AN EFFICIENT CELL-BY-CELL ADAPTIVE MESH REFINEMENT ALGORITMHM ON STRUCTURED GRIDS

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ABSTRACT

As an efficient AMR method on structured grids, a new cell-by-cell refinement algorithm is developed. Key idea of the new cell-by-cell refinement is to use categorized cell types which are originated from overset mesh technique. Thanks to categorized cell types, present method provides efficient cell-by-cell refinement algorithm, while preserving structured index of grid system. Through numerical test cases, the performance of present algorithm is investigated and comparison of accuracy and efficiency shows that present method is much more accurate and efficient than the case without AMR method.

INTRODUCTION

Generally speaking, in CFD society, there are several approaches for improving the accuracy and efficiency of Euler/Navier-Stokes analyses: *h*-(mesh size), *p*-(the order of the interpolant), *hp*-, and *r*-(mesh redistribution) refinement. While *h*-methods improve the accuracy by mesh refinement with a fixed, usually low-order, interpolant, *p*-methods do it by increasing the order of the interpolant with a fixed mesh size. Also, *hp*-methods and *r*-methods enhance the accuracy by combining *h*- and *p*-refinement, and by mesh movement, respectively.

In present research, the previous efforts and search for an accurate and effective way are continued to improve the resolution capability of the Godunov-type schemes and thus to reduce their numerical diffusion. Among above strategies, *h*-refinement method is mainly dealt with and an efficient cell-by-cell adaptive mesh refinement (AMR) algorithm is introduced in following sections.

ADAPTIVE MESH REFINEMENT (AMR) METHODS ON STRUCTURED GRIDS

Although the majority of research on solution adaptive gridding has been for unstructured solution methods [1], there have been meaningful progresses for structured counterpart as well [2-6]. Previous methods for solution-adaptive, structured gridding have involved globally regenerating the grid. Such techniques are referred to as grid movement (see review in ref. [2]). Another approach is to refine sub-domains of a multi-block, structured grid [3-6]. In this case discontinuous grids meet at block-to-block boundaries and appropriate coupling operators must ensure accuracy and conservation. The sub-domain refinement approach is the method that present research is aimed at. Thus, several algorithms related to sub-domain refinement are introduced in this section.

(1) Block-by-Block AMR Algorithm

This algorithm was originally proposed by M. J. Berger [3, 4] and it is one of the most popular AMR algorithms on structured-based grid system. In this approach, the refined regions consist of a small number of rectangular grid patches with finer mesh spacing than the underlying global coarse grid. These rectangular subgrids contain refined points where the error in the coarser grid solution is too high. We use rectangular subgrids so that we can use integration methods for rectangular grids whose convergence properties are well understood. These methods can be made quite efficient on vector and parallel computers.

(2) Cell-by-Cell AMR Algorithm

In previous works of [3, 4], refinement occurs in rectangular blocks and solutions must be obtained on all levels. In contrast, the approach discussed in [5] considers a cell-by-cell refinement and we solve only on the finest resolution that exists for each part of the domain. The solution is obtained on the dynamic mesh containing both coarse and fine elements, while retaining the efficiency of a hierarchical array based data structure.

(3) Add-and-Blank Lines AMR Algorithm

The locally adaptive method proposed in [6] was motivated by the idea of *iblanking* (the terminology *iblank* comes from a variable name used in computer codes). Originally, *iblanking* was a device to insert geometry into a structured grid by extending the grid inside the body, then blanking out the interior portion. The region inside the body is decoupled from the fluid via boundary conditions, which is called as chimera or overset mesh technique.

AN EFFICIENT CELL-BY-CELL AMR ALGORITHM

In previous chapter, three types of AMR on structured grids are reviewed. Keeping the characteristics of each algorithm in mind, a new AMR algorithm is proposed in this section.

In order to understand how the AMR algorithms work in actual, let us consider that one grid system has sub-domain where higher resolution is required. Firstly, let us consider the case in which block-by-block AMR algorithm is used. The refined regions of the block-by-block AMR consist of several rectangular grid patches with finer mesh spacing than the underlying global coarse grid. Figure 1(a) shows a typical configuration of grid refinement

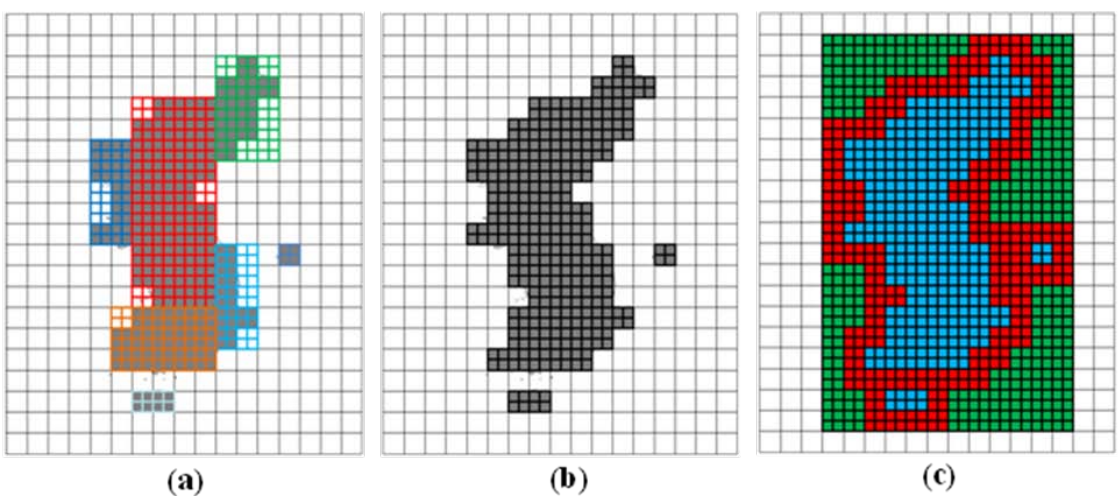

Figure 1. Comparison of refined sub-domain for the Korean Peninsula:
(a) block-by-block AMR method, (b) cell-by-cell AMR method, (c) present method
(blue: normal cell, red: fringe cell, green: hole cell)

using this method. Each patch forms a sub-domain which is independently solved and block-to-block boundaries are used to couple the sub-domains.

In Fig. 1(a), seven rectangular grid patches are involved, but the number of grid patches and formation are not determinate. Fewer grid patches lead the inclusion of more unnecessary fine grids, which decreases computational efficiency. On the other hand, more grid patches need complicated data structure and robust algorithm, although they can express more complex configuration. Thus, in order to use block-by-block AMR, efficient and robust grid patch construction algorithm has to be implemented essentially. It makes block-by-block AMR algorithm be bothersome to use widely.

Next, let us consider cell-by-cell AMR algorithm in ref. [5]. Figure 1(b) is an example of grids refined by using cell-by-cell AMR algorithm. Since grid refining is conducted by the unit of cell, this algorithm has capability to refine complicated configuration effectively. It does not include unnecessarily refined cells, but refines the target cells only, which is very charming characteristics as an AMR algorithm.

In spite of its powerful capability, this algorithm also has fatal weak point at the viewpoint of structured solver. Since meshes with different size exist simultaneously, structured index of mesh can't be used in this case. It means that the most remarkable strong points of structured solver is not available in this method. Thus, this approach needs a cell-based connectivity array data hierarchy like quadtree/octree data structure and it can be considered as a kind of approach on unstructured or Cartesian grids.

In contrast to block-by-block AMR algorithm, the refined regions of present method consist of one rectangular grid patch with finer mesh spacing than the underlying global coarse grid. The outer boundary of grid patch is determined by checking the maximum and minimum coordinates of target region. Then, refined cells in the patch are categorized into three types: *normal cell*, *fringe cell* and *hole cell*. Normal cell is the cell that is going to be solved and hole cell is the cell that is blanked. Fringe cell is the cell abutting to the outer boundary of normal cell and it can be considered as an interface between normal and hole cells. Figure 1(c) describes grid patch and three types of cells. The grid patch consists of bold lined meshes. In Fig. 1(c), blue, red and green cells are normal, fringe, and hole cells, respectively.

This cell categorization simplifies the way to couple sub-domains. Especially, for researchers who have experiences to deal with overset mesh, present grid description may be very familiar conceptually and practically. Thus, present method can be easily accepted and implemented into his/her own existing solver.

NUMERICAL RESULTS

The accuracy and efficiency of a new AMR algorithm are examined through various test cases. For each numerical test, we compare accuracy and efficiency characteristics of present AMR approach with those of conventional solver without AMR. As a time integration method, the first-order Euler simple method and the third-order TVD Runge-Kutta method is used for unsteady calculation.

Among them, the interaction of a vortex with a steady shock wave is introduced here. This shock-vortex interaction (SVI) leads to complicated flow patterns where smooth and discontinuous features, namely acoustic waves and shock waves, are present. The computational domain is defined as Ω=[0; 2]×[0; 1]. The initial condition is given by a steady normal shock wave at x=0.5 with a shock Mach number M_s. The vortex is located initially at the position $(x_c, y_c) = (0.25, 0.5)$ and moves downstream.

Numerical schlieren of shock-vortex interaction (SVI) at t =0.7 can be seen in Fig. 2. As seen in this figure, the resolution of complicated vortex shape is noticeably enhanced by using

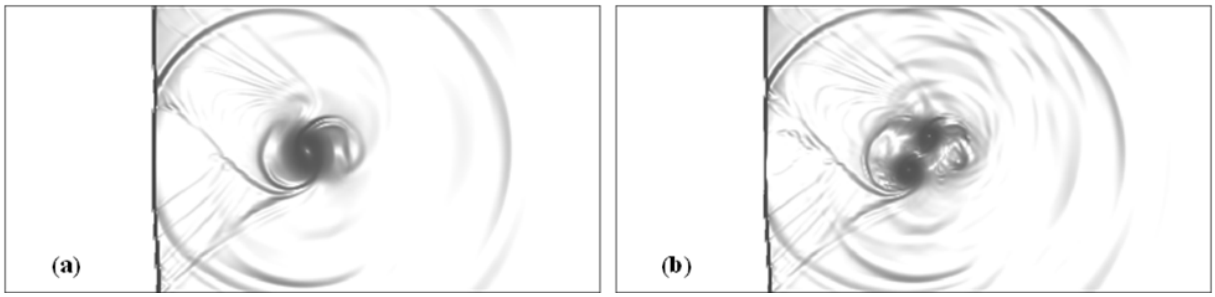

Figure 2. Numerical Schlieren of shock-vortex interaction (SVI) at t=0.7:
(a) 400×200 main grid, (b) 400×200 main grid with AMR (λ_r=2)

present AMR algorithm. Especially, one primary vortex seems to be separated into two small ones, which shows the performance of present AMR algorithm clearly.

CONCLUDING REMARKS

As an efficient AMR method on structured grids, a new cell-by-cell refining algorithm is developed. Key idea of present method is to use categorized cell type which is originated from overset mesh technique. Thanks to categorized cell types, present method provides efficient cell-by-cell refinement algorithm, while preserving structured index of grid system.

Through various numerical tests, the performance of present algorithm is investigated. In test cases, the error decrease per unit cost which is closely related to computational efficiency is used as a useful parameter to assess the efficiency of present method. Comparison of efficiency shows that present methods are much more efficient than the case without AMR method.

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REFERENCES

1. D. J. Mavripilis, *Unstructured Mesh Generation and Adaptivity*, in von Karman Inst. for Fluid Dynamics, Lecture Series 1995.02 (AGARD Publication, 1995).
2. R. Hagmeiger and J. Kok, *Adaptive Generation of Structured Grids*, in von Karman Inst. for Fluid Dynamics, Lecture Series 1996.06 (AGARD Publication, 1996)
3. Marsha J. Berger and Joseph Oliger, “Adaptive Mesh Refinement for Hyperbolic Partial Differential Equations,” *J. Comput. Phys*. 53 (1984) 484-512
4. M. J. Berger and P. Colella, “Local Adaptive Mesh Refinement for Shock Hydrodynamics,” *J. Comput. Phys.* 82 (1989) 64-84
5. J.M. Morrell, P.K. Sweby and A. Barlow, “A cell by cell anisotropic adaptive mesh ALE scheme for the numerical solution of the Euler equations,” *J. Comput. Phys.* 226 (2007) 1152.1180
6. P. A. Durbin and G. Iaccarino, “An Approach to Local Refinement of Structured Grids,” *J. Comput. Phys.* 181 (2002), 639-653