Optimum PSK signal constellation for Multi-Phase B-CDMA Systems

다중 위상 B-CDMA 시스템에서 최적의 PSK 신호 매핑

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Abstract: Although CDMA system can efficiently support multiple users, it suffers from large peak-to-average power ratio (PAPR) as the number of users increases. It requires the use of a highly linear power amplifiers with a large back off. Recently, a new CDMA scheme, called binary CDMA (B-CDMA), has been proposed to alleviate this problem [1]. In B-CDMA systems, the envelope of multi-user CDMA signals is truncated into a small number of levels to reduce the PAPR. The performance of B-CDMA system is mainly limited by two types of noise; the truncation and decision error. The truncation noise can be minimized by Lloyd-max algorithm [2]. In this paper, the optimum PSK signal constellation is analytically designed in multi-phase B-CDMA to minimize the decision errors. Finally, the analytic results are verified by computer simulation.

Keywords: B-CDMA, PAPR

Ⅰ. Introduction

Multi-code CDMA signals have high peak-to-average power ratio (PAPR) as the number of users increase. Therefore, multi-code CDMA transmitter requires highly linear power amplifiers with a large back off, which is power-inefficient and expensive.

Binary CDMA (B-CDMA) is a new modulation method that first truncates the signal amplitude into a small number of levels and then employs PSK-modulation in chip-level for transmission with constant envelope [1]. This enables the use of nonlinear power amplifiers. Thus, the B-CDMA can reduce the power amplifier burden, while keeping advantages of CDMA signaling such as soft capacity and robustness to the interference. However, the performance can significantly be affected by the truncation noise and decision error.

The B-CDMA signal can be generated by various methods, including the pulse-width (PW), multi phase (MP) and code selection (CS) methods [1]. The PW B-CDMA signal is obtained by converting the magnitude of multi-level signal into a finite number of pulse width. Thus, the transmission of PW B-CDMA system increases as the truncation level increases. In practice, the signal is truncated into two levels to accommodate the transmission bandwidth, which may cause substantial performance degradation. The MP B-CDMA signal is generated by transforming the signal amplitude into a finite number of PSK signal constellation. The CS B-CDMA signal is generated by two step process. First, the subset of spreading codes is selected to reduce the number of signal levels. The selected code is modulated using the MP B-CDMA scheme.

The Optimum truncation of the signal amplitude can be achieved using the Lloyd-max algorithm [2]. However, no analytic consideration has been given to the design of MP B-CDMA. Two signal points having the largest distance after the truncation are PSK-mapped such that the distance is still largest on the PSK signal constellation [3]. For given PSK signal constellation, the decision region can be determined so as to minimize the Bayes cost criterion [4]. However this may not be optimum because the bit error rate (BER) performance is more affected by PSK signal mapping rather than the decision region. In this paper, we consider the optimization of PSK mapping points of the MP B-CDMA system so as to minimize the decision error in additive white Gaussian noise (AWGN) channel. Since the CS B-CDMA is the same as the MP B-CDMA except the code selection block [5], the analytic results can also be applied to optimum design of the PSK mapping points in the CS B-CDMA.

Section Ⅱ describes the structure of the MP B-CDMA system. In Section Ⅲ, the noise due to truncation and decision errors is analyzed and approximated. We optimize the PSK mapping points so as to minimize the decision error using an iterative method. The proposed system is verified by computer simulation. Conclusions are summarized in Section Ⅳ.

Ⅱ. System Model

In the MP B-CDMA system, the sum of multiple user data is truncated into a finite number of levels and then modulated using a PSK modulator. Fig. 1 depicts the transceiver structure of a baseband-equivalent MP B-CDMA system with truncation level 4, where b_i and $\mathbf{c}_i$ respectively denote the bit and spreading code of the

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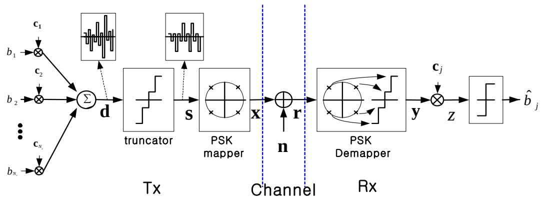

Fig. 1. Transceiver structure of a baseband-equivalent MP B-CDMA

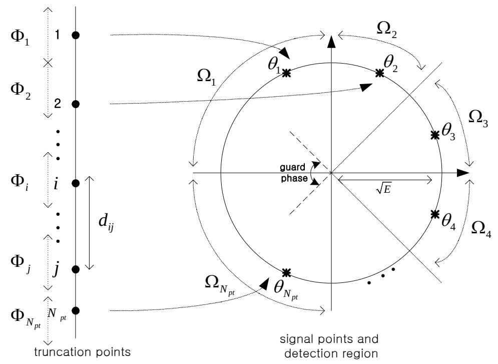

Fig. 2. PSK modulation of the MP B-CDMA

i-th user, and the signal $\mathbf{d}$ denotes the sum of multiple users data, given by

$$\mathbf{d} = \sum_{i=1}^{N_c} b_i \mathbf{c}_i \tag{1}$$

Here, N_c is the total number of users. The multiple user signal $\mathbf{d}$ is truncated into a finite number of levels at a chip-rate. The output of the truncator is represented as

$$\mathbf{s} = f_{tr}(\mathbf{d}) \tag{2}$$

where $f_{tr}(\mathbf{d})$ is the truncation function that maps the signal $\mathbf{d}$ in the truncation region Φ_i onto the signal point m_i as depicted in Fig.2. Then the truncated signal $\mathbf{s}$ is PSK-modulated as

$$\mathbf{x} = f_{map}(\mathbf{s}) \tag{3}$$

where $f_{map}(\mathbf{s})$ denotes the mapping function that maps the truncated signal $\mathbf{s}$ onto the PSK constellation. Fig. 2. depicts the truncation and PSK-mapping region of the MP B-CDMA system, where d_{ij} denotes the distance between the truncated signal point i and j, Ω_i denotes the detection region of the PSK-modulated signal point i, and N_{pt} is the number of signal points after the truncation. Note that the guard phase is required in the MP B-CDMA system to avoid errors between the signal points having a large distance [3].

The received signal can be represented as

$$\mathbf{r} = \mathbf{x} + \mathbf{n} \tag{4}$$

where $\mathbf{n}$ denotes the AWGN term. The PSK demodulator transforms the phase information of $\mathbf{r}$ into the magnitude

$$\mathbf{y} = f^{-1}{}_{map}(\mathbf{r}) \tag{5}$$

where $f^{-1}{}_{map}(\mathbf{r})$ denotes the demapping function that maps the phase of $\mathbf{r}$ into the magnitude. The demodulated signal $\mathbf{y}$ is despreaded using the spreading code $\mathbf{c}_j$ of the j-th user. Then, the user data $\hat{b}_j$ is decoded as

$$\hat{b}_j = \begin{cases} 1 & , if\ \mathbf{y} \bullet \mathbf{c}_j \geq 0 \\ -1 & , if\ \mathbf{y} \bullet \mathbf{c}_j < 0 \end{cases} \tag{6}$$

Ⅲ. Optimum PSK signal constellation for MP B-CDMA

Let s_j, δ_j and e_j be the signal component, truncation noise and noise due to chip error at the j-th chip, respectively. Then, the demodulated signal $\mathbf{y}$ at the j-th chip can be represented as

$$y_j = s_j + \delta_j + e_j \tag{7}$$

Since there is no correlation between δ_j and e_j, the variance of y_j is given by

$$\sigma_y{}^2 = \sigma_\delta{}^2 + \sigma_e{}^2 \tag{8}$$

where $\sigma_\delta{}^2$ and $\sigma_e{}^2$ are the variance of δ_j and e_j, respectively. Assuming that the spreading code of each user has unit power, the variance of $\mathbf{b}$ is equal to the number of multi-codes, N_c. The variance of the truncation noise per each chip, $\sigma_\delta{}^2$, can be represented as [6]

$$\sigma_\delta{}^2 = \sum_{i=1}^{N_{pt}} \int_{\Phi_i} (x - m_i)^2 \frac{1}{\sqrt{2\pi N_c}} \exp\left(-\frac{x^2}{2N_c}\right) dx \tag{9}$$

The optimum m_i and Φ_i minimizing $\sigma_t{}^2$ can be obtained using the Lloyd-max algorithm [2]. The Lloyd-max algorithm can find the optimum truncation points in an iterative manner so as to minimize the truncation noise power. The variance of e_j can be calculated as

$$\sigma_e{}^2 = \sum_{i=1}^{N_{pt}} \sum_{j=1}^{N_{pt}} p(j \mid i) p_s(i) d_{ij}{}^2 \tag{10}$$

where $p_s(i)$ is a probability of the signal point i given by

$$p_s(i) = \int_{\Phi_i} \frac{1}{\sqrt{2\pi N_c}} \exp\left(-\frac{x^2}{2N_c}\right) dx \tag{11}$$

and $p(j \mid i)$ is the error probability that the signal point i is misdetected to j [6]

$$p(j \mid i) = \int_{\Omega_j} \int_0^\infty \frac{1}{2\pi} e^{-\gamma_s \sin^2(\theta_r - \theta_i)} R e^{-(R - \sqrt{2\gamma_s} \cos(\theta_r - \theta_i))^2 / 2} dR d\theta_r \tag{12}$$

Here γ_s is the chip energy to noise power ratio (i.e., E_c / N_o) and θ_i is the phase of the signal point i. Provided that γ_s is high enough, it can be assumed that the most of chip errors correspond to the decision to the adjacent signal points. Thus the variance of the chip error can be approximated as

$$\sigma_e{}^2 = \sum_{i=1}^{N_{pt}} \sum_{j=i_-, i_+} p(j \mid i) p_s(i) d_{ij}{}^2 \tag{13}$$

where i_- and i_+ denote the adjacent signal points of the signal point i. Since (12) is a nonlinear function, we consider an approximate $p(j|i)$ for ease of mathematical analysis. It can be shown that

$$\begin{aligned} p(j|i) &= \int_{\Omega_j} \left[\frac{1}{2\pi} e^{-\gamma_s \sin^2(\theta_r)} \int_0^\infty R e^{-(R-\sqrt{2\gamma_s}\cos(\theta_r))^2/2} dR \right] d\theta_r \\ &= \int_{\Omega_j} \left[\frac{1}{2\pi} e^{-\gamma_s} + \frac{1}{2\pi} e^{-\gamma_s \sin^2(\theta_r)} \sqrt{\gamma_s \pi} \cos(\theta_r) erfc(-\sqrt{\gamma_s}\cos(\theta_r)) \right] d\theta_r \end{aligned} \quad (14)$$

Since the first term $\frac{1}{2\pi} e^{-\gamma_s}$ in (14) can be ignored at high E_b/N_0, (14) can be approximated as

$$\begin{aligned} p(j|i) &\simeq \int_{\Omega_j} \frac{1}{2\pi} e^{-\gamma_s \sin^2(\theta_r)} \sqrt{\gamma_s \pi} \cos(\theta_r) erfc(-\sqrt{\gamma_s}\cos(\theta_r)) d\theta_r \\ &= \int_{\Omega_j} \frac{1}{2\pi} e^{-\gamma_s \theta_r^2} \left[e^{-\gamma_s (\sin^2(\theta_r) - \theta_r^2)} \sqrt{\gamma_s \pi} \cos(\theta_r) erfc\left(-\sqrt{\gamma_s}\cos(\theta_r)\right) \right] d\theta_r \end{aligned} \quad (15)$$

It can be shown for a small θ_r, $p(j|i)$ can be further approximated as

$$p(j|i) \simeq \int_{\Omega_j} \frac{1}{2\pi} e^{-\gamma_s \theta_r^2} \sqrt{\gamma_s \pi} erfc\left(-\sqrt{\gamma_s}\right) d\theta_r \quad (16)$$

As θ_r increases, the difference between the integrand of (12) and (16) increases. However, the difference between the two integrals is negligible. Fig. 3 compares the validity of (12) and (16) when $E_b/N_0 = 11dB$, $N_c = 32$ and $N_{SF} = 128$. It can be seen that the approximation (16) is quite valid in the nominal operating condition.

Let the misdetection region Ω_j of signal point i to signal point j be from ω_j to ω_j' (i.e., $\Omega_j \in (\omega_j, \omega_j')$) in (16). Because the integrand in (16) decreases rapidly as θ_r increases and is not periodic, ω_j' can be set to be infinite. Therefore the variance of e_j can be represented as

$$\sigma_e^2 = \sum_{i=1}^{N_{pt}} \sum_{j=i_-, i_+} \left(\int_{\omega_j}^{\infty} \frac{1}{2\pi} e^{-\gamma_s \theta_r^2} \sqrt{\gamma_s \pi} erfc\left(-\sqrt{\gamma_s}\right) d\theta_r \right) \left(\int_{\Phi_i} \frac{1}{\sqrt{2\pi N_c}} \exp\left(-\frac{x^2}{2N_c}\right) dx \right) d_{ij}^2 \quad (17)$$

We consider the optimum PSK mapping points, $\theta_1, \theta_2, \cdots, \theta_{N_{pt}}$, that minimize σ_e^2. Since σ_e^2 is a convex function of θ_i, the optimum θ_i can be obtained by partially derivating σ_e^2 with regard to θ_i,

$$\frac{\partial \sigma_e^2}{\partial \theta_i} = 0 \quad (18)$$

Then, for a given θ_j, the optimum θ_i can be represented as a function of θ_j

$$\theta_i = f_i(\theta_j), \quad \forall j \neq i \quad (19)$$

For a given value of θ_j, θ_i can be found through an iterative method. It can be seen that θ_i is a function of E_b/N_0. This implies that the optimum mapping points will be different for E_b/N_0.

To verify the performance improvement, we evaluate the performance of the MP B-CDMA system with $N_{pt} = 7$ (i.e., 8-PSK) and one guard phase in an AWGN channel using computer simulation. The MP B-CDMA use a spreading code which is an extended PN sequence with $N_{SF} = 128$ and $N_c = 32$. The optimum θ_i is determined by (18). The

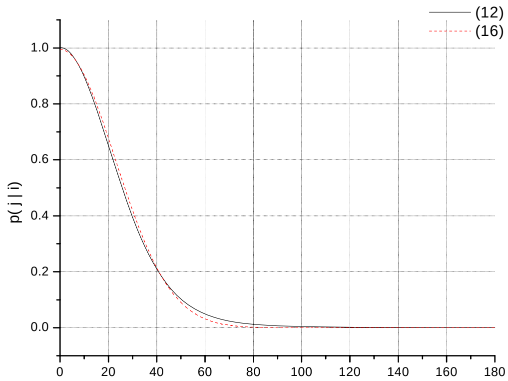

Fig.3. Approximation of $p(j|i)$

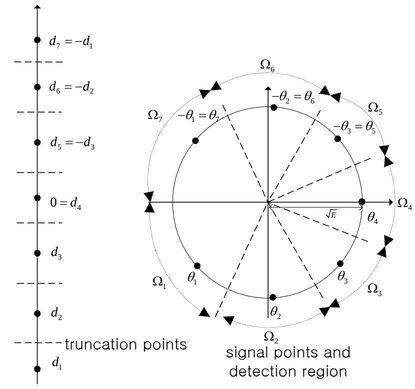

Fig. 4. Proposed 8-PSK signal constellation of the MP B-CDMA

$$\theta_1 = \frac{-\theta_2\gamma_s - 4\pi\gamma_s - 2\sqrt{\gamma_s}\sqrt{3\ln\left(\frac{4d_{17}^2 p_s(1)}{2d_{12}^2 p_s(1) + 2d_{12}^2 p_s(2)}\right) + \theta_2^2\gamma_s + 2\theta_2\pi\gamma_s + \pi^2\gamma_s}}{3\gamma_s}$$

$$\theta_2 = -\frac{4\log\left(\frac{2d_{23}^2 p_s(2) + 2d_{23}^2 p_s(3)}{2d_{12}^2 p_s(1) + 2d_{12}^2 p_s(2)}\right) - \theta_1^2\gamma_s + \theta_3^2\gamma_s}{2(\theta_1 - \theta_3)\gamma_s} \quad (20)$$

$$\theta_3 = \frac{4\log\left(\frac{2d_{34}^2 p_s(3) + 2d_{34}^2 p_s(4)}{2d_{32}^2 p_s(2) + 2d_{23}^2 p_s(3)}\right) + \theta_2^2\gamma_s}{2\theta_2\gamma_s}$$

proposed PSK mapping points $\hat{\theta}_1, \hat{\theta}_2$ and $\hat{\theta}_3$ can be found by iteratively searching (20). Conventional PSK mapping points have equal distance on the signal constellation with one guard phase, that is, $\theta_1 = -135°$, $\theta_2 = -90°$, $\theta_3 = -45°$. The optimum PSK mapping points can be found by exhaustive computer search.

Table.1 summarizes the PSK mapping points by the conventional, exhaustive search and proposed methods. Fig. 5 depicts the BER performance of MP B-CDMA system with the different PSK mapping points. It can be seen that the use of the proposed mapping points can provide the near optimum performance. Note that the proposed point $\hat{\theta}_1$ is close to the optimum point, but $\hat{\theta}_2$ and $\hat{\theta}_3$ are different from the optimum points. This implies that $\hat{\theta}_1$ with the large truncation

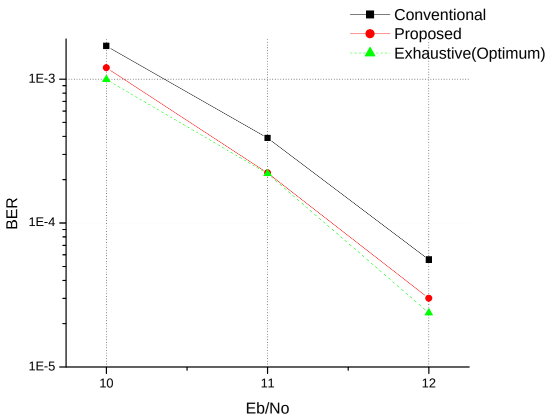

Fig. 5. performance due to different signal mapping

Table 1. PSK Mapping Points

E_b/N_0	Conventional	Exhaustive	Proposed
$10\,dB$	$-135°, -90°, -45°$	$-120°, -85°, -46°$	$-118°, -75°, -30°$
$11\,dB$	$-135°, -90°, -45°$	$-125°, -91°, -45°$	$-124°, -80°, -34°$
$12\,dB$	$-135°, -90°, -45°$	$-127°, -85°, -46°$	$-129°, -84°, -38°$

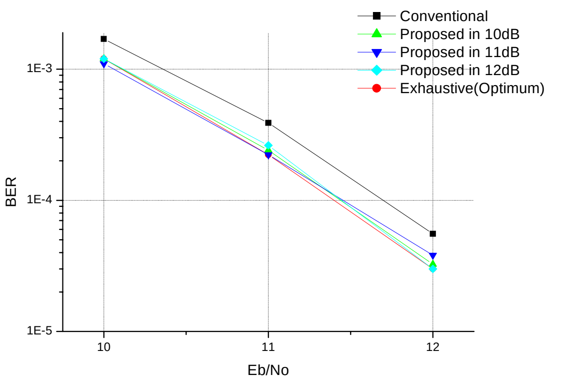

Fig. 6. Proposed Mapping Point

level is the dominant in the BER performance.

To see the effect of different E_b/N_0, the BER performance is compared when the signal mapping points optimized for different E_b/N_0 are used. It can be seen that the signal mapping optimized in the nominal operation condition can be used for other E_b/N_0 condition without significant performance degradation.

Ⅳ. Conclusion

In this paper, the optimum PSK mapping points of MP B-CDMA has been analyzed in AWGN channel. The proposed PSK mapping point is optimum in the sense that the mean square error due to chip error. For ease of calculation, there has been some approximation at high operating E_b/N_0. The simulation result says that the proposed scheme is nearly same with the optimum points which are obtained by exhaustive computer search. The use of the signal points yields about $0.5dB$ performance improvement over the use of conventional ones.

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