Efficient Weight Vector Representation for Closed-Loop Transmit Diversity

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***Abstract*—For a closed-loop transmit (Tx) diversity, the Tx weights are calculated at a receiver, and fed back to a transmitter. As the number of Tx antennas increases, the potential gain of closed-loop Tx diversity may be significant. However, the amount of feedback information, which is the number of Tx weights that should be fed back, linearly increases, and the performance improvement of a closed-loop Tx diversity system may not be as significant as expected due to delay in the feedback process. Thus, an efficient Tx weight representation, which can reduce the amount of feedback information, is needed. In this letter, a Tx weight vector representation is presented, and its performance is analyzed. Analysis shows that this weight vector representation, referred to as *basis selection*, significantly reduces the amount of feedback information with little performance degradation.**

***Index Terms*—Antenna arrays, performance, transmit diversity.**

I. Introduction

THE third-generation mobile communication systems such as the universal mobile telecommunication systems (UMTS) and interim standard 2000 (IS-2000) are currently being standardized [1]. Due to the rapid increase of Internet and multimedia service in wired communication, it is necessary to increase the capacity of these third-generation systems, especially, in downlink (base to mobile). Receive (Rx) diversity with multiple receive antennas at the mobile terminal may be applied to increase downlink capacity, but this may be difficult because of the implementation complexity increase and size limitation of the mobile terminal. Hence, a number of transmit (Tx) diversity schemes have been proposed [2], [3], and schemes using two Tx antennas, such as space–time transmit diversity (STTD), space–time spreading (STS), and transmit antenna array (TxAA), have been included in the UMTS [4] and IS-2000 standard [5].

Tx diversity systems can be classified into open-loop or closed-loop systems, depending on the existence of the feedback signal from the mobile. Open-loop Tx diversity systems [2], [5], which operate without any feedback information from the mobile, are known to offer diversity gain. As the number of Tx antennas for these open-loop systems increases, the average signal-to-noise ratio (SNR) does not change, whereas the variation of SNR in decibel scale decreases. This variation decrease due to diversity gain brings about performance improvement. In contrast to the open-loop systems, the closed-loop Tx diversity systems [4], [6], [7], which operate with feedback information from the mobile, offer not only diversity gain but also beamforming gain [8], [10]. As the number of Tx antennas increases in closed-loop systems, the variation of SNR in decibel scale decreases because of diversity gain, and the average SNR increases due to beamforming gain. The average SNR increases by 3 dB, as the number of Tx antennas doubles [8]. Because of the average SNR increase and variation decrease, the closed-loop Tx diversity systems offer potentially more benefit than the open-loop Tx diversity systems.

Although the closed-loop Tx diversity provides potentially significant gain as the number of Tx antennas increases, the use of numerous Tx antennas may be impractical because of the large amount of feedback information. For two Tx antennas systems in UMTS, the relative weight of the second Tx antenna, which maximizes the SNR, is calculated at a mobile station [4]. The direct extension of this two-Tx-antennas system to M Tx antennas requires the calculation of $M-1$ relative weights and the transmission of these weights periodically to the base station. The transmission of these weights takes nonnegligible time because of the limited uplink channel allocated for feedback information. This feedback delay causes the performance degradation. Thus, an efficient weight representation is desired to reduce the amount of feedback information.

There have been some research efforts on the practical aspects of closed-loop Tx diversity systems [7], [9]–[11]. In [7] and [9], methods to reduce the amount of feedback information have been described. In [10], the effects of feedback delay on the performance of a closed-loop Tx diversity system have been analyzed. The effects of weight digitization on the performance of a closed-loop Tx diversity system have been discussed in [11]. Recently, the authors have proposed a Tx weight representation for closed-loop Tx diversity systems in the third-generation partnership project (3GPP) standardization for UMTS [6]. In this letter, we describe this Tx weight representation method, and analyze its performance. The remainder of this letter is organized as follows. Section II describes a closed-loop transmit diversity system. An efficient weight vector representation method is presented in Section III, and its performance analysis is given in Section IV. The effects of weight digitization and feedback delay are investigated in Section V. Finally, conclusions are made in Section VI.

II. Closed-Loop Tx Diversity System

For closed-loop Tx diversity, the Tx weights are calculated at the receiver, and fed back to the transmitter. At the transmitter, the data symbol is multiplied by these weights before transmission. In this letter, it is assumed that M Tx antennas and L Rx antennas are employed for diversity. The baseband equivalent system model of closed-loop Tx diversity to be considered is

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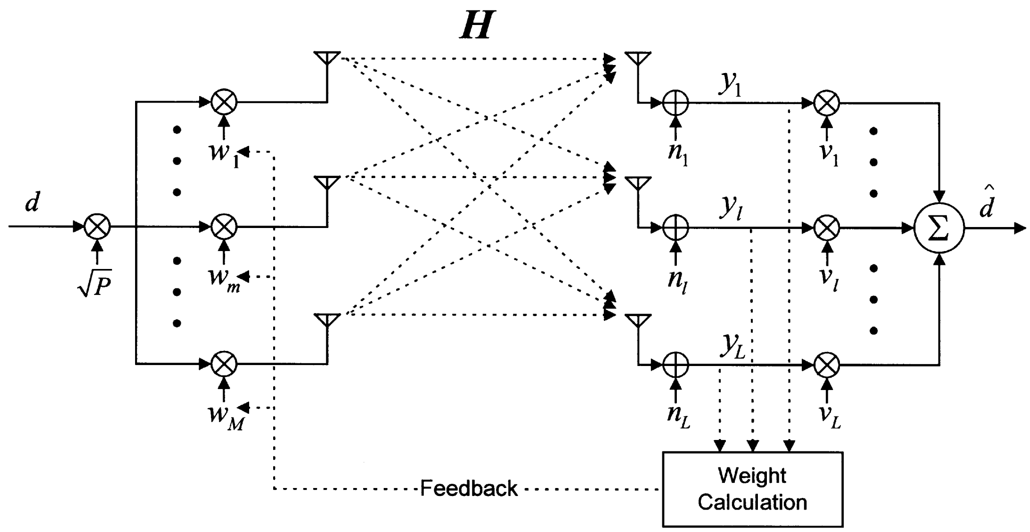

Fig. 1. System model.

shown in Fig. 1. The received signal vector $\boldsymbol{y} = [y_1\, y_2\, \ldots\, y_L]^T$ may be expressed as

$$\boldsymbol{y} = \sqrt{P}(\boldsymbol{H}\boldsymbol{w})d + \boldsymbol{n} \tag{1}$$

where the superscript ^{T} denotes the transpose, P denotes the total transmit power, d is the binary data symbol with the probability $\Pr\{d = +1\} = \Pr\{d = -1\} = 1/2$, and $w = [w_1\, w_2\, \ldots\, w_M]^T$ denotes the Tx weight vector. To maintain the same transmit power for various M, we normalize the weight vector such that $||\boldsymbol{w}||^2 \triangleq \sum_{m=1}^{M} |w_m|^2 = 1$. In this letter, the channels between Tx and Rx antennas are assumed to be slowly time varying and frequency flat, and may be described by the $L \times M$ channel matrix $\boldsymbol{H} = [\boldsymbol{h}_1\, \boldsymbol{h}_2\, \ldots\, \boldsymbol{h}_M]$, where $\boldsymbol{h}_m = [h_{1,m}\, h_{2,m}\, \ldots\, h_{L,m}]^T$ denotes the channel vector from the mth Tx antenna, and $h_{i,j}$ denotes the channel response from the jth Tx antenna to the ith Rx antenna. The channel responses $h_{i,j}$'s are assumed to be independent and identically distributed (i.i.d.) zero-mean circular complex Gaussian random variables with $E[|h_{i,j}|^2] = 1$, where $E[\cdot]$ is an expectation operation. The vector $\boldsymbol{n} = [n_1\, n_2\, \ldots\, n_L]^T$ denotes an additive white Gaussian noise (AWGN) vector, whose elements are i.i.d. zero-mean circular complex Gaussian random variables with $E[|n_i|^2] = \sigma_n^2$.

At the receiver, the received signal vector $\boldsymbol{y}$ is coherently combined with the maximal ratio combining (MRC) weight vector $\boldsymbol{v}^T = \boldsymbol{w}^H\boldsymbol{H}^H$, where the superscript ^{H} denotes the conjugate transpose [10]. Thus, the decision variable $\hat{d}$ may be expressed as

$$\hat{d} = \sqrt{P}(\boldsymbol{w}^H\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{w})d + \boldsymbol{w}^H\boldsymbol{H}^H\boldsymbol{n} \tag{2}$$

where the first term is associated with a desired signal and the second term denotes noise. Note that $E[\boldsymbol{n}\cdot\boldsymbol{n}^H] = \sigma_n^2\boldsymbol{I}_L$, where $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix. Thus, when $\boldsymbol{H}$ and $\boldsymbol{w}$ are given, the variance of noise part of $\hat{d}$ may be found to be $\sigma_n^2\boldsymbol{w}^H\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{w}$. Hence, the receive SNR (RxSNR) may be expressed as

$$\begin{aligned}\gamma &= \frac{P}{\sigma_n^2}\boldsymbol{w}^H\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{w} \\ &= \Gamma\cdot\boldsymbol{w}^H\boldsymbol{R}\boldsymbol{w}\end{aligned} \tag{3}$$

where $\boldsymbol{R} \triangleq \boldsymbol{H}^H\boldsymbol{H}$ is an $M \times M$ Hermitian matrix, and $\Gamma \triangleq P/\sigma_n^2$ denotes the transmit SNR (TxSNR) [8]. In this letter, we assume that $\boldsymbol{H}$ is perfectly known at the receiver. With the channel matrix $\boldsymbol{H}$, the receiver calculates the weight vector $\boldsymbol{w}$, which maximizes the RxSNR in (3). This weight vector $\boldsymbol{w}$ is periodically fed back to the base station through the feedback channel. In real systems, this feedback channel may be erroneous. However, for analytical simplicity, we assume that the feedback channel is an error-free channel. Therefore, the transmitter may be considered to use the same $\boldsymbol{w}$ that is calculated at the receiver.

III. Efficient Weight Vector Representation

In this section, we investigate an efficient weight vector representation scheme. In Section III-A, the optimum Tx weight vector is derived. Based on approximation of optimum Tx weight vector, an efficient weight vector representation is developed in Section III-B.

A. Optimum Transmit Weight Vector

An optimum weight vector may be obtained by finding $\boldsymbol{w}$, which maximizes the RxSNR γ in (3). Hence, for a given $\boldsymbol{R}$, the optimum weight vector $\boldsymbol{w}_{\text{opt}}$ may be described as

$$\boldsymbol{w}_{\text{opt}} = \max_{\boldsymbol{w}:||\boldsymbol{w}||^2=1} \boldsymbol{w}^H\boldsymbol{R}\boldsymbol{w}. \tag{4}$$

Using eigenanalysis, one can easily show that the optimum weight vector $\boldsymbol{w}_{\text{opt}}$ becomes the eigenvector associated with the maximum eigenvalue of the matrix $\boldsymbol{R}$ [10]. Thus, the RxSNR with this optimum weight vector may be expressed as

$$\begin{aligned}\gamma_{\text{opt}} &= \Gamma\cdot\boldsymbol{w}_{\text{opt}}^H\boldsymbol{R}\boldsymbol{w}_{\text{opt}} \\ &= \Gamma\cdot\lambda_{\max}(\boldsymbol{R})\end{aligned} \tag{5}$$

where $\lambda_{\max}(\boldsymbol{R})$ denotes the maximum eigenvalue of the matrix $\boldsymbol{R}$.

For example, let us consider the case of $L = 1$. Using (4), the optimum weight for the mth Tx antenna element $w_m^{(\text{opt})}$ may be obtained as

$$w_m^{(\text{opt})} = \frac{h_{1,m}^*}{\sqrt{\sum_{i=1}^{M} |h_{1,i}|^2}}, \quad m = 1, 2, \ldots, M \tag{6}$$

where the superscript * denotes the complex conjugate. With these optimum weights, the RxSNR for this $L=1$ case may be expressed as

$$\gamma_{\rm opt}^{(L=1)} = \Gamma \cdot \left(\sum_{m=1}^{M} w_m^{(\rm opt)} \cdot h_{1,m} \right)^2 = \Gamma \cdot \sum_{m=1}^{M} |h_{1,m}|^2. \quad (7)$$

From (6) and (7), the Tx weight vector element $w_m^{(\rm opt)}$, associated with a small amplitude channel response, is found to have a small magnitude and to contribute less to RxSNR than the weights associated with a larger amplitude channel response. In approximating the optimum Tx weight vector, the Tx weights with small magnitude may be approximated as zeros without decreasing RxSNR significantly.

B. Efficient Transmit Weight Vector Representation: Basis Selection

A weight vector with M elements may be represented as a linear sum of basis vectors, which span an M-dimensional space [12]. For an M-dimensional space, M basis vectors are required to span the whole M-dimensional space. Let these basis vectors be $\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_M$, which are $M \times 1$ vectors with $||\boldsymbol{b}_i||^2 = 1$. With these basis vectors, the optimal weight vector $\boldsymbol{w}_{\rm opt}$ may be represented as

$$\boldsymbol{w}_{\rm opt} = \sum_{i=1}^{M} c_i \boldsymbol{b}_i \quad (8)$$

where c_i is the complex coefficient associated with the basis vector $\boldsymbol{b}_i$, and may be obtained as

$$c_i = \boldsymbol{b}_i^T \cdot \boldsymbol{w}_{\rm opt}. \quad (9)$$

If we assume that $c_{(i)}$ is the ordered c_i according to its magnitude, i.e., $|c_{(1)}| > |c_{(2)}| > \ldots > |c_{(M)}|$, then the weight vector $\boldsymbol{w}_{\rm opt}$ may be approximated as

$$\tilde{\boldsymbol{w}}_S = \frac{1}{\xi} \sum_{i=1}^{S} c_{(i)} \boldsymbol{b}_{(i)}, \quad S = 1, 2, \ldots, M \quad (10)$$

where $\boldsymbol{b}_{(i)}$ is the basis vector corresponding to $c_{(i)}$, and S denotes the degree of approximation. The normalization factor $\xi = \sqrt{\sum_{i=1}^{S} |c_{(i)}|^2}$ is needed to ensure that $||\tilde{\boldsymbol{w}}_S||^2 = 1$. Note that this approximated weight vector $\tilde{\boldsymbol{w}}_S$ may be viewed as the projection of $\boldsymbol{w}_{\rm opt}$ into S-dimensional subspace [12]. Note also that $\tilde{\boldsymbol{w}}_S$ becomes accurate as S increases, and $\tilde{\boldsymbol{w}}_S = \boldsymbol{w}_{\rm opt}$ when $S = M$.

To represent the weight vector for M Tx antennas system, the basis vectors $\boldsymbol{b}_i$, $i = 1, 2, \ldots, M$, should span the M-dimensional space. A simple set of basis vectors may be obtained from the Cartesian basis, e.g., $\{[1\ 0]^T, [0\ 1]^T\}$ for $M = 2$, and $\{[1\ 0\ 0\ 0]^T, [0\ 1\ 0\ 0]^T, [0\ 0\ 1\ 0]^T, [0\ 0\ 0\ 1]^T\}$ for $M = 4$. For simplicity, we will use these basis vectors from the Cartesian basis for $\tilde{\boldsymbol{w}}_S$ throughout this letter. For the set of basis vectors from the Cartesian basis, only one element in the Tx weight vector is not zero when $S = 1$. This means that only one Tx antenna is used for transmission, and this case may be viewed as a Tx antenna selection case. Besides the basis vector set from the Cartesian basis, many other set of basis vectors may be constructed such as $[1/2\ 1/2\ 1/2\ 1/2]^T$, $[1/2\ -1/2\ 1/2\ -1/2]^T$, $[1/2\ 1/2\ \ -1/2\ \ -1/2]^T$ and $[1/2\ \ -1/2\ \ -1/2\ 1/2]^T$ for $M = 4$. If this set of basis vectors is used, all Tx antennas are used, even when $S = 1$. We refer to the above Tx weight vector representation as *basis selection*. The general algorithm for basis selection is as follows:

1) calculate $\boldsymbol{w}_{\rm opt}$;
2) project $\boldsymbol{w}_{\rm opt}$ to M basis vectors, and calculate the corresponding coefficient c_i;
3) select the basis vectors with S largest coefficients;
4) feed back the selected basis vector information and the corresponding coefficients.

Note that the feedback information consists of the specification of basis vectors in use and associated coefficients.

IV. Performance Analysis

In this section, we analyze the performance of Tx diversity with basis selection for the case of ideal feedback: no weight digitization and no feedback delay. The effects of weight digitization and feedback delay will be discussed in Section V. If we use $\tilde{\boldsymbol{w}}_S$ in (10) as the Tx weight vector, the RxSNR in (3) may be expressed as

$$\tilde{\gamma}_S = \Gamma \cdot \tilde{\boldsymbol{w}}_S^H \boldsymbol{R} \tilde{\boldsymbol{w}}_S. \quad (11)$$

After some manipulation, the conditional bit-error probability (BEP) for given $\tilde{\gamma}_S$ may be expressed as [10]

$$P_b(\tilde{\gamma}_S) = Q(\sqrt{2\tilde{\gamma}_S}) \quad (12)$$

where $Q(\cdot)$ is a Gaussian tail integral defined as $Q(x) = (1/\pi)\int_0^{\pi/2} \exp(-x^2/(2\sin^2\theta))d\theta$ [10]. Note that the RxSNR $\tilde{\gamma}_S$ is a random variable, since it is a function of $\boldsymbol{H}$. Thus, the average BEP may be obtained as

$$\begin{aligned} P_b &= E\left[Q(\sqrt{2\tilde{\gamma}_S})\right] \\ &= \int_0^\infty Q(\sqrt{2\tilde{\gamma}_S}) p(\tilde{\gamma}_S) d\tilde{\gamma}_S \end{aligned} \quad (13)$$

where $p(\tilde{\gamma}_S)$ is the probability density function (pdf) of $\tilde{\gamma}_S$. In the following subsections, we investigate P_b for two cases: $L = 1$ and general L.

A. When the Number of Rx Antennas is 1 ($L = 1$)

When $L = 1$, the coefficient $c_{(i)}$ in (10) may be expressed as

$$c_{(i)} = h_{(1,i)} \quad (14)$$

where $h_{(1,i)}$ is the ordered $h_{1,i}$ according to its magnitude, i.e., $|h_{(1,1)}| > |h_{(1,2)}| > \ldots > |h_{(1,M)}|$. Hence, the weight vector approximated by basis selection may be expressed as

$$\tilde{\boldsymbol{w}}_S^{(L=1)} = \frac{1}{\xi} \sum_{i=1}^{S} h_{(1,i)} \boldsymbol{b}_{(i)} \quad (15)$$

where ξ for this $L = 1$ case may be expressed as $\xi = \sqrt{\sum_{i=1}^{S} |h_{(1,i)}|^2}$. From (11), the RxSNR may be expressed as

$$\begin{aligned}\tilde{\gamma}_S^{(L=1)} &= \Gamma \cdot \left(\tilde{\boldsymbol{w}}_S^H \boldsymbol{R} \tilde{\boldsymbol{w}}_S\right) \\ &= \frac{\Gamma}{\xi^2} \cdot \left(\sum_{i=1}^{S} h_{(1,i)}^* \boldsymbol{b}_{(i)}^H \cdot \sum_{i=1}^{M} h_{1,i}^* \boldsymbol{b}_i \right. \\ &\qquad \left. \cdot \sum_{i=1}^{M} h_{1,i} \boldsymbol{b}_i^H \cdot \sum_{i=1}^{S} h_{(1,i)} \boldsymbol{b}_{(i)}\right) \\ &= \Gamma \cdot \sum_{i=1}^{S} \left|h_{(1,i)}\right|^2. \end{aligned} \tag{16}$$

Note that $\tilde{\gamma}_S^{(L=1)}$ is the sum of ordered random variables $|h_{(1,i)}|^2$. Using the order statistics in [13], the average BEP may be expressed as [17]

$$\begin{aligned}P_b^{(L=1)} &= E\left[Q\left(\sqrt{2\tilde{\gamma}_S^{(L=1)}}\right)\right] \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} E\left[\exp\left(-\frac{\tilde{\gamma}_S^{(L=1)}}{\sin^2\theta}\right)\right] d\theta \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\theta}{\sin^2\theta + \Gamma}\right)^S \\ &\qquad \cdot \prod_{n=S+1}^{M} \left(\frac{\sin^2\theta}{\sin^2\theta + \Gamma\frac{S}{n}}\right) d\theta \end{aligned} \tag{17}$$

where $\prod_{n=x}^{y} (\cdot)$ is defined to be 1 for $x > y$. Note that this error probability takes the same form as that of the hybrid selection/MRC for Rx antenna diversity system with M Rx antennas [13]. For high Γ, the error probability in (17) can be approximated as

$$P_b^{(L=1)} \cong \frac{\prod_{n=S+1}^{M} \frac{n}{S}}{\Gamma^M} \int_0^{\frac{\pi}{2}} \frac{\left(\sin^2\theta\right)^M}{\pi} d\theta. \tag{18}$$

Since we assume that $L = 1$, there are M independent fading signals and the maximum achievable diversity order is M. Note that the BEP in (18) decreases inversely with the Mth power of TxSNR Γ, which implies that the basis selection offers full diversity order of M.

Fig. 2 shows the average BEP of Tx diversity with basis selection as a function of TxSNR Γ for various S, when $M = 8$ and $L = 1$. The average BEPs of Tx diversity with optimum weights ($S = 8$) and the Tx antenna selection ($S = 1$) are also plotted for reference. From this figure, it can be seen that as S increases, the performance of Tx diversity with basis selection improves and approaches that of optimum weights. Compared with the Tx antenna selection at a 10^{-6} BEP, the basis selection provides 2.5-dB gain when $S = 2$, 3.8-dB gain when $S = 3$, and 4.6-dB gain when $S = 4$. Note that the performance improvement diminishes with the increase in S. Note also that, at high Γ, the slope of BEP curve becomes the same for all S. This verifies that the Tx diversity with basis selection does not reduce the diversity order. Fig. 3 compares the average BEP of Tx diversity with basis selection and that of optimum weights for various M, when $L = 1$ and $S = M/2$. When the degree of approximation S is half the number of Tx antennas, the basis selection does not degrade the performance more than 1 dB.

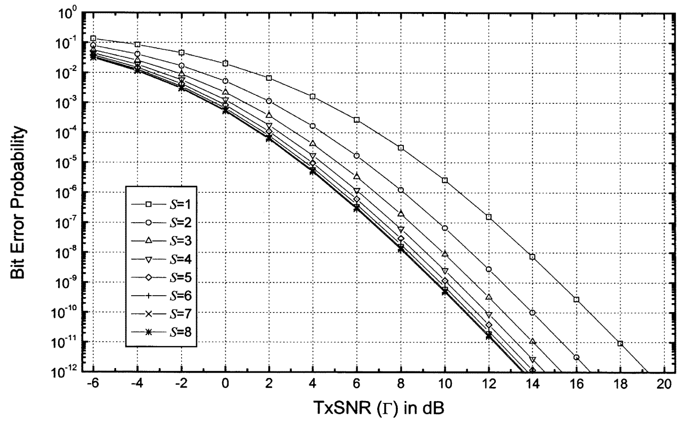

Fig. 2. Average BEP performance for various S as a function of TxSNR Γ, when $M = 8$ and $L = 1$.

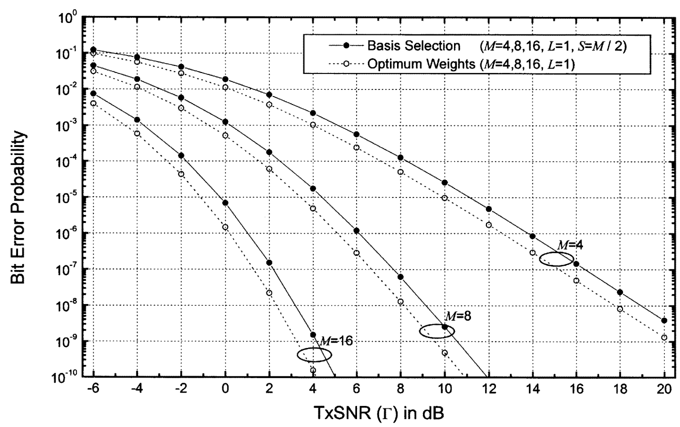

Fig. 3. Average BEP performance for various M as a function of TxSNR Γ, when $L = 1$ and $S = M/2$.

B. When the Number of Rx Antennas is General (General L)

For the general L case, we may easily obtain P_b in (13) through Monte Carlo integration [14]. We obtain P_b based on 10^6 independent realizations of the channel matrix $\boldsymbol{H}$. Fig. 4 shows the average BEP P_b of Tx diversity with basis selection for various L when $M = 4$. Like Fig. 2 for $L = 1$ case, this figure shows that the performance of basis selection improves and approaches that of optimum weights as S increases. This figure also shows that, at high Γ, the slopes of BEP become the same for all S when L is fixed. As L increases, the slope of BEP curves becomes steeper due to the increase in diversity order, and the difference between the performance of Tx diversity with basis selection and that of optimum weights decreases for the same S. For example, at a 10^{-6} BEP, the difference between the performance of optimum weight and that of basis selection with $S = 1$ is about 3 dB when $L = 2$, but decreases to 1.9 dB when $L = 8$. To investigate this performance difference,

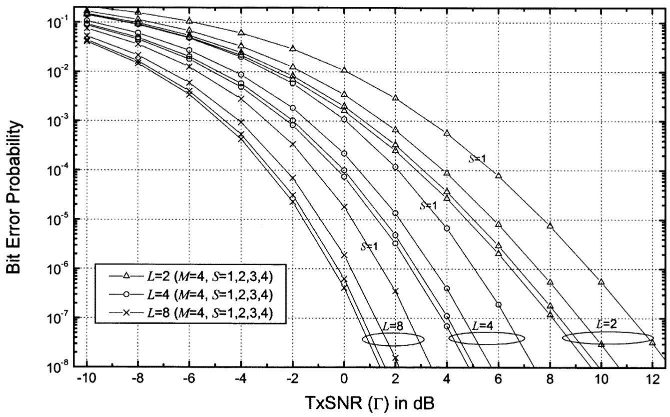

Fig. 4. Average BEP performance for various L as a function of TxSNR Γ, when $M = 4$.

we consider the asymptotic behavior of basis selection when L approaches infinity. Since the eigenvectors of $\boldsymbol{R}$ are the same as those of the normalized matrix $\boldsymbol{R}' = (1/L)\boldsymbol{R}$, the matrix $\boldsymbol{R}'$, instead of $\boldsymbol{R}$, may be used to investigate the asymptotic behavior of basis selection. The elements of the matrix $\boldsymbol{R}'$ may be expressed as

$$r'_{i,j} = \frac{1}{L}\sum_{n=1}^{L} h^*_{n,i} \cdot h_{n,j}, \quad i,j = 1,2,\ldots,M \tag{19}$$

where $r'_{i,j}$ denotes the element of $\boldsymbol{R}'$ at the ith row and the jth column. Note that the diagonal elements $r'_{i,i} = (1/L)\sum_{n=1}^{L}|h_{n,i}|^2$ are the normalized coherent sum of the channel responses, and the off-diagonal elements $r'_{i,j} = (1/L)\sum_{n=1}^{L} h^*_{n,i} \cdot h_{n,j}$, $i \neq j$, are the normalized incoherent sum. Thus, as L approaches infinity, the off-diagonal elements become negligible compared with the diagonal elements, and may be approximated as 0

$$\begin{aligned}\lim_{L\to\infty} r'_{i,j} &= \lim_{L\to\infty}\left[\frac{1}{L}\sum_{n=1}^{L} h^*_{n,i} \cdot h_{n,j}\right] \\ &\cong E\left[h^*_{n,i} \cdot h_{n,j}\right] \\ &= 0\end{aligned} \tag{20}$$

where the approximation is made with the law of large numbers [15]. Consequently, for large L, the matrix $\boldsymbol{R}'$ may be approximated as a diagonal matrix of

$$\boldsymbol{R}' \cong \mathrm{diag}\left(\left[\frac{1}{L}\sum_{i=1}^{L}|h_{i,1}|^2, \frac{1}{L}\sum_{i=1}^{L}|h_{i,2}|^2, \ldots, \frac{1}{L}\sum_{i=1}^{L}|h_{i,M}|^2\right]\right). \tag{21}$$

Since the eigenvalues of a diagonal matrix are the diagonal elements, the maximum eigenvalue of $\boldsymbol{R}'$ may be approximated as

$$\lambda_{\max}(\boldsymbol{R}') \cong \max_m \left[\frac{1}{L}\sum_{i=1}^{L}|h_{i,m}|^2\right]. \tag{22}$$

Moreover, since the eigenvectors of a diagonal matrix are the column vectors of $\boldsymbol{I}_M$, the eigenvector associated with $\lambda_{\max}(\boldsymbol{R}')$ is the $m_{\max}$th column vector of $\boldsymbol{I}_M$, where $m_{\max}$ is the index m which maximizes (22). As mentioned previously, the eigenvectors of $\boldsymbol{R}'$ are the same as those of $\boldsymbol{R}$. Thus, the eigenvector associated with the maximum eigenvalue of $\boldsymbol{R}$, that is, the optimum weight vector $\boldsymbol{w}_{\mathrm{opt}}$ is also the $m_{\max}$th column vector of $\boldsymbol{I}_M$. Note that this optimum weight vector consists of $M - 1$ elements with 0 and one element with 1. Hence, from (10), the Tx weight vector by basis selection equals the optimum weight vector, regardless of S : $\tilde{\boldsymbol{w}}_S = \boldsymbol{w}_{\mathrm{opt}}$. Consequently, as L approaches infinity, the difference between the performance of optimum weight and that of basis selection decreases and becomes zero.

V. Effects of Weight Digitization and Feedback Delay

In this section, we investigate the effects of weight digitization and feedback delay on the performance of Tx diversity with basis selection.

A. Effects of Weight Digitization

Before transmitting feedback information to the transmitter, the weight vector should be digitized. If we assume that Q bits are required for digitizing each element of $\boldsymbol{w}_{\mathrm{opt}}$, then the total number of bits for $\boldsymbol{w}_{\mathrm{opt}}$ is $(M - 1) \cdot Q$ bits. The reason for $(M-1)\cdot Q$, not $M \cdot Q$, is that one of M Tx antennas may be viewed as a reference antenna, and the relative weights for other antennas are needed. Currently, two different values of Q are used for UMTS closed-loop Tx diversity with two Tx antennas. In mode 1, Q is one bit, and two consecutive bits are used for Tx weight. In mode 2, Q is four bits: one bit for gain and three bits for phase. A detailed description of these two modes can be found in [4] and [16].

If we use $\tilde{\boldsymbol{w}}_S$, instead of $\boldsymbol{w}_{\mathrm{opt}}$, for Tx weight vector, the selected basis vectors should be specified. When the approximation is made in a S-dimensional subspace, there are $\binom{M}{S}$ combinations in selecting S basis vectors out of M vectors, and the required number of bits to specify the basis vector combination is $\lceil \log_2 \binom{M}{S} \rceil$, where $\lceil x \rceil$ is the smallest integer greater than or equal to x. The required number of bits for digitizing the S coefficients $c_{(i)}$'s may be expressed as $(S-1)\cdot Q$. Thus, the total required number of bits for $\tilde{\boldsymbol{w}}_S$ may be expressed as

$$Q_{\mathrm{total}} = \left\lceil \log_2 \binom{M}{S} \right\rceil + (S-1)\cdot Q. \tag{23}$$

Table I shows the total required number of bits for digitizing $\boldsymbol{w}_{\mathrm{opt}}$ and $\tilde{\boldsymbol{w}}_S$, when $Q = 4$ bits. Compared to the optimum weight vector $\boldsymbol{w}_{\mathrm{opt}}$, the use of a weight vector by basis selection $\tilde{\boldsymbol{w}}_S$ offers 42% reduction in the total required number of bits, when $M = 4$ and $S = 2$; and 68% reduction when $M = 8$ and $S = 2$. Note that this reduction amount increases as the number of Tx antennas M increases.

To digitize the ordered coefficients $c_{(i)}$'s, we first calculate the relative coefficients. If we set one of $c_{(i)}$'s to the reference coefficient, then additional feedback information about the index of the reference coefficient is needed, since the feedback information about the selected basis vector does not contain the order information. To avoid this, we reorder $c_{(i)}$'s according to their indexes: $c_{[1]}\, c_{[2]} \ldots c_{[S]}$, where the indexes $[i]$'s are the ascent-ordered indexes of (i)'s. For example, if (i)'s are $(1) = 2$, $(2) = 1$, $(3) = 7$, and $(4) = 4$, then $[i]$'s are given as $[1] = 1$,

TABLE I
TOTAL REQUIRED NUMBER OF BITS FOR DIGITIZING $\boldsymbol{w}_{\text{opt}}$ AND $\tilde{\boldsymbol{w}}_S$ ($Q = 4$ BITS)

Number of Tx Antennas (M)	w_{opt}	S	$\tilde{w}_S$
4	12 bits	1	2 bits
		2	7 bits
		3	10 bits
		4	12 bits
8	28 bits	1	3 bits
		2	9 bits
		3	14 bits
		4	19 bits
		5	22 bits
		6	25 bits
		7	27 bits
		8	28 bits

$[2] = 2$, $[3] = 4$, and $[4] = 7$. If we set one of $c_{[i]}$'s to the reference coefficient, then there is no need to feedback the index of this reference coefficient, since the transmitter can extract this index from the selected basis vectors information. For example, if we set the coefficient with the kth index $[k]$ to the reference coefficient, then the transmitter extracts this index by choosing the kth smallest number among the S numbers indicating the selected basis vectors. For simplicity, let $c_{[1]}$ be the reference coefficient. Then the relative coefficients $\hat{c}_{[i]}$ may be expressed as $\hat{c}_{[i]} = c_{[i]}/c_{[1]} \triangleq a_{[i]} \cdot \exp(j\varphi_{[i]})$, $i = 2, \ldots, S$, where $a_{[i]}$ and $\varphi_{[i]}$ denote the relative amplitude and relative phase, respectively. Note that $a_{[1]}$ and $\varphi_{[1]}$ are not needed to be fed back since they are always 1 and 0, respectively.

When S is small, the relative amplitudes $a_{[i]}$'s vary over a small range with high probability since the coefficients $c_{[i]}$'s are associated with the S largest amplitudes. For example, when $M = 8$, $L = 1$, and $S = 2$, although $a_{[2]}$ can vary over $(0, \infty)$, the probability that $a_{[2]}$ is less than two is as high as 0.988. Unlike the relative amplitudes, the relative phases $\varphi_{[i]}$'s vary over $[0, 2\pi)$ with equal probability. Consequently, in this letter, we consider a phase-only digitization scheme, with which the relative amplitudes $a_{[i]}$'s are assumed to be the same and not digitized, and only the relative phases $\varphi_{[i]}$'s are digitized. A candidate set Θ_Q for a digitized relative phase may be written as $\Theta_Q = \{2\pi q/2^Q\}_{q=0}^{2^Q-1}$, where Q denotes the number of bits used to digitize one coefficient. A digitized relative phase for $\varphi_{[i]}$ is determined by choosing an element of Θ_Q nearest to $\varphi_{[i]}$. Let the digitized relative phase for $\varphi_{[i]}$ be $\hat{\varphi}_{[i]}$. Then the digitized Tx weight vector $\tilde{\boldsymbol{w}}_{S,Q}$, which is referred to as *the phase-only digitized (POD) basis selection*, may be expressed as

$$\tilde{\boldsymbol{w}}_{S,Q} = \frac{1}{\sqrt{S}} \left(\boldsymbol{b}_{[1]} + \sum_{i=2}^{S} \exp(j\hat{\varphi}_{[i]}) \boldsymbol{b}_{[i]} \right) \tag{24}$$

where $\boldsymbol{b}_{[i]}$ is the basis vector corresponding to $c_{[i]}$. Note that when $S = M$, $\tilde{\boldsymbol{w}}_{S,Q}$ becomes a POD optimum weight vector.

A bit representation for the digitized relative phase may be obtained using a predetermined mapping table, in which 2^Q elements in Θ_Q are uniquely mapped onto 2^Q combinations of Q bits. Similar mapping tables may be applied to obtain a bit representation for the selected basis vectors. Along with the bits for the selected basis vectors, the bits for the digitized relative phases are fed back to the transmitter. Using these bits and two mapping tables, the transmitter generates the digitized Tx weight vector $\tilde{\boldsymbol{w}}_{S,Q}$, which is the same as that calculated at the receiver. To investigate the effects of weight digitization alone, let the feedback delay be zero. Then the RxSNR in (11) may be modified to

$$\tilde{\gamma}_{S,Q} = \Gamma \cdot \tilde{\boldsymbol{w}}_{S,Q}^H \boldsymbol{H}^H \boldsymbol{H} \tilde{\boldsymbol{w}}_{S,Q}. \tag{25}$$

Substituting $\tilde{\gamma}_{S,Q}$ for $\tilde{\gamma}_S$ in (13) and using Monte Carlo integration, we can easily obtain the average BEP. Fig. 5 shows the average BEP of Tx diversity with the POD basis selection $(S = 2)$ when $M = 8$ and $L = 1$. For comparison, the average BEP of Tx diversity with the POD optimum weights $(S = 8)$ is also plotted. This figure shows that the performance of Tx diversity with the POD basis selection $(S = 2)$ improves as Q increases, and the performance with $Q = 4$ bits is almost the same with that without weight digitization. This indicates that the POD is sufficient for coefficient digitization of basis selection with small S. For the POD optimum weights $(S = 8)$, however, even the performance with $Q = 8$ bits does not approach that without

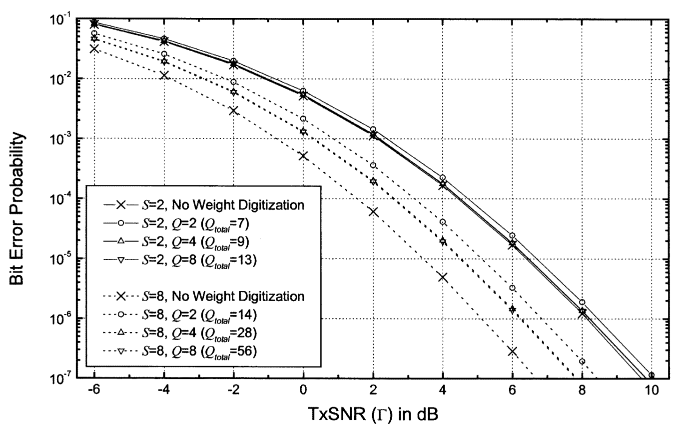

Fig. 5. Average BEP performance for various Q as a function of TxSNR Γ, when $M = 8$ and $L = 1$.

weight digitization. Thus, an amplitude digitization as well as a phase digitization is needed for digitizing the optimum weights.

B. Effects of Feedback Delay

In this subsection, we investigate the effects of feedback delay on the performance of Tx diversity with the POD basis selection described in Section V-A. In real systems, a feedback channel is usually employed with the limited bit rates. For example, in the UMTS, the feedback bit rate for closed-loop Tx diversity is limited to 1500 b/s [4]. The limited feedback bit rate may induce a feedback delay, which is the required time for updating $\tilde{\boldsymbol{w}}_{S,Q}$. Let the feedback bit rate in bits per second be f_b, then the feedback delay τ may be expressed as

$$\tau = \frac{Q_{\text{total}}}{f_b} \tag{26}$$

where Q_{total} is given in (23). Note that the feedback delay τ is proportional to Q_{total}. Thus, the feedback delay of the POD basis selection is smaller than that of the POD optimum weights, since Q_{total} for basis selection, as shown in Table I, is smaller than that for optimum weights when Q is given.

When there is τ seconds feedback delay, the current Tx weight vector is a delayed one that *was* calculated at τ seconds ago. If we represent the current digitized Tx weight vector calculated at τ seconds ago as $\tilde{\boldsymbol{w}}_{S,Q,\tau}$, the RxSNR in (25) may be modified to

$$\tilde{\gamma}_{S,Q,\tau} = \Gamma \cdot \tilde{\boldsymbol{w}}_{S,Q,\tau}^H \boldsymbol{H}^H \boldsymbol{H} \tilde{\boldsymbol{w}}_{S,Q,\tau}. \tag{27}$$

Note that the current Tx weight vector $\tilde{\boldsymbol{w}}_{S,Q,\tau}$ *was* calculated based on the previous channel matrix $\boldsymbol{H}_\tau$, which denotes the channel matrix at τ seconds ago. Using a first-order Markov model, $\boldsymbol{H}_\tau$ may be expressed as [10]

$$\boldsymbol{H}_\tau = \rho \cdot \boldsymbol{H} + \sqrt{1-\rho^2} \cdot \boldsymbol{H}_\varepsilon \tag{28}$$

where ρ is the correlation coefficient between the elements of the current channel $\boldsymbol{H}$ and those of the previous channel $\boldsymbol{H}_\tau$, and may be expressed as $\rho = J_0(2\pi f_D \tau)$, where $J_0(x)$ is the zeroth-order Bessel function and f_D is the Doppler frequency in Hertz [10]. The matrix $\boldsymbol{H}_\varepsilon$ denotes the uncorrelated components between $\boldsymbol{H}$ and $\boldsymbol{H}_\tau$, and consists of i.i.d. circular complex Gaussian random variables of zero mean and unit variance.

The average BEP may be obtained using Monte Carlo integration with respect to $\boldsymbol{H}$ and $\boldsymbol{H}_\varepsilon$. Fig. 6 shows the average BEP of Tx diversity with the POD basis selection $(S = 2)$ as a function of Doppler frequency when $M = 8$, $L = 1$, $\Gamma = 5$ dB, and $f_b = 1500$ b/s. For comparison, the average BEP of Tx diversity with the POD optimum weights $(S = 8)$ is also plotted. Due to the effects of feedback delay, the performance of Tx diversity with the POD basis selection $(S = 2)$ gradually degrades as Doppler frequency increases. For low Doppler frequencies (<1 Hz), the effect of feedback delay is negligible, since the channel varies very slowly. Thus, the Tx diversity with the POD optimum weights outperforms that with the POD basis selection. As Doppler frequency increases, however, the performance of Tx diversity with the POD basis selection becomes better than that with the POD optimum weights. For example, when $Q = 4$ bits, the Tx diversity with the POD basis selection outperforms that with the POD optimum weights when the Doppler frequency is higher than 5 Hz. This is because the feedback delay of the POD basis selection is smaller than that of the POD optimum weights for a given Q.

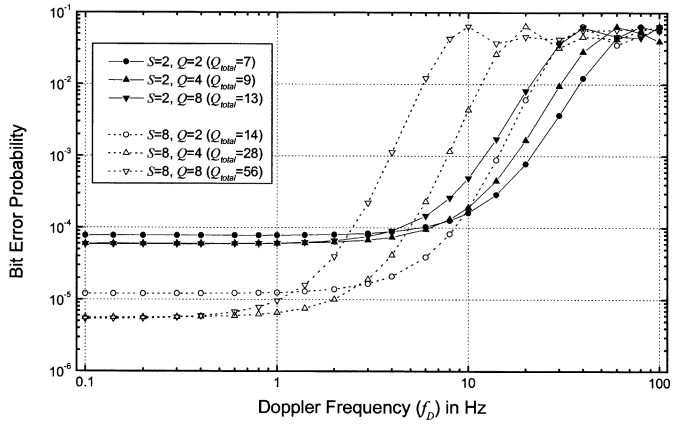

Fig. 6. Average BEP performance as a function of Doppler frequency f_D, when $M = 8$, $L = 1$, $\Gamma = 5$ dB, and $f_b = 1500$ b/s.

VI. Conclusions

In this letter, we have presented an efficient weight vector representation which is appropriate for closed-loop Tx diversity with numerous Tx antennas, and investigated its performance. This representation, called basis selection, significantly reduces the amount of feedback information with small performance degradation. Performance analysis showed that when the degree of approximation is half the number of Tx antennas, the performance degradation by basis selection is less than 1 dB. It was found that the Tx diversity with basis selection offers a full diversity order regardless of the degree of approximation. It was also found that as the number of receive antennas increases, the performance degradation due to basis selection decreases and becomes zero. If four bits are used to represent each coefficient, the basis selection offers 42% reduction in the total required number of bits when the number of Tx antennas $M = 4$ and the degree of approximation $S = 2$; and 68% reduction when $M = 8$ and $S = 2$. It was found that a POD is sufficient for the coefficient digitization of basis selection with small S. Due to the effect of feedback delay, the performance of Tx diversity with the POD basis selection gradually degrades as Doppler frequency increases. Nevertheless, due to the reduction in the total required number of bits, the Tx diversity with the POD basis selection outperforms that with the POD optimum weights at high Doppler frequencies.

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