Do Consumption and Income Have a Long Run Relationship?

Jae-Young Kim*

This paper provides some new empirical evidence on the consumption-income relation which is one of the most thoroughly studied subjects in economics. According to the recent literature in economics the two variables should be cointegrated for many theoretical results in economics, such as the permanent income hypothesis, to be meaningful. Our initial empirical results, however, show that cointegration between income and consumption is not well confirmed for U.S. quarterly data for extended postwar periods. This is an important problem that has to be addressed in the literature. In this paper we conjecture that failure of confirming cointegration for the consumption-income relation is due to nonstationary fluctuations in some relatively short period(s) although the relation prevails in the majority of data period. Our empirical result confirms our conjecture. Two periods of "short-run" nonstationarity are identified for an extended postwar era of the U.S. economy: One is the Volker era in the early 1980's and the other consists of the recent years of unusually low interest rate. Our result has important implication for empirical analysis in economics where consumption and income variables are involved.

Keywords: Consumption-income relation, Cointegration, Short-run nonstationarity, Partial sample cointegration breakdown

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*Associate Professor, School of Economics, Seoul National University, San 56-1 Shilim-dong, Kwanak-gu, Seoul 151-742, Korea, (E-mail) jykim017 @snu.ac.kr. I thank anonymous referees for helpful comments. I also thank Don Andrews, Michael Jerison, and Terry Kinal for helpful comments, and Woong-Yong Park for research assistance. I gratefully acknowledge the financial supports from the Advanced Strategy Program (ASP) of the Institute of Economic Research, Seoul National University, and BK21 program of Korea Research Foundation.
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I. Introduction

The relationship between consumer expenditure and income is one of the most thoroughly studied subjects in economics. Since Keynes (1936) explained the relationship between the two variables, many researchers provided empirical findings as well as theoretical results on it. In this paper we provide some new empirical evidence on the consumption-income relation that one needs to be aware of in studying an economic relation containing these two variables.

In relatively recent years researchers found that nonstationarity in the two variables of consumption and income has some important implication on the relationship between the two variables. Examples are Davidson *et al.* (1978), Hall (1978), Flavin (1981), Campbell (1987), and Pollock and Lekka (2004), among others. In particular, Campbell (1987) noted that cointegration between the two variables of consumption and income has some important implication for the permanent income hypothesis. Generally speaking, existence of cointegration implies that there is a long-run equilibrium relation among the variables involved. Thus, in economic theory it is a well understood hypothesis that the two variables of consumption and income would be cointegrated. Also, in empirical literature in the consumption-income relation the hypothesis has been well accepted as in Davidson *et al.* (1978).

Our initial empirical results, however, show that cointegration between income and consumption expenditure is *not* well confirmed for U.S. quarterly data for extended post-war periods. This is an important problem that has to be addressed in the literature. In this paper we study why the well understood relation is not well confirmed by real data. We propose a hypothesis that failure of confirming cointegration for the consumption-income relation is due to nonstationary fluctuations in some relatively short period(s), or "short-run" cointegration breakdown, although the relation prevails in the majority of data period. We analyze this hypothesis by formal procedures developed in Andrews and Kim (2003).

For the U.S. quarterly observations for the period of 1953:Q1-2004:Q2 we obtained a result that supports our hypothesis. We have identified two periods of short-run cointegration breakdown: One is the Volker era in the U.S. economy when the domestic fiscal and monetary policies underwent some important change that might cause disturbance in the consumers' optimization behavior.

The other period is the period of early 2000's which may be due to an unusually low interest rate policy in the U.S.. Our result has important implication for empirical analysis in economics where consumption and income variables are involved.

Our discussion in this paper goes as follows. Section II provides a brief explanation of our econometric methods. Section III exhibits our empirical results with some explanations. Section IV concludes the paper.

II. The Model and Hypotheses

As is shown in the next section, cointegration between income and consumption is not confirmed for U.S. quarterly data of extended postwar periods. In this section we propose a hypothesis that failure of confirming cointegration for the consumption-income relation is due to short-run nonstationary fluctuations. Such non-stationary fluctuations may be due to some important change in the economy that causes disturbance in the agent's optimization behavior. Examples of such change are policy interventions, change in tastes, oil shocks, *etc.*

For formal analysis of this hypothesis we apply inference procedures in Andrews and Kim (2003). In this section we introduce the model and the hypothesis. For convenience of the reader we provide a brief explanation of the methods that we use in the paper in Appendix.

We are interested in the following relation:

$$C_t = b_0 + b_1 Y_t + u_t \tag{1}$$

where C_t and Y_t are, respectively, logarithms of private consumption expenditure and disposable personal income. If the error term u_t is a stationary process, the two variables C_t and Y_t are cointegrated. If u_t is nonstationary for some periods but stationary in the other periods, then C_t and Y_t are "segmented cointegrated" as is named in Kim (2003). In the case of segmented cointegration we have period (s) of cointegration breakdown.

Now, writing $y_t = C_t$, $x_t = (1, Y_t)'$, $\beta = (b_0, b_1)'$, and assuming that the period of cointgeration breakdown is $(t_0, \cdots, t_0 + m - 1)$, we can formalize the case of possible segmented cointegration for C_t and Y_t

as in the following:

$$y_t = \begin{cases} x'_t\beta_0 + u_t & \text{for } t=1,\cdots,t_0-1,t_0+m,\cdots,T+m \\ x'_t\beta_t + u_t & \text{for } t=t_0,\cdots,t_0+m-1. \end{cases} \tag{2}$$

where $y_t, u_t \in R$ and $x_t, \beta_0, \beta_t \in R^2$. We assume that the errors for the cointegrated periods $\{u_t\text{: } t=1,\cdots,t_0-1,t_0+m,\cdots,T+m\}$, are stationary, of mean zero, and ergodic.

The null and alternative hypotheses are

$$\begin{aligned} &H_0: \begin{cases} \beta_t=\beta_0 \quad \text{for all } t=T+1,\cdots,T+m \text{ and} \\ \{u_t\text{: } t=1,\cdots,T+m\} \text{ are stationary and ergodic} \end{cases} \\ &H_1: \begin{cases} \beta_t \neq \beta_0 \quad \text{for some } t=t_0,\cdots,t_0+m-1 \text{ and/or} \\ \text{the distribution of } \{u_t,\cdots,u_{t+m-1}\} \text{ differs from} \\ \text{the distribution of } \{u_{t_0},\cdots,u_{t_0+m-1}\} \text{ for } t\neq t_0. \end{cases} \end{aligned} \tag{3}$$

Under the null hypothesis, the model is a well-specified cointegrating regression model for all $t=1,\cdots,T+m$. Under the alternative hypothesis, the cointegrating relationship breaks down in the period $\{t=t_0,\cdots,t_0+m-1\}$. The breakdown may be due to (i) a shift in the cointegrating vector from β_0 to β_t, (ii) a shift in the distribution of u_t from being stationary to a unit root, (iii) some other shift in the distribution of u_t or (iv) some combination of the these shifts.

The above alternative hypothesis postulates the case of middle-of-sample cointegration breakdown. Breakdown of cointegration may also occur at the end of sample as well. In the latter case we have the following hypotheses:

$$\begin{aligned} &H_0: \begin{cases} y_t = x'_t\beta_0 + u_t \quad \text{for all } t=1,\cdots,T+m \text{ and} \\ \{u_t\text{: } t\geq 1\} \text{ are stationary and ergodic} \end{cases} \\ &H_1: \begin{cases} y_t = x'_t\beta_0 + u_t \quad \text{for all } t=1,\cdots,T, \text{ and} \\ y_t = x'_t\beta_t + u_t \quad \text{with } \beta_t\neq\beta_0 \text{ for } t=T+1,\cdots,T+m \text{ and/or} \\ \text{the distribution of } \{u_{T+1},\cdots,u_{T+m}\} \text{ differs from that of} \\ \text{error sequences } \{u_1,\cdots,u_T\}. \end{cases} \end{aligned} \tag{4}$$

For notational convenience we consider the case of the end-of-sample cointegration breakdown in our explanation in Appendix. In the case of the middle-of-sample breakdown in the hypotheses in (3) we can construct tests for these hypotheses by moving the observations $\{(y_t, x_t): t=t_0, \cdots, t_0+m-1\}$ to the end of the sample and moving the observations after $t=T+m-1$ up to fill the gap. The observations originally indexed by $t=t_0, \cdots, t_0+m-1$ are subsequently indexed by $t=T, \cdots, T+m$ and the tests defined above can be used to test the hypotheses in (3).

III. Empirical Results

A. Data and Results of Cointegration Tests

We use data of U.S. quarterly observations for the period of 1953:Q1-2004:Q2. We have seasonally adjusted data for private consumption expenditure and disposable personal income from U.S. Bureau of Economic Analysis. Our model is a simple consumption-income relation (1)

$$C_t = b_0 + b_1 Y_t + u_t$$

where C_t and Y_t are, respectively, logarithms of private consumption expenditure and disposable personal income.

First, we apply the residual based tests of Phillips and Ouliaris (1990) for testing the null of no cointegration. The results are shown in Figures 1 and 2. The two figures show that cointegration is not well confirmed for the two variables for U.S. postwar quarterly data if we include data after early 1990's, which is contrary to the general understanding that the two variables are well cointegrated. More detailed explanations are in the following.

Figure 1 shows the values of the "coefficient" test statistic for testing the null of no cointegration in Phillips and Ouliaris (1990). The values are evaluated for data from 1953:Q1 to an end period that runs from 1977:Q4 (making the sample size 100) through 2004:Q2. The horizontal scale is for the ending periods of data. The dotted line in Figure 1 is for the 5% critical value of the test. Figure 2 shows values of t-statistic for testing the null of no cointegration in Phillips and Ouliaris (1990) calculated by the same

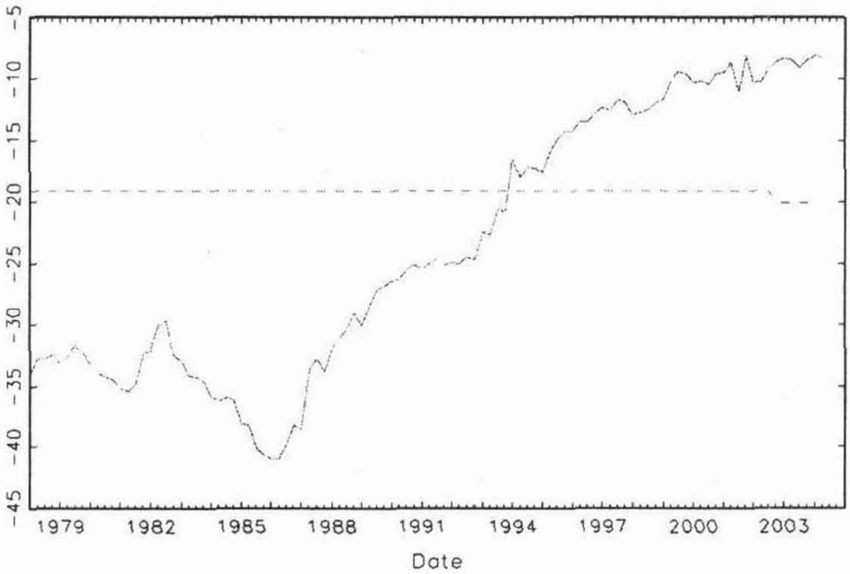

FIGURE 1
RESIDUAL-BASED COINTEGRATION TEST STATISTICS (*Za*)
FOR EACH PERIOD

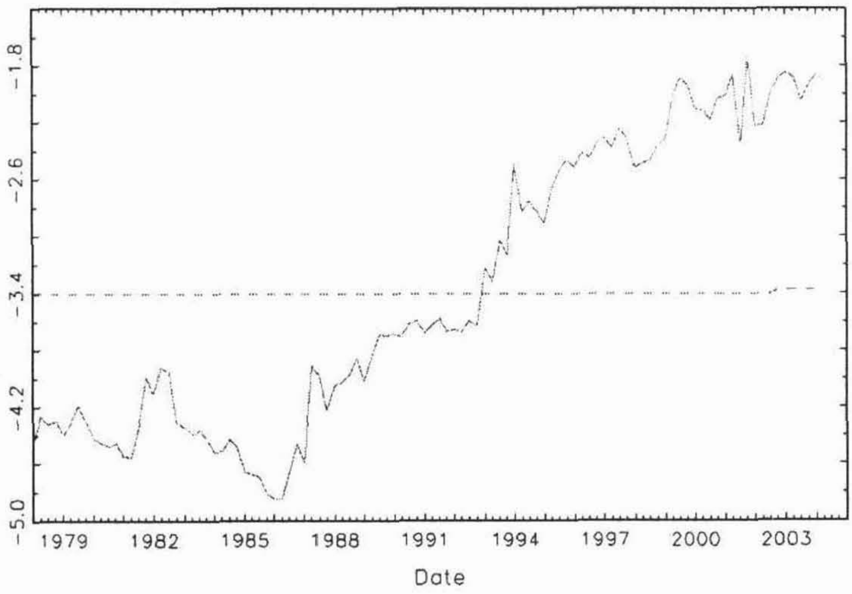

FIGURE 2
RESIDUAL-BASED COINTEGRATION TEST STATISTICS (*Zt*)
FOR EACH PERIOD

way as for Figure 1. As is shown in Figure 1, if we include observations after 1993:Q4 in the data, the coefficient test fails to reject the null of no cointegration at the 5% level. Also, as is shown in Figure 2, if we include observations after 1992:Q4 in the data, the t-test fails to reject the null of no cointegration at the 5% level.

B. Results of Partial Sample Cointegration Breakdown Tests

To test for the existence of and to identify the period of short-run fluctuations we apply the middle-of-sample cointegration breakdown tests. The p-values of P and R tests for the middle-of-sample breakdown are shown in Figure 3 for $m=8$. We have very similar results from the two tests. The dotted line in Figure 3 is the 5% line. We identify the period of cointegration breakdown in Figure 3 as the period in which the p-values falls below the 5% line. The identified periods are reported in Table 1. The first breakdown period, 1980:Q3-1983:Q3 for the P test, may be due to changes in the U.S. domestic fiscal and monetary policies in the period. When there is an important change in economic policy, it can cause a shift or instability in the consumers' optimization behavior, as the Lucas' critique explains. In such a case a "reduced form" relation such as the simple consumption-income relation might change or become unstable. On the other hand, the second breakdown in the period of early 2000's may be due to an unusually low interest rate policy in U.S..

We also check whether cointegration is confirmed in the periods other than the identified breakdown periods. The results are provided in Table 2. As is shown in Table 2 we can reject the null of no cointegration in all the sub-samples identified by the P and R tests.

To some readers, it may be curious why the cointegration tests do not fail to confirm cointegration for data up to 1992 or 1993 but fail to do so for data including observations after 1992 or 1993 while the cointegration breakdown occurs in early 1980.[1] One reason for it might be that the turbulence to the relation in the period of early 1980 alone is not strong enough, or the length of this period is not long enough, for a cointegration relation not to

[1]One of the referees pointed out this important finding. I appreciate the referee for it.

TABLE 1
THE IDENTIFIED PERIODS OF COINTEGRATION BREAKDOWN BY P AND R TESTS FOR MIDDLE-OF-SAMPLE BREAKDOWN

P test	R test
1980:3-1983:3	1980:2-1985:2
2002:2-2004:2	2001:4-2004:2

TABLE 2
RESULTS OF THE RESIDUAL-BASED COINTEGRATION TEST RESULTS

Period	T	statistics		OLS coefficient estimates		
		Z_a	Z_t	β_0	β_1	ρ
For the whole sample						
1953:1~2004:2	206	-8.25	-1.91	-0.25	1.02	0.94
For cointegrated sub-samples identified by P test						
1953:1~1980:2	110	-33.9476	-4.4567	0.1247	0.9691	0.6971
1983:4~2002:1	74	-41.2436	-5.2166	-1.4570	1.1572	0.4701
For cointegrated sub-samples identified by R test						
1953:1~1980:1	109	-33.3314	-4.4218	0.1235	0.9693	0.6978
1985:3~2001:3	65	-28.0102	-4.0063	-1.3263	1.1423	0.5723

Notes: ρ : autoregressive coefficient of residuals ($u_t = \rho u_{t-1} + e_t$)
Z_a: Coefficient test of Phillips-Ouliaris (1990)
Z_t: t-test of Phillips-Ouliaris (1990)
5% critical values: -19.19 for Z_a and -3.40 for Z_t

be confirmed. We know that a conitegration relation is a long run relation that holds even if there is relatively short period(s) of abnormal disturbance to the relation. It is quite possible that there is another period(s) of turbulence to the consumption-income relation around 1993, not captured by the middle-of-sample breakdown tests, which combined with the period of early 1980 results in failure of confirming cointegration for the extended data. In our middle-of-sample breakdown tests the two periods of cointegration breakdown are not dealt with simultaneously as a set of breakdown periods but is dealt with one by one, which may cause low power of the tests and lead to failure of capturing a relatively less important period of breakdown. To investigate this problem more appropriately, we need to develop a method for testing the problem

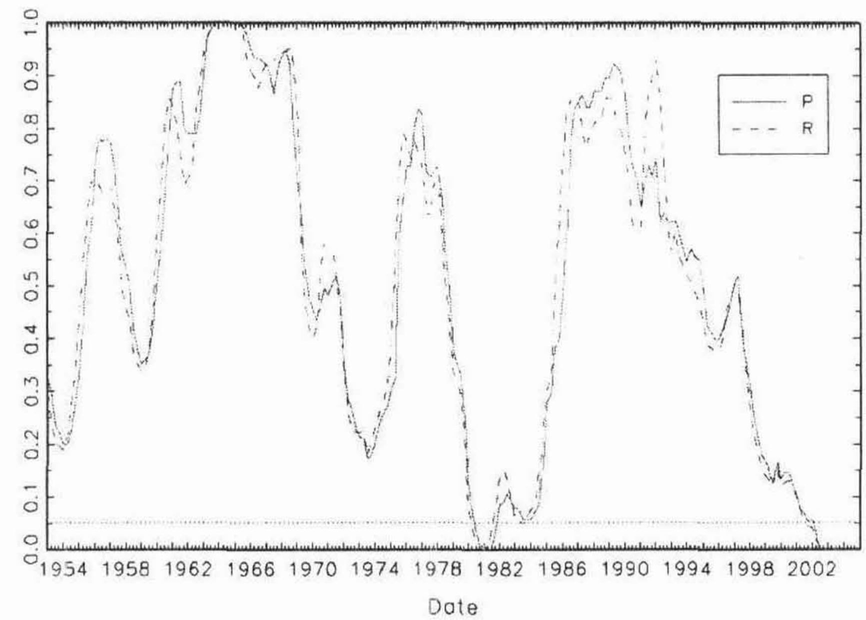

FIGURE 3
P-VALUES FOR *P* TEST & *R* TEST GIVEN m = 8: MIDDLE OF SAMPLE

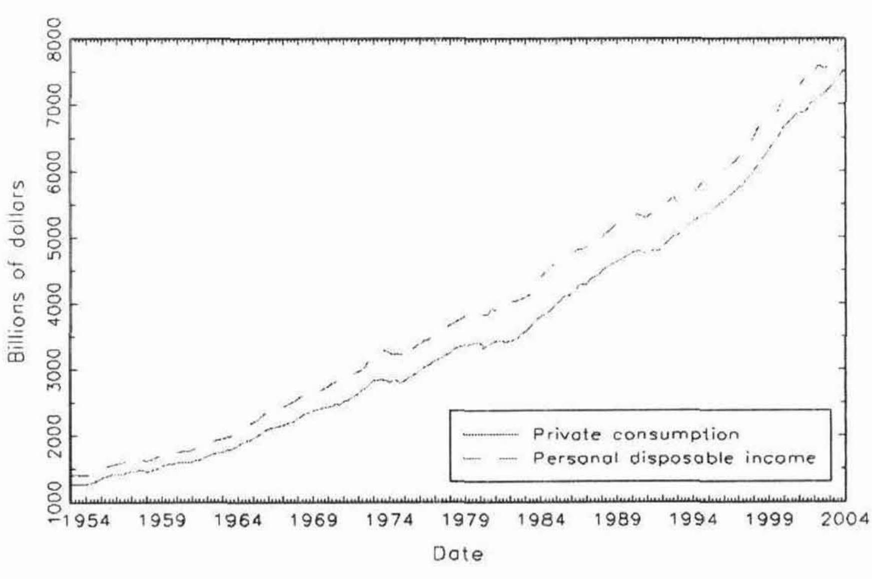

FIGURE 4
REAL PRIVATE CONSUMPTION & PERSONAL DISPOSABLE INCOME
(2000 = 100, SA)

of multiple-period breakdowns that handles the multiple periods of breakdown simultaneously as a set of breakdown periods.

IV. Concluding Remarks

We have found that a well known theoretical relation of consumption and income is not confirmed for U.S. data in an extended postwar era. We have also found that this is due to some short-run nonstationary fluctuations in the relation. This is an important finding that has to be taken into account in practical analysis dealing with the consumption and income relation. Failure to properly incorporate it might cause a misleading inference. For example, failure of confirming the permanent income hypothesis for some data might be due to failure to incorporate short-run non-stationarity.

Our finding in this paper does not provide a completed view on the relation of consumption and income. Rather, it poses a new issue that some short-run nonstationary deviations from a well understood economic relation may cause failure of empirical confirmation of the relation. The method used in this paper can be applied to identify such a short period of nonstationary deviations. It would be worth-while to pursue more comprehensive investigation of the consumption behavior based on a model containing other variables as well as the income level.[2] This subject is currently being studied by the author of this paper. Also, it would be interesting to build a structural mechanism in which such a phenomenon is generated.

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Appendix: The *P*-Test and the *R*-Test

There are two types of tests, *P*-tests and R-tests, developed in Andrews and Kim (2003) that are useful for our analysis.[3] We

[2]One of the referees also suggested this line of research.

[3]There are all six tests developed in Andrews and Kim (2003), three *P*-tests and three *R*-tests. Andrews and Kim (2003) recommends to use the P_c and R_c tests. The *P* and *R* tests used in this paper are, respectively, the

briefly explain them in the following for convenience of the reader. Note that we consider the case of the end-of-sample cointegration breakdown in the following.

Thus, for any $1 \leq r \leq s \leq T+m$, let

$$\mathbf{Y}_{r\text{-}s} = (y_r, \cdots, y_s)',$$
$$\mathbf{X}_{r\text{-}s} = (x_r, \cdots, x_s)', \text{ and}$$
$$\mathbf{U}_{r\text{-}s} = (u_r, \cdots, u_s)'.$$

and

$$P_j(\beta, \Omega) = (\mathbf{Y}_{j-(j+m-1)} - \mathbf{X}_{j-(j+m-1)}\beta)' \Omega (\mathbf{Y}_{j-(j+m-1)} - \mathbf{X}_{j-(j+m-1)}\beta) \text{ and}$$
$$P_j(\beta) = P_j(\beta, I_m)$$

for $j = 1, \cdots, T+1$, where Ω is some nonsingular $m \times m$ matrix and I_m denotes the m dimensional identity matrix.

Let $\hat{\beta}_{1-T+m}$ denote an estimator of β_0 based on the observations $t = 1, \cdots, T+1$. For example, for the LS estimator,

$$\hat{\beta}_{1-T+m} = (\mathbf{X}'_{1-T+m}\mathbf{X}_{1-T+m})^{-1}\mathbf{X}'_{1-T+m}\mathbf{Y}_{1-T+m}$$

$\mathbf{X}'_{1-T+m}\mathbf{X}_{1-T+m}$ is nonsingular.

The first test statistic, P, that we consider is defined by

$$P = P_{T+1}(\hat{\beta}_{1-T+m}) = \sum_{j=1}^{T+m} (y_t - x'_t \hat{\beta}_{1-T+m})^2.$$

Under the null hypothesis, the distribution of $P_{T+1}(\beta_0)$ is the same as that of $P_j(\beta_0)$ for all $j \geq 1$, because $P_j(\beta_0) = \sum_{t=j}^{j+m-1} u_t^2$ and $\{u_t: t \geq 1\}$ is stationary. The estimator $\hat{\beta}_{1-T+m}$, which appears in the statistic P, converges in probability to the true parameter, β_0, under the null hypothesis given suitable assumptions. Hence, the asymptotic null distribution of P is the distribution of $P_1(\beta_0)$. Also, the random variables $\{P_j(\beta_0): j = 1, \cdots, T-m+1\}$ are stationary and ergodic under H_0 *and* H_1. In consequence, the empirical distribution function (df) of $\{P_j(\beta_0): j = 1, \cdots, T-m+1\}$ is a consistent estimator of the df of $P_1(\beta_0)$. Hence, we can consistently estimate the df of

same as the P_c and R_c tests.

$P_1(\beta_0)$ by using the empirical df of $\{P_j(\beta): j=1,\cdots,T-m+1\}$ evaluated at a consistent estimator of β_0.

For an estimator of β_0 in $P_j(\beta_0)$ Andrews and Kim (2003) recommends to use the "leave-$m/2$-out" estimator, $\hat{\beta}_{2(j)}$,

$$\beta_{2(j)} = \text{estimator of } \beta \text{ using observations indexed by } t=1,\cdots,T \text{ with}$$
$$t \neq j,\cdots,j+[m/2]-1$$

for $j=1,\cdots,T-m+1$, where $[m/2]$ denotes the smallest integer that is greater than or equal to $m/2$. For the types of estimators mentioned above, the estimator $\hat{\beta}_{2(j)}$ is consistent for β_0 (uniformly over j) under suitable assumptions, see below.

The empirical df of $\{P_j: j=1,\cdots,T-m+1\}$ for $P_j = P_j(\hat{\beta}_{2(j)})$ is

$$\hat{F}_{P,T}(x) = \frac{1}{T-m+1}\sum_{j=1}^{T-m+1} 1(P_j \leq x).$$

empirical distribution converges in probability (and almost surely) to the df $P_1(\beta_0)$ (under suitable assumptions). In consequence, to obtain a test with asymptotic significance level α, we take the critical value for the test statistic P to be the $1-\alpha$ sample quantile, $\hat{q}_{P,1-\alpha}$, of $\{P_j: j=1,\cdots,T-m+1\}$. By definition,

$$\hat{q}_{P,1-\alpha} = \inf\{x \in R: \hat{F}_{P,T}(x) \geq 1-\alpha\}.$$

One rejects H_0 if $P > \hat{q}_{P,1-\alpha}$. Equivalently, one rejects H_0 if P exceeds $100(1-\alpha)\%$ of the values $\{P_j: j=1,\cdots,T-m+1\}$ - that is, if

$$(T-m+1)^{-1}\sum_{j=1}^{T-m+1} 1(P > P_j) \geq 1-\alpha.$$

The p-value for the P test is

$$pv_P = (T-m+1)^{-1}\sum_{j=1}^{T-m+1} 1(P \leq P_j).$$

Andrews and Kim (2003) provides another test, called R test, that is designed to consider the Locally Best Invariant (LBI) test for the presence of unit root errors. This statistic constructs a test that are asymptotically valid under more general conditions on the errors and regressors. R-test is the form of

$$R = P_{T+1}(\hat{\beta}_{1-(T+m)}, A_m) \text{ and } R_j = P_{T+1}(\hat{\beta}_{2(j)}, A_m).$$

where $[A_m]_{k,l} = \min\{k,l\}$ for $k,l = 1,\cdots,m$.

The estimator $\hat{\beta}_{2(j)}$ used in the sub-sample statistic R_j is chosen for the same reasons as for the P tests. Critical values and p-values for the R test are obtained as in the P-test with (P,P_j) replaced by (R,R_j). The estimator $\hat{\beta}_{1-(T+m)}$ used with the R test could be the LS estimator or some other estimator. It turns out that the R test statistic is a sum of squares of reverse partial sums of residuals

$$R = P_{T+1}(\hat{\beta}_{1-T+m}, A_m) = \sum_{s=t}^{T+m} (y_s - x'_s \hat{\beta}_{1-T+m})^2.$$

The statistic R_j can be written in the same way with $\hat{\beta}_{1-T+m}$ replaced by $\hat{\beta}_{2(j)}$.

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