Business Cycles and Leverage in Collateral Constraints

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This paper develops a simple dynamic stochastic general equilibrium model with collateral constraints to explore the business cycle implications of financial leverage. From the model-based experiments, the degree of leverage is shown to be an important factor in amplifying the effects of collateral constraints. This finding suggests that financial leverage may affect the real economy in non-neutral ways in the course of business fluctuations. Moreover, instead of the interactions between investment and collateral price, the endogenous accumulation of collateral asset is shown to be an alternative channel through which the business cycle effects of the collateral constraints are generated. From the model simulations, we find it difficult to have both significant amplification and significant persistence at the same time. This is due to the different response patterns of investment and consumption, which are consistent with the intertemporal optimizing behaviors.

Keywords: Credit cycles, Collateral constraints, Financial leverage, Amplification, Persistence

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I. Introduction

Before the recent global financial turmoil unfolded, financial industries worldwide pursued high profits by significantly expanding leverage. Investing on margin, leveraged buyouts, or trading of highly leveraged

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derivative products was a usual and widespread practice. After observing the current global financial crisis, a natural question emerges: Is there any structural modeling framework that can systematically explain the aggregate implications of highly leveraged financial sector?

Unfortunately, the conventional RBC model as well as the textbook IS-LM model treat the financial structure as irrelevant under the assumption of a perfect capital market, as documented by Bernanke, Gertler, and Gilchrist (1999). The recent developments in the field, which improve upon this irrelevance assumption, have explicitly introduced credit market imperfections into the dynamic stochastic general equilibrium (DSGE) model. There are at least two distinct modeling approaches in introducing credit market imperfections. The first one assumes that the outcome of investment projects is costly for lenders to observe as in, *inter alia*, Bernanke and Gertler (1989). The second one assumes that borrowers cannot precommit to produce due to the inalienability of human capital as in, *inter alia*, Kiyotaki and Moore (1997).

The goal of this paper is to devise a simple DSGE model to investigate the short-run business cycle effects of financial leverage. More concretely, following the line of research after Kiyotaki and Moore (1997), this paper develops a computable variant of credit cycle model that reveals the mechanism through which leverage in borrowing constraints affects the level of amplification and persistence of business fluctuations. The model is presented in its form as simple as possible without any other friction such as nominal stickiness to avoid any complication. The application of an abstract model in this paper is intended to enhance its clarity in explaining the dynamics despite its lack of empirical flavor.

In the modeling perspectives, one of the features of the present paper distinct from the existing credit cycle literature, such as Kiyotaki and Moore (1997), Cordoba and Ripoll (2004a), and Iacoviello and Minetti (2006) to name a few, is that it introduces the concept of leverage into collateral constraints. The question is whether the total amount of borrowing should be less than or equal to the total amount of the real asset collateral in the setup of the model. In the existing models, the former is assumed not to exceed the latter. However, relaxing this assumption, that is, introducing leverage against collateral in the model, appeals to observations of the financial industry today. The industry has developed ways to stretch the supply of credit given the limits in real asset collateral. This development has opened the door for highly leveraged finances. In reality, the amount of collateral constrains the amount of borrowing but possibly not in a one-to-one fashion. Securitization is an example. The underlying real asset directly backing mortgage also backs, indirectly but effectively, the MBS backed by the mortgage and the CDO backed by the MBS. In essence, securitization can be considered a way of making the underlying collateral perform double-duty. Other examples may include short-selling and margin trading, where one can trade a huge amount of notional principal while devoting none or only a small fraction of it.

The aggregate data are also consistent with these financial-side evolutions. In the US, the average ratio of total debt outstanding (from the flow of funds accounts) to the net stock of fixed assets (from NIPA tables) have not been restricted below unity. The ratio of borrowing to fixed assets has exceeded 1 since 1999. This indicates that the amount of borrowing is affected not only by the amount of collateral but also by the degree of leverage reflecting the contemporary financial technology. In this respect, the aggregate data seem to support the introduction of leverage into the collateral constraints. To make leveraging against collateral theoretically feasible in the model, this paper introduces two assumptions: (1) a certain type of transaction cost is incurred in the process of recovery as in Iacoviello (2005) and Mendicino (2008), and (2) the commitment of the borrower to produce is limited in its effectiveness. Under these assumptions, the ratio of borrowing to collateral can either be bigger or smaller than 1.

Other than the “leverage” interpretation, what makes the present paper distinct from the previous credit cycle literature is that it proposes a different channel through which the business cycle effects of the collateral constraints are generated. In the literature, the “two-way” interaction between investment and collateral price has been focused on as a candidate mechanism that generates amplification and persistence that the conventional RBC models cannot reproduce. It should be noted that in modeling these two-way interactions, the existing studies assume the supply of collateral assets to be perfectly inelastic, abstracting from the endogenous accumulation of collateral assets. However, even narrowly defined capital, such as plant and equipment, certainly serves well as collateral in the real world. There is no need to assume that collateral assets are in a fixed supply. Notably, the rational economic agents who face borrowing constraints may have motive for accumulating collateral assets to expand borrowing capacity in anticipation of future needs. Until now, this endogenous motive for capital accumulation has been ignored, and its implications in generating credit cycle effects have not been properly studied. The present paper shuts off the two-way interaction channel and instead proposes the endogenous collateral accumulation as an alternative channel. This paper shows evidently that the effects of financial leverage can be generated through the workings of this alternative channel.

To examine the business cycle implications of leveraging against collateral, the numerical analysis in this paper raises the following questions: If the leverage in the collateral constraints is possibly non-neutral to business cycle characteristics, how does it matter? If two otherwise homogeneous economies differ only in the degree of leverage, how do some of the important business cycle moments of these two economies differ? How are they affected by the different levels of financial leverage? To answer these questions properly, this paper designs a comparative study that focuses on the differences in the magnitudes of amplification and the persistence of business fluctuations simulated from different degrees of leverage while controlling for other things being equal.

The key implication of the financial leverage revealed from the experiments in this paper is that in a highly leveraged economy, the effects of shocks are likely to appear in amplified magnitudes but do not persist longer than in the economy with low leverage. This is consistent with the two-fold findings from the impulse-response analyses as noted below. The following two findings jointly imply that the more highly leveraged the economy is in the bigger amplitude, investment deviates from its steady-state, and in the smaller amplitude, consumption deviates from its steady-state, resulting in larger amplification and smaller persistence.

(1) The response patterns of investment and consumption to a positive total factor productivity (TFP) impulse are in stark contrast. The magnitude of the responses of investment is much larger than that of consumption. In this respect, investment is a key driver of amplification. Consumption exhibits stronger persistence. Consumption expands with income in the process of capital accumulation (income effect), but more resources may be allocated to lending rather than consumption, taking advantage of the high productivity (substitution effect). The interactions of these two effects yield hump-shaped responses, implying that consumption is a more important determinant of persistence than investment.

(2) Higher leverage is associated with stronger responses of investment and weaker responses of consumption to a positive shock. When the level of financial leverage is higher, the benefits from the enlarging bor-

rowing capacity due to increase in investment are larger. This yields a stronger substitution effect, shifting more resources from consumption to investment. This substitution appears weaker when only a lower level of leveraging is allowed.

The results of the model simulations, being largely consistent with the findings from the impulse-response analyses, can be summarized as follows. Amplification increases in the degree of leverage, whereas persistence decreases in the degree of leverage. Moreover, the sizable amplification emerges only with significantly high leverage, whereas persistence generated by the present model appears to be quantitatively insignificant. However, the present model can be argued to improve the one in Cordoba and Ripoll (2004a) in that the experiments based on the present model generate bigger and better amplification results even if the effects from collateral price changes were not included.

This paper is organized as follows: Section 2 lays out the model. Section 3 studies the model's quantitative implications, and Section 4 presents the conclusion.

II. The Model

A. Economic Environment

The model economy is a discrete-time, infinite-horizon economy populated by two types of agents, households and entrepreneurs. For simplification in aggregation, we assume each to be of a continuum of unit mass.

a) The Representative Household

The representative household maximizes its expected lifetime utility as given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t' - \frac{(l_t')^\eta}{\eta} \right], \tag{1}$$

where E_0 denotes the expectation based on time-0 information, $0<\beta<1$ is the discount factor, c_t' is the consumption in the period t, and l_t' are hours of work in the period t. The parameter η governs the labor supply elasticity.

The household faces the following budget constraint:

$$c'_t + r_{t-1} \cdot b'_{t-1} = w_t \cdot l'_t + b'_t \,, \tag{2}$$

where w_t is the wage rate per hour, b'_t is the amount of borrowing in period t, and r_{t-1} is the gross rate of interest determined in period $t-1$ to be paid in t. All the relative prices, such as w_t and r_{t-1}, are expressed in terms of consumption good, which is the numeraire in this model economy. Note also that b'_t takes on negative values when the household saves or lends.

b) The Representative Entrepreneur

The representative entrepreneur is assumed to have an instantaneous log utility function in consumption as given by

$$E_0 \sum_{t=0}^{\infty} \gamma^t \log c_t \,, \tag{3}$$

where c_t denotes consumption in the period t, and $0<\gamma<1$ is the discount factor. The ex ante heterogeneity in the subjective discount rates between the entrepreneur and household facilitates the credit flows in this model economy and departs from the unconstrained optimum. We assume $\gamma<\beta$ for the purposes to be discussed later.[1]

The entrepreneur produces consumption good using the following Cobb-Douglas technology:

$$y_t = z_t \cdot k_{t-1}^{\alpha} \cdot l_t^{1-\alpha} \,, \tag{4}$$

where k_t is the capital stock in the period t, l_t employment, z_t is TFP, y_t is the amount of outputs, and α is the capital share. Capital stock evolves *via* the law of motion given by:

$$k_t = (1-\delta) \cdot k_{t-1} + i_t \,, \tag{5}$$

where i_t denotes real gross investment in t and $0<\delta<1$ depreciation rate.

[1] If the entrepreneur is patient enough to save much, then the borrowing constraint does not necessarily bind. The assumption of $\gamma<\beta$ ensures the borrowing constraint to bind in the neighborhood of the steady state, as will become clear later.

Other than the technological constraints of (4) and (5), the entrepreneur faces the following budget constraint:

$$c_t + i_t + r_{t-1} \cdot b_{t-1} = y_t - w_t \cdot l_t + b_t , \tag{6}$$

where b_t is the amount of borrowing in the period t.

From (5) and (6), the relative price of capital in terms of consumption good is assumed to be constant and one-for-one. In the existing literature, the interaction between investment and collateral asset price is focused on the candidate mechanism that generates amplification and persistence that the conventional RBC models cannot reproduce. That interaction is considered not uni-directional but rather "two-way." However, if we define capital broadly as the only tangible productive asset, the price of collateral is nothing but Tobin's q. Tobin's q should then deviate from unity to generate the credit cycle effects. Suppressing the possible deviations in the relative price of collateral, this paper essentially shuts off the so-called "two-way interactions" between asset prices and real quantities. As documented earlier in this paper, the existing literature usually models these two-way interactions by assuming a fixed supply of collateral asset. For example, Iacoviello (2005) introduced a fixed amount of real estate endowments for collateral while assuming capital not to be collateralizable. However, in the real world, capital itself functions as collateral, and the quantity of collateral assets cannot be assumed to be merely in a fixed supply.

It should be noted that because the aggregate amount of collateral asset does not vary, the endogenous motive for dynamic accumulation of collateral assets is ignored, and its implications in the business fluctuations have not been properly studied until now. However, the rational economic agents who face borrowing constraints may have motive to accumulate collateral assets to expand borrowing capacity. This paper is distinct from the existing literature in that it tries to capture the role of endogenous capital/collateral accumulation in generating the credit cycle effects.[2]

[2] In this sense, shutting off the two-way interaction channel in this paper is an intentional modeling choice to some extent. This paper does not try to compare the relative significance of the two different channels. This paper does not assert the dominance of the alternative channel over the two-way interaction channel at all. Thus, to construct more general model nesting, both channels are not required here.

Another critical feature of the model economy is that producers, as most likely borrowers, cannot precommit to produce fully efficiently, in the sense that the output produced may be strictly below the production possibilities frontier. Producers may choose to exert a low level of effort in utilizing factors of production, idling in some or simply absconding or walking away from the contracts and leaving behind their undepreciated assets and inefficient amount of output. This partial commitment for the producers due to their inalienability of human capital (Hart and Moore 1994) or moral hazard problem requires the collateralization of the asset holdings of the borrower.

The present model shares the assumption of this lack of commitment with other credit cycle models such as Kiyotaki and Moore (1997). What distinguishes the present model is that it assumes partial commitment, not noncommitment *per se*. In the other models where noncommitments are assumed, producers can leave behind only the collateral asset without any output produced. The amount of borrowing cannot exceed the value of the collateral. In Kiyotaki and Moore (1997), Kiyotaki (1998), Cordoba and Ripoll (2004a, 2004b), or Pintus and Wen (2008), the former equals the latter, whereas in Kiyotaki and Moore (2005, 2008), Iacoviello (2005), Song (2005), Iacoviello and Minetti (2006), or Mendicino (2008), the former is the fraction of the latter due to the liquidity characteristics or the existence of a certain type of transaction cost in the process of recovery.[3]

In this paper, the amount of borrowing is allowed to exceed the amount of collateral. Why does the assumption of $\theta < 1$ need to be relaxed? How can it be justified to introduce leverage in the collateral constraints? Casual observations indicate that there are heavy volumes of unsecured corporate bonds trading that are backed purely by cash-flows without requiring collateral assets. Furthermore, finding cases where the value of collateral posted falls short of the amount of borrowing is not difficult under the condition that the borrower has other sources of repayments. However, this is not the entire story.

Allowing the total amount of borrowing to exceed that of collateral, that is, introducing leverage into the model, appeals to observations from the recent financial industry. The industry has expanded leverage significantly in the pursuit of high profits. Faced with limits in real asset collateral, the industry has developed ways to stretch the supply

[3] Kiyotaki and Moore (1997, p. 221 footnote 10) already point out the need to consider transaction cost in the course of repossession by creditors.

of credit. This development has facilitated opportunities for highly leveraged finances. Investing on margin, leveraged buyouts, or derivative trades has become the usual practice. That famous global investment banks invest 30 to 60 times their equities is a well-known fact. Securitization is an example of a method of making underlying collateral perform double-duty. Other examples may include shortselling and margin trading. For instance, in Korea, holding 40% of the stocks to be sold short was permissible. This implies that about 2.5 times of the underlying stock holdings could be traded. In countries where the so-called "naked" short-selling is permitted, one does not even have to hold the stocks to sell at all. In case of derivative trading such as futures, one can trade huge amounts of notional principal while devoting only a small amount of margin requirements.[4]

The question then is whether these environmental changes in financial technology are neutral to the patterns or characteristics of business fluctuations. If they are non-neutral, the question becomes how their effects can be explained and characterized in the DSGE framework. In dealing with these questions, this paper builds a computable DSGE model as simple as possible to obtain meaningful qualitative business cycle implications of financial leverage. This necessitates relaxing the assumption of $\theta \leq 1$.

The aggregate data appear to support the introduction of leverage into collateral constraints. As shown in Figure 1, in the US, the average ratio of year-end total debt outstanding to the year-end net stock of fixed assets stood at 1.09 for the years 2000-2008. This fact implies that the amount of borrowing is affected not only by the amount of collateral but also by the degree of leverage reflecting contemporary financial technology.[5]

[4] Park (2009) provides an interesting characterization on the relationship between financial development and crisis.

[5] Assuming that the aggregate stock of a collateral limits the aggregate value of borrowing does not appear to be very realistic. In reality, the amount of physical assets cannot and do not limit the enlargement of financial sectors in the economy. The World Bank data show that in the years 2000-2004, the ratio of the total market value of listed stocks and bonds relative to GDP on average amounted to 2.87 in the U.S. and 3.02 in Switzerland. It is not a surprise at all that adding the values of all the other financial claims, that is, the reasonable estimates for the fair value of bank loans, derivatives, and other unlisted financial products, can increase the ratio relative to GDP much higher than around 3 in some countries. Roughly speaking, if the steady state marginal product of capital net of depreciation equals about 8%, and capital income share amounts

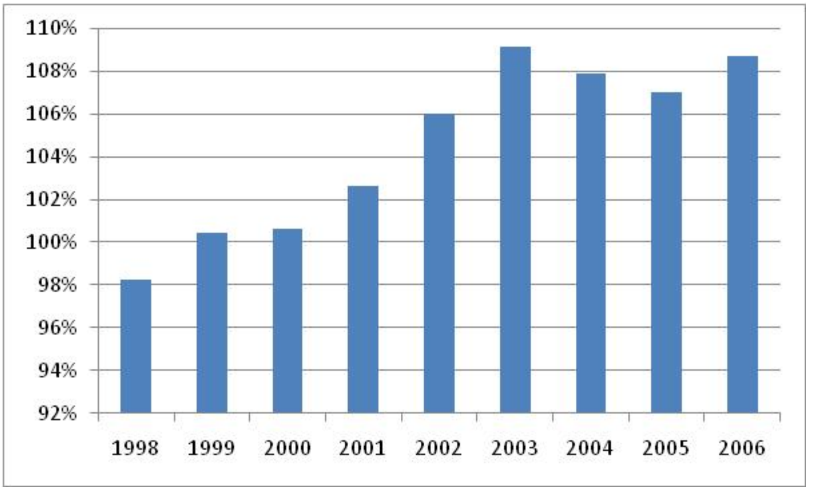

FIGURE 1
RATIO OF BORROWING TO COLLATERAL IN THE US[6]

To make leveraging against collateral technically feasible in the model, the present paper introduces two assumptions: (1) a certain type of transaction cost is incurred in the process of recovery as in Iacoviello (2005) and Mendicino (2008), and (2) the commitment of the borrower to produce is limited in its effectiveness. The original model of Kiyotaki and Moore (1997) is built on the assumption that there is lack of commitment. Under noncommitment, clearly the amount of borrowing cannot exceed the amount of collateral. Hence, if the recovery costs are additionally incurred, the ratio of the former to the latter should be strictly

to about 0.3, then the capital-output ratio in the steady state should be around $2\frac{1}{2}$. As capital is $2\frac{1}{2}$ times larger than the output in the steady state, if the ratio of financial claims to GDP is much higher than 3, then the ratio of financial claims to capital is much higher than 1.

[6] Data sources: The numerator is calculated based on the Flow of Funds Accounts D. 3 Debt Outstanding by Sector (Domestic nonfinancial sectors total *plus* Domestic financial sectors *plus* foreign). The denominator is from the National Income and Product Accounts (NIPA) All Fixed Asset Tables, Table 1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods (Fixed Assets Private *plus* Government). It is evident that the ratio of borrowing to collateral has evolved in the process of financial development and appears to be related mainly to the low-frequency component of macroeconomic variables. The ratio θ as a parameter in the model setup is assumed to be rationalized on this basis, as the present paper focuses on high-frequency business cycle issues.

smaller than 1. However, why cannot we assume partial commitment instead of noncommitment? Under partial commitment, borrowers can commit to produce partially, and then they do not have to pledge collateral as much as or more than the amount they borrow because there are still some positive amount of output produced left behind even when they ultimately walk out of the contract. They can provide lenders with collateral amounting to only a fraction of their borrowing, that is, the amount of borrowing net of the amount of output left that will be produced. Hence, under partial commitment, if there are no recovery costs to be incurred, the ratio of borrowing to collateral should be strictly greater than 1. In contrast, under the assumptions of both recovery costs and partial commitment, the ratio of borrowing to collateral can be either greater or smaller than 1.

In the model economy, there is only one type of collateral asset, that is, capital. Thus, the amount of borrowing should be subject to the following borrowing constraint:[7]

$$r_t \cdot b_t \leq \theta \cdot (1-\delta) \cdot k_t , \tag{7a}$$

where $\theta > 0$ denotes the ratio of maximum leverage allowable. It is noteworthy that θ can be either larger or smaller than unity in this paper.

$$\theta \geq \frac{r_t \cdot b_t}{(1-\delta) \cdot k_t} = \frac{\textit{the amount of borrowing}}{\textit{the amount of collateral}} .$$

B. Competitive Equilibrium

A competitive equilibrium of the model economy is defined as a sequence of capital stock, output, consumption of the entrepreneur, consumption of the household, hours of work, net borrowing of the entrepreneur, rate of interest, wage rate, and the lagrange multiplier with respect to collateralized borrowing constraint, $\{k_t, y_t, c_t, c_t', l_t, b_t, r_t, w_t, \lambda_t\}$, respectively, which satisfies the following properties for every t:

1. Each household chooses $\{c_t', l_t', b_t'\}$ optimally, given $\{r_t, w_t\}$.
2. Each entrepreneur chooses $\{k_t, c_t, l_t, b_t\}$ optimally, given $\{r_t, w_t\}$.

[7] There is no uncertainty on the right-hand side. This makes the occurrence of the event of the default impossible; thus, the external finance premium or default premium becomes irrelevant in this model.

3. The markets for goods, labor, and credit are clear.

$$y_t = c_t + c_t' + i_t, \tag{8}$$

$$l_t = l_t', \tag{9}$$

$$b_t + b_t' = 0. \tag{10}$$

a) Characterization of Optimizing Behaviors

In a competitive equilibrium, the representative household maximizes (1) subject to (2), and the representative entrepreneur maximizes (3) subject to both (6) and (7a), while all the relevant markets are clear in accordance with (8), (9), and (10).

The optimization behavior of the household can be characterized as the following first-order necessary conditions.

$$(l_t')^{\eta-1} = \frac{w_t}{c_t'}, \tag{11}$$

$$\frac{1}{c_t'} = \beta \cdot E_{\mathrm{t}}\left(\frac{r_t}{c_{t+1}'}\right). \tag{12}$$

Next, the optimization behavior of the entrepreneur can be characterized as the following first-order conditions:

$$\frac{1}{c_t} = \gamma \cdot E_{\mathrm{t}}\left[\frac{1}{c_{t+1}}\left(\frac{\alpha \cdot y_{t+1}}{k_t} + 1 - \delta\right)\right] + \lambda_t \cdot \theta \cdot (1-\delta), \tag{13}$$

$$w_t = \frac{(1-\alpha) \cdot y_t}{l_t}, \tag{14}$$

$$\frac{1}{c_t} = \gamma \cdot E_{\mathrm{t}}\left[\frac{r_t}{c_{t+1}}\right] + \lambda_t \cdot r_t. \tag{15}$$

From the steady-state version of (12) and (15), respectively,

$$\beta = \frac{1}{r}, \tag{12sa}$$

$$\frac{1}{r} = \gamma + \lambda \cdot c, \tag{15sa}$$

it follows that the lagrange multiplier should be positive in the neighborhood of the steady state due to the assumption of $\gamma < \beta$,

$$\lambda = \frac{\beta - \gamma}{c} > 0, \tag{16}$$

which in turn implies that the borrowing constraint (7a) always binds in the neighborhood of the steady state.

$$r_t \cdot b_t = \theta \cdot (1 - \delta) \cdot k_t. \tag{7b}$$

Then straightforwardly from (15), it follows that

$$\frac{1}{c_t} \cdot \frac{1}{r_t} > \gamma \cdot E_t\left[\frac{1}{c_{t+1}}\right]. \tag{17}$$

The left-hand side of the Inequality (17) is the discounted marginal utility of borrowing, whereas the right-hand side is the present value of its expected marginal disutility from paying back. Thus, the exact interpretation of the lagrange multiplier λ_t is the degree with which the marginal benefit of borrowing exceeds the marginal cost or, in other words, the marginal value of borrowing.

Now from the Equation (13),

$$\frac{1}{c_t} - \gamma \cdot E_t \frac{1}{c_{t+1}} > \gamma \cdot E_t\left[\frac{1}{c_{t+1}}\left(\frac{\alpha \cdot y_{t+1}}{k_t} - \delta\right)\right]. \tag{18}$$

The left-hand side of the Inequality (18) is the expected opportunity cost or user cost of holding additional capital in terms of utility, whereas the right-hand side is the present value of the expected utility from the net marginal product of capital. The gap between the left-hand and right-hand sides amounts to

$$\lambda_t \cdot \theta \cdot (1-\delta) > 0. \tag{19}$$

Note that the benefit of holding capital is twofold. The right-hand side of (18) is one component. The other component, (19), is the benefit of having more collateral and therefore being allowed to borrow more. One unit of capital that the entrepreneur has can serve as collateral for up to $\theta(1-\delta)$ units, and thus the benefit of having additional capital in terms of utility is given by $\lambda_t\theta(1-\delta)$.

b) Characterization of the Steady State Equilibrium

The steady state is defined as a set of endogenous variables $\{k, y, c, c', i, l, b, r, w, \lambda\}$, which satisfy the following conditions:

$$c' - r \cdot b = w \cdot l - b, \tag{2ss}$$

$$y = z \cdot k^{\alpha} \cdot l^{1-\alpha}, \tag{4ss}$$

$$i = \delta \cdot k, \tag{5ss}$$

$$c + i + r \cdot b = y - w \cdot l + b, \tag{6ss}$$

$$r \cdot b = \theta \cdot (1-\delta) \cdot k, \tag{7ss}$$

$$l^{\eta-1} = \frac{w}{c'}, \tag{11ss}$$

$$\frac{1}{c'} = \frac{\beta \cdot r}{c'}, \tag{12ss}$$

$$\frac{1}{c} = \frac{\gamma}{c}\left(\frac{\alpha \cdot y}{k} + 1 - \delta\right) + \lambda\theta(1-\delta), \tag{13ss}$$

$$w = \frac{(1-\alpha) \cdot y}{l}, \tag{14ss}$$

$$\frac{1}{c} = \left(\frac{\gamma}{c} + \lambda\right) \cdot r. \tag{15ss}$$

There are 10 unknowns and 10 independent equations. The market clearing conditions (9) and (10) are reflected in (2ss) and (11ss). The market clearing condition (8), which is redundant because it is only the sum of (2) and (6), is not considered. The two equations (12sa) and (15sa) introduced early can be derived easily from (12ss) and (15ss).

For analytical tractability and simplicity, we abstract from the decisions regarding labor supply and demand for the following Proposition 1 and Proposition 2:[8]

***Proposition* 1.** *The steady state level of capital, household consumption, investment, and output increases in θ. However, the steady state level of entrepreneurial consumption changes in a non-linear way. That is, it increases in θ for low values of θ and decreases in θ for high values of θ.*

Proof. The steady state level of capital is solved as follows when we abstract from labor:

$$k = \left(\frac{z \cdot \alpha \cdot \gamma}{1 - \theta \cdot (1-\delta) \cdot (\beta - \gamma) - \gamma \cdot (1-\delta)} \right)^{\frac{1}{1-\alpha}},$$

Thus, it follows that

$$\frac{dk}{d\theta} = \frac{1}{1-\alpha} \cdot \frac{(1-\delta) \cdot (\beta - \gamma)}{z \cdot \alpha \cdot \gamma} \cdot \left(\frac{z \cdot \alpha \cdot \gamma}{1 - \theta \cdot (1-\delta) \cdot (\beta - \gamma) - \gamma \cdot (1-\delta)} \right)^{\frac{2-\alpha}{1-\alpha}} > 0.$$

From the steady state relations, it is easy to see that

$$\frac{dc'}{d\theta} = (1-\beta) \cdot (1-\delta) \cdot \left[k + \theta \cdot \frac{dk}{d\theta} \right] > 0,$$

$$\frac{di}{d\theta} = \delta \cdot \frac{dk}{d\theta} > 0,$$

[8] Ignoring the labor-related variables does no critical harm to the propositions and to the subsequent discussions.

$$\frac{dy}{d\theta} = z \cdot \alpha \cdot k^{\alpha-1} \cdot \frac{dk}{d\theta} > 0.$$

Finally, from $c = y - c' - i$,

$$\frac{dc}{d\theta} = \left[z \cdot \alpha \cdot k^{\alpha-1} - \theta \cdot (1-\beta)(1-\delta) - \delta \right] \cdot \frac{dk}{d\theta} - (1-\beta) \cdot (1-\delta) \cdot k.$$

As the second term on the right-hand side of the above equation, $-(1-\beta) \cdot (1-\delta) \cdot k$, can be treated simply as a constant, $dc/d\theta$ is reduced to a linear function of $dk/d\theta$ with the slope of

$$\begin{aligned} & z \cdot \alpha \cdot k^{\alpha-1} - \theta \cdot (1-\beta)(1-\delta) - \delta \\ & = \frac{1-\gamma}{\gamma} \cdot [1 - \theta \cdot \beta \cdot (1-\delta)] \\ & \qquad > 0 \text{ for } \theta < \frac{1}{\beta \cdot (1-\delta)} \text{ and} \\ & \qquad < 0 \text{ for } \theta > \frac{1}{\beta \cdot (1-\delta)}. \end{aligned}$$

This implies that entrepreneurial consumption increases in θ for low values of θ and decreases in θ for high values of θ. ■

High values of θ loosen the borrowing constraint and decrease the amount of downpayment required to buy a unit capital in the steady state. This may increase the benefit of being allowed to borrow more, $\lambda\theta(1-\delta)$, which is the second term on the right-hand side of (13ss), and stimulate capital accumulation, resulting in high level of capital stock. The nonlinearity of consumption in Proposition 1 then implies that more than a desirable amount of economic resources may be devoted to capital accumulation for high enough values of θ while aggregate consumption shrinks.

Proposition 2. *Compared with the unconstrained first-best economy, the credit-constrained economy exhibits under-investment for $\theta \leq 1$. Over-investment can occur only under the condition that $\theta > 1$ is allowed.*

Proof. The steady-state capital stock in the unconstrained economy, k_u, is given by[9]

$$k_u = \left(\frac{z \cdot \alpha \cdot \beta}{1 - \beta \cdot (1 - \delta)} \right)^{1/(1-\alpha)}.$$

Note that

$$sign\ (k_u - k) = sign \left[\frac{\beta}{1 - \beta \cdot (1 - \delta)} - \frac{\gamma}{1 - \theta \cdot (1 - \delta) \cdot (\beta - \gamma) - \gamma \cdot (1 - \delta)} \right],$$

and

$$\frac{\beta}{1 - \beta \cdot (1 - \delta)} - \frac{\gamma}{1 - \theta \cdot (1 - \delta) \cdot (\beta - \gamma) - \gamma \cdot (1 - \delta)}$$
$$= \frac{(\beta - \gamma) \cdot [1 - \theta \cdot \beta \cdot (1 - \delta)]}{[1 - \beta \cdot (1 - \delta)] \cdot [1 - \theta (1 - \delta)(\beta - \gamma) - \gamma \cdot (1 - \delta)]}.$$

Note also that

$$\frac{1}{\beta} < \frac{1 - \gamma \cdot (1 - \delta)}{\beta - \gamma}.$$

Then it is straightforward that

$$k_u - k > 0$$

if and only if $\theta < \dfrac{1}{\beta \cdot (1 - \delta)}$ or $\theta > \dfrac{1 - \gamma \cdot (1 - \delta)}{(\beta - \gamma) \cdot (1 - \delta)}$ (under-investment)

[9] In the unconstrained economy, because there is no borrowing limit, there is no need for any heterogeneity between agents that necessitates credit flow and borrowing limit. The unconstrained "first-best" economy is defined as a simple standard homogeneous agent model where the social planner maximizes the representative agent's expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to the resource constraint in the period t: $c_t + k_t + (1 - \delta) k_{t-1} \leq z_t k_{t-1}^{\alpha}$.

and $k_u - k < 0$

$$\text{if and only if } \frac{1}{\beta \cdot (1-\delta)} < \theta < \frac{1-\gamma \cdot (1-\delta)}{(\beta - \gamma) \cdot (1-\delta)} \text{ (over-investment).}$$

As $\beta \cdot (1-\delta) < 1$, the constrained economy with $\theta \leq 1$ exhibits under-investment, and the constrained economy exhibiting over-investment should have θ strictly larger than 1. ■

For the entrepreneurs, there are two types of assets that they can choose to hold: capital and loan. The coexistence of both assets in equilibrium implies that the rates of return in holding these assets should be equivalent in an ex ante expected sense for each period. Furthermore, in the steady state, the rate of return on holding either asset should be strictly greater than the equilibrium interest rate.

***Proposition* 3.** *In the competitive equilibrium defined above, the rate-of-return equivalence between capital k and loan b should hold. Moreover, in the steady-state, the rate of return on these assets is strictly greater than the equilibrium interest rate.*

Proof. From the first-order conditions (13) and (15), it is straightforward that

$$E_t\left(\mathbf{M}_{t+1}\left(R_{t+1}^k - R_{t+1}^b\right)\right) = 0,$$

where $\mathbf{M}_{t+1} \equiv \gamma \cdot \left(\frac{c_t}{c_{t+1}}\right)$, a pricing kernel,

$$R_{t+1}^k \equiv \frac{\alpha \cdot y_{t+1}}{k_t} + 1 - \delta + \lambda_t \cdot \frac{c_{t+1} \cdot \theta(1-\delta)}{\gamma}, \text{ the return on capital, and} \quad (20)$$

$$R_{t+1}^b \equiv r_t \cdot \left(1 + \lambda_t \cdot \frac{c_{t+1}}{\gamma}\right), \text{ the return on loan.} \quad (21)$$

(12sa) and (15sa) imply that in the steady state, rate-of-return on loan becomes

$$
\begin{aligned}
R^b &= r \cdot \left(1 + \frac{\lambda\, c}{\gamma}\right) \\
&= \frac{1}{\beta} \cdot \left(1 + \frac{\beta - \gamma}{\gamma}\right) \\
&= \frac{1}{\gamma}.
\end{aligned}
$$

From (13ss), it is easy to see that in the steady state

$$
\frac{\alpha \cdot y}{k} + 1 - \delta = \frac{1 - (\beta - \gamma) \cdot \theta \cdot (1 - \delta)}{\gamma}.
$$

Thus, the steady state rate-of-return on capital becomes

$$
\begin{aligned}
R^k &= \frac{1 - (\beta - \gamma) \cdot \theta \cdot (1 - \delta)}{\gamma} + \lambda \cdot \frac{c \cdot \theta (1 - \delta)}{\gamma} \\
&= \frac{1}{\gamma}.
\end{aligned}
$$

Then from (12sa), it follows that

$$
R^k = R^b = \frac{1}{\gamma} > \frac{1}{\beta} = r . \blacksquare
$$

In (20), the return on capital consists of two elements: the marginal product of capital net of depreciation and the term premultiplied by λ_t, which is related to the discounted benefit of being permitted to borrow more. In the same vein, the return on loan in (21) does exceed the normal interest rate. This additional return indicates departure from the first-best allocation.

III. Numerical Experiments

In this section, the dynamic behaviors of the model economy are evaluated qualitatively through numerical experiments. We began with parametrization and then proceeded to solve the model to determine the recursive linear law of motions for endogenous variables expressed in log-linear deviations from the steady state.[10]

The goal of the numerical exercises here is to explore the business cycle implications of leveraging against collateral. The questions to be answered are as follows: Is the leverage in collateral constraints irrelevant or neutral to the business cycle characteristics? If not, how does it matter? More concretely, if two otherwise homogeneous macroeconomy differ only in the degree of leverage, how do the magnitudes of amplification and persistence in business fluctuations of these two economies differ? How are they affected by the different levels of financial leverage? Answering these questions requires a comparative study. In this respect, the proper research strategy should focus on the differences in the key business cycle moments generated from the different values of θ, with other things being controlled to equal.

However, there is a caveat. Applying an abstract and oversimplified model to the issue at hand, such as the one in this paper, has both an advantage and a disadvantage: the former comes from its clarity in revealing dynamics, whereas the latter is its apparent lack of empirical flavor. The present model is too abstract that the numerical exercises based on it do not aim at direct data-matching quantifications *per se*. Instead, the main interest lies in examining its aggregate implications purely from the modeling perspective. This paper deals with whether and how the credit cycle model with the elements of leverage-in-the-collateral constraint improves the one without it.

A. Impulse-Response Analysis

A unit period is a quarter. Following the literature, the parameters β, γ, δ, and α are set to have the values of 0.99, 0.95, 0.03, and 0.3, respectively, whereas the value of η is set to be equal to 1.01 based on Iacoviello (2005). The source of shock is assumed to be represented by the change in z_t, TFP, and the shock process is assumed to follow the AR(1) process of

$$\log z_t = \rho \log z_{t-1} + \varepsilon_t \tag{22}$$

where $\varepsilon_t \sim$ i.i.d.$N(1, \sigma^2)$. The impulse is defined as the 1% increase in z_t from its steady-state value. The value of ρ is assumed to be 0.95.[11]

[10] The log-linearized equilibrium conditions are in the appendix. The MATLAB M-files (version 7.1) used in the experiments are available upon request.

a) Investment as a Key Driver of Amplification

The positive productivity shock increases the marginal product of capital, as evident in (4). This, in turn, increases the demand for capital, thus increasing investment. The responses of investments for different θ's, in terms of percent deviations from the steady state are given in Figure 2. To simplify and clarify the comparisons without loss of generality, only the cases of $\theta=0.5$ and $\theta=1.5$ are presented. The case of $\theta=1$ is not presented because the results are mainly in between. In addition, the behaviors of the present constrained model can be contrasted with those of the unconstrained one defined earlier, if needed.

Figure 2 shows that in the credit-constrained economy, the amount of investment in each period is limited by the borrowing capacity. When θ is larger, both borrowing capacity and investment are larger. As seen in Equation (13), one of the benefits of investment comes from being permitted to borrow more. With the level of financial leverage being higher, the benefits from enlarging borrowing capacity due to the marginal increase in investment are bigger. Thus, larger θ is associated with

[11] The calibrations for the basic parameters are based on the standard business cycle literature; thus, the details are not reproduced here. The present calibration yields the following consumption-to-output and investment-to-output ratios in the steady state for each value of θ's:

Table N1
Steady-State Properties of the Model

θ	c/y	i/y	θ	c/y	i/y
0.5	0.86	0.14	1.0	0.78	0.22
0.6	0.85	0.15	1.1	0.76	0.24
0.7	0.83	0.17	1.2	0.73	0.27
0.8	0.82	0.18	1.3	0.70	0.30
0.9	0.80	0.20	1.4	0.65	0.35

The well-known sources for the long-run investment-to-output ratio from the data include Summers and Heston (1984) and Maddison (1992). The former reports that in the US, it amounts to 24%, whereas in Canada, it accounts for 28% (for the period 1950-1980). The latter reports that, in the US, it amounts to 18.9% for the period of 1950-1969 and 18.7% for the period of 1970-1989. We can also compute the two long-run ratios in the data based on Cooley and Prescott (1994, p. 21), where the long-run investment-to-output and consumption-to-output ratios are about 25% and 75%, respectively. These indicate that the relevant values of θ's should range from 0.8-1.3 to be consistent with the present calibration. In addition, the table above reveals that in the steady state, the ratio of consumption-to-output shrinks and the ratio of investment-to-output expands when the value of θ increases, confirming Proposition 1.

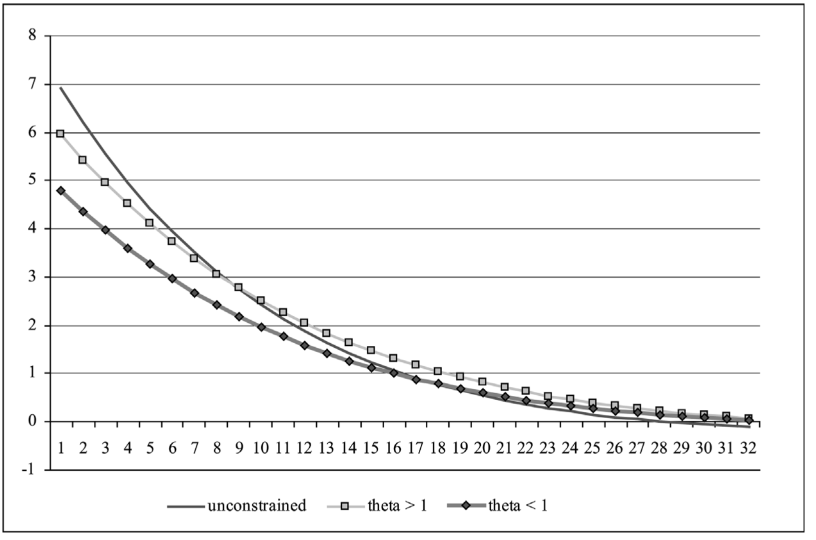

FIGURE 2
RESPONSES OF INVESTMENTS

larger increases in investment, as evident in Figure 2.

Interestingly, limiting the amount of investment in the current period in accordance with (7b) appears to open the door for more persistent responses of investment. Figure 2 illustrates that investments "die out" more slowly with larger θ. However, the persistence generated upon itself does not seem to be quantitatively important. The economic intuitions behind the relationship between the degree of leverage and of persistence are also not clear when we consider investment only. The responses of investment do not appear to shed much light on the issues of persistence. More notably, the speed of returning to the steady state is much faster in case of higher leverage. If capital is accumulated faster during the period when TFP remains high after shock,[12] then marginal product of capital also decreases faster. This, in turn, increases the speed of adjustment. Note in the figure that, initially, the gap between two constrained cases amounts to about 1%, but as time goes by the gap becomes negligible.

From the discussions above, it becomes clear that there is no need

[12] The assumed TFP shock is a one-off in the sense that there is a one-time increase in ε in (22). However, the positive effects of one-time increase in ε last for more than one period, as TFP changes according to (22).

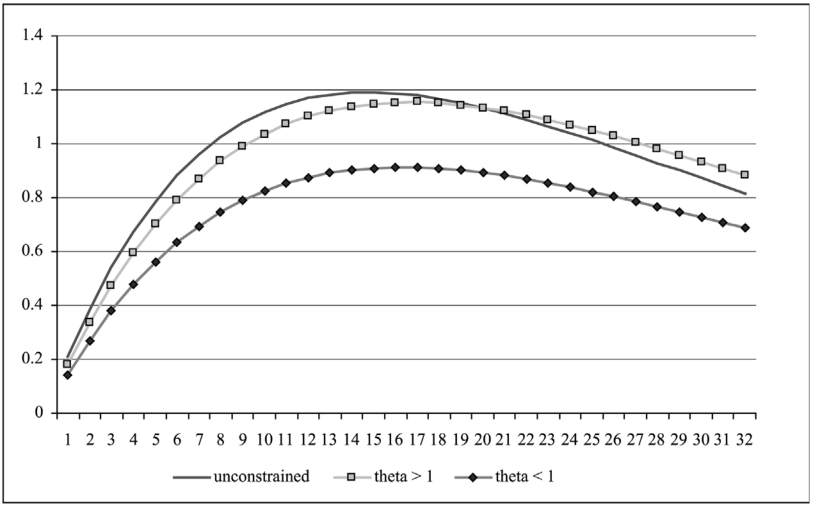

FIGURE 3
RESPONSES OF CAPITAL STOCK

to introduce the changes in the relative price of capital to explain the enlargement of borrowing capacity after a positive shock. In this respect, the two-way interaction is not an exclusive channel facilitating the credit cycle effects. Rather, borrowing capacity can be expanded as an endogenous response to a positive shock consistently with the inter-temporal optimizing behaviors.

b) Consumption as a Key Driver of Persistence

Consumption, in nature, is deeply related to the issue of persistence than investment. The reason is that the amount of consumption in each period fluctuates in the process of intertemporal smoothing. Figure 4 shows the responses of aggregate consumption, which is the sum of both household and entrepreneurial consumption.

Initially, after the positive shock, consumption expands with income. However, the initial responses are not as strong as those of income due to the increase in saving, namely, lending. Confronted with a positive shock, households allocate more resources to lending, rather than current consumption, to take advantage of higher productivity.

There are two different effects on the consumption of the positive TFP shock. These two work in opposite directions. First, consumption increases due to the income effect generated by higher TFP. At the

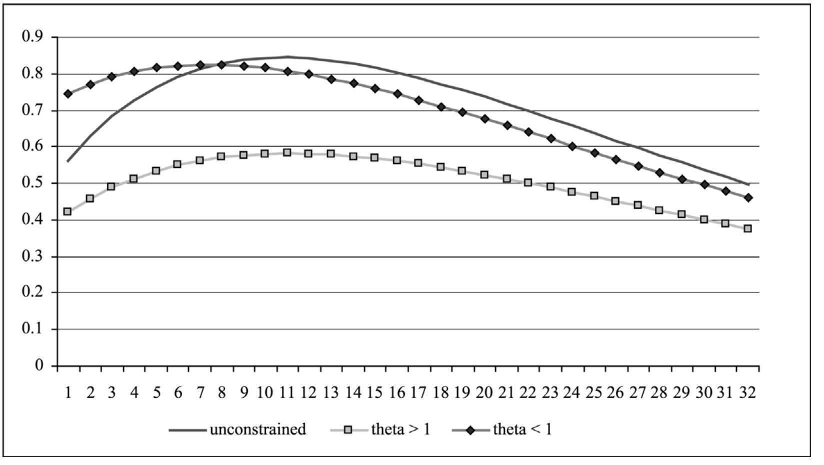

FIGURE 4
RESPONSES OF AGGREGATE CONSUMPTION

same time, the substitution effect works. The positive TFP shock increases the marginal product of capital, which in turn pushes up the interest rate in accordance with the rate-of-return equivalence arguments in Proposition 3. The intertemporal substitution in consumption thus occurs to decrease the current consumption. In the initial phase, the substitution effect is strong, shifting resources to lending rather than consumption. However, after the initial phase, the substitution effect becomes weaker as the interest rate goes back to its steady state level while the income effect becomes stronger for the time being due to the enlargement of productive capacity enabled by capital accumulation as in Figure 3. This reversal yields the hump-shaped or inverted U-shaped responses of consumption in Figure 4, thus the occurrence of persistence. It also seems evident from the figures that the lower the degree of financial leverage is, the larger the magnitude of aggregate consumption responses. This appeals to the intuition that the merit of increasing investment is smaller in the case of low leverage. Higher θ's yield a stronger substitution effect in the initial phase.

c) Summary: Effects of Leveraging on Amplification and Persistence

The responses of investment are in stark contrast with those of consumption to a positive TFP impulse. Investment responds much more strongly in magnitude than in consumption. In this regard, investment is a key driver of amplification. Consumption exhibits the hump-shaped pattern and stronger persistence than investment, implying that consumption is more important in the issue of persistence.

To summarize the discussions above, the higher leverage facilitates stronger responses of investment and weaker responses of consumption. Thus, with higher leverage, the impact of shock is more amplified but does not last longer than with lower leverage. In other words, in a highly leveraged economy, the effects of the shock are likely to appear in amplified magnitudes but disappear more quickly. Higher leverage is likely to be associated with stronger amplification and weaker persistence. This "trade-off" is evident in Figure 5, which shows the responses of output for different θ's.[13]

B. Model Simulation

The cyclical behavior of the model economy is studied through simulations based on the shock process defined in (22). The aim is to compare the magnitudes of amplification and persistence in the present constrained model with those in the unconstrained model or real data.

a) Parametrization of the Shock Process

Parameters ρ and σ in (22) are calibrated jointly in accordance with Cooley and Prescott (1994). Thus, the cyclical behaviors of the unconstrained economy in this paper become close enough to those of the frictionless Cooley-Prescott economy.[14] This enables us to regard the unconstrained economy safely in this paper as a meaningful benchmark.

From the simulations of the unconstrained model, σ should be around 0.54 when ρ is 0.95, and σ should be around 0.49 when ρ is 0.9 to mimic the Cooley-Prescott economy. With the combinations of (ρ, σ) set to (0.95, 0.54) or (0.9, 0.49), the unconstrained economy be-

[13] In the literature, Song (2005) argues for the trade-off between persistence and amplification in a similar manner as this paper. However, the mechanism that generates the trade-off in his paper is not the same as that in this paper. Moreover, in Song (2005), the differences in the responses of consumption and investment are not considered at all. Additionally, his model is still based on the linearities. The readers can also refer to Mendicino (2008) with regard to the relationship between amplification and the value of θ. However, Mendicino (2008) did not consider the issue of persistence at all. Both Song (2005) and Mendicino (2008) ignored the endogenous motive for accumulating collateral assets and did not consider the case of $\theta > 1$.

[14] See Table 1.2 Cyclical Behavior of the Artificial Economy, p. 34 in Cooley and Prescott (1994).

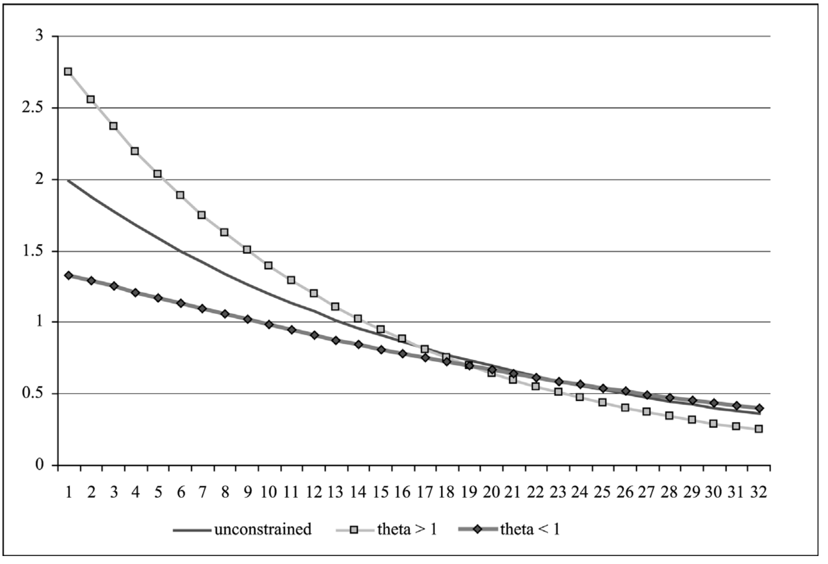

FIGURE 5
RESPONSES OF OUTPUT

TABLE 1
STANDARD DEVIATIONS (HP-FILTERED SERIES)

SD%	output	investment	consumption
Cooley-Prescott	1.351	5.954	0.329
sigma 0.54, rho 0.95	1.368	4.730	0.467
sigma 0.49, rho 0.9	1.386	5.200	0.375

haves in a similar fashion as the Cooley-Prescott economy, as seen in Table 1 and Table 2.

b) Effects of Leveraging on Amplification

To measure amplification, the standard deviation of the output is naturally the first candidate. In the data, the standard deviations of output, investment, and consumption for non-durables are known to be 1.72, 8.24, and 0.86, respectively.[15] These numbers seem huge relative

[15] See Table 1.1 Cyclical Behavior of the U.S. Economy, pp. 30-31 in Cooley and Prescott (1994).

TABLE 2
CROSS-CORRELATIONS (HP-FILTERED SERIES)

Classification	Cross-Correlation of Output with:										
Cooley-Prescott	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	x(+4)	x(+5)
output	-0.049	0.071	0.232	0.441	0.698	1.000	0.698	0.441	0.232	0.071	-0.049
investment	-0.112	-0.007	0.171	0.389	0.664	0.992	0.713	0.470	0.270	0.115	-0.003
consumption	0.232	0.340	0.460	0.592	0.725	0.843	0.502	0.229	0.022	-0.128	-0.234
sigma 0.54, rho 0.95	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	x(+4)	x(+5)
output	-0.060	0.061	0.224	0.437	0.691	1.000	0.691	0.437	0.224	0.061	-0.060
investment	0.014	0.132	0.286	0.484	0.713	0.987	0.629	0.345	0.117	-0.049	-0.165
consumption	-0.269	-0.158	0.004	0.231	0.519	0.884	0.766	0.638	0.501	0.372	0.253
sigma 0.49, rho 0.9	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	x(+4)	x(+5)
output	-0.080	0.043	0.205	0.406	0.672	1.000	0.672	0.406	0.205	0.043	-0.080
investment	-0.006	0.115	0.269	0.454	0.696	0.989	0.613	0.319	0.106	-0.059	-0.177
consumption	-0.359	-0.258	-0.103	0.110	0.409	0.800	0.744	0.654	0.554	0.440	0.324

TABLE 3
STANDARD DEVIATIONS (HP-FILTERED SERIES)

theta	sigma 0.54, rho 0.95			sigma 0.49, rho 0.9		
	Y	I	C	Y	I	C
0.5	0.933	3.322	0.553	0.921	3.752	0.472
0.6	1.012	3.743	0.543	0.998	4.204	0.452
0.7	1.092	4.114	0.530	1.109	4.736	0.440
0.8	1.165	4.413	0.504	1.180	4.998	0.415
0.9	1.261	4.719	0.481	1.306	5.383	0.399
1.0	1.380	4.985	0.467	1.445	5.670	0.396
1.1	1.465	5.016	0.437	1.551	5.690	0.371
1.2	1.611	5.102	0.430	1.624	5.469	0.332
1.3	1.714	4.919	0.395	1.701	5.136	0.299
1.4	1.809	4.582	0.359	1.725	4.554	0.258
1.5	1.870	4.052	0.323	1.781	3.994	0.219
unconst.	1.368	4.730	0.467	1.386	5.200	0.375
DATA	1.72	8.24	0.86	1.72	8.24	0.86

to the ones generated by the constrained models. Table 3 reports the results from the model simulations.

Consistent with the discussions so far, the volatility of output increases in θ. However, only when θ exceeds unity can the constrained model generate larger amplification compared with the unconstrained one. Furthermore, compared with real data, the constrained model can generate larger amplification only when θ exceeds around 1.4. These results may imply that if we assume $\theta \leq 1$ as in the existing literature, then we cannot hope for any amplifying role of the collateral constraints.

Another measure of amplification is the standard deviation of output divided by the standard deviation of TFP, σ_y / σ_z. This measure is proposed by Cordoba and Ripoll (2004a) and is applicable directly regardless of the volatility of underlying shocks because it is essentially standardized. Table 4 compares the results.

Except for the case where there is no autocorrelation of shocks shown in the last two columns, the amplification generated by the constrained model is not significant unless θ exceeds 1 by a large amount. Rather, the degrees of amplification or the values of σ_y / σ_z for the constrained models are smaller than that for the unconstrained model for small θ.

Kocherlakota (2000) shows that the amplification in the credit cycle model depends crucially on factor shares, and if capital share is small, then the amplification may become insignificant. Cordoba and Ripoll (2004a) claims that it is very difficult under standard preferences, technologies, and parameter specifications to have the amplification similar to the typical RBC models.[16] Seemingly, the results of the simulation studies in this paper also cast doubts on the amplifying role of collateral constraints.

However, the model presented in this paper can be argued to improve on Cordoba and Ripoll (2004a) in that the present one generates bigger and better amplification results. In their paper, similar to in Kiyotaki and Moore (1997), both lenders and borrowers are assumed to produce. Hence, only the productivity gap between them the shock is amplified, as properly documented by Pintus and Wen (2008). However, in the model equilibrium of this paper, only borrowers produce. This allows the shock to be amplified by the full capital share, yielding larger amplification than Cordoba and Ripoll (2004a).[17]

[16] Cordoba and Ripoll (2004a) argue that, for example, when capital share is ⅓ and the elasticity of intertemporal substitution is 1, then there is no amplification.

[17] In the literature, Kiyotaki and Moore (1997), Cordoba and Ripoll (2004a), and Mendicino (2008) assume that both lenders and borrowers produce, whereas Iacoviello (2005), Song (2005), and Pintus and Wen (2008) assumed that only

TABLE 4

COMPARISONS OF AMPLIFICATION[18]

theta	σ=0.54 ρ=0.95		σ=0.49 ρ=0.9		σ=1 ρ=0.9		σ=1 ρ=0	
	σ_y	σ_y/σ_z	σ_y	σ_y/σ_z	σ_y	σ_y/σ_z	σ_y	σ_y/σ_z
0.5	0.933	1.342	0.921	1.480	1.884	1.480	2.347	2.444
0.6	1.012	1.451	0.998	1.628	2.007	1.627	2.519	2.634
0.7	1.092	1.580	1.109	1.793	2.242	1.793	2.687	2.790
0.8	1.165	1.722	1.180	1.969	2.485	1.968	2.797	2.915
0.9	1.261	1.876	1.306	2.146	2.627	2.146	2.897	3.013
1.0	1.380	2.036	1.445	2.317	2.878	2.318	2.988	3.087
1.1	1.465	2.195	1.551	2.476	3.151	2.475	2.998	3.143
1.2	1.611	2.345	1.624	2.619	3.346	2.617	3.048	3.185
1.3	1.714	2.489	1.701	2.743	3.471	2.743	3.054	3.216
1.4	1.809	2.622	1.725	2.849	3.612	2.850	3.109	3.237
1.5	1.870	2.744	1.781	2.941	3.615	2.941	3.122	3.252
first-best	1.368	1.984	1.386	2.232	2.803	2.231	2.767	2.859
C-R					3.365	1.467	1.532	1.532
DATA					σ_y= 1.72			

c) Effects of Leveraging on Persistence

The degree of persistence can be measured in terms of the autocorrelations of output. The autocorrelations of the detrended output between the period t and $t\pm1$, $t\pm2$, $t\pm3$, and $t\pm4$ are 85%, 63%, 38%, and 16%, respectively.[19] However, it is not known whether credit-constrained models can match these moments successfully.

borrowers produce.

The models with financial accelerator, such as Bernanke, Gertler, and Gilchrist (1999) can be argued to exhibit amplification results similar to the present paper. However, in the modeling perspectives, there are significant differences between the credit cycle models and financial accelerator models. In the former, credit frictions are introduced in the form of binding collateral constraints, whereas in the latter, they are due to the costly state verification problem. In the text, it is noted that the existing credit cycle models have thus far not been very successful in solving the so-called "small shocks, large cycles" puzzle. The present paper takes up this issue and reveals that "leverage-in-the-collateral-constraint" can be a candidate mechanism that generates amplification.

[18] "First-best" stands for the unconstrained economy defined in footnote 9; it is calibrated in accordance with Cooley and Prescott (1994). "C-R" stands for the results reported in Cordoba and Ripoll (2004a), p. 1036.

[19] See again Table 1.1 Cyclical Behavior of the U.S. Economy, pp. 30-31 in Cooley-Prescott (1994).

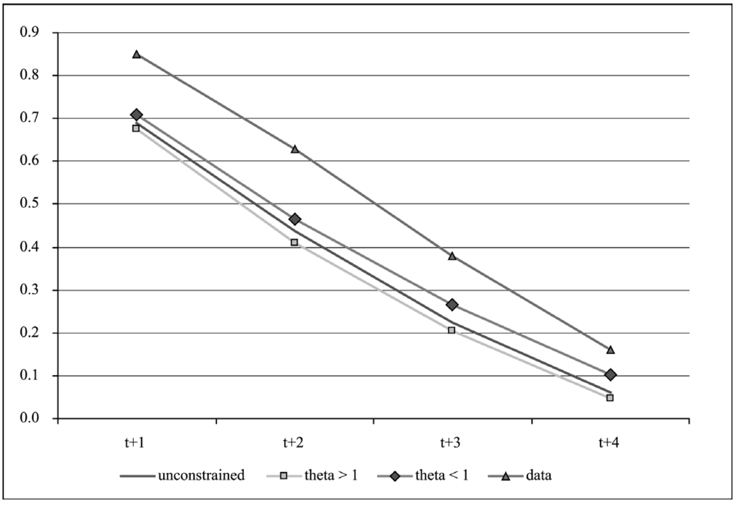

Figure 6
Autocorrelations of Output (HP-Filtered Series): $\sigma=0.54$, $\rho=0.95$

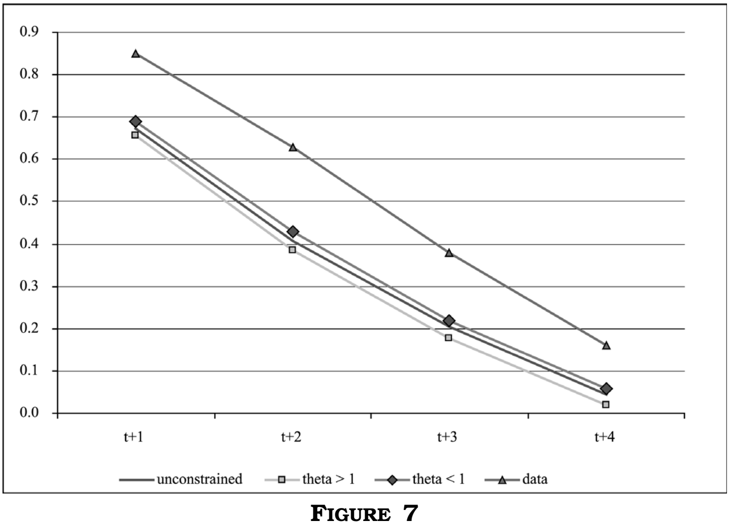

Figure 7
Autocorrelations of Output (HP-Filtered Series): $\sigma=0.49$, $\rho=0.9$

Figures 6-8 present the relevant autocorrelations of the HP-filtered output. From these figures, the larger θ's are likely to be associated with smaller persistence. For $\theta>1$, collateral constraints seem to fail to

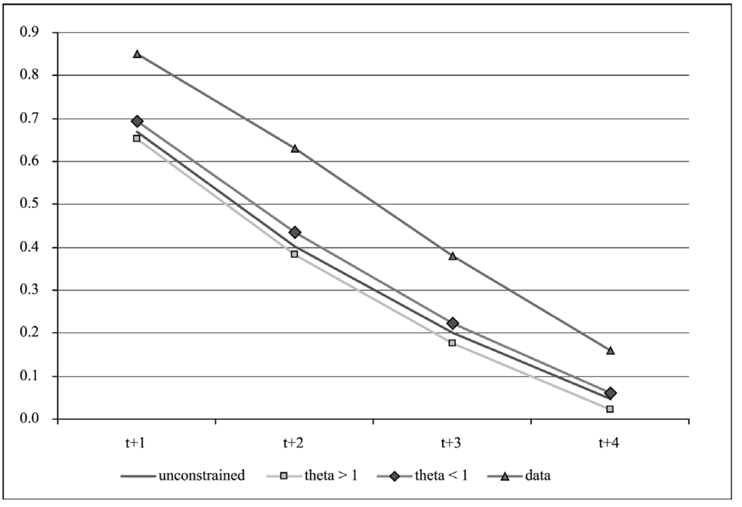

FIGURE 8

AUTOCORRELATIONS OF OUTPUT (HP-FILTERED SERIES): $\sigma = 1$, $\rho = 0.9$

generate persistence, in that serial correlations, when $\theta > 1$, fall behind the ones in the unconstrained economy case. In contrast, for $\theta < 1$, stronger persistence emerges. However, its quantitative significance appears to be doubtful. It is only slightly stronger than in the unconstrained economy case while still falling behind the real data with marked differences.[20]

[20] According to Figure 1, the leverage ratio appears to have been in an increasing trend. However, the results of experiments and simulations reported in this paper do not argue that historically and as a matter of fact, amplification became bigger and persistence became smaller. Instead, they suggest a theoretically derived potential mechanism through which amplification and persistence may be affected. The present paper is a purely model-based one, and the numerical exercise here is not an empirical one but rather a qualitative theoretical experimentation.

As discussed, the present paper aims to improve the existing credit cycle models from a modeling perspective to capture the non-neutral implications of financial leverage on business fluctuations. To this end, a new device, the mechanism of "leverage-in-the-collateral constraint" is introduced into a computable variant of a credit cycle model. As a modeling research, this paper naturally and explicitly acknowledges not only the achievements but also the unresolved limitations and directions for future research. It is in this respect that the present study does not attempt to argue any definitive empirical implication.

To study the possibly related empirical issues, including the explanations on "the great moderation" and trends in the recent consumption-to-output/

IV. Concluding Remarks

Building on the literature after Kiyotaki and Moore (1997), this paper presents a simple DSGE model where the credit cycle effects of financial leverage are generated through the endogenous process of capital accumulation. The model in this paper sheds some light on the macroeconomic consequences of a highly leveraged financial sector. The results of the numerical exercises suggest that the degree of leverage matters critically in amplifying the effects of collateral constraints. The more highly leveraged the economy is, the more amplified the impacts of shocks. The level of financial leverage may affect real economy in non-neutral ways in the course of business fluctuations, suggesting that an economy in recession may experience more severe contractions if it is highly leveraged.

Generating both significant amplification and significant persistence at the same time with the leverage-in-the-collateral constraint mechanism alone is found to be considerably difficult. The trade-off between amplification and persistence emerges. The impulse-response analyses in this paper indicate that the fundamental cause of this trade-off lies in the differences in the response patterns of investment and consumption, which reflect the intertemporal optimizing behaviors of the forward-looking rational agents. How could this trade-off be resolved or mitigated? For now, it seems that the performance of the present model may possibly be enhanced with regard to the persistence issue if the elements of habit formation are added to the present model. Naturally, this will be the next task in future research.

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investment-to-output ratios, we need to apply structural estimations for the models equipped with more realistic features than the model in this paper in order to isolate the effects of the variables of interest properly. The model in this paper is not adequate for this purpose at all because it is designed in its form to be as simple as possible to highlight the subject of this paper. This can be an apparent limitation of the present model.

Appendix 1: The Model

A1.1 The Steady State

The steady state of the model economy is solved analytically. First, as already noted in the text, (12sa) and (16) can be derived easily from (12ss) and (15ss).

From (13ss), it follows that

$$1 = \gamma \cdot \left(\frac{\alpha \cdot y}{k} + 1 - \delta \right) + \theta(1-\delta)(\beta - \gamma)$$
$$\Rightarrow k = \varepsilon_{ky} \cdot y \qquad \text{(A1)}$$
$$\text{where } \varepsilon_{ky} \equiv \frac{\gamma \cdot \mu}{1 - \gamma(1-\delta) - \theta(1-\delta)(\beta-\gamma)}$$

Then from (5ss)

$$i = \delta \cdot \varepsilon_{ky} \cdot y, \qquad \text{(A2)}$$

from (7ss)

$$b = \beta \cdot \theta \cdot (1-\delta) \cdot \varepsilon_{ky} \cdot y, \qquad \text{(A3)}$$

from (6ss) and (14ss)

$$c + \delta \cdot k + \theta \cdot (1-\delta) \cdot \varepsilon_{ky} \cdot y = \alpha \cdot y + \beta \cdot \theta \cdot (1-\delta) \cdot \varepsilon_{ky} \cdot y$$
$$\Rightarrow c = \varepsilon_{cy} \cdot y \qquad \text{(A4)}$$
$$\text{where } \varepsilon_{cy} \equiv \alpha - \varepsilon_{ky} \cdot \left[(1-\beta) \cdot \theta \cdot (1-\delta) + \delta \right],$$

from (16)

$$\lambda = \frac{\beta - \gamma}{\varepsilon_{cy} \cdot y}, \qquad \text{(A5)}$$

from (2ss)

$$c' = (1-\alpha) \cdot y + (1-\beta) \cdot \theta \cdot (1-\delta) \cdot \varepsilon_{ky} \cdot y$$

$$\Rightarrow c' = \varepsilon_{c'y} \cdot y \quad \text{where} \ \ \varepsilon_{c'y} \equiv 1 - \alpha + \varepsilon_{ky} (1 - \beta) \cdot \theta \cdot (1 - \delta). \tag{A6}$$

As the following holds from (11ss) and (14ss)

$$c' = (1 - \alpha) \cdot y \cdot l^{-\eta},$$

it should be that

$$\varepsilon_{c'y} \cdot y = (1 - \alpha) \cdot y \cdot l^{-\eta}$$
$$\Rightarrow l = \left(\frac{\varepsilon_{c'y}}{1 - \alpha} \right)^{-1/\eta}, \tag{A7}$$

and thus from (14ss),

$$w = (1 - \alpha) \cdot \left[\frac{\varepsilon_{c'y}}{1 - \alpha} \right]^{1/\eta} \cdot y. \tag{A8}$$

Finally, from (4ss)

$$y = z \cdot \left(\varepsilon_{ky} \cdot y \right)^{\alpha} \cdot \left[\frac{\varepsilon_{c'y}}{1 - \alpha} \right]^{-(1-\alpha)/\eta},$$
$$\Rightarrow y = \left[z \cdot {\varepsilon_{ky}}^{\alpha} \cdot \left[\frac{\varepsilon_{c'y}}{1 - \alpha} \right]^{-(1-\alpha)/\eta} \right]^{1/(1-\alpha)}. \tag{A9}$$

Equations (A1)-(A6) and (A8) express the variables k, i, b, c, λ, c', and w as functions of y, respectively. Each of the Equations (12sa), (A7), and (A9) gives r, l, and y, respectively.

A1.2 Model Dynamics in Log Linear Deviations from the Steady State

The endogenous variables are $\{k_t, y_t, c_t, c_t', i_t, l_t, b_t, r_t, w_t, \lambda_t\}$. The equilibrium conditions are given by (4), (5), (6), (7b), (8), (11), (12), (13), (14), and (15). To explore the dynamics of the model economy, these equations are log-linearized around the steady state as follows, where the variables with hat denote the percentage deviations from their steady state values. In log-linearizing the expectational equations, the distributional log-normality and conditional homoskedasticity are assumed.

$$\hat{y}_t - \hat{z}_t - \alpha \cdot \hat{k}_{t-1} - (1-\alpha) \cdot \hat{l}_t = 0 \tag{4LL}$$

$$\hat{k}_t - (1-\delta) \cdot \hat{k}_{t-1} - \delta \cdot \hat{i}_t = 0 \tag{5LL}$$

$$c \cdot \hat{c}_t + i \cdot \hat{i}_t - (y - w \cdot l) \cdot \hat{y}_t + r \cdot b \cdot (\hat{r}_{t-1} + \hat{b}_{t-1}) - b \cdot \hat{b}_t = 0 \tag{6LL}$$

$$\frac{r \cdot b}{\theta} \cdot \left(\hat{r}_t + \hat{b}_t\right) - (1-\delta) \cdot k \cdot \hat{k}_t = 0 \tag{7LL}$$

$$-y \cdot \hat{y}_t + c \cdot \hat{c}_t + c' \cdot \hat{c}'_t + i \cdot \hat{i}_t = 0 \tag{8LL}$$

$$\eta \cdot \hat{l}_t - \hat{y}_t + \hat{c}'_t = 0 \tag{11LL}$$

$$E_t \hat{c}'_{t+1} - \hat{c}'_t - \hat{r}_t = 0 \tag{12LL}$$

$$\begin{aligned} &\hat{c}_t + \{\lambda \cdot c \cdot \theta(1-\delta) - 1\} \cdot \mathrm{E}_t\, \hat{c}_{t+1} + \lambda \cdot c \cdot \theta(1-\delta) \cdot \hat{\lambda}_t \\ &+\{1 - (1-\delta) \cdot \gamma - \lambda \cdot c \cdot \theta(1-\delta)\} \cdot (\mathrm{E}_t\, \hat{y}_{t+1} - \hat{k}_t) = 0 \end{aligned} \tag{13LL}$$

$$\hat{w}_t + \hat{l}_t - \hat{y}_t = 0 \tag{14LL}$$

$$\beta \cdot \hat{c}_t - \gamma \cdot E_t \hat{c}_{t+1} + (\beta - \gamma) \cdot \hat{\lambda}_t + \beta \cdot \hat{r}_t = 0 \tag{15LL}$$

Appendix 2: Simulation Results

A2.1 Data [Cooley and Prescott (1994)]

DATA	Standard deviations	Cross-Correlation of Output with:										
		x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	x(+4)	x(+5)
output	**1.72**	0.02	0.16	0.38	0.63	0.85	1.00	0.85	0.63	0.38	0.16	-0.02
investment	8.24	0.04	0.19	0.38	0.59	0.79	0.91	0.76	0.50	0.22	-0.04	-0.24
consumption (nondurables)	0.86	0.22	0.40	0.55	0.68	0.78	0.77	0.64	0.47	0.27	0.06	-0.11

A2.2 Moments based on Model Simulations (HP-filtered series): σ=0.54, ρ=0.95

sigma 0.54, rho 0.95	Standard deviations	Cross-Correlation of Output with:										
Unconstrained economy		x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	x(+4)	x(+5)
capital stock	0.4718	-0.4435	-0.3880	-0.2858	-0.1249	0.1022	0.4088	0.5888	0.6750	0.6883	0.6495	0.5761
output	**1.3677**	-0.0604	0.0611	0.2237	0.4371	0.6907	1.0000	0.6907	0.4371	0.2237	0.0611	-0.0604
investment	4.7304	0.0137	0.1321	0.2860	0.4837	0.7126	0.9871	0.6287	0.3452	0.1166	-0.0491	-0.1654
consumption	0.4674	-0.2693	-0.1584	0.0042	0.2310	0.5186	0.8841	0.7655	0.6379	0.5008	0.3719	0.2535
theta=0.5												
capital stock	0.3458	-0.4220	-0.3533	-0.2406	-0.0787	0.1438	0.4336	0.6050	0.6888	0.7061	0.6738	0.6070
output	**0.9329**	-0.0220	0.1036	0.2670	0.4651	0.7089	1.0000	0.7089	0.4651	0.2670	0.1036	-0.0220
investment	3.3224	0.0722	0.1914	0.3415	0.5180	0.7304	0.9788	0.6269	0.3452	0.1278	-0.0413	-0.1620
consumption	0.5529	-0.1172	0.0092	0.1790	0.3904	0.6559	0.9781	0.7614	0.5663	0.3964	0.2457	0.1204
theta=0.6												
capital stock	0.3715	-0.4233	-0.3632	-0.2581	-0.0952	0.1330	0.4370	0.6144	0.6964	0.7029	0.6584	0.5811
output	**1.0125**	-0.0511	0.0687	0.2295	0.4454	0.6968	1.0000	0.6968	0.4454	0.2295	0.0687	-0.0511
investment	3.7426	0.0378	0.1531	0.3027	0.4991	0.7200	0.9803	0.6160	0.3276	0.0937	-0.0701	-0.1828
consumption	0.5431	-0.1610	-0.0424	0.1233	0.3516	0.6279	0.9687	0.7595	0.5685	0.3873	0.2391	0.1168
theta=0.7												
capital stock	0.4147	-0.4196	-0.3579	-0.2503	-0.0869	0.1401	0.4387	0.6148	0.6964	0.7047	0.6619	0.5852
output	**1.0917**	-0.0427	0.0780	0.2426	0.4503	0.6994	1.0000	0.6994	0.4503	0.2426	0.0780	-0.0427
investment	4.1141	0.0465	0.1621	0.3153	0.5029	0.7214	0.9800	0.6184	0.3323	0.1064	-0.0622	-0.1760
consumption	0.5302	-0.1786	-0.0601	0.1092	0.3320	0.6092	0.9517	0.7692	0.5968	0.4332	0.2872	0.1652
theta=0.8												
capital stock	0.4401	-0.4343	-0.3698	-0.2619	-0.1001	0.1270	0.4292	0.6059	0.6891	0.7018	0.6654	0.5933
output	**1.1646**	-0.0395	0.0847	0.2415	0.4452	0.6961	1.0000	0.6961	0.4452	0.2415	0.0847	-0.0395
investment	4.4134	0.0495	0.1687	0.3136	0.4977	0.7188	0.9814	0.6186	0.3322	0.1109	-0.0499	-0.1685
consumption	0.5043	-0.2076	-0.0871	0.0758	0.2959	0.5780	0.9300	0.7713	0.6148	0.4659	0.3330	0.2107
theta=0.9												
capital stock	0.4587	-0.4409	-0.3863	-0.2834	-0.1235	0.1043	0.4171	0.5971	0.6794	0.6885	0.6436	0.5616
output	**1.2607**	-0.0776	0.0475	0.2132	0.4208	0.6780	1.0000	0.6780	0.4208	0.2132	0.0475	-0.0776
investment	4.7194	0.0032	0.1252	0.2813	0.4713	0.7012	0.9840	0.6064	0.3165	0.0933	-0.0749	-0.1927
consumption	0.4810	-0.2621	-0.1467	0.0197	0.2423	0.5314	0.9055	0.7591	0.6139	0.4704	0.3321	0.2056

sigma 0.54, rho 0.95	Standard deviations	Cross-Correlation of Output with:										
unitary theta		$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
capital stock	0.4874	-0.4468	-0.3910	-0.2898	-0.1318	0.0971	0.4090	0.5879	0.6683	0.6785	0.6403	0.5666
output	**1.3798**	-0.0608	0.0603	0.2143	0.4157	0.6800	1.0000	0.6800	0.4157	0.2143	0.0603	-0.0608
investment	4.9854	0.0163	0.1343	0.2791	0.4640	0.7030	0.9863	0.6153	0.3207	0.1051	-0.0516	-0.1672
consumption	0.4671	-0.2776	-0.1679	-0.0123	0.2047	0.5004	0.8764	0.7583	0.6256	0.4980	0.3765	0.2584
theta=1.1												
capital stock	0.4917	-0.4351	-0.3901	-0.2991	-0.1494	0.0744	0.3869	0.5682	0.6517	0.6646	0.6278	0.5572
output	**1.4649**	-0.0671	0.0462	0.1994	0.4045	0.6663	1.0000	0.6663	0.4045	0.1994	0.0462	-0.0671
investment	5.0160	-0.0008	0.1107	0.2567	0.4482	0.6878	0.9889	0.6095	0.3210	0.1031	-0.0523	-0.1606
consumption	0.4368	-0.2946	-0.1973	-0.0486	0.1663	0.4586	0.8463	0.7416	0.6260	0.5053	0.3892	0.2789
theta=1.2												
capital stock	0.5198	-0.4683	-0.4124	-0.3118	-0.1553	0.0687	0.3731	0.5557	0.6481	0.6721	0.6464	0.5875
output	**1.6113**	-0.0415	0.0760	0.2333	0.4362	0.6892	1.0000	0.6892	0.4362	0.2333	0.0760	-0.0415
investment	5.1021	0.0232	0.1377	0.2873	0.4769	0.7095	0.9912	0.6412	0.3638	0.1484	-0.0126	-0.1274
consumption	0.4300	-0.3150	-0.2106	-0.0551	0.1616	0.4492	0.8202	0.7472	0.6540	0.5490	0.4409	0.3363
theta=1.3												
capital stock	0.4933	-0.4814	-0.4305	-0.3336	-0.1785	0.0454	0.3539	0.5411	0.6379	0.6643	0.6409	0.5813
output	**1.7145**	-0.0549	0.0671	0.2236	0.4310	0.6838	1.0000	0.6838	0.4310	0.2236	0.0671	-0.0549
investment	4.9185	-0.0023	0.1179	0.2686	0.4655	0.7018	0.9944	0.6466	0.3742	0.1566	-0.0029	-0.1228
consumption	0.3955	-0.3315	-0.2263	-0.0722	0.1485	0.4383	0.8178	0.7413	0.6482	0.5397	0.4320	0.3232
theta=1.4												
capital stock	0.4542	-0.4992	-0.4537	-0.3616	-0.2081	0.0172	0.3318	0.5245	0.6265	0.6568	0.6343	0.5740
output	**1.8091**	-0.0680	0.0603	0.2193	0.4270	0.6798	1.0000	0.6798	0.4270	0.2193	0.0603	-0.0680
investment	4.5822	-0.0282	0.0991	0.2542	0.4541	0.6945	0.9969	0.6531	0.3857	0.1704	0.0091	-0.1176
consumption	0.3590	-0.3349	-0.2248	-0.0678	0.1558	0.4496	0.8379	0.7402	0.6351	0.5183	0.4034	0.2870

A2.3 Moments based on Model Simulations (HP-filtered series): $\sigma=0.49,\ \rho=0.9$

sigma 0.49, rho 0.9	Standard deviations	Cross-Correlation of Output with:										
Unconstrained economy		$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
capital stock	0.4860	-0.4808	-0.4297	-0.3283	-0.1682	0.0680	0.3949	0.5840	0.6698	0.6823	0.6422	0.5655
output	**1.3857**	-0.0797	0.0434	0.2054	0.4061	0.6718	1.0000	0.6718	0.4061	0.2054	0.0434	-0.0797
investment	5.2000	-0.0057	0.1151	0.2689	0.4540	0.6956	0.9887	0.6129	0.3192	0.1056	-0.0589	-0.1766
consumption	0.3746	-0.3590	-0.2581	-0.1035	0.1104	0.4090	0.8002	0.7435	0.6545	0.5538	0.4403	0.3244
theta=0.5												
capital stock	0.3648	-0.4411	-0.3796	-0.2714	-0.1052	0.1292	0.4411	0.6178	0.6951	0.6978	0.6494	0.5672
output	**0.9207**	-0.0614	0.0593	0.2193	0.4284	0.6887	1.0000	0.6887	0.4284	0.2193	0.0593	-0.0614
investment	3.7520	0.0322	0.1479	0.2958	0.4840	0.7127	0.9797	0.6055	0.3078	0.0812	-0.0811	-0.1937
consumption	0.4724	-0.1841	-0.0647	0.1012	0.3251	0.6117	0.9633	0.7560	0.5621	0.3894	0.2425	0.1185
theta=0.6												
capital stock	0.4032	-0.4426	-0.3810	-0.2730	-0.1100	0.1220	0.4383	0.6160	0.6935	0.6984	0.6501	0.5660
output	**0.9976**	-0.0668	0.0571	0.2193	0.4212	0.6782	1.0000	0.6782	0.4212	0.2193	0.0571	-0.0668
investment	4.2044	0.0254	0.1447	0.2953	0.4767	0.7026	0.9802	0.5953	0.3017	0.0831	-0.0814	-0.1968
consumption	0.4521	-0.2183	-0.0985	0.0685	0.2868	0.5735	0.9418	0.7568	0.5854	0.4303	0.2864	0.1593

sigma 0.49, rho 0.9	Standard deviations	Cross-Correlation of Output with:										
theta=0.7		$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
capital stock	0.4450	-0.4397	-0.3880	-0.2875	-0.1240	0.1126	0.4342	0.6135	0.6893	0.6880	0.6343	0.5476
output	**1.1090**	-0.0821	0.0350	0.1945	0.4069	0.6731	1.0000	0.6731	0.4069	0.1945	0.0350	-0.0821
investment	4.7363	0.0043	0.1186	0.2683	0.4617	0.6979	0.9816	0.5934	0.2919	0.0640	-0.0963	-0.2043
consumption	0.4402	-0.2588	-0.1510	0.0095	0.2360	0.5337	0.9130	0.7590	0.6021	0.4492	0.3109	0.1886
theta=0.8												
capital stock	0.4780	-0.4388	-0.3811	-0.2774	-0.1181	0.1110	0.4281	0.6063	0.6835	0.6873	0.6410	0.5599
output	**1.1804**	-0.0628	0.0554	0.2106	0.4119	0.6690	1.0000	0.6690	0.4119	0.2106	0.0554	-0.0628
investment	4.9983	0.0232	0.1371	0.2815	0.4642	0.6926	0.9825	0.5921	0.3010	0.0844	-0.0728	-0.1832
consumption	0.4149	-0.2806	-0.1728	-0.0168	0.1988	0.4878	0.8713	0.7548	0.6319	0.5055	0.3821	0.2634
theta=0.9												
capital stock	0.5030	-0.4663	-0.4126	-0.3090	-0.1471	0.0898	0.4169	0.6009	0.6818	0.6889	0.6410	0.5572
output	**1.3057**	-0.0769	0.0456	0.2070	0.4058	0.6691	1.0000	0.6691	0.4058	0.2070	0.0456	-0.0769
investment	5.3832	0.0056	0.1253	0.2771	0.4580	0.6940	0.9852	0.5996	0.3043	0.0912	-0.0721	-0.1874
consumption	0.3992	-0.3332	-0.2296	-0.0730	0.1411	0.4379	0.8284	0.7506	0.6491	0.5399	0.4192	0.2989
unitary theta												
capital stock	0.5467	-0.4645	-0.4043	-0.2986	-0.1378	0.0926	0.4073	0.5926	0.6800	0.6949	0.6572	0.5824
output	**1.4454**	-0.0597	0.0662	0.2254	0.4279	0.6820	1.0000	0.6820	0.4279	0.2254	0.0662	-0.0597
investment	5.6695	0.0183	0.1406	0.2902	0.4762	0.7050	0.9869	0.6183	0.3339	0.1168	-0.0455	-0.1662
consumption	0.3958	-0.3522	-0.2476	-0.0953	0.1167	0.4018	0.7761	0.7453	0.6790	0.5889	0.4846	0.3714
theta=1.1												
capital stock	0.5433	-0.4645	-0.4174	-0.3224	-0.1656	0.0674	0.3902	0.5774	0.6629	0.6730	0.6324	0.5573
output	**1.5509**	-0.0791	0.0390	0.1951	0.4062	0.6753	1.0000	0.6753	0.4062	0.1951	0.0390	-0.0791
investment	5.6902	-0.0116	0.1048	0.2538	0.4511	0.6980	0.9900	0.6209	0.3254	0.1020	-0.0561	-0.1691
consumption	0.3705	-0.3802	-0.2925	-0.1521	0.0584	0.3512	0.7345	0.7256	0.6673	0.5806	0.4816	0.3746
theta=1.2												
capital stock	0.5099	-0.4904	-0.4462	-0.3533	-0.1977	0.0364	0.3662	0.5605	0.6526	0.6701	0.6334	0.5600
output	**1.6242**	-0.0887	0.0342	0.1954	0.4014	0.6696	1.0000	0.6696	0.4014	0.1954	0.0342	-0.0887
investment	5.4688	-0.0300	0.0917	0.2472	0.4414	0.6908	0.9929	0.6253	0.3349	0.1184	-0.0450	-0.1638
consumption	0.3320	-0.4142	-0.3276	-0.1875	0.0198	0.3128	0.7047	0.7106	0.6657	0.5910	0.4950	0.3876
theta=1.3												
capital stock	0.4804	-0.4965	-0.4561	-0.3676	-0.2169	0.0143	0.3451	0.5410	0.6360	0.6569	0.6264	0.5598
output	**1.7013**	-0.0855	0.0333	0.1858	0.3921	0.6607	1.0000	0.6607	0.3921	0.1858	0.0333	-0.0855
investment	5.1361	-0.0374	0.0807	0.2286	0.4257	0.6791	0.9954	0.6260	0.3395	0.1246	-0.0297	-0.1456
consumption	0.2991	-0.4194	-0.3373	-0.2037	0.0010	0.2941	0.6935	0.6968	0.6537	0.5812	0.4931	0.3919
theta=1.4												
capital stock	0.4342	-0.4788	-0.4439	-0.3640	-0.2215	0.0000	0.3238	0.5149	0.6086	0.6318	0.6069	0.5481
output	**1.7249**	-0.0818	0.0260	0.1751	0.3801	0.6509	1.0000	0.6509	0.3801	0.1751	0.0260	-0.0818
investment	4.5544	-0.0467	0.0606	0.2066	0.4052	0.6650	0.9973	0.6260	0.3420	0.1305	-0.0202	-0.1260
consumption	0.2584	-0.3951	-0.3178	-0.1874	0.0160	0.3090	0.7142	0.6879	0.6277	0.5476	0.4591	0.3656

A2.4 Moments based on Model Simulations (HP-filtered series): $\sigma = 1$, $\rho = 0.9$

sigma 1, rho 0.9	Standard deviations	Cross-Correlation of Output with:										
Unconstrained economy		$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
capital stock	1.0004	-0.4490	-0.3966	-0.3002	-0.1467	0.0822	0.4003	0.5834	0.6655	0.6767	0.6370	0.5618
output	**2.8028**	-0.0661	0.0476	0.2006	0.4013	0.6677	1.0000	0.6677	0.4013	0.2006	0.0476	-0.0661
investment	10.5004	0.0047	0.1155	0.2605	0.4468	0.6900	0.9883	0.6078	0.3133	0.0995	-0.0555	-0.1634
consumption	0.7667	-0.3320	-0.2359	-0.0901	0.1186	0.4119	0.7986	0.7395	0.6500	0.5498	0.4421	0.3324
theta=0.5												
capital stock	0.7397	-0.4404	-0.3802	-0.2735	-0.1070	0.1273	0.4389	0.6207	0.7008	0.7058	0.6575	0.5746
output	**1.8841**	-0.0652	0.0604	0.2241	0.4351	0.6930	1.0000	0.6930	0.4351	0.2241	0.0604	-0.0652
investment	7.6857	0.0271	0.1485	0.3008	0.4912	0.7174	0.9800	0.6099	0.3146	0.0859	-0.0803	-0.1979
consumption	0.9644	-0.1862	-0.0631	0.1055	0.3311	0.6157	0.9636	0.7609	0.5694	0.3951	0.2449	0.1163
theta=0.6												
capital stock	0.7906	-0.4299	-0.3773	-0.2785	-0.1205	0.1126	0.4345	0.6133	0.6857	0.6858	0.6338	0.5489
output	**2.0069**	-0.0793	0.0386	0.1960	0.4008	0.6691	1.0000	0.6691	0.4008	0.1960	0.0386	-0.0793
investment	8.4758	0.0068	0.1216	0.2692	0.4553	0.6938	0.9811	0.5880	0.2844	0.0641	-0.0944	-0.2034
consumption	0.9027	-0.2211	-0.1088	0.0513	0.2702	0.5663	0.9435	0.7488	0.5646	0.4042	0.2624	0.1395
theta=0.7												
capital stock	0.9133	-0.4458	-0.3856	-0.2787	-0.1162	0.1162	0.4343	0.6130	0.6910	0.6961	0.6477	0.5636
output	**2.2421**	-0.0699	0.0534	0.2154	0.4176	0.6758	1.0000	0.6758	0.4176	0.2154	0.0534	-0.0699
investment	9.5704	0.0204	0.1395	0.2903	0.4726	0.7004	0.9811	0.5952	0.3012	0.0827	-0.0814	-0.1963
consumption	0.8945	-0.2547	-0.1389	0.0259	0.2446	0.5350	0.9112	0.7607	0.6117	0.4700	0.3329	0.2074
theta=0.8												
capital stock	0.9851	-0.4445	-0.3950	-0.2962	-0.1338	0.1031	0.4268	0.6078	0.6847	0.6843	0.6313	0.5452
output	**2.4849**	-0.0858	0.0305	0.1898	0.4026	0.6702	1.0000	0.6702	0.4026	0.1898	0.0305	-0.0858
investment	10.5342	-0.0027	0.1112	0.2613	0.4562	0.6950	0.9832	0.5948	0.2933	0.0657	-0.0943	-0.2019
consumption	0.8649	-0.2953	-0.1931	-0.0366	0.1883	0.4881	0.8741	0.7575	0.6256	0.4880	0.3571	0.2366
theta=0.9												
capital stock	1.0307	-0.4454	-0.3903	-0.2890	-0.1311	0.0980	0.4172	0.5975	0.6765	0.6820	0.6373	0.5576
output	**2.6266**	-0.0668	0.0505	0.2053	0.4070	0.6657	1.0000	0.6657	0.4070	0.2053	0.0505	-0.0668
investment	10.8194	0.0142	0.1279	0.2729	0.4573	0.6891	0.9847	0.5949	0.3044	0.0883	-0.0685	-0.1786
consumption	0.8109	-0.3171	-0.2156	-0.0644	0.1484	0.4376	0.8252	0.7462	0.6482	0.5377	0.4230	0.3080
unitary theta												
capital stock	1.0548	-0.4736	-0.4230	-0.3222	-0.1621	0.0744	0.4034	0.5898	0.6732	0.6829	0.6375	0.5560
output	**2.8782**	-0.0805	0.0413	0.2023	0.4014	0.6662	1.0000	0.6662	0.4014	0.2023	0.0413	-0.0805
investment	11.3136	-0.0045	0.1151	0.2676	0.4506	0.6903	0.9877	0.6040	0.3101	0.0978	-0.0651	-0.1805
consumption	0.7722	-0.3661	-0.2696	-0.1188	0.0918	0.3874	0.7808	0.7372	0.6596	0.5656	0.4542	0.3383
theta=1.1												
capital stock	1.1139	-0.4739	-0.4170	-0.3141	-0.1553	0.0744	0.3906	0.5785	0.6690	0.6873	0.6529	0.5813
output	**3.1508**	-0.0631	0.0622	0.2210	0.4239	0.6794	1.0000	0.6794	0.4239	0.2210	0.0622	-0.0631
investment	11.5559	0.0070	0.1293	0.2800	0.4684	0.7014	0.9897	0.6244	0.3421	0.1261	-0.0358	-0.1567
consumption	0.7574	-0.3805	-0.2822	-0.1351	0.0730	0.3565	0.7324	0.7278	0.6807	0.6040	0.5085	0.4007

sigma 1, rho 0.9	Standard deviations	Cross-Correlation of Output with:										
theta=1.2		$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
capital stock	1.0761	-0.4743	-0.4306	-0.3386	-0.1841	0.0478	0.3713	0.5613	0.6505	0.6648	0.6283	0.5571
output	**3.3457**	-0.0815	0.0362	0.1922	0.4035	0.6735	1.0000	0.6735	0.4035	0.1922	0.0362	-0.0815
investment	11.2480	-0.0228	0.0936	0.2437	0.4434	0.6944	0.9927	0.6286	0.3360	0.1139	-0.0441	-0.1578
consumption	0.6969	-0.4001	-0.3172	-0.1810	0.0264	0.3178	0.7025	0.7099	0.6645	0.5872	0.4948	0.3921
theta=1.3												
capital stock	0.9814	-0.5003	-0.4596	-0.3699	-0.2169	0.0156	0.3453	0.5423	0.6387	0.6612	0.6295	0.5609
output	**3.4715**	-0.0898	0.0329	0.1941	0.4001	0.6688	1.0000	0.6688	0.4001	0.1941	0.0329	-0.0898
investment	10.4800	-0.0414	0.0805	0.2373	0.4339	0.6872	0.9953	0.6342	0.3475	0.1326	-0.0306	-0.1504
consumption	0.6107	-0.4252	-0.3406	-0.2020	0.0047	0.2986	0.6935	0.7011	0.6589	0.5875	0.4947	0.3903
theta=1.4												
capital stock	0.8987	-0.5020	-0.4706	-0.3872	-0.2366	-0.0050	0.3242	0.5247	0.6232	0.6461	0.6148	0.5507
output	**3.6117**	-0.0942	0.0147	0.1769	0.3934	0.6652	1.0000	0.6652	0.3934	0.1769	0.0147	-0.0942
investment	9.5375	-0.0581	0.0506	0.2098	0.4195	0.6798	0.9974	0.6402	0.3550	0.1319	-0.0317	-0.1385
consumption	0.5371	-0.4173	-0.3415	-0.2015	0.0138	0.3149	0.7175	0.7029	0.6442	0.5574	0.4573	0.3592

A2.5 Moments based on Model Simulations (HP-filtered series): $\sigma=1,\ \rho=0$

sigma 1, rho 0	Standard deviations	Cross-Correlation of Output with:										
Unconstrained economy		$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
capital stock	0.4616	-0.1557	-0.1823	-0.2254	-0.2693	-0.3139	0.4821	0.3804	0.2884	0.2100	0.1556	0.0926
output	**2.7668**	-0.0698	-0.0523	-0.0733	-0.0791	-0.0803	1.0000	-0.0803	-0.0791	-0.0733	-0.0523	-0.0698
investment	11.9681	-0.0613	-0.0415	-0.0604	-0.0636	-0.0622	0.9983	-0.1080	-0.1008	-0.0898	-0.0645	-0.0783
consumption	0.2293	-0.1557	-0.1823	-0.2254	-0.2693	-0.3139	0.4821	0.3804	0.2884	0.2100	0.1556	0.0926
theta=0.5												
capital stock	0.5668	-0.1416	-0.1682	-0.2053	-0.2422	-0.2817	0.5129	0.3935	0.2911	0.2072	0.1424	0.0894
output	**2.3465**	-0.0521	-0.0560	-0.0702	-0.0778	-0.0832	1.0000	-0.0832	-0.0778	-0.0702	-0.0560	-0.0521
investment	14.7775	-0.0395	-0.0410	-0.0521	-0.0565	-0.0582	0.9959	-0.1286	-0.1124	-0.0957	-0.0741	-0.0644
consumption	0.3395	-0.1280	-0.1499	-0.1833	-0.2141	-0.2458	0.7501	0.2727	0.1964	0.1351	0.0905	0.0513
theta=0.6												
capital stock	0.5865	-0.1454	-0.1804	-0.2134	-0.2527	-0.2885	0.5075	0.3957	0.2953	0.2146	0.1431	0.0940
output	**2.5188**	-0.0469	-0.0666	-0.0670	-0.0797	-0.0763	1.0000	-0.0763	-0.0797	-0.0670	-0.0666	-0.0469
investment	15.2823	-0.0344	-0.0515	-0.0488	-0.0585	-0.0517	0.9963	-0.1191	-0.1130	-0.0918	-0.0843	-0.0588
consumption	0.3060	-0.1396	-0.1746	-0.2046	-0.2420	-0.2736	0.6260	0.3447	0.2539	0.1833	0.1189	0.0778
theta=0.7												
capital stock	0.5951	-0.1534	-0.1912	-0.2214	-0.2508	-0.2928	0.5018	0.3887	0.3025	0.2259	0.1513	0.0948
output	**2.6869**	-0.0561	-0.0705	-0.0613	-0.0645	-0.0823	1.0000	-0.0823	-0.0645	-0.0613	-0.0705	-0.0561
investment	15.4693	-0.0440	-0.0555	-0.0434	-0.0442	-0.0591	0.9967	-0.1219	-0.0956	-0.0853	-0.0880	-0.0678
consumption	0.2858	-0.1534	-0.1912	-0.2214	-0.2508	-0.2928	0.5018	0.3887	0.3025	0.2259	0.1513	0.0948
theta=0.8												
capital stock	0.5754	-0.1470	-0.1874	-0.2245	-0.2560	-0.3017	0.5000	0.3847	0.2972	0.2154	0.1412	0.0903
output	**2.7970**	-0.0508	-0.0722	-0.0717	-0.0670	-0.0867	1.0000	-0.0867	-0.0670	-0.0717	-0.0722	-0.0508
investment	15.0907	-0.0402	-0.0589	-0.0553	-0.0480	-0.0650	0.9973	-0.1227	-0.0951	-0.0930	-0.0873	-0.0607
consumption	0.2642	-0.1489	-0.1889	-0.2282	-0.2621	-0.3081	0.3978	0.4197	0.3238	0.2374	0.1587	0.1020

sigma 1, rho 0	Standard deviations	Cross-Correlation of Output with:										
theta=0.9		*x*(-5)	*x*(-4)	*x*(-3)	*x*(-2)	*x*(-1)	*x*	*x*(+1)	*x*(+2)	*x*(+3)	*x*(+4)	*x*(+5)
capital stock	0.5547	-0.1638	-0.1923	-0.2159	-0.2651	-0.3063	0.4905	0.3861	0.2863	0.2223	0.1592	0.0948
output	**2.8972**	-0.0665	-0.0567	-0.0517	-0.0883	-0.0792	1.0000	-0.0792	-0.0883	-0.0517	-0.0567	-0.0665
investment	14.4823	-0.0560	-0.0437	-0.0368	-0.0710	-0.0587	0.9977	-0.1118	-0.1137	-0.0709	-0.0713	-0.0764
consumption	0.2495	-0.1635	-0.1968	-0.2236	-0.2692	-0.3160	0.3119	0.4398	0.3319	0.2537	0.1857	0.1177
unitary theta												
capital stock	0.5225	-0.1463	-0.1856	-0.2279	-0.2728	-0.3137	0.4873	0.3853	0.2913	0.2115	0.1421	0.0823
output	**2.9884**	-0.0634	-0.0683	-0.0736	-0.0797	-0.0801	1.0000	-0.0801	-0.0797	-0.0736	-0.0683	-0.0634
investment	13.6804	-0.0552	-0.0575	-0.0602	-0.0637	-0.0614	0.9982	-0.1090	-0.1023	-0.0907	-0.0805	-0.0713
consumption	0.2323	-0.1438	-0.1859	-0.2309	-0.2789	-0.3246	0.2610	0.4493	0.3448	0.2541	0.1753	0.1079
theta=1.1												
capital stock	0.4825	-0.1504	-0.1898	-0.2300	-0.2774	-0.3132	0.4755	0.3830	0.2905	0.2145	0.1489	0.0978
output	**2.9976**	-0.0539	-0.0660	-0.0699	-0.0840	-0.0720	1.0000	-0.0720	-0.0840	-0.0699	-0.0660	-0.0539
investment	12.4161	-0.0460	-0.0560	-0.0578	-0.0696	-0.0553	0.9986	-0.0972	-0.1040	-0.0851	-0.0771	-0.0616
consumption	0.2133	-0.1501	-0.1900	-0.2329	-0.2818	-0.3248	0.2358	0.4454	0.3457	0.2575	0.1828	0.1228
theta=1.2												
capital stock	0.4405	-0.1542	-0.1921	-0.2382	-0.2806	-0.3197	0.4654	0.3768	0.2938	0.2156	0.1543	0.1050
output	**3.0478**	-0.0531	-0.0632	-0.0776	-0.0736	-0.0747	1.0000	-0.0747	-0.0736	-0.0776	-0.0632	-0.0531
investment	11.2696	-0.0462	-0.0546	-0.0671	-0.0610	-0.0603	0.9990	-0.0957	-0.0905	-0.0907	-0.0728	-0.0599
consumption	0.1923	-0.1546	-0.1934	-0.2400	-0.2870	-0.3305	0.2439	0.4341	0.3424	0.2565	0.1852	0.1283
theta=1.3												
capital stock	0.3767	-0.1449	-0.1903	-0.2344	-0.2867	-0.3398	0.4647	0.3688	0.2799	0.2073	0.1410	0.0896
output	**3.0541**	-0.0562	-0.0704	-0.0701	-0.0852	-0.0850	1.0000	-0.0850	-0.0852	-0.0701	-0.0704	-0.0562
investment	9.9202	-0.0513	-0.0640	-0.0620	-0.0753	-0.0732	0.9994	-0.1012	-0.0979	-0.0797	-0.0774	-0.0609
consumption	0.1600	-0.1446	-0.1907	-0.2382	-0.2915	-0.3490	0.3006	0.4153	0.3188	0.2371	0.1655	0.1073
theta=1.4												
capital stock	0.3369	-0.1667	-0.2027	-0.2429	-0.2978	-0.3375	0.4519	0.3731	0.2859	0.2192	0.1634	0.0976
output	**3.1092**	-0.0757	-0.0600	-0.0665	-0.0878	-0.0710	1.0000	-0.0710	-0.0878	-0.0665	-0.0600	-0.0757
investment	8.6898	-0.0717	-0.0547	-0.0602	-0.0801	-0.0620	0.9996	-0.0830	-0.0975	-0.0740	-0.0657	-0.0797
consumption	0.1387	-0.1665	-0.2037	-0.2443	-0.2994	-0.3404	0.4213	0.3821	0.2941	0.2254	0.1683	0.1021

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