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Adaptive Grouping MMSE Channel Estimation in OFDM-Based Wireless Systems

Dae-Seung Ban, Cheong-Hwan Kim and Yong-Hwan Lee
School of Electrical Engineering and INMC
Seoul Nationl University
Kwanak P.O. Box 34, Seoul 151-600 Korea
e-mail: qks83@ttl.snu.ac.kr; kkins@ttl.snu.ac.kr; ylee@snu.ac.kr

***Abstract*-In this paper, we design an minimum mean-square error (MMSE)-type channel estimation scheme in orthogonal frequency division multiplexing-based wireless systems. By using the averaged channel frequency response (CFR) of pilot symbols in a group as a representative CFR , the proposed MMSE channel estimator can reduce the mean-square error (MSE) of channel estimation by achieving the signal-to-noise ratio (SNR) gain without increasing the computational complexity compared to conventional MMSE channel estimator. The grouping size of pilot symbols is optimized to minimize the MSE according to the channel correlation characteristics. Finally, the performance of the proposed channel estimation scheme is verified by computer simulation.**

Keywords: Channel estimation, grouping minimum mean-square error (MMSE), orthogonal frequency-division multiplexing (OFDM)

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is widely considered as a key technology for next generation wireless communication systems. It can provide high data rate transmission capability with high spectral efficiency even in multi-path delay environments. In practice, effective channel estimation techniques are highly desired for the purpose of improving the system performance since the radio channel is time varying and frequency selective for wideband mobile communication systems.

The channel information can be estimated by means of pilot signaling [1], [2] or blind estimation methods [3]. In order to obtain precise channel estimate, predetermined pilot signals are sent periodically in the time and frequency domain. Then, the channel between the pilot signals can be estimated by using an interpolation technique such as linear interpolation (LI) [4] and minimum mean-square error (MMSE) interpolation [5]. Although the MMSE-type channel estimator provides better performance than other estimators, it may involve implementation problems in practical systems [5].

In this paper, to alleviate the implementation problem and achieve the optimum performance with given complexity, we consider the use of a grouping for MMSE channel estimation. Conventional MMSE channel estimation scheme uses a single pilot signal for each filter tap. The proposed scheme uses a representative channel frequency response (CFR) corresponding to the average of CFRs of pilot symbols in a pilot group. The mean-square error (MSE) performance has trade-off between the SNR gain due to noise suppression and the accuracy of the representative CFR associated with the grouping size, The group size of the pilot symbols is determined by the power delay profile and Doppler frequency spectrum which represent the correlation characteristics in the frequency and time domain, respectively.

The rest of the paper is organized as follows. Section II describes an OFDM system model where pilot symbols are regularly transmitted in the time and frequency domain. The proposed grouping MMSE channel estimation scheme is described in Section III. The performance of the proposed scheme is verified by computer simulation in Section IV. Finally, conclusions are summarized in Section V.

II. SYSTEM MODEL

Consider the transmission of an OFDM signal over a wireless channel whose impulse response is represented as

$$h(t,\tau) = \sum_{l=0}^{L-1} h_l(t)\,\delta(\tau - \tau_l) \tag{1}$$

where L is the number of multi-paths, $h_l(t)$ and τ_l are respectively the complex-valued channel impulse response (CIR) and delay of the l-th path at time t, and $\delta(\cdot)$ is the Kronecker delta function. We assume that $h_l(t)$ is statistically independent for each path and has the same normalized correlation function $r_t(\Delta t)$ for all l. Then, the time-domain correlation of the l-th path CIR can be represented as [5].

$$r_l(\Delta t) = E\{h_l(t+\Delta t)\,h_l^*(t)\} = \sigma_l^2 r_t(\Delta t) \tag{2}$$

where $E\{X\}$ is the mean value of X, the superscript $*$ denotes complex conjugate and σ_l^2 is the average power of the l-th path.

The frequency response of the CIR at time t can be represented as

$$H(t,f) = \int_{-\infty}^{\infty} h(t,\tau) e^{-j2\pi f\tau} d\tau = \sum_{l=0}^{L-1} h_l(t) e^{-j2\pi f \tau_l}. \tag{3}$$

Assuming that the aggregated power of each path has a normalized gain (i.e., $\sum_{l=0}^{L-1}\sigma_l^2 = 1$), the correlation function of the frequency response can be represented as

$$\begin{aligned} r_H(\Delta t, \Delta f) &= E\{H(t+\Delta t, f+\Delta f)\,H^*(t,f)\} \\ &= r_t(\Delta t)\, r_f(\Delta f) \end{aligned} \tag{4}$$

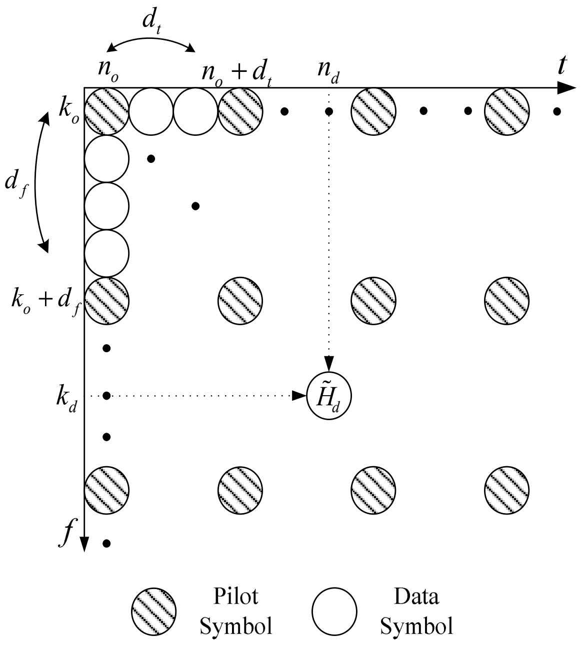

Fig. 1. Rectangular pilot pattern for OFDM Signal.

where $r_f(\Delta f)=\sum_{l=0}^{L-1}\sigma_l^2 e^{-j2\pi\Delta f\tau_l}$. For an OFDM symbol with symbol duration T_s and subcarrier spacing Δf , it can be shown that

$$r_H[n,k]=r_t[n]r_f[k] \tag{5}$$

where $r_t[n]=r_t(nT_s)$ and $r_f[k]=r_f(k\Delta f)$.

The received signal of the k-th subcarrier at the n-th symbol time can be represented as

$$Y[n,k]=H[n,k]X[n,k]+W[n,k],\quad \begin{matrix}1\le n\le N_t\\ 1\le k\le N_f\end{matrix} \tag{6}$$

where $H[n,k]$ is the CFR of the channel at the k-th subcarrier and the n-th symbol time, and $W[n,k]$ is the background noise approximated as zero-mean additive white Gaussian noise (AWGN) with variance σ_W^2 .

We consider the transmission of pilot signal in a rectangle pattern, which can provide nearly optimum performance while requiring low computational complexity as illustrated in Fig. 1 [6]. We assume that the pilot symbols are transmitted with a normalized power, while being separated by d_t and d_f in the time and frequency domain, respectively.

The BS estimates the CIR at the pilot symbol as [7]

$$\begin{aligned}\tilde{H}[n_p,k_p]&=Y[n_p,k_p]/X[n_p,k_p]\\&=H[n_p,k_p]+\tilde{W}[n_p,k_p]\end{aligned} \tag{7}$$

where n_p and k_p denote the symbol and subcarrier index of the pilot symbol, respectively, and $\tilde{W}[n_p,k_p]$ denotes the noise term approximated by zero-mean AWGN with variance σ^2 . Then, the CFR of the data symbol can be estimated by interpolating the CFR of the pilot symbol. In this paper, we consider the use of an MMSE filter for the interpolation [8]. Define the MSE of the channel estimation by

$$\sigma_\varepsilon^2=E\left\{\left|H(n_d,k_d)-\hat{H}(n_d,k_d)\right|^2\right\} \tag{8}$$

where $\hat{H}(n_d,k_d)$ denotes the estimated CFR of the data symbol at the n_d-th symbol time and the k_d-th subcarrier.

III. Proposed Adaptive Grouping MMSE Channel Estimation

For ease of description of the proposed scheme, we briefly describe conventional MMSE channel estimation scheme. The conventional MMSE channel estimator obtains the CFR at the n_d-th symbol time and the k_d-th subcarrier from adjacent N CFRs of the pilot symbol as

$$\hat{H}[n_d,k_d]=\mathbf{c}^H[n_d,k_d]\tilde{\mathbf{H}} \tag{9}$$

where $\mathbf{c}^H[n_d,k_d]$ is the coefficient vector of the MMSE filter, the superscript H denotes complex transpose and $\tilde{\mathbf{H}}$ is the $(N\times 1)$ pilot vector defined as

$$\tilde{\mathbf{H}}=[\tilde{H}[n_1,k_1]\,\tilde{H}[n_2,k_2]\cdots\tilde{H}[n_i,k_i]\cdots\tilde{H}[n_N,k_N]]^T \tag{10}$$

Here, the superscript T denotes transpose, and n_i and k_i denote the index of the i-th pilot in the time and frequency domain, respectively. The optimum coefficient of the MMSE filter can be determined by

$$\mathbf{c}^H[n_d,k_d]=\mathbf{p}^H[n_d,k_d]\mathbf{R}^{-1} \tag{11}$$

where $\mathbf{p}=E[\tilde{\mathbf{H}}\cdot H^*[n,k]]$ is the $(N\times 1)$ cross-covariance vector and $\mathbf{R}=E[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H]$ is the $(N\times N)$ auto-covariance matrix. As the number of filter tap increases, the performance of the MMSE channel estimator improves, but the implementation complexity increases geometrically due to the matrix inverse operation [9],[10].

To alleviate this implementation complexity and achieve the best performance, we consider the use of a representative CFR corresponding to each pilot group as illustrated in Fig. 2., where the representative CFR is achieved by averaging the CFRs of pilot symbols in each group. Assume that the representative CFR is obtained by averaging G_tG_f number of pilot symbols, as illustrated in Fig. 2.

Then, the representative CFR of the (t_i,f_j)-th group can be represented as

$$\begin{aligned}&\tilde{H}_g[t_i,f_j]\\&=\frac{1}{G_tG_f}\sum_{g_t=0}^{G_t-1}\sum_{g_f=0}^{G_f-1}\tilde{H}[n_o+\{(t_i-1)G_t+g_t\}d_t,\\&\qquad k_o+\{(f_j-1)G_f+g_f\}d_f]\\&=\frac{1}{G_tG_f}\sum_{g_t=0}^{G_t-1}\sum_{g_f=0}^{G_f-1}\begin{pmatrix}H[n_o+\{(t_i-1)G_t+g_t\}d_t,\\ k_o+\{(f_j-1)G_f+g_f\}d_f]\end{pmatrix}\\&\quad+\frac{1}{G_tG_f}\sum_{g_t=0}^{G_t-1}\sum_{g_f=0}^{G_f-1}\tilde{W}[n_o+\{(t_i-1)G_t+g_t\}d_t,\\&\qquad k_o+\{(f_j-1)G_f+g_f\}d_f]].\end{aligned} \tag{12}$$

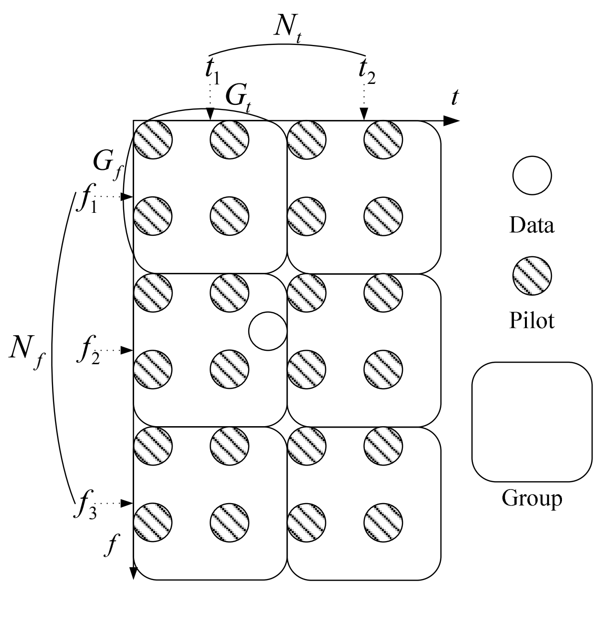

Fig. 2. Rectangular pilot pattern for OFDM Signal.

The CFR of the (n_d, k_d) -th data symbol can be estimated from the representative CFRs of $N_t N_f$ groups as

$$\hat{H}[n_d, k_d] = \sum_{t_i=0}^{N_t-1}\sum_{f_j=0}^{N_f-1} c_g^*[t_i, f_j]\tilde{H}_g[t_i, f_j] = \mathbf{c}^H \tilde{\mathbf{H}}_g \tag{13}$$

where $c_g[t_i, f_j]$ denotes the coefficient of the proposed filter and $\mathbf{c}_g$ is the filter coefficient vector defined as

$$\begin{aligned}\mathbf{c}_g \triangleq [c_g[0,0]\cdots c_g[0,N_f-1]\cdots \\ c_g[N_t-1,0]\cdots c_g[N_t-1,N_f-1]]^T.\end{aligned} \tag{14}$$

Similarly, the group pilot vector $\tilde{\mathbf{H}}_g$ can be represented as

$$\begin{aligned}\tilde{\mathbf{H}}_g \triangleq [\tilde{H}_g[n_o,k_o]\cdots \tilde{H}_g[n_o,k_o+N_f-1]\cdots \\ \tilde{H}_g[n_o+N_t-1,k_o+N_f-1]]^T.\end{aligned} \tag{15}$$

The corresponding MSE of the CFR is represented as [5]

$$\begin{aligned}\sigma_e^2 &= E\left\{\left|H(n_d,k_d)-\hat{H}(n_d,k_d)\right|^2\right\} \\ &= \sigma_h^2 - \mathbf{p}_g^H[n_d,k_d]\mathbf{c}_g[n_d,k_d] - \\ &\quad \mathbf{c}_g^H[n_d,k_d]\mathbf{p}_g[n_d,k_d] + \mathbf{c}_g^H[n_d,k_d]\mathbf{R}_g\mathbf{c}_g[n_d,k_d]\end{aligned} \tag{16}$$

where σ_h^2 denotes the gain of the wireless channel, $\mathbf{p}_g[n_d,k_d]$ $\left(\triangleq E\left\{\tilde{\mathbf{H}}_g H^*[n_d,k_d]\right\}\right)$ is the $(N_t N_f \times 1)$ cross-correlation vector defined as

$$\mathbf{p}_g[n_d,k_d] = \frac{1}{G_t G_f}\begin{bmatrix} \sum_{g_t=0}^{G_t-1}\sum_{g_f=0}^{G_f-1} E\{\tilde{H}[n_o+g_t d_t, k_o+g_f d_f]H^*[n_d,k_d]\} \\ \vdots \\ \sum_{g_t=0}^{G_t-1}\sum_{g_f=0}^{G_f-1} E\{\tilde{H}[n_o+\{(N_t-1)G_t+g_t\}d_t, \\ k_o+\{(N_f-1)G_f+g_f\}d_f]H^*[n_d,k_d]\} \end{bmatrix}. \tag{17}$$

Using the characteristic of the correlation function in (5), the $(N_t N_f \times 1)$ cross-correlation can be rewritten as

$$\mathbf{p}_g[n_d,k_d] = \frac{1}{G_t G_f}\begin{bmatrix} \{\sum_{g_t=0}^{G_t-1} r_t(n_o-n_d+g_t d_t)\cdot \sum_{g_f=0}^{G_f-1} r_f(k_o-k_d+g_f d_f)\} \\ \vdots \\ \{\sum_{g_t=0}^{G_t-1} r_t(n_o-n_d+\{(N_t-1)G_t+g_t\}d_t) \\ \cdot\sum_{g_f=0}^{G_f-1} r_f(k_o-k_d+\{(N_f-1)G_f+g_f\}d_f)\} \end{bmatrix}. \tag{18}$$

Here, $r_t(\cdot)$ and $r_f(\cdot)$ denotes the correlation function in the time and frequency domain, respectively, and $\mathbf{R}_g \triangleq E\left\{\tilde{\mathbf{H}}_g\tilde{\mathbf{H}}_g^H\right\}$ is the $(N_t N_f \times N_t N_f)$ autocorrelation matrix which the (i,j) -th element $R_g(i,j)$ defined as

$$\begin{aligned}R_g(i,j) &= \frac{1}{(G_t G_f)^2} E\left\{\begin{matrix}\sum_{g_t=0}^{G_t-1}\sum_{g_f=0}^{G_f-1}\tilde{H}[n_o+\{(i\backslash N_f-1)G_t+g_t\}d_t, \\ k_o+\{(i-i\backslash N_f-1)G_f+g_f\}d_f] \\ \sum_{g'_t=0}^{G_t-1}\sum_{g'_f=0}^{G_f-1}\tilde{H}^*[n_o+\{(j\backslash N_f-1)G_t+g'_t\}d_t, \\ k_o+\{(j-j\backslash N_f-1)G_f+g'_f\}d_f]\end{matrix}\right\} \\ &= \frac{1}{(G_t G_f)^2}\left(\sum_{g_t=0}^{G_t-1}\sum_{g'_t=0}^{G_t-1} r_t\left(((i\backslash N_f-j\backslash N_f)G_t+g_t-g'_t)d_t\right)\right) \\ &\quad\cdot\left(\sum_{g_f=0}^{G_f-1}\sum_{g'_f=0}^{G_f-1} r_f\left(((i-i\backslash N_f-j+j\backslash N_f)G_f+g_f-g'_f)d_f\right)\right) \\ &\quad+\frac{1}{G_t G_f}\sigma_W^2\delta(i-j)\end{aligned} \tag{19}$$

where the operator $A\backslash B$ denotes the quotient (i.e., $\lfloor A/B \rfloor$). Note that the second term $\sigma_W^2\delta(i-j)/G_t G_f$ is the considered noise component for channel estimation. It means that the representative CFR has lower noise variance than the original CFR does.

Let $\mathbf{c}_{g,\text{opt}}$ be the optimum be the optimum coefficient minimizing the MSE, which is determined as

$$\mathbf{c}_{g,\text{opt}}[n_d,k_d] = \mathbf{R}_g^{-1}\mathbf{p}_g[n_d,k_d]. \tag{20}$$

Then, the corresponding MSE can be represented as

$$\begin{aligned}\sigma_{e,\min}^2 &= \sigma_h^2 - \mathbf{p}_g^H[n_d,k_d]\mathbf{R}_g^{-1}\mathbf{p}_g[n_d,k_d] \\ &= \sigma_h^2 - \mathbf{c}_{g,\text{opt}}^H[n_d,k_d]\mathbf{p}_g[n_d,k_d].\end{aligned} \tag{21}$$

The MSE of the proposed scheme is affected by the grouping size. As the pilot grouping size (i.e., the number of pilot symbol per group) increases, the MSE increases due to inaccurate approximation of the representative CFR. For example, when the channel has low Doppler frequency and/or small delay spread, the use of a large pilot grouping size can achieve large SNR gain, while yielding a marginal loss due to approximation of the

TABLE I
SIMULATION PARAMETERS

Parameters	Value
Number of subcarriers	1024
Carrier frequency	2.375 GHz
Total bandwidth	8.75 MHz
Subcarrier spacing	9.765 KHz
Symbol duration	115.2 μs
Modulation	QPSK
Pilot symbol spacing	Time domain : 2 symbols Frequency domain : 7 subcarriers
Delay profile	Exponential decay
Doppler spectrum	Jake's spectrum
Number of taps	$(N_t, N_f) = (2, 2)$

representative CFR. It may be desirable to optimize the grouping size that minimizes the MSE for given channel correlation in the time and frequency domain. It can be shown from (18) and (19) that the optimum grouping size $G_{t,\text{opt}}$ and $G_{f,\text{opt}}$ can be determined by

$$\left(G_{t,\text{opt}}, G_{f,\text{opt}}\right) = \arg\max_{G_t, G_f} \left\{\mathbf{p}_g^H[n_d, k_d]\mathbf{R}_g^{-1}\mathbf{p}_g[n_d, k_d]\right\}. \tag{22}$$

IV. PERFORMANCE EVALUATION

The performance of the proposed scheme is verified by computer simulation. The system parameters for the simulation are summarized in Table I [11]. For fair comparison of the computational complexity, N_t and N_f are set to 2, respectively (i.e., the number of filter taps is 4). The performance of the system is evaluated in terms of the MSE associated with the channel condition.

Fig. 3 depicts the MSE associated with the pilot grouping size in the time when the RMS delay is $0.3\ \mu s$. It can be seen that the optimum grouping size in the time domain decreases as the user mobility increases, and vice versa. The use of a small $G_{t,\text{opt}}$ is desired in high mobility environments due to low correlation between the pilot symbols, but the use of a large $G_{t,\text{opt}}$ is desired in low mobility environments since the SNR gain due to the pilot grouping is larger than the loss due to inaccurate approximation of the representative CFR.

Fig. 4 depicts the MSE associated with the pilot grouping size in the frequency domain when the velocity is 40 km/h. It can be seen that $G_{f,\text{opt}}$ decreases as the RMS delay increases, and vice versa. This is mainly due to that the channel has frequency selective characteristics.

Fig. 5 compares the MSE of the two estimation schemes according to the velocity when the RMS delay is $0.7\ \mu s$. The optimum grouping size of the proposed scheme by (22) is also depicted. It can be seen that the optimum grouping size increases as the velocity decreases. It can also be seen that the proposed scheme outperforms the conventional MMSE scheme especially when the velocity is low.

Fig. 6 compares the MSE of the two channel estimation schemes according to the RMS delay. It can be seen that

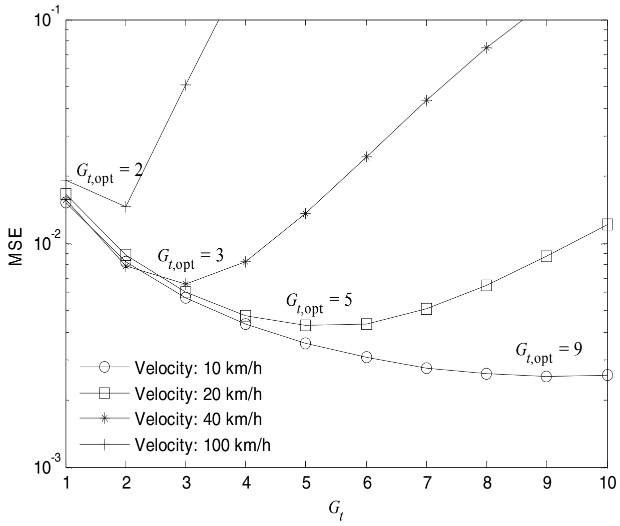

Fig. 3. MSE according to G_t when the RMS delay is $0.3\ \mu s$.

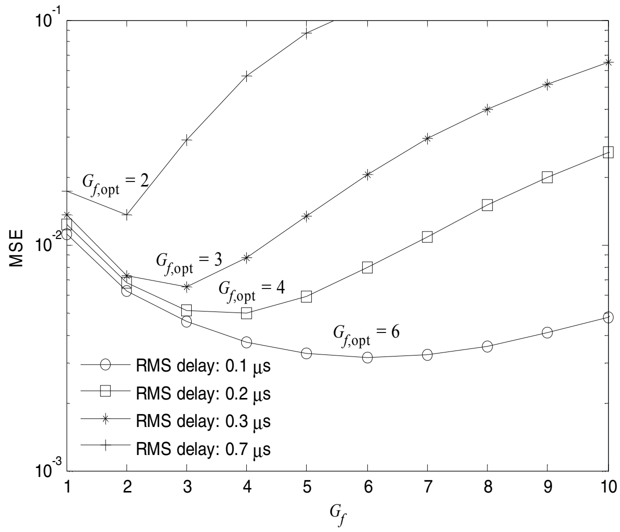

Fig. 4. MSE according to G_f when the velocity is 40 km/h.

the optimum grouping size increases as the RMS delay decreases. It can also be seen that the proposed scheme outperforms the conventional MMSE scheme especially when the channel selectivity is low.

Fig. 7 depicts the MSE in two channel conditions; one is a nominal channel condition where the velocity is 40 km/h and the RMS delay is $0.3\ \mu s$, and the other is a severe channel condition where the velocity is 100 km/h and the RMS delay is $0.7\ \mu s$. It can be seen that the proposed grouping MMSE channel estimation scheme outperforms the conventional MMES scheme when the channel condition is mild. As the channel condition becomes worse, the MSE performance gain over the conventional MMSE scheme becomes marginal. This is mainly because the proposed scheme achieves an SNR gain due to the noise suppression by means of averaging. This implies that this SNR gain may not be noticeable in high SNR environments, and the use of pilot grouping is also less effective in frequency selective and fast time-varying channel environments.

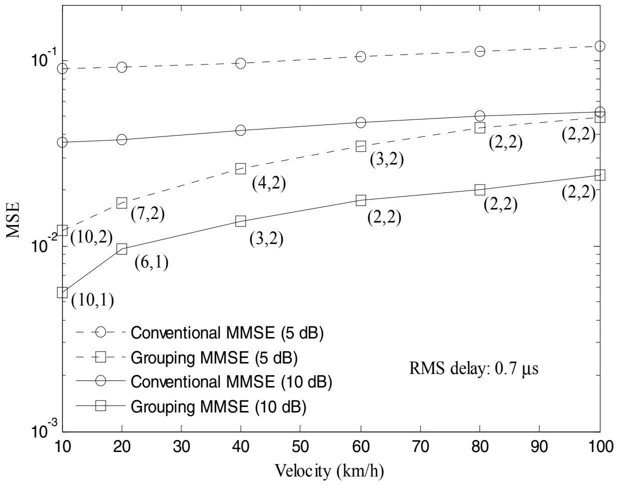

Fig. 5. MSE according to the velocity when RMS delay is 0.7 µs .

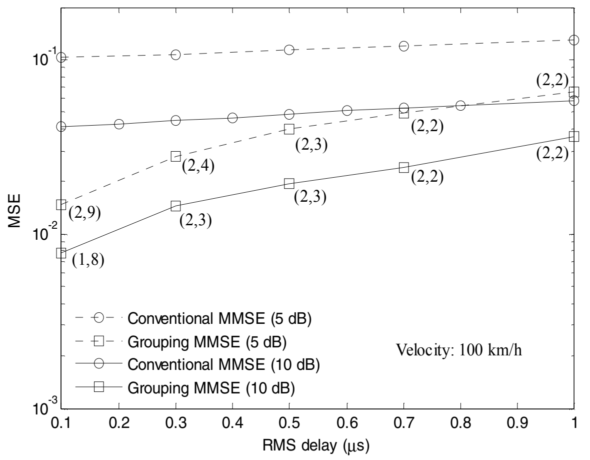

Fig. 6. MSE according to RMS delay when velocity is 100 km/h.

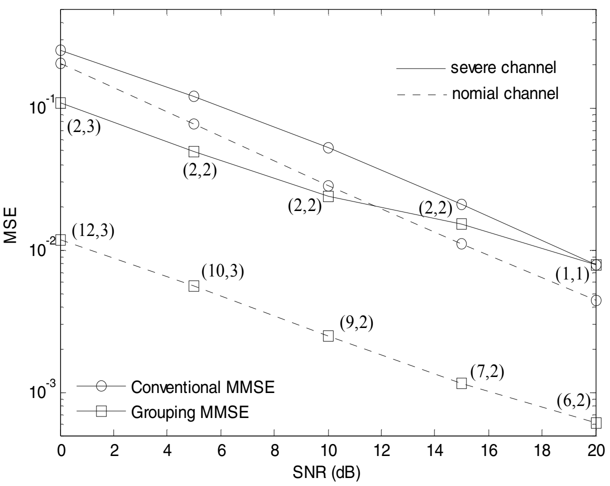

Fig. 7. MSE according to the SNR.

V. CONCLUSIONS

In this paper, we have proposed an MMSE-type channel estimator in an OFDM-based wireless system. The proposed estimation scheme can reduce the channel estimation error by using averaged channel response of pilot symbols as the representative CFR without increasing the computational complexity compared to conventional MMSE channel scheme. The grouping size is optimally determined to minimize the MSE associated with the channel correlation in the frequency and time domain. Finally, the simulation results show that the proposed channel estimation scheme is quite effective when the channel condition is mild.

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