Rapid Acquisition of PN Signals for DS/SS Systems Using a Phase Estimator

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***Abstract*—We propose a new scheme for rapid acquisition of PN signals in direct-sequence spread spectrum (DS/SS) systems by estimating the phase of the received PN signal with the use of an auxiliary signal. The auxiliary signal can be generated by a sum of the phase shifted PN signals. The phase of the incoming PN signal is estimated using the properties of cross correlation between the PN signal and the auxiliary signal. True phase alignment is detected using a conventional serial search scheme, where the initial phase of the local PN generators is set to a value obtained by the phase estimator. The performance of the proposed acquisition scheme is analytically evaluated in terms of the mean acquisition time. Numerical results show that the proposed scheme can achieve acquisition at least two times faster than the conventional scheme in nominal operating condition.**

***Index Terms*—Acquisition, auxiliary signal, DS/SS, PN signal.**

I. Introduction

IT IS FIRST required for reception of direct-sequence spread spectrum (DS/SS) signals to synchronize the phase of the local PN sequence with that of the transmitter [1]. The synchronization is normally achieved by a two-step sequential process of *acquisition* and *tracking*. Acquisition is a coarse synchronization process whereby the phase of the local PN sequence is coarsely aligned to that of the received PN sequence within a locking range of the tracking circuitry. The acquisition scheme is composed of the search scheme and the phase alignment detector. After correct acquisition, the tracking process provides fine synchronization of the two PN sequences. A number of acquisition schemes have been proposed that employ various kinds of search strategies and phase alignment detectors (see [2] and references therein).

Three types of search schemes have been widely considered: serial, parallel, and hybrid schemes [2]. Serial search schemes serially examine the possible code phases of the incoming PN signal. They are generally preferred due to their low implementation complexity, and their acquisition performance has been analyzed extensively in the literature [3]–[5]. In the absence of any *a priori* information about the phase of the incoming PN sequence, the serial search scheme starts the search from a randomly chosen phase and proceeds until true phase alignment is found. If *a priori* information on the phase of the incoming PN signal is available by some means, however, the serial search scheme can reduce the mean acquisition time by starting the search from the most probable region and expanding to less probable region [6]–[8]. In synchronous CDMA cellular systems such as IS-95 [9] and cdma2000 [10], the use of global positioning system (GPS) timing or triangulation with three base stations in the uplink can provide information on the phase of the PN codes, thus reducing the acquisition time. On the other hand, parallel acquisition schemes examine all the phases in parallel using a bank of correlators or matched filters, significantly reducing the acquisition time compared to the serial schemes [11]. However, the implementation complexity of parallel acquisition schemes becomes prohibitive, especially in the case of long PN sequences. To compromise the implementation complexity, hybrid schemes can be used with comparable performance [12].

The phase alignment detection problem can be modeled as testing a simple *in-sync* hypothesis against an alternative *out-of-sync* hypothesis [13]. It is well known that the use of the sequential probability ratio test (SPRT) results in the most efficient phase alignment detection scheme since it requires minimum average time for the hypothesis testing [14]. However, fixed sample size test (FSST) is widely used due to its simplicity of design and implementation. Unlike conventional phase alignment detectors that examine only one code phase at a time, a number of phase alignment detectors have been proposed that simultaneously examine multiple code phases [15], [16]. Corazza [15] showed that the mean acquisition time of a serial search scheme can be reduced by the use of a phase alignment detector employing the MAX/TC criterion, i.e., the maximum value among successive correlator outputs is compared to a threshold to detect phase alignment. Lin and Wei [16] proposed a phase alignment detector that correlates the received PN signal with a sum of M phase shifted PN signals to inspect M phases simultaneously, reducing the mean acquisition time. The rapid acquisition performance of these two schemes results mainly from the fact that the number of false alarms are reduced by jointly inspecting the possible phases.

Recently, a closed-loop acquisition system using an auxiliary signal has been proposed [17]. The auxiliary signal can be generated by a sum of the phase shifted PN signals so that its cross correlation with the PN signal has nonconstant magnitude over the entire PN signal period. In the receiver, the incoming PN signal is correlated with an earlier version and a delayed version of the auxiliary signal. The phase of the local PN signal is updated using the difference between the two cross-correlation outputs. While the phase tracking system updates the phase of the local PN signal, a separate phase alignment detector periodically tests alignment of the phase of the incoming PN signal

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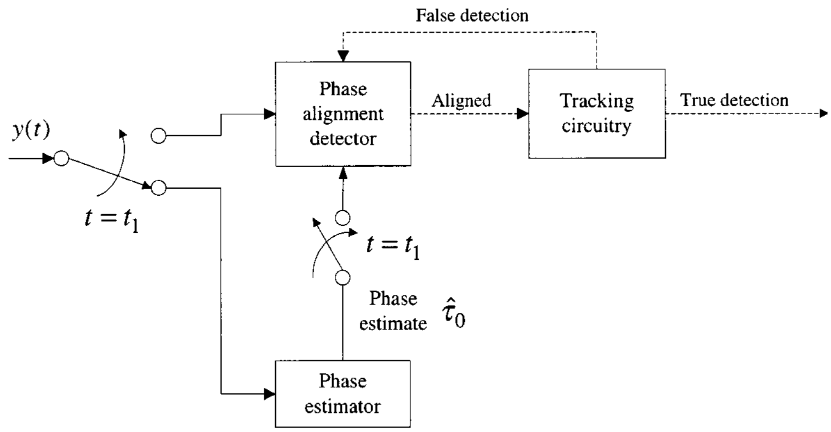

Fig. 1. Block diagram of the proposed acquisition system.

with that of the local PN signal. The acquisition performance can be improved by employing a preloop estimator that estimates the initial phase of the incoming PN signal and initializes the phase tracking system [18]. Although the closed-loop acquisition scheme provides good acquisition performance, the implementation complexity is increased due to the need for an additional closed-loop phase tracking system.

Since the cross correlation between the auxiliary signal and the incoming PN signal has a nonconstant value over the entire PN signal period, it can be directly applied to phase estimation of the incoming PN signal. We consider the use of the maximum likelihood method to estimate the phase of the incoming PN signal with the use of an auxiliary signal. The obtained phase estimate is used to initialize the phase of the local PN code generator, making a conventional serial search scheme rapidly find out true phase alignment.

In Section II, we describe the proposed acquisition system employing the phase estimator. The performance of the proposed acquisition system is analyzed in terms of the mean acquisition time in Section III. The performance of the proposed scheme is verified by computer simulation in Section IV. Finally, conclusions are given in Section V.

II. Proposed Acquisition Scheme

The proposed acquisition scheme consists of a phase estimator and a phase alignment detector as shown in Fig. 1. The proposed scheme first estimates the phase of the incoming PN signal by using a phase estimator based on the observation from time $t = 0$ to $t = t_1$. When a phase estimate is obtained at $t = t_1$, the phase alignment detector starts to search true phase alignment using a conventional serial search scheme, where the initial phase of the local PN code generator is set to a value obtained by the phase estimator. In this paper, we consider the use of a single dwell serial search scheme.

A. PN Signal and the Auxiliary Signal

Although data modulated PN signals can be applied to acquisition process [19], we consider the use of unmodulated pilot signals for the acquisition [9]. We assume for ease of description that the received signal is coherently demodulated. A baseband equivalent representation of the incoming PN signal can be given by

$$y(t) = \sqrt{P}c(t - \tau_0) + n(t) \tag{1}$$

where

- P average power of the received signal;
- τ_0 unknown phase of the incoming PN signal;
- $n(t)$ zero-mean additive white Gaussian noise (AWGN) with two-sided spectral density equal to $N_0/2$; and
- $c(t)$ PN signal expressed as

$$c(t) = \sum_{k=-\infty}^{\infty} c_k h(t - kT_C). \tag{2}$$

Here, c_k denotes the kth chip of a binary PN sequence which is periodic with a period of L, T_C denotes the chip duration, and $h(t)$ is the impulse response of the pulse shaping filter so that

$$\frac{1}{T_C}\int_{-\infty}^{\infty} |h(t)|^2\, dt = 1. \tag{3}$$

Without the loss of generality, it can be assumed that $\tau_0 \in [0, LT_C)$. We consider the use of m-sequences for $\{c_k\}$. Let γ_0 be the chip signal-to-noise ratio (SNR) defined by

$$\gamma_0 \triangleq \frac{PT_C}{N_0/2}. \tag{4}$$

An auxiliary signal can be generated by [17]

$$\alpha(t) = \sum_{i=-((L-3)/2)}^{(L-3)/2} \left[\frac{L-1}{2} - |i|\right] c(t - iT_C). \tag{5}$$

The cross correlation between $c(t)$ and $\alpha(t)$ is

$$\begin{aligned} R_{c\alpha}(\xi) &= \frac{1}{LT_C}\int_0^{LT_C} c(t+\xi)\alpha(t)\, dt \\ &= \sum_{i=-((L-3)/2)}^{(L-3)/2} \left[\frac{L-1}{2} - |i|\right] R_c(\xi + iT_C) \end{aligned} \tag{6}$$

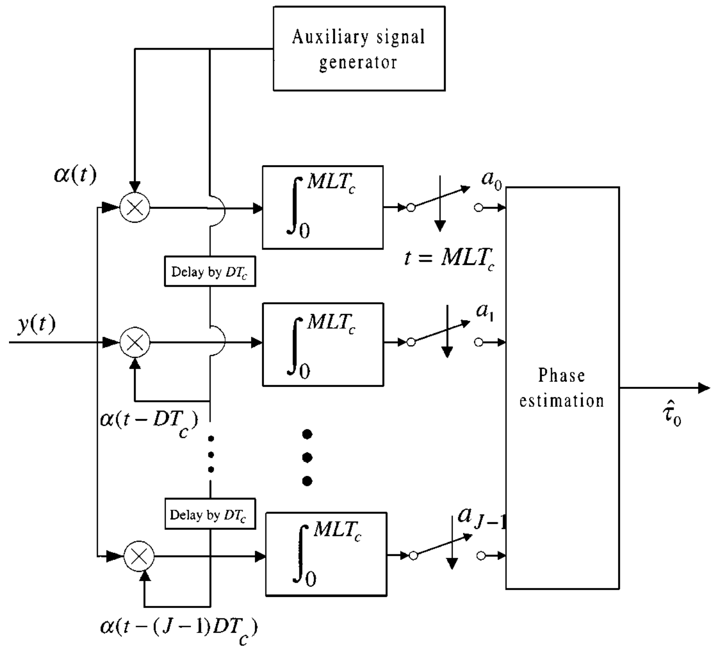

Fig. 2. Block diagram of the proposed phase estimator.

where $R_c(\xi)$ is the autocorrelation function of $c(t)$ given by

$$R_c(\xi) = \frac{1}{LT_C}\int_0^{LT_C} c(t+\xi)c(t)\,dt. \tag{7}$$

Note that the autocorrelation function $R_c(\xi)$ is periodic with period LT_C but it has a constant magnitude of nearly zero for $|\xi| > T_C$. Therefore, it cannot provide explicit information on the difference between the phase of the incoming PN signal and that of the locally generated PN signal.

Assuming that $h(t)$ is a unit-amplitude rectangular pulse having a duration of chip time interval T_C, the cross correlation between $c(t)$ and $\alpha(t)$ is given by

$$R_{c\alpha}(\xi) = \begin{cases} \dfrac{(L-1)(L+3)}{4L} - \dfrac{L+1}{LT_C}|\xi|, & |\xi| \le \dfrac{(L-1)T_C}{2} \\ -\dfrac{(L-1)^2}{4L}, & \dfrac{(L-1)T_C}{2} < |\xi| < \dfrac{(L+1)T_C}{2}. \end{cases} \tag{8}$$

Since $R_{c\alpha}(\xi)$ is periodic with period LT_C and has a nonconstant magnitude over the whole period, the use of the auxiliary signal can provide information on how much the phase of the auxiliary signal differs from that of the incoming PN signal.

B. Phase Estimator

As depicted in Fig. 2, we consider the use of J correlators in parallel to correlate the incoming signal $y(t)$ with the auxiliary signal $\alpha(t)$ for an interval of MLT_C, where J and M are integer numbers. The phase of the auxiliary signal used for the $(j+1)$th correlator is phase shifted by jDT_C with respect to that of the reference auxiliary signal $\alpha(t)$, where $j = 0, 1, \ldots, J-1$, and D is an integer equal to L/J. Let a_j be the sampled output of the $(j+1)$th correlator

$$\begin{aligned} a_j &= \int_0^{MLT_C} y(t)\alpha(t - jDT_C)\,dt \\ &= s_j(\tau_0) + \eta_j \end{aligned} \tag{9}$$

where $s_j(\tau_0)$ and η_j are the signal and the noise terms, respectively, given by

$$s_j(\tau_0) = \sqrt{P}MLT_C R_{c\alpha}(jDT_C - \tau_0) \tag{10}$$

$$\eta_j = \int_0^{MLT_C} n(t)\alpha(t - jDT_C)\,dt. \tag{11}$$

Note that the variance of η_j depends upon the characteristics of the specific PN sequence c_k. However, when the PN sequence has a long period, it can be approximated as a random binary sequence [20]. Assuming the use of long PN sequences as in commercial CDMA systems, it can easily be shown that the variance of η_j is approximated by [18]

$$\begin{aligned} \mathrm{Var}\{\eta_j\} &= \frac{ML(L-1)\left(L^2-4L+1\right)N_0T_C}{24} \\ &\simeq \frac{ML^4N_0T_C}{24}. \end{aligned} \tag{12}$$

Let us define the SNR of a_j by

$$\gamma(a_j) \triangleq \frac{s_j^2(\tau_0)}{\mathrm{Var}\{\eta_j\}} \tag{13}$$

and the normalized phase θ_0 by

$$\theta_0 \triangleq \frac{\tau_0}{LT_C}, \qquad 0 \le \theta_0 < 1. \tag{14}$$

Then, it can be shown that

$$\begin{aligned}\gamma(a_j) &\simeq 24M\,\frac{PT_C}{N_0}\left[\frac{R_{c\alpha}(jLT_C/J - LT_C\theta_0)}{L}\right]^2\\ &= 12M\gamma_0\left[\frac{R_{c\alpha}(LT_C\beta_j)}{L}\right]^2\end{aligned} \tag{15}$$

where $\beta_j = j/J - \theta_0$. It can be seen from (8) that, for a given β_j, $R_{c\alpha}(LT_C\beta_j)/L$ is almost independent of L when $L \gg 1$. This implies that $\gamma(a_j)$ depends on M and γ_0, but not on L.

Since the auxiliary signal $\alpha(t)$ is a sum of PN signal $c(t)$, it has randomness properties similar to $c(t)$. Thus, the noise samples in (11) can be modeled as uncorrelated Gaussian random variables. The maximum-likelihood (ML) estimate of the code phase $\hat{\tau}_0$ is the least square estimate given by

$$\begin{aligned}\hat{\tau}_0 &= \arg\min_{\tau}\sum_{j=0}^{J-1}\left[\sqrt{P}MLT_C R_{c\alpha}(jDT_C-\tau) - a_j\right]^2\\ &= \arg\min_{\tau}\left\{\sum_{j=0}^{J-1}\left[\sqrt{P}MLT_C R_{c\alpha}(jDT_C-\tau)\right]^2\right.\\ &\quad\left. - 2\sqrt{P}MLT_C\sum_{j=0}^{J-1}R_{c\alpha}(jDT_C-\tau)a_j + \sum_{j=0}^{J-1}a_j^2\right\}\end{aligned} \tag{16}$$

where $0 \le \tau < LT_C$. Since it is very complicated to find the solution of (16), we consider a simple method to find $\hat{\tau}_0$ with some approximation.

From the characteristics of $R_{c\alpha}(\xi)$, as J increases, the first term in (16) rapidly approaches a constant independent of τ [21]. Since the third term in (16) is also a constant, an approximate solution of the problem (16) can be obtained by finding τ that maximizes the second term

$$f(\tau) = \sum_{j=0}^{J-1} R_{c\alpha}(jDT_C - \tau)a_j. \tag{17}$$

Since $R_{c\alpha}(\xi)$ is periodic with period LT_C and even symmetric with respect to $\xi = 0$, it can be expressed by the Fourier cosine series

$$R_{c\alpha}(\xi) = \sum_{i=0}^{\infty} r_i\cos\left(\frac{2\pi i}{LT_C}\,\xi\right) \tag{18}$$

where r_is are the Fourier coefficients of $R_{c\alpha}(\xi)$. Therefore, $f(\tau)$ can be represented by

$$f(\tau) = \sum_{j=0}^{J-1}\sum_{i=0}^{\infty} r_i\cos\left[\frac{2\pi i}{LT_C}\,(jDT_C-\tau)\right]a_j. \tag{19}$$

Using the trigonometric identity and exchanging the order of summation, we have

$$\begin{aligned}f(\tau) = \sum_{i=0}^{\infty} r_i\sum_{j=0}^{J-1}&\left(a_j\cos\frac{2\pi ij}{J}\cos\frac{2\pi i\tau}{LT_C}\right.\\ &\left.+ a_j\sin\frac{2\pi ij}{J}\sin\frac{2\pi i\tau}{LT_C}\right).\end{aligned} \tag{20}$$

By letting

$$I_i = \sum_{j=0}^{J-1} a_j\cos\frac{2\pi ij}{J}$$
$$Q_i = \sum_{j=0}^{J-1} a_j\sin\frac{2\pi ij}{J}$$

it can be shown that

$$f(\tau) = \sum_{i=0}^{\infty} r_iG_i\cos\left(\frac{2\pi i\tau}{LT_C} - \phi_i + \delta_i\right) \tag{21}$$

where

$$\begin{aligned}G_i &= \sqrt{I_i^2 + Q_i^2}\\ \phi_i &= \tan^{-1}(Q_i/I_i)\\ \delta_i &= \begin{cases}0, & \text{if } I_i > 0\\ \pi, & \text{otherwise.}\end{cases}\end{aligned}$$

Evaluating the magnitude of r_i's, we can see that the magnitude of r_1 is larger than that of $r_i, i \ge 2$, by more than 20 dB regardless of L. Therefore, an approximate solution of the problem (16) can be obtained by finding τ that maximizes

$$f_1(\tau) = r_1G_1\cos\left(\frac{2\pi\tau}{LT_C} - \phi_1 + \delta_1\right). \tag{22}$$

Thus, an approximate phase estimate $\hat{\tau}_0$ of the incoming PN signal is given by

$$\hat{\tau}_0 = \frac{LT_C}{2\pi}\,(2\pi m + \phi_1 - \delta_1) \tag{23}$$

where m is an integer such that $\hat{\tau}_0 \in [0, LT_C)$. The estimate $\hat{\theta}_0$ of the normalized phase θ_0 can be obtained by

$$\hat{\theta}_0 = m + \frac{1}{2\pi}\,(\phi_1 - \delta_1). \tag{24}$$

It should be noted that the proposed phase estimator does not require the knowledge on the power P of the incoming PN signal, although its performance depends on P. Note also that the shape of $R_{c\alpha}(\xi)$ affects the performance of the proposed phase estimator. If the pulse shaping filter has an impulse response different from the rectangular pulse shape, it may affect the shape of $R_c(\xi)$. Since the shape of $R_{c\alpha}(\xi)$ is affected mainly by the triangular weighting of the auxiliary signal and little by $R_c(\xi)$, the characteristics of the shaping filter $h(t)$ do not seriously affect the performance of the proposed acquisition scheme.

Since ϕ_1 and δ_1 are obtained from a_js whose SNR $\gamma(a_j)$ is independent of L, they are statistically independent of L. Let θ_ϵ be the normalized phase estimation error defined by

$$\theta_\epsilon \triangleq \hat{\theta}_0 - \theta_0 \tag{25}$$

and τ_ϵ be the phase estimation error defined by

$$\tau_\epsilon \triangleq \hat{\tau}_0 - \tau_0 = LT_C\theta_\epsilon. \tag{26}$$

It can be seen that θ_ϵ is statistically independent of L, but dependent upon γ_0 and M.

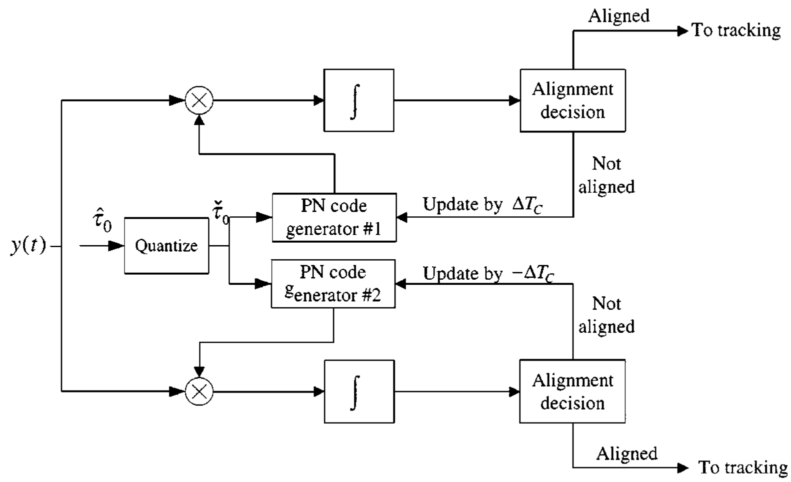

Fig. 3. Block diagram of the phase alignment detector.

C. Phase Alignment Detector

The phase alignment detector begins the search for alignment of the phase of the local PN signal with that of the incoming PN signal using a phase estimate obtained by the phase estimator. The estimated phase is first quantized with respect to the step size ΔT_C for phase update, yielding $\check{\tau}_0$,

$$\check{\tau}_0 = \left[\!\left[\frac{\hat{\tau}_0}{\Delta T_C} \right]\!\right] \Delta T_C \tag{27}$$

where $[\![x]\!]$ denotes an integer nearest to x. The normalized step size Δ is usually set to a value of 1 or 1/2, depending upon the locking range of a tracking circuitry. Note that there are L/Δ phases to be examined for each period of the PN signal. The quantized phase estimate $\check{\tau}_0$ is used to initialize the local PN code generators.

To achieve rapid acquisition, two serial search schemes are employed in parallel, each of which consists of a PN code generator, a correlator, and a phase alignment decision device. Fig. 3 depicts a block diagram of the phase alignment detector used in the proposed acquisition scheme. The phase of each PN code generator is updated in the direction opposite from each other from the initial phase $\check{\tau}_0$. The phase alignment detector correlates the received signal $y(t)$ with the local PN signal for a fixed interval and makes a decision by comparing the correlator output with a threshold. If a phase alignment between the two PN signals is detected by one of the serial search schemes, the tracking circuitry starts fine synchronization process. Otherwise, the PN code generators update their phases by $\pm\Delta T_C$ and phase alignment search continues. If phase alignment is simultaneously declared by both the serial search schemes, the PN signals of the two PN code generators are applied to the tracking circuitry one after another for verification. When all L/Δ phases have been checked without any phase alignment detection, the phase alignment search resumes with the phases of the two local PN code generators reset to the initial phase $\check{\tau}_0$.

III. Performance Analysis

Assume that the phases of the local PN code and the incoming PN code are uT_C and vT_C, respectively, and that the locking range of the phase tracking circuitry is $\pm T_C$. Then, acquisition

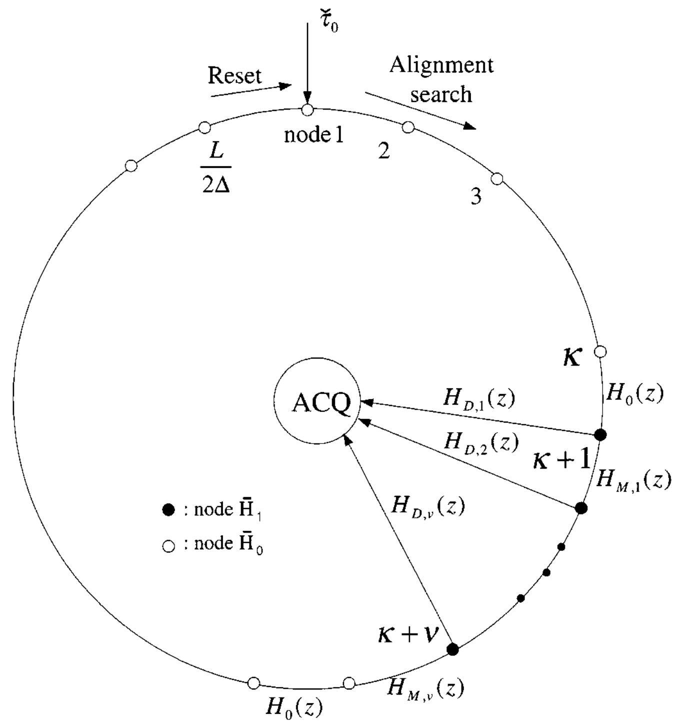

Fig. 4. Flow graph diagram of the proposed acquisition scheme.

can be obtained when the absolute phase offset between the two PN sequences is less than T_C, i.e., $|\delta| \triangleq |u - v| < 1$. The phase alignment detection problem can be modeled as testing a simple hypothesis against a simple alternative one. Let H_1 be the hypothesis that $|\delta| < 1$ and H_0 be the alternative hypothesis that $|\delta| \geq 1$. When the phase of the local PN code generator is updated by ΔT_C, there can be cases of up to $2/\Delta$ or $((2/\Delta)-1)$, corresponding to hypothesis H_1 [3].

To analyze the performance of the proposed acquisition scheme, we can use the flow graph technique [4]. Assume that the difference between the initial phase of the local PN code generator $\check{\tau}_0$ and the nearest H_1 phase is $\kappa\Delta T_C$, and that there are νH_1 phases, where κ and ν are integers. Then, a flow graph diagram of the acquisition scheme can be represented as depicted in Fig. 4, where the alignment search proceeds from node 1 in a clockwise direction. Note that the two serial

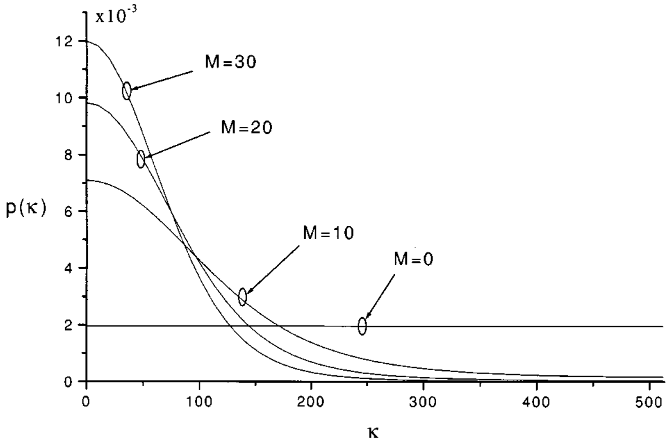

Fig. 5. The pmf of κ when $J = 3, \gamma_0 = -10$ dB, $\Delta = 1$, and $L = 1023$.

search schemes are treated as a single serial search scheme that examines two phases simultaneously. The lth node represents the two phases, $[\tilde{\tau}_0 + (l-1)\Delta T_C]$ and $[\tilde{\tau}_0 - (l-1)\Delta T_C]$, simultaneously examined by the two serial search schemes. At node $\overline{H}_0$ marked by an open circle, none of the two phases is in hypothesis H_1. At node $\overline{H}_1$ marked by a solid circle, one or both of the two phases can be in hypothesis H_1.

Since there are ν H_1 phases among L/Δ phases, the probability that both of the two search schemes are in hypothesis H_1 is equal to $\nu\Delta/L$, assuming that the uncertain region is uniformly distributed before phase estimation. After phase estimation, the probability can be expressed by $(\nu\Delta/L)\rho$, where ρ denotes a factor accounting for the improvement due to phase estimation. Letting $p(\kappa)$ be the probability mass function (pmf) that the first $\overline{H}_1$ node is at the $(\kappa+1)$th node, the value of ρ can be approximately determined from the probability of $p(\kappa)$ at $\kappa = 0$. Fig. 5 depicts $p(\kappa)$ for different values of M, when $J = 3$, $\gamma_0 = -10$ dB, $\Delta = 1$, and $L = 1023$. It can be seen that as M increases, the pmf $p(\kappa)$ is getting more concentrated around $\kappa = 0$. When $M = 10$, 20, and 30, ρ is equal to 3.5, 5, and 6, respectively. Since $(\nu\Delta/L)\rho \ll 1$ in most of practical situations, it can be assumed without the loss of analytical accuracy that only one of the two phases is in hypothesis H_1.

Let P_F be the probability of false phase alignment detection, T_D be the dwell time of the phase alignment detector, and T_P be the penalty time due to a false detection. At node $\overline{H}_0$, there are three possible detection cases by the two search schemes: no false detection with probability $(1-P_F)^2$, one false detection with probability $2P_F(1-P_F)$, and two false detections with probability P_F^2. Since these three cases require detection time T_D, (T_D+T_P), and (T_D+2T_P), respectively, the branch gain from a node $\overline{H}_0$ to the next node ($\overline{H}_1$ or $\overline{H}_0$) can be represented by

$$H_0(z) = (1-P_F)^2 z^{T_D} + 2P_F(1-P_F)z^{T_D+T_P} + {P_F}^2 z^{T_D+2T_P}. \quad (28)$$

At node $\overline{H}_1$, two transitions can happen. One is to reach the acquisition (ACQ) node and the other one is to proceed to the next $\overline{H}_1$ or $\overline{H}_0$ node. Let $H_{M,i}(z), i = 1, \ldots, \nu$, denote the branch gain from the ith $\overline{H}_1$ node to the next $\overline{H}_1$ or $\overline{H}_0$ node, due to a miss detection. The gain $H_{M,i}(z)$ can be calculated by considering that one search scheme that inspects an H_0 phase declares either H_1 or H_0, while the other one that inspects an H_1 phase declares H_0. Since the time spent is T_D for no false detection and (T_D+T_P) for a false detection, $H_{M,i}(z)$ can be represented by

$$H_{M,i}(z) = (1-P_{D,i})\left[(1-P_F)z^{T_D} + P_F z^{T_D+T_P}\right] \quad (29)$$

where $P_{D,i}$ represents the true detection probability of the alignment detector at the ith $\overline{H}_1$ node.

Let $H_{D,i}(z)$ be the branch gain from the ith $\overline{H}_1$ node to the ACQ node. In the case of a true phase alignment detection, one search scheme declares H_1 and proceeds to the ACQ node, while the other one may declare either H_1 or H_0. It should be noted that a false detection can happen either before or after acquisition with equal probability, and that only the former case requires a penalty time of T_P. The gain $H_{D,i}(z)$ can be calculated by considering the following three cases determined by the search scheme that inspects an H_0 phase: no false detection, false detection before acquisition, and false detection after acquisition. Since these three cases require detection time T_D, (T_D+T_P), and T_D, respectively, it follows that

$$H_{D,i}(z) = P_{D,i}\left[(1-P_F)z^{T_D} + \tfrac{1}{2}P_F z^{T_D+T_P} + \tfrac{1}{2}P_F z^{T_D}\right]. \quad (30)$$

To simplify the expression, ν $\overline{H}_1$ nodes can be aggregated into a single *collective* node, denoted by $\overline{H}_{1,c}$ in what follows, with branch gain $H_D(z)$ for true detection and $H_M(z)$ for miss detection given by

$$H_M(z) = \prod_{i=1}^{\nu} H_{M,i}(z) \quad (31)$$

$$H_D(z) = \sum_{i=1}^{\nu} H_{D,i}(z) \prod_{j\geq 1}^{i-1} H_{M,j}(z). \quad (32)$$

The transfer function from the node 1 to the node ACQ can be obtained by

$$H_{ACQ,\kappa}(z) = \Gamma(z){H_0}^{\kappa}(z) \quad (33)$$

where $\Gamma(z)$ is the loop gain from the collective node $\overline{H}_{1,c}$ to the node ACQ given by

$$\Gamma(z) = \frac{H_D(z)}{1 - H_M(z)H_0^{m-1}(z)} \quad (34)$$

and m denotes the total number of nodes after aggregation of $\overline{H}_1$ nodes, equal to $((L/2\Delta) - \nu + 1)$. The overall transfer function is given by

$$H_{ACQ}(z) = \sum_{\kappa=0}^{m-1} H_{ACQ,\kappa}(z)p(\kappa). \quad (35)$$

The mean time $\overline{T}_S$ for phase alignment search is obtained by

$$\overline{T}_S = \left.\frac{\partial H_{ACQ}(z)}{\partial z}\right|_{z=1}. \quad (36)$$

Using the result that $H_0(1) = \Gamma(1) = 1$ and letting

$$H_0^{(1)}(z) \triangleq \frac{\partial H_0(z)}{\partial z} \quad \text{and} \quad \Gamma^{(1)}(z) \triangleq \frac{\partial \Gamma(z)}{\partial z}$$

the mean search time is given by

$$\overline{T}_S = H_0^{(1)}(1)E\{\kappa\} + \Gamma^{(1)}(1) \tag{37}$$

where $E\{\kappa\}$ is the average phase error obtained by

$$E\{\kappa\} = \sum_{\kappa=0}^{m-1} \kappa p(\kappa). \tag{38}$$

Since it takes $H_0^{(1)}(1)$ for the detector to process each $\overline{H}_0$ node, it can be seen that the first term in (37) is the average time for the detector to reach the node $\overline{H}_{1,c}$ from the initial phase $\check{\tau}_0$. The second term in (37) is the average time to reach acquisition from the node $\overline{H}_{1,c}$, which depends only upon the parameters of the phase alignment detector, not upon those of the phase estimator. Including the time for phase estimation, the total mean acquisition time by the proposed scheme is given by

$$\overline{T}_{acq} = MLT_C + \overline{T}_S. \tag{39}$$

The proposed scheme improves the acquisition performance by reducing $\overline{T}_S$ with the use of the proposed phase estimator. As can be seen in (37), $E\{\kappa\}$ that depends upon the performance of the phase estimator is a major factor affecting $\overline{T}_S$. In order to analytically calculate $E\{\kappa\}$, the pmf $p(\kappa)$ needs to be expressed in a closed form. Due to nonlinearity included in the estimation process, it is not easy to analytically obtain $p(\kappa)$. Instead, we can obtain $p(\kappa)$ empirically by Monte Carlo simulation as in Fig. 5. Since

$$\begin{aligned} \kappa &\simeq \left[\!\left[\frac{|\check{\tau}_0 - \hat{\tau}_0|}{\Delta T_C} \right]\!\right] \\ &\simeq \left[\!\left[\frac{|\tau_\epsilon|}{\Delta T_C} \right]\!\right] \\ &= \left[\!\left[\frac{|L\theta_\epsilon|}{\Delta} \right]\!\right] \end{aligned} \tag{40}$$

where $E\{\kappa\}$ is inversely proportional to Δ for a given L. Using the fact that θ_ϵ is statistically independent of L, it can be seen that $E\{\kappa\}$ is proportional to L. Define Y by

$$Y \triangleq \frac{E\{\kappa\}\Delta}{L}. \tag{41}$$

It can be seen that Y is the average phase estimation error normalized by L/Δ. Note that Y is independent of both Δ and L, and it is only a function of M and γ_0. Fig. 6 plots the value of Y as a function of M when γ_0 = −5 dB, −10 dB, and −15 dB. Note that, as M increases, the phase estimator uses more periods of the received PN signal, yielding a smaller phase estimation error. This results in a smaller $\overline{T}_S$ at the expense of increased phase estimation time. However, the decreasing rate becomes smaller as M increases. Thus, the proposed scheme can provide performance improvement over the conventional one when the decrease in $\overline{T}_S$ is larger than the increase in phase estimation time MLT_C.

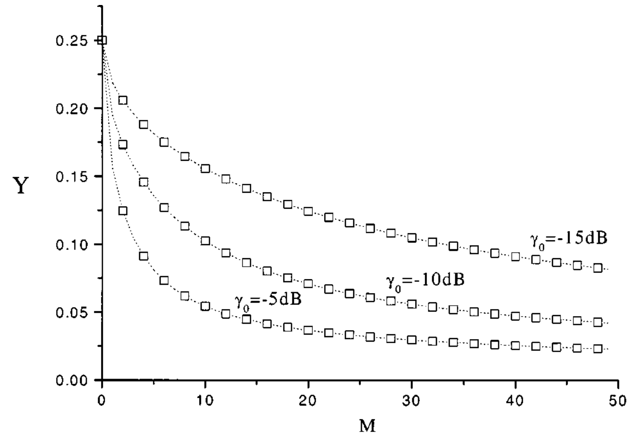

Fig. 6. Normalized average phase estimation error Y.

TABLE I
NORMALIZED AVERAGE PHASE ESTIMATION ERROR Y ACCORDING TO J

γ_0	-5dB		-10dB		-15dB	
J	Exact	Simplified	Exact	Simplified	Exact	Simplified
2	0.148	0.25	0.184	0.249	0.221	0.250
3	0.052	0.053	0.100	0.101	0.156	0.156
5	0.050	0.053	0.098	0.100	0.154	0.155
15	0.050	0.053	0.097	0.100	0.154	0.155
255	0.049	0.052	0.095	0.098	0.150	0.155

IV. NUMERICAL RESULTS AND DISCUSSION

The number J of correlators used for phase estimation is an important design parameter affecting the implementation complexity. The performance of the phase estimator according to J is evaluated in terms of Y in Table I using m-sequences with $L = 255$, where the exact and the simplified results represent the estimate obtained by the ML scheme using (16) and the simplified scheme using (22), respectively. Note that all the values of J except 2 are divisors of L such that $L = MJ$ can be satisfied. As J increases, the performance is improved since more observations are used for phase estimation. Also, the output samples of the correlator become more correlated, making the performance improvement marginal when J is larger than three. Considering the complexity and performance, it can be seen that the use of $J = 3$ is most practical. It can also be seen that the simplified phase estimation scheme can provide performance almost the same as the exact one.

To verify the acquisition performance of the proposed scheme, we consider the use of m-sequences generated by the polynomial $g(x) = x^{10} + x^7 + 1$ and three correlation branches for phase estimation, i.e., $J = 3$. We assume that the chip timing is set to the worst condition. In other words, when Δ is set to 1 or 1/2, the chip timing of the received signal assumed to be staggered by $(1/2)T_C$ and $(1/4)T_C$ with respect to that of the local PN signal, respectively. Let P_D denote the detection probability at the collective node $\overline{H}_{1,c}$, which is equal to $H_D(1)$. The dwell time and the threshold of the phase alignment detector are designed according to P_D and P_F [22]. The penalty time T_P represented by K_PT_C depends

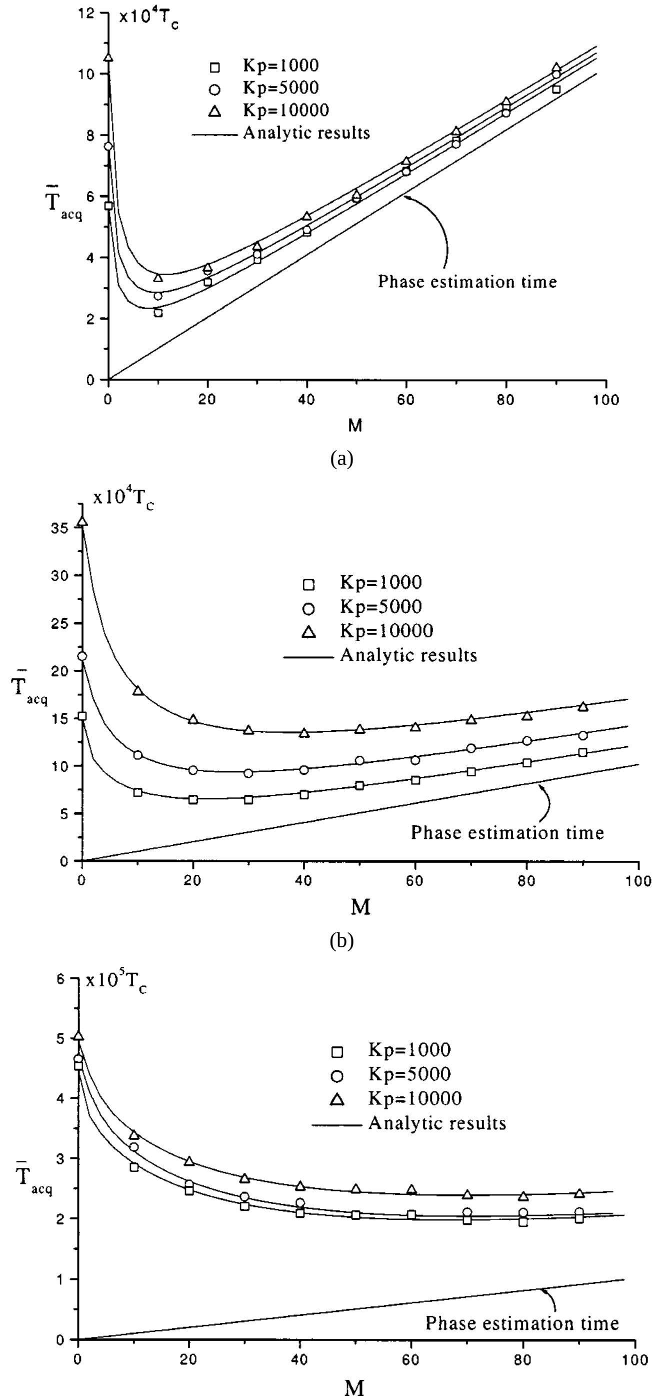

Fig. 7. $\overline{T}_{acq}$ as a function of M for $\Delta = 1$ and $P_D = 0.99$. (a) When $\gamma_0 = -5$ dB. (b) When $\gamma_0 = -10$ dB. (c) When $\gamma_0 = -15$ dB.

on the characteristics of the phase alignment verification and tracking circuitry. To see the effect of T_P on the performance, we consider the case of three values of K_P equal to 1000, 5000, and 10 000.

Fig. 7 depicts $\overline{T}_{\rm acq}$ as a function of M when $P_D = 0.99$, $P_F = 0.01$, $\Delta = 1$, and γ_0 is -5 dB, -10 dB, and -15 dB, where the rectangular symbols represent the simulation results and the lines represent the analytical performance from (39). It can be seen that both the results agree quite well. Note that $M = 0$ corresponds to the case of the conventional scheme that

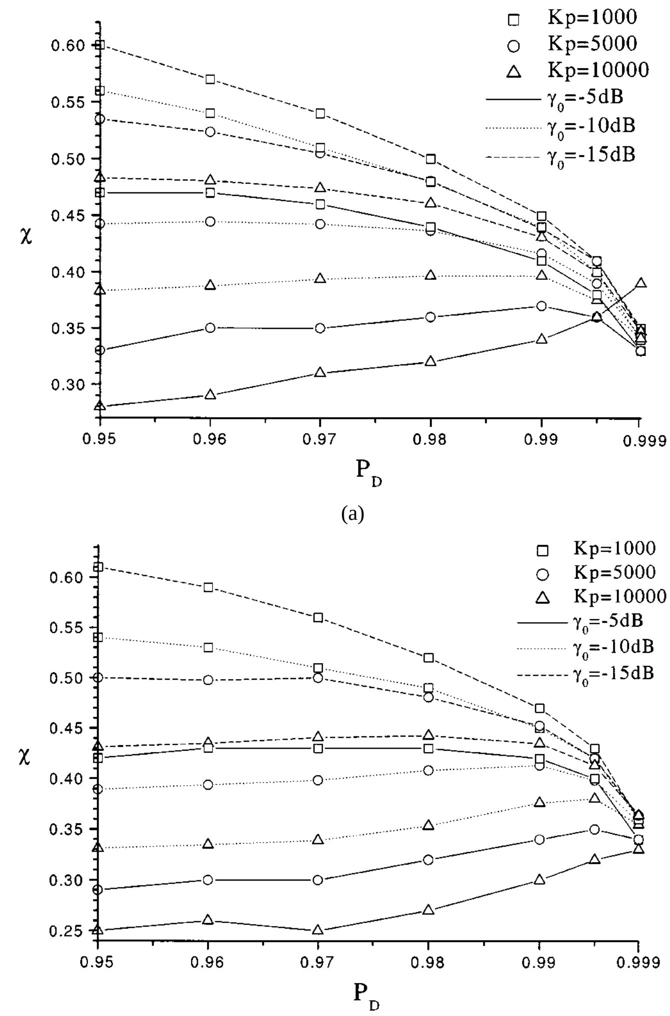

Fig. 8. Value of χ as a function of P_D when $P_D = 1 - P_F$. (a) When $\Delta = 1$. (b) When $\Delta = 1/2$.

employs two serial search schemes in parallel without phase estimation. It can be seen that there is an optimum value of M, $M_{\rm opt}$, that minimizes $\overline{T}_{\rm acq}$. This can be explained as follows. The decrease in phase alignment search time is larger than the increase in phase estimation time when $M < M_{\rm opt}$ but the opposite situation happens when $M > M_{\rm opt}$.

To see the performance improvement of the proposed acquisition scheme over the conventional scheme, we define the relative ratio of the mean acquisition time by

$$\chi \triangleq \frac{\overline{T}_{\rm acq}\big|_{M=M_{\rm opt}}}{\overline{T}_{\rm acq}\big|_{M=0}}. \tag{42}$$

Figs. 8 and 9 plot χ and $M_{\rm opt}$, respectively, as a function of P_D when $P_F = 1 - P_D$. It can be seen that the proposed scheme reduces the mean acquisition time by more than half compared to the conventional scheme. The higher the SNR, the larger performance improvement over the conventional scheme, since the performance of the phase estimator is improved as the SNR increases. It can be seen that the detection probability should be optimized in conjunction with the penalty time T_P. The larger

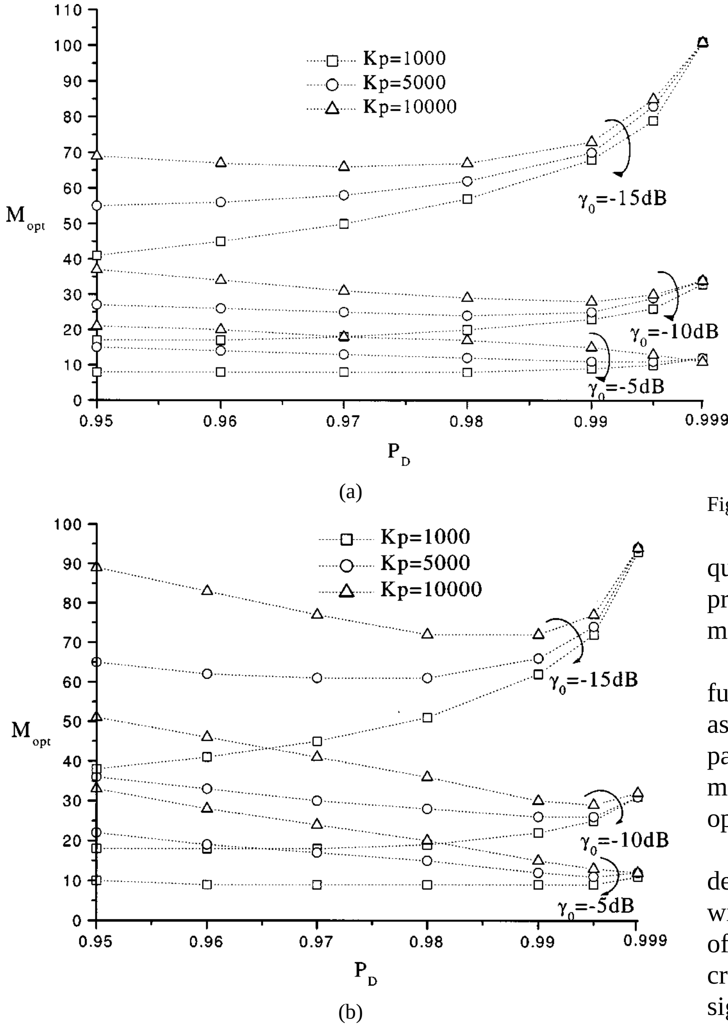

Fig. 9. Optimum value of M as a function of P_D when $P_D = 1 - P_F$. (a) When $\Delta = 1$. (b) When $\Delta = 1/2$.

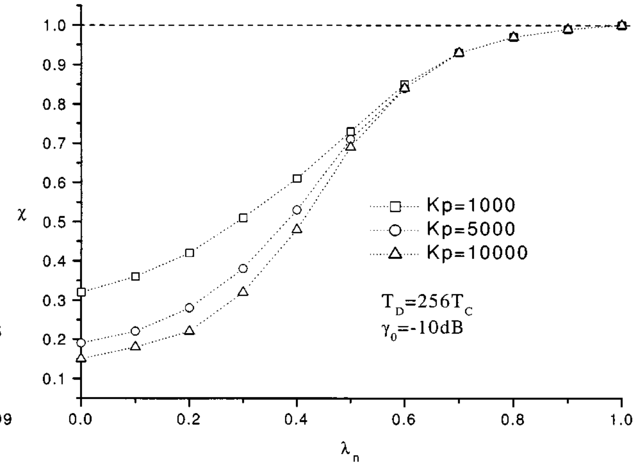

Fig. 10. Improvement factor χ as a function of λ_n.

the penalty time factor K_p, the larger the performance improvement over the conventional scheme. This is due to the fact that the increase in K_P affects only $\overline{T}_S$ of $\overline{T}_{\rm acq}$ in the proposed scheme, whereas it affects the whole acquisition time $\overline{T}_{\rm acq}$ in the conventional scheme. It can be seen that the optimum value of M at low SNR is larger than that at high SNR, since more samples of the received PN signal are needed to obtain a reasonable phase estimate as the SNR decreases.

Fig. 10 plots χ when $T_D = 256T_C$, $\gamma_0 = -10$ dB, $\Delta = 1$ and the normalized threshold λ_n is equal to $\lambda/\sqrt{P T_D}$. Here, λ is the threshold for the phase alignment detector. The increase of the threshold makes the detection probability P_D decrease. When P_D decreases, the phase alignment detector misses the correct phase with high probability, making the use of the phase estimator inefficient. Thus, χ approaches to 1 as λ_n increases. On the other hand, χ decreases as λ_n decreases. Thus, the use of a low threshold increases P_D, which makes the phase estimation improve the acquisition performance.

The proposed scheme employs multiple correlators for phase estimation. Since the phase estimator and the phase alignment detector operate in a sequential manner, the correlators used for phase alignment detection can also be used for phase estimation. In the case of $J = 3$, only one correlator is additionally required by the proposed scheme. Thus, the proposed scheme can provide rapid acquisition without significant increase of implementation complexity.

The performance of the proposed acquisition scheme can be further improved by employing various search schemes such as proposed in [7]. Since the phase estimator proposed in this paper localizes the uncertainty region, the acquisition performance can be further improved by employing a search scheme optimized for the characteristics of the phase uncertainty.

Although the proposed scheme assumes the use of coherent demodulation, it can be easily applied to noncoherent scheme with some modification. In noncoherent receivers, the phase of the incoming PN signal can be estimated using the squared cross-correlation function $R^2_{c\alpha}(\cdot)$. The use of the auxiliary signal used in this paper results in $R^2_{c\alpha}(\cdot)$ to have phase ambiguity of half the period of PN signal since the period of $R^2_{c\alpha}(\cdot)$ is reduced to half that of $R_{c\alpha}(\cdot)$. The phase ambiguity problem can be solved by using an auxiliary signal whose $R^2_{c\alpha}(\cdot)$ has the same period as $R_{c\alpha}(\cdot)$.

V. Conclusion

In this paper, we have proposed a rapid PN acquisition scheme by employing a PN code phase estimator. The proposed scheme first estimates the phase of the incoming PN signal using an auxiliary signal, and then finds out true phase alignment by starting the phase alignment search from the estimated phase. A simplified ML estimator is designed for estimation of the phase of the received PN signal. The acquisition performance of the proposed scheme has been analyzed in terms of the mean acquisition time and verified by computer simulation. Numerical results show that the proposed scheme can provide acquisition at least two times faster than the conventional scheme, while requiring small additional implementation complexity.

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