# Coherent opportunistic beamforming in multiuser wireless systems

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*Abstract* - In this paper, we propose a beamforming scheme that exploits the advantages of opportunistic beamforming and coherent beamforming in multi-user environment. It is analytically shown that the proposed scheme achieves multiuser diversity gain and beamforming gain simultaneously, providing much better performance over the conventional ones. The performance of the proposed scheme is analyzed using an upper bound method. Although the proposed scheme involves an additional feedback delay, the analytical result implies that the use of the proposed scheme is quite effective unless the user mobility is too high. Finally, the analytic results are verified by computer simulation.

#### I. INTRODUCTION

In recent years, the capacity of wireless systems has been increased significantly with the development of two key technologies; the use of multiple antennas known as multiinput multi-output (MIMO) [1,3] and opportunistic scheduling [4,5]. The use of multiple antennas at the transmitter and/or the receiver enables to increase the data rate or lower the transmission error rate for a given signal-tonoise power ratio (SNR). On the other hand, in a multi-user environment, scheduler that considers channel conditions of users can remarkably improve the system efficiency by achieving so-called multi-user diversity (MUD) gain.

With channel knowledge at the transmitter, one can achieve beamforming gain (i.e., SNR and spatial diversity gain) by using coherent beamforming (also called maximum ratio transmission or transmit MRC) [6,7,13]. In frequency division duplex (FDD) systems, the channel knowledge cannot be exploited at the transmitter without feedback signaling through the uplink channel. Since the amount of feedback signaling increases in linear proportion to the number of users, the coherent beamforming is hardly applicable in multi-user environment. Although the use of quantization methods [8,9] can be applied to reduce the feedback signaling burden, it may suffer from the quantization noise as the number of users increases.

Opportunistic beamforming is a multi-antenna technique that can increase the MUD gain with the use of random beamforming [10]. Since it requires only partial channel information (i.e., SNR) from the users, it remarkably reduces the feedback signaling burden. Moreover, it can provide coherent beamforming configuration with probability of one as the number of users increases to infinity [10]. However, it may considerably suffer from performance degradation unless the number of users is sufficiently large, because it exploits only the MUD gain. This problem can be alleviated by employing a receiver antenna selection technique [11]. However, the use of multiple receiver antennas may limit the flexibility of the receiver structure.

In this paper, we consider a new beamforming technique that exploits both the MUD gain and beamforming gain simultaneously in multi-user environment which can be achieved by combining the opportunistic beamforming and coherent beamforming. The proposed scheme requires only slightly larger feedback information than opportunistic beamforming while providing much better performance over the coherent beamforming and opportunistic beamforming. Although the proposed scheme requires additional feedback delay which may degrade the performance in a mobile environment, analytical result indicates that the use of the proposed scheme is still desirable for large cases of environments.

This paper is organized as follows. Section II describes the system model in consideration. Following brief discussion of the conventional beamforming techniques in Section III, the proposed scheme is described and its performance is analyzed in Section IV. The performance is verified by computer simulation in Section V. Finally, Section VI concludes this paper.

#### II. SYSTEM MODEL

Consider a downlink environment with *K* users, where the BS has *M* transmitter antennas and each user has a single receiver antenna. We assume the signal transmission over a wireless channel whose gains are described by independent zero mean complex Gaussian random variables and that all the users have the same average SNR  $\gamma_o$  and experience independent fading. We also assume that the channel information is available at the user terminal with the use of common pilot signal, but it is not available at the BS without feedback signaling through an uplink channel.

When the signal is transmitted with beamforming weight  $\mathbf{w} \in \mathbf{C}^{M \times 1}$ , the received signal of user k can be represented as

$$y_k = \mathbf{h}_k \mathbf{w} \, s + n_k \tag{1}$$



Fig. 1. Procedure of the opportunistic beamforming

where  $\mathbf{h}_k \in \mathbf{C}^{i \times M}$  is the channel gain vector of user *k* whose elements are zero mean complex Gaussian random variables with unit variance, *s* is the transmit symbol with average power of *P* and  $n_k$  is additive white Gaussian noise (AWGN). We assume that the Frobenious norm of  $\mathbf{w}$  is equal to one to preserve the total transmission power (i.e.,  $||\mathbf{w}||=1$ ).

#### **III. CONVENTIONAL BEAMFORMING TECHNIQUES**

#### A. Opportunistic beamforming

We briefly review the opportunistic beamforming [10] for easy understanding of the proposed scheme. Fig. 1 depicts the processing concept of the opportunistic beamforming, where the beamforming weight  $\mathbf{w}$  is generated in a random manner, while preserving the transmission power, i.e.,

$$\mathbf{w} = \left(\sum_{i=1}^{M} |w_i|^2\right)^{-1/2} [w_1 \, w_2 \cdots w_M]^T$$
(2)

where  $w_i$  represents the random weight described as an independent and identically distributed (i.i.d.) zero mean complex Gaussian random variable and the superscript T denotes transpose. Each user estimates the SNR for a given beam and reports it to the BS. Then, the BS selects a user based on the scheduling policy.

In an independent Rayleigh fading channel, the effective channel of user k

$$\tilde{h}_k = \mathbf{h}_k \mathbf{w} \tag{3}$$

can be described by a zero mean complex Gaussian random variable with unit variance. Note that  $\tilde{h}_k$  has the same distribution as that in a single-input single-output (SISO) Rayleigh fading channel. This implies that the opportunistic beamforming does not provide any performance gain over the SISO configuration in Rayleigh fading channel [10].

Assuming that the scheduler in the BS chooses a user in the best channel condition, the effective channel gain of the selected user can be described as

$$\Gamma_{OB} = \max_{k=1,2,\cdots,K} \{ \left| \tilde{h}_k \right|^2 \}.$$
(4)

Thus, the channel capacity  $C_{OB}$  of the opportunistic beamforming in Rayleigh fading channel can be represented

as

$$C_{OB} = E \left\{ \log_2(1 + \gamma_o \Gamma_{OB}) \right\}.$$
(5)

Since we cannot represent (5) in a closed form, we consider the use of an upper bound  $\overline{C}_{OB}$  using Jensen's inequality as

$$\overline{C}_{OB} = \log_2 \left( 1 + \gamma_o E \left\{ \Gamma_{OB} \right\} \right) \ge C_{OB} \,. \tag{6}$$

Since the maximum of K i.i.d. exponential random variables has a mean value equal to the sum of harmonic numbers [14], (6) can be rewritten as

$$\overline{C}_{OB} = \log_2 \left[ 1 + \gamma_o \left( 1 + \sum_{i=2}^{K} 1/i \right) \right]$$
  
$$\approx \log_2 \left[ 1 + \gamma_o \left( \log(1+K) + \zeta - 0.5/(K+1) \right) \right]$$
(7)

where  $\zeta \approx 0.577216$  is the Euler's constant. Although the opportunistic beamforming has the same capacity with the SISO system in Rayleigh fading channel, with the use of different beam weight at each scheduling time, the user may experience different SNR. Thus the opportunistic beamforming can provide a fast fading condition even in a slowly varying channel, making it easy to satisfy the quality-of-service (QoS) requirement for delay sensitive traffic.

### B. Coherent beamforming

The coherent beamforming technique generates the beam weight so that it maximizes the received SNR. The optimum weight  $\mathbf{w}_k^o$  for user *k* can be determined by [16]

$$\mathbf{w}_{k}^{o} = \mathbf{h}_{k}^{H} / \left\| \mathbf{h}_{k} \right\| \tag{8}$$

where the superscript H denotes conjugate transpose. Since the effective channel of user k is described by

$$\tilde{h}_{k} = \mathbf{h}_{k} \mathbf{w}_{k}^{o} = \left\| \mathbf{h}_{k} \right\|, \tag{9}$$

the effective channel gain  $\Gamma_{CB} = |\tilde{h}_k|^2$  can be described by a Chi-square random variable with 2*M* degrees of freedom in Rayleigh fading channel [15]. Thus, the channel capacity  $C_{CB}$  of the coherent beamforming is bounded by

$$C_{CB} = E \left\{ \log_2 (1 + \gamma_o \Gamma_{CB}) \right\}$$
  
$$\leq \log_2 \left[ 1 + \gamma_o \left( 1 + (M - 1) \right) \right]$$
(10)

where (M-1) represents the SNR gain achieved by the coherent beamforming.

From (7) and (10), it can be seen that by using the conventional beamforming schemes, one can achieves either the MUD gain or beamforming gain. In the following, we will consider a new beamforming scheme that exploits both of them simultaneously.

#### IV. PROPOSED BEAMFORMING SCHEME

Fig. 2 illustrates the procedure of the proposed beamforming scheme. Each user estimates the SNR by using common pilot signal assuming the use of a beamforming weight optimized to its channel condition and then reports it to the BS. The BS selects a user based on the reported SNR. Then the selected user reports its optimum beam weight to



Fig. 2. Procedure of the proposed beamforming

the BS. Finally, the BS transmits data of the selected user using the optimum beam weight. Note that the opportunistic beamforming transmits the data using a beam generated with a random weight. Since the scheduler only needs the beam weight of the selected user, the amount of additional feedback signaling burden is marginal, except additional delay.

The optimum beam weight  $\mathbf{w}_k^o$  of user k for a given channel vector  $\mathbf{h}_k$  can be determined by

$$\mathbf{w}_{k}^{o} = \frac{\mathbf{h}_{k}^{H}}{\|\mathbf{h}_{k}\|}.$$
(11)

Assuming the scheduler selects a user with the largest channel gain, the effective channel gain of the selected user can be represented by

$$\Gamma_{MB-O} = \max_{k=1,2,\cdots,K} \{ \|\mathbf{h}_k\|^2 \}.$$
 (12)

Here, the subscript *MB-O* denotes the proposed beamforming scheme with the use of optimum beam weight. The corresponding channel capacity  $C_{MB-O}$  of the proposed scheme can be described by

$$C_{MB-O} = E \left\{ \log_2 \left( 1 + \gamma_o \Gamma_{MB-O} \right) \right\}.$$
<sup>(13)</sup>

Note that  $\Gamma_{MB-O}$  and  $\Gamma_{OB}$  are the maximum of *K* i.i.d. Chi-square random variables with 2*M* and 2 degrees of freedom, respectively. This implies that the proposed scheme provides a capacity larger than that of the opportunistic beamforming.

In a slow fading channel, however, a scheduler that selects a user in the best channel condition may not be desirable for traffics with strict delay requirement because it may select the same user for a long time. Therefore, we consider another realization of the proposed scheme where each user estimates SNR with the use of randomly generated beam weight instead of the optimum beam weight, providing same delay QoS as the opportunistic beamforming.

With the use of random beam weight  $\mathbf{w}_k$ , the effective channel of user k can be written as

$$\tilde{h}_k = \mathbf{h}_k \mathbf{w}_k \,. \tag{14}$$

The MISO channel vector  $\mathbf{h}_k$  can be decomposed by orthonormal bases  $\{\mathbf{O}_{k1}, \mathbf{O}_{k2}, \cdots, \mathbf{O}_{kM} \in \mathbf{C}^{1 \times M}\}$  as

$$\mathbf{h}_{k} = \alpha_{k1} \mathbf{O}_{k1} + \alpha_{k2} \mathbf{O}_{k2} + \dots + \alpha_{kM} \mathbf{O}_{kM} \,. \tag{15}$$

Since in spatially uncorrelated Rayleigh fading channel, each element of  $\mathbf{h}_k$  can be represented as an i.i.d. zero mean complex Gaussian random variable with unit variance,  $\{\alpha_{k1}, \alpha_{k2}, \dots, \alpha_{kM}\}$  can also be modeled as i.i.d. zero mean complex Gaussian random variables with unit variance for any orthonormal bases. Assuming that the first orthonormal basis is  $\mathbf{w}_k^H$  (i.e.,  $\mathbf{O}_{k1} = \mathbf{w}_k^H$ ), the effective channel can be represented as

$$\tilde{h}_{k} = \left(\alpha_{k1}\mathbf{w}_{k}^{H} + \alpha_{k2}\mathbf{O}_{k2} + \dots + \alpha_{kM}\mathbf{O}_{kM}\right)\mathbf{w}_{k}$$
  
=  $\alpha_{k1}$ . (16)

Let Q be the index number of the selected user. Then the effective channel gain of the selected user can be represented as

$$\left|\alpha_{\mathcal{Q}1}\right|^{2} = \max_{k=1,2,\cdots,K} \left\{\left|\alpha_{k1}\right|^{2}\right\}.$$
(17)

Since  $|\alpha_{Q1}|^2$  is the maximum of *K* i.i.d. exponential random variables, it has the same distribution as the effective gain in the opportunistic beamforming. However, the proposed scheme uses the optimum weight  $\mathbf{w}_Q^{\rho} = \mathbf{h}_Q^H / ||\mathbf{h}_Q||$  for the data transmission, producing an effective channel represented as

$$\tilde{h}_{\varrho} = \mathbf{h}_{\varrho} \mathbf{w}_{\varrho}^{o}$$
$$= \sqrt{\Gamma_{\rho B} + |\alpha_{\varrho 2}|^{2} + \dots + |\alpha_{\varrho M}|^{2}}.$$
(18)

Thus, the channel capacity of the proposed scheme can be described by

$$C_{MB-R} = E \left\{ \log_2 \left( 1 + \gamma_o (\Gamma_{OB} + |\alpha_{k,2}|^2 + \dots + |\alpha_{k,M}|^2) \right) \right\}.$$
(19)

Here, the subscript MB-R is used to indicate the proposed scheme with the use of random beam weight in the SNR calculation procedure. It can be shown that

$$C_{MB-R} \leq \log_2 \left( 1 + \gamma_o E \left| \Gamma_{OB} + \left| \alpha_{k,2} \right|^2 + \dots + \left| \alpha_{k,M} \right|^2 \right\} \right)$$
  
$$= \log_2 \left( 1 + \gamma_o \left( 1 + \sum_{i=2}^K \frac{1}{i} + (M-1) \right) \right) \equiv \overline{C}_{MB-R}.$$
 (20)

Note that the proposed scheme achieves both the MUD gain  $\sum_{i=2}^{K} 1/i$  and beamforming gain (M-1) simultaneously.

# *A.* Performance comparison with the conventional beamforming schemes

We compare the performance when the proposed scheme provides the same delay QoS with the opportunistic beamforming. We evaluate the performance in terms of upper bounds  $\overline{C}_{MB-R}$ ,  $\overline{C}_{OB}$  and  $\overline{C}_{CB}$ . Let  $G_{OB}$  and  $G_{CB}$  be the performance gains defined by

$$G_{OB} = \frac{\bar{C}_{MB-R}}{\bar{C}_{OB}} = \frac{\log_2 \left( 1 + \gamma_o \left( 1 + \sum_{i=2}^{K} \frac{1}{i} + (M-1) \right) \right)}{\log_2 \left( 1 + \gamma_o \left( 1 + \sum_{i=2}^{K} \frac{1}{i} \right) \right)}$$
(21)

and

$$G_{CB} = \frac{\overline{C}_{MB-R}}{\overline{C}_{CB}} = \frac{\log_2 \left( 1 + \gamma_o \left( 1 + \sum_{i=2}^{K} \frac{1}{i} + (M-1) \right) \right)}{\log_2 \left( 1 + \gamma_o M \right)}.$$
 (22)

Since  $\partial G_{OB} / \partial \gamma_o \leq 0$  and  $\partial G_{CB} / \partial \gamma_o \leq 0$ , the performance improvement of the proposed scheme over the conventional ones increases as the SNR decreases. Using the Lopital's theorem, it can be seen that the maximum performance gain is obtained as  $\gamma_o \rightarrow 0$ , i.e.,

$$\lim_{\gamma_o \to 0} (G_{OB}) = 1 + \frac{M - 1}{1 + \sum_{i=2}^{K} 1/i}$$
(23)

and

$$\lim_{\gamma_{c} \to 0} \left( G_{CB} \right) = 1 + \frac{\sum_{i=2}^{K} 1/i}{M}.$$
 (24)

On the other hand, it can be shown that no performance gain is achieved as  $\gamma_e \rightarrow \infty$ , i.e.,

$$\lim_{\gamma_o \to \infty} (G_{OB}) = \lim_{\gamma_o \to \infty} (G_{CB}) = 1.$$
(25)

Thus, the proposed scheme has a marginal gain over the conventional schemes in good channel condition, but it is quite effective in poor channel condition. In fact, from the information theoretic point of view, it can be shown that the proposed scheme with the optimum beam weight in the user selection procedure achieves the sum capacity of dirty paper coding (DPC)  $C_{DPC}$  as the average SNR approaches to zero, i.e.,

$$\lim_{\chi_{0} \to 0} \left( C_{MB-O} \right) = C_{DPC} \,. \tag{26}$$

This is due to that the sum capacity of the DPC converges to that of beamforming technique where the BS transmits a single data stream to a user in the best channel condition as the average SNR approaches to zero [12]. Since the DPC provides the maximum system capacity, it can be said that the proposed scheme is the optimum strategy in low SNR environment.

#### B. Performance with feedback information delay

In practice, accurate channel information may not be available at the transmitter. As the user mobility increases, the channel measurement delay can make serious mismatch between the measured channel and the actual one due to the channel variation. Let  $\mathbf{h}_k(t) = [h_{k,1}(t) \ h_{k,2}(t) \ \cdots \ h_{k,M}(t)]$  be the channel vector of user k at time t. As illustrated in Fig. 2, assume that the users report their SNR to the BS at time  $t + \tau_1$ , and finally the desired data is received to the user at time  $t + \tau_2$ . The correlation coefficient of the channel can be written as

$$\rho_{\tau_{1}} = E\{h_{k,i}(t)^{*}h_{k,i}(t+\tau_{1})\}$$

$$\rho_{\tau_{2}} = E\{h_{k,i}(t)^{*}h_{k,i}(t+\tau_{2})\}$$

$$\rho_{\tau_{2}-\tau_{1}} = E\{h_{k,i}(t+\tau_{1})^{*}h_{k,i}(t+\tau_{2})\}$$
(27)

where  $k \in \{1, \dots, K\}$ ,  $i \in \{1, \dots, M\}$  and  $(\cdot)^*$  denotes complex conjugate. The correlation coefficient is usually depends on the mobility of the user. For example, in rich scattering environment, correlation coefficient with the time difference of  $\tau$  can be represented as [17]

$$\rho_{\tau} = J_0(2\pi f_d \tau) \tag{28}$$

where  $J_0(\cdot)$  is the zeroth order Bessel function of the first kind and  $f_d$  is the maximum Doppler frequency. It is possible to completely describe the jointly Gaussian random variables in terms of the first and second order statistics [15]. Thus,  $\mathbf{h}_k(t+\tau_1)$  and  $\mathbf{h}_k(t+\tau_2)$  can be expressed in terms of  $\mathbf{h}_k(t)$  as

$$\mathbf{h}_{k}(t+\tau_{1}) = \rho_{\tau_{1}}\mathbf{h}_{k}(t) + \sqrt{1-\rho_{\tau_{1}}^{2}}\mathbf{z}_{k}$$
(29)

and

$$\mathbf{h}_{k}(t+\tau_{2}) = \rho_{\tau_{2}}\mathbf{h}_{k}(t) + \frac{\rho_{\tau_{2}-\tau_{1}}-\rho_{\tau_{1}}\rho_{\tau_{2}}}{\sqrt{1-\rho_{\tau_{1}}^{2}}}\mathbf{z}_{k} + \sqrt{1-\rho_{\tau_{2}}^{2}} - \left(\frac{\rho_{\tau_{2}-\tau_{1}}-\rho_{\tau_{1}}\rho_{\tau_{2}}}{\sqrt{1-\rho_{\tau_{1}}^{2}}}\right)^{2}}\mathbf{z}_{k}' \quad (30)$$

where  $\mathbf{z}_k, \mathbf{z}'_k \in \mathbf{C}^{1 \times M}$  are zero mean complex Gaussian random vectors whose elements have unit variance and  $\mathbf{h}_k(t)$ ,  $\mathbf{z}_k$  and  $\mathbf{z}'_k$  are independent of each other.

For a selected user Q, the effective channel gain with delay  $\tau_1$  and  $\tau_2$  can be represented as

$$\Gamma_{MB-R}(\tau_1,\tau_2) = E\left\{ \left\| \frac{\mathbf{h}_{\mathcal{Q}}(t+\tau_1)^H}{\|\mathbf{h}_{\mathcal{Q}}(t+\tau_1)\|} \mathbf{h}_{\mathcal{Q}}(t+\tau_2) \right\|^2 \right\}.$$
 (31)

where  $\mathbf{h}_{\varrho}(t+\tau_2)$  can be represented in term of  $\mathbf{h}_{\varrho}(t+\tau_1)$  as

$$\mathbf{h}_{\varrho}(t+\tau_{2}) = \alpha \mathbf{h}_{\varrho}(t+\tau_{1}) + \sqrt{1-\beta^{2}} \mathbf{z}_{\varrho}''.$$
(32)

Here  $\mathbf{z}''_{\varrho} \in \mathbf{C}^{\bowtie M}$  is zero mean complex Gaussian random vector with each element of unit variance and is independent of  $\mathbf{h}_{\varrho}(t+\tau_1)$ , and  $\alpha$  and  $\beta$  are the coefficients calculated from the correlation between  $\mathbf{h}_{\varrho}(t+\tau_1)$  and  $\mathbf{h}_{\varrho}(t+\tau_2)$ , and the variance of  $\mathbf{h}_{\varrho}(t+\tau_2)$ , respectively. After some derivation, it can be shown that

$$\alpha = \frac{\rho_{\tau_1} \rho_{\tau_2} \sum_{i=2}^{K} \frac{1}{i} + \rho_{\tau_2 - \tau_1} M}{\rho_{\tau_1} \sum_{i=2}^{K} \frac{1}{i} + M}$$
(33)

and

$$\beta = \sqrt{\frac{1}{M} \left(\sum_{i=2}^{K} \frac{1}{i}\right) \left(\alpha^{2} \rho_{\tau_{1}}^{2} - \rho_{\tau_{2}}^{2}\right) + \alpha^{2}} .$$
(34)

Then, (31) can be represented in terms of  $\alpha$  and  $\beta$  as



Fig. 3. Performance gain of the proposed beamforming scheme when M=4 and K=8

$$\Gamma_{MB-R}(\tau_{1},\tau_{2}) = E\left\{ \frac{\left\| \mathbf{h}_{\mathcal{Q}}(t+\tau_{1})^{H} \right\| \left( \alpha \mathbf{h}_{\mathcal{Q}}(t+\tau_{1}) + \sqrt{1-\beta^{2}} \mathbf{z}^{*} \right)^{2} \right\}}{\left\| \mathbf{h}_{\mathcal{Q}}(t+\tau_{1}) \right\|^{2}} \left\{ \alpha \mathbf{h}_{\mathcal{Q}}(t+\tau_{1}) + \sqrt{1-\beta^{2}} \mathbf{z}^{*} \right\|^{2} \right\}$$
(35)  
$$= \alpha^{2} E\left\{ \left\| \mathbf{h}_{\mathcal{Q}}(t+\tau_{1}) \right\|^{2} \right\} + \left(1-\beta^{2}\right) E\left\{ \left\| \mathbf{z}^{*} \right\|_{\mathcal{Q}} \frac{\left\| \mathbf{h}_{\mathcal{Q}}(t+\tau_{1})^{H} \right\|^{2}}{\left\| \mathbf{h}_{\mathcal{Q}}(t+\tau_{1}) \right\|} \right\|^{2} \right\}.$$

From (29) and the independence of  $\mathbf{z}''_{Q}$  and  $\mathbf{h}_{Q}(t+\tau_{1})$ , the mean of the effective channel gain can be obtained by

$$E\{\Gamma_{MB-R}(\tau_{1},\tau_{2})\} = \alpha^{2} \left(\rho_{\tau_{1}}^{2} \sum_{i=2}^{K} \frac{1}{i} + M\right) + \left(1 - \beta^{2}\right). \quad (36)$$

Finally, the upper bound  $\overline{C}_{MB-R}(\tau_1, \tau_2)$  with feedback delay  $\tau_1$  and  $\tau_2$  is given by

$$\overline{C}_{MB-R}(\tau_1, \tau_2) = \log_2 \left( 1 + \gamma_o E \left\{ \Gamma_{MB-R}(\tau_1, \tau_2) \right\} \right)$$
$$= \log_2 \left[ 1 + \gamma_o \left( \alpha^2 \left( \rho_{\tau_1}^2 \sum_{i=2}^K \frac{1}{i} + M \right) + \left( 1 - \beta^2 \right) \right) \right].$$
(37)

Note that the MUD gain is reduced by a factor of  $\alpha^2 \rho_{\tau_1}^2$  due to the additional feedback delay while the beamforming gain is reduced by a factor of  $\alpha^2$ .

Similarly, it can easily be shown that the opportunistic beamforming and coherent beamforming with feedback delay  $\tau_1$  have an upper bound represented as

$$\overline{C}_{OB}(\tau_1) = \log_2 \left[ 1 + \gamma_o \left( 1 + \rho_{\tau_1}^2 \sum_{i=2}^K \frac{1}{i} \right) \right]$$
(38)

and

$$\overline{C}_{CB}(\tau_1) = \log_2 \left[ 1 + \gamma_o \left( 1 + \rho_{\tau_1}^2 \left( M - 1 \right) \right) \right].$$
(39)

Assume that  $\tau_2 = 2\tau_1$ , which may be a very practical condition. Then, as the number of users decreases or the number of antennas increases,  $\alpha$  and  $\beta$  can be approximated to  $\rho_{\tau_1}$ , i.e.,  $\beta \approx \alpha \approx \rho_{\tau_1}$ . In this case, it can be easily be shown that

$$\overline{C}_{MB-R}(\tau_1,\tau_2) \ge \overline{C}_{OB}(\tau_1) \quad \text{iff } M - 1 \ge \left(1 - \rho_{\tau_1}^2\right) \sum_{i=2}^{K} \frac{1}{i} \quad (40)$$



Fig. 4. Performance gain associated with K when M=4 and SNR=0dB



Fig. 5. Performance gain associated with M when K=8 and SNR=0dB

and

$$\overline{C}_{MB-R}(\tau_1, \tau_2) \ge \overline{C}_{CB}(\tau_1) \quad \text{for any } \tau_1.$$
(41)

Note that  $\overline{C}_{MB-R}(\tau_1) \ge \overline{C}_{OB}(\tau_1)$  for all  $\rho_{\tau_1}$  provided that  $M-1 \ge \sum_{i=2}^{K} \frac{1}{i}$ .

## V. SIMULATION RESULTS

To verify the design and analysis of the proposed scheme, we evaluate the performance by computer simulation assuming that all users have mutually independent channel with the same average SNR. We assume a MISO configuration with a Rayleigh fading channel. Fig. 3 depicts the performance of the proposed scheme in terms of SNR when M=4 and K=8. It can be seen that the proposed scheme provides significant performance gain especially at low SNR. It can also be seen that the analytic results agree well with the simulation results.

Fig. 4 and Fig. 5 depict the performance gain of the proposed scheme in term of the number of users and transmitter antennas at SNR=0dB, respectively. It can be



(b) When SNR=10dB

Fig. 6. Performance with feedback signaling delay when M=4 and K=16

seen that the performance gain of the proposed scheme over the coherent beamforming increases as the number of users increases by exploiting the MUD gain. On the other hand, as the number of transmitter antennas increases, the proposed scheme has increased performance gain over the opportunistic beamforming by exploiting the beamforming gain.

Fig. 6 depicts the performance associated with feedback signaling delay when  $2\tau_1 = \tau_2$ . It can be seen that the proposed scheme can provides performance gain over the conventional schemes unless the normalized delay is larger than 0.15 (this value is somewhat associated with system parameters, e.g., *M* and *K*). For IS-856 system where  $\tau_1$  is about two time slots (i.e., 3.33ms) [18], this normalized delay 0.15 corresponds to a mobility of about 25km/h at 1.9 GHz carrier frequency. This simulation result indicates that the proposed scheme is quite effective for users in low mobility (e.g., nomadic environments).

#### VI. CONCLUSIONS

We have proposed an improved beamforming scheme by exploiting the advantages of opportunistic beamforming and coherent beamforming. By using a beam weight optimized to the selected user, the proposed scheme can achieve the beamforming gain as well as the MUD gain, without noticeable increase of feedback signaling burden. The performance of the proposed scheme is analytically evaluated using an upper bound method and verified by computer simulation. The simulation results show that the proposed scheme provides the performance gain over the conventional beamforming schemes, particularly significant in low SNR environment. They also show that the proposed scheme is quite effective unless the user mobility is too high in spite of an additional delay for the processing.

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