Capital Income Tax Evasion, Capital Accumulation and Welfare

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We construct an overlapping-generations model where individuals evade capital income tax and carry out the short- and the long-run analyses to abstract the pure effects of policy parameters such as the capital income tax rate and the penalty rate on welfare levels. We show that: (i) undeclared savings may increase both in the short- and the long-run, even when the tax rate (the penalty rate) decreases (increases); (ii) there are trade-offs within each policy and across policies regarding the welfare effects in the short- and the long-run; (iii) both the welfare levels and the government revenue increase in the long-run if the tax rate decreases or the penalty rate increases, as long as the elasticities of such parameters on capital stock are sufficiently large.

Keywords: Tax evasion, Capital accumulation, Overlapping-generations model, Welfare trade-offs

JEL Classification: D91, E22, H26, H31

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[**Seoul Journal of Economics** 2010, Vol. 23, No. 3]

I. Introduction

This paper constructs an overlapping-generations model in which individuals evade a capital income tax imposed on their savings, to compare how the changes in the capital income tax rate and the penalty rate for the tax evasion affect the utility (or welfare) levels in the short- and the long-run.

Since Allingham and Sandmo (1972), extensive studies treating tax evasion have been carried out from static viewpoints, examining the effects of the actions of the government on tax evasion behavior of individuals.[1] Recently, from dynamic macroeconomics viewpoints, Caballe and Panades (1997), Chen (2003), and Lin and Yang (2001) examine the effects of the tax evasion behavior on economic growth. The former two studies incorporate the positive externality of public goods on the productivity of the private sector, developed by Barro (1990). Caballe and Panades (1997), using a Diamond (1965) -type, overlapping-generations framework, deal with the evasion of a labor income imposed on the young individuals, of which penalty is levied when they become old. Chen (2003) examines the optimal income taxation within the framework of an AK-type, endogenous growth model. It shows that the optimal tax rate in the presence of tax evasion is higher than that in the absence of it. In both of these studies, the supply of public goods is financed not only by taxes but also by penalties. Lin and Yang (2001) introduce both a Barro (1990) -type utility function, on which public goods have an externality, and an AK-type production function into a portfolio selection model.

Though the dynamic macroeconomic analyses presented above mainly have paid attention to the interaction between long-run growth and the externality of public goods either on production or on the utility, the shortcomings are twofold: firstly, the difference in the effects on the utility levels in the short- and the long-run has not been inferred; secondly, the interesting results obtained in the above papers are more or less based on the assumption for various externalities on production or on the utility levels. Our motivation is to abstract pure effects (not including the effect attributed to such externalities) on the utility levels, with clarifying the difference of these effects in the short- and the long-run.

[1] Notable surveys on tax evasion theory are those of Cowell (1990), and Andreoni, Erard, and Feinstein (1998).

In general, the main source of economic growth is capital accumulation. The taxation on capital income will, therefore, not only distort the individuals' optimal allocation between consumption in the present and the future periods, but also decelerate capital accumulation. From the standpoint of dynamic macroeconomics, it is crucial to analyze the effects of the change in the capital income tax rate on the utility levels where the capital income tax evasion behavior prevails.

This paper, therefore, introduces the capital income tax evasion behavior of individuals into the overlapping-generations model developed by Diamond (1965), where there is no externality, to compare the effects of the tax rate and the penalty rate on the utility levels in the short- and the long-run. The key elements for determining these effects in this analysis are capital accumulation and the tax evasion behavior, even though the externality neither on the production nor on the utility level exists. To make it sure, the effects on the amount of undeclared savings (*i.e.*, concealed amount of savings) and the level of capital stock would be also investigated.

The main results of this paper are as follows. Firstly, the amount of undeclared savings may increase both in the short- and the long-run, even when the capital income tax rate decreases, or when the penalty rate rises, as opposed to the intuition. These results depend on the volumes of the elasticities of such parameters on capital stock. Secondly, we show that, while a rise in the tax rate necessarily decreases the utility level in the long-run, it can increase the utility level in the short-run. In addition, a rise in the penalty rate can increase the utility level in the long-run, whereas it necessarily decreases the utility level in the short-run. These findings can be stated as the trade-offs within each policy and across policies. Thirdly, in the long-run, both the utility levels and the government revenue increase if the tax rate decreases or the penalty rate increases, as long as the elasticities of such parameters on capital stock are sufficiently large. The above results are all attributed to the tax evasion behavior of individuals.

This paper is organized as follows: Section 2 presents a basic model. In Section 3, we examine both the short- and the long-run effects caused by rises in the capital income tax rate and the penalty rate on capital stock and undeclared savings. Section 4 examines the short- and the long-run effects on the utility levels, and Section 5 provides some insight into policy implications. Lastly, Section 6 contains the conclusion.

II. A Basic Model

The model developed in this paper extends the overlapping-generations model proposed by Diamond (1965) to include circumstances where a capital income tax imposed on individuals can be evaded. The economy begins from an initial period (the first period) and lasts forever. Capital stock in the initial period (k_1) is given. Individuals are identical and live for two periods: the young and the old periods. In every period, there exist the young and the old generations. For the sake of simplicity, we assume that there is no population growth and that the population size of each generation is normalized to one.

A. Maximization of Individuals' Utility

In this subsection, focusing on the individuals who live in the t-th period as the young generation (henceforth called "the t-th generation"), we formulate the optimization behavior of those individuals with an incentive to evade tax.

The individuals supply one unit of labor inelastically to obtain wages, w_t, during their young period. The wages are allocated between consumption during that period, c_{1t}, and savings, s_t.[2] Therefore, the budget constraint in the young period can be expressed as:

$$c_{1t} = w_t - s_t. \tag{1}$$

The savings of the individuals are allocated to consumption in the old period. Savings bear interest at a rate of $r_{t+1} > 0$, on which a capital income tax is imposed at a fixed rate, $0 < \tau < 1$.[3] The net interest rate, which the individuals receive, becomes $(1-\tau) r_{t+1}$.

As it is possible for the individuals to evade the tax by declaring a false amount of savings, *i.e.*, concealing a part of their savings (called "undeclared savings"), x_{t+1}, the amount of capital income tax is calculated based on the amount of savings as declared by the individuals

[2] Subscripts generally indicate the period. However, in regards to the individuals' consumption, the former subscript represents the generation (1 and 2 correspond to the young and the old generations, respectively) while the latter subscript represents the period.

[3] Kim (2000) treats the case where the distribution of the tax base for income and that of the tax rate are stochastically dependent, under the progressive tax rate with the tax exemption.

(called "declared savings"), $s_t - x_{t+1}$. This tax evasion behavior would be detected by a probability of $0 < p_{t+1} < 1$. Following Allingham and Sandmo (1972), when tax evasion is detected, a penalty of the amount, $\theta r_{t+1} x_{t+1}$, will be imposed, where $\theta(>\tau)$ is a fixed penalty rate. Therefore, the budget constraint in the old period becomes:

$$\begin{aligned} c_{2t+1} &= (1-p_{t+1})[(1+r_{t+1})s_t - r_{t+1}\tau(s_t - x_{t+1})] \\ &\quad + p_{t+1}[(1+r_{t+1})s_t - r_{t+1}\tau(s_t - x_{t+1}) - \theta r_{t+1} x_{t+1}]. \end{aligned} \tag{2}$$

In order for an incentive to evade tax to exist, the expected return when the tax can be evaded should be larger than that in the case where the tax cannot be evaded, that is, $\tau > p_{t+1}\theta$.[4] In addition, when there exists an interior solution,

$$s_t > x_{t+1} \tag{3}$$

must hold in every period. We assume that the detection probability of tax evasion is increasing in the amount of concealed capital income and is specified as:

$$p_{t+1} = \delta(r_{t+1} x_{t+1})^{\alpha}, \tag{4}$$

where $\delta > 0$ is a fixed detection probability parameter and $\alpha > 1$.[5] This relation is assumed to be known to all individuals.

Individuals' utility is assumed to be a time-separable, log-linear function consisting of the amount of consumption in the young period and the expected amount of consumption in the old period, $u(c_{1t}, c_{2t+1}) = \log c_{1t} + \log c_{2t+1}$.[6] The individual's problem is to maximize this utility

[4] If $\tau < p_{t+1}\theta$, then undeclared savings take the corner solution: $x_{t+1} = 0$. This case corresponds to Appendix A.

[5] Although the early theoretical studies of tax evasion assume that the detection probability is given, such recent studies as Yitzhaki (1987) and Slemrod (1985) assume that the detection probability depends on the level of undeclared income. In addition, experimental studies like Klepper and Nagin (1989) show that detection probability is endogenous, positively correlated to tax compliance: the detection probability depends on the undeclared income. The assumption regarding the detection probability in this paper is, qualitatively, the same as the one in Yitzhaki (1987), which is $p'(Z) > 0$ and $p''(Z) \geq 0$, where Z represents concealed income.

[6] It should be noted that this does not take the form of expected utility. Such a kind of setting is the same as the one in Chen (2003). In addition, the dis-

subject to (1), (2), and (4). Then, the amount of undeclared savings and savings are:

$$x_{t+1} = \frac{\phi}{r_{t+1}}, \tag{5}$$

$$s_t = \frac{1}{2}\left\{ w_t - \frac{(\tau - \delta\theta\phi^{\alpha})\phi}{1+(1-\tau)r_{t+1}} \right\}, \tag{6}$$

where $\phi \equiv \{\tau/[\delta\theta(1+\alpha)]\}^{1/\alpha} > 0$. From (6), it should be noted that the determination of s_t is affected by the tax evasion. As shown in Appendix A, the first term of (6) is equal to the amounts of savings in the absence of tax evasion ($s_t = w_t/2$), so that the second term implies the effect of tax evasion (in fact, $x_{t+1} = \phi/r_{t+1}$ is included in this second term). Therefore, savings in the presence of tax evasion is lower than that in the absence of it. Intuitively, because individuals can obtain the hidden income from undeclared savings in the old period, they can save less compared to the case in the absence of tax evasion so as to smooth the lifetime consumption levels.

The features of undeclared savings and savings can be stated as follows. First, undeclared savings are decreasing in the interest rate and neutral in regards to the wages. The reason for the former feature is that a rise in the interest rate increases the detection probability of tax evasion. The latter feature is attributed to the fact that the utility function is time-separable.

Second, the savings function is increasing in both the wages and the interest rate. On one hand, the former feature corresponds to the property of a two-period life-cycle model without tax evasion, in which the utility function is log-linear (concretely, the amount of savings is, as we noted, $s_t = w_t/2$).[7] On the other hand, to understand the latter feature, it is beneficial to note that in the absence of tax evasion, savings are independent of the interest rate. Therefore, the rise in the interest rate indirectly raises savings through the tax evasion (the undeclared savings). This process can be conceived as intertemporal consumption smoothing.

In summary, we have the following lemma.

count factor of this utility function is set to one (that is, no discounting), for the sake of simplicity. This specification does not affect the qualitative results.

[7] See Appendix A.

***Lemma* 1.** If individuals' utility function is log-linear and time separable, and the detection probability is increasing in the amount of concealed capital income:

(i) Undeclared savings are decreasing in the interest rate and independent of wages.

(ii) Savings are increasing both in the interest rate and wages.

The effects of the rise in the capital income tax rate on the amounts of savings and undeclared savings can be understood as follows. On one hand, the rise in the tax rate directly increases undeclared savings because the expected return from the tax evasion rises. On the other hand, it does not directly affect the amount of savings because the savings in the absence of tax evasion are independent of the tax rate, $s_t = w_t/2$. As a result, the rise in the tax rate indirectly decreases savings through the tax evasion behavior.

In contrast, as the rise in the penalty rate will directly increase the expected cost to evade tax, the undeclared savings decrease. Therefore, the amounts of savings should be indirectly increased through the tax evasion.

***Lemma* 2.** If individuals' utility function is log-linear and time separable, and the detection probability parameter is increasing in the amount of concealed capital income:

(i) A rise in the capital income tax rate increases undeclared savings and decreases savings.

(ii) A rise in the penalty rate decreases undeclared savings and increases savings.

As we will see in the following sections, these results may reverse in the general equilibrium framework where capital accumulation is considered. For instance, a rise in the capital income tax rate may decrease undeclared savings and a rise in the penalty rate may increase them.[8]

B. Maximization of Firms' Profit

Firms produce goods from capital and labor, using a Cobb-Douglas type, constant returns to scale production technology. Denoting the capital share and the technology parameter as $0<\gamma<1$ and $A>0$, pro-

[8] See Proposition 1 and 2.

duced goods per capita can be expressed as a function of capital stock per capita shown as $y_t = Ak_t^{\gamma}$. Assuming that the price of the goods is normalized to one, that there is no capital depreciation and that production is carried out under a perfect competitive market, the first order conditions for profit maximization can be written as:

$$r_t = A\gamma k_t^{\gamma-1}, \tag{7}$$

$$w_t = A(1-\gamma)k_t^{\gamma}. \tag{8}$$

No tax is imposed on firms: firms' tax evasion behavior is beyond the scope of this paper.

C. Equilibrium

In this subsection, we describe a capital market equilibrium condition which is dependent on k_t and k_{t+1}. We will also present the amount of undeclared savings and the government revenue, for the analyses in the following sections.

The capital market equilibrium in period t is attained when individuals' savings (supply of capital) is equal to firms' demand for capital, that is, $k_{t+1} = s_t$ holds.[9] Therefore, using (6), (7), and (8), the capital market equilibrium condition can be expressed as:

$$k_{t+1} = \frac{1}{2}\left\{ A(1-\gamma)k_t^{\gamma} - \frac{(\tau - \delta\theta\phi^{\alpha})\phi}{1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}} \right\}. \tag{9}$$

Then, substituting (7) into (5), the amount of undeclared savings becomes:

$$x_{t+1} = \frac{\phi}{A\gamma k_{t+1}^{\gamma-1}}. \tag{10}$$

The government imposes a capital income tax on individuals' interest

[9] It is possible for the government to know the amount of individuals' real savings by using the information of the firms' demand for capital in the market. Even if that is the case, the government cannot perfectly prove an accusation of tax evasion by individuals because of the limit of the feasibility for its detection. For this reason, this paper presumes that the detection probability is not equal to one.

income from savings at a constant rate and imposes a penalty on offenders if tax evasion is detected.[10] Therefore, the (expected) government revenue in period $t+1$ can be written as $\tau r_{t+1}(s_t - x_{t+1}) + p_{t+1}\theta r_{t+1} x_{t+1}$. Using (4), (7), and (10), and noting $k_{t+1} = s_t$, it can be rewritten as:

$$T_{t+1} = \tau A\gamma k_{t+1}^{\gamma-1}\psi_{t+1} + \delta\theta\phi^{1+\alpha}, \tag{11}$$

where $\psi_{t+1} \equiv k_{t+1} - (\phi / A\gamma k_{t+1}^{\gamma-1}) > 0$ represents declared savings. Similarly, in the steady state, the capital market equilibrium, the undeclared savings and the government revenue can be obtained by substituting $k_t = k_{t+1} = k$ into (9), (10), and (11) respectively. In the following sections, we omit the time subscripts to express the value of the variables in the steady state.

III. Effects of the Policy Parameters on Capital Stock and Undeclared Savings

In this section, first, we will examine the short- and the long-run effects on capital stock and undeclared savings when the government raises the capital income tax rate after the $t+1$-th period. Next, we will examine the effects of the penalty rate. In this analysis, we assume that once the policy parameters are changed, they will be kept constant.[11]

A. Effects of the Capital Income Tax Rate

When the government raises the capital income tax rate, the short- and the long-run effects on capital stock are as follows:

$$\frac{dk_i}{d\tau} = -\frac{\phi}{\Delta_i}\frac{1+\alpha+(1+\alpha-\tau)A\gamma k_i^{\gamma-1}}{2(1+\alpha)[1+(1-\tau)A\gamma k_i^{\gamma-1}]^2} < 0, \tag{12}$$

[10] As seen in the footnote 11 below, because the detection probability becomes constant, we can ignore the cost for the detection activities.

[11] It might be possible to think that the detection probability parameter δ can be chosen by the government. In our paper, however, we have not treated δ as a policy variable, because, by substituting (5) into (4), the detection probability becomes constant and independent of δ: $p_{t+1} = \tau / [\theta(1+\alpha)]$. Therefore, δ would not be suitable as a policy variable.

where $\Delta_i > 0$, and $i = t+1$ in the short-run; i is omitted in the long-run (the steady state).[12]

From (12), a rise in the tax rate decreases capital stock both in the short- and the long-run. A rise in the tax rate has two negative effects on savings incentive. The first one is a direct effect: a rise in the tax rate discourages the incentive to save, and decreases capital stock. In the short-run, this is the only effect present. The second one is an indirect effect: the above negative effect on capital stock decreases the level of production, which in turn lowers wages, and therefore savings (capital stock) in the long-run.

On the other hand, the effects on undeclared savings are as follows:

$$\frac{dx_i}{d\tau} = \frac{\phi}{\alpha\tau A\gamma k_i^{\gamma-1}}\left[1-\alpha(1-\gamma)\varepsilon_{\tau i}\right], \tag{13}$$

where $\varepsilon_{\tau i} \equiv -\tau dk_i/(k_i d\tau) > 0$, implying the elasticity of the tax rate on capital stock. The sign of (13) is determined by the volume of this elasticity; if $\varepsilon_{\tau i} > (<) 1/[\alpha(1-\gamma)]$, (13) becomes negative (positive). To interpret this further, it is beneficial to understand what each term in the square bracket of (13) implies. The former term represents a positive direct effect, as shown in Lemma 2: a rise in the tax rate stimulates the incentive to evade tax. The latter term exhibits a negative indirect effect: as we have seen in (12), a rise in the tax rate decreases capital stock, and therefore, raises the interest rate. As in Lemma 1, undeclared savings will decrease. These things make it clear that if the rise in the tax rate brings about the significant decrease in the interest rate (that is, if $\varepsilon_{\tau i}$ is sufficiently large), the first positive direct effect will be dominated by the second negative indirect effect. In this case, undeclared savings will decrease.

The results obtained here can be summarized in the following proposition.

***Proposition* 1.** *When the capital income tax rate rises, both in the short- and the long-run,*

(i) *Capital stock decreases.*

(ii) *Undeclared savings decrease (increase) if the elasticity of the tax rate on capital stock is larger (less) than* $1/[\alpha(1-\gamma)]$.

[12] For the detailed calculation, see Appendix B.

B. Effects of the Penalty Rate

The effects of rises in the penalty rate on capital stock are:

$$\frac{dk_i}{d\theta}=\frac{1}{\Delta_i}\frac{\tau\phi}{2(1+\alpha)\theta[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]}>0. \tag{14}$$

This can be decomposed into two positive effects. The first is a direct effect on savings (and capital stock), which is the only effect present in the short-run: a rise in the penalty rate raises the return from savings as shown in Lemma 2. The second is an indirect effect brought about by an increase in the wages. Therefore, capital stock necessarily increases both in the short- and the long-run.

Based on the above results, the effects of the rise in the penalty rate on undeclared savings can be written as follows:

$$\frac{dx_i}{d\theta}=-\frac{\phi}{\alpha\theta A\gamma k_i^{\gamma-1}}[1-\alpha(1-\gamma)\varepsilon_{\theta i}], \tag{15}$$

where $\varepsilon_{\theta i}\equiv\theta dk_i/(k_i d\theta)>0$, implying the elasticity of the penalty rate on capital stock. The sign of (15) is determined by the volume of this elasticity; if $\varepsilon_{\theta i}>(<)1/[\alpha(1-\gamma)]$, (15) becomes positive (negative). Contrary to the case of a rise in the tax rate, the first term in the square bracket of (15) represents the negative effect to lower the incentive for tax evasion directly; the second term does the positive effect to stimulate the incentive, because of a decrease in the interest rate caused by an increase in capital stock.

These results can be summarized as follows.

***Proposition* 2.** *When the penalty rate rises, both in the short- and the long-run,*

(i) *Capital stock increases.*
(ii) *Undeclared savings increase (decrease) if the elasticity of the penalty rate on capital stock is larger (less) than* $1/[\alpha(1-\gamma)]$.

IV. Effects of the Policy Parameters on Utility Levels

To see the effects of the rises in the capital income tax rate and the penalty rate on the utility levels of individuals, we firstly describe the utility as functions of capital stock in the short- and the long-run as:

$$v(k_{i-1}, k_i; \tau, \theta) = \log[A(1-\gamma)k_{i-1}^{\gamma} - k_i] + \log\left[k_i + (1-\tau)A\gamma k_i^{\gamma} + \frac{\alpha\tau\phi}{1+\alpha}\right], \quad (16)$$

where $i=t+1$ in the short-run; i is omitted in the long-run. It should be noted that the term $\alpha\tau\phi/(1+\alpha)$ emerges because of the introduction of the tax evasion behavior.[13]

A. Effects of the Capital Income Tax Rate on Utility Levels

First, differentiating (16) where $i=t+1$ with respect to τ and using (10), we can obtain the effect of a rise in the tax rate on the utility level in the short-run,

$$\frac{dv(k_t, k_{t+1}; \tau, \theta)}{d\tau} = \frac{A\gamma k_{t+1}^{\gamma-1}}{\tilde{c}_{2t+1}}\left\{-\psi_{t+1} + (1-\tau)(1-\gamma)\frac{k_{t+1}}{\tau}\varepsilon_{\tau t+1}\right\}, \quad (17)$$

where $\tilde{c}_{2t+1} \equiv \{(1+\alpha)[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]A(1-\gamma)k_t^{\gamma} + \alpha\tau\phi\}/[2(1+\alpha)]$ and k_{t+1} is the solution of (9) (note that k_t is given).

On one hand, the sign of the first term in the curly bracket of (17) is negative, because this term is equivalent to declared savings, $(s_t - x_{t+1})$, and, from (3), $s_t > x_{t+1}$. This negative effect indicates that the rise in the tax rate directly decreases the utility level, because it reduces the return from declared savings. On the other hand, from (12), the second term represents the positive indirect effect. That is, as the rise in the tax rate decreases capital stock, and in turn, raises the interest rate, the return from savings and therefore the utility level will rise. In conclusion, the effect of the tax rate on the utility level in the short-run depends on the configuration of these effects, or concretely, on the volume of $\varepsilon_{\tau t+1}$.

Next, differentiating (16) where i is omitted with respect to τ, the effect of the tax rate on the utility level in the long-run can be obtained by:

$$\frac{dv(k, k; \tau, \theta)}{d\tau} = \frac{A\gamma k^{\gamma-1}}{\tilde{c}_2}\left\{-\psi + (1-\tau)(1-\gamma)\frac{k}{\tau}\varepsilon_{\tau} - (1-\gamma)[1+(1-\tau)A\gamma k^{\gamma-1}]\frac{k}{\tau}\varepsilon_{\tau}\right\} < 0, \quad (18)$$

where $\tilde{c}_2 \equiv \{(1+\alpha)[1+(1-\tau)A\gamma k^{\gamma-1}]A(1-\gamma)k^{\gamma} + \alpha\tau\phi\}/[2(1+\alpha)]$, and k is the

[13] See (A2) in Appendix A.

solution of (9), where $k_t = k_{t+1} = k$.

We can see that the rise in the tax rate unambiguously decreases the utility level in the long-run. As we can see the terms in the curly bracket of (18), the effect consists of three components: the negative direct effect brought by the decrease in the return from declared savings, the positive indirect effect caused by the increase in the return from savings and negative indirect effect caused by the decrease in the wage. The two former effects (corresponding to the first and the second terms in the curly bracket of (18), respectively) are the same ones in the short-run's case, as seen in (17). In addition, in the long-run, the third negative effect (the third term in the curly bracket of (18)) emerges. It should be noted, however, that the third negative effect necessarily dominates the second positive effect, because the economy is dynamically efficient; the utility level necessarily increases as capital stock increases.[14]

The results obtained here can be summarized in the following proposition.

***Proposition* 3.** *When the capital income tax rate rises,*

(i) *The utility level necessarily decreases in the long-run.*

(ii) *The utility level increases in the short-run if $\varepsilon_{\tau t+1}$ is so large that the positive indirect effect through the rise in the return from savings dominates the negative direct effect.*

In the absence of tax evasion, only the direct effect emerges both in the short- and the long-run, when the tax rate rises. For detailed calculation, see (A3) in Appendix A.

B. Effects of the Penalty Rate on Utility Levels

First, differentiating (16) where $i = t+1$ with respect to θ, we can obtain the effect of the penalty rate on the utility level in the short- run:

$$\frac{dv(k_t, k_{t+1}; \tau, \theta)}{d\theta} = \frac{A\gamma k_{t+1}^{\gamma-1}}{\tilde{c}_{2t+1}} \left\{ -\frac{\phi}{\theta(1+\alpha)A\gamma k_{t+1}^{\gamma-1}} - (1-\tau)(1-\gamma)\frac{k_{t+1}}{\theta}\varepsilon_{\theta t+1} \right\} < 0. \quad (19)$$

Like (17), the interpretation of (19) can be given as follows. The first

[14] The dynamic efficiency in this model is guaranteed by the assumptions that both the population growth rate and the capital depreciation rate are set to zero.

term in the curly bracket of (19) means the direct effect of the penalty rate on the return from undeclared savings. Then, the second term is the indirect effect on the return from savings through the fall in the interest rate (from (14)), with an opposite direction of the indirect effect seen in (17). As both effects are negative, the rise in the penalty rate necessarily decreases the utility level.

Next, the long-run effect is calculated as:

$$\frac{dv(k,k;\tau,\theta)}{d\theta}=\frac{A\gamma k^{\gamma-1}}{\tilde{c}_2}\left\{-\frac{\phi}{\theta(1+\alpha)A\gamma k^{\gamma-1}}-(1-\tau)(1-\gamma)\frac{k}{\theta}\varepsilon_\theta\right.$$
$$\left.+(1-\gamma)[1+(1-\tau)A\gamma k^{\gamma-1}]\frac{k}{\theta}\varepsilon_\theta\right\}. \tag{20}$$

The effect depends on the following three effects: the negative direct effect caused by the decrease in the return from undeclared savings; the negative indirect effect caused by the decrease in the return from savings (from (14)); the positive indirect effect caused by the increase in the wage. The former two negative effects correspond to the ones seen in the short-run's case. It should be noted that, because of the dynamic efficiency, the third positive indirect effect necessarily dominate the second negative indirect effect. Therefore, the total indirect effect becomes positive. Additionally, if ε_θ is so large that this total negative indirect effect dominates the first direct effect, then the sign of (20) becomes positive.

The results obtained here can be stated in the following proposition.

***Proposition* 4**. *When the penalty rate rises,*

(i) *The utility level necessarily decreases in the short-run.*

(ii) *The utility level increases in the long-run if ε_θ is so large that the total positive indirect effect dominates the negative direct effect.*

We shall note, from Proposition 3 and 4, that two kinds of trade-offs for the effects on the utility levels can be acknowledged. One is the trade-off within each policy. When the capital income tax rate rises, the utility level possibly increases in the short-run; however, it necessarily decreases in the long-run. Similarly, the utility level necessarily decreases in the short-run and could increase in the long-run, when the penalty rate rises. The other one is the trade-off across policies. If the government raises the tax rate, the present generation could become better-off and the future generation necessarily becomes worse-off. To the contrary, if it raises the penalty rate, the present generation necessarily becomes worse-off and the future generation could become better-off.

V. Policy Implications

A. Effects on Government Revenue

We will investigate the effects of the rises in the capital income tax rate and the penalty rate on the government revenue to check whether the government has an incentive to raise them. In general, in the absence of tax evasion, the government revenue necessarily increases when the tax rate rises.[15] However, in the presence of tax evasion, it is not clear that the rise in the tax rate or the penalty rate will increase the government revenue.

Thus, differentiating (11) with respect to τ, and noting (12), we can obtain the effects of the rise in the tax rate on the government revenue in the short- and the long-run as follows:

$$\frac{dT_i}{d\tau} = A\gamma k_i^{\gamma-1}\psi_i - A\gamma^2 k_i^{\gamma}\varepsilon_{\tau i}, \tag{21}$$

where $i=t+1$ in the short-run; i is omitted in the long-run. The first term of (21) indicates the direct effects: the rise in the tax rate directly increases the government revenue, other things being equal. The second term is the indirect effect: when the tax rate rises, capital stock decreases, and therefore, decreases the government revenue. As a result, the sign of (21) is determined by configuration of two effects, or more concretely, by the volume of the elasticity of the tax rate on capital stock: if $\varepsilon_{\tau i}$ is sufficiently large (small), the government revenue will decrease (increase) both in the short- and the long-run.

Next, noting (14), the effects of the rise in the penalty rate on the government revenue can be calculated as:

$$\frac{dT_i}{d\theta} = \frac{\tau\phi}{\theta(1+\alpha)} + \frac{\tau A\gamma^2 k_i^{\gamma}}{\theta}\varepsilon_{\theta i} > 0. \tag{22}$$

[15] See (A5) in Appendix A.

The effects seen in (22) can be split into two positive ones: the direct effect by the rise in the penalty rate, other things being equal, and the indirect effect through the increase in capital stock. Unlike the case of the rise in the tax rate, the latter one is positive, and therefore, the rise in the penalty rate necessarily increases the government revenue.

From the above discussion, as long as the government is only concerned with the revenue, it always has an incentive to raise the penalty rate. However, whether it has an incentive to raise the tax rate depends on the volume of the elasticity of the tax rate on capital stock. Therefore, contrary to the case without tax evasion, the revenue might be paradoxically increased by the decrease in the tax rate.

The above findings lead to the conditions that the rise, or the decrease, in the tax rate will simultaneously increase both the utility level and the government revenue. In general, the rise in the tax rate decreases the utility level and increases the government revenue both in the short- and the long-run, where tax evasion does not prevail. However, if we take the tax evasion behavior into account, it might not. We also show the condition that both the utility level and the government revenue increase when the penalty rate rises.

From (17) and (21), the conditions that the rise in the tax rate increases them in the short-run are:

$$\tau+\gamma<1$$

and

$$\frac{\tau}{(1-\tau)(1-\gamma)k_{t+1}}\psi_{t+1}<\varepsilon_{\tau t+1}<\frac{1}{\gamma k_{t+1}}\psi_{t+1}.$$

Similarly, the ones that the decrease in the tax rate increases them are:

$$\tau+\gamma>1$$

and

$$\frac{1}{\gamma k_{t+1}}\psi_{t+1}<\varepsilon_{\tau t+1}<\frac{\tau}{(1-\tau)(1-\gamma)k_{t+1}}\psi_{t+1}.$$

When the former conditions hold, the tax rate should be raised; when the latter ones hold, it should be decreased in the short-run.

In the long-run, as seen in (18), the rise in the tax rate necessarily decreases the utility level. Therefore, the decrease in the tax rate increases both the utility level and the government revenue in the long-run, if (21) is negative:

$$\varepsilon_\tau > \frac{1}{\gamma k}\psi.$$

That is, if the elasticity of the tax rate on capital stock is sufficiently large, the tax rate should be decreased.

Regarding the effects of the penalty rate, the rise in the penalty rate necessarily decreases the utility level in the short-run and increases the government revenue both in the short- and the long-run. Therefore, it is impossible to increase both of them simultaneously by a change in the penalty rate in the short-run. In contrast, in the long-run, if

$$\varepsilon_\theta > \frac{\phi}{(1+\alpha)(1-\gamma)A\gamma k^\gamma[\tau+(1-\tau)A\gamma k^{\gamma-1}]}$$

holds, that is, the elasticity of the penalty rate on capital stock should be sufficiently large, the rise in the penalty rate necessarily increases both of them.

As a result, we arrive at the following proposition.

***Proposition* 5.** *Both the utility levels and the government revenue simultaneously increase:*

(i) *(a) When the capital income tax rate rises (decreases) in the short-run, if*

$$\tau+\gamma \underset{(>)}{<} 1$$

and

$$\frac{\tau}{(1-\tau)(1-\gamma)k_{t+1}}\psi_{t+1} \underset{(>)}{<} \varepsilon_{\tau t+1} \underset{(>)}{<} \frac{1}{\gamma k_{t+1}}\psi_{t+1}.$$

(b) When the capital income tax rate decreases in the long-run, if

$$\varepsilon_{\tau} > \frac{1}{\gamma k}\psi.$$

(ii) *When the penalty rate rises in the long-run, if*

$$\varepsilon_{\theta} > \frac{\phi}{(1+\alpha)(1-\gamma)A\gamma k^{\gamma}[\tau+(1-\tau)A\gamma k^{\gamma-1}]}.$$

B. Extension: Incorporating Public Goods

In the above discussions, the government revenue has not been utilized, because our interest in this paper has been on how the changes in policy parameters affect welfare levels and the government revenue in the short- and the long-run.[16] In this subsection, we consider the case where the supply of the public goods is financed by the government revenue. Here, the public goods are assumed to benefit the old individuals who burden the tax, based on the benefit principle. It is also assumed that the marginal rate of transformation between private and public goods is normalized to one. For simplicity, the discussion is only limited to the long-run analysis (in the steady state).

The individuals' utility function takes the additively-separable form as $u(c_1, c_2, T) = \log c_1 + \log c_2 + \log T$. Using (16), this utility function with public goods can be rewritten as:

$$v(k,T;\tau,\theta) = \log[A(1-\gamma)k^{\gamma}-k] + \log\left[k+(1-\tau)A\gamma k^{\gamma}+\frac{\alpha\tau\phi}{1+\alpha}\right] + \log T. \quad (23)$$

Therefore, the government maximizes (23) by choosing τ and T, subject to (11). The first-order conditions yield the following relation:

$$\frac{MU_T}{MU_{c_2}} = \frac{\tilde{c}_2}{T} = 1 + \frac{\tau k\varepsilon_{\tau}+(1-\tau)(1-\gamma)A\gamma k^{\gamma}\varepsilon_{\tau}}{\tau(\psi-\gamma k\varepsilon_{\tau})}. \quad (24)$$

The left hand side of (24) represents the marginal rate of substitution between consumption for old generation and the public goods (MRS); the right hand side does the marginal cost of public funds (MCPF). (24)

[16] From this point of view, the analyses before this subsection cannot be strictly referred as "a general equilibrium framework." To fill its gap, the provision of public goods is introduced in this subsection.

implies that MRS should be equal to MCPF, as is well known. This MCPF is less than (equal to, or more than) one if $\varepsilon_\tau > (=, \text{ or } <) \psi/\gamma k$.[17] That is, if the elasticity of the tax rate on capital stock is sufficiently large, MCPF in the presence of tax evasion is lower than those in the absence of tax evasion (which is equal to one).[18] This means that the optimal level of public goods (government expenditures) in the presence of tax evasion is larger than that in the absence of tax evasion.

VI. Conclusion

We have analyzed the capital income tax evasion behavior of individuals in order to illustrate both the short- and the long-run effects of changes in the policy parameters such as the capital income tax rate and the penalty rate on the individuals' utility, or the welfare level, from the standpoint of capital accumulation.

The main results obtained in this paper are as follows. First, in regards to the effects on undeclared savings, the effects on capital stock work as a determinant factor, in both cases where the capital income tax rate and the penalty rate rise. More specifically, if the elasticity of the tax rate (penalty rate) on capital stock is sufficiently large (small), undeclared savings decrease (increase) both in the short- and the long-run.

Second, as for the effects of changes in the policy parameters on the utility levels, a rise in the tax rate may have a positive effect in the short-run when the tax evasion behavior is taken into consideration. Similarly, a rise in the penalty rate may bring about a positive effect in the long-run.

Third, in this framework with the capital income tax evasion, both the utility levels and the government revenue increase in the long-run if the tax rate decreases or the penalty rate increases, as long as the elasticities of such parameters on capital stock are sufficiently large.

The previous studies of the tax evasion in a dynamic framework have incorporated various externalities due to the policies of the government. Though those studies have obtained meaningful results concerning policy implications, they have provided little insight into the pure effects of the tax evasion. Our findings shown above indicate that the tax evasion behavior will bring about distinctive effects, or in some case,

[17] It should be noted that this condition corresponds to Proposition 5.(i) (b).
[18] See (A7) in Appendix A.

paradoxical ones, which cannot be seen in the models without the tax evasion.

Although this paper has succeeded in obtaining some interesting policy implications, our simplification of the government behavior may have been excessive. Strategic settings, or game theoretical frameworks could be introduced into individual and government behavior.

(Received 27 June 2008; Revised 24 December 2009; Accepted 27 March 2010)

Appendix A

This Appendix A describes the economy where the capital tax evasion is absent and carries out the comparative statics analysis. Here, the budget constraint in the young period is the same as that in the presence of tax evasion, (1). To the contrary, the budget constraint in the old period becomes:

$$c_{2t+1} = (1 + r_{t+1}) s_t - \tau r_{t+1} s_t. \tag{A1}$$

Individuals maximize the log-linear utility subject to (1) and (A1). Then, the savings function can be easily obtained as $s_t = w_t/2$. In addition, using this savings function, (7) and (8), the capital market equilibrium condition can be expressed as $k_i = A(1-\gamma)k_{i-1}^{\gamma}/2$, where $i = t+1$ in the short-run; i is omitted in the long-run. It is clear that this equilibrium condition is independent of τ: $dk_i/d\tau = 0$ holds both in the short- and the long-run.

The utility level of individuals in the short- and the long-run can be expressed as a function of capital stock:

$$v(k_{i-1}, k_i; \tau) = \log[A(1-\gamma)k_{i-1}^{\gamma} - k_i] + \log[k_i + (1-\tau)A\gamma k_i^{\gamma}]. \tag{A2}$$

Differentiating (A2) with respect to τ and using $dk_i/d\tau = 0$, we can obtain the effects of the rise in the tax rate on the utility levels in the short- and the long-run,

$$\frac{dv(k_{i-1}, k_i; \tau)}{d\tau} = -\frac{A\gamma k_i^{\gamma}}{\tilde{c}_{2i}} < 0, \tag{A3}$$

where $\tilde{c}_{2i} \equiv k_i + (1-\tau)A\gamma k_i^{\gamma}$. Hence, we find that, in the absence of tax evasion, the rise in the tax rate necessarily lowers the utility levels both in the short- and the long-run.

From the savings function, the government revenue in the short- and the long-run can be expressed as:

$$T_i = \frac{\tau A^2 \gamma(1-\gamma) k_{i-1}^{\gamma} k_i^{\gamma-1}}{2}. \tag{A4}$$

Noting $dk_i/d\tau = 0$, the effects of the tax rate on the government revenue in the short- and the long-run become:

$$\frac{dT_i}{d\tau} = \frac{A^2 \gamma(1-\gamma) k_{i-1}^{\gamma} k_i^{\gamma-1}}{2} > 0. \tag{A5}$$

Therefore, the government revenues increase both in the short- and the long-run, when the tax rate rises. It implies that the government always has an incentive to raise the tax rate, as long as it is concerned with the government revenue.

Next, we consider the case where the supply of the public goods is financed by the government revenue. The individuals' utility function takes the additively-separable form as $u(c_1, c_2, T) = \log c_1 + \log c_2 + \log T$. Using (A2), the utility function with public goods can be rewritten as:

$$v(k, T; \tau, \theta) = \log[A(1-\gamma)k^{\gamma} - k] + \log[k + (1-\tau)A\gamma k^{\gamma}] + \log T. \tag{A6}$$

Therefore, the government chooses τ and T so as to maximize (A6) subject to (1) and (A4). The first-order conditions yield the following relation:

$$\frac{MU_T}{MU_{c_2}} = \frac{\tilde{c}_2}{T} = 1, \tag{A7}$$

where $\tilde{c}_2 = k + (1-\tau)A\gamma k^{\gamma}$. The condition corresponds to the Samuelson rule for the optimal expenditures on public goods.

Appendix B

This Appendix B demonstrates how the results of the comparative statics analysis in Section 3 are obtained. First, totally differentiating (9), we have:

$$\Delta_{t+1} dk_{t+1} = \Delta_t dk_t - \frac{1+\alpha+(1+\alpha-\tau)A\gamma k_{t+1}^{\gamma-1}}{2(1+\alpha)[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]^2}\phi d\tau + \frac{\tau\phi}{2(1+\alpha)\theta[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]}d\theta, \tag{A8}$$

where, $\Delta_t \equiv A\gamma(1-\gamma)k_t^{\gamma-1}/2 > 0$ and $\Delta_{t+1} \equiv 1 + \frac{\tau(1-\tau)\alpha A\gamma(1-\gamma)k_{t+1}^{\gamma-2}\phi}{2(1+\alpha)[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]^2} > 0.$ Then, we evaluate (A8) in the steady state:

$$\Delta dk = -\frac{1+\alpha+(1+\alpha-\tau)A\gamma k^{\gamma-1}}{2(1+\alpha)[1+(1-\tau)A\gamma k^{\gamma-1}]^2}\phi d\tau + \frac{\tau\phi}{2(1+\alpha)\theta[1+(1-\tau)A\gamma k^{\gamma-1}]}d\theta, \tag{A9}$$

where $\Delta \equiv \Delta_{t+1} - \Delta_t$ is evaluated in the steady state. Furthermore, the condition for monotonic convergence to the steady state is:

$$0 < \frac{dk_{t+1}}{dk_t} = \frac{\Delta_t}{\Delta_{t+1}} < 1 \Rightarrow \Delta > 0. \tag{A10}$$

Under this condition, (12) and (14) can be obtained from (A8) and (A9). In the same way, (13) and (15) can be also calculated.

References

Allingham, Micheal G., and Sandmo, Agnar. "Income Tax Evasion: A Theoretical Analysis." *Journal of Public Economics* 1 (Nos. 3-4 1972): 323-38.

Andreoni, James, Erard, Brian, and Feinstein, Jonathan. "Tax Compliance." *Journal of Economic Literature* 36 (No. 2 1998): 818-60.

Barro, Robert J. "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* 98 (No. 5 1990): S103-25.

Caballe, Jordi, and Panades, Judith. "Tax Evasion and Economic Growth." *Public Finance* 52 (Nos. 3-4 1997): 318-40.

Chen, Been-Lon. "Tax Evasion in a Model of Endogenous Growth." *Review of Economic Dynamics* 6 (No. 2 2003): 381-403.

Cowell, Frank A. *Cheating the Government: The Economics of Tax Evasion.* Cambridge, Massachusetts: MIT Press, 1990.

Diamond, Peter A. "National Debt in a Neoclassical Growth Model." *American Economic Review* 55 (No. 5 1965): 1126-50.

Kim, Iltae. "Tax-rate Uncertainty, Tax Exemption, and Tax Evasion." *Seoul Journal of Economics* 13 (No. 1 2000): 37-47.

Klepper, Steven, and Nagin, Daniel. "The Anatomy of Tax Evasion." *Journal of Law, Economics, and Organization* 5 (No. 1 1989): 1-24.

Lin, Wen-Zhung, and Yang, C. C. "A Dynamic Portfolio Choice Model of Tax Evasion: Comparative Statics of Tax Rates and Its Implication for Economic Growth." *Journal of Economic Dynamics and Control* 25 (No. 11 2001): 1827-40.

Slemrod, Joel. "An Empirical Test for Tax Evasion." *Review of Economics and Statistics* 67 (No. 2 1985): 232-8.

Yitzhaki, Shlomo. "On the Excess Burden of Tax Evasion." *Public Finance Quarterly* 15 (No. 2 1987): 123-37.