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Interference Alignment-and-Cancellation Scheme Based on Alamouti Codes for the MIMO Interference Channels

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Presented to the Graduate School of Seoul National University in Partial Fulfillment of the Requirements for

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Abstract

Interference Alignment-and-Cancellation Scheme Based on Alamouti Codes for the MIMO Interference Channels

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This dissertation contains the following three contributions to the interesting research topics on Alamouti code, interference alignment (IA), and cooperative communications.

- Interference alignment-and-cancellation (IAC) scheme based on Alamouti codes for the three-user MIMO interference channel
  - Propose interference cancellation (IC) scheme based on Alamouti codes for the K-user MIMO interference channel.
  - Propose IAC scheme based on Alamouti codes for the three-user MIMO interference channel.
  - Analyze the diversity order of the proposed schemes.
- Two-way relaying schemes with Alamouti codes

  - Propose two schemes for the two-way relay channel (TWRC) based on Alamouti codes.

- Analysis on soft-decision-and-forward (SDF) cooperative networks with multiple relays

  - Analyze the diversity order of the conventional scheme for the SDF cooperative networks with multiple relays.

  - Analyze the diversity order of the best relay selection scheme.

First, the methods on how to apply Alamouti code to MIMO interference channels are proposed. The IC method based on Alamouti codes for the multi-access scenario can be used for the K-user interference channel, which enables the receivers to perform symbol-by-symbol decoding by cancelling interfering signals by utilizing Alamouti structure and achieve diversity order of two. Moreover it does not require channel state information at the transmitters (CSIT) unlike the IA scheme. However, it requires more receive antennas than the IA scheme to achieve the same degrees of freedom (DoF). In order to reduce the number of receive antennas, especially for the three-user MIMO interference channel, an IAC scheme based on Alamouti codes is proposed, which keeps the same DoF as that of the IC scheme, but it requires partial CSIT. It is analytically shown that the IC and IAC schemes enable symbol-by-symbol decoding and achieve diversity order of two, while the conventional IA scheme achieves diversity order of one.

In the second part of this dissertation, we propose two schemes for

a TWRC based on Alamouti codes. Our IC method based on Alamouti codes for the K-user interference channel can be used for the TWRC, which enables the nodes to perform symbol-by-symbol decoding and achieve diversity order of two. In order to achieve more diversity gain, we propose a new two-way relaying scheme based on Alamouti codes which utilizes beamforming matrices to align signals at the relay node. From the simulation results, it is shown that the proposed scheme achieves diversity order of four.

Finally, we analyze the best relay selection scheme for the SDF cooperative networks with multiple relays. The term ‘best relay selection’ implies that the relay having the largest end-to-end signal-to-noise ratio is selected to transmit in the second phase transmission. The upper and lower bounds on the average pairwise error probability (PEP) are analyzed and compared with the conventional multiple-relay transmission scheme, where all the relays participate in the second phase transmission. Using the upper and lower bounds on the PEP and the asymptotes of the Fox's H-function, the diversity orders of the best relay selection and conventional relay schemes for the SDF cooperative networks are derived. It is shown that both schemes have the same full diversity order.

Keywords: Alamouti code, degrees of freedom (DoF), diversity order, interference alignment (IA), interference alignment-and-cancellation (IAC), interference cancellation (IC), interference channel, pairwise-error probability (PEP), relay selection, soft-decision-and-forward (SDF) protocol,

two-way relay channel (TWRC).

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Chapter 1. Introduction

1.1. Background

The recent advancement in wireless communication systems has increased their throughput and reliability. As a result, mobile devices become more popular than wired devices. However, as the market of the mobile contents becomes bigger and various services are widely introduced, throughput and reliability become more important research topics. When new services which require high data transmission rate are introduced, the data rate should be restricted for the reliability and thus the services cannot be provided properly. In fact, this dilemma has led to the advance of wireless communication technologies.

Multi-input and multi-output (MIMO) as a new technology has drawn great attention because it provides significant increases in data throughput and reliability without additional bandwidth or increase in transmit power. It achieves the enhancement of performance by spreading the total transmit power over the antennas to obtain multiplexing gain that improves the spectral efficiency and to obtain diversity gain that improves the reliability. Due to these properties, MIMO technology is an important part of modern wireless communication systems.

In the MIMO technology, two kinds of gains have been investigated. One is the diversity gain which is related to the reliability and the other is multiplexing gain which is related to the throughput. Sometimes, the multiplexing gain may be called degrees of freedom (DoF). Cheng and Tse [1] introduced the tradeoff of these two gains when the channel state information (CSI) is not available at the transmitter and proved the existence of the code to achieve so called the fundamental tradeoff.

In general, transmitted signals experience deep fading distortion in wireless communications. To overcome this fading, diversity technique has been adopted, which is a good solution to obtain the diversity gain. Space-time codes (STCs) are the most popular scheme to obtain diversity gain without CSI. Tarokh et al. [2] proposed the design criterion for STCs and Alamouti [3] designed a simple 2×2 space-time block code (STBC). After that, many STBCs [4]–[6] are proposed such as orthogonal and quasi-orthogonal STBCs (OSTBCs, QOSTBCs) and coordinated interleaved STBCs (CISTBCs) and their performances have been analyzed from the viewpoint of diversity gain.

Through MIMO technology such as STCs, the reliability can be enhanced much more than before. However, the deployment of multiple transmit and receive antennas in the limited space of devices is difficult and costly. This makes researchers to consider usage of relays to create a virtual MIMO channels for wireless communication systems, called the cooperative communication, which is considered as a core technology in the next generation wireless communication systems.

The protocols of the cooperative communications are classified according to the operation of relay. In orthogonal transmission (OT) protocol, the relay keeps silent when the transmitter transmits its signal but in non-orthogonal transmission (NT) protocols, the relay transmits while the transmitter transmits. And in amplify-and-forward (AF) protocol, the relay amplifies the received signal and just forward it to the destination. In decode-and-forward (DF) protocol, the relay decodes the received signal and forwards it to the destination.

In general, it is assumed that the relay does not generate new information but it transmits the replica of information received from transmitters and thus the cooperative communication is mainly focused on obtaining the diversity gain. This technique is called the cooperative diversity technique. Since the relay can be thought as a virtual antenna for the source to destination, it can be used to design the distributed STCs (DSTCs) to obtain the cooperative diversity [7].

Through the MIMO technology and cooperative communications, the reliability and throughput can be improved greatly for pear-to-pear (P2P) communications. However, in the wireless communication environment where many mobile devices exist, these techniques may cause strong interference.

In addition, due to introduction of small cell technology such as WiFi, femto-cell, and so on, the radius of a cell becomes smaller and it makes the cell boundary area wider. Usually, the problems related to interference occur in the cell boundary and thus the small cell technology makes the problem of interference difficult.

Even though the research on interference management has been done since 1980's, the optimal scheme has not been introduced yet. Treating the interference as noise, decoding the interference, and orthogonalization are three classical techniques for the interference management. However, the first scheme does not guarantee the reliability under the strong interference and the second one requires too much decoding complexity. The last one guarantees the reliability and low complexity but its throughput becomes low.

Recently, interference alignment (IA) has attracted a great attention as a good solution to resolve the drawbacks of the previous research. Cadambe and Jafar [8] proposed the IA scheme for K-user interference channel, which achieves the maximum DoF. Similarly, they proposed IA scheme for X channel in [9].

However, the IA schemes in [8], [9] have several problems. The first is that the time varying channel should be assumed. It is also assumed that the perfect global CSI is available at the transmitter, which requires lots of feedback information. Thus these two assumptions cannot simultaneously be accepted. And thus, recently, IA is implemented by using multiple antennas or multiple carriers under quasi-static channel assumptions.

In fact, it is limited by the size of device to use multiple antennas and thus there are some research results to use relay [10]–[12], which are called relay-aided IA schemes. Since the relay-aided IA schemes are based on the quasi-static channel, they are more practical than the IA schemes in the time-varying channel [8], [9].

The other problem is that IA scheme is only focused on the throughput which is measured by DoF and the reliability is not considered seriously. Since the reliability is also important in the wireless communications, the enhancement of the error performance of IA can be a good research topic.

1.2. Overview of the Dissertation

This dissertation is organized as follows. In Chapter 2, MIMO, STC, cooperative communication, and IA are overviewed. The basic concepts of STCs are explained and some terminologies are introduced. Also, information-theoretic views of the multiple antenna techniques are illustrated and the cooperative communications are reviewed. The basic idea and concept of IA are introduced.

Chapter 3 is based on the result in [13]. In this chapter, the methods on how to apply Alamouti code to MIMO interference channels is proposed. The interference cancellation (IC) method based on Alamouti codes for the multi-access scenario can be used for the K-user interference channel, which enables the receivers to perform symbol-by-symbol decoding by cancelling interfering signals by utilizing Alamouti structure and achieve diversity order of two. Moreover it does not require channel state information at the transmitters (CSIT) unlike the IA scheme. However, it requires more receive antennas than the IA scheme to achieve the same DoF. In order to reduce the number of receive antennas, especially for the three-user MIMO interference channel, an Interference alignment-and-cancellation (IAC) scheme based on Alamouti codes is proposed, which keeps the same DoF as that of the IC scheme, but it requires partial CSIT.

It is analytically shown that the IC and IAC schemes enable symbol-by-symbol decoding and achieve diversity order of two, while the conventional IA scheme achieves diversity order of one.

Chapter 4 is based on the result in [14]. In this chapter, we propose a method on how to apply Alamouti codes to the two-way relay channel (TWRC). The IC method based on Alamouti codes for the interference channel can be used for the TWRC, which enables the nodes to perform symbol-by-symbol decoding and achieve diversity order of two. In order to achieve more diversity gain, we propose a new two-way relaying scheme based on Alamouti codes which utilizes beamforming matrices to align signals at the relay node. From the simulation results, it is shown that the proposed scheme achieves diversity order of four.

Chapter 5 is based on the result in [15]. In this chapter, the performance of two cooperative communication schemes is derived. The upper and lower bounds on pairwise error probability (PEP) of the conventional multiple-relays transmission scheme is derived by using Gauss' hypergeometric function. And it is shown that the system has full diversity. The diversity order for the best relay selection is obtained from the upper and lower bounds on the PEP and Fox's H-function. From the numerical results, it is shown that the best relay selection scheme outperforms the conventional multiple-relay transmission scheme in terms of bit error rate (BER) and throughput.

And finally, in Chapter 6, the concluding remarks are addressed.

1.3. Terms and Notations

In this section, we arrange the terms and notations used in this dissertation. Tables 1.1 and 1.2 explain the terms and notations.

Table 1.1: Terms used in the dissertation.

Term	Meaning
AF	amplify-and-forward
AWGN	additive white Gaussian noise
BER	bit error rate
CISTBC	coordinated interleaved space-time block code (coding)
CSI	channel state information
CSIT	channel state information at transmitters
DF	decode-and-forward
DMT	diversity-multiplexing gain tradeoff
DoF	degrees of freedom
DSTC	distributed space-time code (coding)
IA	interference alignment
IAC	interference alignment-and-cancellation
IC	interference cancellation
i.i.d.	independent and identically distributed
MGF	moment generating function
MIMO	multiple-input multiple-output
ML	maximum-likelihood
OSTBC	orthogonal space-time block code (coding)
P2P	pear-to-pear
PDF	probability density function
PEP	pairwise error probability
QAM	quadrature amplitude modulation
QOSTBC	quasi-orthogonal space-time block code (coding)
QPSK	quadrature phase-shift keying
SDF	soft-decision-and-forward
SER	symbol error rate
SNR	signal-to-noise ratio
STBC	space-time block code (coding)
STC	space-time code (coding)
TWRC	two-way relay channel

Table 1.2: Notations used in the dissertation.

Notation	Meaning
$\mathbf{A}(a,b)$	Alamouti encoding: $\mathbf{A}(a,b) = \begin{bmatrix} a & -b^* \\ b & a^* \end{bmatrix}$
$\mathbb{C}$	set of complex number
$cv(\cdot)$	complex vectorization
D	destination node
$\mathbf{diag}(a_1, \cdots, a_k)$	diagonal matrix with diagonal entries $a_1, \cdots, a_k$
$\mathbb{E}_X[\cdot]$	expectation of random variable X
$f_X(\cdot)$	PDF of random variable X
$F_X(\cdot)$	CDF of random variable X
${}_2F_1(\cdot,\cdot;\cdot;\cdot)$	Gauss' hypergeometric function
$\mathcal{H}_{p,q}^{m,n}(z)$	Fox's H-function
$H(X,Y)$	harmonic mean of X and Y
$\mathcal{K}_{\mathbf{x}}$	covariance matrix of vector $\mathbf{x}$
$\mathcal{M}_X(s)$	MGF of random variable X in terms of s
P	transmit signal power (or SNR for zero-mean and unit-variance Gaussian noise)
$\Pr(A)$	probability that occurs event A
$\mathbb{R}$	set of real numbers
R	relay node
S	source node
$X \sim \mathcal{CN}(0, \sigma^2)$	X is a complex normal random variable with zero mean and variance $\sigma^2/2$ in both real and imaginary parts, respectively
$\mathbb{Z}$	set of integers
$\mathbf{0}_n$, $\mathbf{I}_n$	zero matrix and the identity matrix of size n
$(\cdot)'$	$\left(\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\right)' = \begin{bmatrix} b_{11} & b_{12}^* & b_{21} & b_{22}^* \\ b_{12} & -b_{11}^* & b_{22} & -b_{21}^* \end{bmatrix}^{\mathrm{T}}$
$(\cdot)^{\mathrm{T}}$	transpose of a matrix or a vector
$(\cdot)^{\dagger}$	conjugate transpose of a matrix or a vector
$\\|\cdot\\|$	Frobenius norm of a matrix or a vector
$\|\cdot\|$	modulus of complex number
$\Re\{\cdot\}$	real part of complex number
$(\cdot)^*$	complex conjugate of complex number

Chapter 2. Preliminaries

In this chapter, we review MIMO, STC, cooperative communications, and IA. MIMO communications can be used for two different goals. One is to obtain higher data transmission rate measured by DoF and the other is to achieve higher reliability known as diversity order. MIMO technique can achieve higher DoF without any further communication resources such as transmit power and frequency spectrum. Diversity techniques such as STCs and selection diversity are focused on the high reliability. Since Alamouti codes [3] were introduced, STCs, which are classified as transmit diversity technique in general, have drawn great interests. However, due to the limitation of the multiple antenna implementation in a mobile node, the virtual multiple antenna scheme can be considered. Cooperative communications are well-known virtual multiple antenna schemes which use relays as the virtual antennas for the source to the destination.

In fact, STCs and cooperative communications are diversity techniques for the P2P communications and they can transmit strong desired signals to the destination node. But the strong desired signals can be a strong interference for the other nodes. Therefore, without interference management, the reliable communication is not possible in such systems. Recently, IA schemes are introduced as good solutions for interference management and it has been proved that IA can achieve the maximum DoF for K-user interference channel [8].

In this chapter, the preliminaries of MIMO communications such as DoF, diversity order, and STCs are explained first and then, the cooperative communications are introduced and its several protocols are also explained. And, the basic concept of IA is described.

2.1. MIMO Communications

The advantage of MIMO communications is the use of multiple antennas at both the transmitter and receiver to improve the performance of the communication systems. Since MIMO can offer significant enhancement in data throughput and reliability without additional bandwidth or increased transmit power, it has drawn great attention in wireless communications. In [16], the capacity of the Gaussian MIMO channels was derived and the advantage of MIMO communications was proved theoretically.

Consider $M \times N$ MIMO communication system, where M antennas and N antennas are equipped at the transmitter and receiver, respectively. Then, the received signal in one time slot is represented as

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{n} \tag{2.1}$$

where $\mathbf{H}$, $\mathbf{x}$, and $\mathbf{n}$ are the $N \times M$ channel matrix, the $M \times 1$ transmitted signal vector, and the $N \times 1$ noise vector, respectively and ρ is the value proportional to SNR. It is assumed that elements of $\mathbf{n}$ are i.i.d. with $\mathcal{CN}(0, 1)$.

If the channel state is known at the transmitter and receiver, the channel capacity in (2.1) is derived as [16]

$$C = \log_2 \det \left(\mathbf{I}_N + \rho \mathbf{H} \mathcal{K}_{\mathbf{x}} \mathbf{H}^\dagger \right)$$

where $\mathcal{K}_{\mathbf{x}}$ is the covariance matrix of $\mathbf{x}$.

If the channel state is not known to the transmitter, the transmitter transmits the data according to $\mathcal{K}_{\mathbf{x}} = \mathbf{I}_M / M$, which implies that the uniform power allocations and independent codes are used for each antenna. Then the channel capacity can be represented as

$$C = \sum_{m=1}^{\min(M,N)} \log_2 \left(1 + \frac{\rho}{M} \lambda_m \right) \tag{2.2}$$

where λ_m's are the eigenvalues of $\mathbf{H}\mathbf{H}^\dagger$.

Definition 2.1 (Degrees of freedom)**.** The degrees of freedom or multiplexing gain r of the channel is defined as

$$r = \lim_{SNR \to \infty} \frac{R(SNR)}{\log(SNR)}$$

where $R(SNR)$ is an achievable rate at the SNR.

In (2.2), it can be easily seen that DoF of the MIMO channel is $\min(M, N)$.

2.2. Space-Time Coding and Selection Diversity

In this section, we review the STCs and their design criteria in order to obtain the transmit diversity together with selection diversity.

Definition 2.2 (Diversity order (gain))**.** The diversity order G_d of the system is defined as

$$G_d = -\lim_{SNR\to\infty} \frac{\log(P_e)}{\log(SNR)}$$

where P_e denotes the error probability of the system.

Comparing with Definition 2.1, the diversity order is directly related to the error probability. In MIMO communications, since the reliability is also the important factor, STCs with higher DoF and diversity order are desirable.

In fact, STCs may reduce the data transmission rate to increase diversity order and thus, the STCs should satisfy the requirement of diversity order and multiplexing gain. In [1], the fundamental tradeoff of the diversity order and multiplexing gain (DMT) is analyzed as follows.

Theorem 2.3 (Diversity and multiplexing tradeoff [1])**.** Assume a code at the transmitter of a MIMO channel with M transmit antennas and N receive antennas. For a given DoF i, $i = 0, 1, \cdots, \min(M, N)$, the maximum diversity order $G_d(i)$ is given as

$$G_d(i) = (M - i)(N - i)$$

if the block length of the code is greater than or equal to $N + M - 1$.

Let $\mathbf{X}$ be a $M \times T$ space-time code matrix, which is transmitted by M transmit antennas in T time slots. Then the received signal matrix is given as

$$\mathbf{Y} = \sqrt{\rho}\mathbf{H}\mathbf{X} + \mathbf{N} \tag{2.3}$$

where $\mathbf{H} \in \mathbb{C}^{N\times M}$ and $\mathbf{N} \in \mathbb{C}^{N\times T}$ denote a channel matrix with Rayleigh fading and a AWGN matrix, respectively.

The pairwise error probability is a useful tool to quantify the system performance of the STC. Tarokh, Seshadri, and Calderbank [2] derived the upper bound on the PEP of STCs in (2.3). First, they consider the conditional PEP that the receiver detects the received signal as $\hat{\mathbf{X}}$ when $\mathbf{X}$ is transmitted. Then, the error (difference) matrix is defined as $D(\hat{\mathbf{X}}, \mathbf{X}) = \hat{\mathbf{X}} - \mathbf{X}$ and the conditional PEP is expressed as

$$\begin{aligned}\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}\right) &= Q\left(\sqrt{\frac{\rho}{2}\|(\hat{\mathbf{X}} - \mathbf{X})\mathbf{H}\|^2}\right) \\ &= Q\left(\sqrt{\frac{\rho}{2}\sum_{n=1}^{N}\sum_{m=1}^{M}\lambda_m|\beta_{m,n}|^2}\right)\end{aligned}$$

where γ is the received SNR and $Q(\cdot)$ is the Gaussian Q-function defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt. \tag{2.4}$$

Let λ_n's be the (non-negative) eigenvalues of $A(\hat{\mathbf{X}}, \mathbf{X}) = D(\hat{\mathbf{X}}, \mathbf{X})D(\hat{\mathbf{X}}, \mathbf{X})^\dagger$ and $\beta_{m,n}$'s be the (m, n)th element of the product of the unitary matrix V and the channel matrix $\mathbf{H}$ with $A(\hat{\mathbf{X}}, \mathbf{X}) = V\Lambda V^\dagger$ for $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$. Using the Chernoff bound, $Q(x) \leq \frac{1}{2}\exp(-\frac{1}{2}x^2)$, the above PEP can be upper-bounded as

$$\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}\right) \leq \frac{1}{2}\exp\left(-\frac{\rho}{4}\sum_{n=1}^{N}\sum_{m=1}^{M}\lambda_m|\beta_{m,n}|^2\right).$$

Since $\beta_{m,n}$ is Gaussian, $|\beta_{m,n}|$ is Rayleigh-distributed with the PDF

$$f(|\beta_{m,n}|) = 2|\beta_{m,n}|\exp(-|\beta_{m,n}|^2).$$

Thus, the PEP can be obtained by averaging the conditional PEP over the channel matrix $\mathbf{H}$

$$\begin{aligned}\Pr\left(\mathbf{X}\to\hat{\mathbf{X}}\right) &= \mathbb{E}_{\mathbf{H}}\left[\Pr\left(\mathbf{X}\to\hat{\mathbf{X}}|\mathbf{H}\right)\right] \\ &\leq \prod_{m=1}^{M}\left(1+\frac{\rho\lambda_m}{4}\right)^{-N}. \end{aligned} \tag{2.5}$$

From (2.5), it can be seen that the performance of STCs depends on the rank and eigenvalues of $\hat{\mathbf{X}}-\mathbf{X}$. If its rank $r<M$, we have $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r > 0$ and $\lambda_{r+1} = \cdots = \lambda_N = 0$. In the high SNR region, (2.5) can be upper-bounded as

$$\Pr\left(\mathbf{X}\to\hat{\mathbf{X}}\right) \leq \frac{4^{rN}}{\left(\prod_{n=1}^{r}\lambda_n\right)^N \rho^{rN}} \propto (G_c\rho)^{-G_d} \tag{2.6}$$

where G_c and G_d denote the *coding gain* and *diversity order*, respectively.

There are two criteria to minimize the upper bound in (2.6). One is the determinant criterion to maximize the determinant of $\mathbf{A}(\hat{\mathbf{X}},\mathbf{X})$, which corresponds to the coding gain, i.e., $G_c = \left(\prod_{n=1}^{r}\lambda_n\right)^{1/r}$. The other is the rank criterion to maximize the rank of $\hat{\mathbf{X}}-\mathbf{X}$, which corresponds to the diversity order, i.e., $G_d = rN$. In high SNR region, the diversity order has dominant effect on the error performance.

The diversity order in Theorem 2.3 can be obtained by selection diversity techniques regardless of code length. Selection diversity techniques are based on singular value decomposition (SVD) of channel matrix, which is explained as

$$\mathbf{Y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{n}$$

$$
\begin{aligned}
&= \sqrt{\rho}\mathbf{U}\Sigma\mathbf{V}^{\dagger}\mathbf{x} + \mathbf{n} \\
\mathbf{U}^{\dagger}\mathbf{Y} &= \sqrt{\rho}\mathbf{U}^{\dagger}\mathbf{U}\Sigma\mathbf{V}^{\dagger}\mathbf{V}\tilde{\mathbf{x}} + \mathbf{U}^{\dagger}\mathbf{n} \\
&= \sqrt{\rho}\Sigma\tilde{\mathbf{x}} + \mathbf{U}^{\dagger}\mathbf{n}
\end{aligned}
$$

where $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^{\dagger}$ by SVD and $\mathbf{x} = \mathbf{V}\tilde{\mathbf{x}}$. In this way, $\min(M, N)$ Gaussian channels can be obtained and each channel coefficient is the non-zero singular value of $\mathbf{H}$. By selecting i beamforming vectors by i largest singular values, diversity order $(M - i + 1)(N - i + 1)$ can be obtained [17].

Recently, a transmit antenna selection scheme for the MIMO communication system is proposed, which is based on the maximum likelihood (ML) decoding. Since the PEP of ML decoding depends on $|\mathbf{H}_{\text{sub}}\mathbf{x}|$, where $\mathbf{H}_{\text{sub}}$ is a sub-matrix of $\mathbf{H}$, the subset of transmit antennas is selected to maximize $|\mathbf{H}_{\text{sub}}\mathbf{x}|$. Even though the complexity of the ML decoding is much higher than that of the zero-forcing, this selection scheme with ML decoding can achieve diversity order $(M - i + 1)N$ [18].

2.3. Cooperative Communications

Multiple antenna communication systems have advantage against the single-antenna systems. However, due to the difficulty of the implementation of the multiple antennas in a mobile, virtual MIMO systems are good candidates for co-located MIMO systems. These virtual MIMO systems can be viewed as cooperative communication systems. In general, cooperative communication networks consist of source (transmitter), relay, and destination (receiver) nodes. Relay nodes help the communication between the source and destination nodes. The first mathematical model of three-node cooperative communication systems was proposed by van der Meullen [19] in 1971. In 1979, Cover and El Gamal [20] proved some capacity theorems on several channels such as degraded channel, feedback channel, and so on. After then, many researchers have studied the relay channels in the information-theoretic point of view.

Along with the theoretical progress, more practical issues have been investigated. In 2004, Nosratinia, Hunter, and Hedayat [21] arranged the basic relaying protocols: AF, DF, and coded-cooperation. In [22], Laneman, Tse, and Wornell proposed the efficient relaying protocols and analyzed the performance of the proposed schemes in terms of outage probability and diversity-multiplexing gain tradeoff [1]. In 2005, Kramer, Gastpar, and Gupta [23] showed various cooperative strategies and then derived the capacity theorems for relay networks. Nabar, Bölcskei, and Kneubüler [24] derived the diversity order for AF and DF protocols using different TDMA schemes and derived the design criteria for (distributed) space-time codes for an AF single-relay fading channel. Yang, Song, No, and Shin [25] analyzed performance of the SDF protocol combined with AF and coded-cooperation. Song, No, and Chung [26] analyzed the BER and found the suboptimal power allocation. And one of main contributions of this dissertation is to treat the relay selection criterion deployed SDF protocol called 'best relay selection' and derived the diversity order of the proposed scheme by Song, Kim, No, and Chung [15]. For Chapter 5, we will investigate the basic operations for AF and DF schemes with single relay.

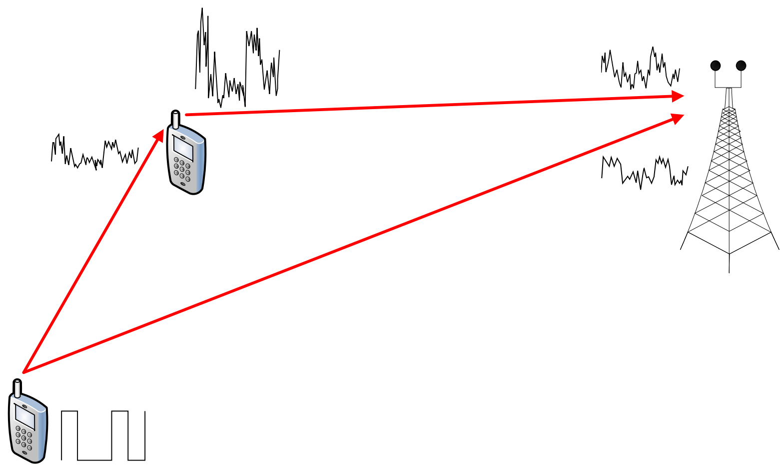

Figure 2.1: Amplify-and-forward protocol.

2.3.1. Amplify-and-Forward Protocol

In this and the following subsections, we only consider schemes with orthogonal time uses, i.e., the source broadcasts the signal into the relay and destination in the first phase, and then, only the relay transmits signal obtained from the source to the destination in the second phase.

First, we assume frequency-flat fading, no CSI at the transmitters and perfect CSI at the receivers, and perfect synchronization.

The signal transmitted from S (source node) during the first time slots is denoted as $x \sim \mathcal{CN}(0, 1)$. The data symbols may be chosen from a complex-valued finite constellation such as QAM or from a Gaussian codebook. And then, the signal received at R (relay node) and D (destination node) in the first time slot is given as

$$y^{[\mathsf{R}]} = \sqrt{P_{\mathsf{SR}}} h^{[\mathsf{SR}]} x + n^{[\mathsf{R}]} \tag{2.7}$$

$$y^{[\mathsf{D1}]} = \sqrt{P_{\mathsf{SD}}} h^{[\mathsf{SD}]} x + n^{[\mathsf{D1}]} \tag{2.8}$$

where P_{SR} and P_{SD} are the average signal energy received at R and D over one symbol period through the link (having accounted for path loss and shadowing between S and D), $h^{[\mathsf{SR}]}$ and $h^{[\mathsf{SD}]}$ are the random, complex-valued, unit-power channel gain, i.e., $h^{[\mathsf{SR}]},\ h^{[\mathsf{SD}]} \sim \mathcal{CN}(0,1)$ between S and R, S and D, and $n^{[\mathsf{R}]} \sim \mathcal{CN}(0,1)$ and $n^{[\mathsf{D1}]} \sim \mathcal{CN}(0,1)$ are AWGN. Note that $P_{\mathsf{SR}} \neq P_{\mathsf{SD}}$ generally due to differences in path loss and shadowing between $\mathsf{S} \to \mathsf{R}$ and $\mathsf{S} \to \mathsf{D}$ links.

As an intermediate step, R normalizes the received signal by a factor of $\sqrt{\mathbb{E}\left[|y^{[\mathsf{R}]}|^2\right]}$ (so that the average energy is unity) and retransmits the signal during the second time slot. D receives a signal transmitted from R during the second time slot according to

$$y^{[\mathsf{D2}]} = \sqrt{P_{\mathsf{RD}}} h^{[\mathsf{RD}]} \frac{y^{[\mathsf{R}]}}{\sqrt{\mathbb{E}\left[|y^{[\mathsf{R}]}|^2\right]}} + n^{[\mathsf{D2}]} \tag{2.9}$$

$$= \sqrt{\frac{P_{\mathsf{SR}} P_{\mathsf{RD}}}{P_{\mathsf{SR}} + 1}} h^{[\mathsf{SR}]} h^{[\mathsf{RD}]} x + n^{[\mathsf{D}]} \tag{2.10}$$

where the effective noise in the second time slot $n^{[\mathsf{D}]}$ is represented as

$$n^{[\mathsf{D}]} = n^{[\mathsf{D2}]} + \sqrt{\frac{P_{\mathsf{RD}}}{P_{\mathsf{SR}} + 1}} h^{[\mathsf{RD}]} n^{[\mathsf{R}]}. \tag{2.11}$$

The effective input-output relation for AF protocol can be expressed as

$$\underbrace{\begin{bmatrix} y^{[\mathsf{D1}]} \\ y^{[\mathsf{D2}]} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \sqrt{P_{\mathsf{SD}}} h^{[\mathsf{SD}]} \\ \sqrt{\frac{P_{\mathsf{SR}} P_{\mathsf{RD}}}{P_{\mathsf{SR}} + 1}} h^{[\mathsf{SR}]} h^{[\mathsf{RD}]} \end{bmatrix}}_{\mathbf{h}} x + \underbrace{\begin{bmatrix} n^{[\mathsf{D1}]} \\ n^{[\mathsf{D}]} \end{bmatrix}}_{\mathbf{n}}. \tag{2.12}$$

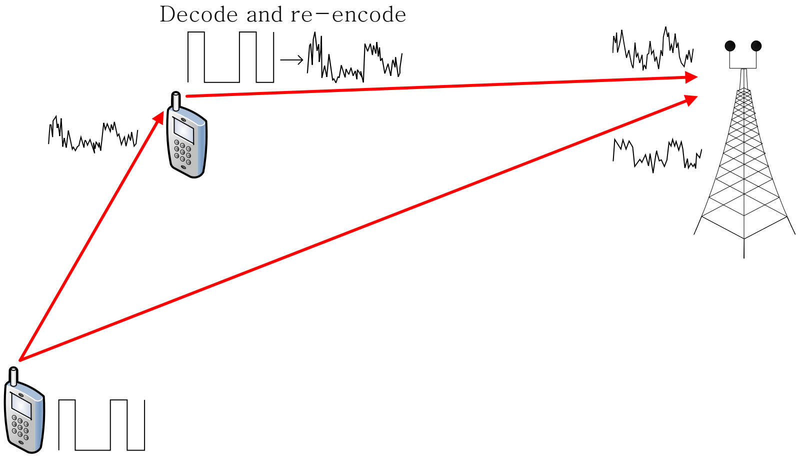

Figure 2.2: Decode-and-forward protocol.

2.3.2. Decode-and-Forward Protocol

In the DF protocol, the signals received at R and D during the first time slot are identical to those for the AF protocol. Unlike AF protocol, R decodes and re-encodes the signal received during the first time slot. Assuming that the signal is decoded correctly and retransmitted, we obtain in the second time slot

$$y^{[\mathsf{D2}]} = \sqrt{P_{\mathsf{RD}}} h^{[\mathsf{RD}]} x + n^{[\mathsf{D2}]}. \tag{2.13}$$

The correct reception at R can be confirmed by cyclic redundancy check (CRC) with high order polynomial and automatic repeat request (ARQ).

And then, the effective input-output relation in the DF protocol can be summarized as

$$\underbrace{\begin{bmatrix} y^{[\mathsf{D1}]} \\ y^{[\mathsf{D2}]} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \sqrt{P_{\mathsf{SD}}} h^{[\mathsf{SD}]} \\ \sqrt{P_{\mathsf{RD}}} h^{[\mathsf{RD}]} \end{bmatrix}}_{\mathbf{h}} x + \underbrace{\begin{bmatrix} n^{[\mathsf{D1}]} \\ n^{[\mathsf{D2}]} \end{bmatrix}}_{\mathbf{n}}. \tag{2.14}$$

2.4. Interference Alignment

In order to satisfy the requirement of wireless communication service, such as high data rate and reliability, MIMO technology becomes common and some small cellular systems are considered such as pico- and femto-cells. Especially, as cell size becomes smaller than before, the spectral efficiency and power consumption can be improved.

However, as the cell size becomes smaller, the cell boundary region becomes wider. This implies that the interference increases and causes the bad effect on the reliable communications specially in the cell boundary region. Therefore, the interference management becomes a hot issue of the MIMO wireless communications.

Conventionally, there are three interference management approaches:

1) Decode:

- Interference is strong.
- The interference signal can be decoded and then subtracted from the desired signal.

2) Treat as noise:

- Interference is weak.

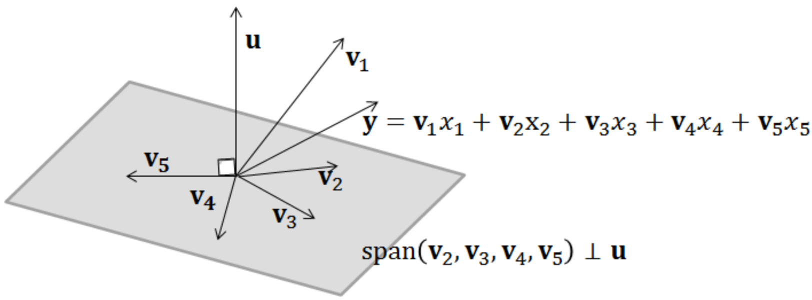

Figure 2.3: Graphical interpretation on IA.

- The interference signal is treated as noise and single user encoding/decoding suffices as in the conventional communication systems.

3) Orthogonalization:

- The strength of interference is comparable to the desired signal.
- This approach is to orthogonalize interferences and desired signals in time, frequency, or code.

When interference signal power is comparable to the desired signal, 'decode' and 'treat as noise' do not guarantee high reliability and 'orthogonalization' reduces the throughput. In this situation, IA scheme is drawing great attention as a solution for the above conventional approaches. The basic idea of IA is to separate the spaces of interference signal from the spaces of desired signal. And by maximizing the overlap of all interference signal spaces at each receiver, we can increases the DoF of the desired signal.

Fig. 2.3 describes the basic concept of IA. In Fig. 2.3, four interference signals are aligned to the two-dimensional subspace and thus, by using

$\mathbf{u}$, the desired signal $\mathbf{v}_1$ can be extracted without interference signal. In more details, $\mathbf{u}$ is orthogonal to the $\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$ and thus

$$\begin{aligned}\mathbf{u}^\dagger\mathbf{y} &= \mathbf{u}^\dagger\mathbf{v}_1 x_1 + \mathbf{u}^\dagger\mathbf{v}_2 x_2 + \mathbf{u}^\dagger\mathbf{v}_3 x_3 + \mathbf{u}^\dagger\mathbf{v}_4 x_4 + \mathbf{u}^\dagger\mathbf{v}_5 x_5 \\ &= \mathbf{u}^\dagger\mathbf{v}_1 x_1.\end{aligned}$$

In this case, orthogonalization can provide 1/5 DoF but IA achieves 1/3 DoF.

Usually, IA is implemented by using multiple antennas, carriers, or time extension. Cadambe and Jafar [8], [9] proposed IA scheme for K-user interference channel and X channel by using time extension. However, it is assumed that the channel is time-varying and the global CSI is available at the transmitters, which implies that each transmitter knows the CSIs of all links between transmitters and receivers. These assumptions are infeasible because for global CSI, each transmitter requires lots of feedback information and the channel state may vary during feedback of global CSI. Therefore, IA under the quasi-static channel is required in the practical point of view and Yetis et. al. [27] derived feasibility condition for IA to use multiple antennas.

In general, IA is closely connected with MIMO technology because many IA schemes are implemented by using multiple antennas. As the number of antennas increases at the node, the dimensions of transmit and received signal spaces also increase. Therefore, if the transmitter or receiver has plenty of dimension of signal spaces, the perfect IA can be implemented easily. However, the numbers of antennas for IA are restricted from the size of device. Then, the virtual antenna system is recommended for IA and thus, recently, some relay-aided IA schemes are introduced [10]–[12], where relays are operated in AF-mode. Since these relay-aided IA schemes are implemented under the quasi-static channel, they can be good alternatives for the conventional IA.

Although most of studies for IA focus on network throughput, that is, DoF, reliability in terms of diversity order is also an important performance measure for communication systems. Since Alamouti code [3] is simple and powerful approach to improve the diversity gain, Li et. al. [29] proposed a fixed-rate transmission scheme over the simplest two-user X channel where each of the two double-antenna transmitters has independent messages for each of the two double-antenna receivers. Each transmitter encodes symbols using Alamouti codes followed by beamformers that align interferences at unintended receivers. Li's scheme achieves a diversity order of two and the maximum DoF for the two-user X channel.

Chapter 3. Interference Alignment-and-Cancellation Scheme Based on Alamouti Codes for the Three-User Interference Channel

3.1. Introduction

Wireless communication systems such as IMT-Advanced [30] require higher spectral efficiency and data rate, that is, 100Mbps for high-speed mobility and 1Gbps for low-speed mobility. With the increasing demands for higher spectral efficiency, the cell size is getting smaller and smaller such as micro-, pico-, and femto-cells. In the heterogeneous cellular networks [31] consisting of various sizes of cells, interference is a major barrier in achieving reliable high data-rate transmission. The interference channel in Fig. 3.1 shows that three transmitter-receiver pairs share a communication channel so that the transmission of information from one transmitter to its corresponding receiver interferes with the communications of the other transmitter-receiver pairs.

To resolve the interference problem, an interference alignment scheme was recently proposed and has become a subject of special interest in the area of wireless communications. Cadambe and Jafar [8] showed that each user in a multi-user interference channel can utilize one half of all the net-

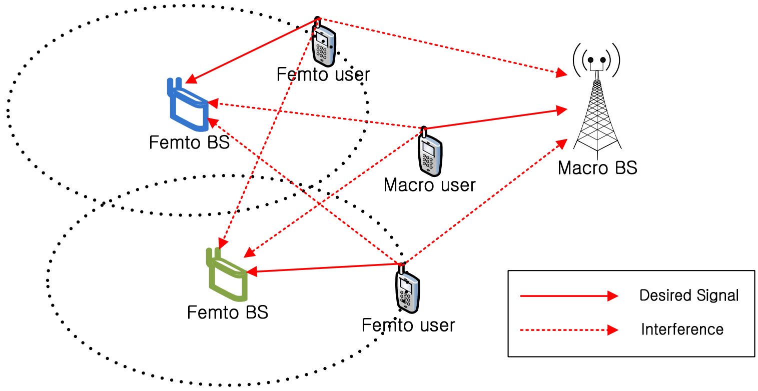

Figure 3.1: Uplink three-user interference channel.

work resources, which corresponds to the maximum degrees of freedom (also known as multiplexing gain). The key idea of this result is the IA which maximizes the overlap of all interference signal spaces at each receiver so that the dimension of the interference-free space for the desired signal is maximized. Especially in the case of three-user MIMO interference channel, it is shown that an IA scheme can achieve the maximum DoF for constant (not time-varying) channel with finite number of dimensions (number of antennas). The idea of IA was further developed by many researchers [32]–[42].

Although most of studies for IA focus on network throughput, that is, DoF, reliability in terms of diversity order is also an important performance measure of communication systems. Since Alamouti code [3] is simple and powerful approach to improve the diversity gain, in this chapter we study how to apply Alamouti code to the MIMO interference channels. We first show that the interference cancellation scheme based on Alamouti codes in the multi-access scenario [43]–[45] can also be used for the K-user MIMO interference channel with $2M$ transmit antennas. It is analytically shown that the proposed IC scheme enables the receivers to perform symbol-by-symbol decoding and achieves diversity order of two. Furthermore, unlike IA schemes for K-user interference channel (except for a blind IA in [47]), the IC scheme does not require CSIT. However, it requires more receive antennas than IA scheme to achieve the same DoF. As an effort to reduce the number of antennas at the receivers, especially for the three-user MIMO interference channel, we propose an interference alignment-and-cancellation scheme based on Alamouti codes, which is motivated by the result in [29]. However, our IAC scheme and Li's scheme in [29] are quite different. Li's scheme designed beamforming matrices for alignment by casting interference into small dimensions in the received signal space. In our IAC scheme, by using the proper beamforming matrices, three Alamouti signals from three transmitters can be combined into two Alamouti signals. Then, the receivers separate the desired symbols to perform symbol-by-symbol decoding with less antennas. Further, Li's work focused on only the simplest multi-antenna case of the X channel, which consists of two double-antenna transmitters and two double-antenna receivers. Proposed IAC scheme considers the three-user interference channel, where each transmitter has arbitrary $2M$ antennas.

We will show that while the proposed IAC scheme achieves the same performance as the IC scheme in terms of DoF and diversity order together with the symbol-by-symbol decoding algorithm, it requires partial CSIT contrary to the IA scheme requiring full CSIT and less receive antennas than the IC scheme.

This chapter is organized as follows. Section 3.2 describes the interference channel model and the IC scheme based on Alamouti codes, where it is shown that a symbol-by-symbol decoding is possible for the K-user interference channel. In Section 3.3, we propose an IAC scheme based on Alamouti codes for the three-user interference channel and derive its DoF and diversity order. The numerical analysis is performed in Section 3.4. Finally, concluding remarks are given in Section 3.5.

3.2. Interference Cancellation Scheme Based on Alamouti Codes

Consider the K-user interference channel, where each transmitter and each receiver have N_T and N_R antennas, respectively. There are K transmitter-receiver pairs and each transmitter wishes to send independent data stream to its corresponding receiver. The channel output at the kth receiver is described as

$$\mathbf{Y}^{[k]} = \sum_{t=1}^{K} \mathbf{H}^{[kt]}\mathbf{X}^{[t]} + \mathbf{N}^{[k]} \tag{3.1}$$

where $\mathbf{Y}^{[k]}$ is the received signal matrix at the receiver k, $k \in \{1, \cdots, K\}$, $\mathbf{X}^{[t]}$ is the transmitted signal matrix from the transmitter t, $t \in \{1, \cdots, K\}$, $\mathbf{H}^{[kt]}$ is the channel matrix from the transmitter t to the receiver k, and

$\mathbf{N}^{[k]}$ denotes the additive white complex Gaussian noise matrix with zero-mean and unit-variance entries at the receiver k. Entries of $\mathbf{H}^{[kt]}$ are assumed to be i.i.d. complex Gaussian random variables. It is also assumed that channel is block fading (or constant), i.e., the channel state does not change during transmission of each code.

In the rest of this section, we will show that an interference cancellation scheme based on Alamouti codes can be applied to the K-user MIMO interference channel.

Theorem 3.1. For the K-user interference channel, when $N_T = 2M$ and $N_R = KM$, $2M$ DoF and two diversity order are achieved using the interference cancellation method for the multi-access scenario [43]–[45] without CSIT.

3.2.1. Proof of Theorem 3.1 for $K = 2$

We first consider two-user interference channel, i.e., $K = 2$, when $N_T = N_R = 2M$. Each transmitter sends $2M$ data symbols to its corresponding receiver in two channel uses. The transmitted $2M \times 2$ block codes are generated by using Alamouti codes as

$$\mathbf{X}^{[t]} = \begin{bmatrix} \mathbf{A}(x_1^{[t]}, x_2^{[t]}) \\ \mathbf{A}(x_3^{[t]}, x_4^{[t]}) \\ \vdots \\ \mathbf{A}(x_{2M-1}^{[t]}, x_{2M}^{[t]}) \end{bmatrix} \tag{3.2}$$

where $x_i^{[t]},\ i \in \{1, \cdots, 2M\},\ t \in \{1, 2\}$, is the i-th data symbol transmitted from the transmitter t. As shown in (3.2), each transmitter encodes $2M$ data symbols using stacked M Alamouti matrices. It is assumed that every transmitter has the average power constraint per channel use P and thus we set $\mathbb{E}\left[|x_i^{[t]}|^2\right] = P/2M$. Without loss of generality, we concentrate on the receiver 1, because the receiver 2 operates similarly and shows the same performance as the receiver 1 due to their symmetric structure. The received signal matrix at the receiver 1 is given as

$$\mathbf{Y}^{[1]} = \mathbf{H}^{[11]}\mathbf{X}^{[1]} + \mathbf{H}^{[12]}\mathbf{X}^{[2]} + \mathbf{N}^{[1]} \tag{3.3}$$

where $\mathbf{N}^{[1]}$ is the AWGN matrix with zero-mean and unit-variance entries. We assume that $\mathbf{H}^{[1t]}$ is a $2M \times 2M$ matrix whose entries are $h_{i,j}^{[1t]}$, and $\mathbf{Y}^{[1]}$ and $\mathbf{N}^{[1]}$ are $2M \times 2$ matrices whose entries are $y_{i,j}^{[1]}$ and $n_{i,j}^{[1]}$, respectively. Then, (3.3) can be vectorized as

$$\begin{bmatrix} \mathbf{y}_1^{[1]} \\ \mathbf{y}_2^{[1]} \\ \vdots \\ \mathbf{y}_{2M}^{[1]} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1,1}^{[11]} & \cdots & \mathbf{H}_{1,M}^{[11]} & \mathbf{H}_{1,1}^{[12]} & \cdots & \mathbf{H}_{1,M}^{[12]} \\ \mathbf{H}_{2,1}^{[11]} & \cdots & \mathbf{H}_{2,M}^{[11]} & \mathbf{H}_{2,1}^{[12]} & \cdots & \mathbf{H}_{2,M}^{[12]} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{2M,1}^{[11]} & \cdots & \mathbf{H}_{2M,M}^{[11]} & \mathbf{H}_{2M,1}^{[12]} & \cdots & \mathbf{H}_{2M,M}^{[12]} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1^{[1]} \\ \vdots \\ \mathbf{s}_M^{[1]} \\ \mathbf{i}_1^{[1]} \\ \vdots \\ \mathbf{i}_M^{[1]} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1^{[1]} \\ \mathbf{n}_2^{[1]} \\ \vdots \\ \mathbf{n}_{2M}^{[1]} \end{bmatrix} \tag{3.4}$$

where $\mathbf{y}_i^{[1]} = \begin{bmatrix} y_{i,1}^{[1]} & y_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}$, $\mathbf{H}_{i,j}^{[1t]} = \begin{bmatrix} h_{i,2j-1}^{[1t]} & h_{i,2j}^{[1t]} \\ h_{i,2j}^{[1t]*} & -h_{i,2j-1}^{[1t]*} \end{bmatrix}$, $\mathbf{n}_i^{[1]} = \begin{bmatrix} n_{i,1}^{[1]} & n_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}$, $i \in \{1, \cdots, 2M\}$, $j \in \{1, \cdots, M\}$, and $\mathbf{s}_m^{[1]}$ and $\mathbf{i}_m^{[1]}$, $m \in \{1, \cdots, M\}$, are the desired symbol vector $\begin{bmatrix} x_{2m-1}^{[1]} & x_{2m}^{[1]} \end{bmatrix}^{\mathrm{T}}$ and the interfering symbol vector $\begin{bmatrix} x_{2m-1}^{[2]} & x_{2m}^{[2]} \end{bmatrix}^{\mathrm{T}}$, respectively. This vectorization is extended from (10) in [46]. Note that $\mathbf{H}_{i,j}^{[1t]}$ is an Alamouti matrix.

Then the receiver successively removes the interfering symbol vectors $\mathbf{i}_m^{[1]}$ by using the interference cancellation method for the multi-access scenario [43]–[45], which utilizes the following well-known properties of the Alamouti structure.

Alamouti Property 1. The Alamouti matrices are closed under conjugate transpose, matrix addition, matrix multiplication, and real scalar multiplication.

Alamouti Property 2. For the Alamouti matrix $\mathbf{A}$, $\mathbf{A}\mathbf{A}^{\dagger} = \mathbf{A}^{\dagger}\mathbf{A} = \frac{1}{2}\|\mathbf{A}\|^2\mathbf{I}_2$.

Define $\mathbf{G}_i^{[1]}(l)$ as

$$\mathbf{G}_i^{[1]}(l) = \begin{bmatrix} \mathbf{I}_2 & \mathbf{F}_i^{[1]\dagger}(l) \end{bmatrix}^{\dagger} = \begin{cases} \begin{bmatrix} \mathbf{I}_2 & -\frac{2\mathbf{H}_{i,M-l+1}^{[12]}(l-1)\mathbf{H}_{2M-l+1,M-l+1}^{[12]\dagger}(l-1)}{\left\|\mathbf{H}_{2M-l+1,M-l+1}^{([12]}(l-1)\right\|^2} \end{bmatrix}^{\dagger} \\ \qquad\qquad \text{for } l = 0, \cdots, M \\ \begin{bmatrix} \mathbf{I}_2 & -\frac{2\mathbf{H}_{i,2M-l+1}^{[11]}(l-1)\mathbf{H}_{2M-l+1,2M-l+1}^{[11]\dagger}(l-1)}{\left\|\mathbf{H}_{2M-l+1,2M-l+1}^{[11]}(l-1)\right\|^2} \end{bmatrix}^{\dagger} \\ \qquad\qquad \text{for } l = M+1, \cdots, 2M-1 \end{cases} \tag{3.5}$$

for $i = 1, \cdots, 2M - l$. Note that the 4×2 matrix $\mathbf{G}_i^{[1]}(l)$ is composed of two Alamouti matrices $\mathbf{I}_2$ and $\mathbf{F}_i^{[1]}(l)$ (due to the Alamouti Property 1). Then, the interfering symbol vector $\mathbf{i}_{M-l+1}^{[1]}$ is removed successively by using $\mathbf{G}_i^{[1]}(l)$ starting from $l = 1$ to $l = M$ as

$$\begin{bmatrix} \mathbf{y}_1^{[1]}(l) \\ \mathbf{y}_2^{[1]}(l) \\ \vdots \\ \mathbf{y}_{2M-l}^{[1]}(l) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1,1}^{[11]}(l) & \cdots & \mathbf{H}_{1,M}^{[11]}(l) & \mathbf{H}_{1,1}^{[12]}(l) & \cdots & \mathbf{H}_{1,M-l}^{[12]}(l) \\ \mathbf{H}_{2,1}^{[11]}(l) & \cdots & \mathbf{H}_{2,M}^{[11]}(l) & \mathbf{H}_{2,1}^{[12]}(l) & \cdots & \mathbf{H}_{2,M-l}^{[12]}(l) \\ & & & \vdots & & \\ \mathbf{H}_{2M-l,1}^{[11]}(l) & \cdots & \mathbf{H}_{2M-l,M}^{[11]}(l) & \mathbf{H}_{2M-l,1}^{[12]}(l) & \cdots & \mathbf{H}_{2M-l,M-l}^{[12]}(l) \end{bmatrix}$$
$$\times \begin{bmatrix} \mathbf{s}_1^{[1]\mathrm{T}} & \cdots & \mathbf{s}_M^{[1]\mathrm{T}} & \mathbf{i}_1^{[1]\mathrm{T}} & \cdots & \mathbf{i}_{M-l}^{[1]\mathrm{T}} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \mathbf{n}_1^{[1]}(l)^{\mathrm{T}} & \cdots & \mathbf{n}_{2M-l}^{[1]}(l)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{3.6}$$

where

$$\mathbf{y}_i^{[1]}(l) = \mathbf{G}_i^{[1]\dagger}(l) \begin{bmatrix} \mathbf{y}_i^{[1]}(l-1) \\ \mathbf{y}_{2M-l+1}^{[1]}(l-1) \end{bmatrix}$$
$$\mathbf{H}_{i,j}^{[1t]}(l) = \mathbf{G}_i^{[1]\dagger}(l) \begin{bmatrix} \mathbf{H}_{i,j}^{[1t]}(l-1) \\ \mathbf{H}_{2M-l+1,j}^{[1t]}(l-1) \end{bmatrix}$$
$$\mathbf{n}_i^{[1]}(l) = \mathbf{G}_i^{[1]\dagger}(l) \begin{bmatrix} \mathbf{n}_i^{[1]}(l-1) \\ \mathbf{n}_{2M-l+1}^{[1]}(l-1) \end{bmatrix}$$

and the index "(l)" in them indicates that they are the results after the l-th interference cancellation. Note that $\mathbf{H}_{i,j}^{[1t]}(l)$'s are still Alamouti matrices due to the Alamouti Property 1 and the last two rows and two columns of the equivalent channel matrix in (3.4) are removed after each successive cancellation. After the Mth interference cancellation, all the interfering symbols are removed and the received symbol equation is obtained as

$$\begin{bmatrix} \mathbf{y}_1^{[1]}(M) \\ \mathbf{y}_2^{[1]}(M) \\ \vdots \\ \mathbf{y}_M^{[1]}(M) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1,1}^{[11]}(M) & \mathbf{H}_{1,2}^{[11]}(M) & \cdots & \mathbf{H}_{1,M}^{[11]}(M) \\ \mathbf{H}_{2,1}^{[11]}(M) & \mathbf{H}_{2,2}^{[11]}(M) & \cdots & \mathbf{H}_{2,M}^{[11]}(M) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{M,1}^{[11]}(M) & \mathbf{H}_{M,2}^{[11]}(M) & \cdots & \mathbf{H}_{M,M}^{[11]}(M) \end{bmatrix} \begin{bmatrix} \mathbf{s}_1^{[1]} \\ \mathbf{s}_2^{[1]} \\ \vdots \\ \mathbf{s}_M^{[1]} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1^{[1]}(M) \\ \mathbf{n}_2^{[1]}(M) \\ \vdots \\ \mathbf{n}_M^{[1]}(M) \end{bmatrix}. \tag{3.7}$$

In order to perform symbol-by-symbol decoding of the desired symbols, successive cancellation of $\mathbf{s}_{2M-l+1}^{[1]}$ should be performed starting from $l = M+1$ to $l = 2M-1$ similar to the above interference cancellation. That is, by using $\mathbf{G}_i^{[1]}(l)$ for $l = M+1, \cdots, 2M-1$ in (3.5), we have

$$\begin{bmatrix} \mathbf{y}_1^{[1]}(l) \\ \mathbf{y}_2^{[1]}(l) \\ \vdots \\ \mathbf{y}_{2M-l}^{[1]}(l) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1,1}^{[11]}(l) & \mathbf{H}_{1,2}^{[11]}(l) & \cdots & \mathbf{H}_{1,2M-l}^{[11]}(l) \\ \mathbf{H}_{2,1}^{[11]}(l) & \mathbf{H}_{2,2}^{[11]}(l) & \cdots & \mathbf{H}_{2,2M-l}^{[11]}(l) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{2M-l,1}^{[11]}(l) & \mathbf{H}_{2M-l,2}^{[11]}(l) & \cdots & \mathbf{H}_{2M-l,2M-l}^{[11]}(l) \end{bmatrix} \begin{bmatrix} \mathbf{s}_1^{[1]} \\ \mathbf{s}_2^{[1]} \\ \vdots \\ \mathbf{s}_{2M-l}^{[1]} \end{bmatrix}$$
$$+ \begin{bmatrix} \mathbf{n}_1^{[1]}(l)^{\mathrm{T}} & \mathbf{n}_2^{[1]}(l)^{\mathrm{T}} & \cdots & \mathbf{n}_{2M-l}^{[1]}(l)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{3.8}$$

where

$$\mathbf{y}_i^{[1]}(l) = \mathbf{G}_i^{[1]\dagger}(l) \begin{bmatrix} \mathbf{y}_i^{[1]}(l-1) \\ \mathbf{y}_{2M-l+1}^{[1]}(l-1) \end{bmatrix}$$
$$\mathbf{H}_{i,j}^{[11]}(l) = \mathbf{G}_i^{[1]\dagger}(l) \begin{bmatrix} \mathbf{H}_{i,j}^{[11]}(l-1) \\ \mathbf{H}_{2M-l+1,j}^{[11]}(l-1) \end{bmatrix}$$

$$\mathbf{n}_i^{[1]}(l) = \mathbf{G}_i^{[1]\dagger}(l) \begin{bmatrix} \mathbf{n}_i^{[1]}(l-1) \\ \mathbf{n}_{2M-l+1}^{[1]}(l-1) \end{bmatrix}.$$

After cancelling out $\mathbf{s}_M^{[1]}, \mathbf{s}_{M-1}^{[1]}, \cdots, \mathbf{s}_2^{[1]}$ from (3.7), the receiver 1 can extract two desired symbols $\mathbf{s}_1^{[1]} = \begin{bmatrix} x_1^{[1]} & x_2^{[1]} \end{bmatrix}^{\mathrm{T}}$ from

$$\mathbf{y}_1^{[1]}(2M-1) = \mathbf{H}_{1,1}^{[11]}(2M-1)\mathbf{s}_1^{[1]} + \mathbf{n}_1^{[1]}(2M-1) \tag{3.9}$$

where $\mathbf{H}_{1,1}^{[11]}(2M-1)$ still keeps the Alamouti structure due to the Alamouti Property 1. Therefore, symbol-by-symbol decoding is performed to estimate the desired symbols $x_1^{[1]}$ and $x_2^{[1]}$ and the other desired symbols can be decoded from (3.7) in a similar successive way. Note that the Alamouti Property 2 makes each cancellation procedure possible and the Alamouti Property 1 preserves the Alamouti structures of the equivalent channels. Due to the symmetry, it is easy to show that the same method can be applied to the receiver 2 to decode its $2M$ desired symbols. Consequently, the total DoF is $4M$ and the IC scheme achieves $2M$ DoF per channel use which is the maximum DoF for the two-user interference channel with $2M$ antennas. Moreover, the proposed IC scheme achieves diversity order of two, i.e, its PEP decays proportional to $1/P^2$, which will be shown in Section 3.3.

3.2.2. Proof of Theorem 3.1 for $K \geq 3$

Consider the $K(\geq 3)$-user interference channel, where each user has $2M$ antennas. Similar to (3.4), we have the equivalent system of the linear equations at the receiver 1 as

$$\begin{bmatrix} \mathbf{y}_1^{[1]} \\ \mathbf{y}_2^{[1]} \\ \vdots \\ \mathbf{y}_{2M}^{[1]} \end{bmatrix} =$$

$$\underbrace{\begin{bmatrix} \mathbf{H}_{1,1}^{[11]} & \cdots & \mathbf{H}_{1,M}^{[11]} & \mathbf{H}_{1,1}^{[12]} & \cdots & \mathbf{H}_{1,M}^{[12]} & \cdots\cdots & \mathbf{H}_{1,1}^{[1K]} & \cdots & \mathbf{H}_{1,M}^{[1K]} \\ \mathbf{H}_{2,1}^{[11]} & \cdots & \mathbf{H}_{2,M}^{[11]} & \mathbf{H}_{2,1}^{[12]} & \cdots & \mathbf{H}_{2,M}^{[12]} & \cdots\cdots & \mathbf{H}_{2,1}^{[1K]} & \cdots & \mathbf{H}_{2,M}^{[1K]} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{H}_{2M,1}^{[11]} & \cdots & \mathbf{H}_{2M,M}^{[11]} & \mathbf{H}_{2M,1}^{[12]} & \cdots & \mathbf{H}_{2M,M}^{[12]} & \cdots\cdots & \mathbf{H}_{2M,1}^{[1K]} & \cdots & \mathbf{H}_{2M,M}^{[1K]} \end{bmatrix}}_{\mathbf{H_{eff}}}$$

$$\times \underbrace{\begin{bmatrix} \mathbf{s}_1^{[1]\mathrm{T}} & \cdots & \mathbf{s}_M^{[1]\mathrm{T}} & \mathbf{i}_1^{[1]\mathrm{T}} & \cdots & \mathbf{i}_M^{[1]\mathrm{T}} & \cdots\cdots & \mathbf{i}_{M(K-2)+1}^{[1]\mathrm{T}} & \cdots & \mathbf{i}_{M(K-1)}^{[1]\mathrm{T}} \end{bmatrix}^{\mathrm{T}}}_{\mathbf{S_{eff}}}$$

$$+\begin{bmatrix} \mathbf{n}_1^{[1]\mathrm{T}} & \mathbf{n}_2^{[1]\mathrm{T}} & \cdots & \mathbf{n}_{2M}^{[1]\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{3.10}$$

where $\mathbf{y}_i^{[1]} = \begin{bmatrix} y_{i,1}^{[1]} & y_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}$, $\mathbf{H}_{i,j}^{[1t]} = \begin{bmatrix} h_{i,2j-1}^{[1t]} & h_{i,2j}^{[1t]} \\ h_{i,2j}^{[1t]*} & -h_{i,2j-1}^{[1t]*} \end{bmatrix}$, $\mathbf{n}_i^{[1]} = \begin{bmatrix} n_{i,1}^{[1]} & n_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}$, and $\mathbf{s}_m^{[1]}$ and $\mathbf{i}_m^{[1]}$ are the desired symbol vector and the interfering symbol vector, respectively. After $2M-1$ cancellations similar to the two-user case, we have (3.11) at the top in the next page, where the index "$(2M-1)$" is omitted for tractability. Equation (3.11) shows that the receiver 1 cannot cancel the interference symbols anymore. Therefore, the desired symbols cannot be obtained by symbol-by-symbol decoding. In order to apply symbol-by-symbol decoding to the IC scheme based on Alamouti code for the K-user interference channel, the number of rows

$$\mathbf{y}_1^{[1]}=\begin{bmatrix}\mathbf{H}_{11}^{[11]} & \cdots & \mathbf{H}_{1M}^{[11]} & \mathbf{H}_{11}^{[12]} & \cdots & \mathbf{H}_{1M}^{[12]} & \cdots\cdots & \mathbf{H}_{11}^{[1(K-2)]} & \cdots & \mathbf{H}_{1M}^{[1(K-2)]} & \mathbf{H}_{11}^{[1(K-1)]}\end{bmatrix}$$
$$\times\begin{bmatrix}\mathbf{s}_1^{[1]\mathrm{T}} & \cdots & \mathbf{s}_M^{[1]\mathrm{T}} & \mathbf{i}_1^{[1]\mathrm{T}} & \cdots & \mathbf{i}_M^{[1]\mathrm{T}} & \cdots\cdots & \mathbf{i}_{M(K-4)+1}^{[1]\mathrm{T}} & \cdots & \mathbf{i}_{M(K-3)}^{[1]\mathrm{T}} & \mathbf{i}_{M(K-3)+1}^{[1]\mathrm{T}}\end{bmatrix}^{\mathrm{T}}+\mathbf{n}_1^{[1]} \quad (3.11)$$

---

of $\mathbf{H_{eff}}$ in (3.10) should be the same as the number of elements of $\mathbf{S_{eff}}$ in (3.10). Consequently, when each transmitter and each receiver have $2M$ and KM antennas, respectively, the IC scheme utilizing symbol-by-symbol decoding can be applied to the K-user interference channel and KM DoF is achieved. In Section 3.3, it will be shown that the proposed IC scheme achieves diversity order of two.

3.3. Interference Alignment-and-Cancellation Scheme for the Three-User MIMO Interference Channel

In this section, we consider the three-user interference channel where each transmitter has $2M$ antennas to utilize Alamouti codes. In the study of interference alignment, the three-user MIMO interference channel is interesting because $3M$ DoF is achieved with constant channel matrices and finite number of dimensions (number of antennas). The result in Section 3.2 implies that if each receiver has $3M$ antennas, the IC scheme utilizing symbol-by-symbol decoding can be applied to the three-user interference channel and $3M$ DoF is achieved. However, it is well known that IA scheme can achieve the same DoF of $3M$ when each transmitter and each receiver have $2M$ antennas. Even though the IC scheme achieves diversity order of two and requires no CSIT, it requires more receive antennas than the IA scheme. In this section, an IAC scheme based on Alamouti code for the three-user interference channel is proposed as a part of efforts to reduce the number of antennas at the receivers while keeping the advantages of IC scheme.

3.3.1. Transmission and Reception Schemes

In order to reduce the number of antennas at the receivers, the IAC scheme utilizes beamforming matrices. The transmitted $2M \times 2$ block code at each transmitter is designed as

$$\mathbf{X}^{[t]} = \mathbf{P}^{[t]} \underbrace{\begin{bmatrix} \mathbf{A}(x_1^{[t]}, x_2^{[t]}) \\ \mathbf{A}(x_3^{[t]}, x_4^{[t]}) \\ \vdots \\ \mathbf{A}(x_{2M-1}^{[t]}, x_{2M}^{[t]}) \end{bmatrix}}_{\mathbf{A}_t} \tag{3.12}$$

where $x_i^{[t]}$ is the i-th data symbol transmitted at the transmitter t and $\mathbf{P}^{[t]}$ is the beamforming matrix of the transmitter t. As shown in (3.12), each transmitter encodes data symbols using stacked M Alamouti codes followed by $\mathbf{P}^{[t]}$. Then, the received signal matrix at the receiver k, $k \in \{1, 2, 3\}$, is given as

$$\mathbf{Y}^{[k]} = \mathbf{H}^{[k1]}\mathbf{P}^{[1]}\mathbf{A}_1 + \mathbf{H}^{[k2]}\mathbf{P}^{[2]}\mathbf{A}_2 + \mathbf{H}^{[k3]}\mathbf{P}^{[3]}\mathbf{A}_3 + \mathbf{N}^{[k]}. \tag{3.13}$$

In (3.13), the term $\mathbf{H}^{[kk]}\mathbf{P}^{[k]}\mathbf{A}_k$ is the desired part and the other two terms are interfering parts at the receiver k. If two interfering parts can be aligned without breaking the Alamouti structure, we can regard this situation as that the receiver receives two Alamouti codes from two transmitters. Then, similar to the IC scheme in Section 3.2, the receiver can cancel interference and perform symbol-by-symbol decoding with less antennas.

Suppose that the beamforming matrices $\mathbf{P}^{[t]}$ are designed to satisfy the following conditions

$$\mathbf{H}^{[12]}\mathbf{P}^{[2]} = \mathbf{H}^{[13]}\mathbf{P}^{[3]} \tag{3.14}$$

$$\mathbf{H}^{[21]}\mathbf{P}^{[1]} = \mathbf{H}^{[23]}\mathbf{P}^{[3]} \tag{3.15}$$

$$\mathbf{H}^{[31]}\mathbf{P}^{[1]} = \mathbf{H}^{[32]}\mathbf{P}^{[2]}. \tag{3.16}$$

Then (3.13) can be rewritten as

$$\mathbf{Y}^{[k]} = \mathbf{H}^{[kk]}\mathbf{P}^{[k]}\mathbf{A}_k + \mathbf{H}^{[kt]}\mathbf{P}^{[t]}\sum_{t\neq k}\mathbf{A}_t + \mathbf{N}^{[k]}. \tag{3.17}$$

Since $\sum_{t\neq k}\mathbf{A}_t$ has the Alamouti structure due to the Alamouti Property 1, (3.17) has the same structure as (3.3). Then the receiver can decode by using the same method in the previous section with $2M$ antennas. However, the conditions (3.14), (3.15), and (3.16) cannot have non-trivial exact solutions $\mathbf{P}^{[t]}$ simultaneously. Therefore, we propose an IAC scheme which uses the beamforming matrices satisfying just two of the three conditions. We choose the identity matrix $\mathbf{I}_{2M}$ for $\mathbf{P}^{[1]}$, and $\mathbf{P}^{[2]}$ and $\mathbf{P}^{[3]}$ are determined to satisfy (3.15) and (3.16). By using these beamforming matrices, the receivers 2 and 3 satisfy (3.17) and thus the number of antennas at each of them is decreased from $3M$ to $2M$. Note that the channel state information at the receiver 1 is not required to the transmitters to design the above beamforming matrices. It can be interpreted that, in the proposed IAC scheme, the receivers with less antennas must feedback their channel state information to all transmitters.

Now, the decoding scheme at each receiver will be proposed. First, for the receiver 1, the IC decoding scheme as shown in Section 3.2 is used. The received signal matrix at the receiver 1 is given as

$$\mathbf{Y}^{[1]} = \mathbf{H}^{[11]}\mathbf{P}^{[1]}\mathbf{A}_1 + \mathbf{H}^{[12]}\mathbf{P}^{[2]}\mathbf{A}_2 + \mathbf{H}^{[13]}\mathbf{P}^{[3]}\mathbf{A}_3 + \mathbf{N}^{[1]}. \tag{3.18}$$

Note that the receiver 1 has $3M$ antennas. We assume that $\mathbf{H}^{[1t]}\mathbf{P}^{[t]}$ is a $3M \times 2M$ matrix whose entries are $h_{i,j}^{[1t]}$, and $\mathbf{Y}^{[1]}$ and $\mathbf{N}^{[1]}$ are $3M \times 2$ matrices whose entries are $y_{i,j}^{[1]}$ and $n_{i,j}^{[1]}$, respectively. Then the equivalent system of the linear equations in (3.18) can be vectorized as

$$\begin{bmatrix} \mathbf{y}_1^{[1]} \\ \mathbf{y}_2^{[1]} \\ \vdots \\ \mathbf{y}_{3M}^{[1]} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_{1,1}^{[11]} & \cdots & \mathbf{H}_{1,M}^{[11]} & \mathbf{H}_{1,1}^{[12]} & \cdots & \mathbf{H}_{1,M}^{[12]} & \mathbf{H}_{1,1}^{[13]} & \cdots & \mathbf{H}_{1,M}^{[13]} \\ \mathbf{H}_{2,1}^{[11]} & \cdots & \mathbf{H}_{2,M}^{[11]} & \mathbf{H}_{2,1}^{[12]} & \cdots & \mathbf{H}_{2,M}^{[12]} & \mathbf{H}_{2,1}^{[13]} & \cdots & \mathbf{H}_{2,M}^{[13]} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & & & \\ \mathbf{H}_{3M,1}^{[11]} & \cdots & \mathbf{H}_{3M,M}^{[11]} & \mathbf{H}_{3M,1}^{[12]} & \cdots & \mathbf{H}_{3M,M}^{[12]} & \mathbf{H}_{3M,1}^{[13]} & \cdots & \mathbf{H}_{3M,M}^{[13]} \end{bmatrix}}_{\mathbf{H_{eff}}}$$

$$\times \underbrace{\begin{bmatrix} \mathbf{s}_1^{[1]\mathrm{T}} & \cdots & \mathbf{s}_M^{[1]\mathrm{T}} & \mathbf{i}_1^{[1]\mathrm{T}} & \cdots & \mathbf{i}_{2M}^{[1]\mathrm{T}} \end{bmatrix}^{\mathrm{T}}}_{\mathbf{S_{eff}}} + \begin{bmatrix} \mathbf{n}_1{}^{\mathrm{T}} & \mathbf{n}_2{}^{\mathrm{T}} & \cdots & \mathbf{n}_{3M}{}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{3.19}$$

where $\mathbf{y}_i^{[1]} = \begin{bmatrix} y_{i,1}^{[1]} & y_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}$, $\mathbf{H}_{i,j}^{[1t]} = \begin{bmatrix} h_{i,2j-1}^{[1t]} & h_{i,2j}^{[1t]} \\ h_{i,2j}^{[1t]*} & -h_{i,2j-1}^{[1t]*} \end{bmatrix}$, and $\mathbf{n}_i^{[1]} = \begin{bmatrix} n_{i,1}^{[1]} & n_{i,2}^{[1]*} \end{bmatrix}^{\mathrm{T}}$, $i \in \{1, \cdots, 3M\}, j \in \{1, \cdots, M\}, t \in \{1, 2, 3\}$. The vectors $\mathbf{s}_m$ and $\mathbf{i}_m$ are the desired symbol vector and interfering symbol vector given as

$$\mathbf{s}_m^{[1]} = \begin{bmatrix} x_{2m-1}^{[1]} & x_{2m}^{[1]} \end{bmatrix}^{\mathrm{T}}, \qquad m \in \{1, \cdots, M\}$$

$$\mathbf{i}_m^{[1]} = \begin{cases} \begin{bmatrix} x_{2m-1}^{[2]} & x_{2m}^{[2]} \end{bmatrix}^{\mathrm{T}}, & m \in \{1, \cdots, M\} \\ \begin{bmatrix} x_{2(m-M)-1}^{[3]} & x_{2(m-M)}^{[3]} \end{bmatrix}^{\mathrm{T}}, & m \in \{M+1, \cdots, 2M\}. \end{cases}$$

Since the number of rows of $\mathbf{H}_{\mathbf{eff}}$ in (3.19) is the same as the number of elements of $\mathbf{S}_{\mathbf{eff}}$ in (3.19), the receiver 1 can cancel all interfering symbols and symbol-by-symbol decoding is possible.

Each of the receivers 2 and 3 receives the signal in the form of (3.17). Therefore, by using $2M$ antennas, they can separate the desired symbols from the interfering symbols to perform symbol-by-symbol decoding, which is similar to the IC scheme for the two-user interference channel in the previous section. Consequently, the total DoF is $6M$ and the DoF per channel use is $3M$ when one receiver has $3M$ antennas and each of the other two receivers has $2M$ antennas.

3.3.2. Diversity Analysis

In this subsection, the diversity order of the proposed IAC scheme is derived by using PEP. We show the achievable diversity order of the proposed IAC scheme in the following theorem.

Theorem 3.2. For the three-user interference channel, when one receiver has $3M$ antennas and each of the other two receivers has $2M$ antennas, the proposed IAC scheme achieves a diversity order of two for each symbol.

3.3.2.1. Proof of Theorem 3.2 for Receiver 1 when $M = 1$

First, consider the case of $M = 1$, i.e., the three-user interference channel with three transmitters and three receivers, where each is equipped with two antennas except that the receiver 1 has three antennas. At the receiver 1 with three antennas, (3.19) can be rewritten as

$$\begin{bmatrix} \mathbf{y}_1^{[1]} \\ \mathbf{y}_2^{[1]} \\ \mathbf{y}_3^{[1]} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1,1}^{[11]} & \mathbf{H}_{1,1}^{[12]} & \mathbf{H}_{1,1}^{[13]} \\ \mathbf{H}_{2,1}^{[11]} & \mathbf{H}_{2,1}^{[12]} & \mathbf{H}_{2,1}^{[13]} \\ \mathbf{H}_{3,1}^{[11]} & \mathbf{H}_{3,1}^{[12]} & \mathbf{H}_{3,1}^{[13]} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1^{[1]} \\ \mathbf{i}_1^{[1]} \\ \mathbf{i}_2^{[1]} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1^{[1]} \\ \mathbf{n}_2^{[1]} \\ \mathbf{n}_3^{[1]} \end{bmatrix}. \tag{3.20}$$

Note that each entry of $\mathbf{H}_{ij}^{[1t]}$ in (3.20) might not be a complex Gaussian random variable, since it is the result of multiplication of Gaussian channel matrix with the beamforming matrix. However, authors in [28] derived that approximating each entry of $\mathbf{H}_{ij}^{[1t]}$ as a complex Gaussian random variable does not change the diversity order. Therefore, we approximate $\mathbf{H}_{ij}^{[1t]}$ as a complex Gaussian matrix to derive the diversity order of the proposed IAC scheme.

Using $\mathbf{G}_i^{[1]}(1) = \begin{bmatrix} \mathbf{I}_2 & \mathbf{F}_i^{[1]\dagger}(1) \end{bmatrix}^\dagger = \begin{bmatrix} \mathbf{I}_2 & -\frac{2\mathbf{H}_{i,1}^{[13]}\mathbf{H}_{3,1}^{[13]\dagger}}{\|\mathbf{H}_{3,1}^{[13]}\|^2} \end{bmatrix}^\dagger$, $i \in \{1, 2\}$, and the Alamouti Property 2, the receiver 1 removes the interfering symbol

$$\mathbf{G}_1^{[1]\dagger}(2)\begin{bmatrix}\mathbf{y}_1^{[1]}(1)\\ \mathbf{y}_2^{[1]}(1)\end{bmatrix} = \mathbf{G}_1^{[1]\dagger}(2)\begin{bmatrix}\mathbf{H}_{1,1}^{[11]}(1)\\ \mathbf{H}_{2,1}^{[11]}(1)\end{bmatrix}\mathbf{s}_1^{[1]} + \mathbf{G}_1^{[1]\dagger}(2)\begin{bmatrix}\mathbf{n}_1^{[1]}(1)\\ \mathbf{n}_2^{[1]}(1)\end{bmatrix}$$

$$\overset{(a)}{\Leftrightarrow} \begin{bmatrix}\mathbf{G}_1^{[1]}(1)\\ \mathbf{G}_2^{[1]}(1)\mathbf{F}_1^{[1]}(2)\end{bmatrix}^{\dagger}\begin{bmatrix}\mathbf{y}_1^{[1]}\\ \mathbf{y}_3^{[1]}\\ \mathbf{y}_2^{[1]}\\ \mathbf{y}_3^{[1]}\end{bmatrix} = \begin{bmatrix}\mathbf{G}_1^{[1]}(1)\\ \mathbf{G}_2^{[1]}(1)\mathbf{F}_1^{[1]}(2)\end{bmatrix}^{\dagger}\begin{bmatrix}\mathbf{H}_{1,1}^{[11]}\\ \mathbf{H}_{3,1}^{[11]}\\ \mathbf{H}_{2,1}^{[11]}\\ \mathbf{H}_{3,1}^{[11]}\end{bmatrix}\mathbf{s}_1^{[1]} + \begin{bmatrix}\mathbf{G}_1^{[1]}(1)\\ \mathbf{G}_2^{[1]}(1)\mathbf{F}_1^{[1]}(2)\end{bmatrix}^{\dagger}\begin{bmatrix}\mathbf{n}_1^{[1]}\\ \mathbf{n}_3^{[1]}\\ \mathbf{n}_2^{[1]}\\ \mathbf{n}_3^{[1]}\end{bmatrix} \tag{3.24}$$

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vector $\mathbf{i}_2$ as

$$\begin{bmatrix}\mathbf{y}_1^{[1]}(1)\\ \mathbf{y}_2^{[1]}(1)\end{bmatrix} = \begin{bmatrix}\mathbf{H}_{1,1}^{[11]}(1) & \mathbf{H}_{1,1}^{[12]}(1)\\ \mathbf{H}_{2,1}^{[11]}(1) & \mathbf{H}_{2,1}^{[12]}(1)\end{bmatrix}\begin{bmatrix}\mathbf{s}_1^{[1]}\\ \mathbf{i}_1^{[1]}\end{bmatrix} + \begin{bmatrix}\mathbf{n}_1^{[1]}(1)\\ \mathbf{n}_2^{[1]}(1)\end{bmatrix} \tag{3.21}$$

where

$$\mathbf{y}_i^{[1]}(1) = \mathbf{G}_i^{[1]\dagger}(1)\begin{bmatrix}\mathbf{y}_i^{[1]}\\ \mathbf{y}_3^{[1]}\end{bmatrix}$$

$$\mathbf{H}_{i,1}^{[1t]}(1) = \mathbf{G}_i^{[1]\dagger}(1)\begin{bmatrix}\mathbf{H}_{i,1}^{[1t]}\\ \mathbf{H}_{3,1}^{[1t]}\end{bmatrix}$$

$$\mathbf{n}_i^{[1]}(1) = \mathbf{G}_i^{[1]\dagger}(1)\begin{bmatrix}\mathbf{n}_i^{[1]}\\ \mathbf{n}_3^{[1]}\end{bmatrix}. \tag{3.22}$$

Let $\mathbf{G}_1^{[1]}(2) = \begin{bmatrix}\mathbf{I}_2 & \mathbf{F}_1^{[1]\dagger}(2)\end{bmatrix}^{\dagger} = \begin{bmatrix}\mathbf{I}_2 & -\frac{2\mathbf{H}_{1,1}^{[12]}(1)\mathbf{H}_{2,1}^{[12]\dagger}(1)}{\|\mathbf{H}_{2,1}^{[12]}(1)\|^2}\end{bmatrix}^{\dagger}$. Then, the second interference cancellation is performed to obtain

$$\mathbf{y}_1^{[1]}(2) = \mathbf{H}_{1,1}^{[11]}(2)\mathbf{s}_1^{[1]} + \mathbf{n}_1^{[1]}(2), \tag{3.23}$$

which is equivalent to (3.24) at the top in this page, where (a) uses the definition of $\mathbf{G}_1^{[1]}(2)$ and (3.22). Equation (3.24) shows that all the interfering signals are removed after two successive cancellations and $\mathbf{H}_{1,1}^{[11]}(2)$ in (3.23) still keeps the Alamouti structure due to the Alamouti Property 1. From the definition of $\mathbf{G}_i^{[1]}(l)$, (3.24) can be rearranged as

$$\hat{\mathbf{G}}^{[1]\dagger}\begin{bmatrix}\mathbf{y}_1^{[1]}\\ \mathbf{y}_2^{[1]}\\ \mathbf{y}_3^{[1]}\end{bmatrix} = \underbrace{\hat{\mathbf{G}}^{[1]\dagger}\begin{bmatrix}\mathbf{H}_{1,1}^{[11]}\\ \mathbf{H}_{2,1}^{[11]}\\ \mathbf{H}_{3,1}^{[11]}\end{bmatrix}}_{\mathbf{H}_{\mathbf{eff}}^{[1]}}\mathbf{s}_1^{[1]} + \hat{\mathbf{G}}^{[1]\dagger}\begin{bmatrix}\mathbf{n}_1^{[1]}\\ \mathbf{n}_2^{[1]}\\ \mathbf{n}_3^{[1]}\end{bmatrix}$$

$$= \underbrace{\hat{\mathbf{G}}^{[1]\dagger}\begin{bmatrix}\mathbf{h}_1^{[11]} & \mathbf{h}_2^{[11]}\end{bmatrix}}_{\mathbf{H}_{\mathbf{eff}}^{[1]}}\mathbf{s}_1^{[1]} + \hat{\mathbf{G}}^{[1]\dagger}\begin{bmatrix}\mathbf{n}_1^{[1]}\\ \mathbf{n}_2^{[1]}\\ \mathbf{n}_3^{[1]}\end{bmatrix} \tag{3.25}$$

where

$$\hat{\mathbf{G}}^{[1]} = \begin{bmatrix}\mathbf{I}_2\\ \mathbf{F}_1^{[1]}(2)\\ \mathbf{F}_1^{[1]}(1)+\mathbf{F}_2^{[1]}(1)\mathbf{F}_1^{[1]}(2)\end{bmatrix} \tag{3.26}$$

and $\mathbf{h}_i^{[11]}$ is defined as the i-th column vector of $\begin{bmatrix}\mathbf{H}_{1,1}^{[11]\dagger} & \mathbf{H}_{2,1}^{[11]\dagger} & \mathbf{H}_{3,1}^{[11]\dagger}\end{bmatrix}^\dagger$. The Alamouti Property 1 shows that $\hat{\mathbf{G}}^{[1]}$ is a 6×2 matrix stacked with three Alamouti matrices and the equivalent channel $\mathbf{H}_{\mathbf{eff}}^{[1]}$ in (3.25) is also an Alamouti matrix. Therefore, (3.25) can be divided into two equations for two symbols of $\mathbf{s}_1^{[1]} = \begin{bmatrix}x_1^{[1]} & x_2^{[1]}\end{bmatrix}$ by multiplying the conjugate transpose of the equivalent channel matrix.

The receiver 1 can decode its desired symbols $x_1^{[1]}$ and $x_2^{[1]}$ by using the symbol-by-symbol decoding as

$$\mathbf{H}_{\mathbf{eff}}^{[\mathbf{1}]}{}^{\dagger}\hat{\mathbf{G}}^{[1]\dagger}\begin{bmatrix}\mathbf{y}_1^{[1]}\\ \mathbf{y}_2^{[1]}\\ \mathbf{y}_3^{[1]}\end{bmatrix}=\mathbf{H}_{\mathbf{eff}}^{[\mathbf{1}]}{}^{\dagger}\mathbf{H}_{\mathbf{eff}}^{[\mathbf{1}]}\mathbf{s}_1^{[1]}+\mathbf{H}_{\mathbf{eff}}^{[\mathbf{1}]}{}^{\dagger}\hat{\mathbf{G}}^{[1]\dagger}\begin{bmatrix}\mathbf{n}_1^{[1]}\\ \mathbf{n}_2^{[1]}\\ \mathbf{n}_3^{[1]}\end{bmatrix}. \tag{3.27}$$

Due to the symmetry, we focus only on the desired symbol $x_1^{[1]}$. Since $\mathbf{H}_{\mathbf{eff}}^{[\mathbf{1}]}{}^{\dagger}\mathbf{H}_{\mathbf{eff}}^{[\mathbf{1}]}$ is the scaled identity matrix from the Alamouti Property 2, we can extract an equation in only $x_1^{[1]}$ from (3.27) as

$$\tilde{r}_1^{[1]}=\mathbf{h}_1^{[11]\dagger}\hat{\mathbf{G}}^{[1]}\hat{\mathbf{G}}^{[1]\dagger}\mathbf{h}_1^{[11]}x_1^{[1]}+\mathbf{h}_1^{[11]\dagger}\hat{\mathbf{G}}^{[1]}\hat{\mathbf{G}}^{[1]\dagger}\begin{bmatrix}\mathbf{n}_1^{[1]}\\ \mathbf{n}_2^{[1]}\\ \mathbf{n}_3^{[1]}\end{bmatrix} \tag{3.28}$$

where $\tilde{r}_1^{[1]}$ denotes the first element of $\mathbf{H}_{\mathbf{eff}}^{[\mathbf{1}]}{}^{\dagger}\hat{\mathbf{G}}^{[1]\dagger}\begin{bmatrix}\mathbf{y}_1^{[1]\dagger} & \mathbf{y}_2^{[1]\dagger} & \mathbf{y}_3^{[1]\dagger}\end{bmatrix}^{\dagger}$ in (3.27).

Let $\mathbb{E}\left[\left|x_1^{[1]}\right|^2\right]=P_1^{[1]}$ and the covariance matrix of $\begin{bmatrix}\mathbf{n}_1^{[1]\dagger} & \mathbf{n}_2^{[1]\dagger} & \mathbf{n}_3^{[1]\dagger}\end{bmatrix}^{\dagger}$ is the identity matrix. Then, the normalized instantaneous receive SNR is expressed as

$$\begin{aligned}\mathrm{SNR}^{[1]}&=P_1^{[1]}\frac{(\mathbf{h}_1^{[11]\dagger}\hat{\mathbf{G}}^{[1]}\hat{\mathbf{G}}^{[1]\dagger}\mathbf{h}_1^{[11]})^2}{\mathbf{h}_1^{[11]\dagger}\hat{\mathbf{G}}^{[1]}\hat{\mathbf{G}}^{[1]\dagger}\hat{\mathbf{G}}^{[1]}\hat{\mathbf{G}}^{[1]\dagger}\mathbf{h}_1^{[11]}}\\ &\overset{(b)}{=}P_1^{[1]}\left(\frac{\|\hat{\mathbf{G}}^{[1]}\|^2}{2}\right)^{-1}\mathbf{h}_1^{[11]\dagger}\hat{\mathbf{G}}^{[1]}\hat{\mathbf{G}}^{[1]\dagger}\mathbf{h}_1^{[11]}\end{aligned} \tag{3.29}$$

where (b) uses $\hat{\mathbf{G}}^{[1]\dagger}\hat{\mathbf{G}}^{[1]}=\left(\frac{\|\hat{\mathbf{G}}^{[1]}\|^2}{2}\right)\mathbf{I}_2$.

Let $\begin{bmatrix} u_1^{[1]} & u_2^{[1]} \end{bmatrix}^{\mathrm{T}} = \sqrt{\left(\frac{\|\hat{\mathbf{G}}^{[1]}\|^2}{2}\right)^{-1}} \hat{\mathbf{G}}^{[1]\dagger} \mathbf{h}_1^{[11]}$. Then (3.29) is rewritten as

$$\mathrm{SNR}^{[1]} = P_1^{[1]} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix}^{\dagger} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix}. \tag{3.30}$$

In order to easily analyze the diversity order of the proposed IAC scheme using PEP, we fix $\mathbf{H}^{[kt]}$ for distinct $k, t \in \{1, 2, 3\}$, and allow $\mathbf{H}^{[kk]}$ to change for all $k \in \{1, 2, 3\}$. Then, $\begin{bmatrix} u_1^{[1]} & u_2^{[1]} \end{bmatrix}^{\mathrm{T}}$ is a jointly circular complex Gaussian random vector. Let

$$\begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} = \underbrace{\begin{bmatrix} u_{1\mathrm{R}}^{[1]} \\ u_{2\mathrm{R}}^{[1]} \end{bmatrix}}_{\mathbf{u}_{\mathrm{R}}} + j \underbrace{\begin{bmatrix} u_{1\mathrm{I}}^{[1]} \\ u_{2\mathrm{I}}^{[1]} \end{bmatrix}}_{\mathbf{u}_{\mathrm{I}}} \tag{3.31}$$

where $\mathbf{u}_{\mathrm{R}}$ and $\mathbf{u}_{\mathrm{I}}$ are the real part and imaginary part of $\begin{bmatrix} u_1^{[1]} & u_2^{[1]} \end{bmatrix}^{\mathrm{T}}$ with covariance matrices $\mathbf{K}_{\mathrm{RR}}$ and $\mathbf{K}_{\mathrm{II}}$, respectively, and cross-covariance matrix $\mathbf{K}_{\mathrm{RI}}$, that is,

$$\begin{aligned} &\mathbb{E}\left[\mathbf{u}_{\mathrm{R}} \mathbf{u}_{\mathrm{R}}^{\mathrm{T}}\right] = \mathbf{K}_{\mathrm{RR}}, \qquad \mathbb{E}\left[\mathbf{u}_{\mathrm{I}} \mathbf{u}_{\mathrm{I}}^{\mathrm{T}}\right] = \mathbf{K}_{\mathrm{II}} \\ &\mathbb{E}\left[\mathbf{u}_{\mathrm{R}} \mathbf{u}_{\mathrm{I}}^{\mathrm{T}}\right] = \mathbf{K}_{\mathrm{RI}}. \end{aligned} \tag{3.32}$$

Since $\begin{bmatrix} u_1^{[1]} & u_2^{[1]} \end{bmatrix}^{\mathrm{T}}$ is a jointly circular complex Gaussian random vector, by using (2b) and (17) in [48], we have the following relations

$$\begin{aligned} &\mathbf{K}_{\mathrm{RR}} = \mathbf{K}_{\mathrm{II}}, \qquad \mathbf{K}_{\mathrm{RI}}^{\mathrm{T}} = -\mathbf{K}_{\mathrm{RI}} \\ &(\mathbf{K}_{\mathrm{RI}})_{jj} = 0, \qquad j = 1, 2. \end{aligned} \tag{3.33}$$

$$\Psi_{\bar{\gamma}}(j\mathbf{w}) = \mathbb{E}\Big[\exp\Big(j\bar{\gamma}^{\mathrm{T}}\mathbf{w}\Big)\Big] = \{\det(\mathbf{I}_2 - 2j\mathrm{diag}(\mathbf{w})\mathbf{K}_{\mathrm{II}})\}^{-1/2}\times$$
$$\{\det\big(\mathbf{I}_2 - 2j\mathrm{diag}(\mathbf{w})\mathbf{K}_{\mathrm{RR}} + 4\mathrm{diag}(\mathbf{w})\mathbf{K}_{\mathrm{RI}}[\mathbf{I}_2 - 2j\mathrm{diag}(\mathbf{w})\mathbf{K}_{\mathrm{II}}]^{-1}\mathrm{diag}(\mathbf{w})\mathbf{K}_{\mathrm{RI}}{}^{\mathrm{T}}\big)\}^{-1/2} \tag{3.35}$$

The average PEP conditioned on interfering channels at the receiver 1 can be written as

$$\begin{aligned} P\Big(x_1^{[1]} \to \hat{x}_1^{[1]}\Big|\mathbf{H}^{[ij]}\Big) &= \mathbb{E}_{\mathbf{H}^{[11]}}\left[Q\left(\sqrt{\frac{d_1^{[1]}\mathrm{SNR}^{[1]}}{2}}\right)\right] \\ &\leq \mathbb{E}_{\mathbf{H}^{[11]}}\left[\frac{1}{2}\exp\left(-\frac{d_1^{[1]}\mathrm{SNR}^{[1]}}{4}\right)\right] \\ &= \mathbb{E}_{\mathbf{H}^{[11]}}\left[\frac{1}{2}\exp\left(-\frac{d_1^{[1]}P_1^{[1]}}{4}\Big(\big|u_1^{[1]}\big|^2 + \big|u_2^{[1]}\big|^2\Big)\right)\right] \end{aligned} \tag{3.34}$$

where $d_1^{[1]} = \big|x_1^{[1]} - \hat{x}_1^{[1]}\big|^2$. The characteristic function (CF) of $\bar{\gamma} = \Big[\big|u_1^{[1]}\big|^2 \quad \big|u_2^{[1]}\big|^2\Big]^{\mathrm{T}}$ will be used to analyze the PEP in (3.34). The joint CF can be derived by using (75d) in [48] as (3.35) at the top in this page, where $\mathbf{w} = \big[w_1 \;\; w_2\big]^{\mathrm{T}}$. Then the last line of (3.34) can be rewritten using the CF in (3.35) as

$$\begin{aligned} &\mathbb{E}_{\mathbf{H}^{[11]}}\left[\frac{1}{2}\exp\left(-\frac{d_1^{[1]}P_1^{[1]}}{4}(\big|u_1^{[1]}\big|^2 + \big|u_2^{[1]}\big|^2)\right)\right] \\ &= \mathbb{E}_{\mathbf{H}^{[11]}}\left[\frac{1}{2}\Psi_{\bar{\gamma}}(j\mathbf{w})\bigg|_{\mathbf{w}=\frac{jd_1^{[1]}P_1^{[1]}}{4}\begin{bmatrix}1 & 1\end{bmatrix}^{\mathrm{T}}}\right] \\ &\overset{(c)}{\approx} \frac{1}{2}\det\left(\frac{d_1^{[1]}P_1^{[1]}}{2}\mathbf{K}_{\mathrm{II}}\right)^{-1/2}\det\left(\frac{d_1^{[1]}P_1^{[1]}}{2}\mathbf{K}_{\mathrm{RR}} - \frac{d_1^{[1]}P_1^{[1]}}{2}\mathbf{K}_{\mathrm{RI}}\mathbf{K}_{\mathrm{II}}^{-1}\mathbf{K}_{\mathrm{RI}}{}^{\mathrm{T}}\right)^{-1/2} \end{aligned}$$

$$= \frac{2}{\left(d_1^{[1]} P_1^{[1]}\right)^2} \det(\mathbf{K}_{\mathrm{II}})^{-1} \det(\mathbf{I}_2 - \mathbf{K}_{\mathrm{II}}^{-1}\mathbf{K}_{\mathrm{RI}}\mathbf{K}_{\mathrm{II}}^{-1}\mathbf{K}_{\mathrm{RI}}{}^{\mathrm{T}})^{-1/2} \tag{3.36}$$

where (c) uses high SNR approximation. Since $\mathbf{K}_{\mathrm{RR}}, \mathbf{K}_{\mathrm{RI}}$, and $\mathbf{K}_{\mathrm{II}}$ are full rank matrices with probability 1 and $\mathbf{H}^{[kt]}$'s are drawn from a continuous distribution, we have

$$\Pr\Big(\det(\mathbf{K}_{\mathrm{II}})^{-1} \det(\mathbf{I}_2 - \mathbf{K}_{\mathrm{II}}^{-1}\mathbf{K}_{\mathrm{RI}}\mathbf{K}_{\mathrm{II}}^{-1}\mathbf{K}_{\mathrm{RI}}{}^{\mathrm{T}})^{-1/2} = 0\Big) = 0. \tag{3.37}$$

Since (3.37) is satisfied, i.e, $\det(\mathbf{K}_{\mathrm{II}})^{-1}\det(\mathbf{I}_2 - \mathbf{K}_{\mathrm{II}}^{-1}\mathbf{K}_{\mathrm{RI}}\mathbf{K}_{\mathrm{II}}^{-1}\mathbf{K}_{\mathrm{II}}{}^{\mathrm{T}})^{-1/2}$ can be regarded as nonzero and $\mathbf{K}_{\mathrm{II}}, \mathbf{K}_{\mathrm{RI}}$, and $\mathbf{I}_2$ are independent of $P_1^{[1]}$, PEP decays proportional to $1/\left(P_1^{[1]}\right)^2$, i.e, the receiver 1 achieves diversity order of two for $x_1^{[1]}$. It can be similarly shown that diversity order of two is achieved for the other desired symbol $x_2^{[1]}$. Note that diversity analysis for the receiver 1 in the proposed IAC scheme can be directly applied to the IC scheme in Section 3.2 and thus the IC scheme in Section 3.2 achieves diversity order of two as well.

3.3.2.2. Proof of Theorem 3.2 for Receivers 2 and 3 when $M = 1$

Now let us analyze the diversity order of the receivers 2 and 3 with two antennas. Without loss of generality, we only consider the receiver 2. Due to the symmetry, the receiver 3 operates similarly and has the same performance as the receiver 2. The received signal matrix at the receiver 2 is given as

$$\begin{aligned}\mathbf{Y}^{[2]} &= \mathbf{H}^{[21]}\mathbf{P}^{[1]}\mathbf{A}_1 + \mathbf{H}^{[22]}\mathbf{P}^{[2]}\mathbf{A}_2 + \mathbf{H}^{[23]}\mathbf{P}^{[3]}\mathbf{A}_3 + \mathbf{N}^{[2]} \\ &\overset{(d)}{=} \mathbf{H}^{[22]}\mathbf{P}^{[2]}\mathbf{A}_2 + \mathbf{H}^{[21]}\mathbf{P}^{[1]}(\mathbf{A}_1 + \mathbf{A}_3) + \mathbf{N}^{[2]} \end{aligned} \tag{3.38}$$

where $\mathbf{A}_t = \mathbf{A}(x_1^{[t]}, x_2^{[t]})$ and the equality (d) follows from the alignment condition in (3.15). Note that $\mathbf{A}_1 + \mathbf{A}_3$ is also an Alamouti matrix due to the Alamouti Property 1. The received signal matrix in (3.38) can be viewed as a received signal through a multi-access channel (MAC), where two transmitters transmit Alamouti codes to the same receiver.

As before, we convert (3.38) to the following vectorized form

$$\begin{bmatrix} \mathbf{y}_1^{[2]} \\ \mathbf{y}_2^{[2]} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1,1}^{[22]} & \mathbf{H}_{1,1}^{[21]} \\ \mathbf{H}_{2,1}^{[22]} & \mathbf{H}_{2,1}^{[21]} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1^{[2]} \\ \mathbf{i}_1^{[2]} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1^{[2]} \\ \mathbf{n}_2^{[2]} \end{bmatrix} \tag{3.39}$$

where $\mathbf{y}_i^{[2]} = \begin{bmatrix} y_{i,1}^{[2]} & y_{i,2}^{[2]*} \end{bmatrix}^{\mathrm{T}}$, $\mathbf{H}_{i,1}^{[2t]} = \begin{bmatrix} h_{i,1}^{[2t]} & h_{i,2}^{[2t]} \\ h_{i,2}^{[2t]*} & -h_{i,1}^{[2t]*} \end{bmatrix}$, $h_{i,j}^{[2t]}$'s are entries of $\mathbf{H}^{[2t]}\mathbf{P}^{[t]}$, and $\mathbf{n}_i^{[2]} = \begin{bmatrix} n_{i,1}^{[2]} & n_{i,2}^{[2]*} \end{bmatrix}^{\mathrm{T}}$. The vectors $\mathbf{s}_1^{[2]}$ and $\mathbf{i}_1^{[2]}$ denote the desired symbol vector $\begin{bmatrix} x_1^{[2]} & x_2^{[2]} \end{bmatrix}^{\mathrm{T}}$ and interfering symbol vector $\begin{bmatrix} x_1^{[1]} + x_1^{[3]} & x_2^{[1]} + x_2^{[3]} \end{bmatrix}^{\mathrm{T}}$ for the receiver 2, respectively. Then we can remove the interference by multiplying the conjugate transpose of the 4×2 matrix $\hat{\mathbf{G}}^{[2]} = \mathbf{G}_1^{[2]}(1) = \begin{bmatrix} \mathbf{I}_2 & \mathbf{F}_1^{[2]\dagger}(1) \end{bmatrix}^{\dagger} = \begin{bmatrix} \mathbf{I}_2 & -\frac{2\mathbf{H}_{1,1}^{[21]}\mathbf{H}_{2,1}^{[21]\dagger}}{\|\mathbf{H}_{2,1}^{[21]}\|^2} \end{bmatrix}^{\dagger}$ as

$$\hat{\mathbf{G}}^{[2]\dagger} \begin{bmatrix} \mathbf{y}_1^{[2]} \\ \mathbf{y}_2^{[2]} \end{bmatrix} \overset{(e)}{=} \underbrace{\hat{\mathbf{G}}^{[2]\dagger} \begin{bmatrix} \mathbf{H}_{1,1}^{[22]} \\ \mathbf{H}_{2,1}^{[22]} \end{bmatrix}}_{\mathbf{H}_{\mathbf{eff}}^{[2]}} \mathbf{s}_1^{[2]} + \hat{\mathbf{G}}^{[2]\dagger} \begin{bmatrix} \mathbf{n}_1^{[2]} \\ \mathbf{n}_2^{[2]} \end{bmatrix} \tag{3.40}$$

where the equality (e) uses the fact that $\hat{\mathbf{G}}^{[2]\dagger} \begin{bmatrix} \mathbf{H}_{1,1}^{[21]\dagger} & \mathbf{H}_{2,1}^{[21]\dagger} \end{bmatrix}^{\dagger}$ is a 2×2 zero matrix due to the Alamouti Property 2. The equivalent channel after cancelling the interference, $\mathbf{H}_{\mathbf{eff}}^{[2]}$ in (3.40) is also an Alamouti matrix due to the Alamouti Property 1. Therefore, the desired symbols $x_1^{[2]}$ and $x_2^{[2]}$ can be derived by symbol-by-symbol decoding.

Define column vectors $\mathbf{h}_i^{[22]}$ as $\begin{bmatrix}\mathbf{h}_1^{[22]} & \mathbf{h}_2^{[22]}\end{bmatrix} = \begin{bmatrix}\mathbf{H}_{1,1}^{[22]\dagger} & \mathbf{H}_{2,1}^{[22]\dagger}\end{bmatrix}^{\dagger}$ in (3.40). As before, the receiver 2 can estimate its desired symbols $x_1^{[2]}$ and $x_2^{[2]}$ by symbol-by-symbol decoding as

$$\mathbf{H}_{\mathbf{eff}}^{[\mathbf{2}]\,\dagger}\hat{\mathbf{G}}^{[2]\dagger}\begin{bmatrix}\mathbf{y}_1^{[2]}\\ \mathbf{y}_2^{[2]}\end{bmatrix} = \mathbf{H}_{\mathbf{eff}}^{[\mathbf{2}]\,\dagger}\mathbf{H}_{\mathbf{eff}}^{[\mathbf{2}]}\mathbf{s}_1^{[2]} + \mathbf{H}_{\mathbf{eff}}^{[\mathbf{2}]\,\dagger}\hat{\mathbf{G}}^{[2]\dagger}\begin{bmatrix}\mathbf{n}_1^{[2]}\\ \mathbf{n}_2^{[2]}\end{bmatrix}. \tag{3.41}$$

Now, we focus only on the desired symbol $x_1^{[2]}$. Since $\mathbf{H}_{\mathbf{eff}}^{[\mathbf{2}]\,\dagger}\mathbf{H}_{\mathbf{eff}}^{[\mathbf{2}]}$ is a scaled identity matrix from the Alamouti Property 2, we can extract the equation of $x_1^{[2]}$ from (3.41) as

$$\tilde{r}_1^{[2]} = \mathbf{h}_1^{[22]\dagger}\hat{\mathbf{G}}^{[2]}\hat{\mathbf{G}}^{[2]\dagger}\mathbf{h}_1^{[22]}x_1^{[2]} + \mathbf{h}_1^{[22]\dagger}\hat{\mathbf{G}}^{[2]}\hat{\mathbf{G}}^{[2]\dagger}\begin{bmatrix}\mathbf{n}_1^{[2]}\\ \mathbf{n}_2^{[2]}\end{bmatrix}. \tag{3.42}$$

Let $\mathbb{E}\left[\left|x_1^{[2]}\right|^2\right] = P_1^{[2]}$ and assume that the covariance matrix of $\begin{bmatrix}\mathbf{n}_1^{[2]\dagger} & \mathbf{n}_2^{[2]\dagger}\end{bmatrix}^{\dagger}$ is the identity matrix. Then, the normalized instantaneous receive SNR is expressed as

$$\begin{aligned}\mathrm{SNR}^{[2]} &= P_1^{[2]}\frac{(\mathbf{h}_1^{[22]\dagger}\hat{\mathbf{G}}^{[2]}\hat{\mathbf{G}}^{[2]\dagger}\mathbf{h}_1^{[22]})^2}{\mathbf{h}_1^{[22]\dagger}\hat{\mathbf{G}}^{[2]}\hat{\mathbf{G}}^{[2]\dagger}\hat{\mathbf{G}}^{[2]}\hat{\mathbf{G}}^{[2]\dagger}\mathbf{h}_1^{[22]}}\\ &\overset{(f)}{=} P_1^{[2]}\left(\frac{\|\hat{\mathbf{G}}^{[2]}\|^2}{2}\right)^{-1}\mathbf{h}_1^{[22]\dagger}\hat{\mathbf{G}}^{[2]}\hat{\mathbf{G}}^{[2]\dagger}\mathbf{h}_1^{[22]}\end{aligned} \tag{3.43}$$

where the equality (f) uses $\hat{\mathbf{G}}^{[2]\dagger}\hat{\mathbf{G}}^{[2]} = \frac{\|\hat{\mathbf{G}}^{[2]}\|^2}{2}\mathbf{I}_2$. Let $\begin{bmatrix}u_1^{[2]} & u_2^{[2]}\end{bmatrix}^{\mathrm{T}} =$

$\sqrt{\left(\frac{\|\hat{\mathbf{G}}^{[2]}\|^2}{2}\right)^{-1}}\hat{\mathbf{G}}^{[2]\dagger}\mathbf{h}_1^{[22]}$. Then (3.43) can be rewritten as

$$\mathrm{SNR}^{[2]} = P_1^{[2]} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix}^{\dagger} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix}. \tag{3.44}$$

Since $\begin{bmatrix} u_1^{[2]} & u_2^{[2]} \end{bmatrix}^{\mathrm{T}}$ is a jointly circular complex Gaussian random vector, PEP can be analyzed similar to (3.31)–(3.37). It is easy to show that the receiver 2 achieves diversity order of two for $x_1^{[2]}$ and diversity order of the other desired symbol $x_2^{[2]}$ can also be derived in the same way, which is also two.

3.3.2.3. Proof of Theorem 3.2 for $M \geq 2$

Let us consider the general case, i.e., an interference channel with three transmitters and three receivers, where each of them is equipped with $2M$ antennas except that the receiver 1 has $3M$ antennas. Similar to the previous case, the interference is cancelled at each receiver by using $\hat{\mathbf{G}}^{[k]}$ as

$$\hat{\mathbf{G}}^{[1]\dagger} \begin{bmatrix} \mathbf{y}_1^{[1]} \\ \mathbf{y}_2^{[1]} \\ \vdots \\ \mathbf{y}_{3M}^{[1]} \end{bmatrix} = \hat{\mathbf{G}}^{[1]\dagger} \begin{bmatrix} \mathbf{H}_{1,1}^{[11]} \\ \mathbf{H}_{2,1}^{[11]} \\ \vdots \\ \mathbf{H}_{3M,1}^{[11]} \end{bmatrix} \mathbf{s}_1^{[1]} + \hat{\mathbf{G}}^{[1]\dagger} \begin{bmatrix} \mathbf{n}_1^{[1]} \\ \mathbf{n}_2^{[1]} \\ \vdots \\ \mathbf{n}_{3M}^{[1]} \end{bmatrix} \tag{3.45}$$

$$\hat{\mathbf{G}}^{[k]\dagger}\begin{bmatrix}\mathbf{y}_1^{[k]}\\ \mathbf{y}_2^{[k]}\\ \vdots\\ \mathbf{y}_{2M}^{[k]}\end{bmatrix}=\hat{\mathbf{G}}^{[k]\dagger}\begin{bmatrix}\mathbf{H}_{1,1}^{[kk]}\\ \mathbf{H}_{2,1}^{[kk]}\\ \vdots\\ \mathbf{H}_{2M,1}^{[kk]}\end{bmatrix}\mathbf{s}_1^{[k]}+\hat{\mathbf{G}}^{[k]\dagger}\begin{bmatrix}\mathbf{n}_1^{[k]}\\ \mathbf{n}_2^{[k]}\\ \vdots\\ \mathbf{n}_{2M}^{[k]}\end{bmatrix},\quad k=2,3. \tag{3.46}$$

Note that the matrices $\hat{\mathbf{G}}^{[k]}$ in (3.45) and (3.46) are obtained through tedious derivation similar to the previous case. Let $\hat{\mathbf{G}}^{[k]}=\begin{bmatrix}\mathbf{A}_1^{[k]\dagger} & \mathbf{A}_2^{[k]\dagger} & \cdots\end{bmatrix}^{\dagger}$, where $\mathbf{A}_j^{[k]}$'s are 2×2 matrices. Define $\mathcal{Z}^{(n)}(\mathbf{A}_j^{[k]})$ as the operation which replaces all $\mathbf{F}_i^{[k]}(l)$ in $\mathbf{A}_j^{[k]}$ by $\mathbf{F}_{i+n}^{[k]}(l-n)$, where $\mathcal{Z}^{(n)}(\mathbf{I}_2)=\mathbf{I}_2$. For example, $\hat{\mathbf{G}}^{[1]}$ in (3.26) is rewritten as

$$\hat{\mathbf{G}}^{[1]}=\begin{bmatrix}\mathbf{A}_1^{[1]}\\ \mathbf{A}_2^{[1]}\\ \mathbf{A}_3^{[1]}\end{bmatrix}=\begin{bmatrix}\mathbf{I}_2\\ \mathbf{F}_1^{[1]}(2)\\ \mathbf{F}_1^{[1]}(1)+\mathbf{F}_2^{[1]}(1)\mathbf{F}_1^{[1]}(2)\end{bmatrix}$$

and $\mathcal{Z}^{(1)}(\mathbf{A}_3^{[1]})=\mathbf{F}_2^{[1]}(0)+\mathbf{F}_3^{[1]}(0)\mathbf{F}_2^{[1]}(1)$. Then we have $\hat{\mathbf{G}}^{[1]}$, $\hat{\mathbf{G}}^{[2]}$, and $\hat{\mathbf{G}}^{[3]}$ as (3.47) and (3.48) at the top of the next page, respectively. Since $\mathbf{F}_i^{[k]}(l)$'s in (3.47) and (3.48) are all Alamouti matrices, all $\mathbf{A}_j^{[k]}$'s are also Alamouti matrices due to the Alamouti Property 1 and the equivalent channels in (3.45) and (3.46) are Alamouti matrices as well. Therefore, it can be easily verified that the proposed IAC scheme for the general case with $2M$ antennas achieves diversity order of two in a similar way to (3.27)–(3.37). Until now, it is shown that the proposed IAC scheme for the three-user interference channel with one $3M$-antenna receiver and two $2M$-antenna receivers achieves both $3M$ DoF and diversity order of two.

$$\hat{\mathbf{G}}^{[1]} = \begin{bmatrix} \mathbf{A}_1^{[1]} \\ \mathbf{A}_2^{[1]} \\ \mathbf{A}_3^{[1]} \\ \mathbf{A}_4^{[1]} \\ \vdots \\ \mathbf{A}_j^{[1]} \\ \vdots \\ \mathbf{A}_{3M}^{[1]} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{F}_1^{[1]}(3M-1) \\ \mathbf{F}_1^{[1]}(3M-2) + \mathcal{Z}^{(1)}(\mathbf{A}_2^{[1]})\mathbf{F}_1^{[1]}(3M-1) \\ \mathbf{F}_1^{[1]}(3M-3) + \mathcal{Z}^{(2)}(\mathbf{A}_2^{[1]})\mathbf{F}_1^{[1]}(3M-2) + \mathcal{Z}^{(1)}(\mathbf{A}_3^{[1]})\mathbf{F}_1^{[1]}(3M-1) \\ \vdots \\ \sum_{l=1}^{j-1} \mathcal{Z}^{(j-l)}(\mathbf{A}_l^{[1]})\mathbf{F}_1^{[1]}(3M-j+l) \\ \vdots \\ \sum_{l=1}^{3M-1} \mathcal{Z}^{(3M-l)}(\mathbf{A}_l^{[1]})\mathbf{F}_1^{[1]}(l) \end{bmatrix} \tag{3.47}$$

$$\hat{\mathbf{G}}^{[k]} = \begin{bmatrix} \mathbf{A}_1^{[k]} \\ \mathbf{A}_2^{[k]} \\ \mathbf{A}_3^{[k]} \\ \mathbf{A}_4^{[k]} \\ \vdots \\ \mathbf{A}_j^{[k]} \\ \vdots \\ \mathbf{A}_{2M}^{[k]} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{F}_1^{[k]}(2M-1) \\ \mathbf{F}_1^{[k]}(2M-2) + \mathcal{Z}^{(1)}(\mathbf{A}_2^{[k]})\mathbf{F}_1^{[k]}(2M-1) \\ \mathbf{F}_1^{[k]}(2M-3) + \mathcal{Z}^{(2)}(\mathbf{A}_2^{[k]})\mathbf{F}_1^{[k]}(2M-2) + \mathcal{Z}^{(1)}(\mathbf{A}_3^{[k]})\mathbf{F}_1^{[k]}(2M-1) \\ \vdots \\ \sum_{l=1}^{j-1} \mathcal{Z}^{(j-l)}(\mathbf{A}_l^{[k]})\mathbf{F}_1^{[k]}(2M-j+l) \\ \vdots \\ \sum_{l=1}^{2M-1} \mathcal{Z}^{(2M-l)}(\mathbf{A}_l^{[k]})\mathbf{F}_1^{[k]}(l) \end{bmatrix}, \quad \text{for } k = 2, 3. \tag{3.48}$$

3.3.3. Extension to K-User MIMO Interference Channel

In order to apply the proposed IAC scheme to the K-user interference channel, consider the K-user interference channel where each transmitter has $2M$ antennas. Note that KM antennas are required for all receivers to apply the IC scheme achieving KM DoF per channel use. To reduce the number of receive antennas by using IAC scheme, the beamforming matrices should satisfy the alignment conditions, e.g.,

$$\mathbf{H}^{[kt]}\mathbf{P}^{[t]} = \mathbf{H}^{[kt']}\mathbf{P}^{[t']}, \qquad t, t' \neq k, \quad 1 \leq t, t' \leq K, \quad 2 \leq k \leq K. \tag{3.49}$$

It is clear that the system of linear equations (3.49) have no solutions if the channel matrices are not $2M \times 2M$ matrices. In other words, the numbers of antennas at the receivers $2, \cdots, K$ should be reduced to $2M$. For this, the beamforming matrices are designed to satisfy the following conditions

$$\begin{aligned}
\mathbf{H}^{[21]}\mathbf{P}^{[1]} &= \mathbf{H}^{[23]}\mathbf{P}^{[3]} = \cdots = \mathbf{H}^{[2\ K-1]}\mathbf{P}^{[K-1]} = \mathbf{H}^{[2\ K]}\mathbf{P}^{[K]} \\
&\vdots \\
\mathbf{H}^{[K-1\ 1]}\mathbf{P}^{[1]} &= \mathbf{H}^{[K-1\ 2]}\mathbf{P}^{[2]} = \cdots = \mathbf{H}^{[K-1\ K-2]}\mathbf{P}^{[K-2]} = \mathbf{H}^{[K-1\ K]}\mathbf{P}^{[K]} \\
\mathbf{H}^{[K\ 1]}\mathbf{P}^{[1]} &= \mathbf{H}^{[K\ 2]}\mathbf{P}^{[2]} = \cdots = \mathbf{H}^{[K\ K-2]}\mathbf{P}^{[K-2]} = \mathbf{H}^{[K\ K-1]}\mathbf{P}^{[K-1]}.
\end{aligned} \tag{3.50}$$

However, the above conditions cannot be satisfied simultaneously and we have no choice but to determine the beamforming matrices satisfying the alignment conditions for only one receiver. Therefore, our IAC scheme for the three-user interference channel may be directly generalized to the K-user interference channel by reducing the number of antennas at only one receiver from KM to $2M$. In fact, there may be other methods which can reduce the number of antennas at each receiver differently for the above K-user interference channel or achieve more diversity gain. We leave these as a future work.

3.4. Simulation Results

In this section, average symbol error rate performance of the IC and IAC schemes are compared with that of the IA scheme for the three-user

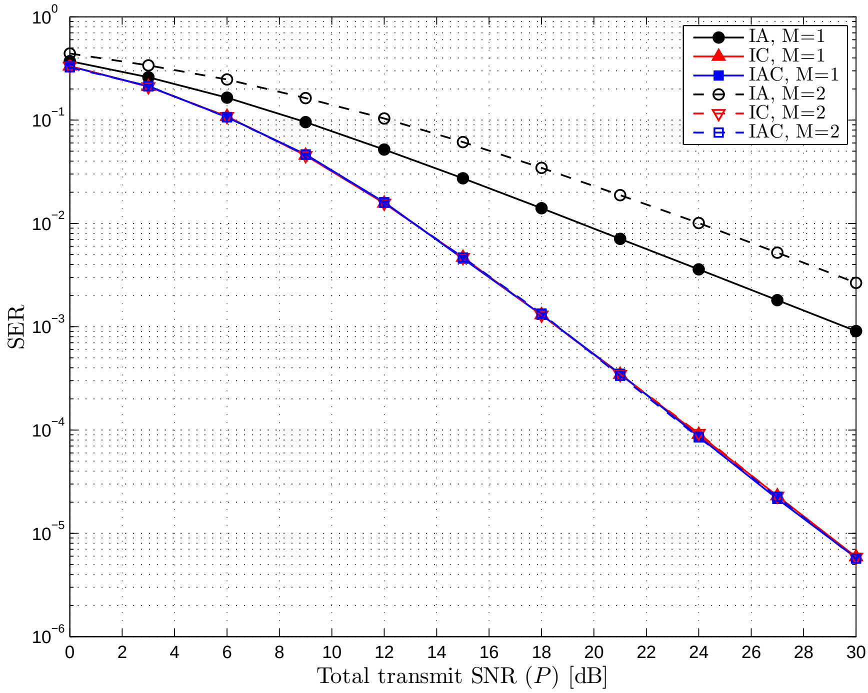

Figure 3.2: SER performance comparison of IA, IC, and IAC schemes for the three-user MIMO interference channel when $M = 1,\ 2$.

MIMO interference channel. It is assumed that the channel is Rayleigh block fading, i.e., the channel state does not change during transmission of each code but varies independently from block to block. All channel coefficients and noise at the receivers are assumed to be complex Gaussian random variables $\mathcal{CN}(0,1)$. Quadrature phase-shift keying is used and the average transmit power per symbol at each transmitter is set to P. Fig. 3.2 compares the SER performance of IA, IC, and IAC schemes for the three-user interference channel when $M = 1,\ 2$. Note that when $M = 1$ (or 2), i.e., each transmitter has two (or four) antennas for IA, IC, and IAC schemes, they achieve three (or six) DoF per channel use(, respectively). Fig. 3.2 shows that the diversity order is the same for each scheme independent of M. The IA scheme achieves diversity order of one for all cases, which can be verified by the slope of the SER curve in high SNR region. When $M = 1$, although the IC scheme requires three antennas at each receiver, Fig. 3.2 shows that it achieves diversity order of two. The IAC scheme also achieves diversity order of two when one receiver has three antennas and the others have two antennas. When $M = 2$, the IC scheme using six antennas at each receiver achieves diversity order of two. The IAC scheme achieves diversity order of two by using six antennas at one receiver and four antennas at each of the other two receivers.

It is clear that the diversity orders shown in Fig. 3.2 match well with the analytical results in the previous sections.

3.5. Conclusions

In this chapter, we propose a method on how to apply Alamouti code to MIMO interference channels. The interference cancellation method based on Alamouti codes for the multi-access scenario can be used for the K-user interference channel, which enables the receivers to perform symbol-by-symbol decoding by cancelling interfering signals by utilizing Alamouti structure and achieve diversity order of two. Moreover it does not require CSIT unlike the IA scheme.

However, the IC scheme requires more receive antennas than the IA scheme to achieve the same DoF. In order to reduce the number of receive antennas, especially for the three-user interference channel, we propose an IAC scheme based on Alamouti codes which utilizes beamforming matrices with partial CSIT to align interfering signals. We also show that while the proposed IAC scheme requires less antennas at the receivers than the IC scheme, it achieves the same performance in terms of DoF and diversity order. In fact, there may be other methods which can achieve more diversity order or reduce the number of receive antennas differently for the K-user interference channel. We leave these as a future work.

Chapter 4. Two-Way Relaying Schemes with Alamouti Codes

4.1. Introduction

In a two-way relay channel as Fig. 4.1, two source nodes simultaneously transmit their messages to each other via a helping relay node. A two-way relaying scheme is proposed in [49] , which requires only two time phases, i.e., MAC and BC phases. The relay receives signals from two nodes simultaneously in the MAC phase. In the BC phase the signals are linearly combined by means of superposition coding, and then, broadcast to two source nodes. Since the source nodes know their own transmitted signal, they subtract its effect to have the desired signal.

In this chapter, we show that the interference cancellation scheme based on Alamouti codes for the inteference channel can also be used for the TWRC. Then, we propose a new scheme using beamforming matrices to achieve more diversity gain.

The rest of this chapter is organized as follows. Section 4.2 describes the TWRC and the two-way relaying scheme I using IC based on Alamouti codes. Section 4.3 address the two-way relaying scheme II utilizing the beamformer to achieve more diversity gain. The simulation results are shown in Section 4.4. Finally, the concluding remarks are given in Section 4.5.

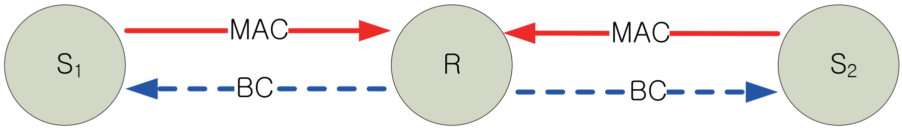

Figure 4.1: Two-way relay channel.

4.2. Two-Way Relaying Scheme I Based on Alamouti Codes

Consider the TWRC, where each node has two antennas. The TWRC consists of two source nodes, S_1 and S_2, which exchange information via a helping relay node, R. It is assumed that there is no direct connection between S_1 and S_2. In this section, we will show that an IC scheme based on Alamouti codes can be applied to the TWRC.

In the MAC phase, each source transmits Alamouti code as

$$\mathbf{X}^{[t]} = \mathbf{A}(x_1^{[t]},\ x_2^{[t]}), \quad t = 1, 2 \tag{4.1}$$

where $x_i^{[t]}$ is the i-th data symbol transmitted from S_t. The received signal matrix at R is given as

$$\mathbf{Y}^{[\mathrm{R}]} = \mathbf{H}^{[\mathrm{R}1]}\mathbf{X}^{[1]} + \mathbf{H}^{[\mathrm{R}2]}\mathbf{X}^{[2]} + \mathbf{N}^{[\mathrm{R}]} \tag{4.2}$$

where $\mathbf{Y}^{[\mathrm{R}]}$ is the received signal matrix at R whose entry is $y_{i,j}^{[\mathrm{R}]}$, $\mathbf{H}^{[\mathrm{R}t]}$ is the channel matrix from S_t to R, and $\mathbf{N}^{[\mathrm{R}]}$ denotes the additive white complex Gaussian noise matrix with zero-mean and unit-variance entries, $n_{i,j}^{[\mathrm{R}]}$, at R. Entries of $\mathbf{H}^{[\mathrm{R}t]}$, $h_{i,j}^{[\mathrm{R}t]}$, are assumed to be i.i.d. complex Gaussian random variable. It is also assumed that channel is block fading (or constant), i.e., the channel state does not change during the transmission of each code.

Equation (4.2) can be converted to the following vectorized form

$$\begin{bmatrix} \mathbf{y}_1^{[\mathrm{R}]} \\ \mathbf{y}_2^{[\mathrm{R}]} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1,1}^{[\mathrm{R1}]} & \mathbf{H}_{1,1}^{[\mathrm{R2}]} \\ \mathbf{H}_{2,1}^{[\mathrm{R1}]} & \mathbf{H}_{2,1}^{[\mathrm{R2}]} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1^{[1]} \\ \mathbf{x}_1^{[2]} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1^{[\mathrm{R}]} \\ \mathbf{n}_2^{[\mathrm{R}]} \end{bmatrix} \tag{4.3}$$

where $\mathbf{y}_i^{[\mathrm{R}]} = \begin{bmatrix} y_{i,1}^{[\mathrm{R}]} & y_{i,2}^{[\mathrm{R}]*} \end{bmatrix}^{\mathrm{T}}$, $\mathbf{H}_{i,1}^{[\mathrm{R}t]} = \begin{bmatrix} h_{i,1}^{[\mathrm{R}t]} & h_{i,2}^{[\mathrm{R}t]} \\ h_{i,2}^{[\mathrm{R}t]*} & -h_{i,1}^{[\mathrm{R}t]*} \end{bmatrix}$, and $\mathbf{n}_i^{[\mathrm{R}]} = \begin{bmatrix} n_{i,1}^{[\mathrm{R}]} & n_{i,2}^{[\mathrm{R}]*} \end{bmatrix}^{\mathrm{T}}$. The vectors $\mathbf{x}_1^{[t]}$ denotes the symbol vector $\begin{bmatrix} x_1^{[t]} & x_2^{[t]} \end{bmatrix}^{\mathrm{T}}$. Then we can separate the equations of $\mathbf{x}_1^{[1]}$ and $\mathbf{x}_1^{[2]}$ from (4.3) using $\hat{\mathbf{G}}_1^{[\mathrm{R}]}$ and $\hat{\mathbf{G}}_2^{[\mathrm{R}]}$ as

$$\hat{\mathbf{G}}_1^{[\mathrm{R}]} = \begin{bmatrix} \mathbf{I}_2 & -\frac{2\mathbf{H}_{1,1}^{[\mathrm{R2}]}\mathbf{H}_{2,1}^{[\mathrm{R2}]\dagger}}{\|\mathbf{H}_{2,1}^{[\mathrm{R2}]}\|^2} \end{bmatrix}^{\dagger} \tag{4.4}$$

$$\hat{\mathbf{G}}_2^{[\mathrm{R}]} = \begin{bmatrix} \mathbf{I}_2 & -\frac{2\mathbf{H}_{1,1}^{[\mathrm{R1}]}\mathbf{H}_{2,1}^{[\mathrm{R1}]\dagger}}{\|\mathbf{H}_{2,1}^{[\mathrm{R1}]}\|^2} \end{bmatrix}^{\dagger}. \tag{4.5}$$

By multiplying the conjugate transpose of the 4×2 matrix $\hat{\mathbf{G}}_i^{[\mathrm{R}]}$, we have

$$\hat{\mathbf{G}}_1^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{y}_1^{[\mathrm{R}]} \\ \mathbf{y}_2^{[\mathrm{R}]} \end{bmatrix} \overset{(a)}{=} \underbrace{\hat{\mathbf{G}}_1^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{H}_{1,1}^{[\mathrm{R1}]} \\ \mathbf{H}_{2,1}^{[\mathrm{R1}]} \end{bmatrix}}_{\mathbf{H}_{\mathbf{eff}}^{[\mathbf{R1}]}} \mathbf{x}_1^{[1]} + \hat{\mathbf{G}}_1^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{n}_1^{[2]} \\ \mathbf{n}_2^{[2]} \end{bmatrix} \tag{4.6}$$

$$\hat{\mathbf{G}}_2^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{y}_1^{[\mathrm{R}]} \\ \mathbf{y}_2^{[\mathrm{R}]} \end{bmatrix} \overset{(b)}{=} \underbrace{\hat{\mathbf{G}}_2^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{H}_{1,1}^{[\mathrm{R2}]} \\ \mathbf{H}_{2,1}^{[\mathrm{R2}]} \end{bmatrix}}_{\mathbf{H}_{\mathbf{eff}}^{[\mathbf{R2}]}} \mathbf{x}_1^{[2]} + \hat{\mathbf{G}}_2^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{n}_1^{[2]} \\ \mathbf{n}_2^{[2]} \end{bmatrix} \tag{4.7}$$

where the equalities (a) and (b) use the fact that $\hat{\mathbf{G}}_i^{[\mathrm{R}]\dagger} \begin{bmatrix} \mathbf{H}_{1,1}^{[\mathrm{R}j]\dagger} & \mathbf{H}_{2,1}^{[\mathrm{R}j]\dagger} \end{bmatrix}^{\dagger}$,

$i \neq j$, is a 2×2 zero matrix. The equivalent channels, $\mathbf{H}_{\mathbf{eff}}^{[\mathrm{R}\mathbf{1}]}$ and $\mathbf{H}_{\mathbf{eff}}^{[\mathrm{R}\mathbf{2}]}$ in (4.6) and (4.7) are Alamouti matrices. Therefore, R can decode four symbols $x_i^{[t]}$ using by symbol-by-symbol Alamouti decoding in [4].

In the BC phase, R broadcast the following Alamouti code.

$$\mathbf{X}^{[\mathrm{R}]} = \mathbf{A}(\hat{x}_1^{[1]} + \hat{x}_1^{[2]},\ \hat{x}_2^{[1]} + \hat{x}_2^{[2]}) \tag{4.8}$$

where $\hat{x}_i^{[t]}$ denotes the estimated symbol at R. The received signal matrices at S_1 and S_2 are given as

$$\mathbf{Y}^{[i]} = \mathbf{H}^{[i\mathrm{R}]}\mathbf{X}^{[\mathrm{R}]} + \mathbf{N}^{[i]}, \quad i = 1, 2. \tag{4.9}$$

Then the sources can decode $\hat{x}_1^{[1]} + \hat{x}_1^{[2]}$ and $\hat{x}_2^{[1]} + \hat{x}_2^{[2]}$ by using Alamouti decoding. Since the sources know the symbol they have transmitted in the MAC phase, they can cancel this contribution and decode the desired symbols.

4.3. Two-Way Relaying Scheme II Based on Alamouti Codes

In order to achieve more diversity gain, our proposed scheme utilizes beamforming matrices. In the MAC phase, the transmitted block code at each source is designed as

$$\mathbf{X}^{[t]} = \mathbf{P}^{[t]}\mathbf{A}(x_1^{[t]},\ x_2^{[t]}). \tag{4.10}$$

As shown in (4.10), each source encodes data symbols using Alamouti code followed by $\mathbf{P}^{[t]}$. Suppose that the beamforming matrices $\mathbf{P}^{[t]}$ are designed to satisfy the following alignment conditions

$$\mathbf{H}^{[\text{R1}]}\mathbf{P}^{[1]} = \mathbf{H}^{[\text{R2}]}\mathbf{P}^{[2]}. \tag{4.11}$$

Then the received signal matrix at R is given as

$$\begin{aligned}\mathbf{Y}^{[\text{R}]} &= \mathbf{H}^{[\text{R1}]}\mathbf{X}^{[1]} + \mathbf{H}^{[\text{R2}]}\mathbf{X}^{[2]} + \mathbf{N}^{[\text{R}]} \\ &= \mathbf{H}^{[\text{R1}]}\mathbf{P}^{[1]}\mathbf{A}(x_1^{[1]} + x_1^{[2]},\ x_2^{[1]} + x_2^{[2]}) + \mathbf{N}^{[\text{R}]}\end{aligned}$$

which can be converted to the following vectorized form as

$$\mathbf{y}^{[\text{R}]} = \begin{bmatrix} \mathbf{y}_1^{[\text{R}]} \\ \mathbf{y}_2^{[\text{R}]} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_{1,1}^{[\text{R1}]} \\ \mathbf{H}_{2,1}^{[\text{R1}]} \end{bmatrix}}_{\mathbf{H}_{\textbf{eff}}^{[\text{R}]}} \begin{bmatrix} x_1^{[1]} + x_1^{[2]} \\ x_2^{[1]} + x_2^{[2]} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1^{[\text{R}]} \\ \mathbf{n}_2^{[\text{R}]} \end{bmatrix} \tag{4.12}$$

where $\mathbf{y}_i^{[\text{R}]} = \begin{bmatrix} y_{i,1}^{[\text{R}]} & y_{i,2}^{[R]*} \end{bmatrix}^{\text{T}}$, $\mathbf{H}_{i,1}^{[\text{R1}]} = \begin{bmatrix} h_{i,1}^{[\text{R1}]} & h_{i,2}^{[\text{R1}]} \\ h_{i,2}^{[\text{R1}]*} & -h_{i,1}^{[\text{R1}]*} \end{bmatrix}$, $h_{i,j}^{[\text{R1}]}$'s are entries of $\mathbf{H}^{[\text{R1}]}\mathbf{P}^{[1]}$, and $\mathbf{n}_i^{[2]} = \begin{bmatrix} n_{i,1}^{[2]} & n_{i,2}^{[2]*} \end{bmatrix}^{\text{T}}$.

In the BC phase, soft decision value at R can be obtained and re-encoded by using Alamouti code as

$$\begin{bmatrix} \hat{x}_1^{[\text{R}]} \\ \hat{x}_2^{[\text{R}]} \end{bmatrix} = {\mathbf{H}_{\textbf{eff}}^{[\text{R}]}}^{\dagger}\mathbf{y}^{[\text{R}]} \tag{4.13}$$

$$\mathbf{X}^{[\text{R}]} = \mathbf{A}(\hat{x}_1^{[\text{R}]},\ \hat{x}_2^{[\text{R}]}). \tag{4.14}$$

The received signal matrices at S_1 and S_2 have the same structure as (4.9), and then the sources can decode by using the same symbol-by-symbol decoding as in the previous section.

4.4. Simulation Results

It is assumed that the channel is Rayleigh block fading. All channel coefficients and noise at S_1, S_2, and R are assumed to be complex Gaussian random variables $\mathcal{CN}(0, 1)$. QPSK is used and the average transmit power per symbol at each node is set to P.

Fig. 4.2 compares the SER performance of two-way relaying schemes I and II. Fig. 4.2 shows that the scheme II outperforms the scheme I. In the MAC phase of the scheme I, R has (4.6) and (4.7) after the cancellation, i.e., multiplying $\hat{\mathbf{G}}_i^{[\mathrm{R}]\dagger}$. Equations (4.6) and (4.7) can be viewed as the received signal for a point-to-point Alamouti scheme, which consists of a double-antenna transmitter and a single-antenna receiver, i.e., (12) in [4]. On the other hand, the scheme II does not require the cancellation in the MAC phase and (4.12) can be viewed as the received signal for the point-to-point Alamouti scheme where both the transmitter and the receiver have two antennas, i.e., (14) in [4]. Therefore, the scheme II achieves a diversity order of four, while the scheme I achieves a diversity order of two, which can be verified by the slope of the SER curve in high SNR region in Fig. 4.2.

4.5. Conclusion

We propose a method on how to apply Alamouti code to the TWRC. The interference cancellation method based on Alamouti codes for the

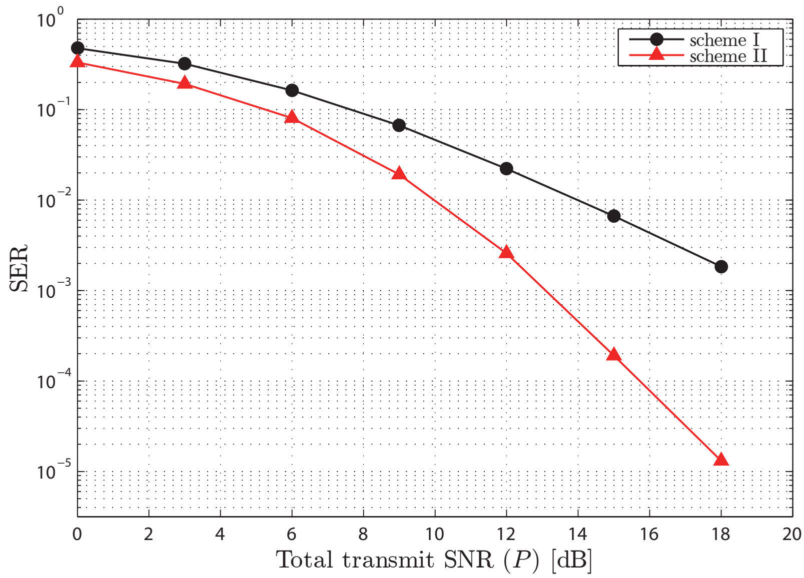

Figure 4.2: Symbol error rate performance comparison of scheme I and scheme II for the two-way relay channel.

interference channel can be used for the TWRC, which enables the nodes to perform symbol-by-symbol decoding and achieve diversity order of two.

In order to achieve more diversity gain, we propose a new two-way relaying scheme based on Alamouti codes which utilizes beamforming matrices to align signals at the relay node. From the simulation results, it is shown that the proposed scheme achieves a diversity order of four. Although we only consider the case where each node has two antennas, the proposed schemes can be easily extended for general $2M$ antennas case.

Chapter 5. Analysis of Soft-Decision-and-Forward Cooperative Networks with Multiple Relays

5.1. Introduction

Next generation wireless communication systems such as IMT-Advanced [30] require higher spectral efficiency and data rate, that is, 100Mbps for high-speed mobility and 1Gbps for low-speed mobility. These demands might be achieved by using multiple antenna technique [16] which increases the channel capacity. However, due to the limitation of implementation on the small-size devices, standardizations such as IEEE 802.16j [51] and Long-Term Evolution (LTE)-Advanced [52] for IMT-Advanced recommend the virtual multiple antenna technique in a distributed sense.

Sendonaris, Erkip, and Aazhang [53], [54] proposed the cooperative diversity using the cooperation between source and relay. In [55], Zhao, Adve, and Lim analyzed outage behaviors such as outage capacity and outage probability of two different AF relay schemes with single antenna, namely, all-participated relay transmission and relay selection. They also proposed the power allocation strategies for those two schemes. Similarly, Ikki and Ahmed proposed and analyzed the best relay selection scheme based on AF protocol with single antenna at each node in [56]. Bletsas *et al.* [57] described the forward channel estimation for the opportunistic relay selection in case of multiple relays. Yang, Song, No, and Shin [58] proposed the ML decoding for AF and soft-decision-and-forward protocols with multiple antennas using Alamouti codes [3]. Since the relay nodes can separate the signals in SDF protocol while they simply amplify the received signals according to the power constraint in AF protocol, SDF becomes substantially different from AF in the case of multiple antennas. In [26], performance on SDF with single relay where each node is equipped with two antennas was analyzed in terms of BER. Furthermore, power allocation scheme between source and relay nodes was proposed so as to maximize the instantaneous end-to-end SNR.

In this chapter, the best relay selection scheme using Alamouti code for the SDF cooperative networks with multiple relays is analyzed. The indications of the performance such as the PEP and the diversity order of the best relay selection scheme are compared with those of the conventional multiple-relay transmission scheme where all the relays participate in the second phase transmission. For these two schemes, we derive the end-to-end SNRs and the PEPs under the ML decoding proposed in [58]. From the derived PEPs for the multiple-relay cooperative networks with SDF protocol, the diversity orders are obtained. Song [59] makes use of approximation of PEP and then derives a diversity order based on this approximation. However, for completeness of the analysis, the tightness of approximation of PEP at high SNR should be examined. Therefore, the upper and lower bounds on the PEP are derived and diversity orders

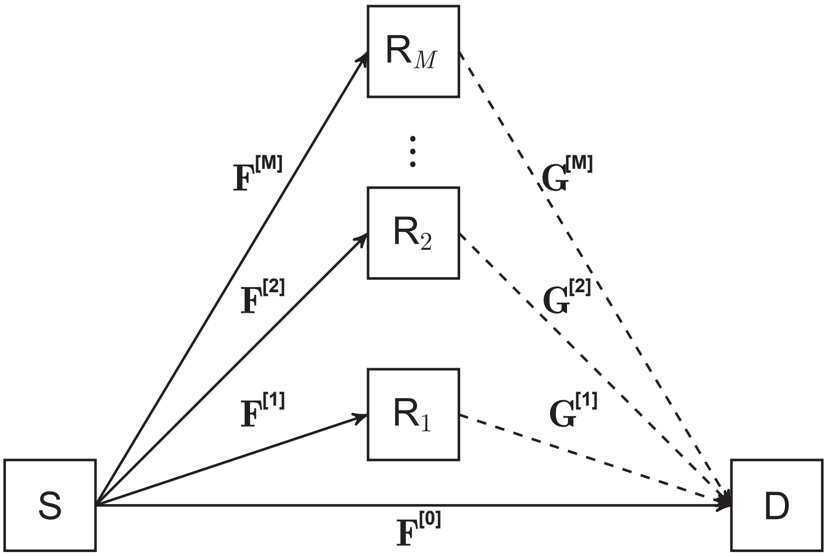

Figure 5.1: Cooperative communication network composed of one source (S), M relays (R_m), and one destination (D) with two antennas in each node.

are obtained based on these bounds instead of approximated PEP in this dissertation. The best relay selection scheme has an advantage over the conventional one in terms of BER and throughput.

The rest of this chapter is organized as follows. Section 5.2 describes the SDF protocol and reviews the results in [26]. Sections 5.3 and 5.4 address the conventional multiple-relay transmission and the 'best relay selection' schemes, respectively. The analytical and simulation results are shown in Section 5.5. Finally, the concluding remarks are given in Section 5.6.

5.2. Soft-Decision-and-Forward Protocol

The system model of SDF protocol [58] with multiple relays where each node is equipped with two antennas is depicted in Fig. 5.1. This cooperative communication system is composed of one source (S), one destination (D), and M relays ($\mathsf{R}_m,\ m = 1, \cdots, M$). In the second phase transmission, the following two transmission methods for the multiple relays are considered: conventional multiple-relay transmission and the best relay selection.

The total transmit power P in the network is defined as the sum of the source power P_1 at S and the total relay power P_2 at R_m's. The channel gains of each link $\mathsf{S} \to \mathsf{D}$, $\mathsf{S} \to \mathsf{R}_m$, and $\mathsf{R}_m \to \mathsf{D}$ are assumed to be Rayleigh-faded, i.e., $f_{ij}^{[0]} \sim \mathcal{CN}(0, \sigma_{\mathsf{SD}}^2)$, $f_{ij}^{[m]} \sim \mathcal{CN}(0, \sigma_{\mathsf{SR}_m}^2)$, and $g_{ij}^{[m]} \sim \mathcal{CN}(0, \sigma_{\mathsf{R}_m\mathsf{D}}^2)$, where $f_{ij}^{[0]}$, $f_{ij}^{[m]}$, and $g_{ij}^{[m]}$, $i, j = 1, 2$, $m = 1, \cdots, M$, denote the path gain from the jth transmit antenna at S to the ith receive antenna at D, from the jth transmit antenna at S to the ith receive antenna at R_m, and from the jth transmit antenna at R_m to the ith receive antenna at D, respectively. These path gains are represented as the channel matrices $\mathbf{F}^{[0]} = [f_{ij}^{[0]}]$, $\mathbf{F}^{[m]} = [f_{ij}^{[m]}]$, and $\mathbf{G}^{[m]} = [g_{ij}^{[m]}]$.

To understand the SDF protocol, we briefly review the SDF protocol with single relay, where each node has two antennas. The transmission is composed of two phases. In the first phase, S transmits the signal using Alamouti code to R and D. Thus, the received signals at R and D are represented, respectively, as

$$\mathbf{Y}^{[\mathsf{R}]} = \sqrt{\frac{P_1}{2}}\mathbf{F}^{[1]}\mathbf{X} + \mathbf{N}^{[\mathsf{R}]}$$
$$\mathbf{Y}^{[\mathsf{D1}]} = \sqrt{\frac{P_1}{2}}\mathbf{F}^{[0]}\mathbf{X} + \mathbf{N}^{[\mathsf{D1}]}$$

where $\mathbf{X} = \mathbf{A}(x_1, x_2)$ is the transmit codeword at S in the first phase, $\mathbf{F}^{[0]}$ and $\mathbf{F}^{[1]}$ denote the channel matrices of $\mathsf{S} \to \mathsf{D}$ and $\mathsf{S} \to \mathsf{R}$, respectively, and $\mathbf{N}^{[\mathsf{R}]}$ and $\mathbf{N}^{[\mathsf{D1}]}$ are the 2×2 AWGN matrices with zero-mean and unit-variance entries. During the intermediate decoding at R, R obtains the soft-decision values from the received signals using the maximal ratio combining as

$$\tilde{\mathbf{x}} \triangleq \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 \end{bmatrix}^{\mathrm{T}} = \lambda \mathbf{F}^{[1]'^{\dagger}} cv(\mathbf{Y}^{[\mathsf{R}]})$$

where

$$cv(\mathbf{Y}^{[\mathsf{R}]}) = \begin{bmatrix} y_{11}^{[\mathsf{R}]} & y_{12}^{[\mathsf{R}]*} & y_{21}^{[\mathsf{R}]} & y_{22}^{[\mathsf{R}]*} \end{bmatrix}^{\mathrm{T}} = \sqrt{\frac{P_1}{2}}\mathbf{F}^{[1]'}\mathbf{x} + cv(\mathbf{N}^{[\mathsf{R}]})$$
$$\lambda = \sqrt{\frac{2}{\|\mathbf{F}^{[1]'}\|^2 (P_1 \|\mathbf{F}^{[1]}\|^2 + 2)}}$$

where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^{\mathrm{T}}$ is the transmitted signal vector at S. And then, R transmits the following codeword into D

$$\mathbf{X}^{[\mathsf{R}]} = \mathbf{A}(\tilde{x}_1, \tilde{x}_2) = \begin{bmatrix} \tilde{x}_1 & -\tilde{x}_2^* \\ \tilde{x}_2 & \tilde{x}_1^* \end{bmatrix}.$$

In the second phase, the received signal at D is expressed as

$$\mathbf{Y}^{[\mathsf{D2}]} = \sqrt{\frac{P_2}{2}}\mathbf{G}^{[1]}\mathbf{X}^{[\mathsf{R}]} + \mathbf{N}^{[\mathsf{D2}]}$$

where $\mathbf{G}^{[1]}$ is the channel matrix of $\mathsf{R} \to \mathsf{D}$ and $\mathbf{N}^{[\mathsf{D2}]}$ denotes the 2×2 AWGN matrix with zero-mean and unit-variance entries. Converting the matrix form into the vector form gives the following alternative expression

$$cv(\mathbf{Y}^{[\mathsf{D2}]}) =$$
$$\frac{\sqrt{P_1 P_2}}{2}\lambda \|\mathbf{F}^{[1]}\|^2 \mathbf{G}^{[1]\prime}\mathbf{x} + \sqrt{\frac{P_2}{2}}\lambda \mathbf{G}^{[1]\prime}\mathbf{F}^{[1]\prime\dagger} cv(\mathbf{N}^{[\mathsf{R}]}) + cv(\mathbf{N}^{[\mathsf{D2}]}).$$

The received signal at D during two phases can be rewritten as an equivalent vector model

$$\underbrace{\begin{bmatrix} cv(\mathbf{Y}^{[\mathsf{D1}]}) \\ cv(\mathbf{Y}^{[\mathsf{D2}]}) \end{bmatrix}}_{\mathbf{y}} = \underbrace{\sqrt{\frac{P_1}{2}} \begin{bmatrix} \mathbf{F}^{[0]\prime} \\ \sqrt{\frac{P_2}{2}}\lambda \|\mathbf{F}^{[1]}\|^2 \mathbf{G}^{[1]\prime} \end{bmatrix}}_{\mathbf{H}} \mathbf{x} + \underbrace{\begin{bmatrix} cv(\mathbf{N}^{[\mathsf{D1}]}) \\ cv(\mathbf{N}^{[\mathsf{D}]}) \end{bmatrix}}_{\mathbf{n}} \tag{5.1}$$

where $cv(\mathbf{N}^{[\mathsf{D}]})$ means the equivalent noise at D in the vector form, given by

$$cv(\mathbf{N}^{[\mathsf{D}]}) = \sqrt{\frac{P_2}{2}}\lambda \mathbf{G}^{[1]\prime}\mathbf{F}^{[1]\prime\dagger} cv(\mathbf{N}^{[\mathsf{R}]}) + cv(\mathbf{N}^{[\mathsf{D2}]}).$$

The ML decoding rule for the SDF protocol can be written as

$$\begin{aligned} \hat{\mathbf{x}} &= \arg\min_{\mathbf{x}} [(\mathbf{y} - \mathbf{H}\mathbf{x})^\dagger \mathcal{K}_{\mathbf{n}}^{-1} (\mathbf{y} - \mathbf{H}\mathbf{x})] \\ &= \arg\min_{\mathbf{x}} \left[\mathbf{x}^\dagger \mathbf{H}^\dagger \mathcal{K}_{\mathbf{n}}^{-1} \mathbf{H}\mathbf{x} - 2\Re\{\mathbf{y}^\dagger \mathcal{K}_{\mathbf{n}}^{-1} \mathbf{H}\mathbf{x}\}\right] \end{aligned}$$

where $\mathcal{K}_{\mathbf{n}} = \mathcal{E}\left[\mathbf{n}\mathbf{n}^\dagger\right] = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \mathcal{K}_{cv(\mathbf{N}^{[\mathsf{D}]})} \end{bmatrix}$ with $\mathcal{K}_{cv(\mathbf{N}^{[\mathsf{D}]})} = \mathbf{I}_4 + P_2/(P_1\|\mathbf{F}^{[1]}\|^2 + 2) \cdot \mathbf{G}^{[1]\prime}\mathbf{G}^{[1]\prime\dagger}$. The ML decoder for SDF protocol [58] chooses $\hat{x}_i$ such as

$$\hat{x}_i = \arg\min_{x_i} \left[\left(\frac{P_1}{2} \|\mathbf{F}^{[0]}\|^2 + \frac{P_1 P_2 \|\mathbf{F}^{[1]}\|^2 \|\mathbf{G}^{[1]}\|^2}{2(P_1 \|\mathbf{F}^{[1]}\|^2 + P_2 \|\mathbf{G}^{[1]}\|^2 + 2)} \right) |x_i|^2 - 2\Re\{\eta_i x_i\} \right] \tag{5.2}$$

(5.2) at the top in this page, where $\begin{bmatrix} \eta_1 & \eta_2 \end{bmatrix} = \mathbf{y}^\dagger \mathcal{K}_{\mathbf{n}}^{-1} \mathbf{H}$.

Using (5.1), the conditional PEP can be written as

$$\begin{aligned} \Pr(\mathbf{x} \to \hat{\mathbf{x}} | \mathbf{H}) &= Q\left(\sqrt{\frac{1}{2} \left\| \mathcal{K}_{\mathbf{n}}^{-\frac{1}{2}} \mathbf{H}(\hat{\mathbf{x}} - \mathbf{x}) \right\|^2} \right) \\ &= Q\left(\sqrt{\frac{\delta_{\mathbf{x}}^2}{2} \left\{ \frac{P_1}{2} \|\mathbf{F}^{[0]}\|^2 + \frac{P_1 P_2 \|\mathbf{F}^{[1]}\|^2 \|\mathbf{G}^{[1]}\|^2}{2(P_1 \|\mathbf{F}^{[1]}\|^2 + P_2 \|\mathbf{G}^{[1]}\|^2 + 2)} \right\}} \right) \\ &= Q\left(\sqrt{\frac{\delta_{\mathbf{x}}^2}{2} \left(\gamma_0 + \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \right)} \right) \end{aligned} \tag{5.3}$$

where $Q(x) = \int_x^\infty e^{-u^2/2}/\sqrt{2\pi} du$, $\delta_{\mathbf{x}} = \|\hat{\mathbf{x}} - \mathbf{x}\|$, and

$$\begin{aligned} \gamma_0 &= \frac{P_1}{2} \|\mathbf{F}^{[0]}\|^2 \sim \mathcal{G}\left(4, \sigma_{\mathsf{SD}}^2 P_1/2\right) \\ \gamma_1 &= \frac{P_1}{2} \|\mathbf{F}^{[1]}\|^2 \sim \mathcal{G}\left(4, \sigma_{\mathsf{SR}}^2 P_1/2\right) \\ \gamma_2 &= \frac{P_2}{2} \|\mathbf{G}^{[1]}\|^2 \sim \mathcal{G}\left(4, \sigma_{\mathsf{RD}}^2 P_2/2\right). \end{aligned}$$

It is shown in [26] that the PEP depends on the instantaneous end-to-end SNR, γ_{eq}, which is the sum of two SNRs, i.e.,

$$\gamma_{\text{eq}} = \gamma_0 + \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}. \tag{5.4}$$

And also, (5.4) can be upper and lower bounded as

$$\gamma_0 + \frac{\gamma_1 \gamma_2}{c(\gamma_1 + \gamma_2)} \le \gamma_{\text{eq}} \le \gamma_0 + \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \tag{5.5}$$

where $c > 1 + (\gamma_1 + \gamma_2)^{-1}$. Since $Q(x)$ is monotonically decreasing for $x \geq 0$, substitution of (5.5) into (5.3) leads to the following inequalities

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) \leq Q\left(\sqrt{\frac{1}{2}\left\{\gamma_0 + \frac{\gamma_1\gamma_2}{c(\gamma_1+\gamma_2)}\right\}\delta_{\mathbf{x}}^2}\right)$$
$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) \geq Q\left(\sqrt{\frac{1}{2}\left\{\gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\right\}\delta_{\mathbf{x}}^2}\right). \tag{5.6}$$

Furthermore, the Q-function is bounded as

$$\sum_{n=1}^{N} a_n \exp\left(-b_{n-1}u\right) \leq Q(\sqrt{u}) \leq \sum_{n=1}^{N} a_n \exp\left(-b_n u\right) \tag{5.7}$$

for $a_n = (\theta_n - \theta_{n-1})/\pi$ and $b_n = 1/(2\sin^2\theta_n)$ for $n = 1, \cdots, N$ with $\theta_0 = 0$ and $\theta_N = \pi/2$. Then, we can bound the PEP by averaging the conditional PEP in (5.6) over $\mathbf{H}$ using the Q-function inequality in (5.7). It gives the upper and lower bounds on the average PEP for the SDF protocol [26] as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \leq \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_n\delta_{\mathbf{x}}^2}{2}\right)\mathcal{M}_{H(\gamma_1,\gamma_2)}\left(\frac{b_n\delta_{\mathbf{x}}^2}{4c}\right) \tag{5.8}$$

and

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \geq \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_{n-1}\delta_{\mathbf{x}}^2}{2}\right)\mathcal{M}_{H(\gamma_1,\gamma_2)}\left(\frac{b_{n-1}\delta_{\mathbf{x}}^2}{4}\right) \tag{5.9}$$

where $\mathcal{M}_X(\cdot)$ is the moment generating function (MGF) for random variable X and $H(a,b) \triangleq 2ab/(a+b)$.

5.3. SDF Protocol with the Conventional Multiple-Relay Transmission

In this section, the PEP and diveristy order of the 'conventional' SDF cooperative networks with multiple relays are derived, where all relays participate in the second phase transmission.

5.3.1. System Model

The signal transmission in the cooperative networks is composed of two phases. In the first phase, S transmits the signals using Alamouti code to $\mathsf{R}_m,\ m = 1, \cdots, M$ and D. The received signals at R_m and D are represented, respectively, as

$$\begin{aligned} \mathbf{Y}^{[\mathsf{R}_m]} &= \sqrt{\frac{P_1}{2}}\mathbf{F}^{[m]}\mathbf{X} + \mathbf{N}^{[\mathsf{R}_m]}, \quad m = 1, \cdots, M \\ \mathbf{Y}^{[\mathsf{D}1]} &= \sqrt{\frac{P_1}{2}}\mathbf{F}^{[0]}\mathbf{X} + \mathbf{N}^{[\mathsf{D}1]} \end{aligned} \tag{5.10}$$

where $\mathbf{X} = \mathbf{A}(x_1, x_2)$ is the transmit codeword for the message vector $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^{\mathrm{T}}$ at S in the first phase, $\mathbf{F}^{[0]}$ and $\mathbf{F}^{[m]}$ denote the channel matrices of $\mathsf{S} \rightarrow \mathsf{D}$ and $\mathsf{S} \rightarrow \mathsf{R}_m$, respectively, and $\mathbf{N}^{[\mathsf{R}_m]}$ and $\mathbf{N}^{[\mathsf{D}1]}$ are the 2×2 AWGN matrices with zero-mean and unit-variance entries. Transforming matrix form into vector form, (5.10) can be rewritten as

$$\begin{aligned} cv(\mathbf{Y}^{[\mathsf{R}_m]}) &= \begin{bmatrix} y_{11}^{[\mathsf{R}_m]} & y_{12}^{[\mathsf{R}_m]*} & y_{21}^{[\mathsf{R}_m]} & y_{22}^{[\mathsf{R}_m]*} \end{bmatrix}^{\mathrm{T}} \\ &= \sqrt{\frac{P_1}{2}}\mathbf{F}^{[m]\prime}\mathbf{x} + cv(\mathbf{N}^{[\mathsf{R}_m]}) \\ cv(\mathbf{Y}^{[\mathsf{D}1]}) &= \begin{bmatrix} y_{11}^{[\mathsf{D}1]} & y_{12}^{[\mathsf{D}1]*} & y_{21}^{[\mathsf{D}1]} & y_{22}^{[\mathsf{D}1]*} \end{bmatrix}^{\mathrm{T}} \end{aligned}$$

$$= \sqrt{\frac{P_1}{2}}\mathbf{F}^{[0]'}\mathbf{x} + cv(\mathbf{N}^{[\mathsf{D1}]}).$$

During the intermediate decoding at R_m, the soft-decision values are obtained from the received signals using the maximal ratio combining as

$$\tilde{\mathbf{x}}^{[m]} \triangleq \begin{bmatrix} \tilde{x}_1^{[m]} & \tilde{x}_2^{[m]} \end{bmatrix}^{\mathrm{T}} = \lambda_m \mathbf{F}^{[m]'^{\dagger}} cv(\mathbf{Y}^{[\mathsf{R}_m]})$$

where

$$\lambda_m = \sqrt{\frac{2}{\|\mathbf{F}^{[m]}\|^2(P_1\|\mathbf{F}^{[m]}\|^2 + 2)}}.$$

And then, R_m transmits the following codeword to D

$$\mathbf{X}^{[\mathsf{R}_m]} = \mathbf{A}(\tilde{x}_1^{[m]}, \tilde{x}_2^{[m]}) = \begin{bmatrix} \tilde{x}_1^{[m]} & -\tilde{x}_2^{[m]*} \\ \tilde{x}_2^{[m]} & \tilde{x}_1^{[m]*} \end{bmatrix}.$$

In the second phase, the re-encoded codewords of M relays are transmitted during M time slots and the received signal at D in the mth time slot is expressed as

$$\mathbf{Y}_m^{[\mathsf{D2}]} = \sqrt{\frac{P_2}{2}}\mathbf{G}^{[m]}\mathbf{X}^{[\mathsf{R}_m]} + \mathbf{N}_m^{[\mathsf{D2}]}, \quad m = 1, \cdots, M$$

where $\mathbf{G}^{[m]}$ is the channel matrix of $\mathsf{R}_m \rightarrow \mathsf{D}$ and $\mathbf{N}_m^{[\mathsf{D2}]}$ denotes the 2×2 AWGN matrix with each element having zero-mean and unit-variance. The signals during the M time slots in the second phase are transmitted from M relays in the orthogonal transmission manner.

Converting the matrix form into the vector form gives the following alternative expression

$$cv\left(\mathbf{Y}_m^{[\mathsf{D2}]}\right) = \frac{\sqrt{P_1P_2}}{2}\lambda_m\|\mathbf{F}^{[m]}\|^2\mathbf{G}^{[m]\prime}\mathbf{x} + \underbrace{\sqrt{\frac{P_2}{2}}\lambda_m\mathbf{G}^{[m]\prime}\mathbf{F}^{[m]\prime\dagger}cv\left(\mathbf{N}^{[\mathsf{R}_m]}\right) + cv\left(\mathbf{N}_m^{[\mathsf{D2}]}\right)}_{cv(\mathbf{N}_m^{[\mathsf{D}]})}$$

where $cv\Big(\mathbf{N}_m^{[\mathsf{D}]}\Big)$ means the equivalent noise at D in the second phase transmission.

The received signals at D in the both phases can be rewritten as an equivalent vector model

$$\underbrace{\begin{bmatrix} cv(\mathbf{Y}^{[\mathsf{D1}]}) \\ cv(\mathbf{Y}_1^{\mathsf{D2}}) \\ \vdots \\ cv(\mathbf{Y}_M^{\mathsf{D2}}) \end{bmatrix}}_{\mathbf{y}} = \underbrace{\sqrt{\frac{P_1}{2}}\begin{bmatrix} \mathbf{F}^{[0]\prime} \\ \sqrt{\frac{P_2}{2}}\lambda_1\|\mathbf{F}^{[1]}\|^2\mathbf{G}^{[1]\prime} \\ \vdots \\ \sqrt{\frac{P_2}{2}}\lambda_M\|\mathbf{F}^{[M]}\|^2\mathbf{G}^{[M]\prime} \end{bmatrix}}_{\mathbf{H}}\mathbf{x} + \underbrace{\begin{bmatrix} cv(\mathbf{N}^{[\mathsf{D1}]}) \\ cv(\mathbf{N}_1^{[\mathsf{D}]}) \\ \vdots \\ cv(\mathbf{N}_M^{[\mathsf{D}]}) \end{bmatrix}}_{\mathbf{n}}. \tag{5.11}$$

The ML decoding rule for (5.11) of the SDF protocol is as follows:

$$\begin{aligned} \hat{\mathbf{x}} &= \arg\min_{\mathbf{x}}[(\mathbf{y}-\mathbf{H}\mathbf{x})^\dagger\mathcal{K}_{\mathbf{n}}^{-1}(\mathbf{y}-\mathbf{H}\mathbf{x})] \\ &= \arg\min_{\mathbf{x}}\left[\mathbf{x}^\dagger\mathbf{H}^\dagger\mathcal{K}_{\mathbf{n}}^{-1}\mathbf{H}\mathbf{x} - 2\Re\{\mathbf{y}^\dagger\mathcal{K}_{\mathbf{n}}^{-1}\mathbf{H}\mathbf{x}\}\right] \end{aligned} \tag{5.12}$$

$$\hat{x}_i = \underset{x_i}{\arg\min}\left[\left(\frac{P_1}{2}\|\mathbf{F}^{[0]}\|^2 + \sum_{m=1}^{M}\frac{P_1P_2\|\mathbf{F}^{[m]}\|^2\|\mathbf{G}^{[m]}\|^2}{2(P_1\|\mathbf{F}^{[m]}\|^2 + P_2\|\mathbf{G}^{[m]}\|^2 + 2)}\right)|x_i|^2 - 2\Re\{\eta_i x_i\}\right]. \tag{5.13}$$

---

where

$$\mathcal{K}_{\mathbf{n}} = \mathcal{E}\left[\mathbf{n}\mathbf{n}^\dagger\right] = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0}_4 & \cdots & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathcal{K}_{cv\left(\mathbf{N}_1^{[\mathsf{D}]}\right)} & \mathbf{0}_4 & \vdots \\ \mathbf{0}_4 & \mathbf{0}_4 & \ddots & \mathbf{0}_4 \\ \mathbf{0}_4 & \cdots & \mathbf{0}_4 & \mathcal{K}_{cv\left(\mathbf{N}_M^{[\mathsf{D}]}\right)} \end{bmatrix}$$

with

$$\mathcal{K}_{cv(\mathbf{N}_\mathbf{m}^{[\mathsf{D}]})} = \mathbf{I}_4 + \frac{P_2}{P_1\|\mathbf{F}^{[m]}\|^2 + 2}\mathbf{G}^{[m]\prime}\mathbf{G}^{[m]\prime\dagger}.$$

The ML decoder in (5.12) can be restated as (5.13) at the top in this page, where $\mathbf{y}^\dagger\mathcal{K}_{\mathbf{n}}^{-1}\mathbf{H} = \begin{bmatrix}\eta_1 & \eta_2\end{bmatrix}$.

5.3.2. PEP and Diversity Order for the Conventional Scheme

Let γ_0, $\gamma_{m,1}$, and $\gamma_{m,2}$ be the SNRs of $\mathsf{S} \to \mathsf{D}$, $\mathsf{S} \to \mathsf{R}_m$, and $\mathsf{R}_m \to \mathsf{D}$ links defined by $\gamma_0 = P_1\|\mathbf{F}^{[0]}\|^2/2$, $\gamma_{m,1} = P_1\|\mathbf{F}^{[m]}\|^2/2$, and $\gamma_{m,2} = P_2\|\mathbf{G}^{[m]}\|^2/2$, respectively. Then, the instantaneous end-to-end SNR for the conventional multiple-relay transmission under ML decoder can be written as

$$\gamma_{\text{eq}} = \gamma_0 + \sum_{m=1}^{M}\frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1} + \gamma_{m,2} + 1} \tag{5.14}$$

where $\gamma_0 \sim \mathcal{G}\left(4, \frac{\sigma^2_{\mathsf{SD}} P_1}{2}\right)$, $\gamma_{m,1} \sim \mathcal{G}\left(4, \frac{\sigma^2_{\mathsf{SR}_m} P_1}{2}\right)$, and $\gamma_{m,2} \sim \mathcal{G}\left(4, \frac{\sigma^2_{\mathsf{R}_m\mathsf{D}} P_2}{2}\right)$, respectively.

Similar to (5.6) in the single relay case, the upper and lower bounds on the conditional PEP can be expressed as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) \le Q\left(\sqrt{\frac{\delta^2_{\mathbf{x}}}{2}\left\{\gamma_0 + \sum_{m=1}^{M}\left(\frac{\gamma_{m,1}\gamma_{m,2}}{c_m(\gamma_{m,1}+\gamma_{m,2})}\right)\right\}}\right)$$
$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}|\mathbf{H}) \ge Q\left(\sqrt{\frac{\delta^2_{\mathbf{x}}}{2}\left\{\gamma_0 + \sum_{m=1}^{M}\left(\frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1}+\gamma_{m,2}}\right)\right\}}\right) \quad (5.15)$$

where $c_m > 1 + (\gamma_{m,1} + \gamma_{m,2})^{-1}$. By averaging the conditional PEP over $\mathbf{H}$, the upper and lower bounds on the PEP for the conventional multiple-relay transmission under ML decoder are derived as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \le \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_n}{2}\delta^2_{\mathbf{x}}\right) \prod_{m=1}^{M}\left[\mathcal{M}_{\gamma_m}\left(\frac{b_n}{4c_m}\delta^2_{\mathbf{x}}\right)\right]$$
$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) \ge \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_n}{2}\delta^2_{\mathbf{x}}\right) \prod_{m=1}^{M}\left[\mathcal{M}_{\gamma_m}\left(\frac{b_n}{4}\delta^2_{\mathbf{x}}\right)\right] \quad (5.16)$$

where $\gamma_m = H(\gamma_{m,1}, \gamma_{m,2})$. Note that Song [59] makes use of approximation of PEP and then derives a diversity order based on this approximation. However, for completeness of the analysis, the tightness of approximation of PEP at high SNR should be examined. Therefore, the upper and lower bounds on the PEP are derived and diversity orders are obtained based on these bounds instead of approximation of PEP in this dissertation.

For the sake of tractability, let us assume that the uniform power allocation is used between the source and relays, i.e., $P_1 = P_2 = P/(M+1)$.

Using the results in [26], the MGFs of γ_0 and γ_m are expressed as

$$\mathcal{M}_{\gamma_0}(s) = \left(1 + \frac{P}{2(M+1)}s\right)^{-4}$$
$$\mathcal{M}_{\gamma_m}(s) = {}_2F_1\left(4, 8; \frac{9}{2}; -\frac{P}{4(M+1)}s\right) \quad (5.17)$$

where

$${}_2F_1(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a} dt$$

for $\Re\{c\} > \Re\{b\} > 0$ and $|\arg(1-z)| < \pi$. Note that the diversity order for $\mathsf{S} \to \mathsf{R}_m \to \mathsf{D}$ link in (5.17) is four [26]. From the above results, we conclude that the diversity order of SDF cooperative communication with conventional multiple-relay transmission is $4(M+1)$, which is maximally achievable since $4(M+1)$ is the total number of distinct paths from S to D.

5.4. SDF Protocol with the Best Relay Selection

In contrast to the conventional multiple-relay transmission, we consider the case when the only one relay participates in the second phase. For this case, it is important to select the relay so as to improve the system performance such as error rate or capacity. It is clear that from (5.3), the instantaneous end-to-end SNR is a good criterion for the relay selection. In this section, the PEP and diversity order of the SDF cooperative networks with the best relay selection are derived.

5.4.1. System Model

The signal transmission of the best relay selection scheme in the first phase is the same as the conventional scheme. In contrast to the conventional multiple-relay transmission, the signal in the second phase of the best relay selection scheme is transmitted from only one relay $\mathsf{R}_{\hat{m}}$ according to the relay selection criterion

$$\hat{m} = \arg\max_{m} \left\{ \frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1} + \gamma_{m,2} + 1} \right\}. \tag{5.18}$$

This criterion assumes that information on $\gamma_{m,1}$ for each relay has to be notified to the destination node. Here, information is only the channel norm between S and R_m. Note that this criterion guarantees the maximum channel capacity as well as the minimum PEP.

Then, the selected $\hat{m}$th relay $\mathsf{R}_{\hat{m}}$ transmits the following codeword to D

$$\mathbf{X}^{[\mathsf{R}_{\hat{m}}]} = \mathbf{A}(\tilde{x}_1^{[\hat{m}]}, \tilde{x}_2^{[\hat{m}]})^{\mathrm{T}} = \begin{bmatrix} \tilde{x}_1^{[\hat{m}]} & -\tilde{x}_2^{[\hat{m}]*} \\ \tilde{x}_2^{[\hat{m}]} & \tilde{x}_1^{[\hat{m}]*} \end{bmatrix}.$$

In the second phase, the destination D receives the signal from the $\hat{m}$th relay as

$$\mathbf{Y}^{[\mathsf{D2}]} = \sqrt{\frac{P_2}{2}}\mathbf{G}^{[\hat{m}]}\mathbf{X}^{[\mathsf{R}_{\hat{m}}]} + \mathbf{N}^{[\mathsf{D2}]}$$

where $\mathbf{G}^{[\hat{m}]}$ is the channel matrix of $\mathsf{R}_{\hat{m}} \rightarrow \mathsf{D}$ and $\mathbf{N}^{[\mathsf{D2}]}$ denotes the 2×2 AWGN matrix with zero-mean and unit-variance entries. Converting the matrix equation into the vector form gives us the following alternative expression

$$cv(\mathbf{Y}^{[\mathsf{D2}]}) =$$
$$\frac{\sqrt{P_1P_2}}{2}\lambda_{\hat{m}}\|\mathbf{F}^{[\hat{m}]}\|^2\mathbf{G}^{[\hat{m}]\prime}\mathbf{x} + \sqrt{\frac{P_2}{2}}\lambda_{\hat{m}}\mathbf{G}^{[\hat{m}]\prime}\mathbf{F}^{[\hat{m}]\prime\dagger}cv(\mathbf{N}^{[\mathsf{R}_{\hat{m}}]}) + cv(\mathbf{N}^{[\mathsf{D2}]}).$$

The received signal at D in both phases can be rewritten as an equivalent vector model

$$\underbrace{\begin{bmatrix} cv(\mathbf{Y}^{[\mathsf{D1}]}) \\ cv(\mathbf{Y}^{[\mathsf{D2}]}) \end{bmatrix}}_{\mathbf{y}} = \underbrace{\sqrt{\frac{P_1}{2}}\begin{bmatrix} \mathbf{F}^{[0]\prime} \\ \sqrt{\frac{P_2}{2}}\lambda_{\hat{m}}\|\mathbf{F}^{[\hat{m}]}\|^2\mathbf{G}^{[\hat{m}]\prime} \end{bmatrix}}_{\mathbf{H}}\mathbf{x} + \underbrace{\begin{bmatrix} cv(\mathbf{N}^{[\mathsf{D1}]}) \\ cv(\mathbf{N}^{[\mathsf{D}]}) \end{bmatrix}}_{\mathbf{n}} \quad (5.19)$$

where $cv(\mathbf{N}^{[\mathsf{D}]})$ means the equivalent noise at D in the vector form given by

$$cv(\mathbf{N}^{[\mathsf{D}]}) = \sqrt{\frac{P_2}{2}}\lambda_{\hat{m}}\mathbf{G}^{[\hat{m}]\prime}\mathbf{F}^{[\hat{m}]\prime\dagger}cv(\mathbf{N}^{[\mathsf{R}_{\hat{m}}]}) + cv(\mathbf{N}^{[\mathsf{D2}]}).$$

The ML decoder for the best relay selection is the same as the one for a single relay case.

5.4.2. PEP and Diversity Order for the Best Relay Scheme

In the best relay selection scheme, the relay $\mathsf{R}_{\hat{m}}$ selected according to the selection criterion in (5.18) transmits the signals with power P_2 in the second phase. For the easy derivation of PEP, it is assumed that the uniform power allocation is used between S and $\mathsf{R}_{\hat{m}}$, i.e., $P_1 = P_2 = P/2$. Let γ_0, $\gamma_{\hat{m},1}$, and $\gamma_{\hat{m},2}$ be the SNRs of S $\rightarrow$ D, S $\rightarrow$ $\mathsf{R}_{\hat{m}}$, and $\mathsf{R}_{\hat{m}}$ $\rightarrow$ D links defined by $\gamma_0 = P\|\mathbf{F}^{[0]}\|^2/4$, $\gamma_{\hat{m},1} = P\|\mathbf{F}^{[\hat{m}]}\|^2/4$, and $\gamma_{\hat{m},2} =$

$P\|\mathbf{G}^{[\hat{m}]}\|^2/4$, respectively. Then, the instantaneous end-to-end SNR for best relay selection can be expressed as

$$\begin{aligned}\gamma_{\text{eq}} &= \gamma_0 + \max_m \frac{\gamma_{m,1}\gamma_{m,2}}{\gamma_{m,1}+\gamma_{m,2}+1} \\ &= \gamma_0 + \frac{\gamma_{\hat{m},1}\gamma_{\hat{m},2}}{\gamma_{\hat{m},1}+\gamma_{\hat{m},2}+1}\end{aligned} \tag{5.20}$$

where $\gamma_0 \sim \mathcal{G}\left(4, \sigma^2_{\mathsf{SD}}P/4\right)$, $\gamma_{\hat{m},1} \sim \mathcal{G}\left(4, \sigma^2_{\mathsf{SR}_{\hat{m}}}P/4\right)$, and $\gamma_{\hat{m},2} \sim \mathcal{G}\left(4, \sigma^2_{\mathsf{R}_{\hat{m}}\mathsf{D}}P/4\right)$, respectively.

Using the similar approach to (5.6) in the single-relay transmission, the conditional PEP of the SDF protocol with the best relay selection can be bounded by

$$\begin{aligned}\Pr(\mathbf{x}\to\hat{\mathbf{x}}|\mathbf{H}) &\le Q\left(\sqrt{\frac{1}{2}\left\{\gamma_0 + \frac{\gamma_{\hat{m},1}\gamma_{\hat{m},2}}{c_{\hat{m}}(\gamma_{\hat{m},1}+\gamma_{\hat{m},2})}\right\}\delta^2_{\mathbf{x}}}\right) \\ \Pr(\mathbf{x}\to\hat{\mathbf{x}}|\mathbf{H}) &\ge Q\left(\sqrt{\frac{1}{2}\left\{\gamma_0 + \frac{\gamma_{\hat{m},1}\gamma_{\hat{m},2}}{\gamma_{\hat{m},1}+\gamma_{\hat{m},2}}\right\}\delta^2_{\mathbf{x}}}\right)\end{aligned} \tag{5.21}$$

for $c_{\hat{m}} > 1 + (\gamma_{\hat{m},1}+\gamma_{\hat{m},2})^{-1}$.

Thus, the average PEP for the SDF cooperative networks with the best relay selection scheme is bounded as

$$\Pr(\mathbf{x}\to\hat{\mathbf{x}}) \le \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_n\delta^2_{\mathbf{x}}}{2}\right)\mathcal{M}_{\gamma_{\max}}\left(\frac{b_n\delta^2_{\mathbf{x}}}{4c_{\hat{m}}}\right) \tag{5.22}$$

$$\Pr(\mathbf{x}\to\hat{\mathbf{x}}) \ge \sum_{n=1}^{N} a_n \mathcal{M}_{\gamma_0}\left(\frac{b_{n-1}\delta^2_{\mathbf{x}}}{2}\right)\mathcal{M}_{\gamma_{\max}}\left(\frac{b_{n-1}\delta^2_{\mathbf{x}}}{4}\right) \tag{5.23}$$

where $\gamma_{\max} \triangleq H(\gamma_{\hat{m},1}, \gamma_{\hat{m},2})$.

Deriving the diversity order directly from the above inequalities is not an easy task. Instead, we will use the following relation:

$$\begin{aligned}\Pr(\gamma_{\max} \le \gamma) &= \Pr(\gamma_1 \le \gamma, \cdots, \gamma_M \le \gamma) \\ &= \Pr\left(\lim_{r\to\infty} [\gamma_1^r + \cdots + \gamma_M^r \le \gamma^r]\right).\end{aligned}$$

Since γ_m's are independent, the MGF of $\gamma_{\max}$ equals to the Mth power of the MGF of γ_m^r, i.e.,

$$\mathcal{M}_{\gamma_{\max}}(s) = \mathcal{M}_{\sum_{m=1}^M \gamma_m^r}(s) = \left[\mathcal{M}_{\gamma_m^r}(s)\right]^M$$

as $r \to \infty$. As γ_m has Fox's H-function distribution [61], so does γ_m^r. From the property of Fox's H-function, the MGF of γ_m^r can be written as in the following theorem.

Theorem 5.1 ([15])**.** The asymptotic MGF of $\gamma_{\max}$ is represented as

$$\mathcal{M}_{\gamma_{\max}}(s) \approx \left\{\frac{1}{2^{10}3^2}\left(\frac{\Omega}{2}\right)^{-4}\left[3 + \frac{95}{2^7}\left(\frac{\Omega}{2}\right)^{-4}\log\left(\frac{\Omega}{2}\right)\right]\right\}^M.$$

From the above result, we can conclude that the diversity order of $\mathsf{S} \to \mathsf{R}_{\hat{m}} \to \mathsf{D}$ is $4M$. Since the diversity order of $\mathsf{S} \to \mathsf{D}$ is four, the diversity order of the cooperative network with the best relay selection under ML decoder becomes $4(M+1)$, which is the same as that of the conventional multiple relay scheme.

5.5. Simulation Results

It is assumed that the channel is Rayleigh-faded and frequency-flat quasi-static, i.e., the channel state does not change within one phase but

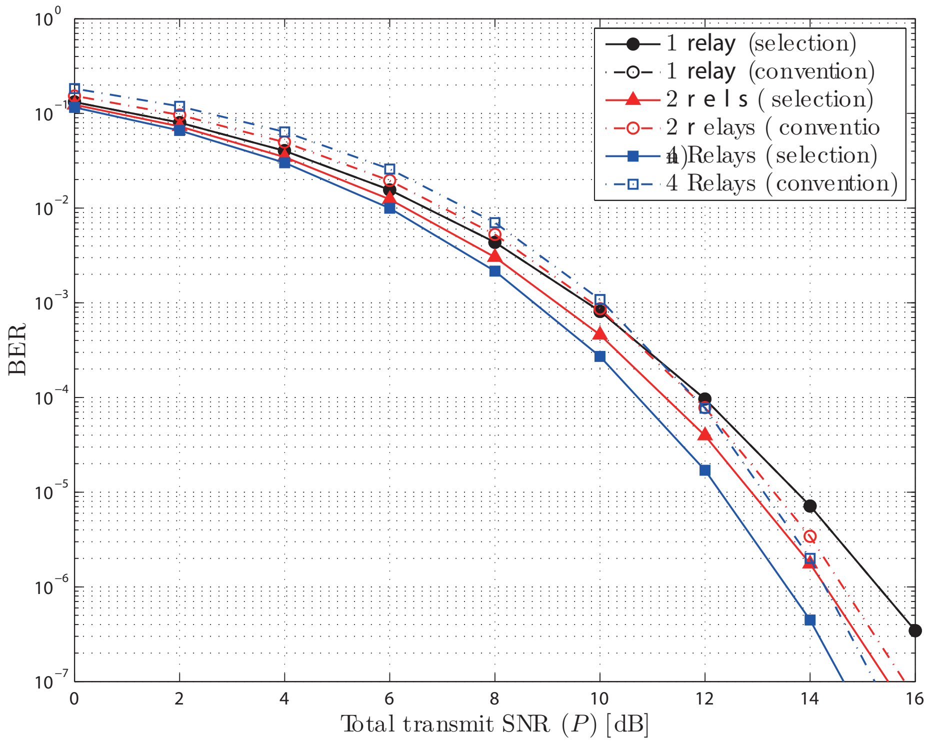

Figure 5.2: Comparison of BERs of SDF with the conventional multiple-relay transmission vs. best relay selection.

varies independently from phase to phase. QPSK is used. For the sake of simplicity, the symmetric channel is considered, i.e., $\sigma^2_{\mathsf{SD}} = \sigma^2_{\mathsf{SR}_m} = \sigma^2_{\mathsf{R}_m\mathsf{D}} = 1$. The total transmit power during two phases is set to P. The cooperative networks with M relays equipped with two antennas both at the transmitter and receiver are considered.

In Fig. 5.2, two schemes with uniform power allocation are compared in terms of BER. From the numerical results, the best relay selection scheme outperforms the conventional multiple-relay transmission in the all SNR region. Furthermore, since less time slots are used for the transmission of

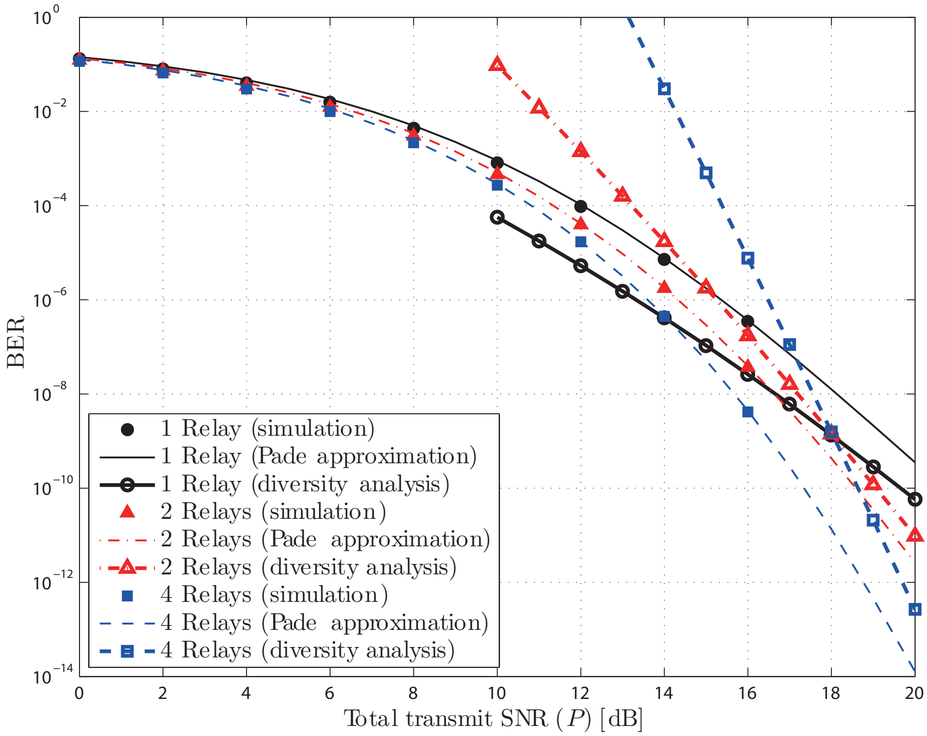

Figure 5.3: BERs and diversity order of SDF cooperative network with the proposed best relay selection (The diversity plots are scaled for the clarity of view).

the best relay selection scheme in the second phase, the best relay selection scheme also has an advantage of the throughput (spectral efficiency) against the conventional one.

Fig. 5.3 shows the analytical and numerical results for the cooperative network with the proposed best relay selection when the uniform power allocation between S and $\mathsf{R}_{\hat{m}}$ is used, i.e., $P_1 = P_2 = P/2$. The 'Pade approximation' results are obtained using Padé approximation technique [63]. And also, diversity orders from PEP are plotted. From this result, we can confirm that the proposed scheme has full diversity order. In fact, Figs. 5.2

and 5.3 are the same as those in [59].

5.6. Conclusion

In this chapter, the performance of the SDF cooperative networks with two different relay-assisted transmission schemes has been analyzed. The PEP of the SDF cooperative networks with the conventional multiple-relays transmission scheme is derived by using Gauss' hypergeometric function. And it has been shown that the conventional scheme has full diversity. Also, The diversity order for the SDF cooperative networks with the best relay selection scheme under ML decoder is obtained by using Fox's H-function. From the numerical results, it has been shown that the best relay selection scheme outperforms the conventional multiple-relay transmission scheme in terms of BER and throughput.

Chapter 6. Conclusion

In this dissertation, IC and IAC schemes based on Alamouti codes are proposed for both the MIMO interference channel and the TWRC. We confirmed that the proposed schemes enable symbol-by-symbol decoding and achieve high diversity gain. In the second part of this dissertation, the performances of cooperative communication with multiple relays using the soft-decision-and-forward protocol have been analyzed in terms of PEP.

In Chapter 2, MIMO, STCs, cooperative communications, and IA have been overviewed. The properties of STC are applicable to the cooperative communications. The performance criteria of the STC were reviewed: Rank criteria, determinant criteria, diversity-multiplexing gain tradeoff and so on. The relaying protocols are explained such as AF and DF. And also the basic concept and recent researches of IA are introduced.

In Chapter 3, we propose a method on how to apply Alamouti code to MIMO interference channels. The interference cancellation method based on Alamouti codes for the multi-access scenario can be used for the K-user interference channel, which enables the receivers to perform symbol-by-symbol decoding by cancelling interfering signals by utilizing Alamouti structure and achieve diversity order of two. Moreover it does not require CSIT unlike the IA scheme. However, the IC scheme requires more receive antennas than the IA scheme to achieve the same DoF. In order to reduce the number of receive antennas, especially for the three-user interference channel, we propose an IAC scheme based on Alamouti codes which utilizes beamforming matrices with partial CSIT to align interfering signals. We also show that while the proposed IAC scheme requires less antennas at the receivers than the IC scheme, it achieves the same performance in terms of DoF and diversity order. In fact, there may be other methods which can achieve more diversity order or reduce the number of receive antennas differently for the K-user interference channel. We leave these as a future work.

In Chapter 4, we propose a method on how to apply Alamouti code to the TWRC. The interference cancellation method based on Alamouti codes for the interference channel can be used for the TWRC, which enables the nodes to perform symbol-by-symbol decoding and achieve diversity order of two. In order to achieve more diversity gain, we propose a new two-way relaying scheme based on Alamouti codes which utilizes beamforming matrices to align signals at the relay node. From the simulation results, it is shown that the proposed scheme achieves a diversity order of four. Although we only consider the case where each node has two antennas, the proposed schemes can be easily extended to general $2M$ antennas case.

In Chapter 5, the performance of the SDF cooperative networks with two different relay-assisted transmission schemes has been analyzed. The PEP of the SDF cooperative networks with the conventional multiple-relays transmission scheme is derived by using Gauss' hypergeometric function. And it has been shown that the conventional scheme has full diversity. Also, the diversity order for the SDF cooperative networks with the best relay selection scheme under ML decoder is obtained by using Fox's H-function. From the numerical results, it has been shown that the proposed best relay selection scheme outperforms the conventional multiple-relay transmission scheme in terms of BER and throughput.

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초 록

본 논문은 알라무티 부호와 협동 통신, 그리고 간섭 정렬에 관한 다음 세 가지 연구 결과를 포함하고 있다.

- 3-사용자 다중입출력 간섭 채널에서 알라무티 부호 기반 간섭 정렬 후 제거 기법
  - K-사용자 다중입출력 간섭 채널에서 알라무티 부호 기반 간섭 제거 기법 제시
  - 3-사용자 다중입출력 간섭 채널에서 알라무티 부호 기반 간섭 정렬 후 제거 기법 제시
  - 제안된 기법들의 다이버시티 차수 분석
- 알라무티 부호 기반 양방향 중계 기법
  - 알라부티 부호를 기반으로 한 두 가지 양방향 중계 기법을 제시
- 여러 개의 중계기를 갖는 연판정 후 전달 기반 협력 통신망
  - 여러 개의 중계기를 갖는 연판정 후 전달 기반 협력 통신망에서 중계기 선택 방식 제시
  - 중계기 선택 방식의 다이버시티 차수 분석

첫째, 다중입출력 간섭 채널에서 알라무티 부호를 활용하는 기법을 제시한다. 다원 접속 채널에서의 알라무티 부호 기반 간섭 제거 기법이 K-사용자 간섭 채널에서도 활용 가능한 것을 보인다. 수신 단에서 알라무티 구조를 이용하여 간섭 신호를 제거함으로써 심볼 단위 복호가 가능하고 다이버시티 차수 2를 얻을 수 있다. 또한 간섭 정렬 기법과 달리 송신 단에서 채널 상태 정보를 필요로 하지 않는다는 이점이 있다.

그러나 알라무티 부호 기반 간섭 제거 기법이 간섭 정렬 기법과 같은 자유도를 달성하기 위해서는 수신 단에서 많은 수의 안테나를 이용해야만 한다. 수신 안테나의 수를 줄이기 위한 노력의 일환으로, 3-사용자 간섭 채널에서의 알라무티 부호 기반 간섭 정렬 후 제거 기법을 제시한다. 제안된 기법은 송신 단에서 부분적 채널 상태 정보를 필요로 하는 대신에 적은 수신 안테나를 이용하여 간섭 제거 기법과 같은 자유도 및 다이버시티 차수를 얻는다. 본 논문에서는 제안된 두 가지 기법에 대해 쌍 오류 확률을 분석하여 기존의 간섭 정렬 기법보다 우수한 다이버시티 차수를 얻을 수 있다는 것을 증명한다.

본 논문의 두 번째 결과로, 알라무티 부호를 기반으로 한 양방향 중계 기법 두 가지를 제시한다. 첫 번째 기법은 K-사용자 간섭 채널에서의 알라무티 부호 기반 간섭 제거 기법을 양방향 중계 채널에 활용한 것이고, 이를 통해 심볼 단위 복호가 가능할 뿐만 아니라 다이버시티 이득을 얻는다. 더욱 많은 다이버시티 이득을 달성하기 위해 두 번째 양방향 중계 기법에서는 빔형성 행렬을 이용하여 중계기에 신호를 정렬시킨다. 컴퓨터 모의실험을 실시하여 두 기법에 대한 비교를 통해, 제안된 두 번째 기법의 다이버시티 이득이 첫 번째 기법보다 우수하다는 결론을 도출한다.

마지막으로, 여러 개의 중계기를 갖는 연판정 후 전달 협동 통신망에서 중계기 선택 방식을 제안하고, 이의 성능을 분석한다. 제안된 중계기 선택 기법은 가장 큰 end-to-end 신호 대 잡음비를 갖는 중계기를 선택하여 전송에 참여시킨다. 중계기 선택 기법의 쌍 오류 확률과 비트 오류 확률을 분석하고, 이를 모든 중계기가 전송에 참여하는 기존 방식의 성능과 비교한다. Fox H-함수의 극한값으로부터 중계기 선택 방식과 기존

방식의 다이버시티 차수를 구한다. 두 시스템에 대한 비교를 통해, 중계기 선택 방식은 비티 오류 확률이나 전송률 측면에서 기존의 방식보다 우수한 성능을 가짐을 확인한다.

주요어: 간섭 정렬 기법, 간섭 정렬 후 제거 기법, 간섭 제거 기법, 간섭 채널, 다이버시티 차수, 쌍 오류 확률, 알라무티 부호, 양방향 중계 채널, 연판정 후 전달 기법, 자유도, 중계기 선택 기법.

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