An Analysis of Social Participation Structure of Middle School Mathematics Classroom in Korea

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ABSTRACT

The research aims to set the foundation for the understanding of the mathematics instruction in the institutional setting of a Korean middle school classroom. To fulfill this purpose, we focused on understanding the types and characteristics of participation structure created by class participants' interaction. One second grade middle school mathematics class was adopted for participant observation and long-term data was collected and analyzed. We have discovered several distinct types of social participation structure based on literature review and the criteria drawn from the comprehensive analysis of the characteristics of the teacher's and students' interaction and behavior.

Key Words : participation structure, interaction, mathematics classroom, participant observation

Ⅰ. Introduction

In spite of increasing changes in society, prevalence of individual values, and the criticism and issues of the institutionalized education, it cannot be doubted that a classroom is the very place where adolescents should progress through, with the knowledge that they gained from interactions in society.

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Likewise, a mathematics classroom is a socio-cultural space created by the emergence of mathematics instruction. A mathematics class is a specific mode of activity and phenomenon taking place in the "instructional triangle", a network of actions and concerns created by a teacher and students, two subjects of the class, and the mathematics itself, which is the objective of the class. Especially, this definition shows the network of actions and concerns that regard the process of learning as its main objective in the context of classroom instruction. The network is a mutually-created social structure of participation and communication, which lets us understand the classroom more deeply and discover learning in the specific mode of activity called classroom instruction. This research is one of the endeavors to understand the classroom as the space of a student's life and learning and improve its quality based on the understanding.

Anyone who has experienced mathematics classes in the context of Korean public education might be able to add a few comments about education or mathematics class. However, how much and how precisely do we understand mathematics in the classroom? In what way do teachers and students communicate and act? How do the communication and actions affect the students' way of life and their value systems? How appropriately do we take the methods to deal with specific domains of mathematics corresponding to the objective of mathematics education? Though people might have some preconceived notions about a mathematics class, they might have a hard time thinking of concrete problems. This is because a mathematics class is too familiar to us to be taken as an object of study and analyzed from different perspectives. To understand a subject, as Geertz(1973) pointed out, we must look into what people involved in the subject are actually doing, rather than reviewing the theories and study results based on it. Therefore, it is necessary to observe what teachers and students are actually doing in the field of the mathematics classroom, analyze the observed things, and interpret why they happen based on

observation and analysis to truly understand the mathematics class.

This article analyzes the structure of interaction in the field of classroom study where a mathematics class takes place. This analysis is expected to overcome a superficial comprehension of the mathematics class and contribute to answering the questions such as: Where should a specific mathematics class focus? How should we organize one, and what route should the students' mathematical learning take? The researchers adopted a second year middle school class and observed in-person the dialogues and actions among a teacher and students. Types of participant structure are discussed based on the observation.

Ⅱ. Theoretical Background

To answer the research question we raised, we needed to perform qualitative observation and analysis of the scenes of classroom instruction and the interaction among participants in the classroom. Qualitative analysis of interaction involves the choice of a constructivist and an interpretationist perspective, which understands the world based on construction and interpretation (Cho, 1999). This perspective seeks understanding (Verstehen) in that the purpose and motivation of participants are attended to, participants themselves define and interpret their own actions in a mutual way, and thereby secure intersubjectivity, rather than trying to explain a social phenomenon as a cause-effect paradigm (Cho, 2005). In the following premises, Blumer's Symbolic Interactionism, a theoretical and methodological foundation of the research, is reviewed briefly.

A. Blumer's Symbolic Interactionism and Study of the Mathematics Class

Blumer's Symbolic Interactionism is based on the following three premises.

> The first premise is that human beings act toward things on the basis of the meanings that the things have for them.
> The second premise is that the meaning of such things is derived from, or arises out of, the social interaction that one has with one's fellows.
> The third premise is that these meanings are handled in, and modified through, an interpretative process used by the person in dealing with the things he encounters. (Blumer, 1969: 2)

The first premise is on the relationship between human and an object, and human behavior toward the object. 'Object' in this premise is a complex system with multiple meanings in that it can be an object, a person, or the whole world including people and objects in a specific context. The nature of this premise is that an object cannot have the same meaning to two different humans, and a human being always acts towards an object with his own meaning reflected in that object. The second premise deals with the origin of meaning that a person imposes on an object, which is created from social interaction with others. After all, meaning is socially constructed and a product of "mutually defining activity (Blumer, 1969: 5)." The third premise is about the association between a human within the context of meaning. Interpretation is intervened when a person encounters an object. This interpretative process is a series of indication and interpretation. Here, indication refers to the process in which a person recognizes the object of his behavior inside his or her mind while interpretation refers to the process of dealing with the recognized meaning.

Based on these premises, Blumer proposes six basic ideas upon which his Symbolic Interactionism is built and which explains the nature of each idea. The six ideas are as follows: ①

human society or human group life, ② social interaction, ③ objects ④ the human being as an acting organism, ⑤ human action, ⑥ interlinkage of action. In brief, society is made up of people's lives, and human life is "a process of ongoing activity in which participants are developing lines of action." (Blumer, 1969: 20). These lines of action are embedded in the process of interaction based on indication and interpretation. Human beings constitute and base their actions and directions on the meaning of objects acquired through the interaction with objects around them.

Now Blumer proposes two ways to draw aside the curtain covering this world of experience, the domain of human life. One is "exploration" and the other "inspection". He says that exploration is a way of making a close relationship with the empirical world as well as a method to draw research questions and data, and to derive the process of analysis and interpretation. Exploration is a process to narrow down the focus of the study. The purpose of an explorative type of research is to build a comprehensive picture of people's lives by moving "toward a clearer understanding of how one's problem is to be posed, learning what are the appropriate data, developing ideas of what are significant lines of relation, and evolving one's conceptual tools in the light of what one is learning about the area of life"(Blumer, 1969: 41). Here, it is very important to get through the hermeneutic cycle involving a researcher's image, beliefs, and concept of the world, which are the objects of the study. In relation to this, Blumer mentions Darwin's two recommendations. First, a researcher should ask all the possible questions on the research of oneself. Second, a researcher should record all the details within his purview. Blumer's inspection refers to a close review of the empirical cases which might come under analysis. The nature of inspection lies in its flexibility and creativity such as when found in a child exploring a strange object. Through the inspection, a researcher "goes to the empirical instances of the analytical element, views them in their different concrete settings, looks at them from different positions,

asks questions of them with regard to their generic character, goes back and re-examines them, compares them with one another, and in this manner sifts out the nature of the analytical element that the empirical instance represent"(Blumer, 1969: 44).

Let us summarize the implications of Blumer's Symbolic Interactionism as a methodology of the study. First, it is necessary to take certain perspectives of the people in question because human beings take action based on the meaning of objects they face. In this vein, a researcher is expected to assume a perspective with cultural relativity and anthropological sensitivity. Second, human life is made up of the processes of interaction involving indication and interpretation. Therefore, human life, an object of research, should be understood as processes of mutual definition and interpretation. Furthermore, those definitions and interpretations should not be understood as the only one possible because they are based on the specific context in which human beings interact with one another. Third, social behavior, whether it is formed between individuals or groups, is a subject of social science. It is also the starting point and destination in the analytical framework. Accordingly, a deep understanding of social behavior is acquired through the understanding of how and in what process such behavior is formed. Fourth, researchers in the circle of Symbolic Interactionism pursue an explanation through defining and interpreting the context in question from the perspective of each individual involved in the network because a complex network of human action takes a lissome form.

Symbolic Interactionism makes a dramatic turn by employing the perspective grasping the students' mathematics learning in a socio-cultural context rather than limiting it to the scope of individual learners. Vöigt(1996) legitimizes his view of mathematical meaning as a product of negotiation based on views of Lakatos and Wittgenstein and admits that he follows the tradition of Symbolic Interactionism. According to Vöigt, interaction, from the viewpoint of Symbolic Interactionism, is

more than a serial exchange of action and counteraction. Furthermore, the relationship among mathematical meanings is formed in the process of continuing negotiations among teachers and students, which in turn forms certain interactional patterns. In other words, negotiation takes added value when considering the object of analysis within mathematical learning and teaching.

Yackel and Cobb(1996), who initially based their position in research on constructivism, accept Symbolic Interactionism realizing that cognitivist research of learning has its limitations and thus they need to expand their view to the interpretive position by exploring the social aspects of learning. As a result, they explain the development of social norm in a classroom, which is a component in the interpretative framework of classroom research. They pay attention to the point that individual reasoning and meaning-making in a classroom cannot be dissociated from his or her participation in the process of meaning negotiation through interaction within the group.

Sierpinska(1998) states that the source of valid knowledge in epistemology based on Symbolic Interactionism lies in the very language of discourse and social practices, rather than in observation of the 'objective world' or empiricism or rationalism assuming innate reason, or a logical structure of the mind emerging in developmental stages. This point of view that knowledge is socially constructed through the process in which discourse and its language are defined and interpreted are based on Symbolic Interactionism.

B. Participation Structure of Mathematics Classroom

There exists a distinctive participation structure in each classroom. A mathematical classroom is not an exception. The atmosphere of a class varies depending on the students even if the same teacher leads the class or if the class leads the teacher. The atmosphere of a classroom has an impact on the participation structure of a teacher and students. The concept of a 'social norm' is closely interrelated with this participation

structure.

Cobb and his co-researchers defined the social norm as 'duties and expectations related to class participation' after they discovered that students and a teacher form a series of participation structure through mutual meaning negotiation as they face conflicts between the teacher's intention or expectation and the students' actual participation(Cobb & Bauersfeld, 1995; Yackel & Cobb, 1996). Examples of the social norm include "explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternative in situations in which a conflict in interpretations or solutions has become apparent"(Cobb & Yackel, 1996: 178). They thought that an interactional process is the fundamental aspect of learning and it is necessary to focus on the social norm on duties and expectations for specific group activities emphasized by a teacher (Cobb, Gravemeijer, Yackel, McClain & Whitenack, 1997).

Cho(2001, 2005) defined types of the social participation structure while analyzing the turn-taking structure among a teacher and students.

Type	Interactional Process
SPS-1	Teacher is the main speaker and most of the students respond to the teacher's elicitation in a uniform way.
SPS-2	Teacher is the main speaker and one or a small group of students respond to the teacher's elicitation.
SPS-3	Teacher is the main speaker and asks a specific student to respond to his or her elicitation. The rest of the students participate indirectly via the student's responses.
SPS-4	Students take part in small group interaction after the teacher talks about something.
SPS-5	A student is the main speaker and the teacher responds to his or her statement.

<Table 1> Types of Participation Structure in Economics Class in Korea

He proposed the following four individual concepts: 'organization of a class', 'turn-taking patterns in dialogue', 'social participation structure', and 'transformation of learning themes'. He states that casual classroom instruction in Korea is made up of a series of teachers' elicitation and students' responses and suggested five types of Social Participation Structure, which is summarized in <Table 1>.

Considering the characteristics of a mathematical class, where students often have to contribute to the class by showing their own solution and explaining it to other students, these types cannot be applied directly to a mathematics class. And these types need to be complemented so that they can grasp the dialogical flow by tapping into the context of turn-taking.

Ⅲ. Methodology

Our research methodology is to review the types of participation structure a teacher and students form. To answer this question, we analyzed the interactional process involving the teacher and students' communication and activity in a second grade middle school classroom through participant observation.

Researchers need to experience the context of specific actions and dialogues through direct participation to understand the interactions among people in a specific group, in this case, the second grade classroom. A qualitative method, which takes both phenomena and their context into account by observing the field for a long period of time, analyzing the obtained data and continuing data collection, is required to fulfill this task. Therefore, we have adopted the qualitative research method in an institutionalized setting to study the life and social interactions of people in a mathematics classroom

A. Qualitative Research in an Institutional Setting

Institutional setting refers to a place where a set of norms

or rules in interaction are deemed to be more rigorous than those of other places. Classrooms in a school, prisons, and offices in a company are typical examples of the institutional setting (Cho, 2005). What are the characteristics of research on an institutional setting? Qualitative research understands the classroom as a field of life where teachers and students produce the fruit of 'life and actions' through mutual, face-to-face interactions. Accordingly, the research focuses especially on specific scenes in an institutional setting named "the classroom."

Anyone can make comments on a classroom in an institutionalized context because he or she has experienced this space in person. The fact that a classroom is very familiar to anyone might hinder the development of research. In other words, the familiarity of the space might drive researchers to a hasty judgment because they think they already know enough about the object of the study. In these respects, the study of a classroom requires a 'deconstructing and estranging' approach to make the familiar strange by looking at things in a doubtful and skeptical way. In other words, we experience the hermeneutic cycle in which we understand the whole by grasping its part through taking the 'deconstructing and estranging' approach, and familiarize the deconstructed and estranged things through understanding the part in relation to the whole (Cho, 2005). In this context, the true understanding of the classroom culture can be achieved not by the theorization based on one's past experiences, but by participant observation of the actual fields-the mathematics classroom in this case, analysis of the participants' situation and interaction, and interpretation of these.

Data analysis in a qualitative study can be understood in different levels. Cho(1999) points out that 'analysis' in "data collection and analysis" is in contrast to 'collection' and includes different concepts like 'description', 'analysis', and 'interpretation.' His concepts of description, analysis, and interpretation follow those of Wolcott(1994). According to Cho, description enables readers to see what the researcher saw while analysis enables

readers to know what the researcher came to know. Finally, interpretation is the task to make readers understand a phenomenon in the same way the researcher understood. Therefore, the term 'analysis' refers to the task of grasping the structure of a phenomenon in a narrow sense. On the other hand, it includes description, analysis in a narrow scope, and interpretation when used in a broad sense. We use the term 'analysis' in the sense that includes description and analysis in a narrow scope and 'interpretation' in the sense that includes explanation of the meaning of a phenomenon based on the analyzed data.

We took the coding method as one of the major tasks in our research. The coding systematically creates certain categories based on recurrent vocabulary, theme, scene, etc. We also used qualitative analytical tools such as taxonomy which binds coded data based on their common characteristics, flow charts which represent a series of actions or events, and networks which show the relationship among various elements or individuals.

B. Venue of Research and Participants

This research is based on the videotaping and observation of the context of the mathematical instruction and participants' interaction with one another. It is not usual that students and teachers take part in a class with someone else observing them. Therefore, the very presence of researchers has the potential to cause unnatural actions of students as well as the teacher. In this sense, there is the need to create an atmosphere which will reveal the natural process of a mathematics class without any bias or distortion. To this purpose, we have employed a mathematics teacher, who clearly understands the aim and process of the study, as an informant and shared various aspects of the research with her before the classroom observation started.

Evergreen Middle School, the venue of this research, is located in Seoul. The research in an institutional setting requires the permission of a teacher and students as well as a principal

of the school due to the very characteristic of the public space. Therefore, we had an interview session with the principal of the school, shared the intention and implication of the study, and got permission to do the research before the research started so that we had no problem in proceeding with the classroom observation and videotaping. We set a built-in video camera in the rear part of the classroom and videotaped all the mathematics classes. When we visited the class, we set another stationery camera at the front of the classroom. In addition, we took digital photographs and taped the classroom with an individual camcorder, changing positions.

Ms. Park, the participating teacher of this research, is a young female teacher with a three-year career, who graduated from a teacher's college and passed the teachers certification exam. She said that she was more passionate during the first year of teaching and was eager to prepare for student-centered classes but lately she found herself taking the central role in the class. In other words, she let students solve the problems and then explained the solution herself rather than allowing students to describe the solution process to other students, which made her unaware of students' thoughts and weaknesses.

Meanwhile she thought that the students' limited mathematical knowledge caused their failure in keeping up with her class. However, she confessed that she had realized that she is the source of the students' unsatisfactory performance while discussing the instructional methods with other teachers in more senior positions. This situation made her make up her mind to go on with her study in graduate school. She also started pondering how to create a classroom atmosphere that encourages student autonomy in mathematics classes. As of 2005, she is teaching the second grade mathematics and working as a homeroom teacher of the third class of the second grade while studying in graduate school. Especially, she is conducting a co-research project on the culture of mathematics classes.

A total of thirty eight students-19 male and 19 female were studying in the third class of the second grade. Ms Park was acquainted with students' family backgrounds and achievement in other subjects because she was in charge of the class as a homeroom teacher. The average mathematics score of the class was slightly below the overall average of the second grade students. However, students had lively participation in the class dynamic, especially showing eagerness in class presentation.

C. Data Collection and Analysis

1. Data Collection

We used the triangular structure of research data, which includes various sources of data such as observation, interview, and documents, collaborative analysis and participant review, and the method of triangular validation based on different data, in order to overcome the criticism of validity and reliability of qualitative research(Creswell, 2003; Merriam, 1998; Strauss & Corbin, 1998). Our data collection is part of a collaborative research on classroom culture. Raw data is the videotapes which contain the entire mathematics classes of the third class of the second grade for one year. Especially, we participated in the observation of the second class of every Tuesday and recorded and videotaped the communication and behavior of the teacher and students with two stationery video cameras and a portable one. We also had interviews with Ms. Park when co-research team meetings were held, which were also recorded. We were able to understand how she thought about her classes and get detailed information about the school and classroom through a regular non-structured interview with her on every Monday and Wednesday. Besides, we gathered the data about the important people who influenced on her decision to become a teacher and her latest field of interest as a teacher through her 'self-report.'

2. Data Analysis

Basically, analysis was based on the collected data. However, we used the constant comparison method. In this method, data collection is continued while accumulated data are being

analyzed, and newly discovered facts are coded and categorized, whose results are compared with the raw data.

Data analysis was mainly concerned with the understanding of the types of participation structure among the teacher and students. We set the criteria like the following based on Cho(2001)'s five social participation structure types. The first criterion is who has the initiative in a certain interaction. The second one involves the turn-taking of dialogues and actions, which accordingly is related to the first criterion. The third one is the scale of interaction: it might be one-to-one or one-to-many. The fourth one is the teacher's position in the field of interaction. The types of participation structure we propose in the following section reflect our actual observation of the classroom rather than represent all the possible types derived from the above criteria. We analyzed the transcript of the recordings, coded each dialogue, and visualized the results in flow charts and networks (Cho, 1999).

Ⅳ. Analysis and Result

We adopted a middle school classroom and analyzed its participation structure to understand the interaction between a teacher and students in a mathematics class. We grasped the characteristics of interaction in a series of discourse among initiators and active/passive participants. This approach is based on the concept of 'interlinkage of action' of Symbolic Interactionism. A social phenomenon within the mathematics class has a complex structure of the interconnection of 'interlinkage of action', which is made up of a series of dialogues and actions.

According to the criteria suggested in the previous section, we categorized the participation structure based on the scale and turn-taking of a dialogue. Especially, initiative of a dialogue and the position of a teacher were also considered. As a result, the

following types of participation structure were proposed.

Type	Initiative	Scale of Participation and Turn-taking Patterns	Role of Teacher
ER-1	Teacher	Teacher elicitation- Response of one student	Leader
ER-2 (ER-2')		Teacher elicitation- Response of all the students	
ER-3		Teacher elicitation- Response of one student - Response of all the students	
ER-4	(Teacher)	Teacher Elicitation- Individual/Small group activities	Leader → Guide
CR-1	Student	Student contribution - Teacher elicitation - Response of all the students/the contributor	Observer → Leader
CR-2		Student contribution - Response of all the students- Teacher intervention	Assistant
CR-3		Student contribution - Response of all the students	Observer

<Table 2> Participation structure of a Korean Middle School Mathematics Class

Now let us summarize these types one by one, paying attention to a few notable characteristics.

A. Teacher-centered Participation Structure

Most of the time, a teacher takes the initiative in her/his interaction with students. This structure is discovered when one or multiple students respond to a teacher's elicitation. The following four types have been identified as belonging to this type of teacher-centered participation structure.

1. ER-1 Type

< #1 Class Subject: the comparison of two infinite recurring decimals>

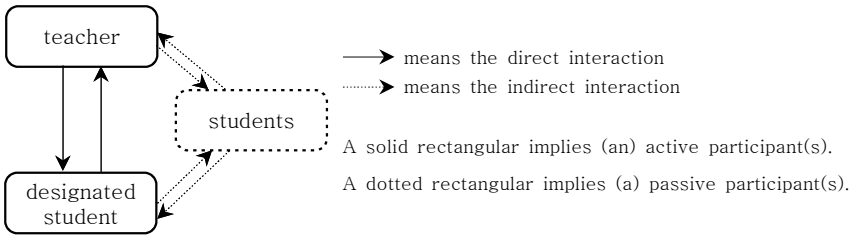
1 Park:	Jae-Hyun! (Writes $2.\dot{3}9$ and $2.\dot{5}\dot{8}$ on the blackboard.) [Designates Student]
2 Jae-Hyun:	(Stands up after being designated by the teacher.) [Prepares to answer the teacher's question]
3 Park:	(Pointing to the blackboard) Which is bigger, the one on the r or l? [Question requiring the student to choose an answer]
4 Jae-Hyun:	Left! [Response: Selection]
5 Park:	The left one? [Repetition of the question]
6	(Interrupting the teacher's utterance) No, no. The right one. [Response: Modification of one's selection]
7 Park:	(Adds the inequality sign like $2.\dot{3}9<2.\dot{5}\dot{8}$ on the blackboard.) Why do you think so? [Subsequent question: Why?]
8 Jae-Hyun:	The tenth digit is bigger. [Response: The reason for his selection]
9 Park:	The tenth digit is bigger? Why do you compare the tenth digit of two numbers?
10 Jae-Hyun:	Because the unit digits are the same. [Response: The reason for his selection]
11 Park:	(Speaks towards the whole class underlining the units digit 2 of $2.\dot{3}9<2.\dot{5}\dot{8}$ on the board.) The largest units digits, here "2", are the same. So we turn to the next digit, here the tenths digit. (Draws an arc-shaped underline below the tenth digit numbers "3" and "5".) So we compare the tenth digits 3 and 5 and 5 is the bigger. [Complementary explanation] (Speaks towards the whole class.) He did a good job, right? [Request for confirmation]

This type of structure is found when the teacher asks for

one student's response, followed by the teacher's next elicitation and the student's response to it. Though this type is often found in the confirmation stage of the class, it is not dominant. In our observation of the type, the dialogue between Ms. Park and one designated student comes to the foreground and other students go to the background. It seems that this type is not directed at all the students at first. However, Ms. Park diverts the target audience from one student to all of the students by raising questions that require them to decide an issue in the middle or conclusion of a dialogue.

This shows that ER-1 is not just one-to-one interaction. Rather, it evokes students' indirect participation through their observation of the dialogue between the teacher and a student. Based on these points, the relationship among the teacher, the designated student, and the other students can be represented in the network like <Figure 1>.

To conclude, the ER-1 type apparently involves a one-to-one interaction between the teacher and one designated student. However, we can interpret that the teacher and multiple students are interacting with each other in an indirect way through the designated student at the front of the classroom.



<Figure 1> Relationship among Participants in ER-1 Type

2. ER-2 Type

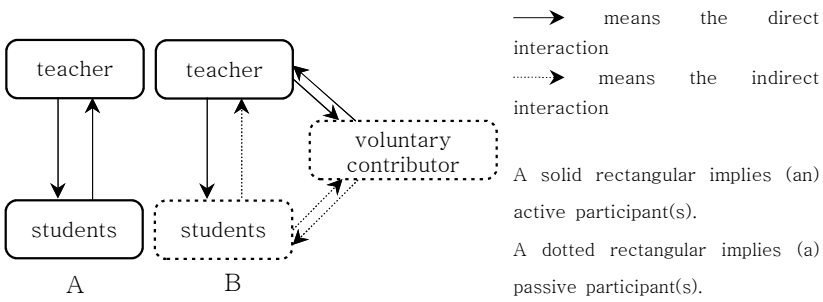
< #2 Class Subject: Two ways to represent a fraction as a definite decimal>

1 Park:	Now let's learn how we can represent $\frac{9}{40}$ into a decimal, a definite one. First of all, we can think of two ways to turn it into a decimal (Indicates 'two' with her fingers.). The first one is .... [Question: What]
2 Students:	Divide the numerator by the denominator! [Response: Method]
3 Park:	Divide the numerator by the denominator. Alright. The second way is to do what to the denominator? [Question - uncompleted]
4. Students:	Use prime factorization... (Slurs the end of the sentence)? [Response: A short answer]
5. Park:	(Tilting her head) Prime factorization? What for? [Repeated question]
6 Students:	... (Hesitation and murmurs among the students)
7 Park:	No, no... here we are trying to turn a fraction into a decimal. [Supplementary explanation]
8 Eun-jung:	Aha! Divided by 10 ... [Response by a voluntary contributor]
9 Jae-hyun:	Represent the denominator as a power of 10. [Response by a voluntary contributor]
10 Park:	You got it. [Acceptance] We can multiply the denominator and the numerator with the same number to make the denominator a power of 10. [Supplementary explanation]
11 Students:	(Some nod their head while others remain unresponsive.)
12 Park:	You can do the prime factorization first as you said. It is all right if you do the job after you have changed the denominator into prime numbers. [Supplementary explanation]

This type of structure is found when the teacher asks for all the students' response, followed by the teacher's next elicitation and the students' response to it. This type shows the direct interaction, on its surface level, between the teacher and students, who involve the indirect interaction with the teacher in ER-1. In other words, multiple students here can be understood as a collectivized individual. In this type of participation structure, students give short answers: they answer Ms. Park's questions using one or two words rather than giving description or explanation. We can find a striking difference between multiple students' short responses in ER-2 with those of the designated student in ER-1, which take the form of sentences. Questions like "What?" or "How?" are used in ER-2 in contrast to "Why?" in ER-1.

On the other hand, it is often the case that one student interacts with the teacher in ER-2. However, there is an important difference between this and ER-1. The student, who participates in a one-to-one interaction with the teacher in ER-2, does not initiate his utterance in reaction to the teacher's elicitation. Rather, he starts his voluntary response when other students do not initiate the expected response or hesitate for a while. For this reason, this student was named 'a voluntary contributor' in this study.

It does not seem that all the students respond to the teacher's words in a uniform way in ER-2 or ER-2'. However, Ms. Park's words and attention are directed towards students in general, some of whom respond to the teacher's explanation or questions while others keep silent, rather than address or communicate with a specific student. Based on these points, the relationship among the teacher, students, and a voluntary contributor can be represented in the network like <Figure 2>.



<Figure 2> Relationship among Participants in ER-2 and ER-2' Types

In <Figure 2>, (A) represents the network for ER-2 while (B) represents the network for ER-2'. Students' silence or hesitancy in ER-2' shows the situation where they cannot think of plausible answers or express their opinion effectively in their own words. This situation gives rise to the context in which a voluntary contributor emerges. This scene is found more often in the geometry classes in the second semester than in the algebra classes in the first semester. Further, it was found that ER-2' is more prevalent than ER-2 in the geometry classes. Specifically, the voluntary contributor was limited to a couple of students when the teacher explained the proof on the characteristics of a figure.

The teacher tended to regard a voluntary contributor's answer as the response of all other students when there was no disagreement. Thus, ER-2 and ER-2' are not discrete. For some students, a voluntary contribution gives a clue to understanding difficult questions. In this vein, the existence of ER-2' type reflects the natural emergence of ZPD(Zone of Proximal Development) in the classroom(Vygotsky, 1978).

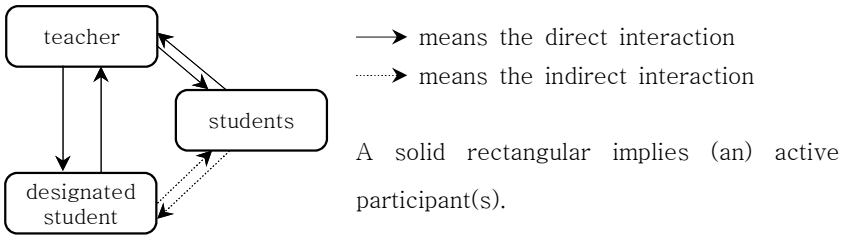
3. ER-3 Type

< #3 Class Subject: Measuring length>

1 Park: Let's do number one together. Student number one! [Instruction]

2 Han-shin:	(Playing with his classmate in the back)
3 Park:	(With a warning voice) Han-shin! [Designation of the student]
4 Han-shin:	Yes! [Preparation]
5 Park:	Would you read question number one? [Instruction]
6 Han-shin:	(Directly picks up his book and starts reading.) Measure the length of the marked part in the following pictures with a millimeter ruler.
7 Park:	Oh, measure with a millimeter ruler! What's your measurement result for number one? [Question: How long(What)?]
8 Students:	(In a loud voice) 32 millimeters! [Response: A short answer]
9 Park:	32 millimeters. (Echoing the students' response, and gesturing with her hand.] And? [Subsequent question: What?]
10 Students:	(Some of the students) 3.2 centimeters. [Response: A short answer]

This type of structure is found when the teacher asks for one student's action like reading aloud a question in the textbook, followed by the teacher's next elicitation of the entire class response to it. In this type, Ms. Park designates a student who is not concentrating on the class. She may proceed to the ER-2 type interaction without taking this step. However, she point out one student to the purpose that the classroom atmosphere may be refreshed. The first part of ER-3 is similar to ER-1 in that the teacher designates one student for a response while its latter part resembles ER-2 in that the teacher's elicitation is directed towards all the students. While one student is reading aloud a question in the textbook, other students are preparing for the answer, expecting a subsequent interaction with the teacher. Based on these points, the relationship among the teacher, the designated student, and the other students can be represented in the network as illustrated in <Figure 3>.



<Figure 3> Relationship among Participants in ER-3 Type

Comparing ER-1 with ER-3, we can find the dotted line relationship in ER-1 between a teacher and students is replaced by the solid line relationship in ER-3. In ER-3, the teacher uses the strategy to assist all students to concentrate by designating one student to read aloud a question. Students do not know who will be designated but they are aware that they are likely to be chosen if they do not focus on the class. As a result, the participation structure like ER-3 has the effect of making students focus on the class for a moment. This type of participation structure was not found as frequently as ER-1 or ER-3. It was used when students are not paying attention in class or they needed to focus on a newly introduced explanation or question. This type of interaction happened at most one or two times per class.

4. ER-4 Type

< #4 Class Subject: Solving problems on rational numbers and decimal numbers>

1 Park:	OK. Who is gonna come out and solve question number two? [Proposing the opportunity for presentation]
2 Woo-jae:	All of it? Everything in number two? [Repeated question]
3 Park:	There is only one question in question number 2 on page 21.
4 Students:	(Some of them raise their hands.) [Volunteering]
5 Park:	Ah! Seung-soo! [Designation of the student]
6 Jae-hyun:	(Raising his hand up high) Number

three, three, three! [Volunteering]

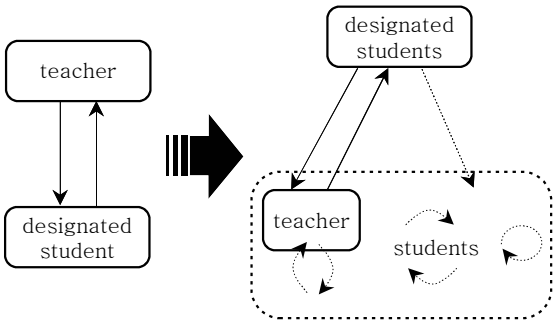
7 Park: Seung-soo, come out and take number 2. For No. 3, Jae-hyun take it. Solve the bracketed one in No. 4, Yosep... (The rest has been omitted. Ms. Park designates some more students after this.) [Naming]

8 Park: Please come out and do your part. [Instruction]

9 Students: (Designated students come out and start solving the problems. The other students start solving the problems in their textbook or chatting with classmates.) (The rest has been omitted.) (While designated students solve the problems, Ms. Park walks around the class and gives individualized instruction by receiving questions from some students and explaining answers to others..) [Individualized activity, Individualized instruction]

This type of structure is found when students interact with one another individually or in a small group after the teacher's instruction. It involves the situation in which students are asked to write their solution on the blackboard. The problem solving activity in Ms. Park's class has two types of 'solving together' and 'showing one's solution'. 'Solving together' refers to the method in which the teacher shows how to solve a problem to all the students. This usually takes the participation structure like ER-2 and ER-2', sometimes taking the types of ER-1 or ER-3. On the other hand, 'showing one's solution' involves the selection of (a) student(s) who will come in front of the classroom and show his or her solution to students through the teacher's designation or students' volunteering. The selected student(s) comes out to the front and solves the problem while other students start their own activity. At this moment, the designated student have the duty and right to demonstrate his or her solution but the other students are not required to focus on a specific task. Thus, students may just relax or chat with other students. Ms. Park

says that she allows this relaxed behavior as long as they are not "extremely noisy." Some students, on the other hand, use this time to ask personal questions of the teacher. Ms Park also takes this time to give individualized instruction to students or provide specific students with the explanation which she thinks don't need to be given publicly. The relationship among the teacher, the designated student(s), and the other students can be represented in the network such as in <Figure 4>.



→ means the direct interaction
⇢ means the indirect interaction
A solid rectangular implies (an) active participant(s). A dotted rectangular implies (a) passive participant(s).
means local interaction and thus does not come to the forefront of the class.
means students' individual activities such as self-directed

<Figure 4> Relationship among Participants in ER-4 Type

In ER-4, the initiative of the interaction still belongs to the teacher. However, there exists a transitionary period when the initiative is being transferred to students during the 'showing one's solution' session. We can view this as 'transition' because the designated student has to explain his or her own solution to other students in addition to writing down the solution on the blackboard. This is the rule shared by Ms. Park and the class. In fact, every designated student understood that he or she must explain the solution to the class and this rule was kept through all the classes we observed. In short, the student-centered participation structure is created after the transfer of the initiative from the teacher to the designated student.

B. Student-centered Participation Structure

Now, let us review the student-centered participant structure, characterized by students' contribution to the class and the teacher's questions and students' response to them, which emerges after the "showing one's solution".

1. CR-1 Type

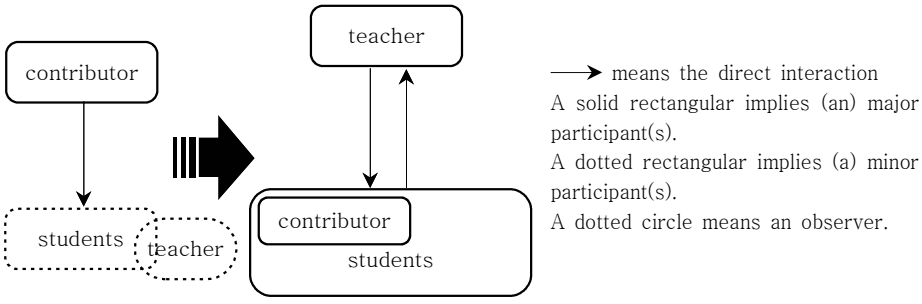
< #5 Class Subject: Representing a fraction as an infinite recurring decimal>

1	Sung-jin:	(Comes out to the blackboard and starts explaining the question he has just solved.) Uh, we've got to turn $\frac{8}{33}$ into an infinite recurring decimal. The numerator divided by the denominator, uh..., we get zero..., 80 and so, put 2 here and get 66, and the remainder is 14..., we need another 0 here, we have 140.... If we keep going, 2424 is repeated. We place two dots over 0.24 because 2424 is constantly repeated. [Presentation: explanation]
2	Park:	OK. Any questions? [Elicitation of questions]
3	Students:	... (No one speaks.) [Silence]
4	Park:	No question? Good job. [Evaluation]
5	Sung-jin:	(Goes back to his seat.) [End of presentation]
6	Park:	(Moving towards the board) Isn't this correct if I do like this? (Writes 0.2424 beside 0.24, which Sung-jin wrote down.) [Question: ironical inquiry]
7	Students:	No! [Response]
8	Park:	No? Why? [Question: Why?]
9	Students:	Minimum...! [Response: why]
10	Park:	Right. We said that we should set just one cycle. However, how many cycles

	did we apply to this number (0.2424)? [Question: incomplete sentence]
11 Students:	Two! [Response]
12 Park:	Right. Two repeated cycles of 2424. So we cannot represent the number like this. No. [Consolidation]

This type of participation structure is found when there is no specific response of students to the student's contribution and thus the teacher intervenes by asking questions to elicit students' response. This type is the most frequent one among the types of student-centered participation structure we have observed.

This type of interaction happens when there is a possibility of dialogue closure right after the student's contribution while there are some important points worth mentioning, as far as the teacher judges, but students fail to recognize this situation. In this case, the teacher's elicitation is directed either towards one specific student or students in general. We can witness the shift of the teacher's role here. During a student's contribution, the teacher takes the role of an observer. However, she takes the initiative of the dialogue as soon as the contribution is concluded. In other words, she shifts her role from an observer to an active participant, who raises questions and encourages students' responses. This type of participation structure can be represented by the following illustration <Figure 5>.



<Figure 5> Relationship among Participants in CR-1 Type

At the initial stage of the structure, the teacher expects students to involve an active and autonomous interaction but this does not happen. The teacher takes an alternative to encourage students' responses by raising questions herself, which creates modified ER-1 or ER-2 structure. However, CR-1 is based on the student's contribution such as his or her solution and explanation of it while ER-1 and ER-2 are initiated by the teacher based on her planned content and materials.

2. CR-2 Type

< #6 Class Subject: Representing a fraction as an infinite recurring decimal>

1 Young-man:	(Comes out to the blackboard and starts explaining the question he has just solved.) First we should check whether $\frac{4}{15}$ is reducible or not. Then 4 divided by 15, ... (3 seconds) multiply 2 here to get 30, add 0 ... (5 seconds) Uh, subtraction like this makes 10, which cannot be divided by 15. So put a period here and add 0, multiply 6 here to get 90, we get the same number again and add 6 here, and ..., we get $0.2666\cdots$, here we have 6 over and over again. So we put a mark over here (Indicating the second 6 in "$0.2666\cdots$") So the answer is $0.2\dot{6}$. [Presentation: explanation]
2 Park:	Ah, good job. Any questions? [Question elicitation]
3 Young-man:	(Goes back to his seat as soon as Ms Park's utterance ends) [End of presentation]
4 Park:	There might be some questions. [Repeated elicitation of questions]
5 Joon-soo:	[Raises his hand] [Volunteering]
6 Park:	Ok, yes, there is a question for

Young-man. Please answer the question before you go back to your seat.. [Intervention: holding the presentation session]

7 Joon-soo: There are two six's in 0.266 there. Shouldn't we put a mark over the first 6 of 66 rather than the second... [Question]

8 Park: (Looking at Young-man) Can you understand, Young-man? [Intervention]

9 Young-man: (Nods his head while coming back to the front.) [Response]

10 Park: Oh, then do it again, please. Go ahead.

11 Young-man: (Looking at his solution on the board) What was it? One more time. [Confirmation request]

12 Joon-soo: (Smiling) So, you have 0.266 there. [Repeated question: why?]

13 Young-man: (Nodding his head) Yeah! [Agreement]

14 Joon-soo: And you put a mark on the last digit. [Question: confirmation]

15 Young-man: Yes. [Agreement]

16 Joon-soo: Not there, I think. Shouldn't we write 0.26 and put a mark right above the 6 there? [Question: method]

17 Young-man: Aha! Now I got it. I think I was wrong. (Modifies his answer on the board from 0.266 to 0.26.) [Response: modification of one's solution]

18 Park: Any other questions? [Elicitation of further question]

19 Students: ... [Silience]

20 Park: No further questions?

21 Young-man: (Goes back and sits down.) [End of presentation]

(This has been omitted.)

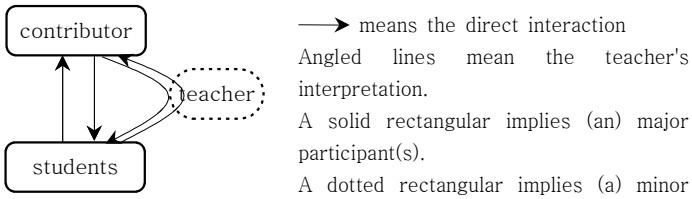
22 Park: Alright. In mathematics, we can reason that, in placing a mark like this, put it over the first cycle. So we may as well have put the mark over the first 6. Your comment was relevant. (This has been

omitted.) [Wrap-up and Evaluation]

This type of participation structure is found when a student makes contribution to the class and an interaction between the contributor and students is made by intervention of the teacher. First, the contributor explains referring to his or her solution that was written on the blackboard. After the explanation, the teacher encourages students to raise questions of the solution. The teacher repeats her request for questions, if the solution is not accurate or the contributor's explanation is considered to be inadequate, to the effect that students have the opportunity to reflect on "something wrong" in the given solution or explanation. Here we can find that Ms. Park is steering the direction of the class even after she hands off the direction of the class to the students. Another participation structure between the contributor and the inquirer is created if a student raises a question on the given solution or explanation to the student who contributed. In this case, an interaction happens mainly among students while the teacher sometimes intervenes to facilitate the interaction process. During this interaction, the content of contribution is modified by mutual negotiation. After these steps, the contribution session is closed. The teacher recapitulates the whole process and one big phase of a class is concluded.

Although the teacher in CR-2 intervenes in the class to facilitate the interaction between the contributor and students, she does not ask questions directly related to the solution and explanation. Ms Park's intervention is limited to the guidance of students' reaction or confirmation of their understanding of the content being discussed. The fact has significance that there exists a process where the teacher leads students to modify inaccurate points of a solution or an explanation by encouraging them to raise questions and communicating with one another rather than pointing out the wrong or inadequate aspects herself. In this vein, a teacher, in contrast to the widespread belief that he or she usually dominates a class, can empower students as well as take the role of facilitator by delivering partial control to them. In this case students can learn adequately from mutual

interaction. This type of participation structure can be represented in the following <Figure 6>.



<Figure 6> Relationship among Participants in CR-2 Type

Note that this structure is typically discovered in the algebra classes of the first semester: the geometry classes of the second semester did not witness students' responses to the teacher's repeated elicitations: it just witnessed students' request for the reiteration of the explanation of the solution.

3. CR-3 Type

< #7 Class Subject: Fraction that can be represented as a finite decimal>

1 Da-hye: (Comes out to the blackboard and reads aloud the question she is about to explain.) Find the value of numerator A if the fraction $\frac{A}{B}$ meets the following conditions. [Confirmation-reading the question] Uh, among the conditions, the second one says the value of B is 1400. Then (Writing down "$\frac{A}{1400}$" on the board) we get $\frac{A}{1400}$, right? [Explanation - Elicitation of response]

2 Students: (Paying attention to Da-hye's explanation and nod their heads.) [Response-passive]

3 Da-hye: And, uh, the third condition says that $\frac{A}{B}$ is represented as a finite decimal.

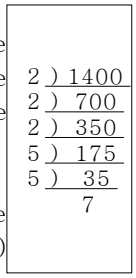
So if we get just 2 or 5 in the denominator when we factorize 1400 into primes, then the result would be a finite decimal, right? [Elicitation of response]

4 Students: Yes! (Some reply with voice while others clap their hands.)

5 Da-hye: Then let me do the prime factorization. (Writes down, the first step of factorization on the board.) [Guidance]

6 Students: 700! [Response]

7 Da-hye: (Writes down)700 after the students' response.)



[Record]

8 Students: 2!, 350! [Response]

9 (This part has been omitted.) (Students continually give response to Da-hye's explanation, keeping pace with the process of factorization. When the process has been finished, the result is shown as follows.)

10 Da-hye: The result is the 2 cubed multiplied by ...? [Questions: incomplete form]

11 Students: 5 squared! [Response]

12 Da-hye: The square of 5 multiplied by ...? [Questions: incomplete form]

13 Students: 7! [Response]

14 Da-hye: 7? (Writes down $2^3 \times 5^2 \times 7$ on the board.) [Request of confirmation]

15 Young-man: (Smiling) Yes! [Response]

16 Da-hye: By the way, we still have 7 here even though we should have just 5 or 7. Then this 7 should be reduced. Right? [Request of confirmation]

17 Young-man: (In a louder voice) Yes! [Response]

18 Da-hye: Then A should be divided by 7 to get a finite decimal. The first condition says that A is a multiple of 11. (Pointing to 7 in $2^3 \times 5^2 \times 7$) A is divided by 7 and a multiple of 11, then A is ..? [Question: the value]

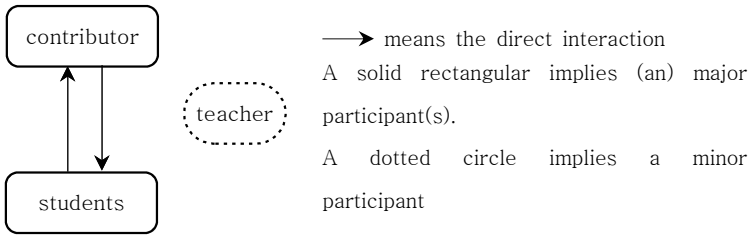
19 Students: 77. [Response]
20 Da-hye: 7 multiplied by 11 makes 77. [End of presentation]
21 Students: Aha! / Wow! (Claps here and there) [Evaluation - Applause]

This type of participation structure is found when a student makes contribution to the class and he or she interacts with other students, without any intervention of the teacher. First, a student reads aloud his solution to students and concentrates them on his or her presentation by asking appropriate questions of them. This is similar to the process where the teacher asks students to read aloud and let them understand questions. The contributor in CR-3, taking after Ms. Park's techniques, guides other students to the proper problem-solving steps. Using this strategy, he or she explicates the next steps in the process of problem solving and shows the whole process while encouraging students' responses. Students show satisfaction and agreement by vocalizing their approval and applauding when the contributor concludes the presentation with the final sentence.

The student's contribution in CR-3 is fundamentally different from that found in CR-1 or CR-2. The contribution in CR-3 is characterized by dialogical interaction from its initial step while the one in CR-1 and CR-2 involves unilateral presentation of the solution to other students. It is also noteworthy that Ms. Park does not involve this participation at all. Ms Park just takes the role of an observer who pays attention to students' interaction, consequent revision of the solution, and their recount of the solution.

This kind of student-centeredness is typical of CR-3. In the wrap-up session of the students' contribution and interaction, Ms Park leads students to exert their autonomy in learning by encouraging them to evaluate their activities on their own. This scene, where students play an active role through all the steps of a dialog, is very noteworthy in this class and can be understood as a result of Ms Park's belief in the importance of the promotion of student autonomy. Based on these aspects, this

type of participation structure can be represented such as in <Figure 7>. In CR-3, students become major participants all through the dialog and the teacher just observes the interaction.



<Figure 7> Relationship among Participants in CR-3 Type

Ⅴ. Conclusion and Final Remark

The research aims to set the foundation for the understanding of the mathematics instruction in the institutional setting of a Korean middle school classroom. To fulfill this purpose, we focused on understanding the types and characteristics of participation structure created by class participants' interaction. Because understanding the structure is starting point of mathematics learning in classroom. So, we observed the second year middle school classroom for one year, analyzed the interaction between the teacher and students, and derived seven types of social participation structure from the result. These types were largely divided into teacher-centered and student-centered ones depending on the initiative of the dialogue. The transitional structure characterized by the shift of initiative from teacher to student was also observed. We named these types as ER-1, ER-2(ER-2'), ER-3(teacher-centered participation structure), ER-4(transitional structure), and CR-1, CR-2, CR-3(student-centered participation structure), respectively. The characteristics of each type are summarized as follows.

First, the ER-1 type is a one-to-one dialogue found when the teacher asks for a specific student's response, followed by the

teacher's next elicitation and the student's response to it. Apparently, this involves the interaction between the teacher and one student, who emerges at the front of the classroom. However, we can recognize that the teacher interacts with all the students indirectly by designating a student as an agent.

Second, the ER-2 type is a one-to-many interaction found when the teacher asks for multiple students' response. In this type of interaction, a voluntary contributor sometimes emerges on the front of the class. It is noteworthy that the existence of ER-2' type characterized by the emergence of this voluntary contributor. In this type, the voluntary contributor keeps the interaction between the teacher and students go on smoothly when students come into the ZPD(Zone of Proximal Development), where they are faced with the problems or questions they cannot solve themselves. This indicates that the teacher and students are collaborating to make their own participation structure optimal for classroom learning.

Third, ER-3 is one of the teacher's strategies to enhance students' concentration. In this type of participation structure, the teacher asks a question of one student, who is not paying attention to the class. This one-to-one interaction is then developed into a one-to-many interaction involving the teacher and all the students. This type can be understood as a series of ER-1 and ER-2 types.

Fourth, ER-4 is the transitional participant structure where interaction shifts from the teacher's elicitation to a student's contribution to the class. In this type of structure, the teacher comes to the back of the class and students come to the front, who will lead the presentation at the subsequent session. Students other than the contributor turn their attention from the teacher to themselves and their local interactions or self-study is observed.

Fifth, the CR-1 type is the participation structure found

when a contributor does not have an expected response from students and the teacher intervenes in this situation, emerging to the foreground of the interaction. It can be interpreted as a failure of the teacher's initial intention to create the atmosphere for students' autonomous participation. Even though the latter part of this structure is similar to ER-1 or ER-2 in that the teacher takes the control of the interaction, this was classified as student-centered interaction due to the fact that the subject and content of the dialog are based on the designated student's contribution.

Sixth, the CR-2 type refers to the participation structure found when a student makes contribution to the class, other students ask questions about his or her presentation, and the contributor responds to the questions, during which the teacher intervenes as a facilitator and interpreter among students. In this structure, the teacher takes the role of an assistant to facilitate the interaction. Students usually participate actively in the class.

Finally, the CR-3 type refers to the participation structure found when one student contributor interacts with other students as if he took the role of a teacher. In this structure, the teacher remains in the background all through the interaction and just observes students' dialog.

We have reviewed types of the participation structure observed in the second grade middle school mathematics classes. We can find that there is a close link between each type of participation structure and the teacher's intention. Factors such as the material being studied in the class and the difficulty level of a mathematical problem will affect the teacher's intention: the teacher tries to create the optimal participation structure taking these aspects into consideration. For example, CR-3 was frequently observed in the proof process of characteristics of a figure in the second semester geometry class while the subsequent participation structure assumes the types of ER-1 or ER-2 for the same content. We have established the criteria to

grasp the teacher's intention for the optimal strategy by examining how various types of participation structure are organized in a class.

Our research activities such as observation, data collection, analysis, and interpretation have been limited to the second grade mathematics classes in Evergreen middle school. Therefore it cannot be guaranteed that other mathematics classes will witness the same types of participation structure. Thus, following research is needed to understand more deeply the participation structure of mathematics classes. Specifically, the following research needs to examine the various classroom participation structures depending on teachers' age, regional characteristics of the target school and classes, etc, comparing research results with the structure types and their implications suggested in this research.

Acknowledgement

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