Transmit Beamforming with Reduced Channel Information in OFDM Based Wireless Systems

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Abstract: In this paper, we consider transmit beamforming that works with reduced channel state information in OFDM based multiple-input single-output (MISO) wireless systems. The proposed scheme significantly reduces the amount of feedback signaling burden by generating beamforming weights using information on the previous beamforming weights and channel correlation. The feedback signaling overhead is further reduced with the use of clustering and interpolation techniques. Simulation results show that the proposed scheme outperforms conventional beamforming techniques, while using the same amount of feedback signaling overhead.

Keywords: Transmit beamforming, feedback, multiple antennas, OFDM

Ⅰ. INTRODUCTION

Multiple-input multiple-output (MIMO) systems can considerably improve the performance of wireless communication systems. They can easily be applied to orthogonal frequency division multiplexing (OFDM) schemes. The performance improvement due to the use of multiple antennas is deeply associated with the accuracy of channel state information (CSI). When the CSI is available at the transmitter, the system performance can significantly be improved by employing a transmit beamforming technique that can provide a diversity gain as well as array gain [1]–[3]. Optimum beamforming techniques, however, require accurate CSI at the transmitter, which may cause heavy feedback signaling overhead.

To employ beamforming techniques in MIMO-OFDM systems, the transmitter may require the CSI of each subcarrier, increasing the amount of feedback signaling overhead in linear proportion to the number of subcarriers. This problem can be elevated by the use of codebook based quantization methods, where the receiver determines the optimum beamforming vector from a finite set of vectors, called codebook, and sends the index of the corresponding vector to the transmitter using a feedback channel in the uplink [4]–[6]. The overhead can further be reduced by exploiting the frequency correlation [7], [8]. The size of the codebook can be reduced by recursively encoding the codeword with the use of previous ones [7]. The amount of feedback signaling information can further be reduced by employing a so-called clustering technique that combines adjacent subcarriers into a cluster and uses a same beamforming vector for these subcarriers [8]. Performance degradation due to the use of a large cluster size can be alleviated by interpolating the beamforming vectors in the frequency domain [8]. However, it may require additional phase information.

In this paper, we consider the reduction of the feedback signaling burden for the transmit beamforming by exploiting the channel correlation in the frequency domain. The beamforming vectors can be generated in a sequential manner using the correlation characteristics between the adjacent subcarriers. For instance, when the adjacent subcarriers are correlated to each other, the beamforming vector of the current subcarrier can be represented in terms of that of the adjacent subcarrier and a vector corresponding to the difference between the two adjacent subcarriers. Since the vector corresponding to the difference will have a span smaller than the original one, the resulting quantization error will be smaller than that of the original one when the same amount of feedback signaling is considered. Further reduction can be achieved by using interpolation techniques with the use of a phase-dependent codebook.

This paper is organized as follows. Section II describes the system model in consideration. The proposed feedback reduction scheme is presented in Section III and the performance is verified by computer simulation in Section IV. Finally, Section V concludes the paper.

Notation: Bold upper and lower letters denote matrices and vectors, respectively. $(\cdot)^T$ and $(\cdot)^*$ denote the transpose and the conjugate transpose, respectively. $E\{\cdot\}$ denotes the expectation operator. $U(N,m)$ denotes a set of N $(m\times 1)$ -dimensional vectors with unit norm. $\lceil x \rceil$ denotes the smallest integer larger than or equal to x.

Ⅱ. SYSTEM MODEL

Consider an OFDM based wireless system with n_t transmit antennas and a single receive antenna (i.e., multiple-input single-output (MISO) system) as illustrated in Fig. 1. We

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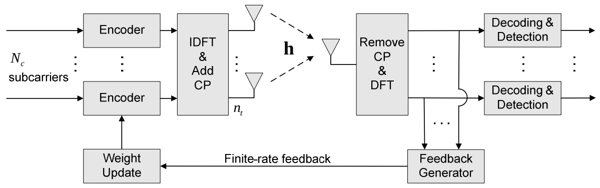

Fig. 1. Block diagram of an OFDM based MISO system with transmit beamforming.

assume that the system utilizes N_c subcarriers where the index of each subcarrier is denoted by k. When a signal $s(k)$ is transmitted using an ($n_t \times 1$)-dimensional beamforming weight vector $\mathbf{w}(k)$, the received signal can be expressed as

$$r(k) = \mathbf{h}(k)\mathbf{w}(k)s(k) + n(k), \quad k = 0, ..., N_c - 1 \tag{1}$$

where $\mathbf{h}(k)$ is the ($1 \times n_t$)-dimensional channel vector whose entities are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance, and $n(k)$ is additive white Gaussian noise (AWGN). We assume that the CSI is perfectly estimated at the receiver and sent back to the transmitter through a limited feedback channel without delay and error.

Ⅲ. PROPOSED TRANSMIT BEAMFORMING SCHEME

Assuming that the channel has correlation in the frequency domain, the channel vector of subcarrier k can be represented in terms of the channel vector of subcarrier $(k-1)$ as

$$\mathbf{h}(k) = \rho\mathbf{h}(k-1) + \sqrt{1-|\rho|^2}\,\mathbf{e} \tag{2}$$

where the correlation coefficient ρ is defined by

$$\rho = E\left\{ \frac{\mathbf{h}(k)\mathbf{h}^*(k-1)}{\sqrt{\|\mathbf{h}(k)\|^2 \|\mathbf{h}(k-1)\|^2}} \right\} \tag{3}$$

and $\mathbf{e}$ is an $(1 \times n_r)$-dimensional vector whose entries are i.i.d. zero mean complex Gaussian random variables and unit variance. Note that the entries of $\mathbf{e}$ are independent of those of $\mathbf{h}(k-1)$.

Since the optimum beamforming vector can be determined by [6]

$$\mathbf{w}(k) = \frac{\mathbf{h}^*(k)}{\|\mathbf{h}(k)\|}, \tag{4}$$

(2) can be rewritten as

$$\mathbf{w}(k) = \rho^* \frac{\|\mathbf{h}(k-1)\|}{\|\mathbf{h}(k)\|}\mathbf{w}(k-1) + \sqrt{1-|\rho|^2}\frac{\|\mathbf{e}\|}{\|\mathbf{h}(k)\|}\mathbf{z} \tag{5}$$

where $\mathbf{z} = \mathbf{e}^* / \|\mathbf{e}\|$.

Unless the number of transmit antennas is too small, it can be shown that $\|\mathbf{h}(k-1)\| / \|\mathbf{h}(k)\| \approx \|\mathbf{e}\| / \|\mathbf{h}(k)\| \approx 1$ (refer to Appendix)[1]. Then, (5) can further be simplified to

[1] Simulation results show this assumption is quite valid if the number of transmit antennas is larger than 3.

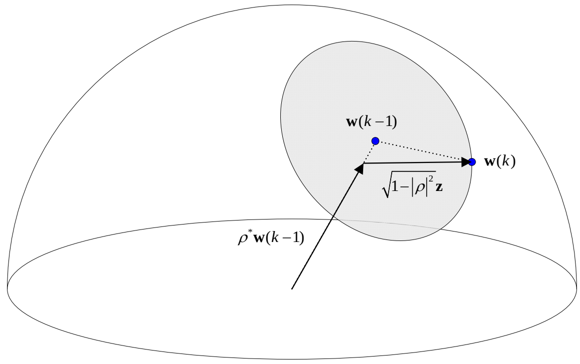

Fig. 2. Representation of $\mathbf{w}(k)$ using $\mathbf{w}(k-1)$.

$$\mathbf{w}(k) \approx \rho^*\mathbf{w}(k-1) + \sqrt{1-|\rho|^2}\,\mathbf{z}. \tag{6}$$

Note that for a given beamforming vector $\mathbf{w}(k-1)$, the beamforming vector for subcarrier k can be represented in terms of the correlation coefficient ρ and random vector $\mathbf{z}$. Since $\sqrt{1-|\rho|^2}$ is less than 1, the effect of quantization error for $\mathbf{z}$ becomes smaller, improving the accuracy with the use of same amount of feedback signaling. Moreover, since the correlation coefficient ρ is not fast varying, the amount of additional feedback signaling overthed for the correlation coefficient may be marginal. In fact, the random vector $\mathbf{z}$ indicates the direction of $\mathbf{w}(k)$ from $\mathbf{w}(k-1)$ as illustrated in Fig. 2.

The random vector $\hat{\mathbf{z}}$ can be quantized as

$$\hat{\mathbf{z}} = \arg\max_{\mathbf{z}\in \boldsymbol{W}} \left| \mathbf{h}(k) \left(\frac{\rho^*\mathbf{w}(k-1) + \sqrt{1-|\rho|^2}\,\mathbf{z}}{\left\| \rho^*\mathbf{w}(k-1) + \sqrt{1-|\rho|^2}\,\mathbf{z} \right\|} \right) \right| \tag{7}$$

where $\boldsymbol{W}$ denoted the codebook comprising N $(n_t \times 1)$-dimensional unit norm vectors. It can be seen that the beamforming vector is normalized to have unit norm. The receiver sends a $\lceil \log_2 N \rceil$-bit index of $\hat{\mathbf{z}}$ to the transmitter. After receiving the index, the transmitter recovers the beamforing vector for subcarrier k as

$$\mathbf{w}(k) = \frac{\rho^*\mathbf{w}(k-1) + \sqrt{1-|\rho|^2}\,\hat{\mathbf{z}}}{\left\| \rho^*\mathbf{w}(k-1) + \sqrt{1-|\rho|^2}\,\hat{\mathbf{z}} \right\|}. \tag{8}$$

The codebook can be generated as [6], [9]

$$\boldsymbol{W} = \arg \max_{\boldsymbol{X}\in U(N,n_t)} \delta(\boldsymbol{X}). \tag{9}$$

Here, $\delta(\boldsymbol{X})$ is defined as

$$\delta(\boldsymbol{X}) = \min_{1\le i\le j\le N} \|\mathbf{x}_i - \mathbf{x}_j\| \tag{10}$$

where $\mathbf{x}_n$ denotes the n-th entry of $\boldsymbol{X}$. Note that in (9), the codebook is generated by maximizing the minimum Euclidean distance between any pair of two vectors in the codebook unlike conventional codebooks such as Grassmannian

TABLE I
SIMULATION CONDITION

Number of subcarriers (N_c)	64
Number of transmit antennas (n_t)	4
Number of receive antennas (n_r)	1
Cluster size (K)	8
RMS delay spread	167 ns
Link adaptation	Ideal (i.e., using the Shannon's capacity curve)

codebook [6].

The performance can further be improved by employing a frequency interpolation technique. Assume that K subcarriers are combined into a cluster. The receiver can generate a beamforming weight corresponding to the representative subcarrrier for each cluster (e.g., the center subcarrier in each cluster). Then the beamforming weight of other subcarriers can be estimated using those of the representative subcarriers. For example, assuming that the beamforming vector for subcarrier mK, where $m = 1, 2, ..., N_c / K - 1$, is reported from the receiver, the beamforming vector of other subcarriers can be estimated by linear interpolation as

$$\hat{\mathbf{w}}(mK+l) = \frac{\left(1-\frac{l}{K}\right)\mathbf{w}(mK) + \frac{l}{K}\mathbf{w}(mK+K)}{\left\| \left(1-\frac{l}{K}\right)\mathbf{w}(mK) + \frac{l}{K}\mathbf{w}(mK+K) \right\|} \quad (11)$$

where m is the cluster index and $0 \le l \le K-1$. Note that the proposed scheme does not require any additional information (e.g., the phase information in [8]).

Assuming the use of two different codebooks known at both the transmitter and the receiver, the algorithm can be realized as follows.

1. Initialization: Select an initial state codebook for c enter subcarrier of the first cluster. The beamformi ng vector corresponding to the initial subcarrier is quantized into a b_1-bit codeword using a codebook comprising N_1 vectors generated by a convention al codebook method.
2. Update of the beamforming vector corrseponding to the center subcarrier for each cluster: After the in itialization, the beamforming vector corrseponding t o the center subcarrier for each cluster is calculate d according to (7). The codebook comprising N_2 vectors is used to quantize the random vector $\mathbf{z}$ i nto a b_2-bit codeword.
3. Estimation of the beamforming vector for other subcarr iers: The transmitter calculates the beamforing vect or corresponding to the center subcarrier of each c luster by (8) and then the beamforming vectors for other subcarriers ofeach cluster by an interpolation method (e.g., (11)).

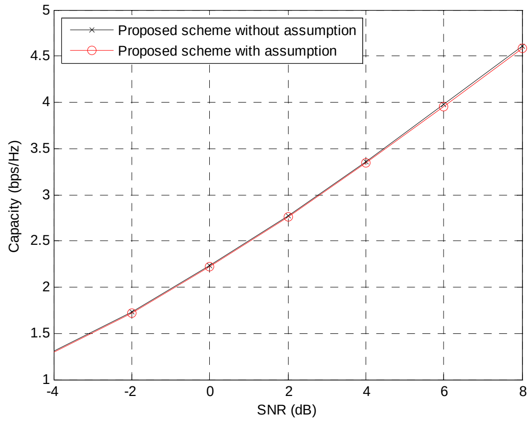

Fig. 3. Capacity with and without the assumption in (5) and (6).

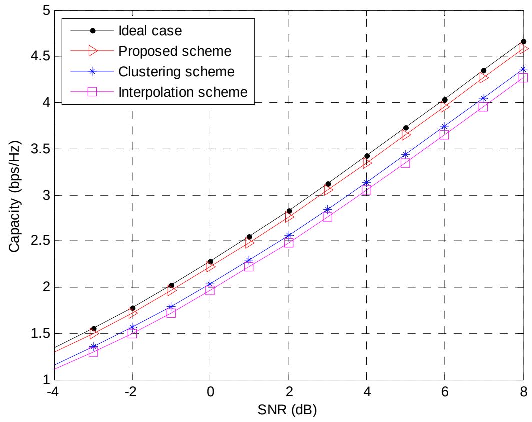

Fig. 4. Capacity with the use of the proposed scheme.

Ⅳ. PERFORMANCE EVALUATION

The performance of the proposed scheme is verified by computer simulation. The simulation condition is summarized in Table 1.

Fig. 3 compares the capacity of the proposed scheme with and without the assumption (5) and (6). The proposed scheme without the assumption means that the transmitter uses all the quantized random vectors, and the norm values of the channel and random vectors by (5), whereas the proposed scheme with the assumption means that the transmitter only uses the quantized random vector by (6). It can be seen that performance gab between the two assumptions is neglegible.

Fig. 4 depicts the performance of the proposed scheme in terms of the capacity when b_1 and b_2 are quantized into 6 bits, respectively. For comparison, we consider two conventional schemes, clustering and interpolation-based scheme in [7], [8]. The clustering scheme uses the beamforming vector of the center subcarrier as the representative vector of the cluster and the interpolation-based scheme aaditionally uses

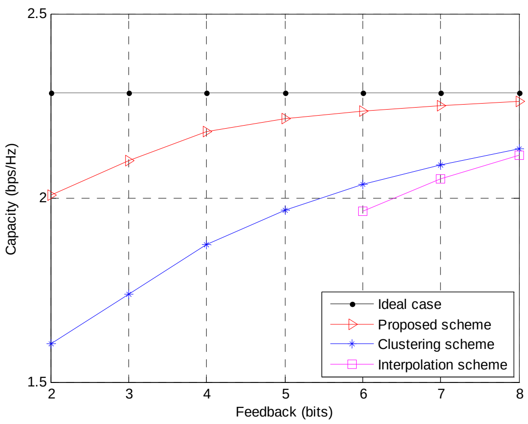

Fig. 5. Capacity comparison of beamforming techniques.

phase rotaion information to interpolate the beamforming vectors. We assume that the clustering scheme uses 6 bits for the codebook and the interpolation-based scheme uses 4 bits for the codebook and 2 bits for the phase information. For reference, we also consider the ideal case corresponding to the case when the transmitter has perfect channel information of all subcarriers with full precision. It can be seen that the proposed scheme outperforms conventional schemes and that the interpolation-based scheme has the worst performance due to the use of a reduced size codebook for additional phase information.

Fig. 5 depicts the capacity of the proposed scheme associated with the amount of feedback overhead, which represents the amount of feedback signals required for the information of each cluster after the initialization. The proposed scheme uses $b_1 = 8$ bits codewords for the initialization and the interpolation scheme uses 2 bits for the phase information. It can be seen that the proposed scheme outperforms other schemes with the use of the same bit size. This impies that the proposed scheme needs less amount of feedback to provide the same performance, compared to other conventional schemes.

Ⅴ. CONCLUSION

In this paper, we have proposed a new transmit beamforming scheme with less amount of feedback signaling in OFDM based wireless MISO systems. The proposed scheme can reduce the amount of feedback signaling burden by generating the beamforming vector using information on the previous beamforming vector, channel correlation and quantized random vector. The simulation results show that the proposed scheme can estimate the channel information more accurately than conventional schemes using the same amount of feedback signaling burden, improving the system capacity. This implies that the proposed scheme can reduce the amount of feedback signaling burden to provide the same performance as conventional schemes.

APPENDIX

PROOF OF THE NORM CONVERGENCE

Consider two column vectors, $\mathbf{a} = [a_1, \ldots, a_{n_t}]^T$ and $\mathbf{b} = [b_1, \ldots, b_{n_t}]^T$, whose entries are zero-mean complex Gaussian random variables with unit variance. Letting $R_1 = \|\mathbf{a}\|$, the pdf of R_1 can be expressed as [10]

$$f_{R_1}(R_1) = \frac{2}{\Gamma(n_t)} R_1^{2n_t - 1} e^{-R_1^2} \qquad \text{(A-1)}$$

where $\Gamma(\cdot)$ denotes the Gamma function. The pdf of $R_2 = \|\mathbf{b}\|$ is the same as that of R_1. Letting $R = R_1 / R_2 = \|\mathbf{a}\| / \|\mathbf{b}\|$, it can be shown that the pdf of R is

$$\begin{aligned} f_R(R) &= \int_{-\infty}^{\infty} |x|\, f_{R_1,R_2}(xR, x)dx \\ &= \int_0^{\infty} x f_{R_1}(xR) f_{R_2}(x)dx \\ &= \frac{4}{(\Gamma(n_t))^2} R^{2n_t-1} \int_0^{\infty} x^{4n_t-1} e^{-(R^2+1)x^2} dx \\ &= 2R^{2n_t-1}(1+R^2)^{-2n_t} \frac{\Gamma(2n_t)}{(\Gamma(n_t))^2}. \end{aligned} \qquad \text{(A-2)}$$

Thus, it can be seen that

$$\lim_{n_t \to \infty} f_R(R) = \begin{cases} \infty, & \text{if } R = 1, \\ 0, & \text{otherwise.} \end{cases} \qquad \text{(A-3)}$$

This implies that $R = \|\mathbf{a}\| / \|\mathbf{b}\| \approx 1$ as n_t increases.

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