Fast Slot Synchronization for Intercell Asynchronous DS/CDMA Systems

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***Abstract*—Slot synchronization is a critical step for fast and reliable cell search in intercell asynchronous direct sequence-code division multiple access systems. To increase reliability, observations over a number of slots may be combined. In this paper, combining schemes of multiple observations are studied for slot synchronization. The optimal combining rule is determined based on detection theory. It is found that two known combining schemes correspond to special cases of the optimal combining. These schemes may not work well in typical environments, since the schemes are optimized for specific environments. To improve slot synchronization performance in typical environments, a new combining scheme is proposed in this paper. The performance of the proposed combining scheme as well as other combining schemes is analyzed for Rayleigh fading channels with frequency offset. Numerical analysis shows that the proposed combining scheme significantly outperforms other combining schemes in typical environments.**

***Index Terms*—Acquisition, cell search, differentially coherent combining, intercell asynchronous direct sequence-code division multiple access system, slot synchronization.**

I. INTRODUCTION

INTER-CELL asynchronous direct sequence-code division multiple access (DS/CDMA) has been chosen as a radio access technology for next generation mobile radio systems [1], [2]. This system assigns different scrambling codes to different cell sites and eliminates the need for external timing reference source. Consequently, this system is generally expected to require a longer cell search time than intercell synchronous systems [3]. Cell search refers to the process of a mobile station establishing synchronization to the scrambling code associated with the best cell site. A three-step cell search scheme has been proposed in [3] and refined in the standardization process. The three steps in this scheme are: 1) slot synchronization; 2) frame synchronization and code group identification; and 3) scrambling code identification. The first slot synchronization step is addressed in this paper.

In an intercell asynchronous system, a synchronization code is periodically transmitted at the beginning of every slot for one symbol duration, as shown in Fig. 1 [2]. The slot synchronization is for estimating the starting position of the synchronization code, which corresponds to the slot boundary. The uncertainty region for slot synchronization is confined to one slot duration. Thus, the starting position of the synchronization code may be determined from observations over one slot duration. However, decisions based on observations over a single slot may be unreliable, when the signal-to-noise-plus-interference ratio (SNIR) is low or if fading is severe. Reliable slot synchronization is required to minimize cell search time. In order to increase reliability, observations may be made over a number of slots and combined. This is possible, since observations associated with the relatively same positions in different slots contain the same synchronization information.

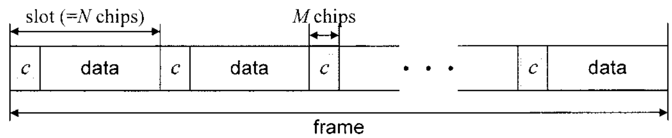

Fig. 1. Slot structure (c: synchronization code).

To minimize cell search time, multiple observations should be effectively combined. Two combining schemes have been proposed in [3] and [4]. The scheme in [3] combines observations after squaring. This scheme has been referred to as noncoherent combining in [5], and will be referred to as such in this paper. Although this noncoherent combining scheme reduces fading effects due to its inherent diversity property, it results in noncoherent combining loss [6]. The other scheme investigated in [4] combines observations without squaring or any preprocessing. In this paper, this scheme is referred to as no-preprocessing combining. It has been found that this scheme outperforms the noncoherent combining scheme in static and very slow fading environments. However, its performance degrades severely with Doppler spread and frequency offset [4]. For the second step of cell search, the use of the coherent combining scheme has been proposed in [2], and has been found to significantly reduce cell search time compared with the noncoherent combining scheme [5]. However, this coherent combining scheme cannot be employed in the first slot synchronization step, since the phase estimation required for coherent combining is not available during slot synchronization.

In this paper, the combining schemes of multiple observations are investigated for slot synchronization. The optimal combining rule is determined using the detection theory for multiple observations [7]. The noncoherent combining and no-preprocessing combining schemes are found to correspond

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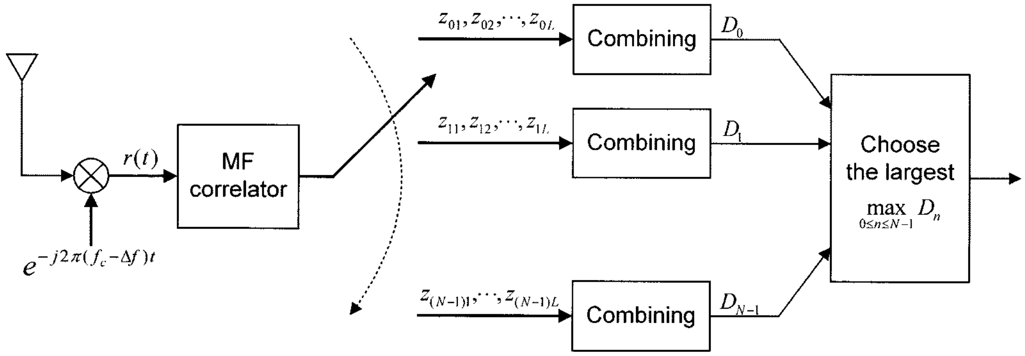

Fig. 2. Structure of a slot synchronization block based on multiple observations.

to special cases of the optimal combining. To improve slot synchronization performance in typical environments, a new combining scheme of multiple observations is proposed in this paper. The performance of the proposed combining scheme is analyzed and compared with that of other combining schemes for Rayleigh fading channels with frequency offset.

This paper is organized as follows. Section II introduces the system model. Combining schemes of multiple observations are studied in Section III, and a new combining scheme is proposed in Section IV. In Section V, the detection and mean acquisition time performance of the proposed combining, optimal combining, noncoherent combining, and no-preprocessing combining schemes is analyzed. In Section VI, numerical results are presented and the performance of the proposed combining scheme is compared with that of noncoherent combining and no-preprocessing combining schemes. Finally, conclusions are drawn in Section VII.

II. System Model

The slot structure for an intercell asynchronous system considered in this paper is depicted in Fig. 1. One slot is N chips long, and c is the synchronization code which is MT_c long in time where T_c is the chip duration. The complex baseband equivalent of the received signal over the first slot interval $t \in [0, NT_c)$ may be expressed as

$$r(t) = \sqrt{S}\alpha(t)e^{j2\pi\Delta f t}\left[\sum_{m=0}^{M-1} c[m]p(t-(m+\tau)T_c) + \sum_{m=M}^{N-1} d[m]p(t-(m+\tau)T_c)\right] + w(t) \quad (1)$$

where S is the transmit signal power, τ is the received code phase offset in chips, Δf is frequency offset between transmitter and receiver, and $p(t)$ is 1 over $t \in [0, T_c)$ and 0, otherwise. $c[m]$ and $d[m]$ are, respectively, the mth chip of the synchronization code c and the data symbol spread by a scrambling code. $w(t)$ is a complex additive white Gaussian noise (AWGN) process with one-sided power spectral density N_0, and it represents noise plus interference. The sum of interfering signals is assumed to be complex Gaussian distributed. $\alpha(t)$ denotes the multiplicative Rayleigh fading channel [8]. According to Jakes' fading model [9], $\alpha(t)$ is a complex Gaussian random process with the autocorrelation function given as

$$E[\alpha(t_1)\alpha^*(t_2)] = J_0(2\pi f_D|t_1 - t_2|) \quad (2)$$

where $E[\cdot]$ denotes the statistical expectation, $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, and f_D is the maximum Doppler spread.

The structure of a slot synchronization block is depicted in Fig. 2 where L and N, respectively, denote the number of slots to be combined and slot length in chips. N is the duration of a code phase uncertainty region. A synchronization code matched filter (MF) is employed to collect N observations corresponding to N different code phases for each slot interval. For each phase, L observations are collected from MF outputs for L slot duration and combined to form a decision variable, and the code phase corresponding to the largest decision variable is selected. If the selected code phase is the correct phase, the slot synchronization is completed, otherwise, the above test is resumed after penalty time.

A MF output for the nth code phase in the kth slot may be expressed as

$$z_{nk} = \frac{1}{M}\sum_{m=0}^{M-1} r_{nk}[m]c[m], \quad k = 1, 2, \ldots, L, \quad n = 0, 1, \ldots, N-1 \quad (3)$$

where $r_{nk}[m] = r((kN + n + m)T_c)$ denotes the $(n+m)$th sample of the received signal in the kth slot. There are two kinds of hypotheses: code alignment (H_1) corresponding to a correct code phase, and code misalignment (H_0) corresponding to $(N-1)$ incorrect code phases. For a given hypothesis, the MF output in (3) may be calculated as

$$z_{nk} = \begin{cases} \sqrt{S}\alpha_{nk}\frac{\sin(M\pi\varepsilon)}{M\sin(\pi\varepsilon)}e^{j\pi\varepsilon(M-1)} \\ \cdot e^{j2\pi\varepsilon(kN+n)} \\ +\frac{1}{M}\sum_{m=0}^{M-1} w_{nk}[m]c[m], & \text{under } H_1 \\ \frac{1}{M}\sum_{m=0}^{M-1} w_{nk}[m]c[m], & \text{under } H_0 \end{cases} \quad (4)$$

where α_{nk} denotes a fading coefficient for the nth code phase in the kth slot, and it is assumed to be constant over the correlation time MT_c; $\alpha_{nk} = \alpha((kN + n + m)T_c)$ for $0 \le m \le M-1$. $w_{nk}[m] = w((kN + n + m)T_c)$ is a sample of the AWGN process $w(t)$, and $\varepsilon = \Delta f T_c$ denotes the normalized frequency offset. For both H_1 and H_0 phases, the MF output z_{nk} follows a complex Gaussian distribution, since the signal term as well as the noise plus interference term is complex Gaussian distributed.

III. Study on Combining Schemes of Multiple Observations

In this section, we investigate combining schemes of multiple observations for slot synchronization. In Section III-A, the optimal combining scheme is determined for a general environment using detection theory for multiple observations [7], and in Section III-B for two specific environments.

A. Study on Optimal Combining

The maximum-likelihood (ML) detection theory for multiple observations may be utilized to determine the optimal combining rule. The ML criterion is to choose a code phase that maximizes the likelihood function. In the slot synchronization case, the likelihood function is the joint probability density function (PDF) of a set of N MF output vectors $\{\mathbf{z}_n = [z_{n1}, z_{n2}, \ldots, z_{nL}]^T : 0 \le n \le N-1\}$, conditioned on the received code phase. Since the elements of $\mathbf{z}_n$ are jointly complex Gaussian random variables, the PDF of $\mathbf{z}_n$ for a given hypothesis H_i may be written as [10]

$$f_{\mathbf{z}_n}(\mathbf{z}_n|H_i) = \frac{1}{\pi^L \det(\mathbf{R}_i)} \exp\left[-\mathbf{z}_n^H \mathbf{R}_i^{-1} \mathbf{z}_n\right], \quad n = 0, 1, \ldots, N-1 \tag{5}$$

where $\mathbf{R}_i = E\left[\mathbf{z}_n \mathbf{z}_n^H | H_i\right]$ is the covariance matrix of $\mathbf{z}_n$ for a given hypothesis H_i and $(\cdot)^H$ denotes the Hermitian transpose operator. The (k, ℓ) element of $\mathbf{R}_i$ is determined using (4) and is found as

$$E\left[z_{nk} z_{n\ell}^* | H_i\right] = \begin{cases} S \cdot J_0(2\pi f_D (k-\ell) N T_c) \\ \quad \cdot \frac{\sin^2(M\pi\varepsilon)}{M^2 \sin^2(\pi\varepsilon)} \cdot e^{j2\pi\varepsilon N(k-\ell)} \\ \quad + \sigma_w^2 \delta[k-\ell], & i = 1 \\ \sigma_w^2 \delta[k-\ell], & i = 0 \end{cases} \tag{6}$$

where $\sigma_w^2 = N_0/MT_c$ is the noise plus interference power in a MF output, and $\delta[k]$ is the delta function, defined as 1 for $k = 0$ and 0 otherwise. Using (5), the decision variable for the log-likelihood ratio test is found as [7]

$$D_n = \mathbf{z}_n^H \left(\mathbf{R}_0^{-1} - \mathbf{R}_1^{-1}\right) \mathbf{z}_n. \tag{7}$$

This equation describes the optimal combining rule and indicates that channel information is required to construct the matrices $\mathbf{R}_0^{-1}$ and $\mathbf{R}_1^{-1}$. In practice, this information is not available in the slot synchronization process and, thus, the optimal combining is difficult to implement.

B. Special Cases of the Optimal Combining

The matrix $\mathbf{R}_0^{-1} - \mathbf{R}_1^{-1}$ in (7) may be simplified under certain environments. In this subsection, the optimal combining is investigated for two special cases.

1) *Extremely fast fading channels*: For an extremely fast fading channel, the MF outputs for the same code phase become uncorrelated, since α_{nk} in (4) varies fast enough over one slot duration. Consequently, the covariance matrices are expressed as $\mathbf{R}_1 = \left[S \cdot (\sin^2(M\pi\varepsilon)/M^2 \sin^2(\pi\varepsilon)) + \sigma_w^2\right] \cdot \mathbf{I}$ and $\mathbf{R}_0 = \sigma_w^2 \mathbf{I}$ where $\mathbf{I}$ denotes the L-dimensional identity matrix. The optimal combining rule (7) in this case is simplified to

$$D_n = \mathbf{z}_n^H \mathbf{z}_n = \sum_{k=1}^{L} |z_{nk}|^2 \tag{8}$$

which is the noncoherent combining scheme studied in [3] and [5]. To form a decision variable in this scheme, the MF outputs are squared before summation. Note that this combining scheme is the same as the optimal combining scheme of statistically uncorrelated observations, which has been found in the general detection theory [7].

2) *Static channels without frequency offset*: When the channel is static without frequency offset, the signal parts of MF outputs z_{nk} $(k = 1, 2, \ldots, L)$ associated with the same code phase do not vary, and they are statistically perfectly correlated. In this case, it can be shown that the covariance matrices are expressed as $\mathbf{R}_1 = S \cdot \mathbf{E} + \sigma_w^2 \mathbf{I}$ and $\mathbf{R}_0 = \sigma_w^2 \mathbf{I}$, where $\mathbf{E}$ denotes a matrix with all elements equal to unity. Using the matrix inversion lemma [11], $\mathbf{R}_1^{-1}$ can be calculated, and the optimal combining rule in (7) reduces to the no-preprocessing combining scheme presented in [4]

$$D_n = \mathbf{z}_n^H \mathbf{E} \mathbf{z}_n = \sum_{k=1}^{L} \sum_{\ell=1}^{L} z_{nk}^* z_{n\ell} = \left|\sum_{k=1}^{L} z_{nk}\right|^2. \tag{9}$$

To form a decision variable in this scheme, MF outputs are summed without any preprocessing, which is followed by a squaring operation.

IV. Proposed Combining Scheme

As presented in Section III-B, the noncoherent combining and no-preprocessing combining schemes are optimal for specific environments. The former is optimal for an extremely fast fading channel, while the latter is for a static channel without frequency offset. However, both of these schemes may not perform well in typical environments. The noncoherent combining scheme may not be effective in static or slow fading channels due to noncoherent combining loss. The performance of the no-preprocessing combining scheme may severely degrade due to phase fluctuations over time, when the frequency offset is not negligible and/or the fading is not slow enough.

Differential detection has been widely used for the detection of a differential phase-shift keying (DPSK) signal [12]. This does not require phase estimation and obtains phase reference information from the symbol preceding the symbol to be detected. Recently, the differential detection technique without combining has been proposed for code acquisition of a direct sequence-spread spectrum (DS/SS) to reduce the effects of background noise in a static channel without frequency offset [13]. We propose employing differential processing prior to the combining of multiple observations for slot synchronization. The differential preprocessing reduces the effects of phase fluctuations due to frequency offset and fading, and allows differential processing outputs to be combined coherently without severe degradation.

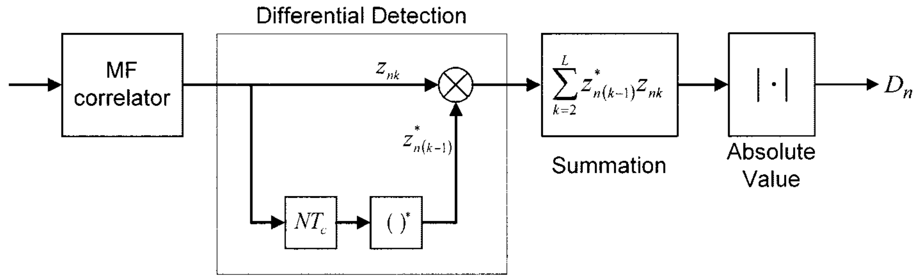

Fig. 3. Differentially coherent combining scheme.

Assuming that the nth code phase corresponds to the H_1 phase, the signal component of $z^*_{n(k-1)} z_{nk}$ after differential detection of the MF outputs may be expressed as

$$s^*_{n(k-1)} s_{nk} = S \cdot \alpha^*_{n(k-1)} \alpha_{nk} \frac{\sin^2(M\pi\varepsilon)}{M^2 \sin^2(\pi\varepsilon)} e^{j2\pi\varepsilon N}, \quad k = 2, 3, \ldots, L \tag{10}$$

where $s_{nk} = \sqrt{S}\alpha_{nk}(\sin(M\pi\varepsilon)/M\sin(\pi\varepsilon))e^{j\pi\varepsilon(M-1)} e^{j2\pi\varepsilon(kN+n)}$ denotes the signal component of z_{nk} in (4). Note that the phase rotation $e^{j2\pi\varepsilon N}$ due to frequency offset is independent of slot index k in (10), and that the phase fluctuation due to fading would be significantly reduced by the operation $\alpha^*_{n(k-1)}\alpha_{nk}$, if the fading is slow enough not to vary significantly over one slot interval. The differential detection outputs $z^*_{n(k-1)} z_{nk}$ for $k = 2, 3, \ldots, L$ are summed together. This summation may be viewed as a coherent combining since the signal components in (10) are nearly phase aligned by differential processing. The term $e^{j2\pi\varepsilon N}$ in (10) splits the signal components into real and imaginary parts. To capture both signal components, an absolute value operation may be employed after summation. Hence, the resulting combining scheme is a combination of the differential preprocessing, summation, and absolute value operation. The structure of the proposed combining scheme, referred to as the differentially coherent combining scheme is depicted in Fig. 3, and the decision variable for the nth code phase is expressed as

$$D_n = \left| \sum_{k=2}^{L} z^*_{n(k-1)} z_{nk} \right|. \tag{11}$$

V. Performance Analysis

In this section, the detection and mean acquisition time performance of the differentially coherent combining scheme, as well as, the optimal combining, noncoherent combining, and no-preprocessing combining schemes is analyzed for Rayleigh fading channels. The detection probability is defined as the probability that the decision variable associated with the correct phase has the largest value. The subscript n representing the nth phase will be omitted in representing a decision variable D_n for simplicity in performance analysis. The detection probability is given as

$$P_d = \int_{-\infty}^{\infty} f_D(x|H_1) \cdot [F_D(x|H_0)]^{N-1} dx \tag{12}$$

where $f_D(x|H_1)$ and $F_D(x|H_0)$ represent, respectively, the PDF of a decision variable D for the H_1 phase and the cumulative distribution function (CDF) of D for the H_0 phase. Applying the flow graph method [14], we can determine the mean acquisition time for all four combining schemes discussed in this paper with respect to the corresponding detection probability as

$$E[T_{ACQ}] = \left(\frac{LN + K}{P_d} - K \right) \cdot T_c \tag{13}$$

where K denotes the penalty time in chips caused by false alarm. The PDFs and CDFs of decision variables are required to calculate (12) and (13). These are derived for each combining scheme in Section V-A–D.

A. PDF and CDF for the Optimal Combining Scheme

Since the decision variable D in (7) is of a Hermitian quadratic form, the PDF and CDF of D may be derived using a characteristic function. The characteristic function of D for a given hypothesis H_i $(i = 0, 1)$ is expressed as [15]

$$\begin{aligned}\Phi_D(j\omega|H_i) &= E\left[\exp\left\{j\omega \mathbf{z}^H \left(\mathbf{R}_0^{-1} - \mathbf{R}_1^{-1}\right)\mathbf{z}\right\} |H_i\right] \\ &= \prod_{k=1}^{L} \frac{1}{1 - j\omega\lambda_{ik}}\end{aligned} \tag{14}$$

where λ_{ik}s are the eigenvalues of the matrix $\mathbf{R}_i\left(\mathbf{R}_0^{-1} - \mathbf{R}_1^{-1}\right)$. When all the eigenvalues are distinct, the product in (14) can be expressed as a summation using partial fraction expansion

$$\Phi_D(j\omega|H_i) = \sum_{k=1}^{L} \frac{\gamma_{ik}}{1 - j\omega\lambda_{ik}}$$

where

$$\gamma_{ik} = \prod_{j=1, j\neq k}^{L} \frac{1}{1 - \frac{\lambda_{ij}}{\lambda_{ik}}}. \tag{15}$$

Taking the inverse transform of (15) using the Residue theorem [16], the PDF may be derived as

$$f_D(x|H_i) = \sum_{\substack{k=1 \\ \lambda_{ik}>0}}^{L} \frac{\gamma_{ik}}{\lambda_{ik}} \exp\left(-\frac{x}{\lambda_{ik}}\right) u(x) - \sum_{\substack{k=1 \\ \lambda_{ik}<0}}^{L} \frac{\gamma_{ik}}{\lambda_{ik}} \exp\left(-\frac{x}{\lambda_{ik}}\right) u(-x) \tag{16}$$

where $u(x)$ is the unit step function, defined to be 1 for $x \geq 0$ and 0, otherwise. Integrating (16) with respect to x, the CDF may be calculated as

$$F_D(x|H_i) = \sum_{\substack{k=1 \\ \lambda_{ik}>0}}^{L} \gamma_{ik}\left[1 - \exp\left(-\frac{x}{\lambda_{ik}}\right)\right] u(x) + \sum_{\substack{k=1 \\ \lambda_{ik}<0}}^{L} \gamma_{ik}\left[u(x) + \exp\left(-\frac{x}{\lambda_{ik}}\right) u(-x)\right]. \quad (17)$$

When multiple poles exist in (14), partial fraction expansion is also possible and the corresponding PDF and CDF may easily be derived.

B. PDF and CDF for the Noncoherent Combining Scheme

Since the decision variable D in (8) is also of a Hermitian quadratic form, the PDF and CDF may be derived in a similar manner as for the optimal combining scheme. The characteristic function for a given hypothesis H_i $(i = 0, 1)$ is given as

$$\begin{aligned}\Phi_D(j\omega|H_i) &= E\left[\exp\left\{j\omega \mathbf{z}^H \mathbf{z}\right\}|H_i\right] \\ &= \prod_{k=1}^{L} \frac{1}{1 - j\omega\lambda_{ik}} = \sum_{k=1}^{L} \frac{\gamma_{ik}}{1 - j\omega\lambda_{ik}} \end{aligned} \quad (18)$$

where λ_{ik}s are the eigenvalues of the matrix $\mathbf{R}_i$ which are assumed to be distinct in (18). Since $\mathbf{R}_i$ is nonnegative definite, all of the eigenvalues are nonnegative. From this fact and (16), the PDF may be expressed as

$$f_D(x|H_i) = \sum_{k=1}^{L} \frac{\gamma_{ik}}{\lambda_{ik}} \exp\left(-\frac{x}{\lambda_{ik}}\right) u(x) \quad (19)$$

and the corresponding CDF may be described as

$$F_D(x|H_i) = \sum_{k=1}^{L} \gamma_{ik}\left[1 - \exp\left(-\frac{x}{\lambda_{ik}}\right)\right] u(x). \quad (20)$$

C. PDF and CDF for the No-Preprocessing Combining Scheme

Since the sum of MF outputs without any preprocessing is a complex Gaussian random variable, the decision variable in (9) is central chi-square distributed with two degrees of freedom. Hence, the PDF may be expressed as

$$f_D(x|H_i) = \frac{1}{\lambda_i} \exp\left(-\frac{x}{\lambda_i}\right) u(x) \quad (21)$$

where

$$\lambda_i = \begin{cases} S \cdot \frac{\sin^2(M\pi\varepsilon)}{M^2 \sin^2(\pi\varepsilon)} \\ \quad \cdot \Big(L + 2\sum_{k=1}^{L-1}(L-k)J_0(2\pi f_D kNT_c) \\ \quad \cdot \cos(2\pi\varepsilon kN)\Big) + L\sigma_w^2, & i = 1 \\ L\sigma_w^2, & i = 0. \end{cases} \quad (22)$$

The corresponding CDF may be calculated as

$$F_D(x|H_i) = \left[1 - \exp\left(-\frac{x}{\lambda_i}\right)\right] u(x). \quad (23)$$

D. PDF and CDF for the Differentially Coherent Combining Scheme

For a given observation vector $\mathbf{z} = [z_1, z_2, \ldots, z_L]^T$, the decision variable D in (11) may be rewritten as

$$D = \left|\sum_{k=2}^{L} z_{k-1}^* z_k\right| \triangleq |D_I + jD_Q| \quad (24)$$

where $D_I = \mathbf{z}^H \mathbf{Q}_I \mathbf{z}$, $D_Q = \mathbf{z}^H \mathbf{Q}_Q \mathbf{z}$, and

$$\mathbf{Q}_I = \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 1 & \ddots & \vdots \\ 0 & 1 & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

and

$$\mathbf{Q}_Q = \frac{1}{2}\begin{bmatrix} 0 & -j & 0 & \cdots & 0 \\ j & 0 & -j & \ddots & \vdots \\ 0 & j & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -j \\ 0 & \cdots & 0 & j & 0 \end{bmatrix}. \quad (25)$$

Using (5), the joint characteristic function of the real variables D_I and D_Q for a given hypothesis H_i $(i = 0, 1)$ is found as

$$\begin{aligned}\Phi_{D_I D_Q}(\mu, \nu|H_i) &= E[\exp\{j(\mu D_I + \nu D_Q)\}|H_i] \\ &= \int_{-\infty}^{\infty} \exp\left\{j\mathbf{z}^H\left(\mu\mathbf{Q}_I + \nu\mathbf{Q}_Q\right)\mathbf{z}\right\} f_{\mathbf{z}}(\mathbf{z}|H_i) d\mathbf{z} \\ &= \frac{1}{\pi^L \det(\mathbf{R}_i)} \cdot \int_{-\infty}^{\infty} \exp\left\{-j\mathbf{z}^H\left[\mathbf{R}_i^{-1} - (\mu\mathbf{Q}_I + \nu\mathbf{Q}_Q)\right]\mathbf{z}\right\} d\mathbf{z}. \end{aligned} \quad (26)$$

Since $\mathbf{Q}_I$ and $\mathbf{Q}_Q$ are Hermitian symmetric, (26) may be simplified to [17]

$$\Phi_{D_I D_Q}(\mu, \nu|H_i) = [\det(\mathbf{I} - j\mathbf{R}_i(\mu\mathbf{Q}_I + \nu\mathbf{Q}_Q))]^{-1}. \quad (27)$$

The joint PDF $f_{D_I D_Q}(d_I, d_Q|H_i)$ of D_I and D_Q may be obtained by taking the inverse transform of (27). Using a pair of transforms via the change of variables $\{d_I = x\cos\theta, d_Q = x\sin\theta\}$ and $\{\mu = \rho\cos\phi, \nu = \rho\sin\phi\}$, the joint PDF may be expressed in a polar form as

$$f_{D\Theta}(x, \theta|H_i) = \frac{x}{(2\pi)^2}\int_0^{\infty}\int_{-\pi}^{\pi} \rho\Phi_{D_I D_Q}(\rho\cos\phi, \rho\sin\phi|H_i) \cdot e^{-j\rho x\cos(\theta-\phi)} d\phi d\rho. \quad (28)$$

By averaging the joint PDF in (28) with respect to θ over $[0, 2\pi)$, the PDF of the decision variable D for a given hypothesis H_i may be derived as

$$f_D(x|H_i) = \frac{x}{2\pi}\int_0^{\infty}\int_{-\pi}^{\pi} \rho\Phi_{D_1 D_Q}(\rho\cos\phi, \rho\sin\phi|H_i) \cdot J_0(\rho x) d\phi d\rho. \quad (29)$$

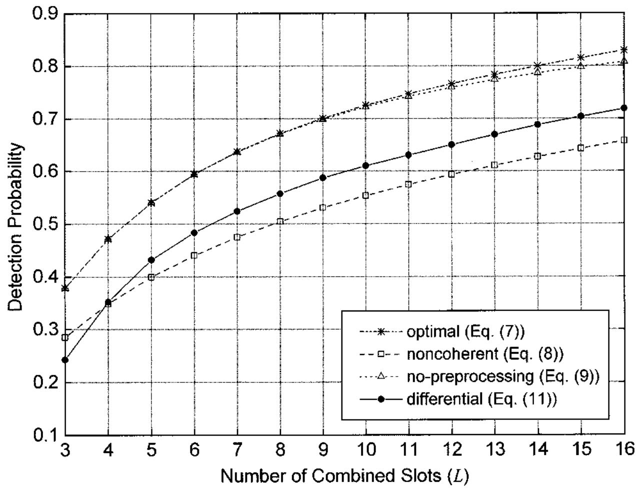

Fig. 4. Detection probability versus number of combined slots ($\mathrm{SNIR/chip} = -20$ dB, $f_D = 18.5$ Hz, and $\Delta f = 0$).

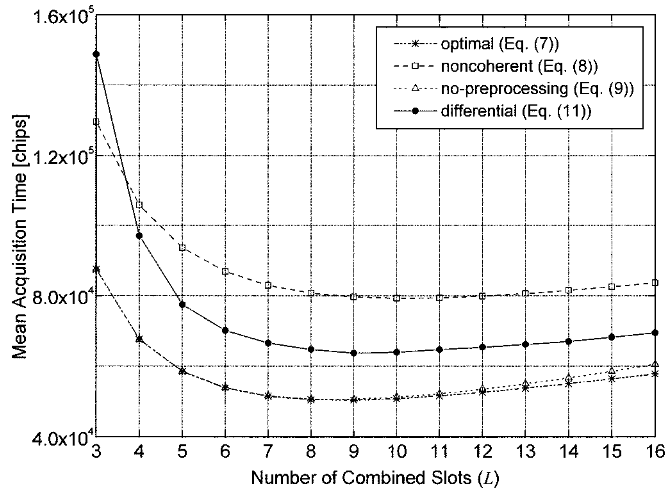

Fig. 5. Mean acquisition time versus number of combined slots ($\mathrm{SNIR/chip} = -20$ dB, $f_D = 18.5$ Hz, and $\Delta f = 0$).

Using the property $d\{x^p J_p(x)\}/dx = x^p J_{p-1}(x)$ [18], where $J_p(\cdot)$ is the pth-order Bessel function of the first kind, the corresponding CDF may be obtained as

$$\begin{aligned} F_D(x|H_i) &= \int_{-\infty}^{x} f_D(\xi|H_i)d\xi \\ &= \frac{1}{2\pi}\int_0^{\infty}\int_{-\pi}^{\pi} \Phi_{D_I D_Q}(\rho\cos\phi, \rho\sin\phi|H_i) \\ &\quad \cdot \left(\int_{-\infty}^{x} \rho\xi J_0(\rho\xi)d\xi\right) d\phi d\rho \\ &= \frac{1}{2\pi}\int_0^{\infty}\int_{-\pi}^{\pi} \Phi_{D_I D_Q}(\rho\cos\phi, \rho\sin\phi|H_i) \\ &\quad \cdot J_1(\rho x)d\phi d\rho. \end{aligned} \tag{30}$$

VI. Numerical Results

In this section, the performance of the combining schemes described in Section V is evaluated and compared. The detection probability of each combining scheme is calculated using (12) and the corresponding PDF and CDF equations: (16) and (17) for optimal combining; (19) and (20) for noncoherent combining; (21) and (23) for no-preprocessing combining; and (29) and (30) for differentially coherent combining. Equation (13) indicates that the mean acquisition time is inversely proportional to the detection probability for a given number of combined slots L, and the combining scheme with the greater detection probability provides shorter mean acquisition time. In the performance evaluation, the synchronization code length (M) and slot length (N) are assumed to be 256 chips and 2560 chips, respectively. The chip rate and carrier frequency are assumed to be 4.096 Mcps and 2 GHz, respectively. These parameters are from [1].

The effects of the number of combined slots L on the detection probability and mean acquisition time are shown, respectively, in Figs. 4 and 5, when $\mathrm{SNIR/chip} = -20$ dB, $f_D = 18.5$ Hz, and $\Delta f = 0$. The detection probability is found to increase with L increasing for all combining schemes. The performance of the noncoherent combining is poor because of noncoherent combining loss. As expected, the differentially coherent combining is shown to outperform the noncoherent combining for L greater than 3. The no-preprocessing combining is shown to be near the optimum, since the channel varies very slowly in this case. Fig. 5 shows that there exists an optimum L that minimizes the mean acquisition time. The optimum L is 9 for the differentially coherent combining in Fig. 5. The reason for this is that an increase in L not only increases the detection probability, but also increases the processing time in proportion to L. Therefore, the mean acquisition time increases if the increase in the processing time is more significant than the increase in the detection probability.

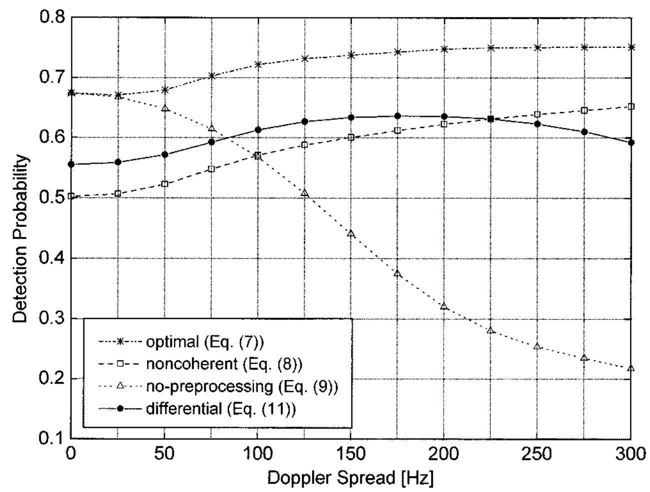

Fig. 6. Detection probability versus Doppler spread ($L = 8$, $\mathrm{SNIR/chip} = -20$ dB, and $\Delta f = 0$).

Fig. 6 shows the effects of Doppler spread f_D on the detection performance, when $L = 8$, $\mathrm{SNIR/chip} = -20$ dB, and $\Delta f = 0$. As f_D increases, the performance of the no-preprocessing combining scheme is found to degrade significantly, while that of the noncoherent combining scheme is found to slightly improve for a given range of Doppler spread. The reason for this is that an increase in f_D makes the MF outputs become statistically less correlated as explained in Section III-B. The detection probability of the differentially coherent combining scheme is observed to increase as f_D increases from 0 to 175 Hz, and to decrease when f_D exceeds 175 Hz. This may be ex-

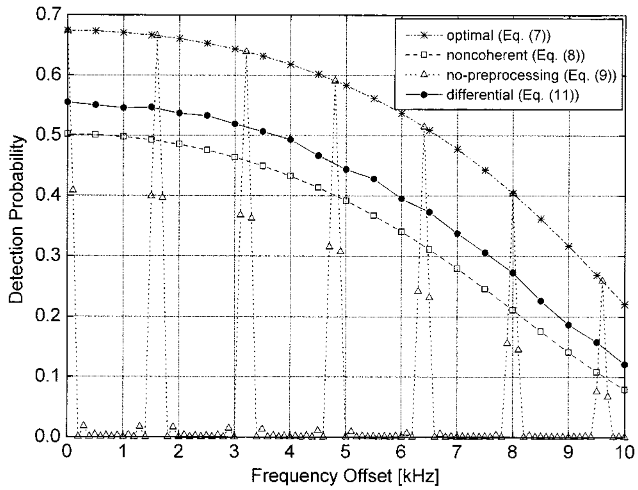

Fig. 7. Detection probability versus frequency offset ($L = 8, \mathrm{SNIR/chip} = -20$ dB, and $f_D = 0$).

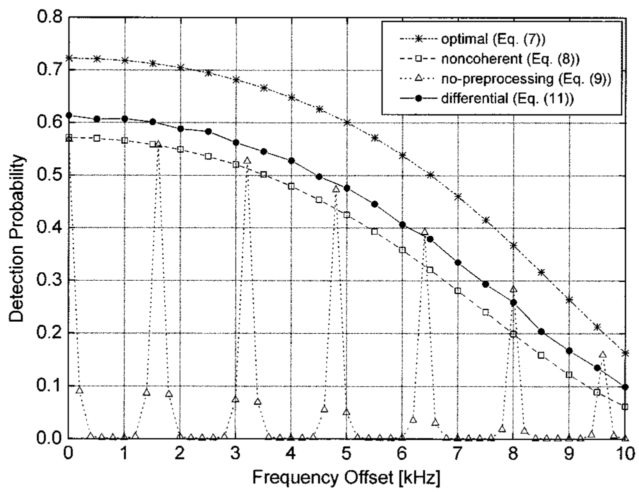

Fig. 8. Detection probability versus frequency offset ($L = 8, \mathrm{SNIR/chip} = -20$ dB, and $f_D = 100$ Hz).

plained as follows. As f_D increases, diversity gain increases, while SNIR gain decreases due to increased phase misalignment. For f_D smaller than 175 Hz, the increase in diversity gain is more significant than the decrease in SNIR gain, whereas, for f_D greater than 175 Hz, the increase in diversity gain is not significant. The differentially coherent combining scheme is shown to outperform the noncoherent combining and no-preprocessing combining schemes for a typical range of Doppler spread.

The effects of frequency offset Δf on the detection performance are shown in Fig. 7, when $L = 8$, $\mathrm{SNIR/chip} = -20$ dB, and $f_D = 0$. The no-preprocessing combining scheme is shown to be very sensitive to frequency offset, and the detection probability oscillates with Δf, since the signal component oscillates with Δf. The differentially coherent combining and noncoherent combining schemes are shown to be relatively robust to frequency offset. The differentially coherent combining scheme is found to always outperform the noncoherent combining scheme. The combined effects of frequency offset and Doppler spread on the detection performance are depicted in Fig. 8, when $L = 8$, $\mathrm{SNIR/chip} = -20$ dB, and $f_D = 100$ Hz. The performance of the no-preprocessing combining scheme is the worst, except for some specific values of frequency offset. From Figs. 7 and 8, it is seen that the differentially coherent combining scheme is superior to the noncoherent combining and no-preprocessing combining schemes for a typical range of Doppler spread and frequency offset.

VII. CONCLUSION

In this paper, we have investigated combining schemes of multiple observations for slot synchronization in intercell asynchronous DS/CDMA systems, and proposed a new differentially coherent combining scheme. The detection and mean acquisition time performance of the proposed differentially coherent combining scheme as well as the optimal combining, noncoherent combining, and no-preprocessing combining schemes has been analyzed for Rayleigh fading channels with frequency offset. It has been found that the performance of the differentially coherent combining scheme is superior to that of the noncoherent and no-preprocessing combining schemes for typical environments, and is relatively robust to channel variations.

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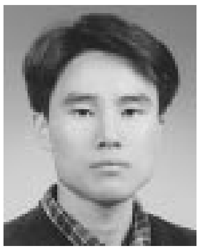

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