Infectious Disease Mortality Prediction

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When mortality statistics are reported for infectious diseases, they commonly reflect the ratio for the entire population impacted from it. This causes an underestimation since the frail members of the population are impacted at a higher rate. With the remaining healthy members, the mortality rate becomes skewed. With this project, we study predicting mortality under varying frailty conditions to account for the hidden heterogeneity's impact on the parameter estimates.

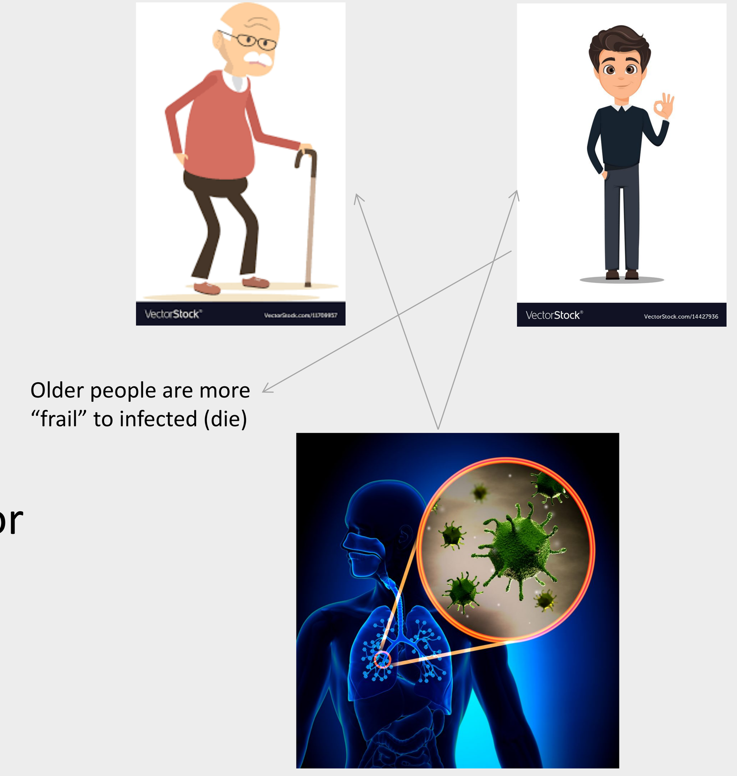
Survival analysis

- A branch of statistics for analyzing the expected period of times till one or more of events happen, like a death in biological organisms or failure in mechanical systems.
- It attempts to answer questions such as:
  - the proportion of a population which may survive past a particular time.
  - of those that survive, at what rate can they die or fail?
  - do specific circumstances or characteristics change the likelihood of survival?
- We are using survival analysis to model varying time to death of patients from pneumonia due to their varying frailty.

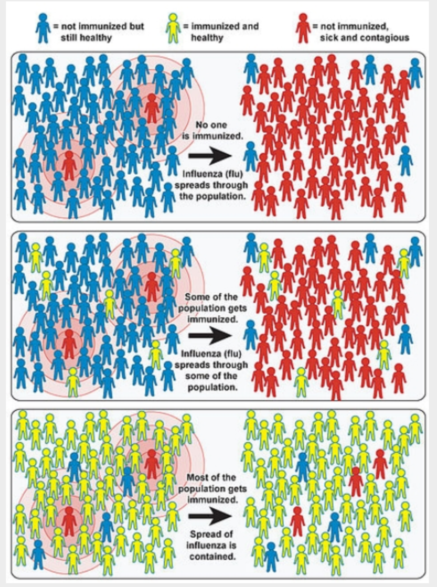
Frailty

- A type of an unobserved random effect shared by subjects within a subgroup.

Frail individuals will die early; late survivors will tend to be healthier.
Frailty represents an effect on the survival not measured when collecting information on individual subjects.
When frailty effects are ignored, the resulting survival estimates could be misleading. Corrections for this overdispersion are needed for accurate predictions.



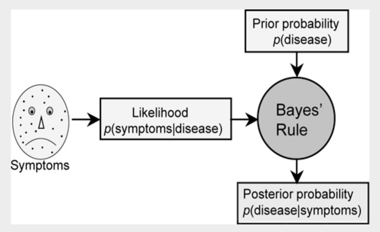
Compartmental Disease Models



Unvaccinated individuals become frail to the virus

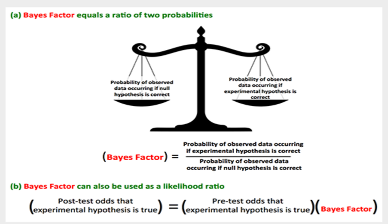
Modeling Tools

- **Bayes Factor**
- Bayes factor is the likelihood ratio of two marginal distributions.
- Related to Bayes theorem and Bayesian analysis.



For given data D if we have two different models M_1 and M_2 then the Bayes factor-

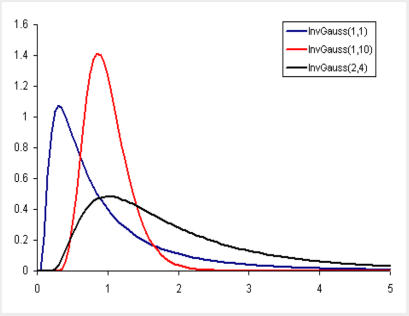
$$BF = \frac{P(D|M_1)}{P(D|M_2)}$$



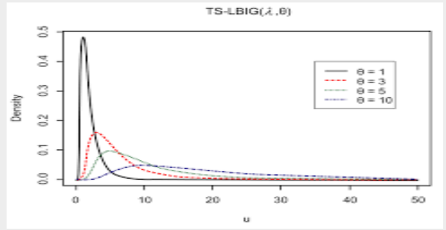
- **Inverse Gaussian Distribution**
- Inverse Gaussian is originated based on Brownian Motion.
- It is commonly used for lifetime models in survival analysis.

In this study, we use the following pdf form of inverse Gaussian distribution, which is based on the pdf form used by Tweedie (1957a).

$$f(\lambda;\mu,\delta) = \left(\frac{\delta}{2\pi}\right)^{1/2} (\lambda)^{-3/2}\, e^{\frac{-\delta(\lambda-\mu)^2}{2\mu^2\lambda}}$$



- **Length Biased Distribution**
- Length bias (or length time bias) caused by overestimation of survival length.
- Length biased distributions are often employed to develop correct models in lifetime data analysis.



- **Weighted Distribution**
- Weight functions allows to allocate more "weight" or influence on some elements of a set.
- Weight functions can help to improve fit when estimating unknown parameters or choose a curve to represent a model.
- Weight can also be used to minimize bias.

For a non-negative continuous random variable λ with probability density function $f(\lambda)$, the pdf of the weighted random variable $w(\lambda)$ is given by

$$f_w(\lambda) = \frac{w(\lambda) f(\lambda)}{E[w(\lambda)]}$$

Our final model is -

$$f(\lambda;\mu,\delta,p) = \left(\frac{\delta}{2\pi}\right)^{1/2} (\lambda)^{-3/2}\, e^{\frac{-\delta(\lambda-\mu)^2}{2\mu^2\lambda}} \left[(1-p) + p\,\frac{w(\lambda)}{E[w(\lambda)]}\right]$$

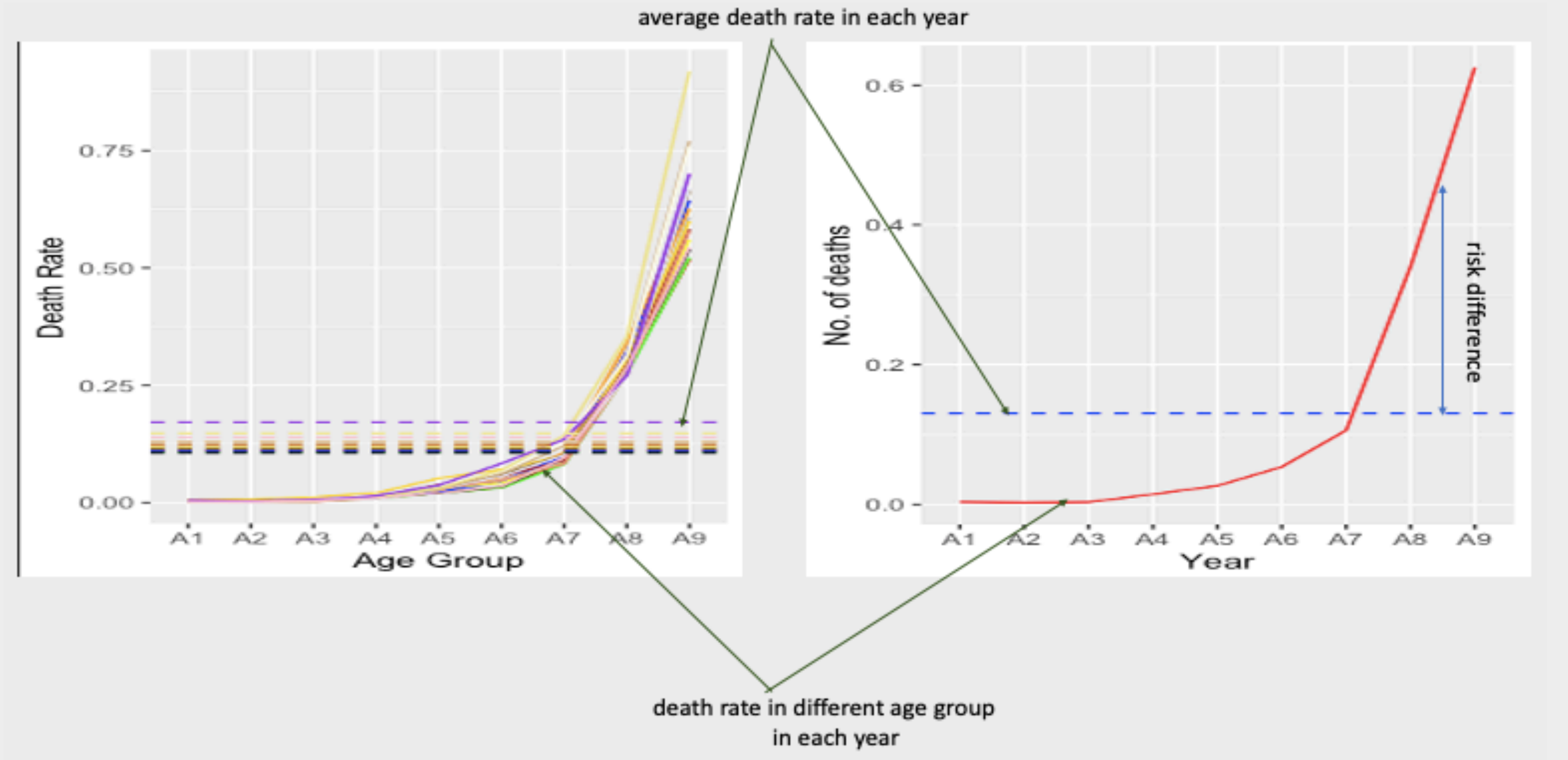
Some commonly used weight functions:

$w(\lambda) = \lambda$, $w(\lambda) = \lambda^2$, $w(\lambda) = \lambda^t$, or $w(\lambda) = e^{t\lambda}$

An Application

Leading causes of death (ICD-10)	Characteristics	Reference period	ages, Both sexes, Canada, place of residence	to 14 years, Both sexes, Canada, place of residence	death, 15 to 24, Both sexes, Canada, place of residence	death, to 34 years, Both sexes, Canada, place of residence	35 to 44 years, Both sexes, Canada, place of residence	death, 45 to 54 years, Both sexes, Canada, place of residence
			Number					
		2000	4,966	7	10	18	49	101
		2001	4,776	6	8	13	51	103
		2002	4,725	14	15	17	44	103
		2003	4,957	25	9	17	60	99
		2004	5,729	9	10	22	56	111
		2005	5,845	17	13	16	72	132
		2006	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		2007	5,452	14	21	17	71	157
		[illegible]	[illegible]	[illegible]	14	30	63	187

Morality by Age



Conclusions

- In the year 2000:
  - 1 in 3 patients of the age group (75-84) died of Pneumonia, whereas only 1 in 1000 patients of the age group (0-14) died. To be exact, patients of the age group (75-84) were 287 times more likely to die compared to the younger patients in age group (0-14).
- In the year 2016:
  - 1 in 3 patients of the age group (75-84) died of Pneumonia, whereas only 5 in 1000 patients of the age group (0-14) died. To be exact, patients of the age group (75-84) were 54 times more likely to die compared to the younger patients in age group (0-14).
- We attribute the improvement in the older age group's mortality rate to the improved health care for the elderly patients during the last decade.
- With accurate mortality prediction healthcare resources may be allocated more effectively.