A Hybrid Channel Estimation Scheme for OFDM Systems

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Abstract: Accurate channel information is indispensable for coherent reception of OFDM signal. Although a Wiener-type channel estimation filter (CEF) is known optimum, it is not easily employable due to large implementation complexity. In practice, a moving average (MA)-type CEF is often employed, but it may not provide robust performance to the variation of channel condition. In this paper, we propose a hybrid CEF that takes advantages of both the Wiener and MA CEF, by alternatively employing the CEF according to the channel condition. Simulation results show that the proposed hybrid CEF scheme provides near optimum performance, while significantly reducing the implementation complexity compared to the long tap Wiener CEF.

Keywords: channel estimation, OFDM, Wiener, moving average.

I. INTRODUCTION

Coherent detection is often employed to improve the detection performance of orthogonal frequency division-multiplexing (OFDM) receiver. It is well acknowledged that the detection performance directly depends on the accuracy of channel information. In the OFDM system, predetermined pilot symbols scattered in time and frequency domain are often used to aid the estimation of time-variant and frequency-selective channel characteristics in the receiver [1-3].

It is known that two-dimensional (2-D) Wiener-type channel estimation filter (CEF) is optimum since it minimizes the mean square error (MSE) of the channel estimate [2]. However, it is not easily employable due to large implementation complexity. To reduce the implementation complexity, the use of two 1-D Wiener filters was considered at the expense of small performance loss [3]. However, it is still too complex to be employed unless the tap size of Wiener filters is small. As a result, a simple interpolator filter such as linear, Lagrange and Spline interpolation scheme is often employed for the CEF [4]. However, since most of these filters have fixed parameters regardless of the channel condition, they experience severe performance degradation under certain channel condition [5].

Recently, a channel estimation scheme composed of a linear interpolator and a moving average (MA) filter, called a cascaded MA, has been proposed as the CEF for simplicity of implementation [6]. In this scheme, the received signal is interpolated using a linear interpolator to obtain instantaneous channel impulse response (CIR) and then low-pass filtered using an adaptive MA filter to suppress the excess noise. This scheme provides good performance in nominal channel condition (e.g., low Doppler frequency and delay spread). However, it may suffer from performance degradation in severe channel condition (e.g., high Doppler frequency and delay spread) due to the properties of the MA filter. In this paper, we propose a hybrid CEF that takes advantages of both the Wiener and cascaded MA CEF, by alternatively employing these filters according to the channel condition. To avoid performance degradation in bad channel condition, we employ the Wiener CEF that is realizable with the use of a small number of taps, while providing near optimum performance. In normal channel condition, we employ the cascaded MA CEF.

Following Introduction, Section II describes the OFDM system and channel model. After a brief introduction of the cascaded 2-D MA CEF, we propose the hybrid CEF and evaluate the performance in Section III. Finally, conclusions are summarized in Section IV.

II. SYSTEM AND CHANNEL MODEL

Consider an OFDM transmitter, where the K subcarrier symbols at the n-th symbol time, $\{X[n,k]; k=0,1,2,\cdots, K-1\}$, are converted into a time domain signal using an inverse fast Fourier transform (FFT). A cyclic prefix (CP) is inserted to preserve the orthogonality between the subcarriers and to eliminate the interference between the adjacent OFDM symbols. We assume that the pilot symbol is regularly inserted in a rectangular shape (i.e., apart by d_t and d_f symbols in the time and frequency grid, respectively).

We assume the transmission over a wireless channel whose CIR is represented as

$$h(t,\tau)=\sum_{l=0}^{L-1} h_l(t)\delta(\tau-\tau_l) \qquad (1)$$

where L is the number of multipaths, $\delta(\cdot)$ is Kronecker delta function, τ_l and $h_l(t)$ are the delay and complex-valued CIR at time *t* of the *l*-th path, respectively. We assume that $h_l(t)$ has the same normalized correlation function $r_t(\triangle t)$ for all *l*. Then, the time-domain correlation of the *l*-th path CIR can be represented as [3]

$$r_l(\triangle t) = E\{h_l(t+\triangle t)h_l^*(t)\} = \sigma_l^2 r_t(\triangle t) \quad (2)$$

where $E\{X\}$ denotes the expectation of X, * denotes the complex conjugate and σ_l^2 denotes the average power of the *l*-th path. The frequency response of the CIR at time *t* can be written by

$$H(t,f) = \int_{-\infty}^{\infty} h(t,\tau)\,\mathrm{e}^{-j2\pi f\tau}d\tau = \sum_{l=0}^{L-1} h_l(t)\,\mathrm{e}^{-j2\pi f\tau_l}. \quad (3)$$

Assuming $h_l(t)$ is statistically independent for each path with the normalized average path power (i.e., $\sum_{l=0}^{L-1}\sigma_l^2 = 1$), the correlation function of the frequency response can be represented as [3]

$$r_H(\triangle t, \triangle f) = E\{H(t+\triangle t, f+\triangle f)H^*(t,f)\} = r_t(\triangle t)r_f(\triangle f) \quad (4)$$

where $r_f(\triangle f) = \sum_{l=0}^{L-1}\sigma_l^2\,\mathrm{e}^{-j2\pi\triangle f\tau_l}$. In an OFDM symbol with symbol period T_s and subcarrier spacing $\triangle f_c$, the correlation function can be represented as $r_H[n,k] = r_t[n]r_f[k]$ where $r_t[n] = r_t(nT_s)$ and $r_f[k] = r_f(k\triangle f_c)$.

In the receiver, the CP is removed before the FFT process. Assuming ideal synchronization at the receiver, the received symbol of the *k*-th subcarrier at the *n*-th symbol time can be represented by

$$Y[n,k] = X[n,k]H[n,k] + Z[n,k] \quad (5)$$

where $H[n,k]$ is the frequency response of the channel at the *k*-th subcarrier and the *n*-th symbol time, and $Z[n,k]$ is the background noise plus interference term, which can be approximated as zero mean additive white Gaussian noise (AWGN) with variance σ_Z^2.

III. PROPOSED HYBRID CEF

The CIR can be estimated using the received pilot symbols as

$$\tilde{H}[n_p,k_p] = Y[n_p,k_p]/X[n_p,k_p] = H[n_p,k_p] + \tilde{Z}[n_p,k_p] \quad (6)$$

where n_p and k_p denote the symbol and subcarrier index of the pilot symbol, respectively, and $\tilde{Z}[n_p,k_p]$ denotes the noise term. Assuming that $X[n_p,k_p]=1$ without the loss of generality, $\tilde{Z}[n_p,k_p]$ can be assumed zero mean AWGN with variance σ_Z^2.

In order to obtain the CIR at the *n*-th symbol time and the *k*-th subcarrier, we exploit adjacent N_w instantaneous CIRs at the pilot symbol for Wiener filtering as

$$\hat{H}_w[n,k] = \boldsymbol{\alpha}_w^H[n,k]\tilde{\mathbf{H}} \quad (7)$$

where

$$\tilde{\mathbf{H}} = [\tilde{H}[n_0,k_0]\,\tilde{H}[n_1,k_1]\cdots\tilde{H}[n_{N_w-1},k_{N_w-1}]]^T. \quad (8)$$

and $\boldsymbol{\alpha}_w[n,k]$ is the coefficient vector of the Wiener CEF. The optimum coefficient of the Wiener CEF can be represented as [2]

$$\boldsymbol{\alpha}_w^H[n,k] = \boldsymbol{\theta}^H[n,k]\boldsymbol{\Phi}^{-1} \quad (9)$$

where the superscript H denoted the complex transpose, $\boldsymbol{\Phi} = E[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H]$ is the $(N_w \times N_w)$ auto-covariance matrix and $\boldsymbol{\theta}[n,k] = E[\tilde{\mathbf{H}}H^*[n,k]]$ is the $(N_w \times 1)$ cross-covariance vector.

As mentioned before, however, the Wiener CEF is not often employed due to its implementation complexity since it needs N_w multiplications per symbol and matrix inversion of an (N_w x N_w) matrix. Thus, it may not be practical unless the number of the tap size N_w is small. To alleviate this problem, the CEF can be designed by cascading an MA filter with a simple linear interpolator [6]. We briefly introduce this scheme in the following.

In order to obtain the CIR corresponding to the data symbol, the CIR estimated from the pilot symbol is linearly interpolated in the frequency domain and then in the time domain as

$$\tilde{H}[n_p,k_p+k] = \tilde{H}[n_p,k_p] + \frac{k}{d_f}(\tilde{H}[n_p,k_p+d_f] - \tilde{H}[n_p,k_p]),\ 0<k<d_f \quad (10)$$

$$\tilde{H}[n_p+n,k] = \tilde{H}[n_p,k] + \frac{n}{d_t}(\tilde{H}[n_p+d_t,k] - \tilde{H}[n_p,k]),\ 0<n<d_t.$$

Note that linear interpolation needs single multiplication.

The CIR estimated using the simple linear interpolator is further processed to reduce the excessive noise by using a 2-D MA filter with $N_t(=2M_t+1)$ and $N_f(=2M_f+1)$ taps in the time and frequency domain as [6]

$$\hat{H}[n,k] = \frac{1}{N_tN_f}\sum_{m_1=-M_t}^{M_t}\sum_{m_2=-M_f}^{M_f}\tilde{H}[n+m_1d_t, k+m_2d_f]. \quad (11)$$

Note that the MA CEF only needs single multiplication.

Denote the Doppler spectrum and power delay profile of the channel by $S_{H_1}(w_1)$ and $S_{H_2}(w_2)$, respectively. They correspond to the Fourier transforms of the time and frequency domain correlation functions, $r_t(\triangle t)$ and $r_f(\triangle f)$, respectively. Then, the optimum tap size $\hat{N}_t$ and $\hat{N}_f$ can be determined by [6]

$$\hat{N}_t = \frac{(d_t\,d_f)^{1/6}}{d_t}\left(\frac{\overline{w}_2^{(4)}}{\overline{w}_1^{(4)}}\right)^{1/8}\left(\frac{144\sigma_Z^2}{\left(\overline{w}_1^{(2)}\overline{w}_2^{(2)}+\sqrt{\overline{w}_1^{(4)}\overline{w}_2^{(4)}}\right)}\right)^{1/6}$$

$$\hat{N}_f = \frac{(d_t\,d_f)^{1/6}}{d_f}\left(\frac{\overline{w}_2^{(4)}}{\overline{w}_1^{(4)}}\right)^{-1/8}\left(\frac{144\sigma_Z^2}{\left(\overline{w}_1^{(2)}\overline{w}_2^{(2)}+\sqrt{\overline{w}_1^{(4)}\overline{w}_2^{(4)}}\right)}\right)^{1/6} \quad (12)$$

where $\overline{w}_1^{(n)}$ and $\overline{w}_2^{(n)}$ are respectively the *n*-th order moment of the Doppler spectrum and power delay profile defined as

$$\overline{w}_1^{(n)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} w_1^n S_{H_1}(w_1)dw_1$$
$$\overline{w}_2^{(n)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} w_2^n S_{H_2}(w_2)dw_2. \quad (13)$$

The moment information indicates how fast the channel varies in the time and frequency domains. It can be seen that the optimum 2-D MA CEF can be designed considering the signal to interference ratio (SIR) $1/\sigma_z^2$, pilot spacing, and the second- and fourth-order moment of the Doppler spectrum and power delay profile of the channel. It can also be seen that the lower SIR, the larger number of taps is required to suppress the interference. As the moments of w_1 or w_2 decrease (i.e., the channel has low Doppler or delay spread), the optimum tap size increases, and vice versa. Note that the SIR and moment information can accurately be estimated using the autocorrelation property without difficulty [7].

To investigate the performance of the cascaded MA CEF, Fig. 1 depicts the MSE in nominal and severe channel condition, where τ_{rms} and f_d denote the root mean squared (rms) delay spread and maximum Doppler frequency, respectively. Optimum tap size of the cascaded MA CEF designed by (12) is also shown in Fig. 1. The simulation condition is summarized in Table 1. For performance comparison, we consider the use of a simple linear interpolator and a (21x21)-tap 2-D Wiener CEF (we represent this as "Long tap Wiener" later). It can be seen that the cascaded MA CEF provides good performance at low SIR and nominal channel condition. However, as the SIR increases and/or the channel condition becomes worse, the performance gap between the cascaded MA and Wiener CEF increases. This is mainly due to that the Wiener CEF still uses a large number of taps with optimized tap coefficients, while the MA CEF uses a small number of taps.

In order to provide good performance, the Wiener CEF needs a large number of filter taps when the channel is slowly time-variant and less frequency-selective. On the other hand, when the channel is fast time-variant and frequency-selective, it does not require a large number of filter taps, making it easily implementable (we represent this as 'Short tap Wiener' afterwards). Meanwhile, the cascaded MA CEF provides good performance especially in nominal channel condition without large implementation complexity. Consequently, if we employ these two CEFs alternatively, we can obtain good performance in a wide range of channel condition.

Fig. 2 depicts the MSE performance of (5×5)-tap Wiener (short tap Wiener) and the cascaded MA CEF according to f_d for given τ_{rms}. It can be seen that the use of the cascaded MA CEF is better than the use of short tap Wiener CEF at low f_d and τ_{rms}, and vice versa. Similar tendency can also be seen when τ_{rms} varies for given f_d.

For proper operation of the proposed hybrid scheme, it is necessary to choose one of these two filters in response to the channel condition. Since the channel estimation performance is directly related to the MSE, it is reasonable to select the CEF that has lower MSE. It was shown that the MSE of the cascaded MA CEF is [6]

$$\sigma_{e,MA}^2 \approx \frac{1}{\left(d_t d_f\right)^2}\left(c_{0,IF}\overline{w}_1^{(2)}\overline{w}_2^{(2)} + c_{1,IF}\overline{w}_1^{(4)} + c_{2,IF}\overline{w}_2^{(4)}\right) + \frac{\sigma_Z^2}{N_t N_f} \quad (14)$$

where

$$c_{0,IF} = d_t^2 d_f^2\left(d_t^2(N_t^2+1)-2\right)\left(d_f^2(N_f^2+1)-2\right)/288$$
$$c_{1,IF} = d_t^2 d_f^2\left(d_t^2(N_t^2+1)-2\right)^2/576 \quad (15)$$
$$c_{2,IF} = d_t^2 d_f^2\left(d_f^2(N_f^2+1)-2\right)^2/576.$$

The MSE of the Wiener CEF is [2]

$$\sigma_{e,w}^2 = 1-\boldsymbol{\theta}^H[n,k]\boldsymbol{\alpha}_w[n,k]-\boldsymbol{\alpha}_w^H[n,k]\boldsymbol{\theta}[n,k]+\boldsymbol{\alpha}_w^H[n,k]\boldsymbol{\Phi}\boldsymbol{\alpha}[n,k] \quad (16)$$

It was shown that the optimum tap size of the MA CEF, the coefficient of the Wiener CEF and moment information can easily be estimated from the autocorrelation of received pilot signals [7]. Thus, the MSE can easily be estimated, making the selection of the CEF practical without any difficulty.

To evaluate the system performance of the proposed scheme, Fig. 3 depicts the packet error rate (PER) performance in response to the variation of f_d for given τ_{rms}. It can be seen that proposed scheme provides near optimum receiver performance in a wide rage of f_d by switching the CEF in response to f_d. Although we have shown the performance according to f_d, similar results can be obtained with the variation of τ_{rms}.

As another measure, the computational complexity is compared in Table 2. Here, we do not consider the complexity required to calculate the SIR and correlation since it is commonly necessary for all the CEFs. It can be seen that the proposed scheme can significantly reduce the implementation complexity compared to long tap 2-D Wiener CEF and two 1-D Wiener CEFs. Thus, the proposed scheme is quite practical considering the implementation complexity and receiver performance.

IV. CONCLUSIONS

In this paper, we have proposed a hybrid CEF that employs a cascaded MA CEF or Wiener CEF with a small tap size in response to the variation of the channel condition. When the channel is in nominal condition, the cascaded MA CEF is effectively employed considering the computational complexity. On the other hand, when the channel condition becomes worse, the Wiener CEF with a small tap size is employed without significant performance degradation. Simulation results show that the proposed hybrid scheme can provide good receiver performance in a wide range of channel condition, while significantly reducing the implementation complexity compared to the use of long-tap Wiener CEF.

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Table 1. Simulation condition.

Parameters	Values
Total bandwidth	100 MHz
Symbol duration	20.48 μs (+5 μs: guard interval)
Number of subcarriers	2048
Packet size	Number of symbol=8, number of subcarrier=64
Pilot occupation	6.25 % (d_t =8, d_f =4)
Carrier frequency	5.8 GHz
Channel coding	Zig-Zag coding (code rate 1/2)
Channel	Rayleigh (Classic spectrum)
Power delay profile	Exponential
Hopping	Equi-distant frequency hopping

Table 2. Computational complexity.

Type (filter tap size)		Multiplications /symbol	Matrix inverse
2-D Wiener (21 by 21)		441 ($N_t N_f$)	441 by 441
Two 1-D Wiener (21 by 21)		23.625 ($N_t + N_f/d_t$)	21 by 21
Hybrid scheme	Short tap two 1-D Wiener (5 by 5)	5.625 ($N_t + N_f/d_t$)	5 by 5
	Cascaded MA	2.969 $(1+(d_t^2-1)/(d_t d_f))$	None

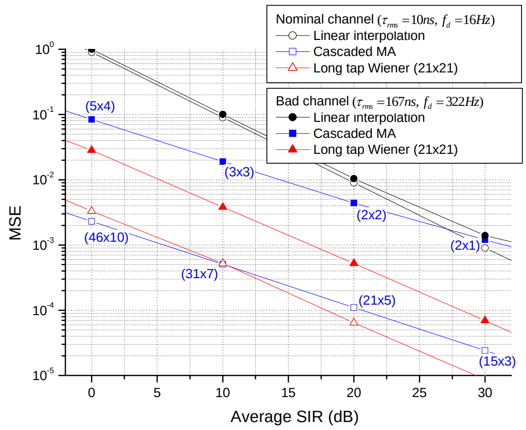

Fig. 1. MSE of the cascaded MA CEF under various channel conditions

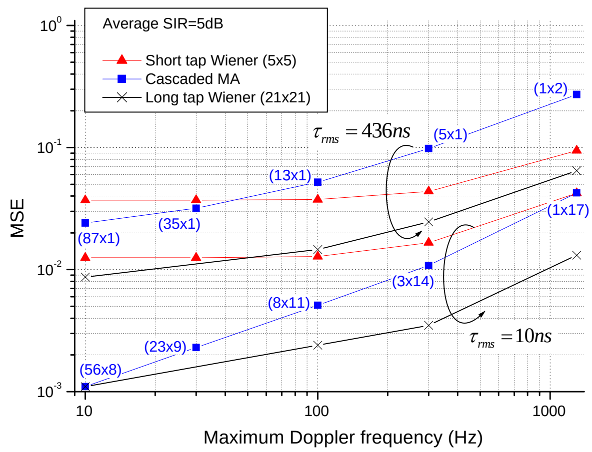

Fig. 2. MSE of the cascaded MA and short tap Wiener CEF when f_d varies

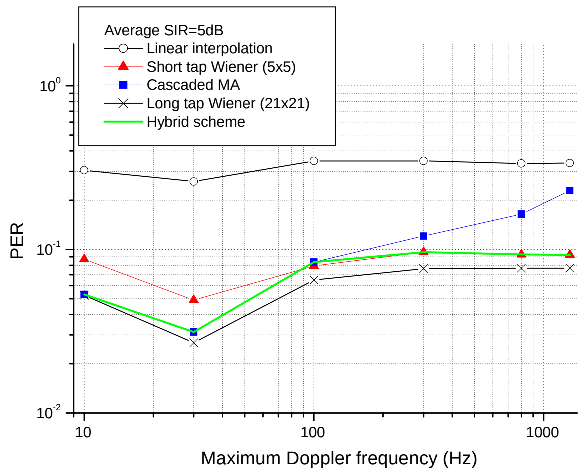

Fig. 3. PER performance of the hybrid scheme when $\tau_{rms} = 167ns$