Joint Carrier Frequency Synchronization and Channel Estimation for OFDM Systems Via the EM Algorithm

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***Abstract*—A joint carrier frequency synchronization and channel estimation scheme is proposed for orthogonal frequency-division multiplexing (OFDM) system. In the proposed scheme, carrier frequency synchronization and channel estimation are performed iteratively via the expectation–maximization (EM) algorithm using an OFDM preamble symbol. Moreover, we analytically investigate the effect of frequency offset error on the mean square error (MSE) performance of channel estimator. Simulation results present that the proposed scheme achieves almost ideal performance for both channel and frequency offset estimation.**

***Index Terms*—Channel estimation, expectation–maximization (EM), frequency synchronization, orthogonal frequency-division multiplexing (OFDM).**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) is an attractive technique to support high-rate data transmission over frequency-selective fading channels. However, it is known to be very sensitive to the frequency synchronization and channel estimation errors [1]. Carrier frequency offset induced by the mismatches of local oscillators in transmitter and receiver causes intercarrier interferences (ICI), which may result in significant performance degradation. Moreover, the coherent detection of OFDM signals requires channel estimation to mitigate amplitude and phase distortions in a fading channel. For the differential detection, however, it is known that the channel estimation is not needed, although it results in the 3-dB loss of signal-to-noise ratio (SNR) [2]–[4].

Several carrier frequency synchronization schemes for OFDM systems are reported in the literature [5]–[9]. In [5] and [6], synchronization is achieved using the redundancy of the cyclic prefix. The synchronization schemes proposed in [7] and [8] employs the maximum length sequence in frequency domain. In [9], the pseudo noise (PN)-based preamble in time-domain is used to achieve the frequency synchronization. Moreover, various channel estimation schemes for OFDM system are also studied in the papers [2]–[4]. Especially, [4] studied and analyzed the performance of the minimum mean square error (MMSE) and deterministic maximum likelihood (ML) estimators.

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The joint frequency offset and channel estimation issue is also highlighted in [10] and [11]. To obtain the ML solutions for both frequency offset and channel impulse response (CIR), the prohibitive computational complexity is required [12]. Therefore, in [10], the ML estimate for only frequency offset was obtained based on the least square (LS) CIR estimate. In [11], an adaptive approach (i.e., steepest descent algorithm) was employed to avoid the complexity of joint ML estimation. However, it is known that the expectation-maximization (EM) algorithm [13] can provide the ML solutions in an iterative manner for the joint ML estimation problem [12], [14]. In this paper, we propose the joint carrier frequency synchronization and channel estimation scheme based on the EM algorithm employing an OFDM preamble symbol to perform joint frequency synchronization and channel estimation iteratively. Expectation and maximization steps in this scheme provide both channel and frequency offset estimates, respectively.

The rest of this paper is organized in the following order. In Section II, we describe the OFDM system to be considered. In Section III, the joint carrier frequency synchronization and channel estimation scheme via EM algorithm is proposed. Section IV presents the Cramer–Rao bound (CRB) for channel and frequency offset estimator and analyzes how the estimation error of carrier frequency offset affects the mean square error (MSE) performance of channel estimator. Section V presents simulation results for the MSE performance of the proposed algorithm over frequency selective fading channels. Concluding remarks are given in Section VI.

II. System Model

Let N be the number of subcarriers and P be the number of modulated subcarriers. Note that N-P subcarriers at the edges of the spectrum (i.e., virtual sub carriers) are not used and the modulated subcarriers can be indexed by the numbers from $-P/2$ to $P/2$. Assuming ideal synchronization in time [15], the received signal vector over a frequency-selective fading channel can be expressed as

$$\mathbf{r} = \mathbf{\Omega}_{\upsilon}\mathbf{FADh} + \mathbf{w} \tag{1}$$

where the normalized frequency offset υ is presented in the matrix $\mathbf{\Omega}_{\upsilon}$ given by

$$\mathbf{\Omega}_{\upsilon} = \operatorname{diag}\{\Omega_{\upsilon}(0), \Omega_{\upsilon}(1), \ldots, \Omega_{\upsilon}(N-1)\} \tag{2}$$

and

$$\Omega_{\upsilon}(n) = \exp\left(j\frac{2\pi\upsilon}{N}n\right). \tag{3}$$

In (1), $\mathbf{A} = \mathrm{diag}\{a_{-P/2}, \ldots, a_0, \ldots a_{P/2}\}$ denotes a diagonal transmitted symbol matrix in the preamble and $[\mathbf{F}]_{n,p} = (1/\sqrt{N})\exp(j(2\pi p/Nn))$, where $0 \leq n < N$, and $|p| \leq P/2$.

Moreover, the CIR vector $\mathbf{h} = [h(0)\ h(1)\ \cdots\ h(L-1)]^T$ is a wide-sense stationary process with L multipaths [16] and the discrete Fourier transform (DFT) matrix is given as $[\mathbf{D}]_{p,l} = \exp(-j(2\pi p/N)(l-1))$, where $1 \leq l \leq L$. The additive white Gaussian noise vector $\mathbf{w} = [w(0)\ w(1)\ \cdots\ w(N-1)]^T$ has the covariance matrix of $\sigma_w^2 \mathbf{I}_N$, where $\mathbf{I}_N$ denotes a $N \times N$ identity matrix.

III. Joint Frequency Synchronization and Channel Estimation

The EM algorithm is an iterative two-step algorithm that consists of the expectation step and the maximization step. The EM algorithm iterates until the estimate converges [13], [14].

Assuming the coarse frequency offset estimate υ_c is obtained, the log-likelihood function can be expressed as

$$L\left(\upsilon_e \mid \upsilon_e^i\right) = \mathrm{Re}\left\{\mathbf{r}^H \mathbf{\Omega}_{\upsilon_e} \mathbf{\Omega}_{\upsilon_c} \mathbf{S} \boldsymbol{\mu}_h^i\right\} \tag{4}$$

where $(\cdot)^H$ denotes conjugated transpose and we define $\mathbf{S} = \mathbf{FAD}$ for simplicity. Moreover, υ_e in (4) denotes the frequency offset estimation error $\upsilon - \upsilon_c$, and υ_e^i is the estimate of υ_e at the ith iteration. Assuming each component of the CIR vector $\mathbf{h}$ in (1) is an independent complex Gaussian random variable with zero mean [21], the channel estimate at the ith iteration $\boldsymbol{\mu}_h^i$ can be obtained by

$$\begin{aligned}\boldsymbol{\mu}_h^i &= E\left[\mathbf{h} \mid \mathbf{r}, \upsilon_e^i\right] \\ &= \mathbf{K}_h \mathbf{S}^H \mathbf{\Omega}_{\upsilon_c}^H \mathbf{\Omega}_{\upsilon_e^i}^H \mathbf{r}\end{aligned} \tag{5}$$

where $\mathbf{K}_h$ is defined by

$$\mathbf{K}_h = \mathrm{Re}\left(\sigma_w^2 \mathbf{R}_h^{-1} + \mathbf{S}^H \mathbf{S}\right)^{-1} \tag{6}$$

and $\mathbf{R}_h = E[\mathbf{h}\mathbf{h}^H]$. Consequently, the result in (5) is equivalent to the MMSE estimate [4]. The derivation of the previous equations is summarized in Appendix I. In the expectation step, the conditional first moment of CIR given $\mathbf{r}$ and υ_e^i in (5) should be evaluated.

Note that the CIR length L should be known to construct $\mathbf{S}$ in (4). The exact value of L is usually not available, and we assumed that the receiver has the estimate of L, which can be treated as a predetermined parameter considering the maximum expected CIR length [22]. When the estimate of L is greater than the CIR length, we can estimate all multipaths in the CIR, while the large dimension of the matrix $\mathbf{S}$ induces unnecessary calculation in (5). However, it might be effective that the estimate of L is less than the exact CIR length in less dispersed multipath channel, where the last taps of CIR are relatively small and negligible. Here, we simply assume the case where the estimate of L is equal to the CIR length.

The maximization step, which is used for the $(i+1)$th estimate υ_e^{i+1}, can be represented by

$$\upsilon_e^{i+1} = \arg\max_{\upsilon_e} L\left(\upsilon_e \mid \upsilon_e^i\right). \tag{7}$$

The log-likelihood function in (4) can be modified into the following equation using $\mathbf{S}_\mu^i = [S_\mu^i(0)\ S_\mu^i(1)\ \cdots\ S_\mu^i(N-1)]^T$ shown as

$$L\left(\upsilon_e \mid \upsilon_e^i\right) = \sum_{n=0}^{N-1} \mathrm{Re}\left\{r^*(n) S_\mu^i(n) \Omega_{\upsilon_c}(n) \exp\left(j\frac{2\pi\upsilon_e}{N}n\right)\right\} \tag{8}$$

where $\mathbf{S}_\mu^i = \mathbf{S}\boldsymbol{\mu}_h^i$. However, it is too complicated to obtain an exact solution of υ_e^{i+1} to maximize the likelihood function in (8) [17]. Therefore, we assume υ_e is sufficiently small to approximate $\exp(j(2\pi\upsilon_e/N)n)$ by Taylor series expansion to the second-order term:

$$\exp\left(j\frac{2\pi\upsilon_e}{N}n\right) \approx 1 + j\frac{2\pi\upsilon_e}{N}n - \frac{1}{2}\left(j\frac{2\pi\upsilon_e}{N}n\right)^2.$$

Then, the likelihood function $L(\upsilon_e \mid \upsilon_e^i)$ can be given in the quadratic form of υ_e^i shown as

$$\begin{aligned}L\left(\upsilon_e \mid \upsilon_e^i\right) \approx &-\upsilon_e^2\left(\frac{2\pi^2}{N^2}\sum_{n=0}^{N-1} n^2 \mathrm{Re}\left\{r^*(n) S_\mu^i(n) \Omega_{\upsilon_c}(n)\right\}\right) \\ &- \upsilon_e\left(\frac{2\pi}{N}\sum_{n=0}^{N-1} n \mathrm{Im}\left\{r^*(n) S_\mu^i(n) \Omega_{\upsilon_c}(n)\right\}\right) \\ &+ \sum_{n=0}^{N-1} \mathrm{Re}\left\{r^*(n) S_\mu^i(n) \Omega_{\upsilon_c}(n)\right\}\end{aligned} \tag{9}$$

and the $(i+1)$th estimate υ_e^{i+1} to maximize (9) is obtained by

$$\upsilon_e^{i+1} = -\frac{N}{2\pi}\frac{\sum_{n=0}^{N-1} n\mathrm{Im}\left\{r^*(n) S_\mu^i(n) \Omega_{\upsilon_c}(n)\right\}}{\sum_{n=0}^{N-1} n^2\mathrm{Re}\left\{r^*(n) S_\mu^i(n) \Omega_{\upsilon_c}(n)\right\}}. \tag{10}$$

In (5) and (10), it is seen that the proposed algorithm iteratively provides the CIR estimate in expectation step and the frequency synchronization in maximization step via the EM algorithm.

The computational complexity of the proposed scheme can be addressed as follows. Assuming $\mathbf{K}_h\mathbf{S}^H$ in (5) is precomputed, the expectation step requires $(L+2)N$ complex products and $L(N-1)$ complex additions. In the maximization step, $(L+2)N$ complex products and $(L-1)N$ complex additions are required to evaluate $\mathbf{S}_\mu^i$, and $\{r^*(n) S_\mu^i(n) \Omega_{\upsilon_c}(n); 0 \leq n < N\}$. Moreover, in (10), we need $3N$ real products and $2(N-1)$ real additions. Note that a complex product requires four real products and two real additions, whereas a complex addition amounts to two real additions. Then, the overall number of real products and real additions in the expectation step can be given as $4(L+2)N$ and $4(\mathrm{LN} + N - 1)$, respectively. Also, the overall operations in the maximization step are $(4L+11)N$ real products and $2(\mathrm{LN} + 3N - 2)$ real additions.

IV. Performance Analysis

In this section, the CRB is derived for both the CIR and the frequency offset estimator of the OFDM system, and the effect of frequency offset error on the MSE performance of the channel estimator is analyzed.

The CRB for the CIR estimation with ideal carrier frequency synchronization $(\upsilon = 0)$ is given by

$$\mathrm{CRB}_h = \sum_{l=1}^{L} \sigma_w^2 [(\mathbf{S}^H \mathbf{S})^{-1}]_{l,l}. \tag{11}$$

The CRB for the carrier frequency offset estimation can be represented by

$$\mathrm{CRB}_\upsilon = \frac{\sigma_w^2}{2\mathbf{x}^H [\mathbf{I}_N - \mathbf{S}(\mathbf{S}^H\mathbf{S})^{-1}\mathbf{S}^H]\mathbf{x}} \tag{12}$$

where $\boldsymbol{\eta} = \mathrm{diag}\{0, 2\pi/N, 4\pi/N, \ldots, 2\pi(N-1)/N\}$, and $\mathbf{x} = \boldsymbol{\eta}\mathbf{S}\mathbf{h}$. The detailed derivation of (11) and (12) is given in Appendix II.

The frequency synchronization error influences the channel estimation accuracy. In the proposed scheme, the imperfect carrier frequency synchronization also induces the CIR estimation error as shown in (5). Therefore, we analytically investigate the MSE performance of CIR estimator affected by the estimation error of carrier frequency offset.

The MSE of the proposed CIR estimate in (5) can be represented by

$$\mathrm{MSE}_h^i = E\left[\left|\boldsymbol{\mu}_h^i - \mathbf{h}\right|^2\right]. \tag{13}$$

Substituting (5) into (13) and taking into account (1), we obtain

$$\begin{aligned}\mathrm{MSE}_h^i &= E[|(\mathbf{K}_h\mathbf{S}^H(\mathbf{I}_N + \boldsymbol{\xi}_{e^i})\mathbf{S} - \mathbf{I}_L)\mathbf{h}|^2] \\ &\quad + E\left[\left|\mathbf{K}_h\mathbf{S}^H\boldsymbol{\Omega}_{e^i}^H\mathbf{w}\right|^2\right]\end{aligned} \tag{14}$$

where $e^i = \upsilon_e - \upsilon_e^i$, and $\mathbf{I}_L$ is a $L \times L$ identity matrix. Moreover, $\boldsymbol{\xi}_{e^i} = \mathrm{diag}\{\xi_{e^i}(0), \xi_{e^i}(1), \ldots, \xi_{e^i}(N-1)\}$ can be evaluated by $\boldsymbol{\xi}_{e^i} = \boldsymbol{\Omega}_{e^i} - \mathbf{I}_N$.

In case of ideal frequency synchronization $(e^i = 0)$, $\boldsymbol{\xi}_{e^i}$ in (14) would be zero. Assuming the components of $\mathbf{h}$ vary independently, we can obtain the MSE of CIR estimate in ideal frequency synchronization case shown as

$$\begin{aligned}\mathrm{MSE}_h^{\mathrm{ideal}} &= \sum_{l=1}^{L}\left[\boldsymbol{\Gamma}_{\mathrm{ideal}}^H\boldsymbol{\Gamma}_{\mathrm{ideal}}\right]_{l,l}[\mathbf{R}_h]_{l,l} \\ &\quad + \sum_{n=1}^{N}\sigma_w^2\left[\boldsymbol{\Gamma}_{\mathrm{noise}}^H\boldsymbol{\Gamma}_{\mathrm{noise}}\right]_{n,n}\end{aligned} \tag{15}$$

where $\boldsymbol{\Gamma}_{\mathrm{ideal}} = \mathbf{K}_h\mathbf{S}^H\mathbf{S} - \mathbf{I}_L$, and $\boldsymbol{\Gamma}_{\mathrm{noise}} = \mathbf{K}_h\mathbf{S}^H$. In the presence of frequency offset, however, the additional MSE due to frequency synchronization error is generated shown as

$$\begin{aligned}\mathrm{MSE}_h^i &= \mathrm{MSE}_h^{\mathrm{ideal}} \\ &\quad + \sum_{l=1}^{L}\left[2\mathrm{Re}\left\{\boldsymbol{\Gamma}_{\mathrm{ideal}}^H\boldsymbol{\Gamma}_{\mathrm{error}}\right\} + \boldsymbol{\Gamma}_{\mathrm{error}}^H\boldsymbol{\Gamma}_{\mathrm{error}}\right]_{l,l}[\mathbf{R}_h]_{l,l}\end{aligned} \tag{16}$$

where $\boldsymbol{\Gamma}_{\mathrm{error}} = \mathbf{K}_h\mathbf{S}^H\boldsymbol{\xi}_{e^i}\mathbf{S}$.

Fig. 1(a) shows the required SNR to achieve $\mathrm{MSE}_h^i = 10^{-1}$ and $\mathrm{MSE}_h^i = 10^{-2}$ in (16) according to the frequency synchronization error e^i. It is seen that the required SNR for both $\mathrm{MSE}_h^i = 10^{-1}$ and $\mathrm{MSE}_h^i = 10^{-2}$ grows exponentially as the frequency synchronization error increases. For $e^i = 0.1$, the additional SNR more than 10 dB is needed to achieve $\mathrm{MSE}_h^i = 10^{-1}$ compared with the required SNR for ideal frequency synchronization. Moreover, we cannot even obtain the MSE_h^i of 10^{-2} when e^i is greater than 0.03. Fig. 1(b) presents the MSE performance evaluated by (16) versus SNR for different values of frequency synchronization error e^i. It is seen that the MSE performance is significantly degraded as frequency synchronization error increases slightly.

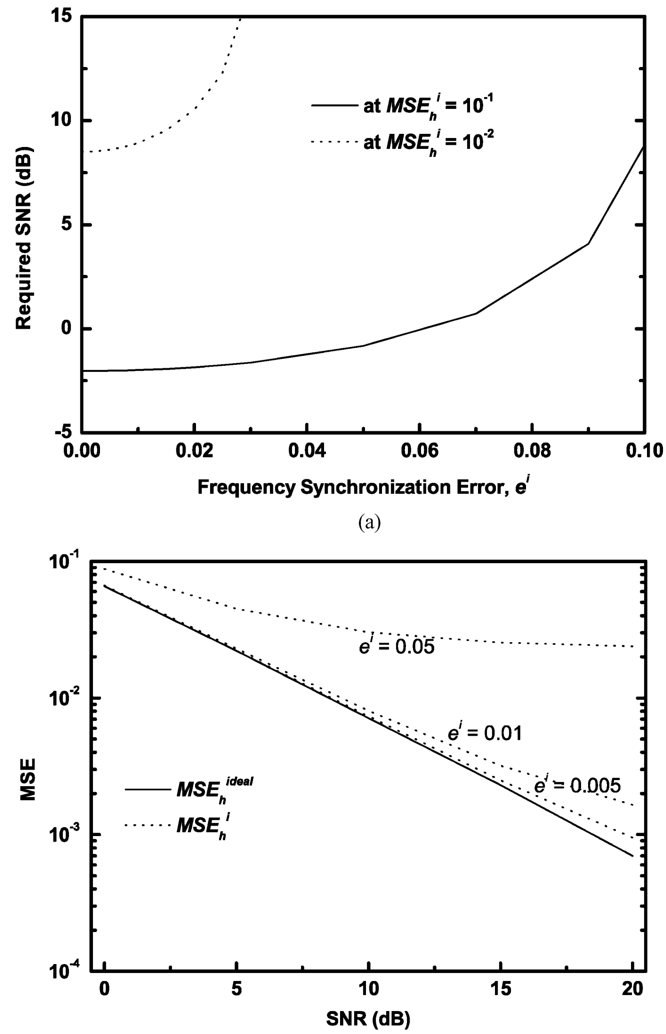

Fig. 1. MSE performance analysis based on (16) at $N = 64$ and $L = 4$. (a) Required SNR to achieve $\mathrm{MSE}_h^i = 10^{-1}$ and $\mathrm{MSE}_h^i = 10^{-2}$ in the presence of e^i. (b) Comparison of MSE_h^i and $\mathrm{MSE}_h^{\mathrm{ideal}}$ for different values of e^i.

V. Simulation Results

System parameters in our simulations follow the IEEE 802.11a standard [20], where the DFT size N and the number of modulated sub carriers P equal 64 and 52, respectively. We consider frequency-selective fading channels with exponential power delay profile given by $E[|h(l)|]^2 \propto e^{-(l-2)/4}, 1 \leq l \leq L$.

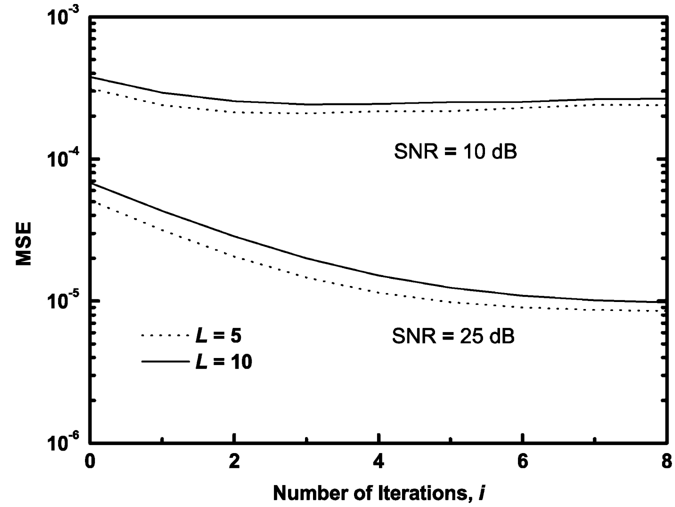

Fig. 2. MSE performance of frequency offset estimator versus number of iterations at normalized frequency offset $\upsilon = -0.3$.

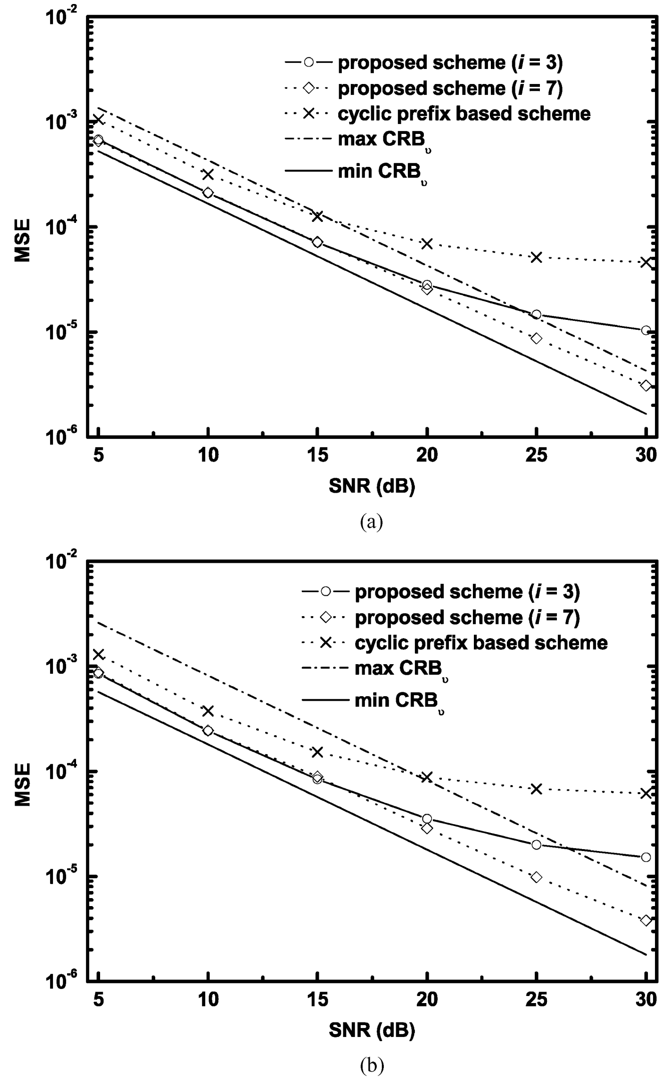

Fig. 3. Comparisons of MSE performance for frequency offset estimate versus SNR at the normalized frequency offset $\upsilon = 0.2$. (a) $L = 5$. (b) $L = 10$.

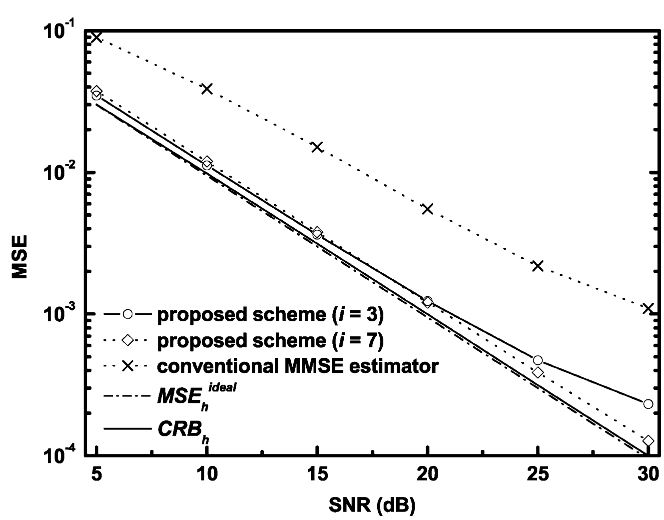

Fig. 4. MSE performance of channel estimator versus SNR at normalized frequency offset $\upsilon = -0.1$ and $L = 5$.

Each multipath is modeled as a zero-mean complex Gaussian random variable; so, it varies according to Rayleigh distribution in mobile environments [21]. Moreover, we obtained the coarse frequency offset estimate υ_c using the redundancy generated by the cyclic prefix [6]. To start the iteration of proposed algorithm, we set the initial estimate $\upsilon_e^0 = 0$.

In Fig. 2, the MSEs of the frequency offset estimator versus the number of iterations were depicted. It is shown that the MSE performance of the proposed frequency offset estimator converges after three iterations at the SNR of 10 dB, while it is improved continually until the number of iterations increases to seven for the SNR values greater than 10 dB.

Fig. 3 compares the MSE performance of proposed algorithm with CRB_υ in (12). Note that the CRB varies according to the channel state [18], and the minimum and maximum CRB_υ in the figures are evaluated by 10^5 simulation runs. The MSE performance of proposed algorithm is quite comparable to CRB_υ in all ranges of SNR. However, in high SNR above 20 dB, the proposed scheme needs seven iterations to guarantee the improved performance over the maximum CRB_υ for both cases of $L = 5$ and 10. Moreover, the SNR gain of the proposed scheme is more than 5 dB compared with the conventional cyclic prefix based estimator [6] at the MSE of 10^{-4}. It is also observed that the error floor of the conventional scheme is significantly suppressed by the proposed algorithm in high SNR.

In Fig. 4, the simulated MSE performance of channel estimator in (5) are presented and compared with the CRB_h in (11) and $\text{MSE}_h^{\text{ideal}}$ in (15). The MSE performance of conventional MMSE estimator [4] is obtained after the frequency synchronization using the cyclic prefix. It is seen that the channel estimator of the proposed algorithm shows almost ideal performance and the proposed joint frequency synchronization and channel estimation algorithm outperforms the conventional scheme in all ranges of SNR. Especially for high SNR greater than 20 dB, seven iterations should be operated to achieve almost ideal MSE performance, as shown in Fig. 4.

VI. CONCLUSION

In this paper, we proposed a joint carrier frequency synchronization and channel estimation scheme for OFDM systems based on the EM algorithm. In the proposed scheme, expectation and maximization steps provide both channel and carrier frequency offset estimates iteratively using an OFDM preamble symbol. The simulation results show that the proposed algorithm achieves almost ideal performance compared with the CRB for both channel and frequency offset estimations.

APPENDIX I

In this Appendix, we highlight the derivation of expectation step for proposed algorithm. In (1), the likelihood function can be given as

$$\Lambda(\mathbf{r};\mathbf{h},\upsilon) = \frac{1}{(\pi\sigma_w^2)^N}\exp\left(-\frac{1}{\sigma_w^2}\left(\mathbf{r}-\mathbf{\Omega}_\upsilon\mathbf{Sh}\right)^H\left(\mathbf{r}-\mathbf{\Omega}_\upsilon\mathbf{Sh}\right)\right). \quad (17)$$

In the ith iteration of proposed algorithm given υ_c and υ_e^i, the log-likelihood function for υ_e can be obtained by logarithm of (17). Discarding the terms without dependency on υ_e, we can obtain (4). To evaluate the channel estimate in ith iteration $\boldsymbol{\mu}_h^i$, we consider the following probability density functions:

$$\begin{aligned} p\left(\mathbf{r}\mid\mathbf{h},\upsilon_e^i\right) &\sim N_c\left(\mathbf{\Omega}_{\upsilon_c}\mathbf{\Omega}_{\upsilon_e^i}\mathbf{Sh},\sigma_w^2\mathbf{I}_N\right)\\ p(\mathbf{h}) &\sim N_c(\mathbf{0},\mathbf{R}_h)\\ p\left(\mathbf{h}\mid\mathbf{r},\upsilon_e^i\right) &\sim N_c\left(\boldsymbol{\mu}_h^i,\mathbf{\Xi}_h\right) \end{aligned} \quad (18)$$

where $\mathbf{\Xi}_h$ denotes the covariance matrix of $\mathbf{h}$ conditioned on $\mathbf{r}$ and υ_e^i. Moreover, $\mathbf{0}$ denotes the $L\times 1$ vector with all zero entries. Bearing in mind that the channel statistic is independent of the frequency offset and employing the relation of

$$p\left(\mathbf{h}\mid\mathbf{r},\upsilon_e^i\right) = \frac{p\left(\mathbf{r}\mid\mathbf{h},\upsilon_e^i\right)p(\mathbf{h})}{p\left(\mathbf{r}\mid\upsilon_e^i\right)} \quad (19)$$

we can obtain

$$\begin{aligned} \boldsymbol{\mu}_h^i &= \frac{1}{\sigma_w^2}\mathbf{\Xi}_h\mathbf{S}^H\mathbf{\Omega}_{\upsilon_c}^H\mathbf{\Omega}_{\upsilon_e^i}^H\mathbf{r}\\ \mathbf{\Xi}_h &= \left(\mathbf{R}_h^{-1}+\frac{1}{\sigma_w^2}\mathbf{S}^H\mathbf{S}\right)^{-1}. \end{aligned} \quad (20)$$

If $\mathbf{K}_h = (1/\sigma_w^2)\mathbf{\Xi}_h$ is defined, (20) yields (8) and (9).

APPENDIX II

The CRB of the channel and frequency offset estimator is evaluated in this Appendix, where we modify the work in [18] for the OFDM system. In the case of ideal frequency synchronization, $\upsilon = 0$ in (17), and we define a $2L\times 1$ vector $\boldsymbol{\rho}_h = (\mathbf{h}_I,\mathbf{h}_Q)$, where $\mathbf{h} = \mathbf{h}_I + j\mathbf{h}_Q$. Then, the Fisher information matrix [19] can be given as

$$[\mathbf{\Phi}_h]_{l,m} = -E\left[\frac{\partial^2\ln\Lambda(\mathbf{r};\boldsymbol{\rho}_h)}{\partial\rho_h(l)\partial\rho_h(m)}\right] \quad (21)$$

where $1\le l,m\le 2L$. Substituting (17) into (21), we can obtain

$$\mathbf{\Phi}_h = \frac{2}{\sigma_w^2}\begin{bmatrix}\mathbf{S}^H\mathbf{S} & \mathbf{0}\\ \mathbf{0} & \mathbf{S}^H\mathbf{S}\end{bmatrix}. \quad (22)$$

Note that all entries of $\mathbf{S}^H\mathbf{S}$ in (22) are real value. Then, the CRB for the channel estimation can be obtained by the inversion of $\mathbf{\Phi}_h$ given by

$$\mathbf{\Phi}_h^{-1} = \frac{\sigma_w^2}{2}\begin{bmatrix}(\mathbf{S}^H\mathbf{S})^{-1} & \mathbf{0}\\ \mathbf{0} & (\mathbf{S}^H\mathbf{S})^{-1}\end{bmatrix} \quad (23)$$

and the CRB for the estimation of the lth multipath is obtained by

$$\mathrm{CRB}_h^l = \sigma_w^2[(\mathbf{S}^H\mathbf{S})^{-1}]_{l,l}. \quad (24)$$

For the CRB of frequency offset estimator, we define a $(2L+1)\times 1$ vector $\boldsymbol{\rho} = (\mathbf{h}_I,\mathbf{h}_Q,\upsilon)$, including the frequency offset υ [18], and evaluate the Fisher information matrix shown as

$$\mathbf{\Phi} = \frac{2}{\sigma_w^2}\begin{bmatrix}\mathbf{S}^H\mathbf{S} & \mathbf{0} & -\mathrm{Im}\{\mathbf{S}^H\mathbf{x}\}\\ \mathbf{0} & \mathbf{S}^H\mathbf{S} & \mathrm{Re}\{\mathbf{S}^H\mathbf{x}\}\\ \mathrm{Im}\{\mathbf{x}^H\mathbf{S}\} & \mathrm{Re}\{\mathbf{x}^H\mathbf{S}\} & \mathbf{x}^H\mathbf{x}\end{bmatrix} \quad (25)$$

where $\boldsymbol{\eta} = \mathrm{diag}\{0,2\pi/N,4\pi/N,\ldots,2\pi(N-1)/N\}$, and $\mathbf{x} = \boldsymbol{\eta}\mathbf{Sh}$. The CRB for frequency offset estimation is given by

$$\begin{aligned} \mathrm{CRB}_\upsilon &= [\mathbf{\Phi}^{-1}]_{2L+1,2L+1}\\ &= \frac{\sigma_w^2}{2\mathbf{x}^H[\mathbf{I}_N-\mathbf{S}(\mathbf{S}^H\mathbf{S})^{-1}\mathbf{S}^H]\mathbf{x}}. \end{aligned} \quad (26)$$

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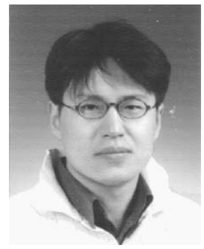

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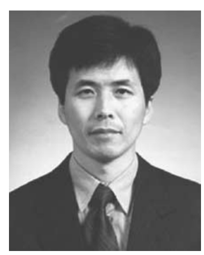

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