Hard Decision Combining-based Cooperative Spectrum Sensing in Cognitive Radio Systems

*Seung-Hwan Lee, **Yong-Hwan Lee
*Seoul Nat. Univ., ** Seoul Nat. Univ.

SUMARY

In this paper, we propose a hard decision combining-based cooperative spectrum sensing scheme to maximize the detection probability in cognitive radio systems. We maximize the detection probability by finding the optimum number of cooperating users for a given false alarm probability. To this end, we analytically derive a closed-form expression for the detection and false alarm probability in terms of the number of cooperating users. It is shown that the detection probability of cooperative sensing is maximized when the optimum number of users with good reporting channel condition and high interference to noise ratio are selected. The analytic results are verified by computer simulation.

I. Introduction

As the demand for wireless communications increases, the spectrum scarcity has been become a major issue for service providers to deploy new services or enhance the capacity of existing applications. Recently, cognitive radio has been under active consideration to deal with conflict between the spectrum demand and inefficient spectrum utilization [1], [2]. Cognitive users need to detect the presence of licensed (interference) users in frequency band to be utilized. The use of a spectrum sensing technique enables to detect spectral holes and opportunistically use under-utilized frequency bands.

A number of single user spectrum sensing methods have been proposed [3], [4]. The use of a matched filter can provide optimum performance with prior knowledge on the primary user [3]. Cyclostationary feature detection can detect the signal in very low interference to noise power ratio (INR) condition, but it still requires some prior knowledge on the primary user [3]. Energy detection is simple to implement, but it may suffer from the presence of fading or shadowing [4]. Without prior knowledge on the primary user, the sensing performance of a single user spectrum sensing method may significantly deteriorate in deep fading environments [4].

To overcome this problem, the use of cooperative spectrum sensing has been proposed to achieve so-called multi-user diversity (MUD) gain [5], [6]. In [5], the base station (BS) receives the information of individual measurements (e.g., energy of the received signal) from all unlicensed (secondary) users to make a decision by comparing the sum of measured energy to a pre-determined threshold, called soft-decision combining. However, it may not be applicable to practical systems due to a large amount of feedback signaling overhead. In order to minimize the feedback signaling overhead, the spectrum is detected in a cooperative manner using single-bit decisions (i.e., occupancy state of the spectrum) of all users, called hard-decision combining [6]. This method can provide asymptotically optimum performance as the number of users goes to infinity provided that the INRs of all users are the same [6]. However, this assumption is not realistic since the INR of individual users may vary significantly depending on the user location. Moreover, most of previous works assume that the decision information is perfectly reported to the BS without an error.

In this paper, we consider cooperative spectrum sensing in a hard-decision combining mode, where each user experiences different INR and the signal to noise ratio (SNR). We maximize the detection probability by finding the optimum number of cooperating users for a desired false alarm probability. We prove that the detection probability of cooperative sensing is maximized by selecting the optimum number of users with high INR and SNR. To this end, we represent the detection and false alarm probability in a closed form according to the number of cooperating users. For ease of mathematical tractability, we consider the use of an OR fusion rule by means of energy detection.

The remainder of this paper is organized as follows. Section II describes the system model in consideration. Section III proposes optimum cooperative sensing scheme considering the effects of imperfect reporting channel and INR. Section IV verifies the performance of the proposed scheme by computer simulation. Finally, conclusions are given in Section V.

II. System Model

A. *Single User Channel Sensing*

Consider a cognitive radio system where K active users share a wideband channel comprising M non-overlapped subchannels. Let $h_{k,m}$ be the channel between the primary user and secondary user k at subchannel m. Then, the spectrum sensing of user k at subchannel m can be represented as a simple the hypothesis test as

$$\begin{aligned} \mathcal{H}_0 &: r_{k,m} = n, \\ \mathcal{H}_1 &: r_{k,m} = h_{k,m}s_m + n, \end{aligned} \quad (1)$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the hypotheses corresponding to the absence and presence of primary user, respectively, $r_{k,m}$ is the received signal of secondary user k through subchannel m, s_m is the signal transmitted from the primary user through subchannel m, and n denotes zero mean Gaussian noise with variance σ_n^2.

We consider the use of an energy detector. For each subchannel m, the test statistic of secondary user k for the hypothesis test can be given by [4]

$$Y_{k,m} = \sum_{v=0}^{V-1} \left| r_{k,m}(v) \right|^2 , \quad (2)$$

where V denotes the number of samples. Then, the decision rule of user k is

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$$\varphi_{k,m} = \begin{cases} 0, & Y_{k,m} < \lambda, \\ 1, & Y_{k,m} \geq \lambda, \end{cases} \quad (3)$$

where λ is a threshold value to be determined. Let $\eta_{k,m}$ be the INR of secondary user k at subchannel m, defined by

$$\eta_{k,m} = \frac{E_s \left|h_{k,m}\right|^2}{\sigma_n^2}, \quad (4)$$

where E_s denotes the average power of the signal transmitted from the primary user. Then, the false alarm and the detection probability of secondary user k at subchannel m in AWGN channel can be represented as, respectively, [4]

$$P_f^{k,m} = \Pr\left(Y_{k,m} > \lambda \middle| \mathcal{H}_0\right) = Q\left(\frac{\lambda - V\sigma_n^2}{\sigma_n^2\sqrt{2V}}\right), \quad (5)$$

$$P_d^{k,m} = \Pr\left(Y_{k,m} > \lambda \middle| \mathcal{H}_1\right) = Q\left(\frac{\lambda - V\sigma_n^2\left(1+\eta_{k,m}\right)}{\sigma_n^2\sqrt{2V\left(1+2\eta_{k,m}\right)}}\right), \quad (6)$$

where $Q(\cdot)$ is the complementary cumulative distribution function of a zero mean Gaussian random variable with unit variance.

B. *Cooperative Channel Sensing*

For collaborate sensing, the BS receives single-bit decision $\{\varphi_{k,m}\}$ of each user. Letting $h'_{k,m}$ be the channel between the secondary BS and secondary user k at subchannel m, the SNR of secondary user k at subchannel m is defined by

$$\gamma_{k,m} = \frac{E_{s'} \left|h'_{k,m}\right|^2}{\sigma_n^2}, \quad (7)$$

where $E_{s'}$ denotes the average transmit power of the secondary user. Let $\tilde{\varphi}_{k,m} (\in \{0,1\})$ be the received value of $\varphi_{k,m}$. Assuming that $L (\leq K)$ users are in cooperation, the test statistic based on hard-decision at subchannel m can be given by

$$\begin{aligned} \mathcal{H}_0 &: \sum_{k=0}^{L-1} \tilde{\varphi}_{k,m} < N, \\ \mathcal{H}_1 &: \sum_{k=0}^{L-1} \tilde{\varphi}_{k,m} \geq N, \end{aligned} \quad (8)$$

When the OR rule (i.e., $N=1$) is applied, the corresponding false alarm and detection probability at subchannel m are represented as, respectively, [6]

$$P_f^m(L) = 1 - \prod_{k=1}^{L}\left(1 - P_f^{k,m}\right), \quad (9)$$

$$P_d^m(L) = 1 - \prod_{k=1}^{L}\left(1 - P_d^{k,m}\right). \quad (10)$$

III. Proposed Scheme

In cognitive radio systems, high detection probability may yield low interference to the primary user and low false alarm probability may improve the spectrum efficiency. Therefore, a good sensing algorithm should have a high detection probability and a low false alarm probability. Our objective is to maximize the detection probability with a constraint on the false alarm probability as

$$\max_L P_d^m(L) \text{ s.t. } P_f^m(L) \leq \alpha . \quad (11)$$

Since (11) is a function of *L*, we should find the optimum number of users that maximizes the detection probability for a given false alarm probability. To this end, we analyze the detection probability according to the number of cooperating users with and without the presence of channel errors.

A. *Optimization without Channel Error*

Let $\bar{P}_f^m \left(= \alpha' \leq \alpha\right)$ be the desired false alarm probability at subchannel m. Assuming that all the decisions are reported to the BS without channel errors (i.e., $\tilde{\varphi}_{k,m} = \varphi_{k,m}$), it can be shown from (9) that the required false alarm probability of the secondary user *k* is given by

$$\bar{P}_f^{k,m} = 1 - \sqrt[L]{1-\alpha'} . \quad (12)$$

It can be shown from (5) that the threshold value for the corresponding false alarm probability is determined by

$$\lambda = Q^{-1}\left(1 - \sqrt[L]{1-\alpha'}\right)\sigma_n^2\sqrt{2V} + \sigma_n^2 V . \quad (13)$$

Thus, the detection probability of the secondary user k at subchannel *m* in AWGN channel is given by

$$P_d^{k,m} = Q\left(\frac{1}{\sqrt{1+2\eta_{k,m}}}\left(Q^{-1}\left(1-\sqrt[L]{1-\alpha'}\right) - \sqrt{\frac{V}{2}}\eta_{k,m}\right)\right). \quad (14)$$

It can be seen that the larger $\eta_{k,m}$, the larger $P_d^{k,m}$. Thus, it is desirable for the maximization of the detection probability to select users with high INR for the cooperation. Without loss of generality, assuming that $\eta_{1,m} > \eta_{2,m} > \ldots > \eta_{K,m}$, the detection probability at subchannel *m* can be represented as

$$P_d^m(L) = 1 - \prod_{k=1}^{L}\left(1 - Q\left(\begin{array}{c}\frac{1}{\sqrt{1+2\eta_{k,m}}} \\ \times\left(Q^{-1}\left(1-\sqrt[L]{1-\alpha'}\right) - \sqrt{\frac{V}{2}}\eta_{k,m}\right)\end{array}\right)\right). \quad (15)$$

As the number L increases, $P_d^{k,m}$, $k \leq L-1$, decreases due to the term $Q^{-1}\left(1-\sqrt[L]{1-\alpha'}\right)$ in (14). This implies the threshold λ increase since $\bar{P}_f^{k,m}$ decreases as L increases. Hence, we can conclude that the cooperation of all K users for the spectrum sensing does not necessarily achieve the optimum detection probability. The optimum number $\hat{L}$ for the cooperation can be determined by (15) for a given $\eta_{k,m}$.

B. *Optimization with channel errors*

Assume that some decisions are reported to the BS with channel errors (i.e., $\tilde{\varphi}_{k,m} \neq \varphi_{k,m}$ for some *k*). Assuming that the channel of the secondary user *k* has bit error probability P_e^k, the detection and false alarm probability at subchannel *m* can be represented as, respectively,

$$P_d^m(L) = 1 - \prod_{k=1}^{L}\left[\left(1 - P_d^{k,m}\right)\left(1 - P_e^k\right) + P_d^{k,m}P_e^k\right], \quad (16)$$

$$P_f^m(L) = 1 - \prod_{k=1}^{L} \left[\left(1 - P_f^{k,m}\right)\left(1 - P_e^k\right) + P_f^{k,m} P_e^k \right]. \quad (17)$$

Assume that each user sends its decision through a subchannel in the best condition to minimize the reporting error. Letting $\hat{m}_k$ be the index of the subchannel in the best condition of the secondary user k, i.e.,

$$\hat{m}_k = \arg\max_m \left\{ \gamma_{k,m} \right\}, \quad (18)$$

the probability density function of $\gamma_{k,\hat{m}_k}$ can be represented as

$$f(x) = \frac{M}{\overline{\gamma}_k} e^{-\frac{x}{\overline{\gamma}_k}} \left(1 - e^{-\frac{x}{\overline{\gamma}_k}}\right)^{M-1}, \quad (19)$$

where $\overline{\gamma}_k$ is the average SNR of user k, given by $\overline{\gamma}_k = \sum_{m=1}^{M} \gamma_{k,m}$. Assuming that decision of each user are reported through BPSK signal, the error probability for a given $\gamma_{k,\hat{m}_k}$ can be shown that

$$P_{e|\gamma_{k,\hat{m}_k}}^k = Q\left(\sqrt{2\gamma_{k,\hat{m}_k}}\right). \quad (20)$$

Then, the average reporting error probability in Rayleigh fading channel is

$$\begin{aligned} P_e^k &= \int_0^\infty P_{e|\gamma_{k,\hat{m}_k}}^k f\left(\gamma_{k,\hat{m}_k}\right) d\gamma_{k,\hat{m}_k} \\ &= \sum_{m=0}^{M-1} \binom{M-1}{m} (-1)^{M-m-1} \frac{M}{2(M-m)} \quad (21) \\ &\quad \times \left(1 - \sqrt{\frac{\overline{\gamma}_k}{M - m + \overline{\gamma}_k}}\right). \end{aligned}$$

Taking account of the channel error rate, the required false alarm probability of the secondary user k is given by

$$\overline{P}_f^{k,m} = \frac{P_e^k - 1 + \sqrt[L]{1-\alpha'}}{2P_e^k - 1}, \quad (22)$$

and the corresponding threshold value is determined as

$$\lambda_k = Q^{-1}\left(\frac{P_e^k - 1 + \sqrt[L]{1-\alpha'}}{2P_e^k - 1}\right) \sigma_n^2 \sqrt{2V} + \sigma_n^2 V. \quad (23)$$

Then, the detection probability of the secondary user k in AWGN channel is given by

$$P_d^{k,m} = Q\left(\frac{1}{\sqrt{1+2\eta_{k,m}}} \times \left(Q^{-1}\left(\frac{P_e^k - 1 + \sqrt[L]{1-\alpha'}}{2P_e^k - 1}\right) - \sqrt{\frac{V}{2}} \eta_{k,m} \right) \right). \quad (24)$$

Let Φ_L be a set of L users selected for the cooperation. For given $\eta_{k,m}$ and $\overline{\gamma}_k$, the users can be selected as follows.

Step I: Initialize the maximum detection probability $\hat{P}_d^m$

$\hat{P}_d^m = P_d^m(1) = P_d^{1,m} + P_e^1 - 2P_d^{1,m} P_e^1$ and $L = 2$.

Step II: Initialize Φ_L

$\Phi_L = \varnothing$, $\Theta_1 = \{1, 2, ..., K\}$ and $i = 1$.

Step III: Select a user with the smallest $(1 - P_d^{k,m})$ $\times (1 - P_e^k) + P_d^{k,m} P_e^k$ and then update Φ_L as

For $\pi_i = \arg\min_{k \in \Theta_i} (1 - P_d^{k,m})(1 - P_e^k) + P_d^{k,m} P_e^k$

$\Phi_L \leftarrow \Phi_L \cup \pi_i$ and $\Theta_{i+1} = \Theta_i - \pi_i$.

Step IV: If $|\Phi_L| < L$, then $i = i + 1$ and go to step II.

Else calculate the $P_d^m(L)$ as

$P_d^m(L) = 1 - \prod_{k=1, k \in \Phi_L}^{K} (1 - P_d^{k,m})(1 - P_e^k) + P_d^{k,m} P_e^k$

Step V: If $P_d^m(L) > \hat{P}_d^m$, then $\hat{P}_d^m = P_d^m(L)$, $L = L + 1$ and go to step II. Else stop.

The maximum detection probability for a given $\Phi_{\hat{L}}$ can be represented as

$$\hat{P}_d^m = 1 - \prod_{k=1}^{\hat{L}} \left(1 - Q\left(\frac{1}{\sqrt{1+2\eta_{k,m}}} \times \left(Q^{-1}\left(1 - \sqrt[\hat{L}]{1-\alpha'}\right) - \sqrt{\frac{V}{2}} \eta_{k,m} \right) \right) \right) \quad (25)$$

for $k \in \Phi_{\hat{L}}$. In summary, the overall procedure can be described as follows:

1. Each user reports the INR and average SNR to the BS.
2. The BS determines a set of optimum users from (24).
3. Selected users make their decision by (3) with a threshold determined by (23) and report their decision to the BS.
4. Finally, the BS makes the final decision on subchannel *m* by (8).

IV. Performance Evaluation

We verify the performance of the proposed scheme by computer simulation. We assume that a TV broadcast station is the primary user, an IEEE 802.22 wireless regional area network (WRAN) BS is the secondary station and customer-premises equipments (CPEs) is the secondary users. We also assume that users are uniformly distributed within the coverage 5km of the secondary BS and the secondary BS is 10km away from the primary user, unless explicitly stated otherwise. To verify the validation of the proposed scheme, we compare the performance of the proposed scheme with that of all-user cooperation scheme [5]. The performance of single user sensing scheme is also referred to for comparison. The desired false alarm probability is set to 0.01. To satisfy constraint for the false alarm probability, we assume that all schemes use the decision threshold in (23).

Fig. 1 depicts the detection probability of the proposed scheme according to the number of cooperating users when

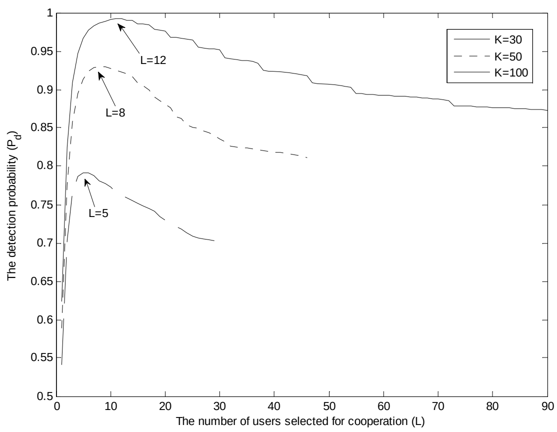

Fig. 1. The detection probability according to L .

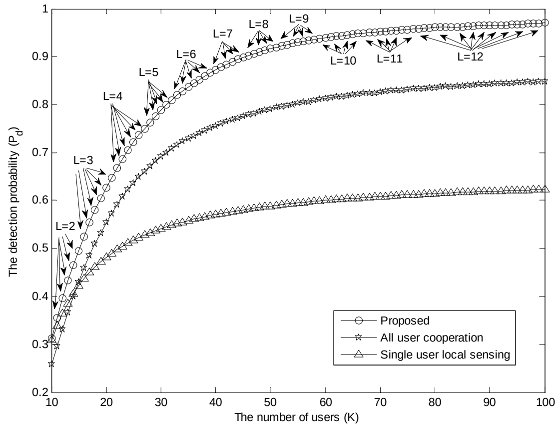

Fig. 2. The detection probability according to K .

$K = 30, 50$ and 100. It can be seen that the detection probability increases rapidly and then decreases as L increases. This verifies that the cooperation among good users can outperform that among all users. It can also be seen that as the number of users in the network increases, the optimum detection probability $\hat{P}_d^m$ increases. This is mainly due to the fact that the number of users with high $\eta_{k,m}$ and $\overline{\gamma}_k$ increases as K increases.

Fig. 2 depicts the detection probability of the proposed scheme in terms of the number of users in the network. It can be seen that the proposed scheme significantly outperforms other schemes. It can also be seen that the single user sensing scheme slightly outperforms the all-user cooperation scheme when $K < 14$. This is mainly due to the fact that the number of optimum cooperating users is 1 or 2 when $K < 14$.

V. Conclusions

We have considered an optimum cooperative spectrum sensing in cognitive radio systems based on hard-decision of each user. The proposed scheme maximizes the detection probability by optimally finding cooperating users for a given false alarm probability. The analytic and simulation results show that cooperative sensing among users with high INR and SNR can provide higher detection probability than that among all users in the network, and that single user sensing may outperform cooperative sensing when the number of users in the network is small.

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