Seoul Journal of Business
Volume 8, Number 1 (June 2002)

Risk-Pooling Effects of Emergency Shipping in a Two-Echelon Distribution System

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ABSTRACT

This paper analyzes the optimal replenishment policy in a one-warehouse N-retailer distribution system operating in a periodic-review mode where emergency shipping from a central warehouse or a retailer to an individual customer is allowed to satisfy retailer backorders at the end of the cycle. We show that, under the assumption that it costs the same whether emergency shipping is made by the warehouse or by a retailer, it is optimal that the warehouse carries no inventory. Under the same assumption, the form of the optimal system-replenishment policy is shown to be a base-stock policy, and the necessary and sufficient condition of optimality, which is analogous to that of the Newsboy problem, is presented. The further analysis shows that, with emergency shipping allowed, the warehouse orders less from its supplier.
Key words: emergency shipping, periodic review, distribution system

1. Introduction

The recent environmental changes such as global competition, shorter product life cycle, time-based competition, etc. require a firm to build a flexible and speedy supply chain to remain competitive in its market. With the help of rapid developments in information and communication technologies, they have been able to make their supply chain leaner and speedier. Some of those efforts are VMI (vendor-managed inventory), QR (quick

* This study was supported by the Institute of Management Research of Seoul National University

response), ECR (efficient consumer response), AR (accurate response), Dell's Direct Business Model, etc. All these efforts are aiming at maximizing customer service at the minimum costs. That is, given real-time information on inventories at various locations in a supply chain and efficient transportation systems, they want to timely provide customers with what they want by keeping only the minimum level of inventories. There are many ways to lower inventory levels without sacrificing customer service such as dynamic allocation, dynamic routing, transshipments, postponement of product differentiation, etc. This paper analyzes the emergency shipping described below.

The other day I visited one of the discount stores in my area, Service Merchandise to buy a humidifier. I decided to buy one of the models exhibited in the store, filled out an order form, and submitted it to a cashier. But the cashier told me that the model I chose was out of stock. After checking its availability at other places (i.e., the company distribution center and other retail stores), she suggested that she could ship the model that I wanted to me directly without additional shipping and handling charge. I did so and the humidifier arrived in about a week. Through this experience, I got to know that this type of emergency shipping from a distribution center or a retail store with available inventory is very common in the U.S retail industry. This paper models emergency shipping in a multi-echelon distribution system to find out its risk-pooling effects.

This paper examines the multi-echelon distribution system that consists of one warehouse and multiple retailers and allows the emergency shipments from the warehouse or the retailers to an individual customer at the end of each replenishment cycle. In particular, part of each system-replenishment quantity can be kept at the warehouse for emergency shipments. If at the end of cycle a retailer is out of stock and the warehouse has available stock, demand is met from the warehouse inventory. If the warehouse can't satisfy all the backorders, the retailers with available inventory at the end of cycle ship their inventory directly to customers in need. In case when system-wide demand during a replenishment cycle is greater than initial system-wide inventory, the difference is backordered. The system with emergency shipments requires fewer inventories to attain a specified service level (= portion of demand met from inventory on-hand both at the retailers and at the warehouse) than the equivalent inventory system without them. But it incurs additional shipping cost. The optimal replenishment policy balances reduced inventory-holding and backorder costs with additional shipping cost, i.e., minimizes the sum of expected inventory-holding, backorders, and shipping costs.

The major results of this paper are (1) that, under the assumption that it costs the same whether emergency shipping is made by the warehouse or by a retailer, it is optimal that the warehouse carries no inventory, (2) that the optimal system-replenishment policy is a base-stock policy, (3) that the necessary and sufficient condition of the optimal system-replenishment quantity can be interpreted in the Newsboy-problem context, and (4) that, with emergency shipping allowed, the warehouse orders less from its supply.

This paper is organized as follows. Section 2 describes the model and its assumptions, and section 3 examines the previous study on related issues. Section 4 identifies the optimal location of inventory and derives the form of the optimal replenishment policy in a one-warehouse N-retailer distribution system. Section 4 also interpret it in the Newsboy problem context, and then, determine the effects of emergency shipping on the optimal system-replenishment quantity.

2. Model and Assumptions

This paper studies a one-warehouse N-retailer system facing stochastic demand and operating in a periodic-review mode. In the specific system examined, the warehouse places a system-replenishment order every period, and receives it after a fixed leadtime L. At that time the warehouse makes allocation decision; that is, the warehouse retains part of the system-replenishment quantity, and allocates the rest to the N retailers. The allocations to the retailers are delivered after a fixed leadtime l.

At the end of each cycle, retailer backorders are met with the warehouse inventory or available inventory at other retailers through emergency shipments to customers. If total retailer backorders at the end of cycle is greater than the warehouse inventory plus total excess inventory at the retailers, the difference is backordered. The paper develops a model that identifies the form of the optimal system-replenishment policy. The additional assumptions are as follows:

(i) Period demand is i.i.d. across periods and among the retailers.

(ii) Inventory-holding and backorder costs are a linear function of net inventory. Unit inventory-holding and backorders cost per period are h and p, respectively.

(iii) Unit shipping costs to an individual customer (t) are identical whether shipping from the warehouse or from any of the retailers, and are a linear function of quantities shipped.

(iv) $p + h - t > 0$. That is, emergency shipping is economical. If at the end of cycle, a retailer experiences backorders and there are available inventory at the warehouse or at any other retailer, it is economical to ship available units to customers in need.

(v) Equal allocation is optimal; that is, at the time of allocation, we replenish every retailer up to the same amount. This is true if we relax the non-negativity constraints on retailer allocations, which is called the "allocation assumption" (Eppen & Schrage, 1981).

3. Previous Study

There are many articles on replenishment and allocation policies of a one-warehouse N-retailer system. Both Schwarz et al. (1985) and Badinelli and Schwarz (1988) investigate the so-called "portfolio" motive for holding warehouse safety-stock inventory in a continuous-review system operating under a (Q, R) inventory-replenishment policy. The results of these two papers indicate that the value of using warehouse inventory to rebalance retailer inventories between system replenishments is very small. This observation has many things to do with their service rule, FCFS (first come, first served). Schwarz (1989) examines the value of warehouse risk-pooling over outside-supplier leadtimes in a periodic-review system in which the warehouse holds no inventory and "static" allocations (= allocations to all the retailers are made at the same time) are made to all the retailers. He assumes that having the warehouse between the supplier and the retailers increases supplier-to-retailer leadtime. He extracts an explicit "price" of risk-pooling; i.e., extra pipeline inventory-holding cost from increased leadtime.

Jönsson and Silver (1987) analyze a periodic-review system with total redistribution of inventory among retailers one period before the end of the order cycle, and compare the expected backorders of this system with that of the system without redistribution. Computational tests show that the system with redistribution can provide the same service level (as the system without redistribution) with a considerably reduced inventory investment. McGavin, Schwarz, and Ward (1993) develop an infinite-retailer model and use it to determine two-interval allocation heuristics for N-retailer systems. Simulation tests suggest that the infinite-retailer heuristic policies are near-optimal for as few as two retailers, and that the risk-pooling benefits of allocation policies with two well-chosen intervals are comparable to those of base-stock policies with four equal intervals. Kumar, Schwarz, and Ward (1995) study the risk-pooling effect of a "dynamic" inventory-allocation policy (= allocations are made sequentially at each retailer) in a periodic-review system with one warehouse and N retailers. In their model a delivery vehicle visits retailers along a fixed route. They compare static and dynamic allocation policies. Through simulation experiments they conclude that dynamic allocations yield significantly lower holding and backordering costs per replenishment cycle than static allocations.

4. Risk-Pooling Effect of Emergency Shipping

In this section, we formulate the system-replenishment problem that minimizes the sum of expected inventory-holding, backorders, and shipping costs in a single cycle. We use the following notation for the formulation.

Notation

p= backorders cost per unit per period

h= holding cost per unit per period
t= shipping cost per unit
L= outside-supplier's leadtime
l= leadtime between the warehouse and any retailer
y= order-up-to level at the time of system-replenishment order
$s= \frac{y}{N}$
α= portion of s assigned to each retailer at the time of allocation
αs= allocation to each retailer
$(1-\alpha)s= \dfrac{\text{stock kept at the warehouse at the time of allocation}}{N}$
δ_i= random period demand at retailer i, i=1,...,N, with p.d.f. $\phi(.)$ and c.d.f. $\Phi(.)$
$\delta_i^N= \sum_{i=1}^{N}\delta_i$, with p.d.f. $\phi^N(.)$ and c.d.f. $\Phi^N(.)$

Under the allocation assumption, it is easily shown to be true that a myopic policy is optimal in the emergency-shipping case (See Federgruen and Zipkin (1984) and Kumar et al. (1995)). Henceforce, we focus our attention on the myopic problem. When emergency shipments are allowed, system backorders each cycle depend on system-wide available inventory (warehouse inventory plus excess inventory at the retailers) and total retailer backorders at the end of cycle. If it costs the same whether we directly ship from the warehouse to an individual customer or from any of the retailers, it is optimal that the warehouse allocates all units to the retailers at the beginning of each cycle. We can explain this as follows; while if a unit at the warehouse is directly shipped to a customer through emergency shipment, it always costs t, a unit at a retailer will satisfy demand without any additional cost if cycle demand at that retailer exceeds its initial allocation or at the cost of t if the unit should be shipped to another retailer to meet shortage. Therefore, it is always better for the retailers to carry all the inventories. We present the following lemma without the proof.

(*Lemma 1*) If it costs the same whether we directly ship from the warehouse to an individual customer or from any of the retailers, the optimal α is 1.

According to the lemma 1, unless the emergency unit shipping cost from the warehouse is significantly lower than that from a retailer (our model assumes that they are the same), we should locate inventory at the retailers rather than at the warehouse to minimize expected total emergency shipping costs. Since, considering the existence of efficient transportation systems, it is very unlikely that the difference in unit shipping costs is significant, the lemma 1 has practical implications.

Since it is optimal for the warehouse to allocate all the units to the retailers, our problem becomes a transshipment problem. For simplicity of the presentation, without loss of generality, we assume that the leadtimes (L and l) are both zero. If s is assigned to each retailer at the time of allocation, the expected total costs per cycle of the transshipment problem is

$$TC = p\int_{Ns}^{\infty}(\delta - Ns)\phi^N(\delta)d\delta + h\int_0^{Ns}(Ns - \delta)\phi^N(\delta)d \\ + t(N\int_s^{\infty}(\delta - s)\phi(\delta)d\delta - \int_{Ns}^{\infty}(\delta - Ns)\phi^N(\delta)d\delta) \tag{1}$$

In (1), the first and second terms represent expected system backorders and inventory-holding cost after emergency shipments, respectively, and the third expected system emergency-shipping cost. When system-wide cycle demand exceeds initial system inventory, we can't satisfy all of retailer backorders through direct shipping at the end of cycle. This adjustment is included in the third term.

Given (1), we can prove the following lemma.

(*Lemma 2*) When emergency shipments are allowed at the end of each cycle, the optimal system-replenishment policy is a base-stock policy.

Proof: By taking the second derivative of (1) with respect to s, we get

$$\frac{\partial^2 TC}{(\partial s)^2} = (p + h - t)N^2\phi^N(Ns) + tN\phi(s) \tag{2}$$

It is clear that (2)>0 for $\forall\, s$ since we assume that $p + h - t > 0$. The optimal system-replenishment policy is a base-stock policy, since (1) is a convex function of s. [ ]

Let x to be the system inventory position at the beginning of cycle. According to the lemma 2, we order up to Ns^* if $x < Ns^*$, order nothing if $x \geq Ns^*$, where s^* minimizes (1).

The lemma 2 says that a base-stock policy is optimal. The following lemma presents the necessary and sufficient condition of the optimal s.

(*Lemma 3*) s is optima if and only if s satisfies

$$\Phi(s) = \frac{p(1-\Phi^N(Ns)) + (t-h)\Phi^N(Ns)}{t} \tag{3}$$

<u>Proof:</u> By taking the first derivative of (1), we get

$$\frac{\partial TC}{\partial s} = -(p-t)N(1-\Phi^N(Ns)) + hN\Phi^N(Ns) - tN(1-\Phi(s)) \qquad (4).$$

By setting (4) to be zero and solving for $\Phi(s)$, we can get (3). [ ]

We can interpret the necessary and sufficient condition of the optimal system order-up-to level (3) in the Newsboy problem context. According to (3), the cost of under-stocking and that of over-stocking are $p(1-\Phi^N(Ns))+(t-h)\Phi^N(Ns)$ and $(t-p)(1-\Phi^N(Ns))+h\Phi^N(Ns)$, respectively. These costs are conditional expectations on if system inventory can meet system demand. In particular, the cost under-stocking is p if system hasn't enough inventory to meet all the demand since the unit should be backordered, and $(t-h)$ if it has enough inventory since the unit will be shipped from any retailer with available inventory to an individual customer in need. The same interpretation is possible for the cost of over-stocking; i.e., it is $(t-p)$ if system hasn't enough inventories to meet all the demand, h if it has.

It is also interesting to compare the optimal s in the emergency-shipping case with that of the no emergency-shipping case. The following lemma identifies the effect of emergency shipping on the magnitude of system-replenishment quantity.

(*Lemma 4*) When emergency shipping is allowed, the warehouse orders less from its supplier.

<u>Proof:</u> Without emergency shipping, the optimal s satisfies

$$\Phi(s) = \frac{p}{p+h} \tag{5}$$

Subtracting the right-hand side of (5) from that of (3), we get

$$\frac{(p+h-t)(p(1-\Phi^N(Ns^*)) - h\Phi^N(Ns^*))}{t(p+h)} \tag{6}$$

The sign of (6) is determined by that of $p(1\text{-}\Phi^N(Ns^*))\text{-}h\Phi^N(Ns^*)$ since $p + h - t > 0$. Rearranging (3), we get

$$\Phi(s^*) = \Phi^N(ns^*) + \frac{p(1-\Phi^N(Ns^*)) - h\Phi^N(Ns^*)}{t} \tag{7}$$

Since $\Phi(s^*) < \Phi^N(Ns^*)$, $\frac{p(1-\Phi^N(Ns^*))-h\Phi^N(Ns^*)}{t}$ should be negative. Therefore, (6) should be negative. This proves the lemma. [ ]

According to the lemma 1 and 4, emergency shipping contributes to improving the supply chain competency at various aspects. Given a reasonable set of cost parameters, allowing emergency shipment lowers both inventory-related costs and system inventory level without sacrificing customer service level. Also it shortens the average time to respond to customer demand.

5. Concluding remarks

A distribution system carries various items, and some important items are efficiently managed by a modern inventory system such as VMI. But most of items should be controlled by a conventional inventory system. By using new information and communication technologies and efficient transportation systems, we can maximize the efficiency of conventional inventory control methods. This paper analyzes the two-echelon distribution system with emergency shipment to derive its managerial implications to the SCM (supply chain management). According to our analysis, emergency shipment can improve the supply chain competency of a firm at various aspects. It can make a supply chain leaner by ordering less from the supplier, and make a supply chain speedier with prompt response to customer demand through emergency shipping to a customer in case of stock-out. Also, according to our analysis, when emergency shipments are allowed, given a reasonable set of cost parameters, it not only saves costs but also increases customer service level to locate inventory close to customers.

The most probable extension of this paper is to incorporate different unit shipping costs for the warehouse and each retailer, which will give more reality to the model but complicate the analysis. Another possible extension would be to analyze the transshipment problem in details, which will give more ideas how to make the optimal transshipment decisions.

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