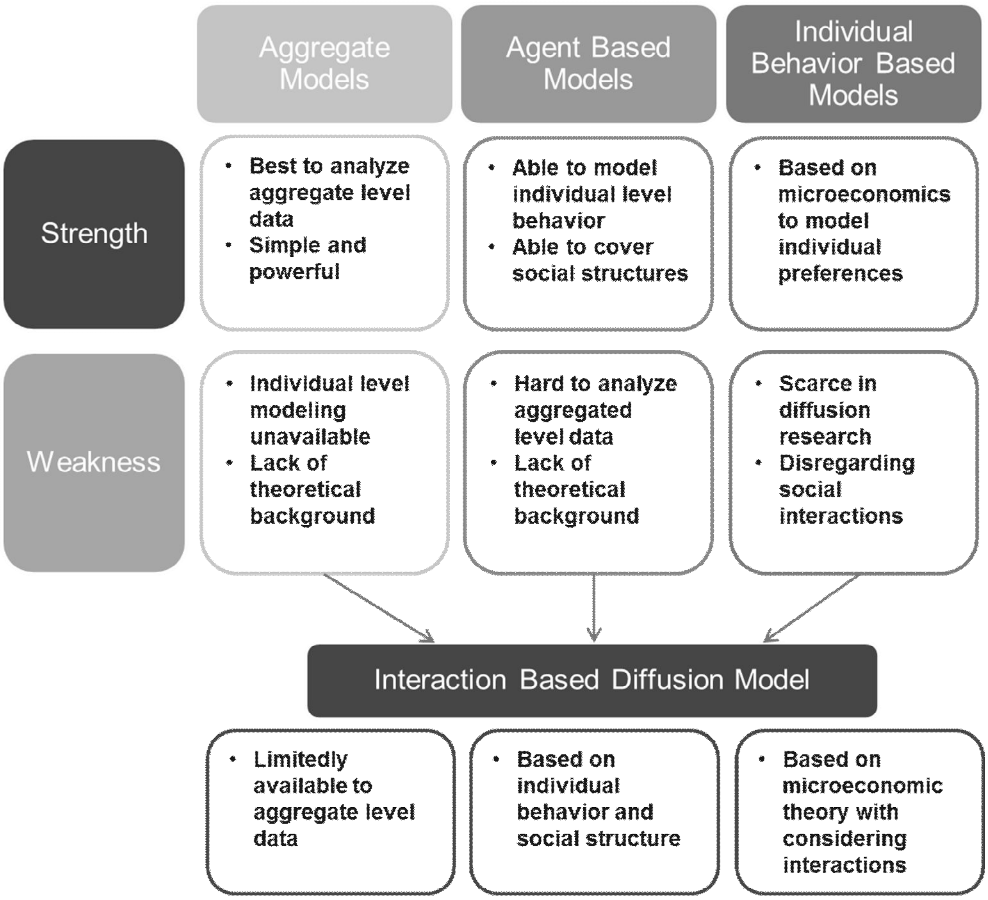

Figure 3. Relation of this study to previous research studies

Chapter 3. Interaction-Based Diffusion Model

In this chapter, a new model called the interaction-based diffusion model (IBDM) is introduced. This model is unique in its construction of utility model, its social network structure, and its diffusion process. The following sub-chapters show the details of the IBDM specifications.

3.1 Utility Model

The IBDM is a microeconomic approach. That is, it assumes that each person makes a decision of adoption by maximizing his/her utility. Previous utilitarian studies, such as that by Delre et al. (2007), suppose the presence of threshold utility; thus, they have received numerous criticisms that they are not actually different from threshold models (Kiesling et al. 2012). In contrast to other researchers, Cho (2007) developed a utility model without a threshold, as seen in equation (10), Section 2.2.3. The IBDM follows the basic framework of Cho (2007), but differs from Cho (2007) in that it encompasses the social utility of Brock and Durlauf (2001).

To make the model simpler, this study adjusts the mathematical formula of social utility. Brock and Durlauf (2001) proposed a quadratic conformity function for social utility, S_i, as per the following equation, which was already introduced in Section 2.3.

$$S_i = -\mathrm{E}_i\left[\sum_{j \neq i} \frac{J_{ij}}{2}(\omega_i - \omega_j)^2\right] \quad \cdots\cdots \text{Eq. (22)}$$

where ω_i is indicator function of agent i's choice, $E_i(\cdot)$ denotes the expectation measure, and J_{ij} represents the 'interaction weight,' which relates the choice of agent i to that of agent j. Brock and Durlauf (2001) defined the value of ω_i as '1' for adoption and '-1' for non-adoption.

As seen in equation (22), the social utility of Brock and Durlauf (2001) is defined to have a negative value always. This means that an agent is not affected by a neighbor agent if both of their choices are the same; however, if their choices are different, the neighbor agent negatively affects the focal agent. In other words, under this assumption, agents try to make a decision that conforms with the consensus of society, as well as to avoid being idiosyncratic. Using the fact that utility only matters with relative orders, Brock and Durlauf (2001) simplified the equation as follows:

$$S(\omega_i, \boldsymbol{\omega}_{-i}) = \sum_{j \neq i} J_{ij} \cdot \omega_i \cdot \mathrm{E}_i[\omega_j] \quad \cdots\cdots \text{Eq. (23)}$$

Note that there is no consideration of dynamics in equation (23). Since the diffusion model is a time series model, the equation should contain a time dimension. Brock and Durlauf (2001) proposed the use of the expectation of the $(t-1)$ period for determining the social utility in period t; that is, they proposed the use of $\mathrm{E}_{i,t-1}[\omega_j]$ for other agents' choices. Although this approach allows a time-variant social utility, it still requires initial values for expectations (i.e., $\mathrm{E}_{i,0}[\omega_j]$) to solve the equation. In this manner, it is hard to derive a tractable model without further assumptions. Even though such an assumption of rational expectation is natural to describe individual behaviors from the perspective of economics, a simpler method is preferred in the ABM (Farmer and Foley, 2009). Many ABM research studies, including those that consider other agents' choices (such as Dugundji and Gulyas (2008), Cho et al. (2012), and Pombo-Romero et al. (2013)) assume that the choices of others are fully observable in society and that previous choices affect current decision making. Though it would be better to accept rational expectations in the model, this study follows the same assumptions as previous research studies. Such an assumption is more appropriate for focusing on the interaction effect on diffusion, rather than on the individual adoption strategy. Therefore, the equation can be rewritten in the following form:

$$S_t(\omega_i, \boldsymbol{\omega}_{-i}) = \sum_{j \neq i} J_{ij} \cdot \omega_{i,\,t} \cdot \omega_{j,\,t-1} \quad \cdots\cdots \text{Eq. (24)}$$

A model with the assumption of conformity is classified as a 'social influence model' in economics (Young, 2009). In contrast, most diffusion research studies are 'contagion models'; that is, they assume that an agent's decision of adoption is affected by others who have already adopted the innovation. Kats and Shapiro (1985) introduced the concept of a 'positive network externality' on the basis that the utility of an agent depends on the number of adopters. Their study also stated that such an assumption is valid for durable goods. Since the social influence model is expected to show an extreme diffusion pattern, an alternative contagion model is required to cover typical innovation diffusion research studies.

An alternative model sets ω_i to have {1, 0}: '1' for adoption and '0' for non-adoption. Following the approach of Cho (2007) and Dugundji and Gulyas (2008), this model assumes that non-adopters derive zero utility from the innovation. Thus, the social utility from interactions among non-adopters is set to zero. In addition, the model supposes that information of other agents' choices is fully accessible by any agent, as is the case above. In other words, the choices of agents made at time $t-1$ can be observed by any agent at time t. In sum, the model allows positive social utility only for the case of interaction among adopters, allowing nothing for other cases. Therefore, the finalized mathematical formula for the social utility of agent i is as follows. This alternative model is the default model of IBDM.

$$S_{i,t} = \sum_{j|\omega_{j,t-1}=1} J_{ij} \quad \cdots\cdots \text{Eq. (25)}$$

Brock and Durlauf (2001) and subsequent research studies, such as those by Brock and Durlauf (2002), Dugundji and Gulyas (2008), and Pombo-Romero et al. (2013), assumed the interaction weight J_{ij} to be constant in order to derive the equilibrium, the choice probability, and the likelihood of estimation. Such an assumption of global interaction, however, disregards the heterogeneity of individual interactions. Hence, the IDBM uses a J_{ij} that is not constant across agents to reflect the heterogeneity of interaction.

Combining Cho's (2007s specification, as shown in equation (10), Section 2.2.3, and Brock and Durlauf's (2001) model, as shown in equation (16), Section 2.3, the utility of the innovation for agent i can be expressed as in the following equation:

$$U_i = \begin{cases} V_i + \sum_{j|\omega_j=1} J_{ij} - \mu \cdot P & \text{if} \quad \omega_i = 1 \\ 0 & \text{if} \quad \omega_i = 0 \end{cases} \quad \cdots\cdots \text{Eq. (26)}$$

where U_i represents the total utility from the innovation and V_i represents the observable deterministic utility from attributes of the innovation. P is the purchasing price of the innovation. μ is defined as the exchange rate between utility and the monetary unit of price. Since P is a monetary variable and U_i represents ordinal utility, μ can be interpreted as an exchange rate between utility and the monetary unit. Even though Cho (2007) assumed that μ has a different value for each agent (i.e., μ_i), from the perspective of an exchange rate, it can be a global constant. Since the representative utility V_i reflects the heterogeneous 'taste' of each individual, it encompasses the effect of μ_i, which can be interpreted as the resistance to the innovation. For instance, it is hard to identify an agent's willingness to adopt if he/she has a high V_i and a high μ_i at the same time. Hence, the IBDM assumes μ as a global constant, using only V_i as a heterogeneous utility to reflect the utility from the innovation solely. Note that the heterogeneous V_i contains the heterogeneity of individual price sensitivity by normalizing μ_i. In this manner, V_i can be interpreted as the innovativeness of each consumer. Goldsmith and Newell (1997) claimed that innovativeness is closely related to the price sensitivity of each consumer, and Chatterjee and Eliashberg (1990) measured consumer innovativeness in their function of innovation performance and price. Thus, V_i can be defined, in a broad sense, as consumer innovativeness.

In sum, the IBDM assumes that an agent adopts an innovation when $U_i > 0$: that is, $V_i + \sum_{j|\omega_j=1} J_{ij} \geq \mu \cdot P$. In other words, in the IBDM, an agent decides to adopt an innovation when the sum of the direct utility from the attributes of the innovation and the social utility from other agents' choices exceeds the purchase price of the innovation.

Along with the heterogeneity assumption for taste, the model allows for the heterogeneity of social interactions. In contrast to Brock and Durlauf (2001), this model assumes the interaction weight J_{ij} to have a probability distribution. It is natural to expect a higher interaction effect from close neighbors, while distant agents exert only a low-level influence. Thus, interaction weight is supposed to be a function of social distance, as follows:

$$J_{ij} = \frac{1}{\sigma_j \sqrt{2\pi}} \cdot \exp\left(-\frac{(\Delta_{ij} / k)^2}{2\sigma_j^2} \right) \cdots\cdots \text{Eq. (27)}$$

where Δ_{ij} denotes the number of links between agent i and agent j in the social network—a measure that can be interpreted as the social distance between i and j. The model assumes a normal distribution for J_{ij}, since a normal distribution draws a symmetric curve from the mean. The position of agent i in the social network is denoted by the number '0,' and the weights of various same-distance neighbors should be equal. Another advantage of adopting a normal distribution is its functional property: that is, that it decreases quickly by distance. If a linear function, such as a triangular distribution, were applied, the influence of the second- and third-closest people would still be too strong.

As seen in equation (27), the IBDM also assumes the presence of heterogeneity of variance σ_j^2 and a global constant k. σ_j^2 can be interpreted as the degree of variance of the interaction influence of agent j on other agents. In other words, an agent with a higher σ_j^2 affects a wider number of agents, while one with a lower σ_j^2 affects only its closest neighbors. The presence of σ_j^2 is essential because it represents the heterogeneity of the interaction weight. Assuming both heterogeneities in the model, however, causes a squared randomness, which can harm the model's robustness[7]. Hence, the variance of interaction influence σ_j^2 is assumed to be proportional to the taste V_i, as per the following equation:

$$\sigma_j^2 \sim s \cdot V_j \quad \cdots\cdots \text{Eq. (28)}$$

where s is defined as the 'correlation of social interaction with taste' and is supposed to be a global constant. This assumption is based on Rogers' (2003) generalizations, which state the relation between innovativeness and social interaction as follows[8]:

Generalization 7-18: Earlier adopters have more social participation than later adopters.

Generalization 7-23: Earlier adopters have greater exposure to interpersonal communication channels than later adopters.

Generalization 8-10: The interconnectedness of an individual in a social system is positively related to the individual's innovativeness.

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[7] Roughly, if there is only one random variable, then it is easy to derive a robust simulation result by averaging the cases resulting from varying that random variable. However, if there are two random variables, a squared number of cases must be derived in order to guarantee the same robustness in the former situation. The details are explained in Chapter 4.

[8] Source: Rogers (2003).

Note that V_i can be interpreted, in a broad sense, as consumer innovativeness. Particularly, the generalization 8-10 exactly justifies equation (28), while other generalizations support the presence of variance in interaction weights.

The model also assumes the presence of a global constant k, which is the denominator of social distance. If the value of k is high, then the social distances among agents would be contracted in calculation of J_{ij}. Thus, the level of social utility would be increased across all agents. Since k affects social utility globally, k can be interpreted as the 'network externality of the innovation.' It is expected that innovations like mobile communications, the Internet, and social network services have a large value for k, but that innovations like automobiles and white goods have a low level for k. In addition, note that k represents a global effect of individual interactions on diffusion patterns (i.e., the homogeneous portion of social interactions).

In sum, the assumptions made for the interaction weight J_{ij} can be visualized as in Figure 4.

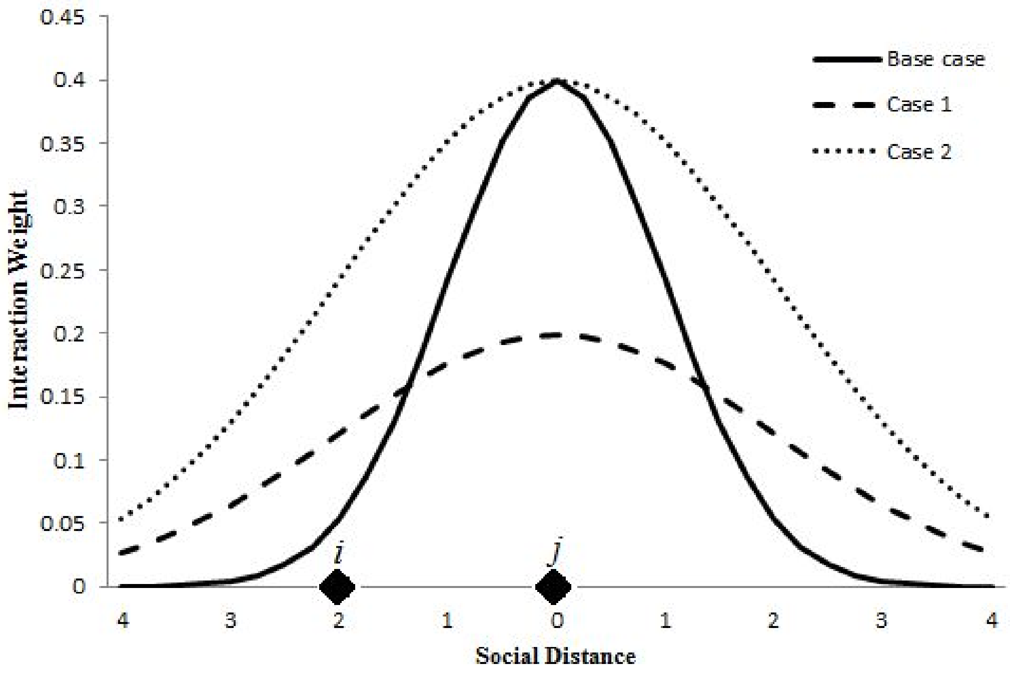

Figure 4. Diagram of interaction weights in the IBDM

As shown in Figure 4, agent i is assumed to have a social distance of '2' from agent j. The figure shows a normal distribution curve, along with the specification in equation (27). It is easy to understand that if i were closer, then i would receive more influence from j. In the base case, it is assumed that $k=1$ and $\sigma_j^2=1$. Case 1, expressed as a dashed line, assumes that $k=1$ and $\sigma_j^2=2$, which implies that j has a greater taste for innovation. In this case, j affects i, as well as other agents distant to j, more than it does in the base case. Case 2, expressed as a dotted line, assumes $k=2$ and $\sigma_j^2=1$, which implies that the innovation is more network friendly. The size of the interaction weight increases with the contraction of social distance. Thus, i receives the same influence as it would if i had a distance of '1' in the base case.

The assumption of such a distribution for interaction weight differentiates the IBDM from other ABMs. Many ABMs assume only the neighborhood effect, in which an agent is affected only by contractual agents. Although this seems to hold true when considering typical products, through the advent of the Internet and social network services, it is becoming natural for agents to be affected by distant agents in many ways, such as through blogs or social network services. The distribution assumption also enables the continuous value of social interactions. Through the adjustment of σ_j^2, J_{ij} can have any value in real numbers, and the social distance Δ_{ij} can be interpreted as a non-integer value through the presence of k. It is expected that the continuous function of interaction makes the model more flexible.

From these utility specifications, it is clear that the IBDM differs from models used in previous diffusion research studies. First of all, the IBDM is a utilitarian approach, which enables economic explanations of diffusion and secures a concrete theoretical background for the model. As mentioned in Section 2.2.2, there is still a lack of utilitarian approaches in diffusion research, and existing ones are not distinguished from simple threshold models. Note that the IBDM does not assume any arbitrary threshold for adoption. Secondly, the IBDM reflects social interaction in its diffusion model. Most previous literatures have considered only aggregated-level network externalities, even though the word-of-mouth effect is considered the most effective channel for innovation diffusion. Thirdly, the IBDM allows for the heterogeneity of interactions, which is also important for explaining diffusion patterns (Bohlmann et al., 2010). Other interaction-based models, including the one used in the original study by Brock and Durlauf (2001), have assumed only global or local constants for interactions. The IBDM, however, encompasses, not only heterogeneity, but also homogeneity in interactions. Lastly, the IBDM assumes a distributional function for social interaction, which enables interactions with distant agents.

Since the IBDM uses the concept of social distance to explain the interaction weights of agents, the specifications of social networks are essential for deriving diffusion curves from the model. In the following sub-chapter, the social network structure used in the model is explained in detail.

3.2 Structure of Social Network

Rogers (1962) pointed out that the nature of social systems is an important factor in the diffusion rate and defined the degree of network interconnectedness as one of the nature. Network interconnectedness is determined by the structure of the social network. After Schelling (1971) proposed the simple line-and-box structure, many scholars in various research fields developed numerous structures. These structures usually define a decision-making unit (i.e., agent) as a node and the interconnectedness between agents as an edge. Edges are also called as ‘links.’ Such links can be defined by neighborhood

(Schelling, 1971) or friendship (Granovetter, 1973). Some empirical ABM research studies on agriculture (Schreinmachers et al., 2010), pharmaceuticals (Pombo-Romero et al., 2013) and energy (Sopha et al., 2011) have defined social links by geographic or spatial distance; however, most ABM research studies use ideal social structures for analysis, and few (e.g., Zhang et al., 2011) utilize individual survey results to construct more realistic social networks. Nevertheless, the exact identification of network structure is not necessary for the aggregate diffusion model (Toubia et al., 2009).

Among such structures, the major structures used in ABM research can be categorized into four groups: random network, cellular automata, small-world networks, and power-law networks. Simple diagrams for these networks are presented in Figure 5.

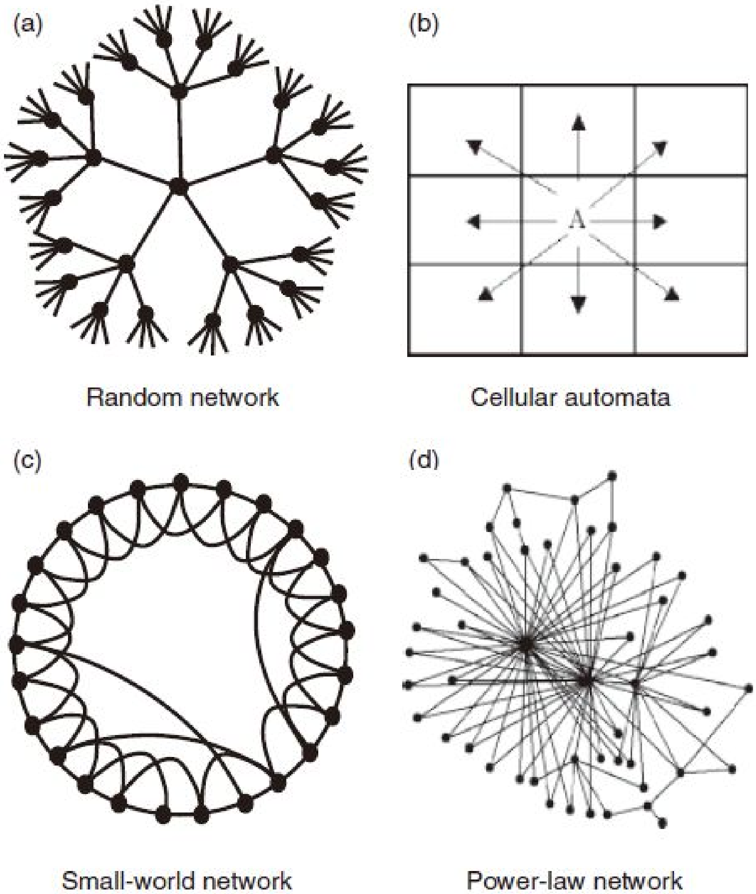

Figure 5. Four different network structure topologies[9]

The first structure, the random network, was developed by the mathematicians Erdös and Rényi (1959). The random network assumes that all vertices in the networks have a common independent connecting probability to build an edge among them. Random networks are very simple and intuitive; thus, they were introduced in early ABM studies, such as Valente (1995). However, physicists Barabási and Albert (1999) criticized the

[9] Source: Bohlmann et al. (2010).

application of random networks to actual networks for the reason that random networks do not exhibit clustering (Figure 5 (a)). In other words, in random networks, there is no chance of an agent being a neighbor of a neighbor agent's neighbor—a situation that can be easily found in the real world. Such a property is critical to diffusion research; thus, random networks are not good candidates.

The most popular network structure in ABMs is the cellular automata network. Mathematician Von Neumann (1951) developed an earlier version of such a network. Since then, it has been widely used in computer science. Cellular automata networks incorporate a structure of two-dimensional lattices in a two-dimensional grid (Figure 5 (b)). In a cellular automata topology, a vertex inside the network is connected to four or eight neighbors, and agents at the edges and corners at the border of the network are connected to two or three neighbors. This characteristic of cellular automata has been criticized by Anderson (1999) because such networks allow only a limited number of connections (i.e., four or eight) and the same number of connections for all agents in inner network. The presence of edges and corners is also problematic, since there is no such thing in real social networks and since it makes the network asymmetric. For instance, if initial adopters are positioned at edges and corners, then the diffusion will be slower than if such agents were positioned in the center. There is another problem with the cellular automata topology in that the diameter of network is too large in proportion to the total number of agents. That is, the average distance between an arbitrary two agents is larger than in other topologies, so the market penetration of innovation is relatively slower in a large network. Even though some studies have proposed alternative structures for cellular automata (for instance, the hexagonal lattices of Cho et al. (2012) and the complex lattice of Goldenberg et al. (2001)), the fundamental problem of the topology has not yet been overcome. However, despite its limitations, the cellular automata network is still the most commonly used in ABM research (Kiesling et al., 2012).

More realistic network models have been proposed by physicists. Watts and Strogatz (1998) developed a topology called 'small-world networks,' which incorporated the desirable properties of clustering of cellular automata and the small characteristic path lengths of random networks (Figure 5 (c)). Based on ring-shaped regular lattices, small-world networks assume a random rewiring of some edges. The topology shows, not only a high degree of local clustering, but also a short diameter for the total network. The details of small-world networks are described later.

Barabási and Albert (1999) proposed a topology called the power-law network to reflect the presence of hub agents (Figure 5 (d)). Unlike other networks, the power-law network is constructed on the basis of two key assumptions: growth and preferential attachment. The first assumption, growth, allows for an increase in the total number of agents. Preferential attachment means that more connected agents, called hubs, are more likely to receive additional new links. It is known that such a topology fits best with real world networks, such as the World Wide Web, research citation networks, and power grids (Barabási and Albert, 1999). Nevertheless, the IBDM uses the small-world network of Watts and Strogatz (1998) rather than that of Barabási and Albert (1999) because the power-law network is more complex, implies the growth of market potential (which is not typical of diffusion research studies), and allows for dynamic changes in society by time, which can affect the diffusion rate rather than the attribute of innovation. In other words, the power-law model is excellent for describing the real world, but it also breaks out of the scope of diffusion research.

The Watts and Strogatz (1998) model, hereafter referred to as the S-W (small world) network model, is constructed from a ring lattice with n vertices and k edges per vertex. Figure 6 (a) shows the case of $n = 20$ and $k = 4$. Then, with a random probability ρ, each edge is rewired to any vertex other than the original one (Figure 6 (b)). If ρ is 1, the graph becomes totally random. By the axiom of probability, ρ lies on the region $0 \le \rho \le 1$.

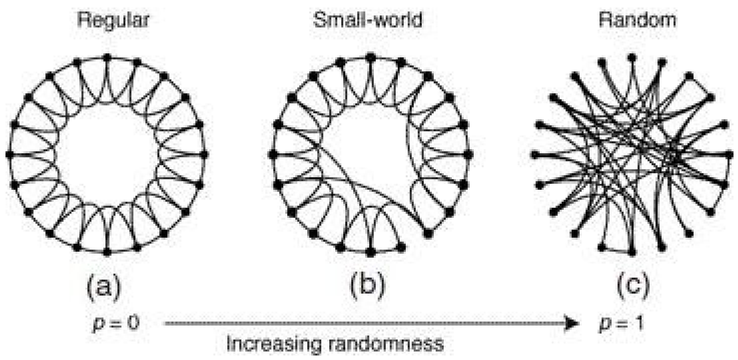

Figure 6. Random rewiring procedure in the S-W network[10]

Watts and Strogatz (1998) determined that the characteristic path length $L(\rho)$—or,

[10] Source: Watts and Strogatz (1998), p. 441.

in other words, the average path length—converges to $n/2k$ when $\rho \to 0$ and $\ln(n)/\ln(k)$ when $\rho \to 1$. For example, in case of $n = 1000$ and $k = 4$, the average path length $L(\rho)$ is 250 when the network is regular but reduces to around 4.98 when it is random. Watts and Strogatz (1998) also calculated the clustering coefficient $C(\rho)$, defined as the probability of two randomly selected neighbors being connected to one another. It is found that $C(\rho)$ converges to $3/4$ when $\rho \to 0$ and k/n when $\rho \to 1$. This implies that a regular network is highly clustered, while a random network is minimally clustered.

Watts and Strogatz (1998) found that, in such a topology, a small ρ can lead to an immediate drop in the average path length while the clustering coefficient is still high (Figure 7). This is because a shortcut created by ρ contracts, not only the distance that it connects, but also distances of neighborhoods in general. In contrast, a removed shortcut from a node reduces only the clustering coefficient linearly, since the nominator of the clustering coefficient is defined as the number of edges. Such a fact implies that a transition to a small world is almost undetectable at the local level. From the perspective of innovation diffusion, this means that, in S-W, assuming the same social interactions based on local clusters (such as cellular automata) can lead to an aggregated-level small-world effect. This is the reason S-W is a better candidate for the ABM diffusion model than cellular automata.

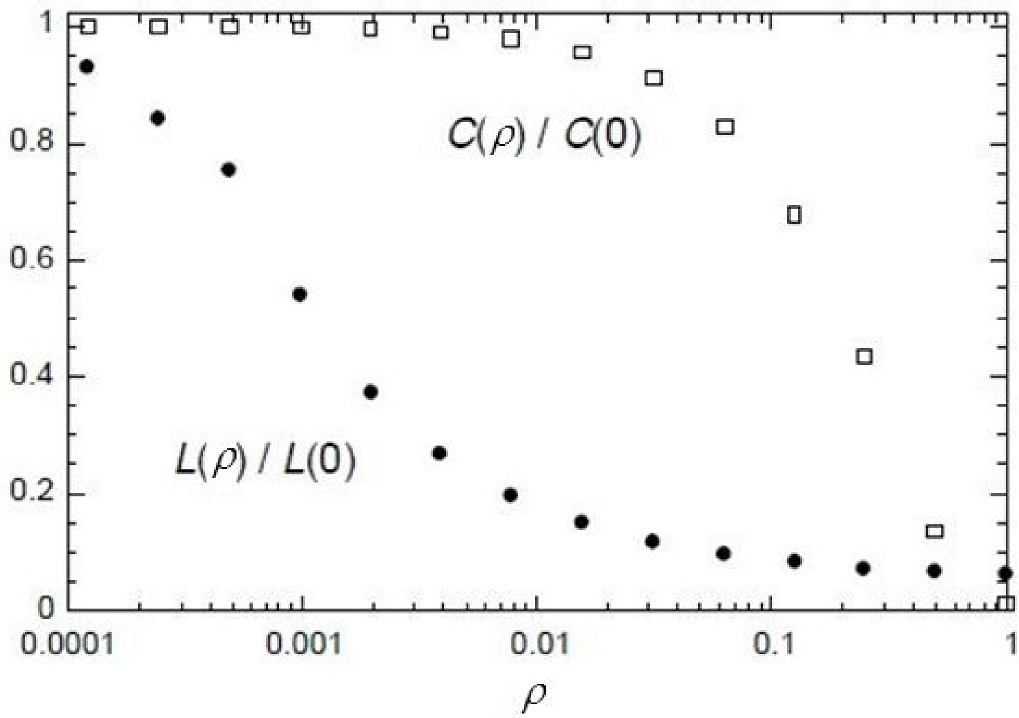

Figure 7. Average path length and clustering coefficients due to rewiring probability[11]

Nevertheless, only a few research studies have applied the S-W model to the diffusion model. Bohlmann et al. (2010) compared the effects of network structures in innovation diffusion in all four categories of topologies with ABM simulation. Choi et al. (2010) examined the role of network structure in innovation diffusion by applying the S-W structure. Their study showed the most similar modeling with IBDM, but their social utility was simply assumed to be aN_{t-1}, where N_{t-1} represents the number of adopters in the previous period and a is the coefficient for this number. In other words, the

[11] Source: Watts and Strogatz (1998), p.441.

interaction is assumed to be homogenous. Hence, there is a poor S-shape in the diffusion curve and an extreme polarization effect from ρ . Pombo-Romero et al. (2013) also used an S-W network to analyze imitative behaviors in new drug market.

The IBDM differs from previous research studies in its specification of interaction. Figure 8 shows a simple diagram for the interaction weight of agent j on other agents. Based on the definition of equation (27), Section 3.1. and Figure 4, Section 3.1, the interaction weight decreases with the social distance from agent j .

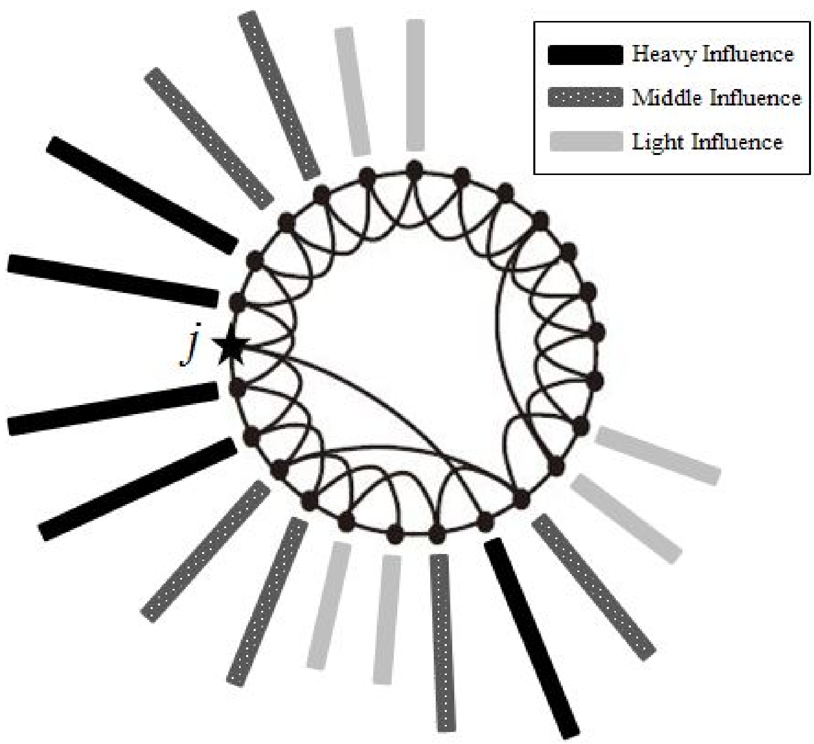

Figure 8. Conceptual diagram of interaction weight in an IBDM

In Figure 8, the position of j in the network is represented by a star. Solid bars represent the level of the interaction weight through the length of the bars. Only three levels are expressed in the diagram—and, for distinguishability, the longest bar is expressed as a black and solid, the middle-level bar is gray-dotted, and the shortest bar is a light gray solid. In the diagram, j is assumed to have five closest neighbors, including one distant agent in the right-bottom area. Due to the presence of this shortcut, agent j can exert significant influence on other distant agents in right-bottom area. In addition, as seen in the diagram, social distance is defined as the minimum edge needed to connect two vertices.

Based on the S-W topology and the Section 3.1, the IBDM can express the diffusion pattern in a social network. In the following sub-chapter, the last ingredient to complete the model—the diffusion process—is discussed.

3.3 Diffusion Process

In previous sections, the specifications for the utility and social networks for the IBDM are discussed. To draw a diffusion pattern, however, the specifications for the diffusion process should be introduced.

The diffusion model is a time-series model, since its objective is to analyze market penetration, or the sales of an innovation over time. Social utility is naturally dynamic because it depends on the dispersion of previous adopters (see equation (25), Section 3.1). Thus, it is possible to draw a diffusion curve without dynamic non-social utilities, such as representative utility or price. Hence, the IBDM keeps the taste of each agent V_i constant over time.

Nevertheless, the IBDM uses the exponential decay function for price because a dynamically decreasing price can generate innovators through a natural diffusion process. It is natural to assume the presence of agents who adopt an innovation independently of other agents. Note that price is only a representative example of such variables. Other variables, such as advertising, marketing efforts, and technological advances can also be applied to the model to encompass innovation effects.

The exponential decay function is common in economic cost projections (Belkaoui, 1986) and is applied to describe the price function in numerous diffusion research studies (Bass, 1980; Bayus, 1993; Jain, 1992; Jain and Rao, 1990; Kamakura and Balasubramanian, 1988; Lee et al. 2006; Thompson and Teng, 1984). The price function used in the IBDM is as follows:

$$P_t = P_0 \exp(-\alpha t) \quad \text{Eq. (29)}$$

where P_t represents the purchasing price of an innovation at time t and P_0 is the initial price. α is the learning coefficient of price. A large α leads to a rapid decrease in price over time, while a small α leads to a gradual decrease in price.

With this dynamic price, the IBDM can explain the diffusion process in society through the following procedure. Note that the taste of each agent V_i is distributed randomly in the S-W network.

At $t = 0$, there are no previous adopters; thus, there is no social utility at this time. Only agents who have a high enough degree of taste (i.e. $V_i \geq \mu \cdot P_0$) decide to adopt the innovation.

At $t = 1$, the decreased price $P_1 < P_0$ allows more adopters who have $V_i \geq \mu \cdot P_1$. Adopters at time 0, represented as j, exert their influence on neighbor agents i through the weight $J_{ij}(\Delta_{ij} / k)$ (see equation (27), section 3.1). Thus, agent i, with $V_i + \sum_{j|\omega_{j,t=0}=1} J_{ij} \geq \mu \cdot P_1$, decides to adopt the innovation.

At $t \geq 2$, the cumulative number of adopters exerts its influence on neighbor potential adopters, as in $t = 1$. Moreover, a decreased price facilitates additional adopters. This procedure continues until all market potential is covered.

Although this shows a step-by-step procedure, only one rule is applied here: agents adopt an innovation whenever they have $V_i + \sum_{j|\omega_{j,\,t-1}=1} J_{ij} \geq \mu \cdot P_t$. Note that, with a continuously distributed V_i and a decreasing P_t, there will be no stagnation in diffusion. If not, a stagnation may occur due to a low level of social utility, which cannot overcome the price level, or isolated agents in the network, who cannot receive social utility from previous adopters.

The diffusion process described in this chapter is similar to that of the probit model, which was introduced by David (1969) and Davies (1979). The probit model assumes that a consumer i decides to adopt a product whenever his/her characteristic level z_i exceeds the time variant threshold z_t^*. Karshenas and Stoneman (1995) stated that, in a firm-level diffusion, the time-variant threshold level can be a function of installation cost, profitability, or number of adopters in market. In consumer-level diffusion, purchasing price is a natural candidate for characteristic level.

The IBDM has a structure similar to that of probit models. Individual taste V_i and the price level $\mu \cdot P_t$ can be interpreted as characteristic levels of each agent and of the time-variant threshold, respectively. It is noteworthy that $\mu \cdot P_t - \sum_{j|\omega_{j,\, t-1}=1} J_{ij}$ is the actual threshold in the IBDM and that it is a function of price and of the number of adopters. The probit model has the advantage of explaining, from the perspective of economics, why some consumers adopt an innovation earlier than others. Based on the same utility structure, the probit model draws a diffusion pattern from a time-variant threshold. However, the IBDM is still distinguishable from the probit model through its specification of threshold: That is, social interaction plays a key role in threshold definition, and it is not a simple function for a specific variable. Another difference comes from the fact that agents with higher V_i do not always adopt the innovation earlier. Hence, the IBDM can be classified as an extension of the probit model.

From such a discrete-time diffusion process, a simulation of IBDM can be developed. In the following chapter, the result of this simulation and its interpretation are discussed.

Chapter 4. Interpretation of the Interaction-Based Diffusion Model

In this chapter, the simulation results are discussed to facilitate an understanding of the interpretation of the parameters of the IBDM. Through the simulation, the effects of price coefficients and of social interaction parameters are investigated. Furthermore, the study shows the similarities between the IBDM and the Bass (1969) model and claims that the IBDM explains the diffusion pattern better because it is based on utility theory. In the following sub-chapters, the basic specifications used in the simulation are explained in detail; then, the study shows the various simulation results and extracts the major implications of the IBDM. Finally, the study draws comparisons with other diffusion models and highlights the distinctiveness of the IBDM.

4.1 Specification of Simulation

The first step in implementing a simulation of the IBDM is to draw Watts and Strogatz' (1998) Small-World (S-W) network for the model. As seen in equation (27), Section 3.1, the social interaction in the IBDM depends on social distances among agents. Thus, if there are a total of N agents, then an $N \times N$ distance matrix should be calculated for the simulation. The computation of $N \times N$ is tough for relatively large

N, so MATLAB[12] is used to implement the simulation. Taylor and Highham (2008) provided an efficient tool, called 'CONTEST,' to draw numerous social networks, including the S-W network. This study adjusted the code of CONTEST to draw the S-W network for the IBDM. Figure 9 shows the S-W topologies derived from the code by assuming $N = 100$ and $K = 4$ edges per vertex.

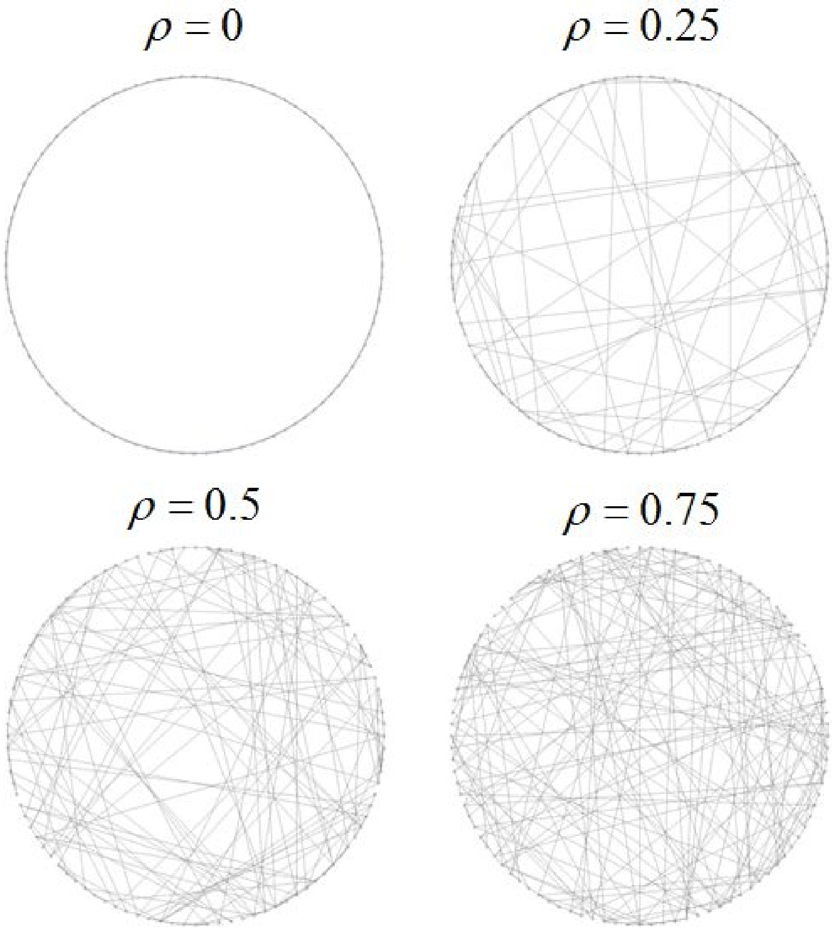

Figure 9. Small-world networks resulting from the rewiring probability

[12] MATLAB version R2013a (8.1.0.604) was used for the following simulations in this chapter.

The degree of interconnectedness of an S-W network depends on the rewiring probability ρ (see Section 3.2). As shown in Figure 9, an S-W network is just a ring lattice when $\rho = 0$; however, as ρ increases, many 'bridges' over distant nodes are generated. Thus, the average path length of the structure is contracted.

For the first step, the taste of each agent V_i is assumed to be distributed uniformly over agents (i.e., $V \sim Unif[0, 1]$). This assumption is common in ABM research studies, which set heterogeneous threshold levels for each agent. Moreover, in a microeconomic approach, Cho (2007) used the same assumption to draw a dynamic diffusion field for the model. ABM research studies using cellular automata topologies often assume that agents with similar innovativeness are clustered (Cho et al., 2012) or draw many random cases for normalization (Bohlmann et al., 2010). Nevertheless, the limitations of cellular automata (i.e., that it has a relatively large diameter for the number of agents and that it allows an asymmetric structure in the border) cannot be overcome in such ways. The S-W network also varies in structure—even for the same N, the same K, and the same ρ—because it is drawn via a probabilistic procedure. However, the position of innovative agents only minimally affects the diffusion because S-W is essentially symmetric and has an almost constant diameter.

The diffusion curve of the IBDM can be drawn by following the diffusion process described in Section 3.3. The exchange rate between utility and the monetary unit of price μ in equation (26), Section 3.1, is normalized to unity because it only measures the exchange rate between utility and the monetary unit. Thus, the diffusion starts whenever the normalized price P falls below 1. Therefore, it can be said that $0 \le P_t < 1$ in simulation. The specifications of the IBDM are replicated as follows to illustrate the entirety of the model (see Section 3.1 for details of equations):

$$U_{it} = \begin{cases} V_i + \sum_{j|\omega_j=1} J_{ij} - P_t & if \quad \omega_i = 1 \\ 0 & if \quad \omega_i = 0 \end{cases} \quad \text{Eq. (30)}$$

$$V \sim Unif[0,1] \quad \text{Eq. (31)}$$

$$P_t = P_0 \exp(-\alpha t) \quad \text{Eq. (32)}$$

$$J_{ij} = \phi\left(\Delta_{ij} / k, 0, \sigma_j\right) \quad \text{Eq. (33)}$$

where ϕ denotes the Gaussian function or normal distribution function.

$$\sigma_j^2 \sim s \cdot V_j \quad \text{Eq. (34)}$$

As shown in the above equations, the IBDM has four parameters in the utility specification: the initial price P_0, which determines the number of first adopters; the learning coefficient of price α, which determines the rate of diffusion by price decay;

the network externality of innovation k, which determines the level of internal influence on diffusion; and the correlation of social interaction with taste s, which determines the rate of diffusion via the internal influences of innovators.

There are three parameters for social networks: the total number of agents N, the average number of links per agent K, and the rewiring probability ρ. Even though K significantly affects the degree of network interconnectedness, it is fixed as $K = 4$ for simplicity, in keeping with other research studies using S-W networks (e.g., Choi et al., 2010; Pombo-Romero et al., 2013). It can be claimed that such an assumption is not strong because other methods, such as cellular automata, even assume a constant number of connection for all agents (Bohlmann et al., 2010). Furthermore, Newman and Watts (1999) showed that K can be normalized by following equation.

$$N_2 = N_1 \cdot \frac{K_2}{K_1},\ \rho_2 = \rho_1 \cdot \frac{K_1^2}{K_2^2} \quad \text{Eq. (35)}$$

According to Newman and Watts (1999), the renormalization of a network using equation (35) derives the same average path length. Thus, it is not necessary to investigate the effects of various K in diffusion research. However, other topological parameters should be examined to improve the robustness of the model.

Firstly, the proper level of N must be investigated. It is obvious that too small a network will draw randomly different diffusion patterns. Following the law of large numbers, a sufficiently large N is required to derive a robust simulation result. Figure

10 shows 10 draws from the same network of $N = 1,000$. Drawing from the same network implies the same value for all four utility parameters and the same connections for all agents, but different allocations of V_i in the network. Each parameter value is assumed as follows: $\rho = 0.25$, $P_0 = 1$, $\alpha = 0.01$, $k = 1$, and $s = 1.5$. The simulation is done via a MATLAB code written by the author for the IBDM.

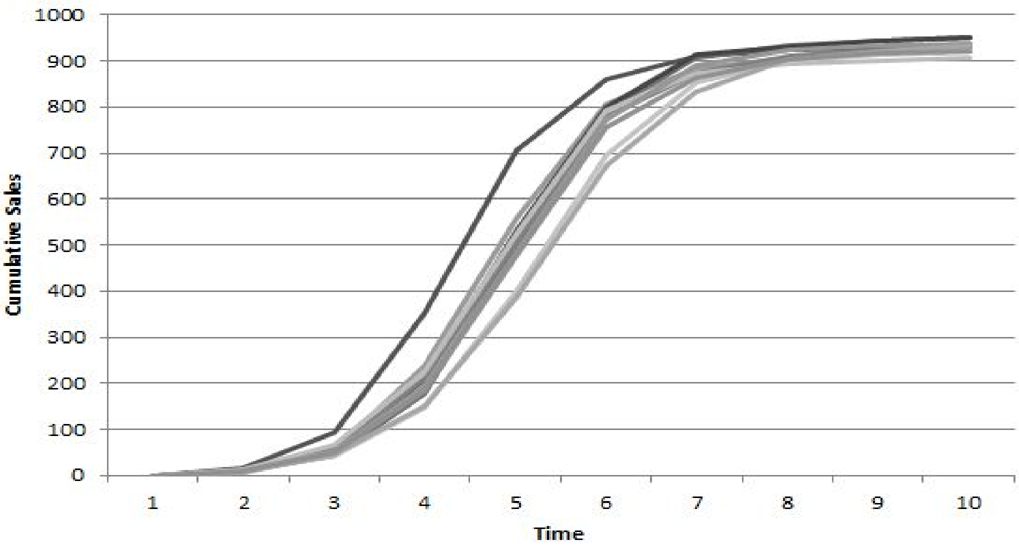

Figure 10. 10 cases drawn when N=1,000

As seen in Figure 10, some of the 10 diffusion curves drawn by the IBDM are significantly different from the others. This can harm the robustness of the model; thus, a larger N was examined for the simulation (Figure 11).

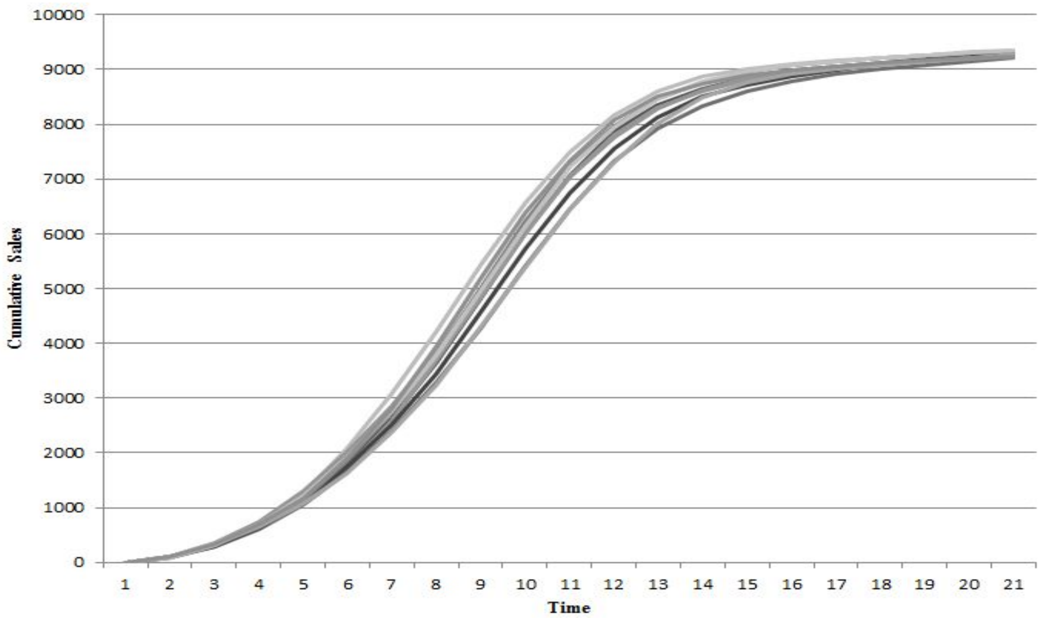

Figure 11. 10 cases drawn when N=10,000

As shown in Figure 11, some differences are still found among the curves; however, it seems sufficiently robust. Hence, hereafter, the simulation assumes $N = 10,000$, draws 10 curves from different allocations of V_i, and takes the average sales of 10 diffusion curves for the final simulation result. Note that all curves in Figure 11 and even ones in Figure 10 are shaped in a smooth S-curve, which is desirable for describing the proposed shape of Rogers (1962) and the standard shape of Bass (1969) (see Figure 1, Section 2.1, and Figure 2, Section 2.2.1). This shows the fitness of the IBDM for describing diffusion patterns.

Secondly, the proper level of rewiring probability ρ is investigated. As explained in Section 3.2, the rewiring probability affects both the average path length of a network and the cliquishness of a network, thus affecting the diffusion pattern. Figure 12 shows five different cases by ρ. Other parameters are assumed to be the same as in previous cases.

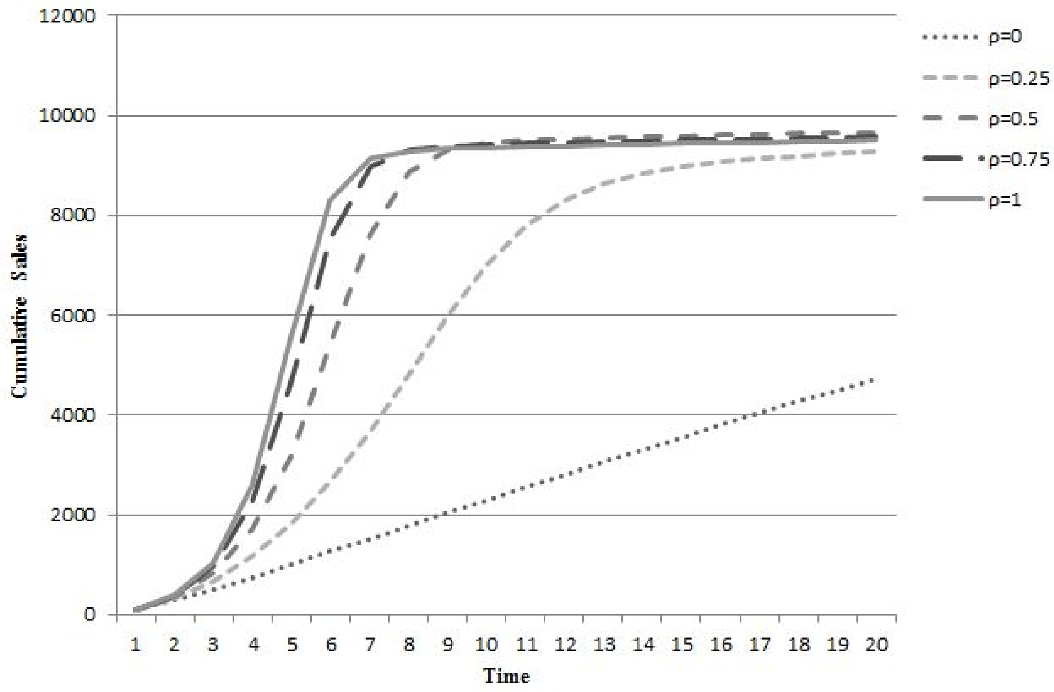

Figure 12. Diffusion patterns resulting from the rewiring probability

As seen in Figure 12, the diffusion rate increases with the increase in ρ. This is natural because a large ρ contracts the network; thus (i.e., in such a small society), the word-of-mouth effect becomes stronger. It is noticeable that when $\rho \geq 0.5$, the differences among curves become smaller in contrast to those that occur when ρ is rather small. This implies that a large ρ is not necessarily required to explain innovation diffusion. Watts and Strogatz (1998) noted that a large ρ reduces clustering in the network, which is not desirable for describing innovation diffusion. In addition, note that the curve in the case of $\rho = 0$ seems linear. This is because the degree of interconnectedness is constant when $\rho = 0$ and because the network converges to a regular lattice model. In other words, as the diameter of network gets larger, the influence of social interaction gets smaller. This justifies the use of an S-W network rather than cellular automata in the IBDM. Another noticeable characteristic is that $\rho = 1$ shows a low level of final market penetration in comparison to others. This is because a large ρ causes many isolated agents. This is also the reason the IBDM does not cover the whole market potential, even though there is enough time. Through the continuous decay of price, these uncovered agents will adopt the innovation when $V_i > P_t$.

The reason $\rho = 0.25$ is used for the study is as follows: As Bohlmann et al. (2010) notes, a society structure with characteristics of both a random network and a regular lattice is appropriate for diffusion research. Newman and Watts (1999) claimed that one should work with the value $\rho \approx (K/2)^{-2}$ to see clean network behavior. For $K = 4$, the recommended value is $\rho = 0.25$. Table 2 shows the average path length or diameter of the network for each case. It also shows the reason $\rho = 0.25$ is selected. Hence, hereafter, the rewiring probability ρ is fixed as 0.25.

Table 2. Average path length of the S-W network due to the rewiring probability

ρ	0	0.25	0.5	0.75	1
Average Path Length	1250	9	8	7	7

4.2 Simulation Results

In this sub-chapter, an alternative model called the conformity model is investigated first. Then, various simulations of the default model are implemented to examine the effects of four utility parameters of the IBDM. As stated in sub-chapter 3.1, the Brock and Durlauf (2001) model assumes a social conformity among agents. It is possible to construct an alternative model with such an assumption: the so-called conformity model. The conformity model shares the same assumptions as the default IBDM (e.g., social networks, heterogeneous individual taste of innovation). On the other hand, other decision variables, such as price, are ignored, and the social utility specification is totally different.

The conformity model is based on equation (24) with choice indicator ω, which is defined to be 1 in the case of adoption and -1 otherwise. The specification for interaction weight J_{ij} is also different. In the case of conformity, it is hard to say that people with a favorable taste for innovation have a broader influence in society (i.e., $\sigma_j^2 \sim s \cdot V_j$), since agents affect each other even when they have not adopted the innovation. Thus, the variance of the interaction weight is also assumed to follow an independent uniform distribution with a scaling coefficient of s. In sum, the equations used in the conformity model are as follows:

$$U_{it} = \begin{cases} V_i + \sum_{j \neq i} J_{ij} \cdot \omega_{j,\,t-1} & if \quad \omega_i = 1 \\ -\sum_{j \neq i} J_{ij} \cdot \omega_{j,\,t-1} & if \quad \omega_i = -1 \end{cases} \quad \cdots\cdots \text{Eq. (36)}$$

$$V_i \sim \mathrm{Unif}[0,1] \quad \cdots\cdots \text{Eq. (37)}$$

$$J_{ij} = \phi\left(\Delta_{ij} / k, 0, \sigma_j\right) \quad \cdots\cdots \text{Eq. (38)}$$

$$\sigma_j^2 \sim s \cdot \mathrm{Unif}[0,1] \quad \cdots\cdots \text{Eq. (39)}$$

Note that an agent will gain social utility even when he/she has not yet adopted an innovation. In this model, an agent decides to adopt an innovation whenever $V_i + \sum_{j \neq i} J_{ij} \cdot \omega_{j,\,t-1} \geq -\sum_{j \neq i} J_{ij} \cdot \omega_{j,\,t-1}$. The right-hand side can be larger in the case that most agents do not adopt the innovation.

Simulation results are derived from an average of 10 cases with $N = 10{,}000, K = 4, \rho = 0.1$. Here, ρ is assumed to have a lower value than that of default model, since the effects of interactions are greater in the conformity case. The network externality of innovation is also assumed to be 1.

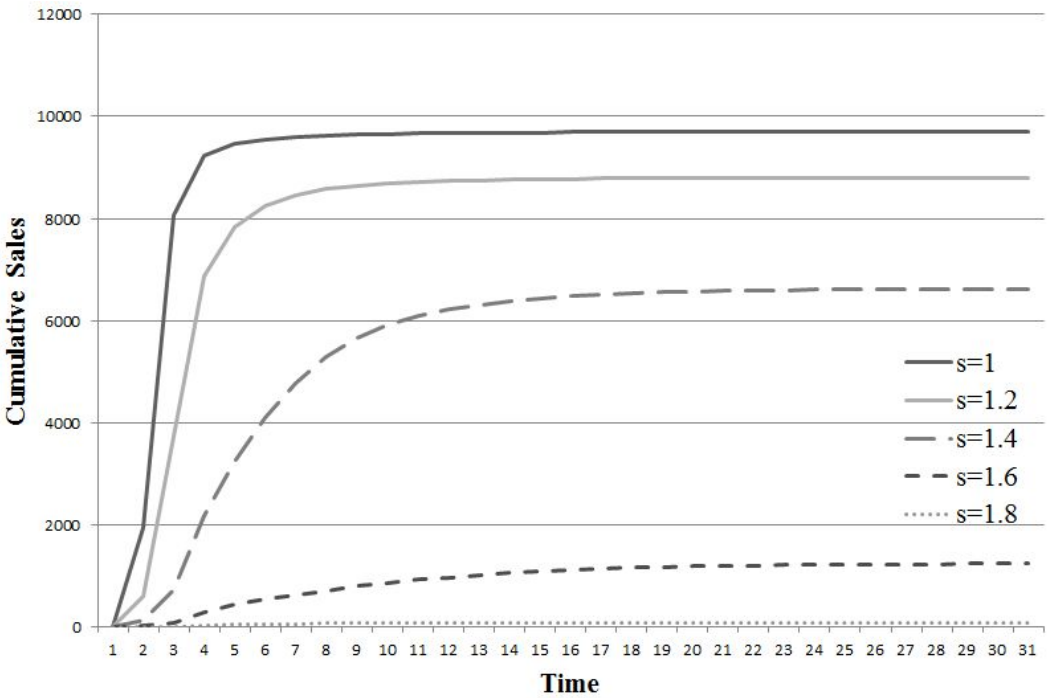

Figure 13. Cumulative sales in the conformity model

As seen in Figure 13, different diffusion patterns are drawn through various values of the scaling factor s. This pattern seems extreme: it is explosive for a small s and stagnant for a large s. Such extremeness is natural for conformity models. Young (2009) also states that the curve must either decelerate initially or accelerates at a super-exponential rate. It is also notable that the diffusion is more likely to discontinue as s increases. A large s implies broader interactions in society. Since there are only a few adopters in the earlier period of diffusion, most agents find it easier to conform to refuse the innovation as the social influence grows larger. Thus, in the case of a few innovators, strong interactions lead to a discontinuity of diffusion. Rogers (2003) pointed out that homogeneous groups tend to avoid changes and are reluctant to adopt new innovations. Granovetter (1973) also stated that it is natural to assume that the resistance to an innovation is greater than the desire to adopt it; thus, to trigger a chain reaction, a larger number of people should adopt an innovation in its early stages.

Although a conformity model can draw interesting diffusion patterns, such an extreme shape is not appropriate for general diffusion research. This is why previous research studies, such as Delre et al. (2007) and Choi et al. (2010), assumed only a contagious model. Hence, this study also defines a contagious model as a default model.

The basic assumption of a social network is as follows (see Section 4.1): a total number of agents $N = 10,000$, an average number of links per agents $K = 4$, and a rewiring probability $\rho = 0.25$. Moreover, the study sets a base case of parameters: an initial normalized price of innovation $P_0 = 0.99$ (to guarantee that diffusion starts from $t = 1$), a learning coefficient of price $\alpha = 0.01$, a network externality of innovation $k = 1$, and a correlation of social interaction with taste $s = 1.5$. Such values of parameters are adopted for the purpose of showing the shape of the diffusion curve clearly.

The scenarios used for the simulation are as follows (Table 3).

Table 3. List of scenarios used in the simulation

Social Network Parameters			
$N = 10{,}000$	$K = 4$	$\rho = 0.25$	
Base Case			
$P_0 = 0.99$	$\alpha = 0.01$	$k = 1$	$s = 1.5$
Case 1	Case 2	Case 3	Case 4
$P_0 = 0.99$	$\alpha = 0.02$	$k = 1$	$s = 1.5$
$P_0 = 0.97$	$\alpha = 0.01$	$k = 1.1$	$s = 1.7$
$P_0 = 0.95$	$\alpha = 0.005$	$k = 1.2$	$s = 1.9$

Four simulation cases are set to have three scenarios, each of which includes the base case. In each case, parameters other than the focused parameter are fixed, as in the base case. In all cases, the study draws ten different societies in the same network (i.e., the fixed network) and a different allocation of V_i. It takes the average sales volume as the final simulation result.

4.2.1 Effect of Price Coefficients

Bass (1969) defined the coefficient p in the following equation (40) as an 'innovation coefficient' and interpreted it as the degree of the external influences on the diffusion rate. The details of the equation are explained in Section 2.2.1.

$$\frac{dN(t)}{dt} = p(\bar{N} - N(t)) + \frac{q}{\bar{N}} N(t)(\bar{N} - N(t)) \quad \text{Eq. (40)}$$

Bass (1969) classified promotion efforts, such as advertising, as part of such external influences; however, in latter research into GBM (Bass et al., 1994), another function, called 'marketing efforts,' is added to the model to capture the effects of price and advertising. Hence, a natural question arises: What does p represent in GBM? As seen in equation (41) of GBM, the marketing efforts function $x(t)$ affects, not only the external influence represented by p, but also the internal influence represented by q.

$$\frac{dN(t)}{dt} = x(t)[p + \frac{q}{\bar{N}} N(t)](\bar{N} - N(t)) \quad \text{Eq. (41)}$$

Although Bass et al. (1994) calculated the effect of $x(t)$ on p and q, it is noteworthy that this shows that GBM cannot sufficiently separate the effects of external influences and internal influences. This is why Russell (1980) criticized aggregate diffusion models. Fourt and Woodlock (1960) showed that the assumption of a constant proportion of adopters in the population leads to the same functional form as Bass' external influence. Russell (1980) claimed that the assumption of price decay is enough to describe the first term of the Bass diffusion function (i.e., external influence). In addition, Bain (1964) and Wells (1977) showed a strong relationship between income class and sales data.

Russell (1980) also criticized the fact that the classification of innovators and imitators in the Bass model is not based on any economic theories. In other words, it is natural to assume that both innovators and imitators maximize the same utility function (from the perspective of microeconomics). Thus, the IBDM introduces a price variable to capture both the innovation effect and the economic interpretation of the model. Note that the meaning and the implication of the price coefficient of the IBDM and the innovation coefficient of the Bass model are different; however, they show a similarity in functional form and in the shape of their diffusion curves. Figure 14 shows the simulation results from three scenarios in Case 1 (Table 3).

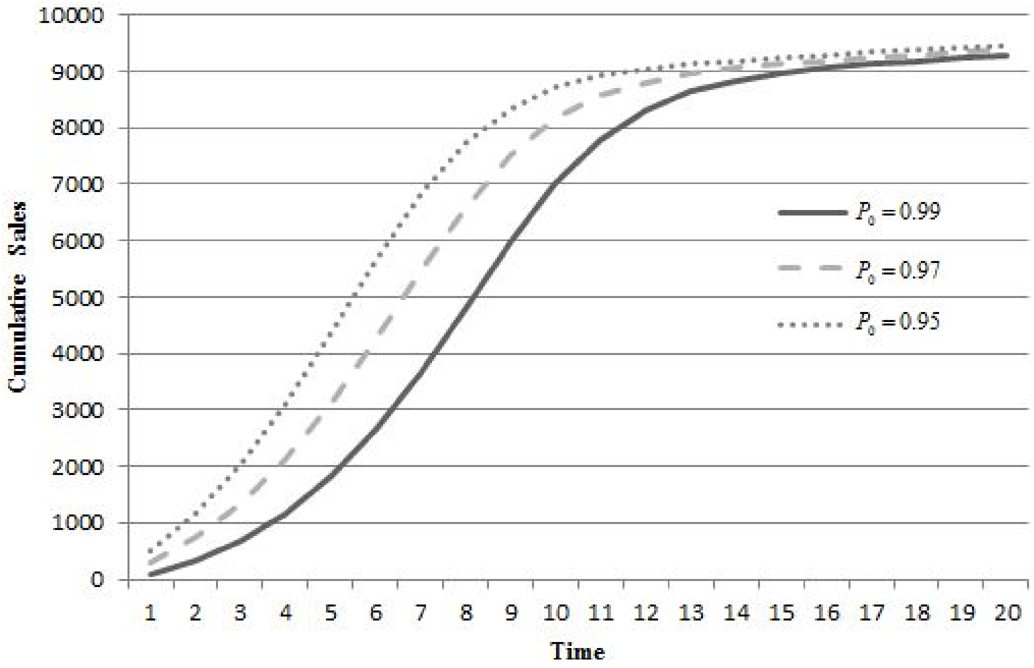

Figure 14. Simulation results of Case 1

The results of the base case are represented by a solid line, and results from scenarios $P_0 = 0.97$ and $P_0 = 0.95$ are represented by a dashed line and a dotted line, respectively. These three results show the shift in the diffusion curve to the left as P_0 decreases. That is, the decrease in P_0 shortens the time to adoption for all agents, but does not accelerate the rate of diffusion. It is noteworthy that such a result is very similar to that shown in the Bass (1969) model, as seen in Figure 15.

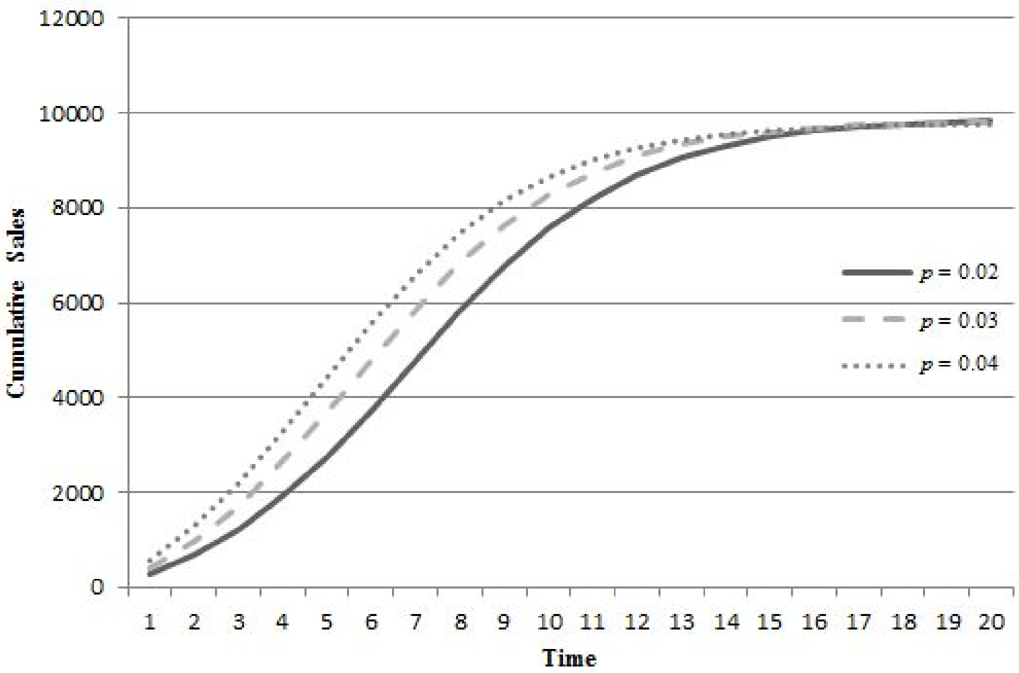

Figure 15. Bass diffusion curves due to the innovation coefficient

For the simulation of Bass (1969) model, the parameters are assumed to be $p = 0.03$ and $q = 0.38$, following the approach of Sultan et al. (1990), who reported the average of parameters in such values. The market potential $\bar{N}$ is defined to be 10,000 to match the result to the IBDM. Note that the increase in p shifts curves to the left, just as P_0 of the IBDM does. That is, P_0 takes the role of p in the IBDM, but in negative way. Furthermore, it is noticeable that the effect of p seems unrealistic in latter periods. It can be found that a lower p covers more adopters eventually. Such a result harms the interpretation of p, which represents the effects of external influences. This implies that increased mass media promotion efforts can lead to a smaller final market penetration of innovation. This is not intuitive, and it is hard to accept from the perspective of a marketing strategy. In contrast, the IBDM does not show any such crosses; thus, it is more intuitive and suitable for explaining real-world marketing strategies.

The second case concerns the learning coefficient of price, α (see equation (32)). The simulation results are shown in Figure 16.

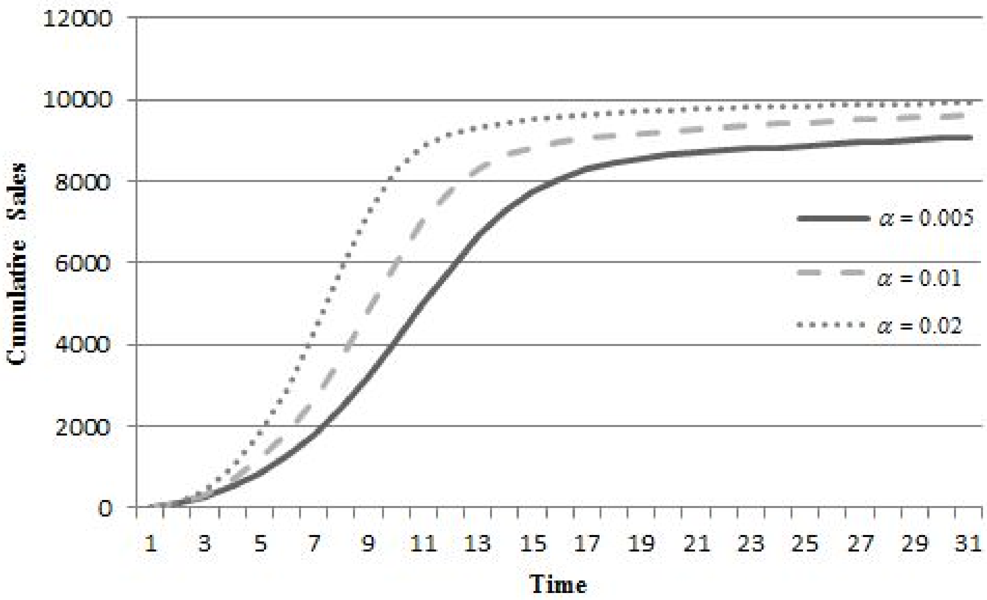

Figure 16. Simulation Results of Case 2

The dashed line, the solid line, and the dotted line show the results of the base cases $\alpha = 0.005$, and $\alpha = 0.02$, respectively. It can be found that a larger α accelerates the diffusion and shifts the curve upward. Its nonlinear effect is due to the specification of price dynamics, which are expressed as an exponential function. The latter aspect—the larger α, which shifts the curve upward—is more interesting. This implies that the learning rate of innovation is the most critical aspect for covering the whole market potential. In other words, instead of other marketing efforts, rather than initial price or factors of internal influences, α has the greatest effect on final market penetration. Though other parameters may have effects on final market coverage, these effects are not as strong as that of α. The reason this happens can be explained by the nature of each parameter. As seen above, P_0 exerts a linear effect on diffusion, while α affects diffusion non-linearly. α also lifts up the whole curve in order to cover more agents; however, this type of effect from P_0 is relatively small.

The following additional simulations compare the two price strategies: low initial price/low drop rate and high initial price/high drop rate. To create a fair comparison, the overall average prices of two strategies are set to be similar. Case 2-1 assumes $P_0 = 1$ and $\alpha = 0.02$, and Case 2-2 assumes $P_0 = 0.94$ and $\alpha = 0.015$. Other assumptions are the same as in the base case. Note that the difference of 0.06 in P_0 is significant because the price is normalized in the simulation; thus, the difference implies a 6% reduction in the initial price. Table 4 shows the results.

Table 4. Simulation results for two different price strategies by time[13]

Time	Price		Cumulative Sales		Sales		Cumulative Revenue	
	Case 2-1	Case 2-2	Case 2-1	Case 2-2	Case 2-1	Case 2-2	Case 2-1	Case 2-2
1	**1.00**	0.94	0.0	**601.9**	0	**601.9**	0.0	**565.8**
2	**0.98**	0.93	199.3	**2688.5**	199.3	**2086.6**	195.4	**2498.0**
3	**0.96**	0.91	1074.0	**6762.2**	874.7	**4073.7**	1035.8	**6214.1**
4	**0.94**	0.90	3735.2	**8820.2**	**2661.2**	2058	3542.0	**8063.5**
5	**0.92**	0.89	7326.5	**9302.1**	**3591.3**	481.9	6857.2	**8490.1**
6	**0.90**	0.87	8899.3	**9434.4**	**1572.8**	132.3	8280.3	**8605.5**
7	**0.89**	0.86	9308.6	**9493.4**	**409.3**	59	8643.3	**8656.2**
8	**0.87**	0.85	9443.0	**9538.3**	**134.4**	44.9	**8760.2**	8694.2
9	**0.85**	0.83	9509.4	**9574.8**	**66.4**	36.5	**8816.7**	8724.6
10	**0.84**	0.82	9565.4	**9607.0**	**56**	32.2	**8863.5**	8751.0
11	**0.82**	0.81	9610.0	**9637.4**	**44.6**	30.4	**8900.0**	8775.6
12	**0.80**	0.80	9649.2	**9663.5**	**39.2**	26.1	**8931.5**	8796.4
13	0.79	**0.79**	9680.9	**9686.5**	**31.7**	23	**8956.4**	8814.5
14	0.77	**0.77**	**9711.5**	9708.7	**30.6**	22.2	**8980.0**	8831.7
15	0.76	**0.76**	**9743.5**	9732.2	**32**	23.5	**9004.2**	8849.6
Mean	**0.87**	0.85			**649.6**	648.8		

Revenue is calculated through a simple multiplication: that is, $R_t = P_t \cdot S_t$, where R_t

[13] The numbers are expressed in boldfaced type if one is relatively larger than another.

represents revenue at time t and S_t represents sales at time t. Note that the price in Case 2-2 is lower than in Case 2-1 during most of the period; thus, it is intuitively less profitable. Furthermore, Case 2-2, where the price of the good is over 4% lower than in Case 2-1, has most of the sales by period 4. Nevertheless, there is no significant difference in cumulative revenue between the two strategies. Rather, it is reasonable to say that Case 2-2 is better for enterprises due to the presence of depreciation. Figure 17 is helpful for understanding what is explained above.

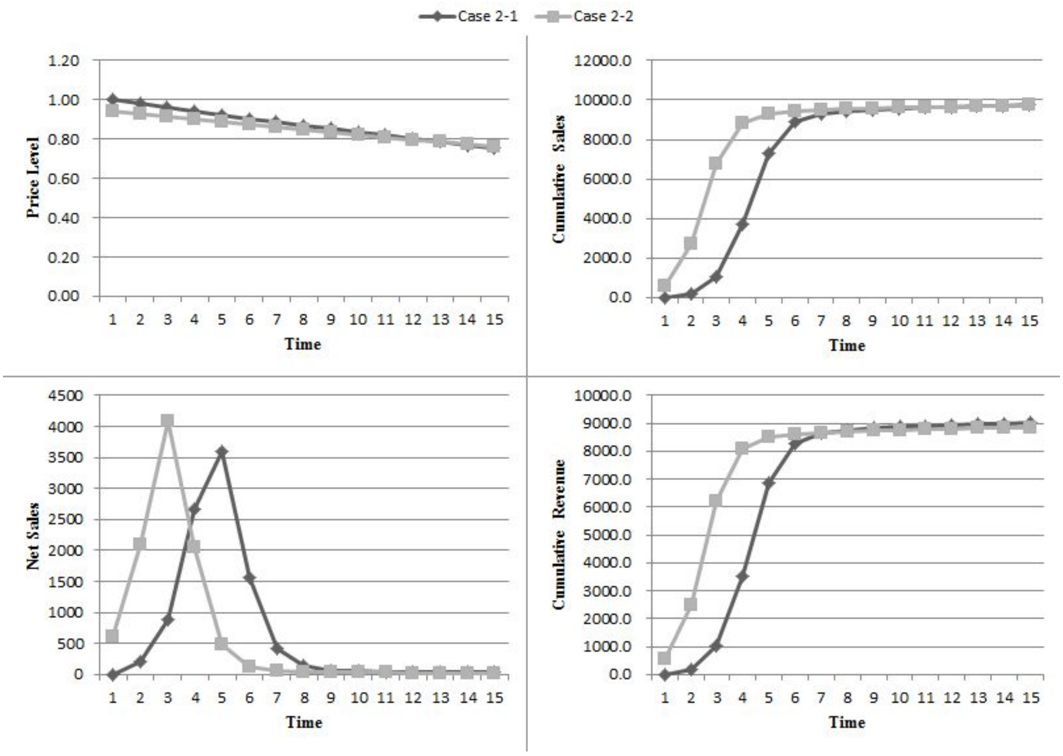

Figure 17. Comparison between two price strategies

4.2.2 Effect of Social Interactions

The effect of internal influences on diffusion is more critical than that of external influences. As shown in Figure 2, Section 2.2.1, according to Mahajan et al. (1995), the share of internal influences on the diffusion rate is much larger than that of external influence for a few periods after the initiation of diffusion. This shows the importance of word-of-mouth or individual interactions in the diffusion model. Nevertheless, the effect from individual interactions is simply assumed to be a constant, not only in an aggregate model (e.g., Bass (1969)), but also in recent ABM research studies (Kiesling et al. 2012). The IBDM was established on the basis of heterogeneous social interaction; thus, it is designed to interpret the effects of internal influences in innovation diffusion.

The IBDM assumes two different parameters for internal influences: the network externality of innovation, k, and the correlation of social interaction with taste, s. The former parameter captures the effect of the network attribute of innovation. This parameter is useful for linking the aspect of social interactions with an innovation itself. It enables an analysis of a marketing strategy from the perspective of individual interaction, while most previous models simply separated the attribute of innovation from the word-of-mouth effect. Figure 18 shows the simulation results of three scenarios in Case 3 (Table 3).

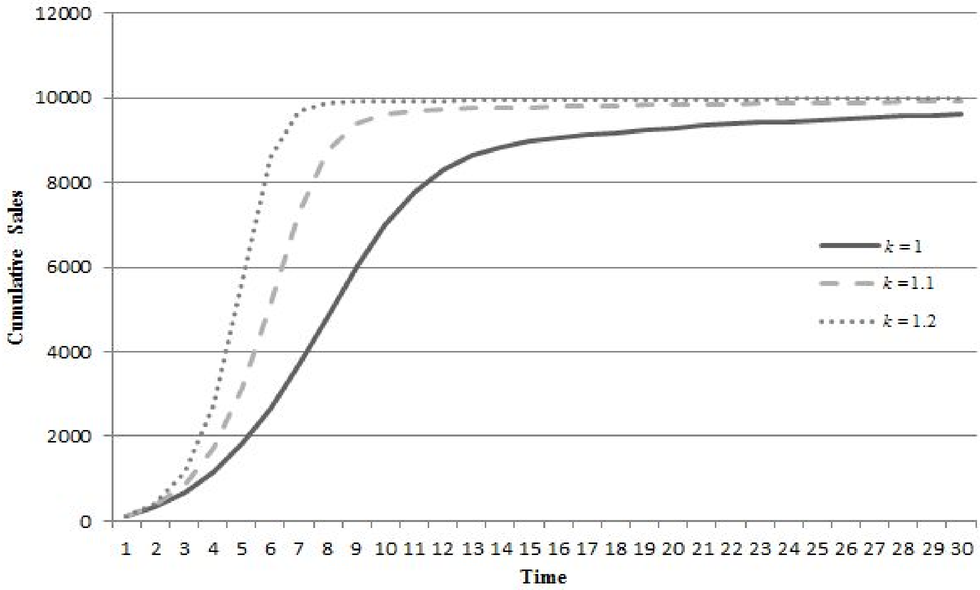

Figure 18. Simulation results of Case 3

The base case, the cases of $k=1.1$ and of $k=1.2$ are represented as the solid line, the dashed line, and the dotted line, respectively. As k gets larger, the diffusion curve bends more to the left-upper side (i.e., a larger k accelerates the rate of diffusion more). In other words, more network-friendly innovations spread with more rapidly increasing speeds than less network-friendly ones.

Before comparing the result to the Bass model, it is necessary to discuss the results of the last case in relation to s (Figure 19).

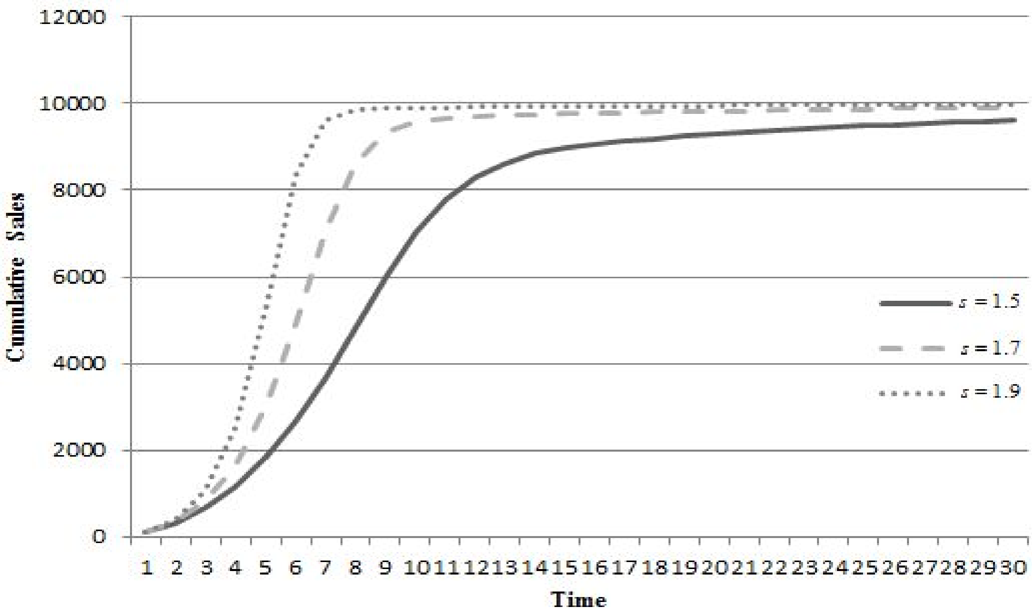

Figure 19. Simulation Results of Case 4

The solid line, the dashed line, and the dotted line represent, respectively, the base case, the case of $s = 1.7$, and the case of $s = 1.9$. The figure also shows an acceleration effect as in the previous figure. Since s refers to the correlation between social interaction and taste, it can be interpreted that, in a society with a larger s, innovative adopters exert a more active and wide influence on potential adopters. Thus, s can be explained to the 'degree of the activeness of innovators.'

Note that the change in the diffusion curve by s is similar to that of k. Since s depends on the nature of the social network, while k is determined by the network attribute of innovation, this implies that a similar diffusion pattern can be derived in a society with a large s by adjusting k to be larger. Such an accelerating effect of internal influences is also found in the Bass model (Figure 20).

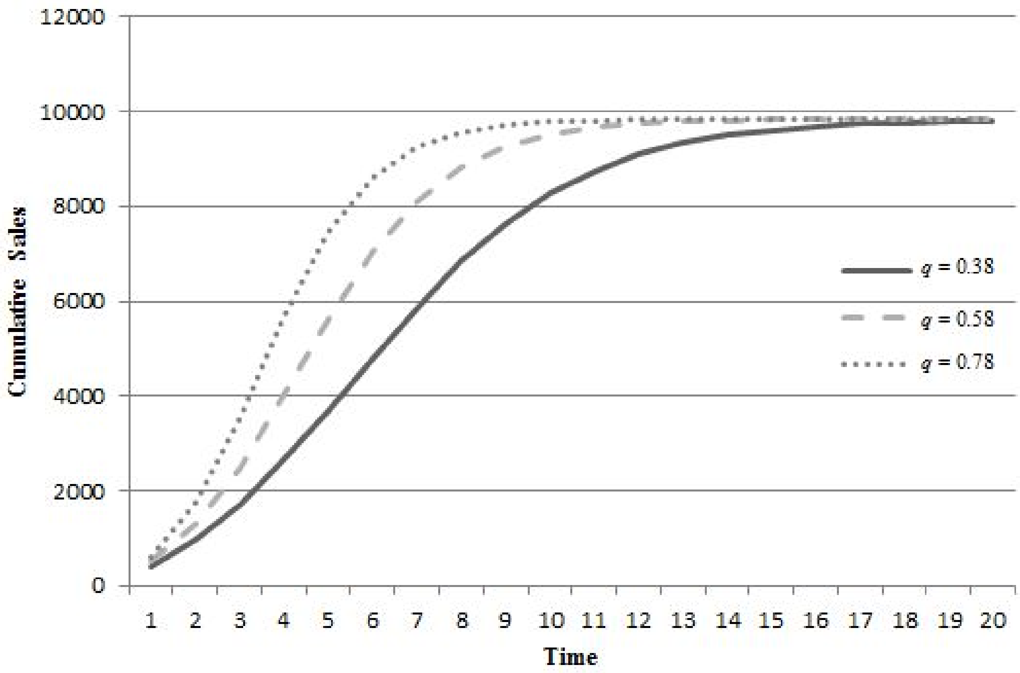

Figure 20. Bass diffusion curves due to the imitation coefficient

In Figure 20, $\bar{N} = 10,000$ and $p = 0.03$ are assumed, and only the imitation coefficient q changes by 0.2. The figure also shows an acceleration effect, along with k and s, in the IBDM. The comparison supports the similarity between the Bass model and the IBDM. Furthermore, by showing the nature of internal influences in innovation diffusion, it provides an answer for the question of why the Bass model fits.

Mahajan et al. (1995) provided a diagram of the adopter shares resulting from each influence (see Figure 2, Section 2.2.1). The same diagram can be drawn for the IBDM, but a further definition of shares is required for that. The question is, who can be counted as adopters due to the price effect, since agents are always affected by a mixture of influences? A natural candidate is the number of people for whom $P_t \le V_i < P_{t-1}$ at time t. However, there may be some agents with $P_t \le V_i < P_{t-1}$, but $V_i + \sum_{j|\omega_{jt}=1} J_{ij} > P_{t-1}$. These agents should be excluded from the counts. To achieve the results, the parameters of the IBDM are assumed to be as follows: $P = 1$, $\alpha = 0.05$, $k = 1$, and $s = 1$. These numbers are selected to enlarge the share of the price effect in order to show the diagram clearly (Figure 21).

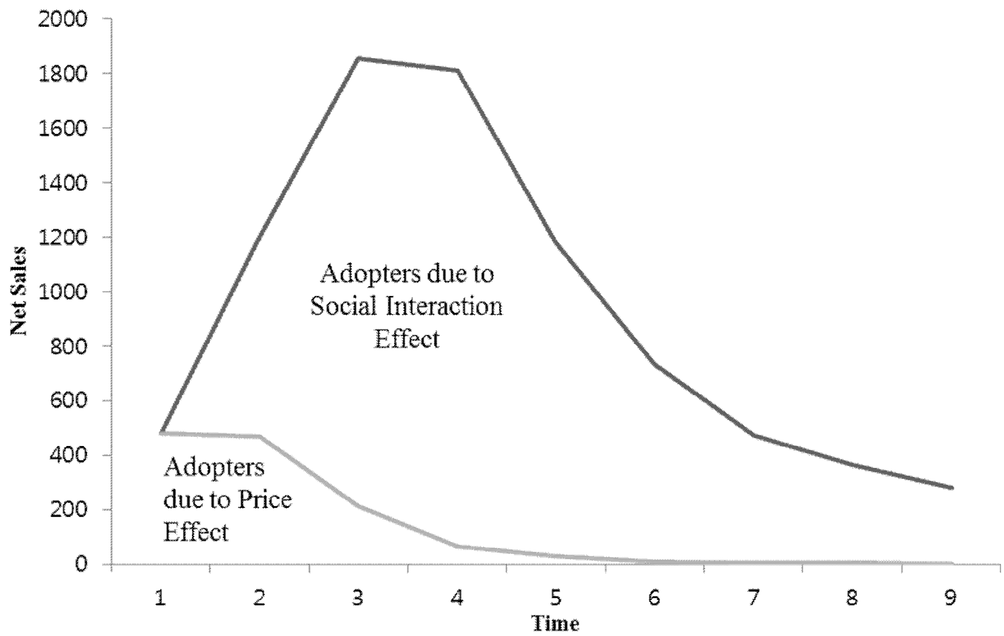

Figure 21. Adopters due to both influences in the IBDM

Like the Bass model, the IBDM also exhibits a large share of internal influences from its middle periods. This implies that dynamic prices create only a small share of the diffusion rate and that the innovation spreads out automatically, largely through individual interactions.

4.3 Summary

Through the simulation results of the IBDM, presented above, it can be verified that the IBDM is working well and that it has desirable properties. Before we explore anything else, the classification of the IBDM should be explained. The IBDM is (1) an ABM model, (2) a microeconomic model, (3) a small-world network model, (4) an individual-behavior-based diffusion model, and (5) an interaction-based model. As mentioned in Chapter 2, of numerous ABM research studies, few utilize a utilitarian framework or small-world networks, and nearly none are based on heterogeneous social interactions.

In Section 2.4, four desirable properties of diffusion models for future research studies were discussed: an individual-level approach, a consideration of social network topologies, a concrete theoretical background, and a proper reflection of social interactions. As shown above, the IBDM satisfies all of these properties through a simple method of modeling. Note that the IBDM does not depend heavily on individual specifications. This can be easily seen from the fact that four parameters of the IBDM are defined in an aggregated level. This means that the IBDM is suitable for describing aggregated-level market data and that it establishes a suitably functioning procedure to link individual-level behavior to aggregated-level behavior.

The adopter categorization proposed by Rogers (1962) can be applied to the IBDM (see Figure 1, Section 2.1). Mahajan, Muller, and Srivastava (1995) developed a method to derive the sizes of adopter categories from a diffusion model. As shown in the following Figure 22, they proposed regarding the peak time T^* as the mean in the Rogers categorization. Then, they suggested calculating two points of inflection in the net sales curve in order to get T_1 and T_2. These three specific times can divide the time dimension into four periods. Agents who adopt the innovation before T_1 are classified as 'Early Adopters,' those who adopt between T_1 and T^* are classified as the 'Early Majority,' and so on. 'Innovators' are defined as those agents who adopt the innovation during the initial period. Mahajan et al. (1995) derived a specific range for each category using the range of parameters reported by Bass (1969).

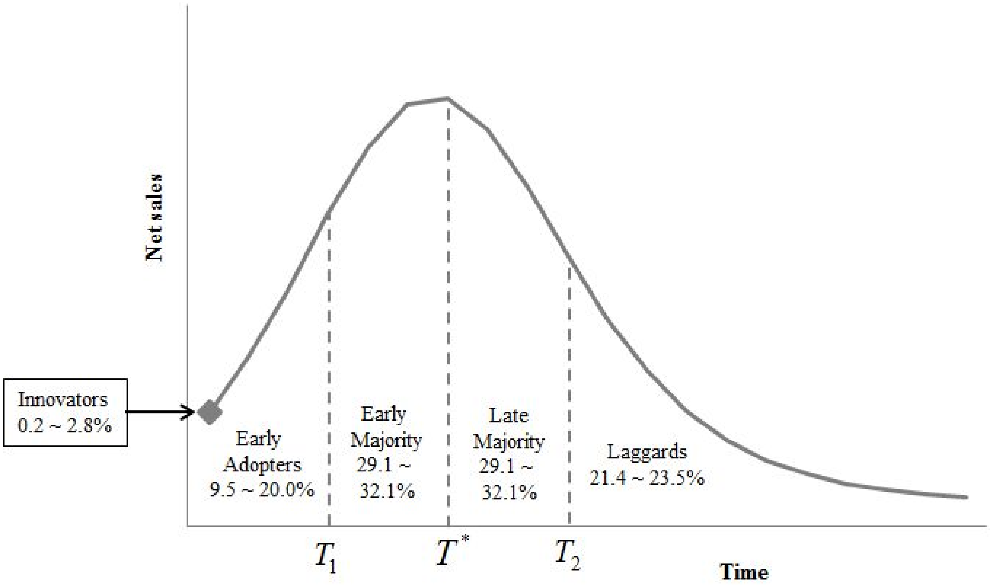

Figure 22. Adopter categories based on the Bass model[14]

In keeping with Mahajan et al. (1995), the portion of each adopter category in IBDM is calculated from the simulation results. Note that the portion of innovators depends heavily on the level of P_0. The Bass model also has a fixed portion of innovators, p. Hence, the following three cases are examined to derive adopter portions: the base case ($P_0 = 0.99$), the case of $P_0 = 0.98$, and the case of $P_0 = 0.97$. Through the definition of IBDM, the expected portions of innovators in each case are around 1%, 2%, and 3%, respectively.

[14] Source: Mahajan, Muller, and Srivastava (1995).

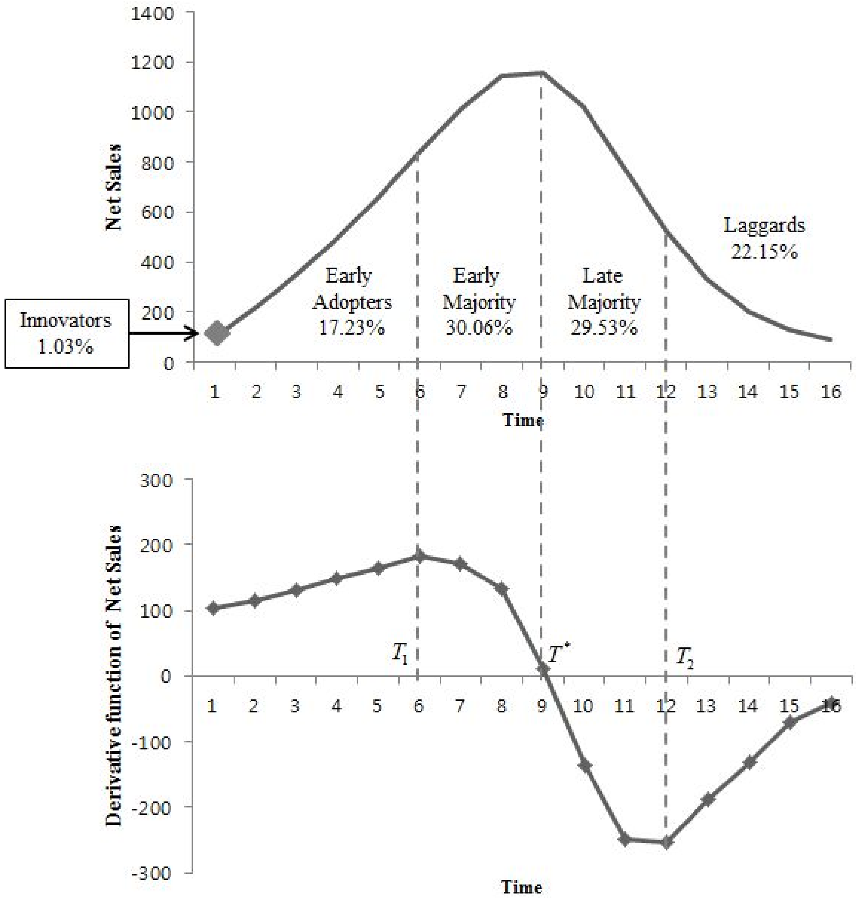

Figure 23. Adopter categories in the IBDM: The base case

Figure 23 shows the results of the base case. Its numbers are similar to those of Bass, rather than those of Rogers (1962). This means that real innovation diffusion can be explained effectively by Bass' hazard function, rather than by a normal distribution. In the meantime, all three models (i.e., Bass, Rogers, and the IBDM) exhibit a common aspect of symmetricity in adopter categories. One may wonder why the portion of early adopters seems to be larger than the reported number. This is because the portion is calculated in a discrete manner, while the diagram shows a continuous area; thus, only adopters in periods 2 through 5 are classified as early adopters. The portion of laggards also seems to be smaller than the actual number because this is just a part of the whole diffusion curve; thus, most adopters in the long tail do not appear in the diagram.

Table 5 shows the portions of the adopter categories from three cases, including the base case.

Table 5. Adopter categories of the IBDM for three cases

Cases	Innovators	Early Adopters	Early Majority	Late Majority	Laggards
$P_0 = 0.99$	1.03%	17.23%	30.06%	29.53%	22.15%
$P_0 = 0.98$	2.03%	14.45%	29.98%	30.60%	22.95%
$P_0 = 0.97$	3.01%	10.51%	29.06%	32.65%	24.77%

It is noteworthy that, even though the portion of innovators changes across cases, the portions of majority adopters remain in fixed ranges—around 30%. It is interesting that the portion of early adopters decreases with the increase in innovators, while that of laggards increases. According to Mahajan et al. (1990), the same phenomenon appears in the Bass model. This is because the percentage of adoptions due to external influences decreases across categories as the innovation coefficient increases. In other words, if there are significantly many innovators in a society, then the imitation effect due to these innovators is more critical than the innovation effect. Decreased innovation effect is much critical to the number of early adopters. Thus, there will be fewer early adopters and more laggards.

By comparison with the Bass model, the IBDM answers the question of why the Bass model fits without decision variables—a question that has been important for understanding the Bass model (Bass et al., 1994). From its utility specifications, the IBDM shows that the initial price level shifts the curve linearly and that internal influences accelerate the curve. The Bass model fits well because it is based on a good mathematical function to have characteristics of linear external effect and non-linear internal effect.

Bass (1995) proposed a guideline to develop a new diffusion model, as follows:

1. The model should explain why the Bass model fits without including decision variables.
2. Different sets of decision variables should produce curves with similar shapes.
3. The model should encompass the properties of internal influence used in the Bass model.

The IBDM satisfies all of these conditions, as shown above. However, Bass (1995) proposed three more conditions:

4. The model should track the irregular deviations of the actual data from the smooth curve.
5. The model should be flexible enough to draw various shapes.
6. The model should yield a closed-form solution.

The latter three conditions focus on the practicability of the model. A simulation analysis cannot determine whether the IBDM satisfies those conditions. Thus, an investigation fitting the IBDM to real market data is required.

Chapter 5. Empirical Availability of the Interaction-Based Diffusion Model

In this chapter, the practicability of the IBDM is investigated. One may argue that the desirable properties of the IBDM can be derived only with proper parameter values. Since simulations are limited in their ability to show the flexibility of the model, fitting the model to real market data is necessary. An analysis of aggregated-level market data is impractical at this step, however, because the IBDM is based on individual specifications, which are not observable in such data. Hence, the adjustment of the IBDM for fitting and the correlated fitting procedure are discussed below. Then, the analysis results of the real-market data are presented.

5.1 Adjustment of the Model for Fitting

Fitting an ABM model to aggregated data is usually impractical. Since ABMs derive their results from individual specifications and from the topologies of social networks, it is unrealistic to analyze aggregated-level data, which have no information about such things. Hence, in order to analyze innovation diffusion at an aggregated level, a simplification of two factors is required: the effects of social networks and the effects of individual heterogeneity.

As shown in Sections 3.2 and 4.1, a network with the fixed parameters N, K, and ρ can draw different topologies due to randomness resulting from the rewiring probability. For instance, there are numerous different networks with a fixed $\rho = 0.25$ because parameters cannot determine which agents are linked to which other agents. Thus, there may be more hub agents or fewer hub agents in different networks with the same specification. Because of this, an average network is required for analysis.

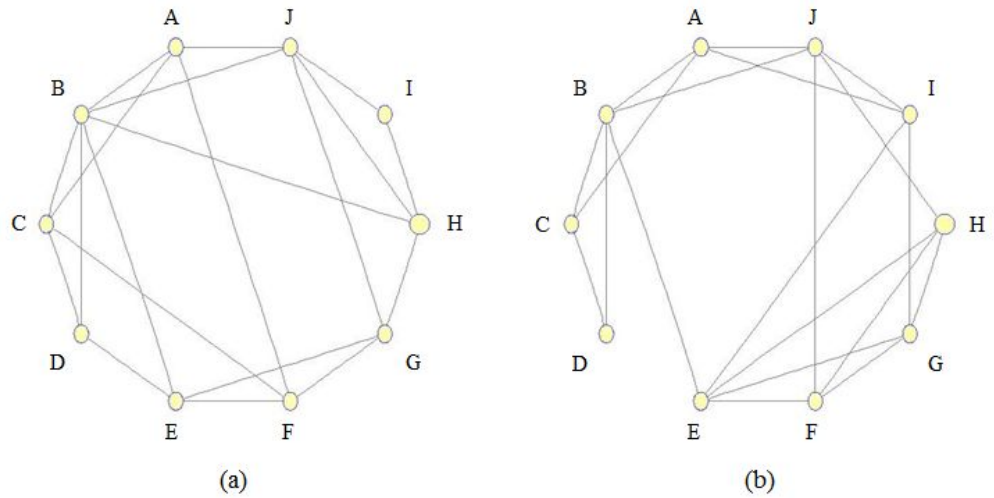

Figure 24. Two small-world networks with the same specifications

Figure 24 shows two different S-W networks with the same specifications: $N = 10$, $K = 4$, and $\rho = 0.25$. However, for example, agent I in network (a) has only 2 neighbors, while the same agent I in network (b) has 4 neighbors. Likewise, most of the agents in (b) have neighbors different from those of their counterparts in (a). Thus, if agent I were the innovator, the diffusion in (a) would be slower than in (b). This shows why fitting to an average network is necessary. Note that this is also the reason why simulations in Chapter 4 fixed the network (i.e., fixed the connections among agents) to derive diffusion curves.

The IBDM reflects the effects of the topology of a social network in social utility. Remember that the interaction weight between agents (J_{ij}) is defined as the function of the social distance between them (Δ_{ij}). However, as stated above, this social distance is not fixed for various networks. Barrat and Weigt (2000) showed the probability density function of Δ_{ij} (hereafter denoted by d_{ij} for simplification), where $N = 2,000$ and $K = 3$ (Figure 25).

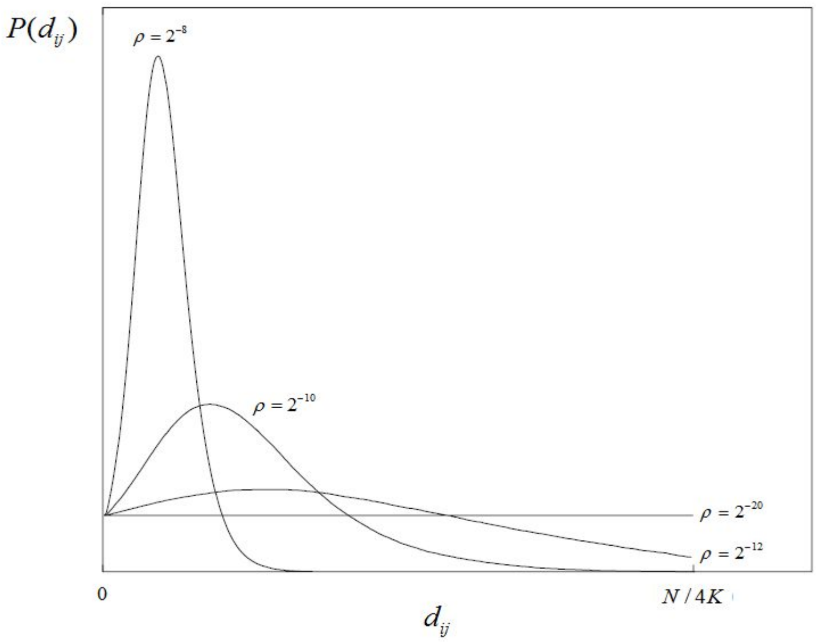

Figure 25. Probability distribution of social distance[15]

Note that when $\rho = 2^{-12} \sim 0$, the probability density converges to a line. Barrat and Weigt (2000) clarified that this diagram is drawn by averaging 500 different networks. Furthermore, the study stated that there is no closed-form solution for $P(d_{ij})$ because too many irregular cases can be drawn in an S-W network. It is also noticeable that $P(d_{ij})$ varies with the value of the rewiring probability; that is, $P(d_{ij})$ is conditional on ρ. In other words, there is a joint probability density $P(d, \rho)$, as shown in the following Figure 26.

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[15] Source: Barrat and Weigt (2000).

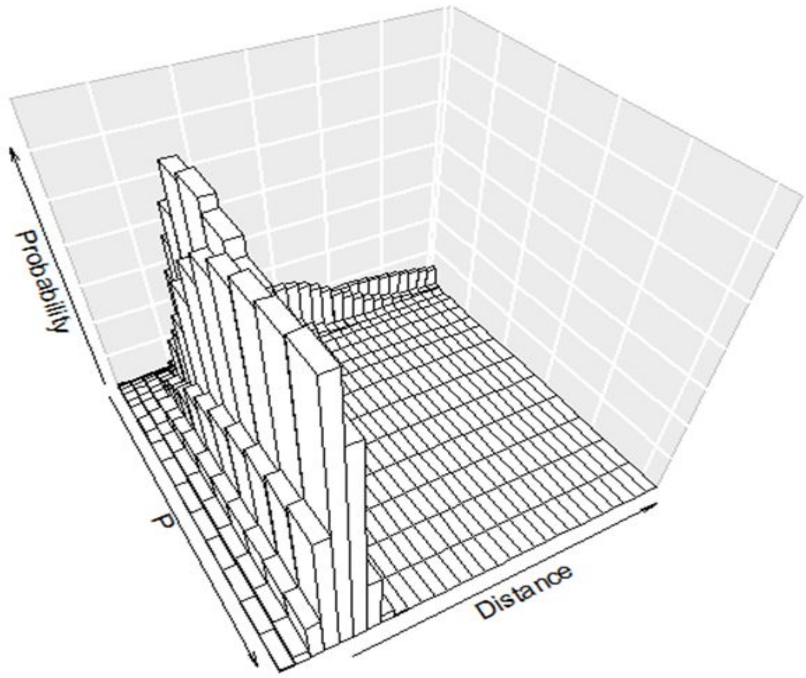

Figure 26. Joint probability distribution of the social distance and rewiring probability

Figure 26 was drawn by a MATLAB code written by the author for this study. This approximate $P(d,\rho)$ was calculated by taking the average of 1,000 cases with $N=1,000$ and $K=4$. It would be better to average more cases with a larger N; however, it takes an extremely long time to compute d_{ij} for all pairs of agents in a network. The range of d_{ij} is 0 to $N/2K=250$, per the definition of Watts and Strogatz (1998). To utilize this $P(d,\rho)$ for the IBDM, the estimation of $P(d,\rho)$ in a closed-function form is required. However, as Barrat and Weigt (2000) stated, there is no closed solution for the joint density $P(d,\rho)$. As seen in Figure 27, the obtained average

$P(\rho \mid d = D)$ seems irregular.

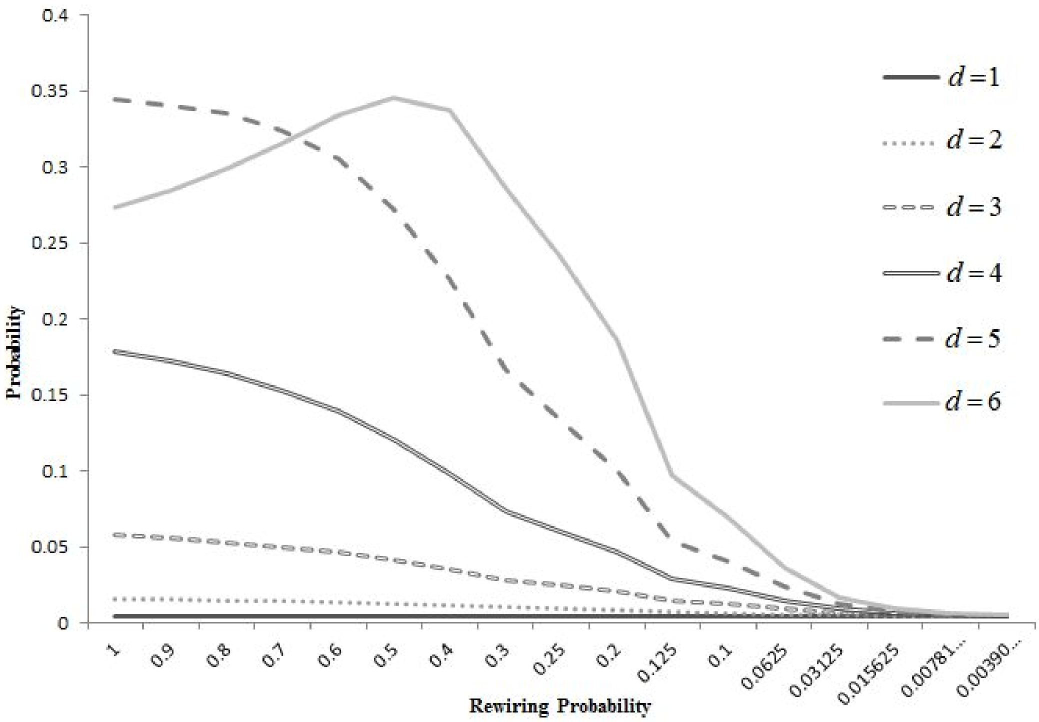

Figure 27. Conditional probability distribution of a rewiring probability

It is noteworthy that $P(\rho \mid d = 1) = K / (N - 1)$ is constant for ρ. This is because the total number of edges does not change due to the rewiring process. The probability $P(d = D, \rho = \rho^{*})$ refers to the number of pairs with distance D in a society with ρ^{*}. It is hard to find a functional form similar to, not only $P(d, \rho)$, but also $P(\rho \mid d = D)$. Thus, Taylor's expansion is used to estimate the polynomial form of $P(\rho \mid d = D)$, per the following equation.

$$P(\rho \mid d = D) = a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + a_4\rho^4 + \varepsilon \quad \text{Eq. (42)}$$

The estimation is done by applying ordinary least squares (OLS) in program R[16]. Table 6 shows the estimation results from the first six distances.

Table 6. Estimation results for the conditional probability of social distance

	a_0	a_1	a_2	a_3	a_4	R-squared
$d=1$	0.004***	0.000*	0.000**	0.000***	0.000***	0.462
$d=2$	0.004***	0.028***	-0.023***	0.005***	0.001***	1.000
$d=3$	0.004***	0.080***	0.027***	-0.104***	0.049***	1.000
$d=4$	0.004***	0.150***	0.459***	-0.744***	0.310***	1.000
$d=5$	0.004***	0.232***	1.822***	-3.124***	1.413***	0.999
$d=6$	0.000	0.718***	1.875**	-5.264***	2.955***	0.995

Note: *, **, and *** indicate statistical significance at the 5, 1, and 0.1% levels, respectively.

For all six distances, R-squared is nearly 1, except for in the case of $d = 1$, which is a linear function. The estimated $P(\rho \mid d = D)$ is essential for modeling aggregated-level

[16] R version 3.0.1 was used for following estimations in this chapter.

social utility, since there is no way to identify social structures from aggregated-level data. The approach to modeling this is as follows.

Suppose there is a cumulative number of adopters, Y_{t-1}, in a social network at time t. Then, there are also groups of agents who have a distance of '1,' '2,' '3,' or so on to those Y_{t-1} adopters. Let I_D be the number of agents whose minimum distance to adopters is D. An I_D number of agents will receive an average social utility of $\overline{J}_t = \phi(D/k, 0, s \cdot \overline{V}_t)$, where $\overline{J}_t$ and $\overline{V}_t$ indicate the average values of Y_{t-1} adopters.

I_D can be derived from $P(d = D, \rho = \rho^*)$ for a certain ρ. One may consider that roughly $Y_{t-1} \cdot P(d = 1, \rho = \rho^*) \cdot (N - Y_{t-1})$ people have a distance of 1 to Y_{t-1} adopters. $P(d = D, \rho = \rho^*)$ refers to the portion of agents in a network who have a distance D to a certain agent. Since there are $N - Y_{t-1}$ potential adopters waiting for a link to Y_{t-1} adopters, each adopter would have a $P(d = 1, \rho = \rho^*) \cdot (N - Y_{t-1})$ number of agents. If we assume that all adopters have disjoint neighbors, then the number of agents whose minimum distance to Y_{t-1} adopters is 1 is $I_1 = Y_{t-1} \cdot P(d = 1, \rho = \rho^*) \cdot (N - Y_{t-1})$.

However, each set of neighbors is clearly not disjoint. One agent can be connected to one adopter and another adopter at the same time. Thus, intersections of these sets must be excluded from the counting. This problem can be solved through two simple mathematic propositions called the inclusion-exclusion principle and De Morgan's law. By applying these principles, the exact number of I_D can be derived, as follows:

$$I_D(\rho) = \left(1-\left(1-P(\rho \mid d = D)\right)^{Y_{t-1}}\right)\cdot\left(N - Y_{t-1} - \sum_{d=1}^{D-1} I_d\right) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \text{Eq. (43)}$$

Note that I_D is a function of the rewiring probability ρ. This reflects the property that social utility is dependent on the topology of a social network. This fact is significant because it shows the aggregated-level parameterization of a social network. The robustness of equation (43) is shown in Figure 28, which follows.

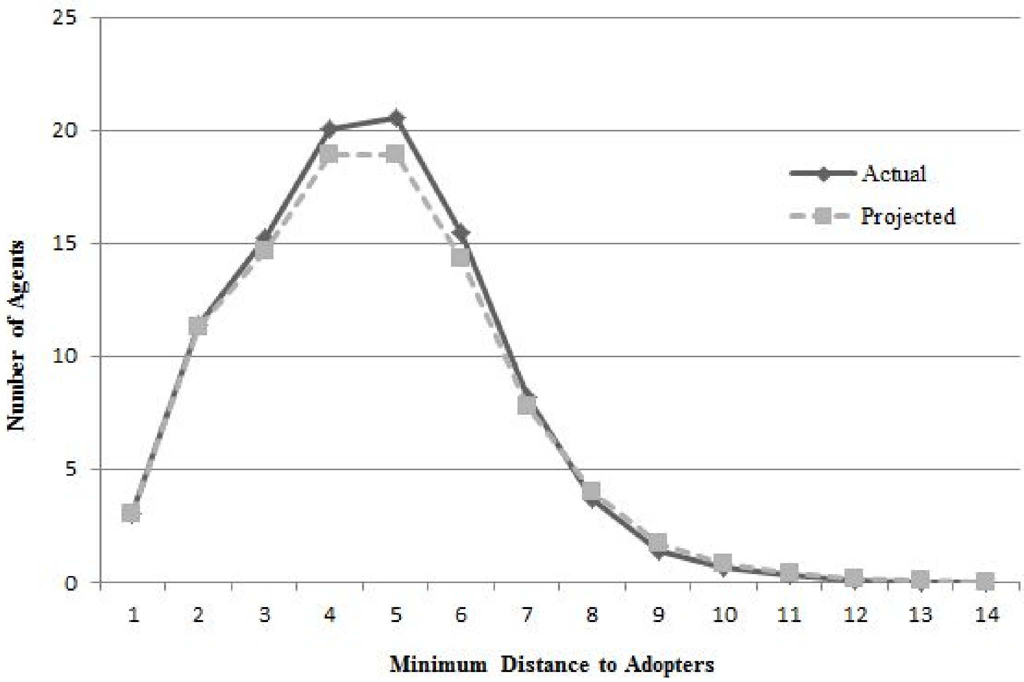

Figure 28. Number of agents due to a minimum distance to adopters

Figure 28 shows the simulation results for the case of $N = 100$, $K = 4$, and

$\rho = 0.1$. It is assumed that there are three total adopters (i.e., 3% of the potential adopters in the society). The actual data are calculated by investigating all path lengths among agents for all possible allocations of the three adopters[17]. Projected numbers are calculated simply using equation (43). As shown in Figure 28, the equation forecasts the actual value well, with 8.98% of mean absolute percentage error (MAPE). Note that the actual value is calculated from only one fixed network. It is expected that the projected value is the same as the value of averaging all possibilities of the network with the same specifications. Hence, it can be said that the result shows the exactness of equation (43).

The objective of fitting the model is to derive a net sales function with the parameters of the IBDM. As in the Bass (1969) model, net sales can be separated into two terms: the effect of external influences and the effect of internal influences. In the IBDM, the effect of external influences is replaced by the decay in price. Recall the utility specification of the IBDM.

$$U_i = \begin{cases} V_i + \sum_{j|\omega_j=1} J_{ij} - \mu \cdot P_t & if \quad \omega_i = 1 \\ 0 & if \quad \omega_i = 0 \end{cases} \quad \text{......Eq. (44)}$$

Unlike in the simulation, μ cannot be normalized in the fitting because there is actual market data for price P. From equation (44), one can assume that adopters

[17] There are $_{100}C_3 = 161,700$ possibilities ~~of~~ for adopter allocation. The a~~A~~ctual data ~~takes~~ take the average value of all ~~those~~ these possibilities.

resulting from the price effect at time t have $\mu \cdot P_t \leq V_i < \mu \cdot P_{t-1}$. Using the fact that V_i is uniformly distributed, the number of adopters due to the price effect can be calculated via $\mu(P_{t-1} - P_t) \cdot (N - Y_{t-1})$. However, as discussed in Section 4.2.2, there are some agents with $\mu \cdot P_t \leq V_i < \mu \cdot P_{t-1}$ and $V_i + \sum_{j|\omega_{jt}=1} J_{ij} \geq \mu \cdot P_{t-1}$. These agents should be removed; however, it is impossible to determine the exact number of them at the aggregated level due to the complexity of the S-W network. Thus, the number of agents is assumed to be a constant a, though it is not constant by time. This a can also be interpreted as the average resistance to the innovation adoption.

To calculate the effect of internal influences on net sales, it is necessary to understand the diffusion field in the below diagram. Suppose the social utility is simply a constant J. Then, among the agents with $V_i < \mu \cdot P_t$, there will be some agents who have $V_i + J > \mu \cdot P_t$. These agents are the adopters resulting from internal influences. Since V_i is uniformly distributed, J % of adopters will be generated from an $(N - Y_{t-1})$ number of potential adopters (Figure 29).

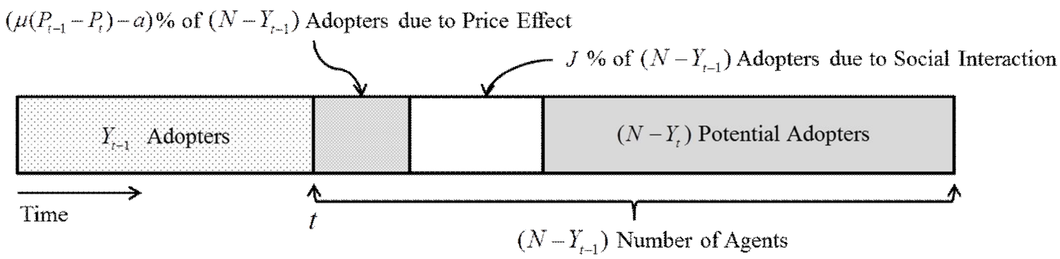

Figure 29. Conceptual diagram of diffusion in the IBDM

Recall that the IBDM assumes a distribution for J. To derive an aggregated-level social utility, J is assumed to be $\bar{J} = \phi(d/k, 0, s \cdot \bar{V})$, where $\bar{V}$ represents the mean taste of adopters. Since $\bar{J}$ is a function of social distance d, the value of $\bar{J}$ varies with the social distance. For instance, I_1 potential adopters will receive $\phi(1/k)$—a number that is larger than $\phi(2/k)$ of I_2 agents. In addition, note that there are two global parameters for $\bar{J}$: k and s. Unlike in the original IBDM, these now have a perfect correlation through $s = (1/k)^2$. Thus, in the fitting model, k is normalized to 1.

In sum, net sales in the IBDM can be expressed by the following equation:

$$S_t = (\mu(P_{t-1} - P_t) - a) \cdot (N - Y_{t-1}) + \sum_D \left(I_D(\rho) \cdot \bar{J}(D, s) \right) + \varepsilon \quad \text{Eq. (45)}$$

It is notable that equation (45) has a similar function to that of the Bass model, even though there is no mathematical similarity in the functions' derivations. Equation (45) can be rewritten to facilitate a clear view of the functional form.

$$S_t = f_1(P_t) \cdot (N - Y_{t-1}) + \sum_D \left(\bar{J}_D \cdot f_2(Y_{t-1}) \cdot f_3(N - Y_{t-1}) \right) + \varepsilon \quad \text{Eq. (46)}$$

It is easy to find the similarities between equation (46) and the Bass model in discrete time, as shown below.

$$S_t = p(N - Y_{t-1}) + \frac{q}{N} \cdot Y_{t-1}(N - Y_{t-1}) \quad \text{Eq. (47)}$$

This realization answers the question of why the IBDM has properties similar to those of Bass in Chapter 4. There is another significance to equation (45), in that an aggregated-level function is derived from individual-level modeling. Such an attempt is rarely found in previous diffusion research studies.

In the following sub-chapter, the empirical availability of the IBDM is investigated by applying equation (45) to real market data.

5.2 Analysis of Real Market Data

To examine the empirical availability and flexibility of the IBDM, fitting to real market data is necessary. In this sub-chapter, the details of the fitting and the analysis results are discussed.

5.2.1 Fitting Procedure

Equation (45) seems to be an econometric model; however, it is still incomplete. Since the specifications of I_D and S_t are recursive, it is difficult to generate a likelihood for S_t. Furthermore, the effects of the social network parameter ρ and of other utility parameters are difficult to identify. For this reason, a classical estimation approach is impractical thus far.

Though the function is incomplete from the perspective of econometrics, fitting to the function to data is still possible. This study applies an algorithm called 'brute force' to fit the data. According to Lohninger (1999), the brute-force algorithm seeks to determine the optimal parameters from all possible combinations of a model. As the term 'brute force' implies, this is a rough approach to fitting; thus, it cannot be called a statistical estimation. This study implements R code for the brute-force nonlinear squared error written by Grothendieck (2013).

To run the algorithm and generate parameter combinations, proper ranges for parameters are needed. As seen in equation (45), four parameters must be estimated: the coefficient of the price effect μ, the base coefficient of the innovation effect a, the rewiring probability of network ρ, and the correlation between social utility and taste s. Firstly, the range of a is set as $(0, 3)$. The number 3 is selected to allow enough range for a. The range of μ is set to be $(\mu^* - 0.1, \mu^* + 0.1)$, where $\mu^* = (1 - S_1 / N) / P_1$. μ^* indicates the optimal value to fit the net sales of the initial period. This value can be derived from the fact that, in the initial period, only agents with $V_i > P_1$ adopt the innovation; thus, $\Pr(V_i > P_1) = S_1 / N$, and, by the uniform distribution of V and the definition of utility, $\Pr(V_i > P_1) = 1 - \mu^* \cdot P_1$. The range of ρ is defined to be $(0, 0.5)$, since this is sufficient to describe the real society, as discussed in Sections 3.2 and 4.1. Lastly, the range of s is set to be $(0, 3)$, just as a (Table 7).

Table 7. Specification of initial parameter grid for the brute-force algorithm

	a	μ	ρ	s
Min. Value	0	$\mu^* - 0.1$	0	0
Max. Value	3	$\mu^* + 0.1$	0.5	3

Note: $\mu^* = (1 - S_1 / N) / P_1$

A total of 10,000 random combinations of parameters in the grid are examined using an R code written by the author. The data required for the IBDM involve only two measures: sales and price. Addition adjustments for data are required, however, due to the specification of equation (45). Firstly, estimated prices, rather than the original data, are used for the fitting. This is because net sales in the early stages of the IBDM depend heavily on price. If there is a stagnation or increase in price, then there will be no adopters due to the price effect. Furthermore, a constant price can cause a negative number of adopters due to the presence of a. Thus, price is estimated using the following equation, as it is in the original IBDM.

$$P_t = P_0 \exp(-\alpha t) + \varepsilon \quad \text{Eq. (48)}$$

Secondly, the normalized number of sales, rather than the original one, is used for the fitting. This is because I_D is calculated based on the case of $N = 1,000$, as discussed in Section 5.1. Barrat and Weigt (2000) stated that the average path length and social distance are functions of N. That is, if N is different from 1,000, then the function $P(\rho \mid d = D)$ will be changed, as will I_D. Nevertheless, normalizing sales seems reasonable, since such an approach should not harm the fitness of the model from the perspective of economics. Since the study uses finalized diffusion data, the normalizing factor is determined by the final market penetration of the innovation.

5.2.2 Analysis Results

Country-level data for mobile/cellular telephone subscriptions, provided by the International Telecommunication Union (2012), are used to test the fitness of the IBDM. These data provide panel data concerning the total number of subscriptions, the number of subscriptions per 100 inhabitants, and the revenue from mobile networks by country. The first group of data is used as the net sales and cumulative sales of innovation, the second group of data is used as the market penetration to normalize the market potential to 1,000, and the third group of data is used to derive annual price data by dividing revenue by the net sales data.

From numerous countries, three countries are selected for analysis: Korea (KOR), Germany (DEU), and the United States of America (USA). Unlike, for example, Japan and France, these countries do not have any missing data, and they exhibit high levels of final market penetration. The data on the revenue are adjusted for inflation based on 2010 values. The data on the inflation rates were provided by the International Monetary Fund (2014). The finalized raw data are presented in Appendix I.

First, the price for each country is estimated as in Table 8. The estimation is done using an R non-linear least square (NLS) code. The prices are normalized to be less than 1.

Table 8. Estimation results of normalized annual price for mobile subscriptions

	P_0	α	R-squared
USA	0.685***	0.048***	0.436
Germany	1.097***	0.158**	0.551
Korea	1.020***	0.121***	0.794

Note: **, and *** indicate statistical significance at the 1 and 0.1% levels, respectively.

The results show a high fitness in data of Korea, but a low fitness in the USA. This is because the period for the USA is much longer (1984 to 2009) than that for Korea (1993 to 2006). The following table shows a comparison of the three models—the IBDM, the Bass model, and the GBM—for the three countries.

Table 9. Mean absolute percentage error of each model

MAPE	IBDM	Bass	GBM
USA	20.27%	61.72%	16.46%
Germany	41.87%	89.30%	53.91%
Korea	52.30%	60.30%	55.42%

The Bass model and the GBM are estimated using R NLS code. According to Bass et al. (1994), the NLS performs better than the OLS in estimating the Bass model. It is noteworthy that, in all cases, the IBDM fits better than the classical Bass model. Furthermore, the fitting performance of the IBDM is similar to that of the GBM. This shows the flexibility of the IBDM in following irregularities of real data. The following figures show the fitting curve of each model to data.

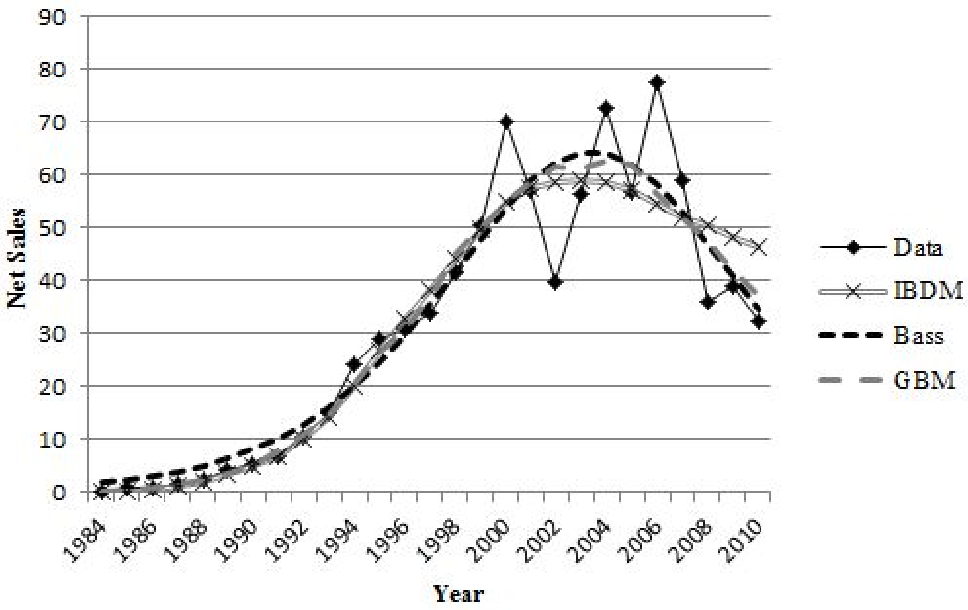

Figure 30. Fitting curves to real market data: USA

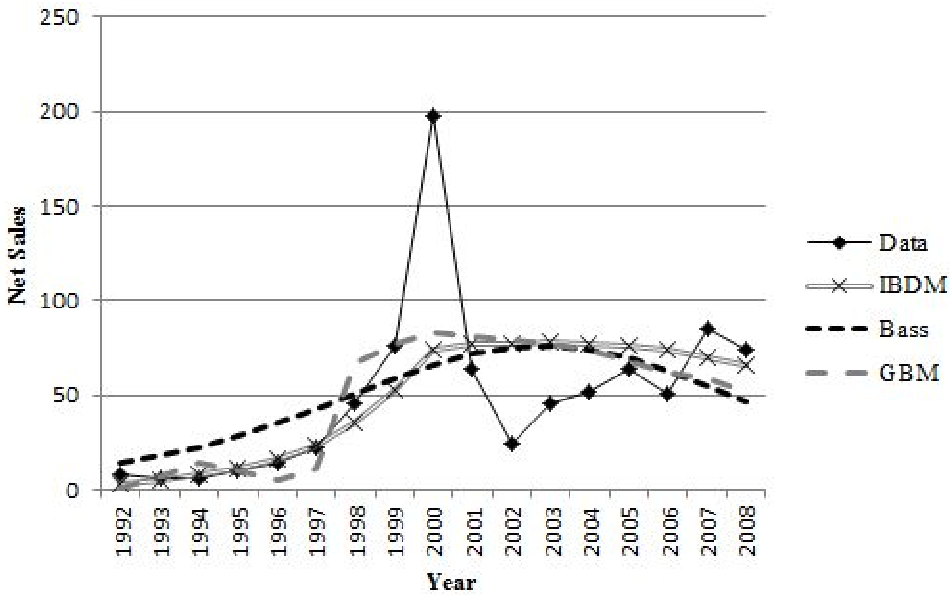

Figure 31. Fitting curves to real market data: Germany

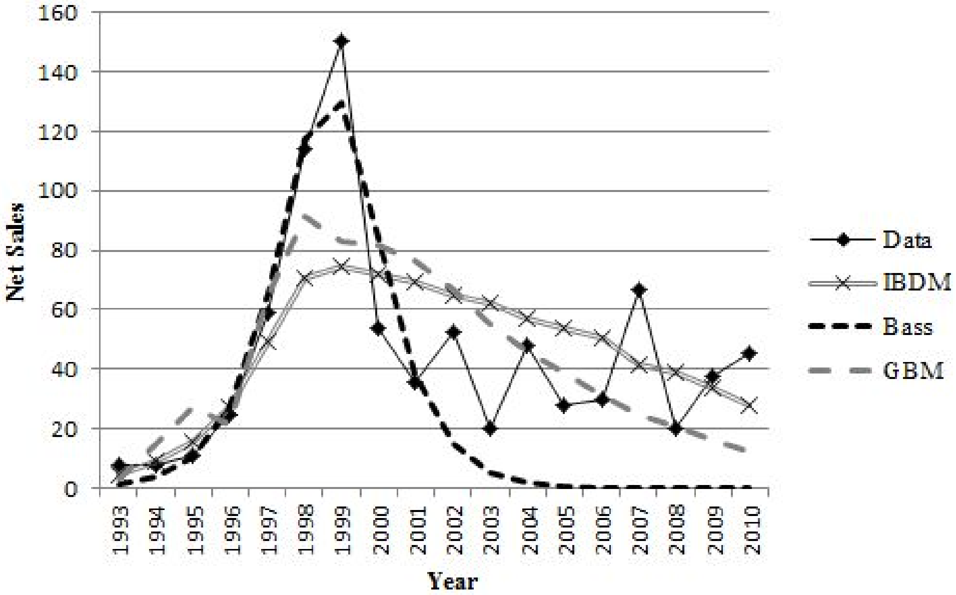

Figure 32. Fitting curves to real market data: Korea

The significance of fitting comes from the fact that the IBDM, which is an ABM, can fit the aggregated-level market data. As shown in the above figures, the IBDM draws, not only various shapes of diffusion, but also some shocks to follow the irregular deviations of the real data.

Even though the estimated parameters resulted from the brute-force algorithm, they are not rigorous or statistically significant. Thus, the numbers in the following Table 10 are presented only for the repeatability of the model—not for the interpretation of parameters. To draw implications from the parameters, the model should be advanced in a closed form, which will facilitate statistical estimation.

Table 10. Estimated parameters of the IBDM drawn from the brute-force algorithm

	a	μ	ρ	s
USA	2.347	1.542	0.165	0.419
Germany	1.230	1.058	0.424	0.472
Korea	0.159	1.141	0.470	0.661

5.3 Summary

In this chapter, the empirical availability of the IBDM has been investigated. Through this attempt to fit the model to real market data, some notable outcomes have been identified. First of all, the aggregated level of net sales function could be drawn from the model. This is important because it shows that individual-level modeling may be able to derive an aggregated-level model without any individual-level data or specifications. This implies that the ABM and the aggregate diffusion model may converge to a generalized model and that the IBDM is one candidate for this.

The analysis of the real market data shows whether the IBDM satisfies the latter three conditions of the future diffusion model proposed by Bass (1995). It is revealed that the IBDM can track the irregular deviations of actual data from a smooth curve and that it is flexible enough to draw various shapes. However, the last condition could not be met:

That is, there is not yet a closed-form solution for the IBDM.

There are many possible modifications that could advance the IBDM from an econometric perspective. Firstly, the IBDM is not identifiable because of the presence of ρ and $P(d, \rho)$. It is has been shown that even a fixed number of ρ cannot enable an econometric estimation. To avoid this problem simply, the whole term can be removed. However, there are many ways to determine the social network exogenously. One may try to draw the full map of a whole social network using big data from social network services or to experiment on a real small world. There is also a chance that using another network topology will be advantageous in terms of deriving the closed-form solution. Another limitation is that the IBDM requires a smooth price function to work well. The best way to neutralize the effect of price should be investigated in future studies.

Despite its limited estimation availability, it is notable that the IBDM derives a functional form of net sales that is very similar to that of the Bass model. This, together with the simulations, shows why the Bass model fits real market data well. It is expected that further developments in the IBDM will establish a concrete theoretical background for innovation diffusion models.

Chapter 6. Conclusion

6.1 Concluding Remarks

Numerous research studies have described innovation diffusion. The Bass model and its extensions exhibit good explanatory power for real market data, but they do not encompass individual-level specifications for innovation adoption and social interaction. Agent-based models are specialized to individual-level modeling, but they cannot analyze solely aggregated-level market data. Furthermore, neither is established on a concrete theoretical background. To overcome such limitations, a new model called the interaction-based diffusion model is proposed for innovation diffusion research.

The model is established on the basis of the theoretical utility model and is designed to analyze aggregated-level market data using individual-level modeling. The properties of the model are found to be desirable for explaining innovation diffusion on the basis of heterogeneous social interactions. In addition, from the examination of the effect of social network topologies on diffusion patterns, the use of a small-world network is justified, and the correlation between social networks and diffusion patterns is discussed in detail.

Even though the econometric model is thus far incomplete, it is able to derive aggregated-level net sales functions from the IBDM. It is notable that the model derives a functional form similar to that of contagion diffusion models used in traditional diffusion model, such as Bass (1969), from the basis of economic models, such as the Brock and

Durlauf (2001) model and the probit model. This provides an answer to the question of why similarities can be found in the simulation experiments. The fitting to the real market shows that the model has good flexibility. However, it also shows the obstacles preventing the derivation of a closed-form econometric model.

It is noteworthy that this study follows recent diffusion research trends. According to Peres et al. (2010), there have been numerous shifts in the field's research focus , such as the shifts from word-of-mouth as a driver to consumer interdependencies as drivers, from aggregate models to individual-level models, and from fully connected networks to partially connected small-world networks. This study follows such trends in diffusion research.

6.2 Contributions and Limitations

The most important contribution of this study is that it establishes a new diffusion model based solely on economic theories. There are still few researches constructing diffusion models from microeconomic foundations, despite the many review studies emphasizing the necessity for such studies (Bass, 2004; Kiesling et al., 2012; Peres et al., 2010; Russell, 1980). Theory-based models, including individual-behavior-based models, also have limitations, such as dependencies on previous models (Lee et al, 2006), a disregard of social interaction (Cho, 2007), and overly simple specifications for interactions (Dugundji and Gulyás, 2008). In contrast to previous studies, this study constructs a diffusion model based on microeconomic theories, while considering heterogeneous social interactions in the utility specification.

Rogers (2003) defined the diffusion of innovation using four key concepts: innovation, communication channels, social systems, and time. However, diffusion research studies have not yet been able to cover all of these concepts. Particularly, innovation itself cannot be regarded as an important factor in diffusion models, even though such models are literally about the diffusion of innovation. Since the IBDM is based on a utility function, it can easily reflect attributes of innovation, such as reservation prices.

The uniqueness of this study comes from its specification of social interaction. Whereas previous research studies considered only consumer interactions, such as network externalities or the word-of-mouth effect, the IBDM allows for the heterogeneity of interdependencies among consumers using a distributional interaction weight. Such a specification enriches certain implications, such as the differences in the effects of more and less heterogeneous social interactions on diffusion, the effects of attributes of innovation on interactions, and the differences in interaction aspects according to social network structures. Some studies have considered individual heterogeneity or varying interaction effects by social distance; however, no studies have explored heterogeneous social interactions.

The contribution of this study can be explained through its benefits with regard to both aggregate-level models and ABMs. For aggregate approaches, such as that of Bass (1969), the study proposes a theoretical explanation. Particularly, Bass (1969) and its extensions separated consumers into innovators and imitators. As Russell (1980) pointed out, however, it is more natural for consumers to have the same utility structure, from the perspective of economics. The IBDM overcomes this limitation of aggregate models. With regard to agent-based models, the study presents a new framework that links individual-level modeling to aggregated-level market behaviors. Furthermore, the study also considers the heterogeneity of social interaction to be an important factor.

A great advantage of the IBDM comes from the fact that it is built on the basis of a utility model. This means that the model is easily expandable via adjustments to some of its utility specifications. For instance, one could add another variable, such as advertising, to the model without hesitation. Note that the method of applying the price variable to the Bass model is controversial. In contrast, the IBDM can reflect any variable that can be expressed in utility models (i.e., most economic variables can be reflected). Such expandability contributes to the diffusion research by allowing for as many models as needed.

The study also highlights the importance of heterogeneous social interactions. By implementing the interaction-based model of Brock and Durlauf (2001), the study generalizes the assumption and describes the diffusion without any further assumptions. The study also emphasizes the use of the small-world network topology of Watts and Strogatz (1998) in agent-based model. It is discussed that a small-world network is appropriate for describing the actual social network and analyzing innovation diffusion.

The last contribution of this study is that it suggests a new framework to develop individual-to-aggregated-level modeling. It has been always unclear whether the sum of individual behaviors can lead to aggregated-level phenomena. The IBDM provides an explanation for this using simple modeling.

However, some limitations remain. First, the econometric model is still incomplete. As Bass (1995) noted, a closed-form solution is required for the practicability of the model. It is expected that, by adopting different specifications, it may become possible to derive a closed-form function for diffusion. The fact that the basic assumption of diffusion is the same as that of the Bass model is also a limitation. That is, the fact that the IBDM also allows only the first purchase of innovation means that the model disregards the gap between knowledge and persuasion and requires market-level data instead of brand-level data, as Bass (1969) does. Reflecting the significant findings of previous diffusion research studies is important for generalizing the IBDM.

6.3 Future Research Topics

There are many options for advancing the IBDM, since it is still in an early stage of development. First of all, adopting a more realistic social network topology, such as the power-law network of Barabási and Albert (1999), is recommended. The limitation of a small-world network is that it allows for only a fixed number of agents; thus, the presence of market potential must to be assumed. Since a power-law network allows for changes in the total number of agents, it can relax the assumption of market potential and better describe innovation diffusion. Implementing the full social network map is another option. If a network can be identified from outside the diffusion, a closed-form solution for the IBDM can be derived.

Further specifications for utility are also possible. For instance, the representative utility V_i, which is assumed to be uniformly distributed in the default model, may be a function of explanatory variables, such as discrete choice models (e.g., $V_i = \beta \cdot \mathbf{x}_j$). With individual survey data, it is possible to estimate β from the survey. Furthermore, a brand-level diffusion model, which represents one of the most important challenges of diffusion research (Peres et al., 2010), can be derived with the help of a multinomial interaction-based model (Brock and Durlauf, 2002). With such a brand-level diffusion model, one can simulate the market share of each brand by time and answer interesting questions concerning, for example, the conditions in which a brand can beat competitors or the pricing strategy that is most effective for a second-mover.

Another important task related the IBDM concerns the derivation of a closed-form solution for the model. Nevertheless, the IBDM's ability to fit with aggregated-level data can be considered a great progress, since this accomplishment has been unreachable for most ABM methodologies. If the IBDM can be expressed as a complete econometric model, it will derive as many implications as other diffusion models, and its framework will be able to cover most diffusion research studies.

Bibliography

Alkemade, F., & Castaldi, C. (2005). Strategies for the diffusion of innovations on social networks. *Computational Economics*, 25(1-2), 3-23.

Amini, M., Wakolbinger, T., Racer, M., & Nejad, M. G. (2012). Alternative supply chain production–sales policies for new product diffusion: An agent-based modeling and simulation approach. *European Journal of Operational Research*, 216(2), 301-311.

Anderson, P. (1999). Perspective: Complexity theory and organization science. *Organization science*, 10(3), 216-232.

Bain, A. D. (1963). The growth of demand for new commodities. *Journal of the Royal Statistical Society*. Series A (General), 285-299.

Bain, A. D. (1964). *The growth of television ownership in the United Kingdom since the war: a lognormal model*. Cambridge: Cambridge University Press.

Barabási, A. L., & Albert, R. (1999). Emergence of scaling in random networks. *science*, 286(5439), 509-512.

Barrat, A., & Weigt, M. (2000). On the properties of small-world network models. *The European Physical Journal B-Condensed Matter and Complex Systems*, 13(3), 547-560.

Bass, F. M. (1969). A new product growth for model consumer durables, *Management Science*, 15(1), 215-227.

Bass, F. M. (1980). The relationship between diffusion rates, experience curves, and demand elasticities for consumer durable technological innovations. *Journal of Business*, S51-S67.

Bass, F. M. (1995). Empirical generalizations and marketing science: A personal view. *Marketing Science*, 14(3_supplement), G6-G19.

Bass, F. M. (2004). Comments on "a new product growth for model consumer durables the bass model". *Management science*, 50(12_supplement), 1833-1840.

Bass, F. M., Krishnan, T. V., & Jain, D. C. (1994). Why the Bass model fits without decision variables. *Marketing science*, 13(3), 203-223.

Bayus, B. L. (1987). Forecasting sales of new contingent products: An application to the compact disc market. *Journal of Product Innovation Management*, 4(4), 243-255.

Bayus, B. L. (1993). High-definition television: assessing demand forecasts for a next generation consumer durable. *Management Science*, 39(11), 1319-1333.

Bayus, B. L., Kim, N., & Shocker, A. D. (2000). Growth models for multiproduct interactions: Current status and new directions. *New-product diffusion models*, 141-163.

Belkaoui, A. (1986). *The learning curve*. A Management Accounting Tool.

Bemmaor, A. C. (1994). *Modeling the diffusion of new durable goods: Word-of-mouth effect versus consumer heterogeneity*. In Research traditions in marketing (pp. 201-229). Springer Netherlands.

Bemmaor, A. C., & Lee, J. (2002). The impact of heterogeneity and ill-conditioning on diffusion model parameter estimates. *Marketing Science*, 21(2), 209-220.

Berger, T. (2001). Agent-based spatial models applied to agriculture: a simulation tool for technology diffusion, resource use changes and policy analysis. *Agricultural economics*, 25(2-3), 245-260.

Bernheim, B. D. (1994). A theory of conformity. *Journal of political Economy*, 841-877.

Bewley, R., & Fiebig, D. G. (1988). A flexible logistic growth model with applications in telecommunications. *International Journal of Forecasting*, 4(2), 177-192.

Bohlmann, J. D., Calantone, R. J., & Zhao, M. (2010). The Effects of Market Network Heterogeneity on Innovation Diffusion: An Agent-Based Modeling Approach. *Journal of Product Innovation Management*, 27(5), 741-760.

Bonabeau, E. (2002). Agent-based modeling: Methods and techniques for simulating human systems. *Proceedings of the National Academy of Sciences of the United States of America*, 99(Suppl 3), 7280-7287.

Brock, W. & Durlauf, S. N. (2000). Interaction-Based Models. *Handbook of Econometrics*, 5, 3297-3380.

Brock, W. A. (1993). Pathways to randomness in the economy: emergent nonlinearity and chaos in economics and finance. *Estudios Economicos*, 3-55.

Brock, W. A., & Durlauf, S. N. (2001). Discrete choice with social interactions. *The Review of Economic Studies*, 68(2), 235-260.

Brock, W. A., & Durlauf, S. N. (2002). A multinomial-choice model of neighborhood effects. *American Economic Review*, 298-303.

Carrillo-Hermosilla, J. (2006). A policy approach to the environmental impacts of technological lock-in. *Ecological economics*, 58(4), 717-742.

Chatterjee, R. A., & Eliashberg, J. (1990). The innovation diffusion process in a heterogeneous population: A micromodeling approach. *Management Science*, 36(9), 1057-1079.

Cho, Y., (2007). *Diffusion of innovations derived from the perspective of decision making units* (Doctoral Dissertation). Seoul National University. Seoul. South Korea.

Cho, Y., Hwang, J., & Lee, D. (2012). Identification of effective opinion leaders in the diffusion of technological innovation: A social network approach. *Technological Forecasting and Social Change*, 79(1), 97-106.

Choi, H., Kim, S. H., & Lee, J. (2010). Role of network structure and network effects in diffusion of innovations. *Industrial Marketing Management*, 39(1), 170-177.

Chow, G. C. (1967). Technological change and the demand for computers. *The American Economic Review*, 1117-1130.

David, P. A. (1969). *A Contribution to the Theory of Diffusion*. Research Center in Economic Growth Stanford University.

Davies, S. (1979). *The diffusion of process innovations*. Cambridge: Cambridge University Press.

Delre, S. A., Jager, W., Bijmolt, T. H. A., & Janssen, M. A. (2007). Targeting and timing promotional activities: An agent-based model for the takeoff of new products. *Journal of business research*, 60(8), 826-835.

Desmarchelier, B., Djellal, F., & Gallouj, F. (2013). Environmental policies and eco-innovations by service firms: An agent-based model. *Technological Forecasting and Social Change*, 80(7), 1395-1408.

Dugundji, E. R., & Gulyás, L. (2008). Sociodynamic discrete choice on networks in space: impacts of agent heterogeneity on emergent outcomes. *Environment and planning. B, Planning & design*, 35(6), 1028.

Durlauf, S. N. (1993). Nonergodic economic growth. *The Review of Economic Studies*, 60(2), 349-366.

Eliashberg, J., & Chatterjee, R. (1985). *Stochastic issues in modeling the innovation diffusion process*. Wharton School, University of Pennsylvania, Marketing Department.

Erdös, P., & Rényi, A. (1959). On random graphs I. *Publicationes Mathematicae*. Debrecen, 6, 290-297.

Farmer, J. D., & Foley, D. (2009). The economy needs agent-based modelling. *Nature*, 460(7256), 685-686.

Fliegel, F. C., & Kivlin, J. E. (1966). Attributes of innovations as factors in diffusion. *American Journal of Sociology*, 235-248.

Föllmer, H. (1974). Random economies with many interacting agents. *Journal of mathematical economics*, 1(1), 51-62.

Fruchter, G. E., & Van den Bulte, C. (2011). Why the Generalized Bass Model leads to odd optimal advertising policies. *International Journal of Research in Marketing*, 28(3), 218-230.

Garcia, R. (2005). Uses of Agent-Based Modeling in Innovation/New Product Development Research. *Journal of Product Innovation Management*, 22(5), 380-398.

Garcia, R., & Jager, W. (2011). From the Special Issue Editors: Agent-Based Modeling of Innovation Diffusion. *Journal of Product Innovation Management*, 28(2), 148-151.

Gerrard, P., & Cunningham, J. B. (2003). The diffusion of internet banking among Singapore consumers. *International Journal of Bank Marketing*, 21(1), 16-28.

Goldenberg, J., & Efroni, S. (2001). Using cellular automata modeling of the emergence of innovations. *Technological Forecasting and Social Change*, 68(3), 293-308.

Goldenberg, J., Libai, B., & Muller, E. (2001). Talk of the network: A complex systems look at the underlying process of word-of-mouth. *Marketing letters*, 12(3), 211-223.

Goldenberg, J., Libai, B., & Muller, E. (2001). Talk of the network: A complex systems look at the underlying process of word-of-mouth. *Marketing letters*, 12(3), 211-223.

Goldenberg, J., Libai, B., Solomon, S., Jan, N., & Stauffer, D. (2000). Marketing percolation. *Physica A: Statistical Mechanics and its Applications*, 284(1), 335-347.

Goldsmith, R. E., & Newell, S. J. (1997). Innovativeness and price sensitivity: managerial, theoretical and methodological issues. *Journal of Product & Brand Management*, 6(3), 163-174.

Granovetter, M. S. (1973). The strength of weak ties. *American journal of sociology*, 1360-1380.

Granovetter, M., & Soong, R. (1983). Threshold models of diffusion and collective behavior. *Journal of Mathematical sociology*, 9(3), 165-179.

Greenhalgh, T., Robert, G., Macfarlane, F., Bate, P., & Kyriakidou, O. (2004). Diffusion of innovations in service organizations: systematic review and recommendations. *Milbank Quarterly*, 82(4), 581-629.

Gregg, J. V., Hossell, C. H., & Richardson, J. T. (1964). *Mathematical trend curves: an aid to forecasting*. Oliver & Boyd.

Grothendieck, G. nls2: non-linear regression with brute force. *R package version 0.2; 2013*.

Haider, M., & Kreps, G. L. (2004). Forty years of diffusion of innovations: utility and value in public health. *Journal of health communication*, 9(S1), 3-11.

Heath, B., Hill, R., & Ciarallo, F. (2009). A survey of agent-based modeling practices (January 1998 to July 2008). *Journal of Artificial Societies and Social Simulation*, 12(4), 9.

Heckbert, S., Baynes, T., & Reeson, A. (2010). Agent-based modeling in ecological economics. *Annals of the New York Academy of Sciences*, 1185(1), 39-53.

International Monetary Fund. (2014). *World Economic Outlook Database*. Retrieved November 7, 2014, from http://www.imf.org/

International Telecommunication Union. (2012). *World Telecommunication/ICT Indicators*. International Telecommunication Union.

Jain, D. (1992). *Marketing mix effects on the diffusion of innovations*. Working Paper,.

Jain, D. C., & Rao, R. C. (1990). Effect of price on the demand for durables: Modeling, estimation, and findings. *Journal of Business & Economic Statistics*, 8(2), 163-170.

Jansson, J. (2011). Consumer eco-innovation adoption: assessing attitudinal factors and perceived product characteristics. *Business Strategy and the Environment*, 20(3), 192-210.

Jiang, Z., & Jain, D. C. (2012). A generalized Norton-Bass model for multigeneration diffusion. *Management Science*, 58(10), 1887-1897.

Kamakura, W. A., & Balasubramanian, S. K. (1988). Long-term view of the diffusion of durables A study of the role of price and adoption influence processes via tests of nested models. *International Journal of Research in Marketing*, 5(1), 1-13.

Karshenas, M., & Stoneman, P. (1995). Technological diffusion. *Handbook of the economics of innovation and technological change*, 265-297

Katz, M. L., & Shapiro, C. (1985). Network externalities, competition, and compatibility. *The American economic review*, 424-440.

Kiesling, E., Günther, M., Stummer, C., & Wakolbinger, L. M. (2012). Agent-based simulation of innovation diffusion: a review. *Central European Journal of Operations Research*, 20(2), 183-230.

Kim, S., Lee, K., Cho, J. K., & Kim, C. O. (2011). Agent-based diffusion model for an automobile market with fuzzy TOPSIS-based product adoption process. *Expert Systems with Applications*, 38(6), 7270-7276.

Kinnunen, J. (1996). Gabriel Tarde as a founding father of innovation diffusion research. *Acta sociologica*, 39(4), 431-442.

Krishnan, T. V., & Jain, D. C. (2006). Optimal dynamic advertising policy for new products. *Management Science*, 52(12), 1957-1969.

Laciana, C. E., & Rovere, S. L. (2011). Ising-like agent-based technology diffusion model: Adoption patterns vs. seeding strategies. *Physica A: Statistical Mechanics and its Applications*, 390(6), 1139-1149.

Lee, J., Cho, Y., Lee, J. D., & Lee, C. Y. (2006). Forecasting future demand for large-screen television sets using conjoint analysis with diffusion model. *Technological Forecasting and Social Change*, 73(4), 362-376.

Lee, K., Kim, S., Kim, C. O., & Park, T. (2013). An agent-based competitive product diffusion model for the estimation and sensitivity analysis of social network structure and purchase time distribution. *Journal of Artificial Societies and Social Simulation*, 16(1), 3.

Lilien, G. L., Rao, A. G., & Kalish, S. (1981). Bayesian estimation and control of detailing effort in a repeat purchase diffusion environment. *Management science*, 27(5), 493-506.

Lin, H. F. (2011). An empirical investigation of mobile banking adoption: the effect of innovation attributes and knowledge-based trust. *International Journal of Information Management*, 31(3), 252-260.

Lohninger, H. (1999). *Teach/me: Data Analysis*. Springer.

Luce, R. D., & Suppes, P. (1965). *Preference, utility, and subjective probability* (pp. 249-410). Wiley.

Macy, M. W., & Willer, R. (2002). From factors to actors: Computational sociology and agent-based modeling. *Annual review of sociology*, 143-166.

Mahajan, V., & Muller, E. (1979). Innovation diffusion and new product growth models in marketing. *The Journal of Marketing*, 55-68.

Mahajan, V., & Peterson, R. A. (1978). Innovation diffusion in a dynamic potential adopter population. *Management Science*, 24(15), 1589-1597.

Mahajan, V., & Peterson, R. A. (1979). Integrating time and space in technological substitution models. *Technological Forecasting and Social Change*, 14(3), 231-241.

Mahajan, V., & Peterson, R. A. (1985). *Models for innovation diffusion*. CA: Sage.

Mahajan, V., Muller, E., & Bass, F. M. (1990). New product diffusion models in marketing: A review and directions for research. *The Journal of Marketing*, 1-26.

Mahajan, V., Muller, E., & Srivastava, R. K. (1990). Determination of adopter categories by using innovation diffusion models. *Journal of Marketing Research*, 37-50.

Makse, T., & Volden, C. (2011). The role of policy attributes in the diffusion of innovations. *The Journal of Politics*, 73(01), 108-124.

Mansfield, E. (1961). Technical change and the rate of imitation. *Econometrica: Journal of the Econometric Society*, 741-766.

Marschak, J. (1960), ‘Binary choice constraints on random utility indications’, in K. Arrow, ed., *Stanford Symposium on Mathematical Methods in the Social Sciences*, Stanford University Press, Stanford, CA, pp. 312–329.

Maya Sopha, B., Klöckner, C. A., & Hertwich, E. G. (2011). Exploring policy options for a transition to sustainable heating system diffusion using an agent-based simulation. *Energy Policy*, 39(5), 2722-2729.

McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior. *Frontiers in Econometrics*, 105-142.

Meade, N. (1985). Forecasting using growth curves-an adaptive approach. *Journal of the Operational Research Society*, 1103-1115.

Meade, N., & Islam, T. (1995). Forecasting with growth curves: An empirical comparison. *International journal of forecasting*, 11(2), 199-215.

Meade, N., & Islam, T. (2006). Modelling and forecasting the diffusion of innovation–A 25-year review. *International Journal of Forecasting*, 22(3), 519-545.

Newman, M. E., & Watts, D. J. (1999). Renormalization group analysis of the small-world network model. *Physics Letters A*, 263(4), 341-346.

Norton, J. A., & Bass, F. M. (1987). A diffusion theory model of adoption and substitution for successive generations of high-technology products. *Management science*, 33(9), 1069-1086.

Ostlund, L. E. (1974). Perceived innovation attributes as predictors of innovativeness. *Journal of Consumer Research*, 23-29.

Pegoretti, G., Rentocchini, F., & Marzetti, G. V. (2012). An agent-based model of innovation diffusion: network structure and coexistence under different information regimes. *Journal of economic interaction and coordination*, 7(2), 145-165.

Peres, R., Muller, E., & Mahajan, V. (2010). Innovation diffusion and new product growth models: A critical review and research directions. *International Journal of Research in Marketing*, 27(2), 91-106.

Perez, L., & Dragicevic, S. (2009). An agent-based approach for modeling dynamics of contagious disease spread. *International journal of health geographics*, 8, 50.

Pombo-Romero, J., Varela, L. M., & Ricoy, C. J. (2013). Diffusion of innovations in social interaction systems. An agent-based model for the introduction of new drugs in markets. *The European Journal of Health Economics*, 14(3), 443-455.

Rahmandad, H., & Sterman, J. (2008). Heterogeneity and network structure in the dynamics of diffusion: Comparing agent-based and differential equation models. *Management Science*, 54(5), 998-1014.

Rand, W., & Rust, R. T. (2011). Agent-based modeling in marketing: Guidelines for rigor. *International Journal of Research in Marketing*, 28(3), 181-193.

Rebaudo, F., & Dangles, O. (2013). An agent-based modeling framework for integrated pest management dissemination programs. *Environmental Modelling & Software*, 45, 141-149.

Rogers, E. M. (1962). *Diffusion of innovations* (1st Edition). The Free Press, New York, NY

Rogers, E. M. (2003). *Diffusion of innovations* (5th Edition). Simon and Schuster.

Russell, T. (1980). Comments on" The Relationship Between Diffusion Rates, Experience Curves, and Demand Elasticities for Consumer Durable Technological Innovations". *Journal of Business*, S69-S73.

Schelling, T. C. (1971). Dynamic models of segregation. *Journal of mathematical sociology*, 1(2), 143-186.

Schlesinger, M., & Parisi, D. (2001). The agent-based approach: A new direction for computational models of development. *Developmental review*, 21(1), 121-146.

Schramm, M. E., Trainor, K. J., Shanker, M., & Hu, M. Y. (2010). An agent-based diffusion model with consumer and brand agents. *Decision Support Systems*, 50(1), 234-242.

Schreinemachers, P., Potchanasin, C., Berger, T., & Roygrong, S. (2010). Agent-based modeling for ex ante assessment of tree crop innovations: litchis in northern Thailand. *Agricultural Economics*, 41(6), 519-536.

Schumpeter, J. A. (1934). *The theory of economic development: An inquiry into profits, capital, credit, interest, and the business cycle* (Vol. 55). Transaction Publishers.

Schwarz, N., & Ernst, A. (2009). Agent-based modeling of the diffusion of environmental innovations—an empirical approach. *Technological forecasting and social* *change*, 76(4), 497-511.

Srivastava, R. K., Mahajan, V., Ramaswami, S. N., & Cherian, J. (1985). A multi-attribute diffusion model for forecasting the adoption of investment alternatives for consumers. *Technological Forecasting and Social Change*, 28(4), 325-333.

Sultan, F., Farley, J. U., & Lehmann, D. R. (1990). A meta-analysis of applications of diffusion models. *Journal of marketing research*, 70-77.

Tanner, J. C. (1978). Long-term forecasting of vehicle ownership and road traffic. *Journal of the Royal Statistical Society*. Series A (General), 14-63.

Tarde, G. (1895). *La logique sociale*. Second Edition. Paris: Alcan.

Taylor, A., & Higham, D. J. (2009). CONTEST: A controllable test matrix toolbox for MATLAB. *ACM Transactions on Mathematical Software* (TOMS), 35(4), 26.

Thiriot, S., & Kant, J. D. (2008). Using associative networks to represent adopters' beliefs in a multiagent model of innovation diffusion. *Advances in Complex Systems*, 11(02), 261-272.

Thompson, G. L., & Teng, J. T. (1984). Optimal pricing and advertising policies for new product oligopoly models. *Marketing Science*, 3(2), 148-168.

Toubia, O., Goldenberg, J., & Garcia, R. (2009). Diffusion forecasts using social interactions data. *MSI Special Report*, 09-210.

Train, K. E. (2009). *Discrete choice methods with simulation*. Cambridge university press.

Valente, T. W. (1995). *Network models of the diffusion of innovations* (Vol. 2, No. 2). Cresskill, NJ: Hampton Press.

Von Neumann, J. (1951). The general and logical theory of automata. *Cerebral mechanisms in behavior*, 1, 41.

Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. *nature*, 393(6684), 440-442.

Wells, J. (1977). The diffusion of durables in Brazil and its implications for recent controversies concerning Brazilian development. *Cambridge Journal of Economics*, 259-279.

Young, H. P. (2009). Innovation diffusion in heterogeneous populations: Contagion, social influence, and social learning. *The American economic review*, 1899-1924.

Zaffar, M. A., Kumar, R. L., & Zhao, K. (2011). Diffusion dynamics of open source software: An agent-based computational economics (ACE) approach. *Decision support systems*, 51(3), 597-608.

Zenobia, B., Weber, C., & Daim, T. (2009). Artificial markets: A review and assessment of a new venue for innovation research. *Technovation*, 29(5), 338-350.

Zhang, T., Gensler, S., & Garcia, R. (2011). A Study of the Diffusion of Alternative Fuel Vehicles: An Agent-Based Modeling Approach. *Journal of Product Innovation Management*, 28(2), 152-168.

Zenobia, B., Weber, C., & Daim, T. (2009). Artificial markets: A review and assessment of a new venue for innovation research. *Technovation*, 29(5), 338-350.

Zhang, T., Gensler, S., & Garcia, R. (2011). A Study of the Diffusion of Alternative Fuel Vehicles: An Agent-Based Modeling Approach. *Journal of Product Innovation*

Management, 28(2), 152-168.

Appendix

Table 11. Raw data of mobile communication subscriptions in three countries

Year	Total Number of Subscriptions (in Million)			Subscription per 100 inhabitants			Revenue (in Trillion)			Annual Unit Price		
	US	Germany	Korea	US	Germany	Korea	US	Germany	Korea	US	Germany	Korea
1984	0.09	-	-	0.04	-	-	0.18	-	-	1943.23	-	-
1985	0.34	0.00	-	0.14	0.00	-	0.48	-	-	1416.76	-	-
1986	0.68	0.02	0.01	0.28	0.03	0.02	0.82	-	-	1207.05	-	-
1987	1.23	0.05	0.01	0.50	0.06	0.02	1.15	-	-	935.93	-	-
1988	2.07	0.10	0.02	0.83	0.13	0.05	1.96	-	-	947.12	-	-
1989	3.51	0.16	0.04	1.40	0.21	0.09	3.34	-	-	952.14	-	-
1990	5.28	0.27	0.08	2.09	0.34	0.19	4.55	-	-	861.05	-	-
1991	7.56	0.53	0.17	2.95	0.67	0.38	5.71	-	-	755.44	-	-
1992	11.03	0.97	0.27	4.27	1.21	0.62	7.85	3.05	-	711.79	3134.46	-
1993	16.01	1.77	0.47	6.14	2.19	1.07	10.89	4.27	0.84	680.36	2408.64	1777.51
1994	24.13	2.49	0.96	9.16	3.06	2.16	14.23	4.90	1.46	589.61	1966.03	1520.33
1995	33.79	3.73	1.64	12.69	4.55	3.67	19.08	8.88	2.31	564.77	2384.71	1408.33
1996	44.04	5.51	3.18	16.35	6.71	7.08	26.00	16.93	5.74	590.33	3071.03	1804.58
1997	55.31	8.28	6.88	20.29	10.05	15.21	33.00	21.41	7.82	596.61	2587.39	1137.52
1998	69.21	13.91	14.02	25.08	16.89	30.81	36.78	11.69	9.98	531.36	840.46	711.62
1999	86.05	23.45	23.44	30.81	28.47	51.24	48.00	16.03	12.94	557.83	683.59	551.90
2000	109.48	48.20	26.82	38.75	58.53	58.31	62.00	21.15	16.63	566.32	438.84	620.32
2001	128.50	56.13	29.05	45.00	68.13	62.85	74.69	23.01	18.64	581.22	410.05	641.82
2002	141.80	59.13	32.34	49.16	71.73	69.67	81.52	23.40	19.87	574.90	395.74	614.31
2003	160.64	64.80	33.59	55.15	78.56	72.05	127.00	24.47	19.18	790.60	377.69	570.84
2004	184.82	71.32	36.59	62.85	86.43	78.12	140.00	26.17	20.30	757.50	366.98	554.90
2005	203.70	79.27	38.34	68.63	96.04	81.50	160.00	25.93	21.02	785.47	327.11	548.24
2006	229.60	85.65	40.20	76.64	103.78	85.04	130.81	25.54	21.33	569.71	298.22	530.56
2007	249.30	96.23	44.37	82.47	116.62	93.41	141.46	28.02	22.09	567.44	291.17	497.98

(Continued)

Year	Total Number of Subscriptions (in Million)			Subscription per 100 inhabitants			Revenue (in Trillion)			Annual Unit Price		
	US	Germany	Korea	US	Germany	Korea	US	Germany	Korea	US	Germany	Korea
2008	261.30	105.52	45.61	85.68	127.95	95.54	152.36	27.06	22.88	583.08	256.47	501.65
2009	274.28	105.00	47.94	89.14	127.42	99.96	155.98	18.29	23.22	568.67	174.15	484.35
2010	285.13	104.56	50.77	91.86	127.04	105.36	165.94	18.18	23.02	581.97	173.84	453.44

Note: Revenue and Unit price is adjusted for inflation and presented in country's own currency (US: Dollar, Germany: Euro, Korea: 1,000 KRW).

Abstract (Korean)

인터넷과 무선통신, 사회연결망서비스 등의 등장으로 사회 네트워크가 발전하면서, 예전보다 소비자들은 서로 더 자주, 신속하게 정보를 교환하고 서로의 제품 구매에 영향을 미치고 있다. 그러나, 기존에 널리 이용되어 온 배스 모형을 비롯한 여러 확산 예측 모형들은 이론적 기반이 취약할 뿐만 아니라, 시장 수준에서의 분석만을 행하고 있기 때문에 이러한 소비자간 상호작용의 효과를 통합적이고 한정적으로 반영하고 있어 최근의 사회 현상을 제대로 설명하지 못하는 한계가 있다. 그 대안으로 등장한 행위자 기반 모형들은 개인 단위 분석을 가능하게 한 장점은 있으나, 여전히 이론적 기반이 취약하고 총 시장 수준 자료를 통한 분석은 요원한 실정이다.

이 연구에서는 소비자들이 서로의 선택으로부터 영향을 받는 사회적 효용함수가 있다고 가정하고, 이와 같은 사회적 상호작용으로부터 오는 효용을 개개인의 효용구조에 직접적으로 반영시켜 이것이 확산 곡선에 어떤 영향을 미치는 지를 '상호작용 기반 확산 모형'이라 명명된 새로운 모형을 통해 보고자 한다. 또한 기존의 대표적인 확산 모형과의 비교를 통해 실제로 본 모형이 최근의 사회 현상을 설명하는 데 적합한 지 살펴보고자 한다. 더불어, 혁신 확산을 잘 설명하기 위해서는 위해서는 기존에 널리 사용되었던 세포자동자 격자구조보다는 작은 세상 연결망 사회구조가 더 적합함을 밝힌다.

이 연구를 통해, 개인의 효용과 상호작용에 기반한 경제학적 확산 모형을

얻을 수 있었을 뿐만 아니라, 기존 확산 모형과 달리 상호작용의 이질성까지 반영할 수 있는 확산 모형을 구축할 수 있었다. 또한 본 연구는 개인 단위의 모형이 시장 전체 수준의 수요 예측에 활용될 수 있는 새로운 접근방식을 제안하고 있으며, 실제 자료의 분석에 대해서도 모형이 충분히 활용가능 할 수 있음을 보였다. 이 연구에서 제시하는 모형은 경제학적 이론에 기반을 두었기 때문에 연구 대상에 따른 확장이 용이하다는 장점이 있으며, 이러한 일반적인 모형 구축은 향후 확산 과정에 대한 이해를 더욱 확장시켜 줄 수 있을 것으로 기대한다.

주요어 : 혁신 확산, 행위자 기반 모형, 상호작용 기반 모형, 작은 세상 연결망, 수요 예측
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Diffusion of Innovation in Small-world Networks with Social Interactions

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