Optimum Pilot Pattern for MMSE Channel Estimation in Single-Carrier MIMO Systems

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Abstract - **The minimum mean squared error (MMSE) channel estimator (CE) can provide receiver performance better than the least square (LS) CE. However, the MMSE CE usually uses a pilot pattern optimally designed for the LS CE. In this paper, we derive an optimum pilot pattern for the MMSE CE in single-carrier MIMO systems assuming that both the transmitter and receiver know the average channel information, such as the channel correlation matrix and signal to interference and noise power ratio. Analytic and simulation results show that the MMSE CE with the use of the proposed pilot pattern can reduce the MSE compared to the use of one optimized for the LS CE.**

I. INTRODUCTION

In practice, the channel impulse response (CIR) is estimated using known pilot symbols. There have been proposed a number of channel estimator (CE) schemes such as the least square (LS) and minimum mean squared error (MMSE) estimators [1]. The MMSE CE can estimate the CIR better than the LS CE if the receiver knows the channel characteristics and signal to interference and noise power ratio (SINR) [1].

To improve the accuracy of the CIR estimation, it is desirable to use an appropriate pilot pattern particularly in multiple transmit antenna systems [2,3]. The optimum pilot pattern for multiple transmit antenna system was derived for the LS CE by minimizing the MSE of the estimated CIR, where the pilot sequence of each transmit antenna should satisfy ideal auto-correlation and zero cross-correlation properties [4,5,6]. The use of a simple binary sequence such as Walsh code can satisfy this condition in flat fading channel [4], while the use of complex-valued poly-phase sequences is required in frequency selective fading channel [5,6].

The optimum pilot pattern for the MMSE CE, however, has not yet been reported to the best of authors' knowledge. In this paper, we design an optimum pilot pattern for the MMSE CE assuming that the statistical characteristics of the channel and the SINR of the received signal are known to both the transmitter and the receiver. This assumption can be valid by using the channel reciprocal property in time division duplex (TDD) systems. In a frequency division duplex (FDD) system, the required channel information can be obtained using a feedback loop without requiring a large signaling overhead.

Section Ⅱ describes the model of a single-carrier (SC) multiple-input multiple-output (MIMO) system. The optimum pilot pattern is derived for the MMSE CE in Section Ⅲ. The performance of the proposed scheme is evaluated in Section Ⅳ. Finally, conclusions are summarized in Section V.

II. SYSTEM MODEL

Suppose an SC MIMO system comprising *M* transmit and *N* receive antennas. Let $\boldsymbol{s}_m$ be the pilot symbol pattern of the *m*-th transmit antenna represented as

$$\boldsymbol{s}_m = [s_m(0)\ s_m(1) \cdots s_m(K-1)] \tag{1}$$

where $s_m(k)$ denotes the *k*-th pilot symbol of the *m*-th antenna and K is the code length. Denote $\boldsymbol{h}_{m,n}$ by the CIR between the *m*-th transmit antenna and the *n*-th receive antenna represented as

$$\boldsymbol{h}_{m,n} = [h_{m,n}(0)\ h_{m,n}(1) \cdots h_{m,n}(L-1)]^T \tag{2}$$

where *L* is the number of multipaths and the superscript *T* denotes the transpose.

The received symbol at the *n*-th receive antenna can be represented as

$$\boldsymbol{r}_n = \boldsymbol{S}\boldsymbol{H}_n + \boldsymbol{z}_n \tag{3}$$

where $\boldsymbol{S} = [\boldsymbol{S}_1\, \boldsymbol{S}_2 \cdots \boldsymbol{S}_M]$, $\boldsymbol{H}_n = [\boldsymbol{h}_{1,n}\ \ \boldsymbol{h}_{2,n}\ \ \cdots \boldsymbol{h}_{M,n}]^T$ and $\boldsymbol{z}_n$ is the background noise and interference approximated as additive white Gaussian noise (AWGN) with zero mean and $E\{\boldsymbol{z}_n \boldsymbol{z}_n^H\} = \sigma_z^2 I_{K-L+1}$. Here, $E\{x\}$ denotes the expectation of *x*, I_{K-L+1} denotes the identity matrix of size $(K-L+1)$ and $\boldsymbol{S}_m$

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is represented as

$$\boldsymbol{S}_m = \begin{bmatrix} s_m(L-1) & s_m(L-2) & \cdots & s_m(0) \\ s_m(L) & s_m(L-1) & \cdots & s_m(1) \\ \vdots & \vdots & \ddots & \vdots \\ s_m(K-1) & s_m(K-2) & \cdots & s_m(K-L) \end{bmatrix}. \quad (4)$$

III. OPTIMUM PILOT PATTERN FOR MMSE CE

Since the channel estimation is independently performed at each receiver antenna, we can omit the index n for simplicity of description. We assume that the correlation statistics of the channel and the SINR of the pilot symbol are known. Then, the CIR can be estimated using the MMSE method [1]

$$\hat{\boldsymbol{H}}_{MMSE} = (\boldsymbol{R}_{\boldsymbol{h}}^{-1} + \boldsymbol{S}^H \boldsymbol{S}/\sigma_z^2)^{-1} \boldsymbol{S}^H \boldsymbol{r}/\sigma_z^2 \quad (5)$$

where $\boldsymbol{R}_{\boldsymbol{h}} (= E\{\boldsymbol{H}\boldsymbol{H}^H\})$ is the correlation matrix of the CIR and $\boldsymbol{A}^H$ denotes the Hermitian of matrix $\boldsymbol{A}$. Here, we consider $\boldsymbol{R}_{\boldsymbol{h}}$ as a full rank matrix assuming that all multipaths have non-zero average power.

The CIR can be estimated using an LS method [1]

$$\hat{\boldsymbol{H}}_{LS} = (\boldsymbol{S}^H \boldsymbol{S})^{-1} \boldsymbol{S}^H \boldsymbol{r}. \quad (6)$$

Note that the MMSE CE becomes the LS CE if the SINR is high or no information on the channel is given, i.e., $\boldsymbol{R}_{\boldsymbol{h}}^{-1} = \boldsymbol{O}$, where $\boldsymbol{O}$ denotes the null matrix.

Let ε_{MMSE} and ε_{LS} be the MSE of the MMSE and LS CE, respectively. Then, it can be shown that

$$\begin{aligned} \varepsilon_{MMSE} &= \frac{1}{LM} E\left\{(\boldsymbol{H}_{MMSE} - \hat{\boldsymbol{H}}_{MMSE})^H (\boldsymbol{H}_{MMSE} - \hat{\boldsymbol{H}}_{MMSE})\right\} \\ &= tr\left[(\boldsymbol{R}_{\boldsymbol{h}}^{-1} + \boldsymbol{S}^H \boldsymbol{S}/\sigma_z^2)^{-1}\right]/LM \end{aligned} \quad (7)$$

and

$$\begin{aligned} \varepsilon_{LS} &= tr[(\boldsymbol{S}^H \boldsymbol{S}/\sigma_z^2)^{-1}]/LM \\ &\geq \sigma_z^2/(K-L+1) \end{aligned} \quad (8)$$

where $tr[\boldsymbol{A}]$ denotes the trace of matrix $\boldsymbol{A}$.

The optimum pilot pattern for the LS CE can be obtained by minimizing ε_{LS}. The minimum ε_{LS} is achieved when $\boldsymbol{S}^H \boldsymbol{S} = cI_{LM}$, where c is a constant [5]. Under a constraint on the total transmit power,

$$tr[\boldsymbol{S}^H \boldsymbol{S}] \leq P, \quad (9)$$

the minimum ε_{LS} can be obtained by

$$\begin{aligned} \varepsilon_{LS} &= \sigma_z^2/c \\ &\geq (\sigma_z^2 LM)/P. \end{aligned} \quad (10)$$

Similarly, the optimum pilot pattern for the MMSE CE can be obtained as follows. The CIR correlation matrix $\boldsymbol{R}_{\boldsymbol{h}}$ can be represented using the singular value decomposition (SVD) as [7]

$$\boldsymbol{R}_{\boldsymbol{h}} = \boldsymbol{Q}\Lambda_{\boldsymbol{h}}\boldsymbol{Q}^H \quad (11)$$

where $\boldsymbol{Q}$ is a unitary matrix (i.e., $\boldsymbol{Q}\boldsymbol{Q}^H = \boldsymbol{I}$) and $\Lambda_{\boldsymbol{h}}$ is a diagonal matrix with diagonal elements $\lambda_{\boldsymbol{h},1}, \lambda_{\boldsymbol{h},2}, \cdots, \lambda_{\boldsymbol{h},LM}$. Letting $\boldsymbol{U} = (\boldsymbol{R}_{\boldsymbol{h}}^{-1} + \boldsymbol{S}^H \boldsymbol{S}/\sigma_z^2)$, we have $\boldsymbol{U}^H = \boldsymbol{U}$. Thus, $\boldsymbol{U}$ can be represented as [7]

$$\boldsymbol{U} = \boldsymbol{P}\Lambda_U \boldsymbol{P}^H \quad (12)$$

where $\boldsymbol{P}$ is a unitary matrix and Λ_U is a diagonal matrix with diagonal elements $\lambda_{U,1}, \lambda_{U,2}, \cdots, \lambda_{U,LM}$.

Let $\Lambda_{\boldsymbol{t}} = \Lambda_U - \Lambda_{\boldsymbol{h}}^{-1}$, where the diagonal elements are $\lambda_{t,1}, \lambda_{t,2}, \cdots, \lambda_{t,LM}$. Then, it can be seen that

$$\begin{aligned} tr\left[\boldsymbol{S}^H \boldsymbol{S}\right] &= \sigma_z^2 tr\left[\boldsymbol{U} - \boldsymbol{R}_{\boldsymbol{h}}^{-1}\right] \\ &= \sigma_z^2 tr\left[\boldsymbol{P}\Lambda_U \boldsymbol{P}^H - \boldsymbol{Q}\Lambda_{\boldsymbol{h}}^{-1}\boldsymbol{Q}^H\right] \\ &= \sigma_z^2 tr\left[\Lambda_U - \Lambda_{\boldsymbol{h}}^{-1}\right] \\ &= \sigma_z^2 \sum_{i=1}^{LM} \lambda_{t,i} \leq P \end{aligned} \quad (13)$$

using the property $tr[\boldsymbol{A}\text{-}\boldsymbol{B}] = tr[\boldsymbol{A}]\text{-}\ tr[\boldsymbol{B}]$ and $tr[\boldsymbol{A}\boldsymbol{B}] = tr[\boldsymbol{B}\boldsymbol{A}]$. The MSE of the MMSE scheme ε_{MMSE} can be rewritten as

$$\begin{aligned} \varepsilon_{MMSE} &= tr\left[\Lambda_U^{-1}\right]/LM \\ &= tr\left[(\Lambda_{\boldsymbol{t}} + \Lambda_{\boldsymbol{h}}^{-1})^{-1}\right]/LM \\ &= \frac{1}{LM}\sum_{i=1}^{LM}(\lambda_{t,i} + \lambda_{\boldsymbol{h},i}^{-1})^{-1}. \end{aligned} \quad (14)$$

Using the Kuhn-Tucker condition [8], we can find $\lambda_{t,i}$ minimizing ε_{MMSE} by

$$\lambda_{t,i} = (\nu - \lambda_{\boldsymbol{h},i}^{-1})^+ \quad (15)$$

where $x^+ \equiv \max(x, 0)$ and

$$\sum_{i=1}^{LM} \lambda_{t,i} = P / \sigma_z^2 \,. \qquad (16)$$

Here, ν is determined so as to satisfy (16). Note that (15) is similar to the 'water-pouring' solution [9]. This implies that the MMSE CE scheme can improve the CIR estimation accuracy by adjusting the pilot pattern based on the channel correlation and SINR of the pilot symbol. As an example, the described optimum pilot pattern can easily be generated by $\boldsymbol{S} = \sigma_t \boldsymbol{G} \Lambda_t^{1/2} \boldsymbol{Q}^H$, where σ_t is a constant and $\boldsymbol{G}$ is a unitary matrix which enables $\boldsymbol{S}$ to be represented as a form of (1).

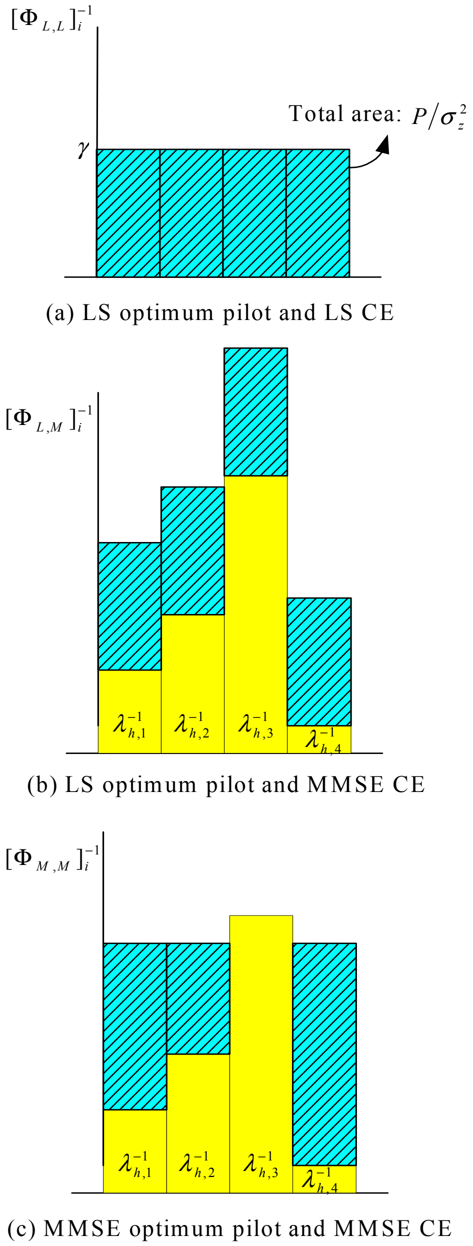

Fig. 1. Inverse of the elements in the MSE matrix

Let $\sigma_s^2 (= P/LM)$ be the average power of the transmitted pilot symbol. Then, the average SINR of each pilot symbol can be represented as

$$\gamma = \sigma_s^2 / \sigma_z^2 \,. \qquad (17)$$

Let $\Phi_{L,L}$ be the MSE matrix of the LS CE with the optimum pilot pattern for the LS CE [5]. Similarly, $\Phi_{L,M}$ and $\Phi_{M,M}$ be the MSE matrix of the MMSE CE with the use of a pilot pattern optimized for the LS and MMSE CE, respectively. Then, the *i*-th diagonal element of these MSE matrices can be represented as

$$\begin{aligned} &[\Phi_{L,L}]_i = \gamma^{-1} \\ &[\Phi_{L,M}]_i = (\gamma + \lambda_{h,i}^{-1})^{-1} \\ &[\Phi_{M,M}]_i = \begin{cases} \nu^{-1} \,, & \lambda_{t,i} > 0 \\ \lambda_{h,i} \,, & \lambda_{t,i} = 0 \end{cases} . \end{aligned} \qquad (18)$$

When $\lambda_{t,i} > 0$ for all i=1, 2,…, LM, it can be shown that

$$[\Phi_{M,M}]_i = (\gamma + \bar{\lambda}_h)^{-1} \qquad (19)$$

where $\bar{\lambda}_h = (LM)^{-1} \sum_{i=1}^{LM} \lambda_{h,i}^{-1}$.

As an example, Fig. 1 depicts $[\Phi]_i^{-1}$ of the MSE matrix Φ when LM = 4. Since $[\Phi_{L,M}]_i^{-1}$ is larger than $[\Phi_{L,L}]_i^{-1}$ by $\lambda_{h,i}^{-1}$ for all i, $1 \le i \le LM$, it can be seen that $\sum_{i=1}^{LM} [\Phi_{L,M}]_i < \sum_{i=1}^{LM} [\Phi_{L,L}]_i$. The MSE difference between the two CE schemes decreases as γ increases since the effect of $\lambda_{h,i}^{-1}$ decreases compared to that of γ. Note that, although the total area of (b) and (c) in Fig. 1 is the same (i.e., $\sum_{i=1}^{LM} [\Phi_{L,M}]_i^{-1} = \sum_{i=1}^{LM} [\Phi_{M,M}]_i^{-1}$), the variance of $[\Phi_{M,M}]_i^{-1}$ is smaller than that of $[\Phi_{L,M}]_i^{-1}$. Thus, the MMSE scheme with the use of a pilot pattern optimized for the MMSE CE can estimate the CIR better than the use of one optimized for the LS CE. The MSE difference between the use of two pilot patterns increases as γ decreases and/or the eigen-value spread of the channel increases.

IV. PERFORMANCE EVALUATION

To verify the design, the MSE of the MMSE CE is evaluated with the use of the proposed pilot pattern. For performance comparison, we assume that M=2, L=2 and no correlation between the CIRs with different delay (i.e., $E\{h_{m_1,n}^*(l_1) h_{m_2,n}(l_2)\} = 0$ for $m_1, m_2 = 1, 2$, $n = 1, 2, \cdots, N$ and $l_1 \ne l_2$, $l_1, l_2 = 0, 1$). We also assume that $E\{|h_{m,n}(0)|^2\} = \alpha$, $E\{|h_{m,n}(1)|^2\} = 1 - \alpha$ for

$m=1,2$ and $n=1, \quad 2,\cdots,N$, and that there is some correlation between the CIRs of each transmit antenna with the same delay (i.e., $E\{h^*_{m_1,n}(l)h_{m_2,n}(l)\} = \beta E\{|h_{m,n}(l)|^2\}$ for $m_1 \neq m_2$, $m_1, m_2 = 1,2$, $l=0,1$ and $n=1,2,\cdots,N$), where α and β are given constants such that $0 \le \alpha, \beta \le 1$. We consider two simple channel models: one is the channel with high correlation ($\beta = 0.8$) between the transmitter antennas, implying a case of Ricean fading or outdoor environment. The other is the channel with low correlation ($\beta = 0.2$), implying a case of Rayleigh fading or indoor environment.

Fig. 2 depicts the MSE of the proposed pattern as a function of the average SINR γ when $\alpha = 0.8$, where $\varepsilon_{L,L}$, $\varepsilon_{L,M}$ and $\varepsilon_{M,M}$ are the MSE of the MSE matrix $\Phi_{L,L}$, $\Phi_{L,M}$ and $\Phi_{M,M}$, respectively. As γ decreases, the use of the MMSE optimum pilot pattern provides a large performance improvement over the use of the LS one. Note that performance gain with the proposed scheme increases as β increases since the eigen-value spread of the CIR correlation matrix becomes larger.

V. CONCLUSIONS

In this paper, we design an optimum MIMO pilot pattern that minimizes the MSE of the estimated CIR in an SC MIMO system when a linear MMSE estimation method is employed for channel estimation. It can be seen that the optimum pilot pattern for the MMSE CE is determined in terms of the correlation matrix of the CIR and the SINR of the pilot symbol. The proposed pilot pattern provides performance better than the conventional one, i.e., pilot pattern optimally designed for the LS CE. The performance improvement increases as the SINR of the pilot symbol decreases and/or the eigen-value spread of the channel increases.

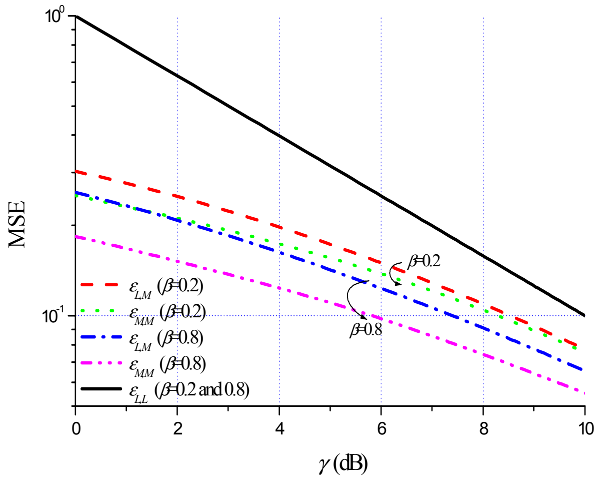

Fig. 2. MSE of the proposed pilot pattern

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