Dual Time Series of Annual Earnings Based on the Direction of Sales Changes

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Abstract

This study characterizes annual earnings as a mixture of two randomwalk processes along two states of sales change, sales-increase and salesdecrease, thereby providing new insights into the earnings response coefficient (ERC). The dual earnings process is based on the premise that sales changes in the opposite direction convey different information about firms' future cash flows or earnings. Building on the extant ERC models, this study shows that the ERC is significantly larger in sales-increase periods than in sales-decrease periods and its magnitude increases as firms experience the increase of sales in multiple consecutive years.

Keywords: earnings response coefficient, sales changes, earnings growth, earnings persistency, sticky cost

INTRODUCTION

The earnings response coefficient (ERC hereafter) measures the degree that the capital market revises its earnings forecasts based on the firm's current abnormal earnings. In the revision of earnings forecasts, time-series properties of earnings play an important role. The ERC literature often assumes a random walk process of annual earnings. This study reevaluates the traditional assumptions on the time-series properties of earnings by considering the earnings' asymmetric behavior based on the direction of sales changes.

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INY HWANG

Asymmetric behavior of earnings has actively been documented in recent management accounting research. Anderson, Banker, and Janakiraman (2003) documents that selling, general, and administrative (SG&A) costs increase on average 0.55 percent per 1 percent increase in sales but decrease only 0.35 percent per 1 percent decrease in sales. Balakrishnan and Gruca (2008), using data from Ontario hospitals, find that operating costs for the hospital as a whole are sticky and that the stickiness of costs is greater for functions that relate to an organization's core competency — higher stickiness in costs pertaining to patient care relative to costs in other functions. Several other studies also find the asymmetric cost behavior — hence asymmetric earnings process — in various contexts such as magnitude of sales changes and industrial factors (Subramaniam and Weidenmier 2003), cross-country differences (Banker and Chen 2006a), and strength of corporate governance (Chen, Lu, and Sougiannis 2008).

The literature also suggests that the sticky cost behavior can be a crucial influence on earnings time-series properties. Anderson, Banker, Huang, and Janakiraman (2007) shows that sticky costs represent deliberate retention of SG&A resources based on managers' expectations that revenue will increase in the future. Thus, the asymmetric cost behavior conveys information about managers' expectation of future earnings and, as a result, the market is likely to adjust their earnings expectation disparately between sales-increase versus sales-decrease years due to the asymmetric nature of cost behavior. Moreover, Banker and Chen (2006b) find that earnings forecast errors incorporating cost stickiness have greater relative information content than forecast errors based on financial statement information in explaining abnormal stork returns. In sum, I believe that the direction of sales change is useful information in the market's equity valuation and that the ERC should be analyzed differently in sales-decrease years as oppose to sales-increase years.

This study characterizes annual earnings as a mixture of two random-walk processes along two states of sales change, salesincrease and sales-decrease, thereby providing new insights into cross-sectional and time-series variation in ERC. The dual random walk assumption is based on the premise that sales increases and sales decreases convey different insights about firms' current financial performance as well as disparate implications about firms' future cash flows or earnings. Anderson et al. (2003) shows that cost stickiness is not a random shock but the asymmetric adjustment of resource results from the manager's optimal behavioral response to changes in volume. That is, the cost stickiness occurs if managers deliberately maintain unutilized resources because of downward adjustment costs or their selfinterested behaviors when sales declines. Thus, there is a systematic discontinuation in annual earnings process depending on the sales change directions, indicating earnings cannot be a single random walk process.

Building a valuation model based on the dual random walk assumption, this study demonstrates that ERCs are larger in salesincrease years than in sales-decrease years. The empirical results show that the ERCs are significantly larger in sales-increase periods than in sales-decrease periods and the magnitude of ERCs in sales-increase years noticeably jumps up as the number of consecutive sales-increase years in the past increases. In addition, I find that for the earnings of a sales-down year, the earnings of the most recent sales-decrease year is a better predictor than the earnings of the preceding year which is a sales-up year.

Section 2 describes the stochastic process of earnings and develops hypotheses. Section 3 presents the empirical results. Section 4 discusses the alternative explanations about the relation between sales status and information content of earnings. Finally, Section 5 summarizes and concludes the paper.

DUAL EARNINGS PROCESS BASED ON SALES CHANGES AND ERCS

The theoretical ERC is the price change induced by a one-dollar shock to current earnings and is equal to one plus the present value of the revisions in expected future earnings caused by this shock. To investigate whether earnings changes have different implications in sales-increase years versus sales-decrease years, the earnings process is decomposed into two random walk processes, one for sales-increase years and the other for salesdecrease years:

 $Earnings_t = Earnings_{t-1*} + \varepsilon_t \tag{1}$

INY HWANG

$$Earnings_s = Earnings_{s-1^*} + \varepsilon_s \tag{1}$$

where $t-1^*$ is the most recent year in which sales increased and $s-1^*$ is the most recent year in which sales decreased.

$$a_t = X_t - E_{t-1}(X_t) = X_t - X_{t-1^*}$$
(2)

$$b_{s} = Y_{s} - E_{s-1}(Y_{s}) = Y_{s} - Y_{s-1^{*}}$$
⁽²⁾

Since earnings follow distinct random walks for sales-increase years and sales-decrease years, a_t revises the earnings expectations for sales-increase years, and b_s does for sales-decrease years.

$$\Delta E_t(X_{t+k}) = \xi_k a_t = a_t \times prob(\xi_k = 1) \text{ for } k = 1, 2, 3, \dots$$
(3)

$$\Delta E_{s}(Y_{s+l}) = \psi_{l}b_{s} = b_{s} \times prob(\psi_{l}=1) \text{ for } l = 1, 2, 3, \dots$$
(3)

where $\xi_k=1$ if year k is a sales-increase year and $\psi_l=1$ if year l is a sales-decrease year and also $\xi_k+\psi_k=\xi_l+\psi_l=1$ for all integer k's and l's. Then, the present value of the revisions in earnings expectations over an infinite horizon is as follows:

$$\sum_{\substack{\text{sales-}\\\text{increase}\\\text{gears}}} \left(\frac{1}{1+r}\right)^k E_t(X_{t+k}) = \sum_{\substack{\text{sales-}\\\text{increase}\\\text{gears}}} \left(\frac{1}{1+r}\right)^k \zeta_k a_t = \sum_{k=1}^{\infty} \left[\left(\frac{1}{1+r}\right)^k a_t \times prob(\zeta_k=1) \right]$$
(4)

$$\sum_{\substack{\text{sales}-\\\text{decrease}\\\text{gears}}} \left(\frac{1}{1+r}\right)^l E_t(\mathbf{Y}_{s+l}) = \sum_{\substack{\text{sales}-\\\text{decrease}\\\text{gears}}} \left(\frac{1}{1+r}\right)^l \psi_l b_s = \sum_{l=1}^{\infty} \left[\left(\frac{1}{1+r}\right)^l b_s \times prob(\psi_l=1) \right]$$
(4)'

Next, it is assumed that sales oscillate between two states: sales increase state and sales decrease state with some fixed probabilities.¹ Also, sales generating process is assumed to follow a two-state Markov chain with transition probabilities below:

$$M = \begin{bmatrix} m & 1 - m \\ 1 - n & n \end{bmatrix}$$

where m(n) is the probability that sales will increase (decrease) in

An examination of the sales-generating process in the return-earnings literature appears in Dechow, Kothari, and Watts (1998) who derive earnings by assuming a random walk process of sales. In contrast, we turn to the process of sales change to extract economic implications of sales by relating the sales change direction to earnings time-series properties.

period t+1 given a sales-increase (sales-decrease) in period t. Then, the theoretical ERCs for sales-increase and sales-decrease years are

$$\frac{(1+r)\left(\frac{1-n+r}{r}\right)}{(1-m)+(1-n)+r} \text{ and } \frac{(1+r)\left(\frac{1-m+r}{r}\right)}{(1-m)+(1-n)+r},$$

respectively. (The proof is shown in the appendix.)

It is clear that the ERC has larger values in sales-increase periods than in sales-decrease periods if m > n (the probability of two consecutive periods in the sales-increase state is greater than the probability of two consecutive periods in the sales-decrease state). It is intuitive to anticipate higher ERCs in sales-increase periods because the market for a stock is sustained only if market participants believe that the value of a firm's parameter m is much larger than that of n. That is, if a firm fails to convince investors that sales will rise more often than they fall (large m and small n), the market for the firm's share will dwindle and it will be delisted from the exchange.

Another reasons to have higher ERCs in sales increase periods pertains to firms' growth perspectives. Sales increases occur more often for growing firms and firms whose product lines are largely comprised of products in early stage of the life cycle. Next, as Collins and Kothari (1989) argue, the price reaction for these firms would be greater than that implied by the time-series persistence of earnings because persistence estimates are likely to be deficient in accurately reflecting current growth opportunities. If an upward change in sales is correlated with growth opportunities, ERCs will be greater in sales-increase years. Finally, cost stickiness may weaken the return-earnings relation in sales-decrease periods. Anderson et al. (2007) argue that in periods when sales decline, an increase in the SG&A cost signal (the ratio of SG&A costs to sales) may reflect either managers' deliberate decisions to maintain slack resources because they expect that demand would be restored in the near future or simply managers' inefficiency to control costs. As a result, cost stickiness intervenes in the return-earnings association.

Thus, investors are expected to consider sales-increase a dominant state for normal firms and hence the market will revise earnings expectation more completely based on current earnings innovation in sales-increase years.

H1: ERC estimates are greater in sales-increase years than in sales-decrease years.

Hypothesis 1 is tested by the following model:

A question that follows naturally is how earnings multiples will turn out if the analysis is extended from one period change (ΔS_t) to multiple consecutive period changes ($\Delta S_t, \Delta S_{t-1}, \Delta S_{t-2}, ...$). The first derivation indicates that the higher probability of sales-increase the investors place, the larger the difference between the two ERCs:

$$\frac{\partial}{\partial m} \left[\frac{\frac{1+r}{r}(1+r-n)}{(1-m)+(1-n)+r} - \frac{\frac{1+r}{r}(1+r-m)}{(1-m)+(1-n)+r} \right] = \frac{\frac{1+r}{r}(2+r-2n)}{\left[(1-m)+(1-n)+r \right]^2} > 0 \quad (7)$$

Investors are anticipated to revise their belief on m in the positive direction if a firm has been reporting sales-increases for more consecutive periods, and therefore hypothesize as follows:

H2: The difference in magnitude between ERC estimates in the sales-increase state and the sales-decrease state is larger if firms have stayed in the sales-increase state for more consecutive periods.

EMPIRICAL RESULTS – COEFFICIENT COMPARISONS

The sample data are collected between 1983 and 2007 based on the following criteria: (i) all of the necessary financial data are available from the COMPUSTAT database and (ii) the annual stock return data are available from the CRSP database. Table 1 presents the frequency of sales increases and sales decreases. Overall the sales-increase state is relatively common, appearing in about 70% of all firm years.

	Number	Number		Number	Number
Fiscal	of Firms	of Firms	Fiscal	of Firms	of Firms
Year	with Sales	with Sales	Year	with Sales	with Sales
	Increase	Decrease		Increase	Decrease
All years	33,948	13,797	1995	1,612	482
1983	1,605	436	1996	1,573	528
1984	1,359	294	1997	1,526	543
1985	997	602	1998	1,428	671
1986	947	595	1999	1,471	620
1987	1,123	420	2000	1,623	540
1988	1,240	350	2001	1,159	1,019
1989	1,192	420	2002	1,195	1,061
1990	1,177	558	2003	1,743	657
1991	1,025	738	2004	2,041	432
1992	1,253	572	2005	1,918	544
1993	1,408	597	2006	1,962	529
1994	1,611	488	2007	300	101

Table 1. Frequency of sales increases and sales decreases

Results on Hypothesis 1:

To test the first hypothesis, two models are used: a basic model and an inclusive model. The basic model considers only the key variables as follows:

$$\operatorname{RET}_{it} = \beta_{0U} \operatorname{SU}_{it} + \beta_{0D} \operatorname{SD}_{it} + \beta_{1U} \operatorname{AE}_{SU}_{it} + \beta_{1D} \operatorname{AE}_{SD}_{it} + \beta_{2} \operatorname{RET}_{it-1}$$
(5)

where the dependent variable, $\text{RET}_{i,t}$, is measured by cumulative monthly stock returns for 12-month window starting 4 months after the prior year's fiscal year end; $\text{SU}_{i,t}$ ($\text{SD}_{i,t}$) is a dummy variable that is 1 (0) if sales increased in the current period and 0 (1) if sales decreased in the current period; $\text{AE}_{\text{SU}_{i,t}}$ is abnormal earnings (measured by $\Delta EPS_t / price_{t-1}$) times a sales-increase dummy, $\text{SU}_{i,t}$; and $\text{AE}_{\text{SD}_{i,t}}$ is abnormal earnings times the salesdecrease dummy.² The leading period return, $\text{RET}_{i,t-1}$, is included to

²⁾ The theoretical framework in the hypothesis development assumes that the probability of sales increase is greater than that of sales decrease. This assumption may conflict with the random walk process of earnings. To deal with

. ERCs in Sales-Increase versus Sales-Decrease Years (Hypothesis 1)	$eta_{ ext{ou}}+eta_{ ext{oD}}+eta_{ ext{1U}} ext{AE}_{ ext{SU}_{ ext{it}}}+eta_{ ext{1D}} ext{AE}_{ ext{SD}_{ ext{it}}}+eta_{ ext{2}} ext{RET}_{ ext{it-1}}$
Table 2.	$RET_{i,t} = /$

$$\begin{aligned} \operatorname{RET}_{\mathrm{it}} &= \beta_{0\mathrm{U}} + \beta_{0\mathrm{D}} + \beta_{1\mathrm{U}} \operatorname{AE}_{-} \mathrm{SU}_{\mathrm{it}} + \beta_{1\mathrm{D}} \mathrm{AE}_{-} \mathrm{SD}_{\mathrm{it}} + \sum_{k=2}^{6} \beta_{k\mathrm{U}} \mathrm{AE}_{-} \mathrm{SU}_{\mathrm{it}} * \operatorname{Control} k_{\mathrm{it}} + \sum_{k=2}^{6} \beta_{k\mathrm{D}} \mathrm{AE}_{-} \mathrm{SD}_{\mathrm{it}} + \beta_{7} \operatorname{RET}_{\mathrm{it}^{-1}} \end{aligned}$$
(6)
$$\begin{aligned} \operatorname{RET}_{\mathrm{it}} &= \beta_{0} + \beta_{1} \mathrm{AE}_{\mathrm{it}} + \beta_{2} \operatorname{RET}_{\mathrm{it}^{-1}} \end{aligned}$$
(7)
$$\end{aligned} \end{aligned}$$
(7)
$$\end{aligned}$$

(-3.76) (-0.01)- 0.0110 (-0.54) (8.21)(5.90)(0.38)(-1.64)(1.52)(2') 43,636 obs. Coefficient (t-value) 13.13% 0.6402*** - 0.0350*** 0.9673*** - 0.0001 0.0146 0.0634 0.0077 (15.10)- 0.0143 (-0.69) (0.30)(1') 45,111 obs. Coefficient (t–value) 10.36% 0.7148 0.0047 Expect-ed Sign + + + ī ī ī I AE * Persistence AE * Leverage AE * Growth Adjusted R² Variables AE * Beta Return_1 AE * Size Intercept Panel B AE (-0.10)-2.32) (-0.40)(-2.97)(-3.15)(-1.44)(8.27)(1.59)-4.81) (7.69) (1.04)(0.82)(0.89)(7.47)(5.95)(2) 43,661 obs. Coefficient (t-value) 22.66%<.0001 0.9633*** 0.2152^{**} 0.0193 0.1068*** 0.3632^{**} 1.2367*** 0.3974*** 0.0184 0.0060 0.0423*** 0.0303 0.0072 0.0391 0.0208 - 0.0201 (13.03)(-1.01)(0.81)(-5.06) (14.29)(1) 45,155 obs. Coefficient (t-value) 19.69% <.0001 0.9597 0.1131^{***} 0.4251 - 0.0142 0.0175 Expect-ed Sign + + + + I I ī ī ī I I. HO: AE SU > AE SD AE_SU * Persistence AE_SD * Persistence AE_SU * Leverage AE_SD * Leverage AE_SU * Growth AE_SD * Growth AE_SU * Beta AE_SD * Beta AE_SU * Size AE_SD * Size Adjusted R² Variables Return_1 Panel A AE_SU AE_SD SD SU

*** significant at 1% level, **significant at 5% level, *significant at 10% level (All tests are two-sided.)

INY HWANG

3

help overcome the errors-in-variables problem in the context that prices lead earnings (Kothari and Sloan 1992).

To examine how the association between earnings and returns differs between sales-increase and decrease years, empirical tests examine the equal magnitude of the coefficient estimates of AE_SU_{i,i} (abnormal earnings in sales-increase years) and AE_SD_{i,i} (abnormal earnings in sales-decrease years). That is, hypothesis 1 relates to whether β_{1U} is greater than β_{1D} . Because of the panel data setting, the Fama-MacBeth method (Fame and MacBeth 1973) is used to control yearly effects. 45,155 observations are used in the regression analysis after eliminating outliers at the 5% level based on Cook's D and studentized residual. Results represented in the third column in Panel A of Table 2 show that the coefficient estimate of AE_SU is 0.9597 which is more than twice of the estimate of AE_SD, 0.4251. The null that the ERC is equal between sales-increase years and sales-decrease years is rejected at 1% level.

Prior studies have identified factors that influence variation in ERCs of different firms. They include persistence (Kormendi and Lipe 1987; Easton and Zmijewski 1989; Ou and Penman 1989; Ali and Zarowin 1992), growth opportunities (Penman 1996; Collins and Kothari 1989), risk (Billings 1999; Collins and Kothari 1989), leverage (Dhaliwal et al. 1991), and information environment (size – Bernard and Thomas 1990; number of analysts' forecasts – Teoh and Wong 1993). Therefore, the next model includes interaction terms based on previously identified ERC determinants:

$$RET_{i,t} = \beta_{0U} + \beta_{0D} + \beta_{1U}AE_SU_{i,t} + \beta_{1D}AE_SD_{i,t}$$

$$+ \beta_{2U}AE_SU_{i,t} * Persistence_{i,t} + \beta_{2D}AE_SD_{i,t} * Persistence_{i,t}$$

$$+ \beta_{3U}AE_SU_{i,t} * Growth_{i,t} + \beta_{3D}AE_SD_{i,t} * Growth_{i,t} \qquad (6)^{3}$$

$$+ \beta_{4U}AE_SU_{i,t} * Beta_{i,t} + \beta_{4D}AE_SD_{i,t} * Beta_{i,t}$$

$$+ \beta_{5U}AE_SU_{i,t} * Leverage_{i,t} + \beta_{5D}AE_SD_{i,t} * Leverage_{i,t}$$

$$+ \beta_{6U}AE_SU_{i,t} * Size_{i,t} + \beta_{6D}AE_SD_{i,t} * Size_{i,t} + \beta_{7}RET_{i,t-1}$$

this concern due to the correlation between sales and income, an alternative measure for abnormal earnings — change in return on sales (income before extraordinary items over sales) — is tested. The empirical results generally hold and do not change the insights.

³⁾ The two dummy variables SU and SD are used to make it straightforward to compare the effect of each control variable between sales-increase and salesdecrease periods.

INY HWANG

In the inclusive model, Persistence_{i,t} is a dummy variable that is set to be 1 if the ratio of earnings to price measured at the beginning of each period falls in the middle six groups of Compustat population's distribution of earnings to price, and 0 otherwise. Growth_{i,t} is the ratio of market value of common equity to book value of common equity (MB).⁴ As a risk proxy, Beta_{i,t} is obtained by regressing 60 prior monthly returns on the value weighted market returns. Leverage_{i,t} is long-term debt divided by the sum of long-term debt, preferred stock, and common equity. Finally, Size_{i,t} is the natural log of the market value of common equity.

Similarly to the basic model, the Fama-MacBeth method is used with 43,661 observations after eliminating outliers. The fourth column of Panel A of Table 2 shows that the ERC of the salesincrease state is greater than that of the sales-decrease state after including the five control variables. The coefficient estimates of AE_SU and AE_SD are 0.9633 and 0.3632, respectively. The test for the equality of AE_SU and AE_SD indicates that the ERC is significantly larger in sales-increase years than sales-decrease years. This finding indicates that the information gap between price and earnings becomes narrower in sales increase years. That is, information content of earnings is greater with respect to stock prices when sales increase. These results are consistent with the hypothesis that investors revise their future earnings expectations more sensitively with respect to current earnings changes in salesincrease years.

To further examine the information content of earnings in the two different states, this study compares adjusted R^2 from equations (5) and (6) with adjusted R^2 from the traditional ERC model that does not consider the direction of sales change. Panel B of Table 2 presents the results from the traditional approach. The coefficient estimate of each independent variable (for instance, 0.7148 for AE) lies between the estimates of the independent variable in relation to sales-increase and sales-decrease periods (0.9597 for AE_SU and 0.4251 for AE_SD). In addition, the adjusted R^2 's when the sales direction is considered (Panel A of Table 2) are almost twice greater than the adjusted R^2 's in the traditional models (Panel B of

⁴⁾ When book value of equity was negative, MB was set equal to zero. To avoid undue influence of very large values of MB on the regression estimates, MB > 5 values are set equal to 5.

Table 2) for both basic and inclusive models.⁵

Robustness Check for Analyses on Hypothesis 1:

As shown in the previous section, the return-earnings relation in sales-decrease years is not as strong as that in sales-increase years. As explained while developing hypothesis 1, this study propose that the low information content in sales-decrease periods stems from the earnings time-series property associated with the growth and sticky cost behavior aspects. In this section, explored is what may cause investors to react differently to earnings surprises in sales increase versus sales decrease years: positive versus negative earnings.

A well-documented issue with regard to the analysis of returnearnings relations is the influence of loss firms (Hayn 1995; Franzen 2002). Hayn (1995) hypothesizes that losses are less informative than profits about the firm's future prospects and shows that when only profitable firm years are considered, stock price movements are much more strongly linked to current earnings. To examine the effects of losses on the dual-series assumption, the estimation is conducted by excluding loss firm years.⁶

The third and fourth columns of Panel A of Table 3 show that the coefficient estimate of AE_SU is significantly larger than that of AE_SD for both basic and inclusive models. Adjusted R^2 continues to remain substantially higher for the models with sales-direction dummies than for the traditional models. This finding indicates that excluding loss firm years from the sample does not affect the rejection of the equal magnitude of ERCs between sales increase and decrease years, ensuring that the results are not driven by the loss firm story. Market investors seem to be aware of both the implication of losses as well as the association between economic conditions and the impact of earnings changes on future earnings.

⁵⁾ Easton, Harris, and Ohlson (1992) analyze the contemporaneous association between prices and earnings to mitigate the measurement error problem. Following the literature, the level model specification is adapted and the results are presented in Table 4. The same equations (1) and (2) are estimated and the results are consistent with the return models.

⁶⁾ In the sample about 24% of observations are loss-reporting firm years.

$RET_{i,t} = \beta_0 + \beta_1 AE_{i,t} + \beta_2 RET_{i,t-1}$	$_{\rm t} + \beta_2 {\rm RET}$, i,t-1								(5')	•
$\text{RET}_{i,t} = \beta_0 + \beta_1 \text{AE}_{i,t} + \sum_{k=2}^{6} \beta_k \text{AE}_{-} \text{SU}_{i,t} * \text{Control } k_{i,t} + \beta_7 \text{RET}_{i,t-1}$	$_{\mathrm{t}}+\sum_{k=2}^{6}eta_{k}\mathrm{A}.$	E_SU _{i,t} * Cc	introl $k_{\rm i,t}$	+ $\beta_7 \text{RET}_{i,t}$ -	1					(9)	
Panel A		(1) 34,810 obs.	0 obs.	(2) 33,627 obs.	7 obs.	Panel B		(1') 34,829 obs.) obs.	(2') 33,647 obs.	7 obs.
Variables	Expect-ed	1 Coefficient	ient	Coefficient	ient		Expect-ed	Coefficient	ent	Coefficient	ient
	Sign	(t-value)	(ən	(t-value)	(ən		Sign	(t-value)	le)	(t-value)	(ər
SU		0.0280	(1.24)	0.0266	(1.15)	Intercept		0.1440 (0.65)	(0.65)	0.0179	(0.77)
SD	I	- 0.0506**	(-2.21)	- 0.0364	(-1.54)						
AE_SU	+	1.3735***	(10.17)	1.0564^{***}	(6.08)	AE	+	1.0121^{***}	(9.35)	0.8116^{***}	(5.63)
AE_SD	+	0.5802	(6.94)	0.4315^{*}	(2.01)						
AE_SU * Persistence	+			2.0827^{***}	(13.32)	AE * Persistence	+			2.2129^{***}	(15.32)
AE_SD * Persistence	+			1.5679^{***}	(6.94)						
AE_SU * Growth	+			0.2713^{***}	(3.59)	AE * Growth	+			0.1314^{***}	(3.33)
AE_SD * Growth	+			0.0701	(1.45)						
AE_SU * Beta	I			- 0.1549	(-1.58)	AE * Beta	I			- 0.0648	(-0.86)
AE_SD * Beta	I			0.1109	(0.74)						
AE_SU * Leverage	I			- 0.6310***	(-3.22)	AE * Leverage	I			- 0.2671*	(-1.94)
AE_SD * Leverage	I			- 0.1400	(-0.28)						
AE_SU * Size	I			- 0.1089***	(-5.63)	AE * Size	I			- 0.0654*** (-4.60)	(-4.60)
AE_SD * Size	I			- 0.0092	(-0.22)						
Return_1	I	- 0.0394** (-3.01)	(-3.01)	- 0.0463***	(-3.31)	Return_1	I	- 0.0254 [*] (-1.89)	(-1.89)	- 0.0391** (-2.75)	(-2.75)
Adjusted R ²		18.35%	2%	24.92%	2%	Adjusted R ²		9.77%	0	16.73%	%
HO: AE SU > AE	E SD			<.0001	01						
	·		.						-		

*** significant at 1% level, **significant at 5% level, *significant at 10% level

INY HWANG

(c) (c)

 $\operatorname{RET}_{\operatorname{it}} = \beta_{\operatorname{0U}} + \beta_{\operatorname{0D}} + \beta_{\operatorname{1U}}\operatorname{AE}_{-}\operatorname{SU}_{\operatorname{it}} + \beta_{\operatorname{1D}}\operatorname{AE}_{-}\operatorname{SD}_{\operatorname{it}} + \sum_{k=2}^{6}\beta_{k}_{\operatorname{U}}\operatorname{AE}_{-}\operatorname{SU}_{\operatorname{it}} * \operatorname{Control} k_{\operatorname{it}} + \sum_{k=2}^{6}\beta_{k}_{\operatorname{D}}\operatorname{AE}_{-}\operatorname{SD}_{\operatorname{it}} * \operatorname{Control} k_{\operatorname{it}} + \beta_{7}\operatorname{RET}_{\operatorname{it-1}} + \beta_{7}\operatorname{RET}_{\operatorname{RET}} + \beta_{7}\operatorname{RET}_{\operatorname$

 $\text{RET}_{i,t} = \beta_{0\text{U}} + \beta_{0\text{D}} + \beta_{1\text{U}}\text{AE_SU}_{i,t} + \beta_{1\text{D}}\text{AE_SD}_{i,t} + \beta_2\text{RET}_{i,t-1}$

Table 3. Estimation Excluding Loss Firms

Price _{i,t} = β_0	$+\beta_1 EPS_{i}$	$_{t} + \beta_{2}$ Price	i,t–1				
Panel A		(8) 45,41	5 obs.	Panel B		(8') 45,44	6 obs.
Variables	Expect-ed	Coeffic	eient	Variables	Expect-ed	Coeffic	ient
	Sign	(t–val	ue)		Sign	(t–val	ue)
SU		1.9131***	(8.50)	Intercept		1.2541***	(5.66)
SD	-	1.6613***	(6.16)				
EPS_SU	+	2.3643***	(16.55)	EPS	+	1.3622***	(4.63)
EPS _SD	+	1.2211***	(15.80)				
Price_1	+	0.8234***	(38.85)	Price_1	+	0.9620***	(43.93)
Adjusted R ²		95.18	3%	Adjusted R ²		87.07	7%
HO: AE_SU	> AE_SD						

Table 4. Level Model

Price_{i,t} = $\beta_{0U} + \beta_{0D} + \beta_{1U}$ EPS_SU_{i,t} + β_{1D} EPS_SD_{i,t} + β_{2} Price_{i,t-1} Price_{i,t} = $\beta_{0} + \beta_{1}$ EPS_{i,t} + β_{0} Price_{i,t}

*** significant at 1% level, **significant at 5% level, *significant at 10% level

Results on Hypothesis 2:

The second hypothesis is that the difference in magnitude between ERC estimates in the sales-increase state and in salesdecrease state becomes larger as firms have stayed in the salesincrease state for more consecutive periods. That is, a longer salesincrease trend makes investors revise their assessment about m, the belief that the firm will stay in the sales-increase state, in the positive direction. Table 5 shows how ERCs change as the sales increase/decrease trend extends for more periods. Panel A presents the results from the sales increase trend — the earnings response coefficient (estimate for the AE variable) and the adjusted R^2 become larger as the number of consecutive sales increase years to the current year increases. In contrast, Panel B shows the opposite results for sales decrease observations. In short, firms with patterns of increasing sales have larger earnings multiples than other firms, while the gap between ERCs in sales-increase states and in sales-decrease states spreads when a firm has been reporting sales-increases for more periods.

I able o. Ifelia of		oaies changes anu familings-retum relations (nypounesis 2)	Veruiti Nelaujus	(Trypounesis 2)		
		Panel A			Panel B	
	Number of Con	Number of Consecutive Sales Increase Years by	crease Years by	Number of Cons	Number of Consecutive Sales Decrease Years by the	ase Years by the
Variables		the Current Year			Current Year	
	One Year	Two Years	Three Years	One Year	Two Years	Three Years
	(6,183 obs.)	(3,978 obs.)	(2,620 obs.)	(6,459 obs.)	(2,330 obs.)	(858 obs.)
T	0.0168^{***}	0.0215^{***}	-0.0075	-0.1048	-0.1048***	-0.0677***
intercept	(3.35)	(3.87)	(-1.21)	(-24.25)	(-13.42)	(-5.49)
Ľ V	0.5605^{***}	0.9241^{***}	1.3743^{***}	0.3466***	0.3891^{**}	0.2922^{***}
AE	(11.17)	(10.37)	(9.57)	(7.83)	(5.95)	(3.56)
∧ ⊡*Domoioton oo	0.8936^{**}	0.9194^{***}	0.8589^{***}	0.1671***	0.0286	0.3809^{**}
AL" FEISISIEILCE	(8.45)	(7.73)	(5.87)	(2.98)	(0.25)	(2.01)
ム Fr*C ******	-0.0004	0.0226^{***}	0.0017	-0.0021^{*}	-0.0085**	-0.0047
TH ALOWIT	(-0.24)	(3.67)	(0.30)	(-1.67)	(-2.35)	(-0.66)
A ⊡*⊓	0.0663*	0.026	-0.2780^{***}	0.0798***	-0.0387	-0.0340
AD DCIA	(1.86)	(0.05)	(-3.46)	(2.62)	(-0.82)	(-0.63)
▲ □*1 00 000	-0.0712	-0.4647***	-0.1939	0.0628^{*}	-0.0563	-0.0016
AE LEVELAGE	(-3.45)	(-4.93)	(96.0-)	(1.74)	(-1.42)	(-0.03)
く ビード マード マード マード マード マード マード マード マ	-0.0347***	-0.0086	-0.0836***	-0.0259***	-0.0165	0.0429^{**}
AE SIZE	(-3.39)	(-0.59)	(-4.18)	(-3.27)	(-1.40)	(2.51)
Dottine 1	-0.0351	-0.0364	-0.0174^{*}	-0.0221^{***}	-0.0340**	-0.014
TTITITITITI	(-4.54)	(-5.86)	(-1.66)	(-3.73)	(-2.40)	(-0.11)
Adjusted \mathbb{R}^2	9.11%	11.67%	13.53%	6.34%	3.93%	4.53%

Table 5. Trend of Sales Changes and Earnings-Return Relations (Hypothesis 2)

16

INY HWANG

SUMMARY AND CONCLUDING REMARKS

The research extends the ERC literature by relating timeseries properties of earnings to the direction of sales change. By considering economic conditions that have not been studied, it provides new insights on the relation between the earnings generating process and the market valuation process. This study assumes that earnings follow two separate random walks depending on sales changes and that the market reacts to earnings innovation in different ways when revising future expectations because investors understand that earnings in different economic states have different processes and hence different information about future performance. The empirical tests reject the null that the magnitude of ERC is equal regardless of sales movement, indicating that investors revise expectations differently for salesincrease and sales-decrease periods. The dual-series of earnings based on sales change direction is also supported by the forecast error comparison between the single-series model and the dualseries model.

The findings in this research are meaningful not only from a valuation perspective, but also from a managerial contracting perspective. Executive compensation contracts generally consist of market-based and accounting-based performance measures, and researchers have studied on the weights on market versus accounting measures in compensation contracts. Given that earnings in sales-increase and sales-decrease periods convey unequal amounts of information with respect to prices, it may be a fruitful area to conduct research on compensation design.

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APPENDIX

 X_{t+k} : Earnings process along sales-increase years (ξ_k =1) Y_{s+l} : Earnings process along sales-decrease years (ψ_l =1)

(i) ERC for a Sales-Increase Period

As expressed in Equation (4), the ERC for a sales-increase period is:

$$\sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k a_t \times prob(\xi_k=1)$$

I introduce a trick to make it easier to solve out the above expression. Let ξ^1 , ξ^2 , ξ^3 , ... represent respectively the years of the first sales-increase realization, the second sales-increase realization, the third sales-increase realization, and so on. For instance, among the infinitive number of possible combinations of sales increases and decreases, let's consider one series of such sales changes:

<i>t</i> + <i>k</i>	t – 2	 t	<i>t</i> + 1	<i>t</i> + 2	<i>t</i> + 3	<i>t</i> + 4	<i>t</i> + 5	
Sales change	Most Recent Increase	 Increase	1 st Increase	Decrease	2 nd Increase	Decrease	3 rd Increase	
Earnings	X_{t-1}	 X _t	X_{t+1}	Y_{s+l}	X_{t+2}	Y_{s+l}	X_{t+3}	
ξ_k			$\xi_{k=1} = 1$	$\xi_{k=2}=0$	$\xi_{k=3}=1$	$\xi_{k=4}=0$	$\xi_{k=5} = 1$	

Then, $\xi^1 = t+1$ ($\xi_{k=1} = 1$), $\xi^2 = t+3$ ($\xi_{k=3} = 1$), $\xi^3 = t+5$ ($\xi_{k=5} = 1$), ...

Now I express the above expression in another way using ξ^i instead of ξ_k .

$$\begin{split} &\sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^{k} a_{t} \times prob(\xi_{k}^{z}=1) \sum_{j=1}^{\infty} \sum_{i\geq j}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{j}=t+i) \\ &= \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{1}=t+i) + \sum_{i=2}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{2}=t+i) \\ &+ \sum_{i=3}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{3}=t+i) + \sum_{i=4}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{4}=t+i) + \dots \end{split}$$

The first term of the last expression, $\sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i a_i \times prob(\zeta^1 = t + i)$, can be rewritten as follows:

$$\left(\frac{1}{1+r}\right)a_{t} \times m + \left(\frac{1}{1+r}\right)^{2}a_{t} \times (1-m)(1-n) + \left(\frac{1}{1+r}\right)^{3}a_{t} \times (1-m)n(1-n) + \left(\frac{1}{1+r}\right)^{4}a_{t} \times (1-m)n^{2}(1-n) + \left(\frac{1}{1+r}\right)^{5}a_{t} \times (1-m)n^{3}(1-n) + \dots$$

Then, the expected present value of a_t in the first sales-increase year is:

$$\sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{1} = i) = \frac{a_{t}m}{1+r} + \frac{a_{t}(1-m)(1-n)}{(1+r)^{2}} \sum_{i=2}^{\infty} \left(\frac{n}{1+r}\right)^{i-2}$$
$$= \frac{m}{1+r} a_{t} + \frac{1}{1+r} \frac{(1-m)(1-n)}{1+r-n} a_{t}$$
$$= \left(\frac{1}{1+r}\right) \left(m + \frac{(1-m)(1-n)}{1+r-n}\right) a_{t}$$
$$= \left(\frac{1}{1+r}\right) \left(\frac{1-n+m}{1-n+r}\right) a_{t}$$

The second term, $\sum_{i=2}^{\infty} \left(\frac{1}{1+r}\right)^i a_i \times prob(\xi^2 = t + i)$, can be rewritten as follows:

$$\left(\frac{1}{1+r}\right)^{2} a_{t} \times m^{2} + \left(\frac{1}{1+r}\right)^{3} a_{t} \times m(1-m)(1-n) \times {}_{2}C_{1}$$

$$+ \left(\frac{1}{1+r}\right)^{4} a_{t} \times m(1-m)n(1-n) \times {}_{2}C_{1} + \left(\frac{1}{1+r}\right)^{4} a_{t} \times (1-m)^{2}(1-n)^{2} \times ({}_{3}C_{1}-{}_{2}C_{1})$$

$$+ \left(\frac{1}{1+r}\right)^{5} a_{t} \times m(1-m)n^{2}(1-n) \times {}_{2}C_{1} + \left(\frac{1}{1+r}\right)^{5} a_{t} \times (1-m)^{2}n(1-n)^{2} \times ({}_{4}C_{1}-{}_{2}C_{1})$$

$$+ \left(\frac{1}{1+r}\right)^{6} a_{t} \times m(1-m)n^{3}(1-n) \times {}_{2}C_{1} + \left(\frac{1}{1+r}\right)^{6} a_{t} \times (1-m)^{2}n^{2}(1-n)^{2} \times ({}_{5}C_{1}-{}_{2}C_{1})$$

$$+ \dots$$

Then, the expected present value of a_t in the second salesincrease year is:

$$\sum_{i=2}^{\infty} \left(\frac{1}{1+r}\right)^i a_t \times prob(\xi^2 = t+i) =$$

INY HWANG

$$\begin{split} &\frac{a_t m^2}{(1+r)^2} + \frac{a_t m (1-m)(1-n)}{(1+r)^3} \sum_{i=2}^{\infty} 2 \times \left(\frac{n}{1+r}\right)^{i-2} + \frac{a_t (1-m)^2 (1-n)^2}{(1+r)^4} \sum_{i=3}^{\infty} (i-2) \times \left(\frac{n}{1+r}\right)^{i-3} \\ &= \frac{m^2}{(1+r)^2} a_t + \frac{1}{(1+r)^2} \frac{2m (1-m)(1-n)}{1+r-n} a_t + \frac{1}{(1+r)^2} \frac{(1-m)^2 (1-n)^2}{(1+r-n)^2} a_t \\ &= \left(\frac{1}{1+r}\right)^2 \left(m + \frac{(1-m)(1-n)}{1+r-n}\right)^2 a_t = \left(\frac{1}{1+r}\right)^2 \left(\frac{1-n+m}{1-n+r}\right)^2 a_t \end{split}$$

The third term, $\sum_{i=3}^{\infty} \left(\frac{1}{1+r}\right)^i a_i \times prob(\zeta^{3} = t + i)$, can be rewritten as follows:

$$\begin{split} &\left(\frac{1}{1+r}\right)^{3}a_{t}\times m^{3} + \left(\frac{1}{1+r}\right)^{4}a_{t}\times m^{2}\left(1-m\right)\left(1-n\right)\times_{3}C_{2} \\ &+ \left(\frac{1}{1+r}\right)^{5}a_{t}\times m^{2}\left(1-m\right)n(1-n)\times_{3}C_{2} + \left(\frac{1}{1+r}\right)^{5}a_{t}\times m(1-m)^{2}\left(1-n\right)^{2}\times_{3}C_{2} \\ &+ \left(\frac{1}{1+r}\right)^{6}a_{t}\times m^{2}(1-m)n^{2}(1-n)\times_{3}C_{2} + \left(\frac{1}{1+r}\right)^{6}a_{t}\times m(1-m)^{2}n(1-n)^{2}\times(_{3}C_{2}\times 2) \\ &+ \left(\frac{1}{1+r}\right)^{6}a_{t}\times(1-m)^{3}(1-n)^{3}\times(_{5}C_{2}-_{3}C_{2}-_{3}C_{2}\times 2) \\ &+ \left(\frac{1}{1+r}\right)^{7}a_{t}\times m^{2}(1-m)n^{3}(1-n)\times_{3}C_{2} + \left(\frac{1}{1+r}\right)^{7}a_{t}\times m(1-m)^{2}n^{2}(1-n)^{2}\times(_{3}C_{2}\times 3) \\ &+ \left(\frac{1}{1+r}\right)^{7}a_{t}\times(1-m)^{3}n(1-n)^{3}\times(_{6}C_{2}-_{3}C_{2}-_{3}C_{2}\times 3) \end{split}$$

Then, the expected present value of a_t in the third sales-increase year is:

$$\begin{split} \sum_{i=3}^{\infty} & \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{3} = t+i) = \\ & \frac{a_{t}m^{3}}{(1+r)^{3}} + \frac{a_{t}m^{2}(1-m)(1-n)}{(1+r)^{4}} \sum_{i=2}^{\infty} 3 \times \left(\frac{n}{1+r}\right)^{i-2} \\ & + \frac{a_{t}m(1-m)^{2}(1-n)^{2}}{(1+r)^{5}} \sum_{i=3}^{\infty} 3 \times (i-2) \times \left(\frac{n}{1+r}\right)^{i-3} \\ & + \frac{a_{t}(1-m)^{3}(1-n)^{3}}{(1+r)^{6}} \sum_{i=4}^{\infty} \left(_{i+1}C_{2} - 3 \times (i-1)\right) \times \left(\frac{n}{1+r}\right)^{i-4} \\ & = \frac{m^{3}}{(1+r)^{3}} a_{t} + \frac{1}{(1+r)^{3}} \frac{3m^{2}(1-m)(1-n)}{1+r-n} a_{t} + \frac{1}{(1+r)^{3}} \frac{3m(1-m)^{2}(1-n)^{2}}{(1+r-n)^{2}} a_{t} \\ & = \frac{1}{(1+r)^{3}} \frac{(1-m)^{3}(1-n)^{3}}{(1+r-n)^{3}} a_{t} = \left(\frac{1}{1+r}\right)^{3} \left(m + \frac{(1-m)(1-n)}{1+r-n}\right)^{3} a_{t} \end{split}$$

$$= \left(\frac{1}{1+r}\right)^3 \left(\frac{1-n+m}{1-n+r}\right)^3 a_t$$

As a consequence,

$$\begin{split} \sum_{k=1}^{\infty} & \left(\frac{1}{1+r}\right)^{k} a_{t} \times prob(\xi_{k} = 1) = \sum_{j=1}^{\infty} \sum_{i\geq j}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{j} = t+i)(\xi^{j} = t+i) \\ & = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{i} = t+i) + \sum_{i=2}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{2} = t-i) \\ & + \sum_{i=3}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{3} = t+i) + \sum_{i=4}^{\infty} \left(\frac{1}{1+r}\right)^{i} a_{t} \times prob(\xi^{4} = t+i) + \dots \\ & = \left(\frac{1}{1+r}\right) \left(\frac{1-n+m}{1-n+r}\right) a_{t} + \left(\frac{1}{1+r}\right)^{2} \left(\frac{1-n+m}{1-n+r}\right)^{2} a_{t} + \left(\frac{1}{1+r}\right)^{3} \left(\frac{1-n+m}{1-n+r}\right)^{3} a_{t} \\ & + \left(\frac{1}{1+r}\right)^{4} \left(\frac{1-n+m}{1-n+r}\right)^{4} a_{t} + \dots \\ & = \sum_{j=1}^{\infty} \left(\frac{1}{1+r}\right)^{j} \left(\frac{1-n+m}{1-n+r}\right)^{j} a_{t} = \frac{m+\frac{(1-n)}{r}}{(1-m)+(1-n)+r} a_{t} \end{split}$$

Therefore, ERC (sales-increase year) = $\frac{(1+r)\left(\frac{1-n+r}{r}\right)}{(1-m)+(1-n)+r}.$

(ii) ERC for a Sales-Decrease Period

By following the same steps as in (i),

$$\begin{split} &\sum_{l=1}^{\infty} \left(\frac{1}{1+r}\right)^{l} b_{s} \times prob(\psi_{1}=1) = \sum_{j=1}^{\infty} \sum_{i \ge j}^{\infty} \left(\frac{1}{1+r}\right)^{i} b_{s} \times prob(\psi^{j}=s+i) \\ &= \sum_{j=1}^{\infty} \left(\frac{1}{1+r}\right)^{j} \left((1-n) + \frac{n(1-m)}{r+1-m}\right)^{j} b_{s} \\ &= \frac{(1-n) + \frac{(1-m)}{r}}{1+r-(m-n)} b_{s}, \text{ and ERC (sales-decrease year)} \\ &= \frac{(1+r)\left(\frac{1-m+r}{r}\right)}{(1-m)+(1-n)+r} \,. \end{split}$$