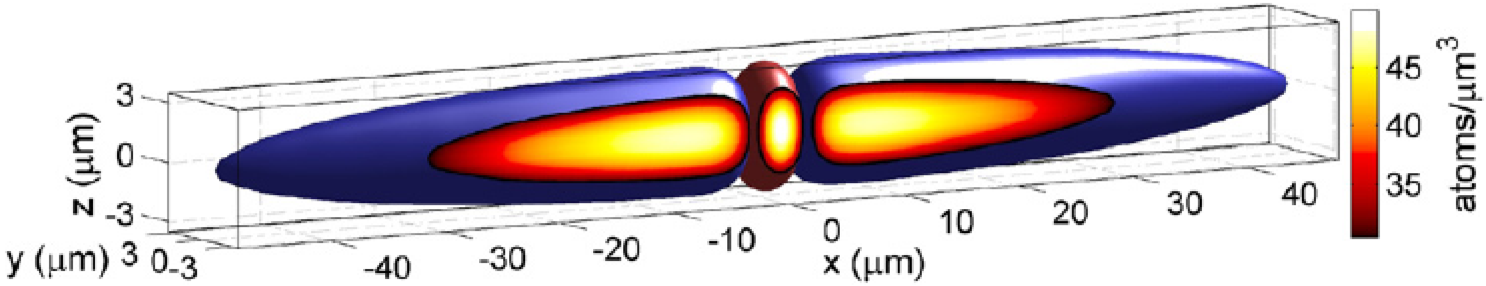
Atomic Dark-Bright Solitons: Theory and Experiments



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Outline

- **Bose-Einstein condensates (BECs)**

  - Binary BECs
  - Gross-Pitaevskii mean-field description

- **Dark-bright solitons in binary BECs**

  - Single and multiple dark-bright solitons
  - SU(2) rotations: “beating” dark-dark solitons

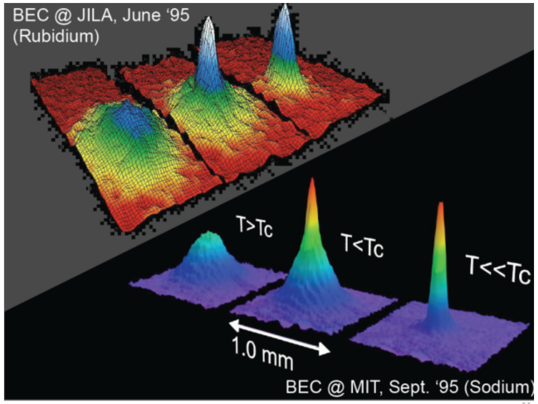
- **Dark-bright solitons at finite temperatures**

  - Dissipative Gross-Pitaevskii equations
  - Different temperature-dependent damping regimes

- **Conclusions and outlook**

Bose-Einstein condensates (BECs)

- **BEC**: State of matter in which a macroscopic number of particles share the *same* quantum state

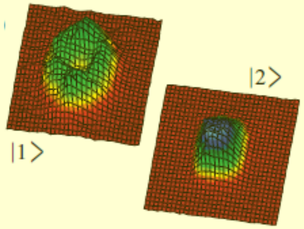


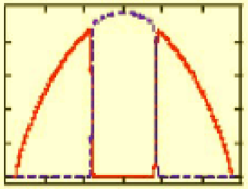
- ***Theoretical prediction*: Bose-Einstein (1925)**

- ***Experimental observation*:**
  Cornell-Wieman-Ketterle-Hulet (1995)
  in ultracold atoms of ^{87}Rb, ^{23}Na and ^{7}Li. **Nobel Prize (2001)**

Binary Bose-Einstein condensates

***First experimental observation*:**
magnetically-trapped spin states of ^{87}Rb BEC (JILA group, PRL 1997)





Binary BECs in the mean-field picture

Two coupled Gross-Pitaevskii equations

(for different hyperfine states of the same atom species):

$$i\hbar\partial_t\psi_1 = \left(-\frac{\hbar^2}{2m}\nabla^2\psi_1 + V_1^{(ext)} + g_{11}|\psi_1|^2 + g_{12}|\psi_2|^2\right)\psi_1$$

$$i\hbar\partial_t\psi_2 = \left(-\frac{\hbar^2}{2m}\nabla^2\psi_2 + V_2^{(ext)} + g_{12}|\psi_1|^2 + g_{22}|\psi_2|^2\right)\psi_2$$

intra-species **atomic collisions** ***inter-species*** **atomic collisions**

Typically, e.g., for different spin states of ^{87}Rb: $g_{11} \approx g_{12} \approx g_{22}$

Binary BECs in *highly anisotropic (quasi-1D)* harmonic traps

$$i\partial_t\psi_1 = \left(-\frac{1}{2}\partial_z^2\psi_1 + V(z) + |\psi_1|^2 + |\psi_2|^2 - 1\right)\psi_1$$

$$i\partial_t\psi_2 = \left(-\frac{1}{2}\partial_z^2\psi_2 + V(z) + |\psi_1|^2 + |\psi_2|^2 - \mu\right)\psi_2$$

$$V(z) = \frac{1}{2}\Omega^2 z^2$$

$$\Omega \equiv \omega_z / \omega_\perp << 1$$

Dark-bright solitons in homogeneous BECs

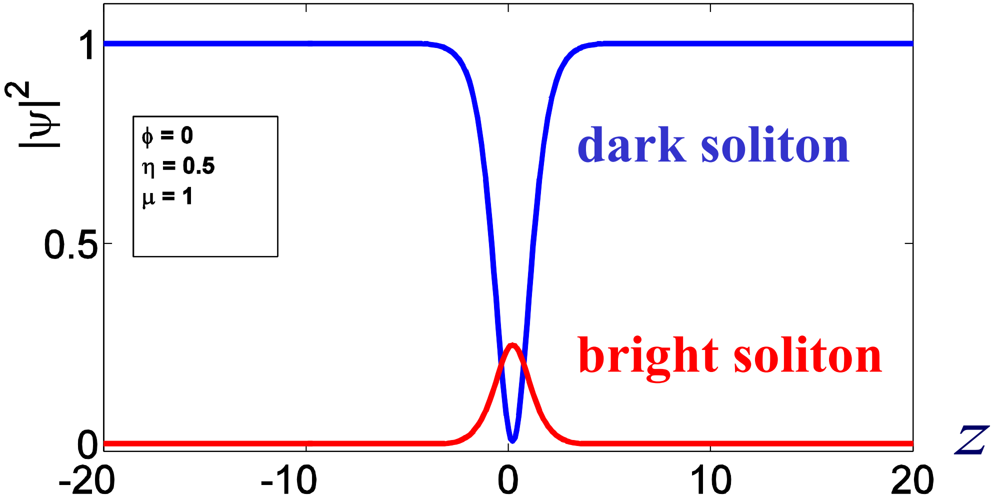
"*Symbiotic*" solitons

The bright soliton (supported only by attractive interactions) exists *only* due to the interspecies interaction with the dark soliton

$$\psi_1(z,t) = \cos\varphi \tanh\zeta + i\sin\varphi, \qquad \text{dark soliton}$$

$$\psi_2(z,t) = \eta\,\mathrm{sech}\,\zeta \exp\{\mathrm{i}[\mathrm{k}z + \theta(t)]\} \qquad \text{bright soliton}$$

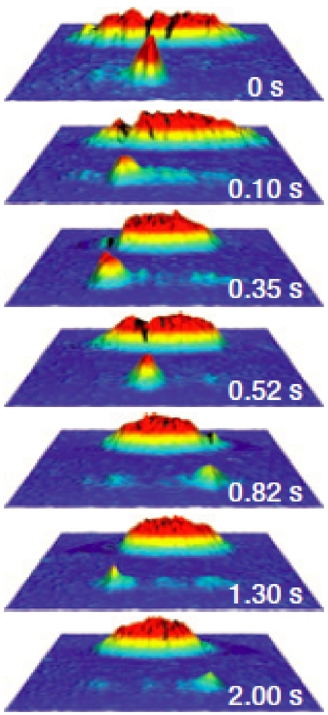
$$\zeta = D[z - z_0(t)], \;\; \dot{z}_0 = k = D\tan\varphi, \;\; \theta(t) = \frac{1}{2}[D^2 - k^2 + 2(\mu - 1)]t$$



Observations of dark-bright solitons

Hamburg experiment

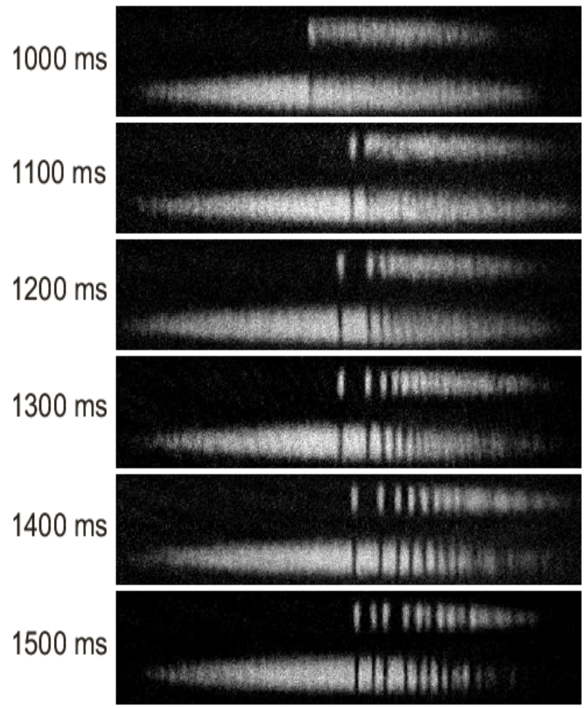
S. Stellmer *et al.*, Nat. Phys. 2008

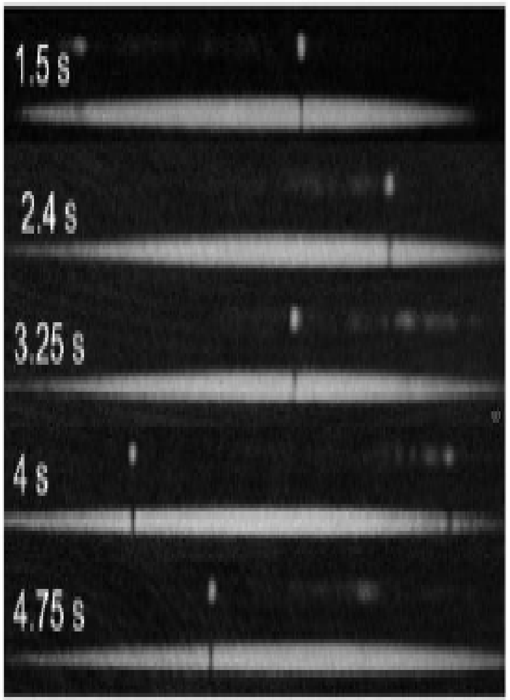


Washington experiments

C. Hammer *et al.*, PRL 2011

S. Middelkamp *et al.*, PLA 2011

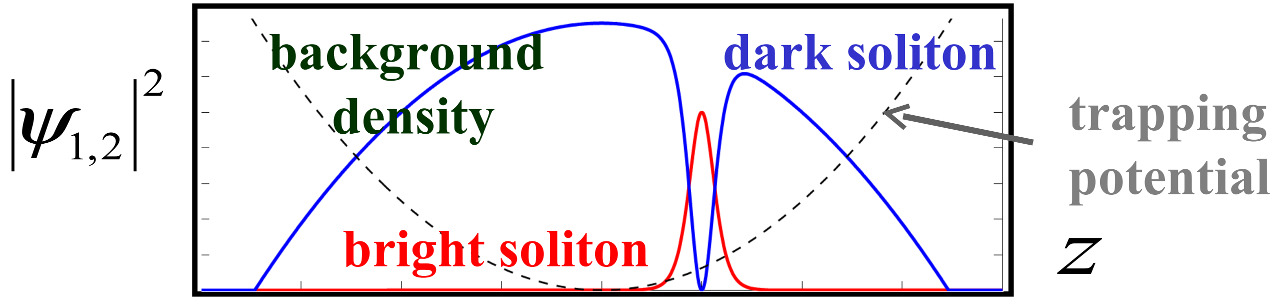




- ***Hamburg*:** ***Phase-imprinting*** **of a dark soliton in state |1, 0> and *filling* the density dip with atoms in state |2, 0 >**

- ***Washington*:** **Generation of dark-bright solitons by *counterflow* of two components in the |1, -1> and |2, -2> states**

Dark-bright solitons in the trap



Adiabatic dynamics of dark-bright solitons in the trap - <u>steps to follow:</u>

- **Find an equation for the *background* wavefunction**
- **Find a perturbed NLS system for the *dark-bright soliton* wavefunction**
- **Assume: the soliton's functional form remains the *same***

  but* the soliton parameters are *unknown functions of time
- **Evolution of *renormalized Hamiltonian* ⇨ evolution of soliton parameters**

$$E_{DB} = \int_{-\infty}^{+\infty} \left(|\partial_z \psi_1|^2 + |\partial_z \psi_2|^2 + (|\psi_1|^2 + |\psi_2|^2 - 1)^2 - 2(\mu - 1)|\psi_2|^2 \right) dz$$

$$= \frac{4}{3} D^3 + \frac{N_b}{\sqrt{\mu}} \left(\frac{1}{2} D^2 \sec^2 \varphi - \frac{\Delta}{\mu} \right)$$

Perturbation theory – Hamiltonian approach

BEC wavefunction carrying a dark soliton: $\psi_1 = \Phi(z)\exp(-i\mu t)\upsilon(z,t)$

$$\text{Thomas – Fermi approximation}: |\Phi|^2 = \mu - V(z)$$

dark soliton

Coupled GPEs as coupled perturbed NLSEs

$$i\frac{\partial \upsilon}{\partial t} + \frac{1}{2}\frac{\partial^2 \upsilon}{\partial z^2} - \left(|\upsilon|^2 + |u|^2 - 1\right)\upsilon = Q_d(\upsilon) \equiv \frac{1}{2\mu^2}\left(2V\left(1-|\upsilon|^2\right)\upsilon + \frac{dV}{dz}\frac{\partial \upsilon}{\partial z}\right)$$

$$i\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial z^2} - \left(|\upsilon|^2 + |u|^2 - \mu\right)u = Q_b(\upsilon) \equiv \frac{1}{\mu^2}V\left(1-|\upsilon|^2\right)u$$

Approximate **dark-bright soliton solution for** $Q_d \neq 0$, $\mathbf{Q_b} \neq 0$ **as in the unperturbed case but with:**

$$\varphi \to \varphi(t), \quad D \to D(t), \quad \dot{z}_0(t) \to D(t)\tan\varphi(t)$$

Use of the *dark-bright soliton energy* to find the evolution of the unknown time-dependent soliton parameters D, φ, z_0

Oscillations of dark-bright solitons

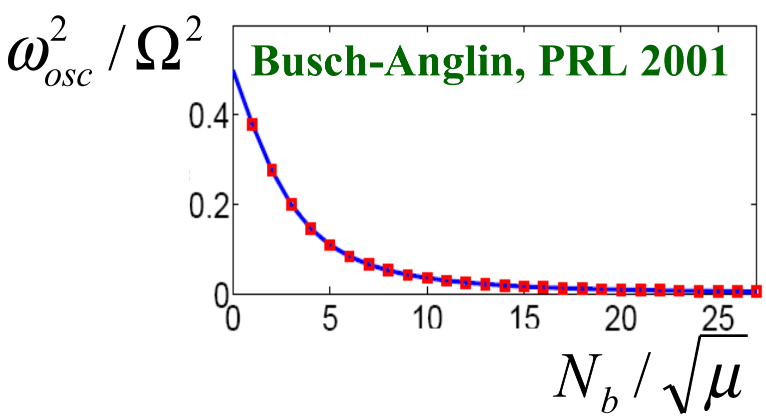
Evolution of the dark-bright soliton energy:

$$\frac{dE}{dt} = 4D\dot{D} + N_b \sec^2\varphi(\dot{D} + D\dot{\varphi}\tan\varphi) = -2\,\mathrm{Re}\left[\int_{-\infty}^{+\infty}\left(Q_d^*\frac{\partial \upsilon_d}{\partial t} + Q_b^*\frac{\partial \upsilon_b}{\partial t}\right)\right]dz$$

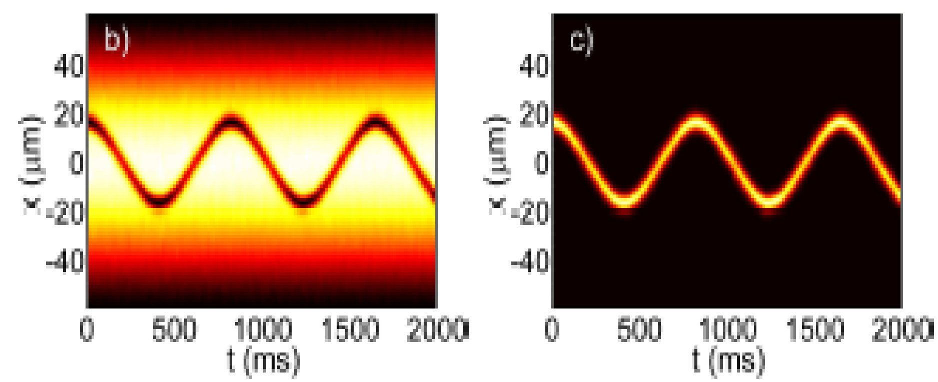
$$\dot{z}_0 = D\tan\varphi, \quad D^2 = \cos^2\varphi - \frac{N_b}{2\sqrt{\mu}}D$$

Equation of motion for the dark-bright soliton center:

$$\frac{d^2 z_0}{dt^2} + \omega_{osc}^2 z_0 = 0, \quad \omega_{osc} = \frac{\Omega}{\sqrt{2}}\left[1 - \frac{r}{4\sqrt{1+(r/4)^2}}\right]^{1/2}; \quad r \equiv \frac{N_b}{\sqrt{\mu}}$$



Busch-Anglin, PRL 2001



S. Middelkamp *et al.*, Phys. Lett. A 2011

Multiple dark-bright solitons

Ansatz for two counter-propagating dark-bright solitons:

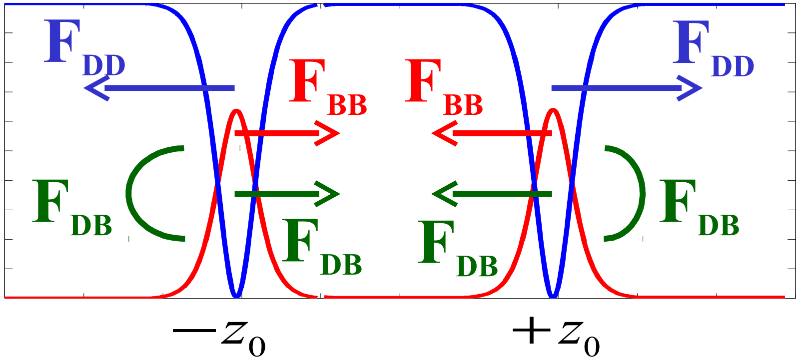
$$\psi_1(z,t) = (\cos\varphi \tanh\zeta_- + i\sin\varphi)(\cos\varphi\tanh\zeta_+ - i\sin\varphi),$$

$$\psi_2(z,t) = \eta\,\mathrm{sech}\,\zeta_-\, e^{i[+kz+\theta(t)]} + \eta\,\mathrm{sech}\,\zeta_+\, e^{i[-kz+\theta(t)]} e^{i\Delta\theta}, \quad \zeta_\pm = D[z \pm z_0(t)]$$

Energy of the dark-bright soliton pair: $E = 2E_1 + E_{DD} + E_{BB} + 2E_{DB}$

Equation of motion for the soliton coordinate:

$$\frac{dE}{dt} = 0 \Rightarrow \ddot{z}_0 = F_{\mathrm{int}} \equiv F_{DD} + F_{BB} + 2F_{DB}$$

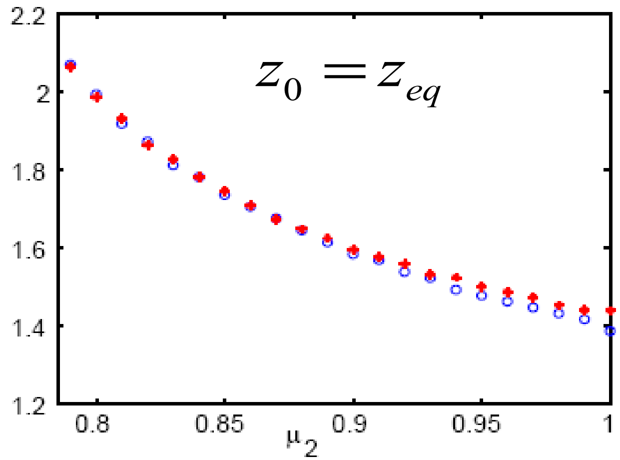


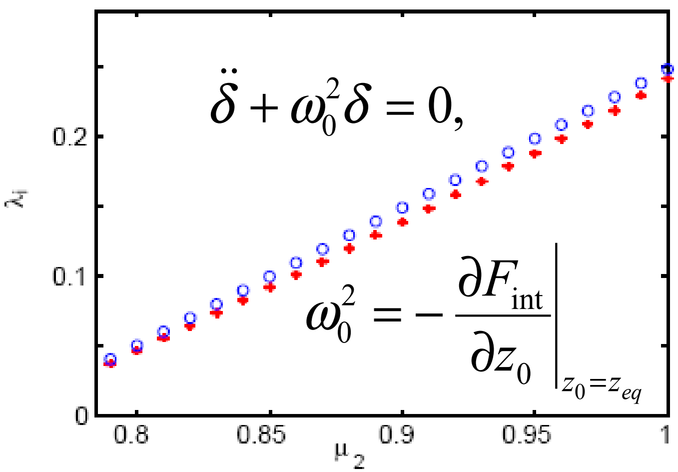
F_{DD} : always *repulsive*

F_{BB} (and part of F_{DB}) : *attractive* $(\Delta\theta = \pi)$ *or repulsive* $(\Delta\theta = 0)$

Statics and dynamics of multiple dark-bright solitons

- **Existence of *stationary* dark-bright soliton pairs ($\Delta\theta = \pi$) even in the absence of the trap $\Rightarrow$ equilibrium distance $z_0 = z_{eq}$; small-amplitude oscillations around the equilibrium points**





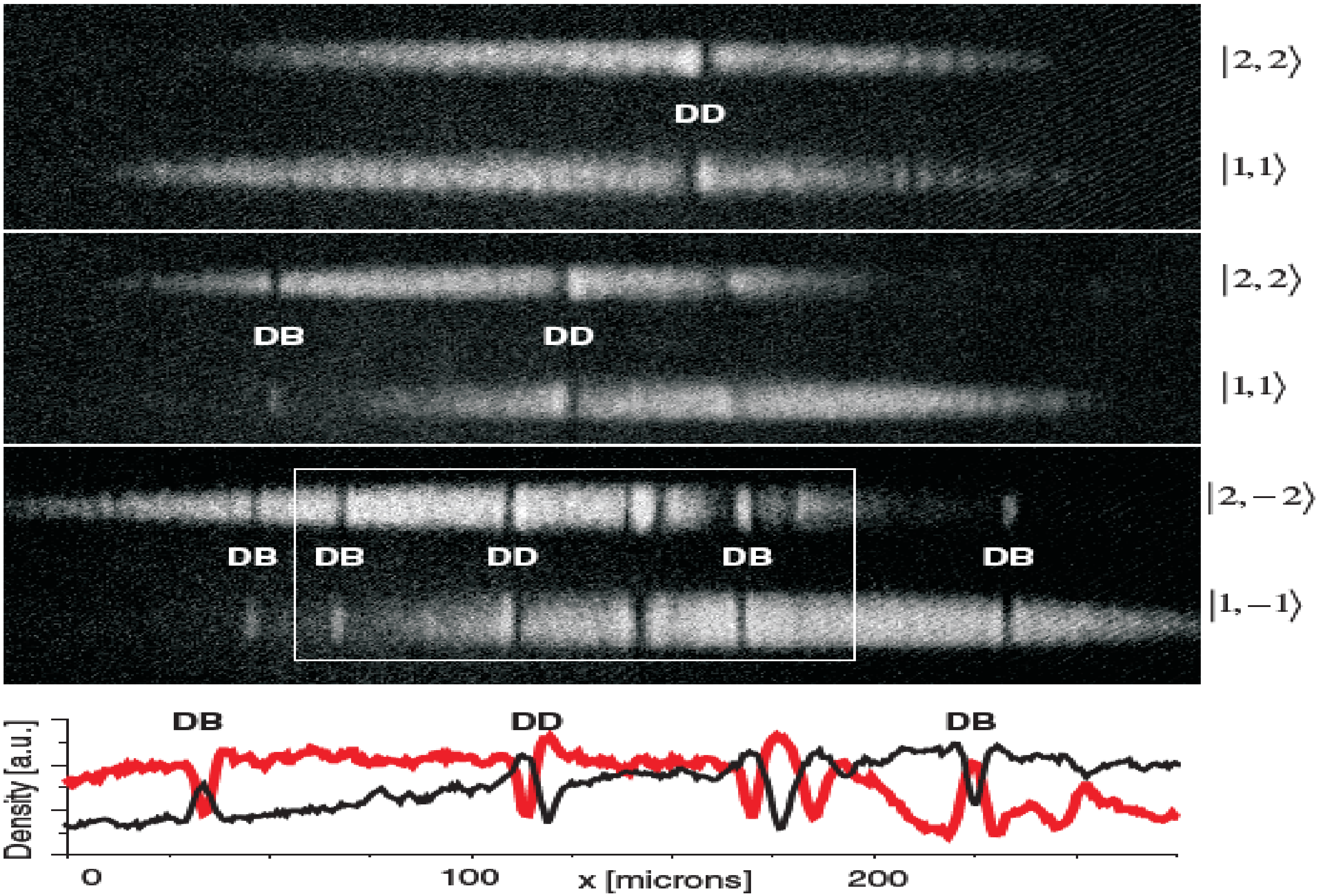
Dark-bright solitons in the trap: $\ddot{z}_0 = F_{tr} + F_{\rm int}$

- **Small-amplitude oscillations around the equilibrium points:**

$$\ddot{\delta} + \omega_1^2 \delta = 0, \quad \omega_1^2 = \omega_{osc}^2 + \left.\frac{\partial F_{\rm int}}{\partial z_0}\right|_{z_0 = z_{eq}} \quad \rightarrow \quad BdG\ analysis$$

Recent observations of dark-dark solitons

Washington experiments



D. Yan, J.J. Chang, C. Hamner, M. Hoefer, P.G. Kevrekidis, P. Engels, V. Achilleos, D.J. Frantzeskakis, and J. Cuevas, J. Phys. B 2012

SU(2) rotated (beating) dark-bright solitons

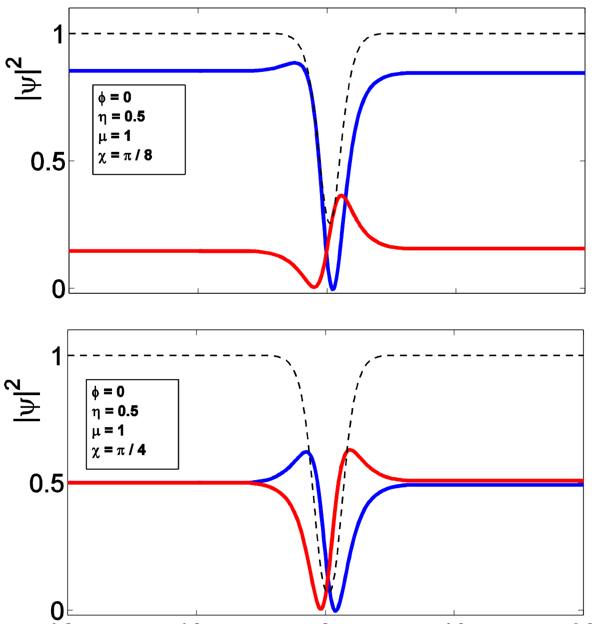
$$If \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \textit{ is a solution then } \begin{pmatrix}\psi_1'\\ \psi_2'\end{pmatrix} = \begin{pmatrix}\cos\chi & \sin\chi\\ \sin\chi & \cos\chi\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \textit{ is also a solution}$$

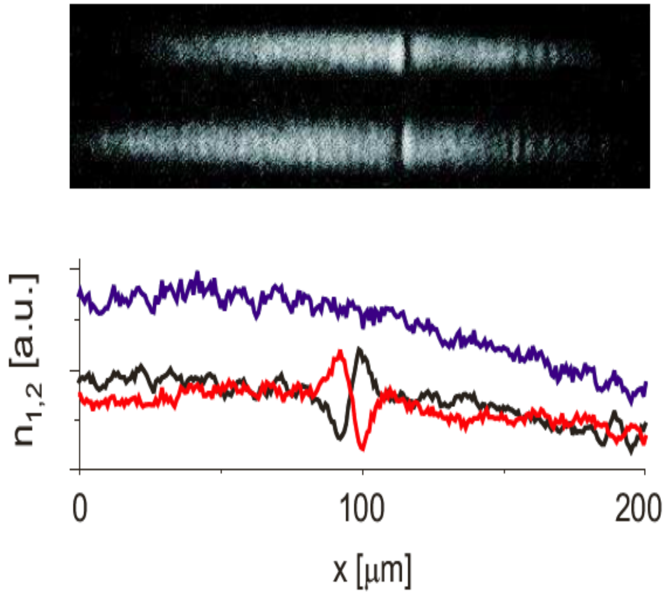
$$\psi_1(z,t) = \sqrt{\mu}\cos\chi(\cos\varphi\tanh\zeta + i\sin\varphi) - \eta\sin\chi\,\mathrm{sech}\,\zeta\,\exp\{i[kz+\theta(t)]\},$$

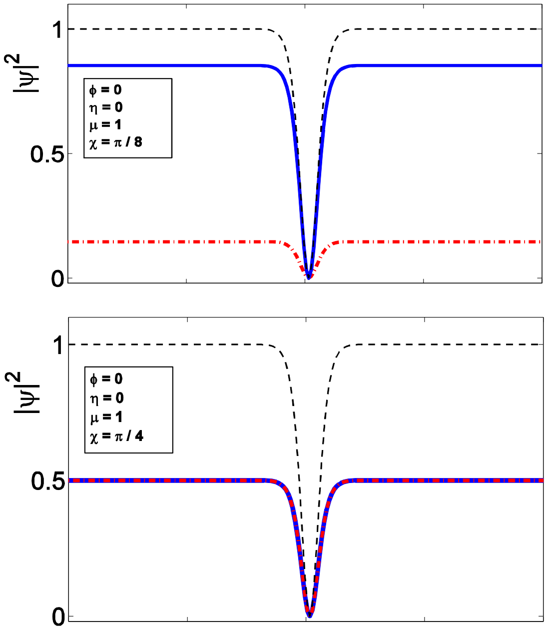
$$\psi_2(z,t) = \sqrt{\mu}\sin\chi(\cos\varphi\tanh\zeta + i\sin\varphi) + \eta\cos\chi\,\mathrm{sech}\,\zeta\,\exp\{i[kz+\theta(t)]\}$$

$$\zeta = D[z - z_0(t)],\ \dot{z}_0 = k = D\tan\varphi,\ \theta(t) = \frac{1}{2}(D^2 - k^2)t$$

Limits: $\chi = 0$ ⇨ dark-bright soliton; $\eta = 0$ ⇨ dark-dark soliton







Properties of the beating dark-dark solitons

Total density: $n_{tot} = n_1 + n_2 = \mu - D^2 \operatorname{sech}^2\zeta$ ⇨ **time-independent**

Individual densities (across the soliton trajectory, $\zeta = 0$):

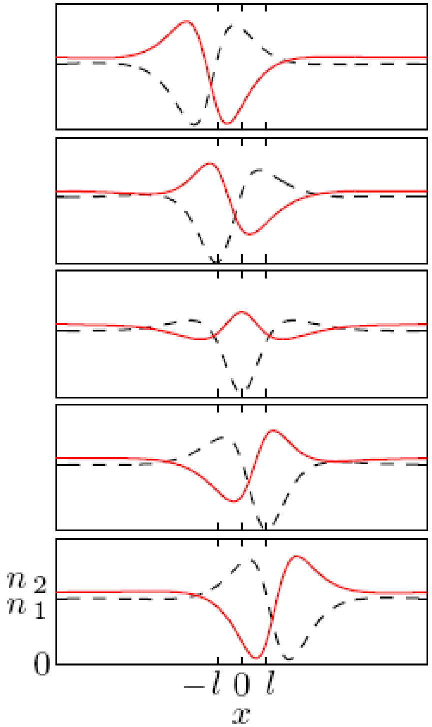
$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \mu \begin{pmatrix} \cos^2\chi \\ \sin^2\chi \end{pmatrix} \sin^2\varphi + \eta^2 \begin{pmatrix} \sin^2\chi \\ \cos^2\chi \end{pmatrix} \mp \sqrt{\mu}\sin(2\chi)\sin\varphi \sin(\omega_0 t)$$

⇨ **oscillate with a frequency:**

$$\frac{1}{2}k^2 < \omega_0 \equiv \frac{1}{2}(k^2 + D^2) \equiv \frac{1}{2}(\mu - \eta^2 \sec^2\varphi) < \frac{1}{2}\mu$$

$D = 0$ ⇨ plane wave

$\eta = 0$ ⇨ dark-dark soliton

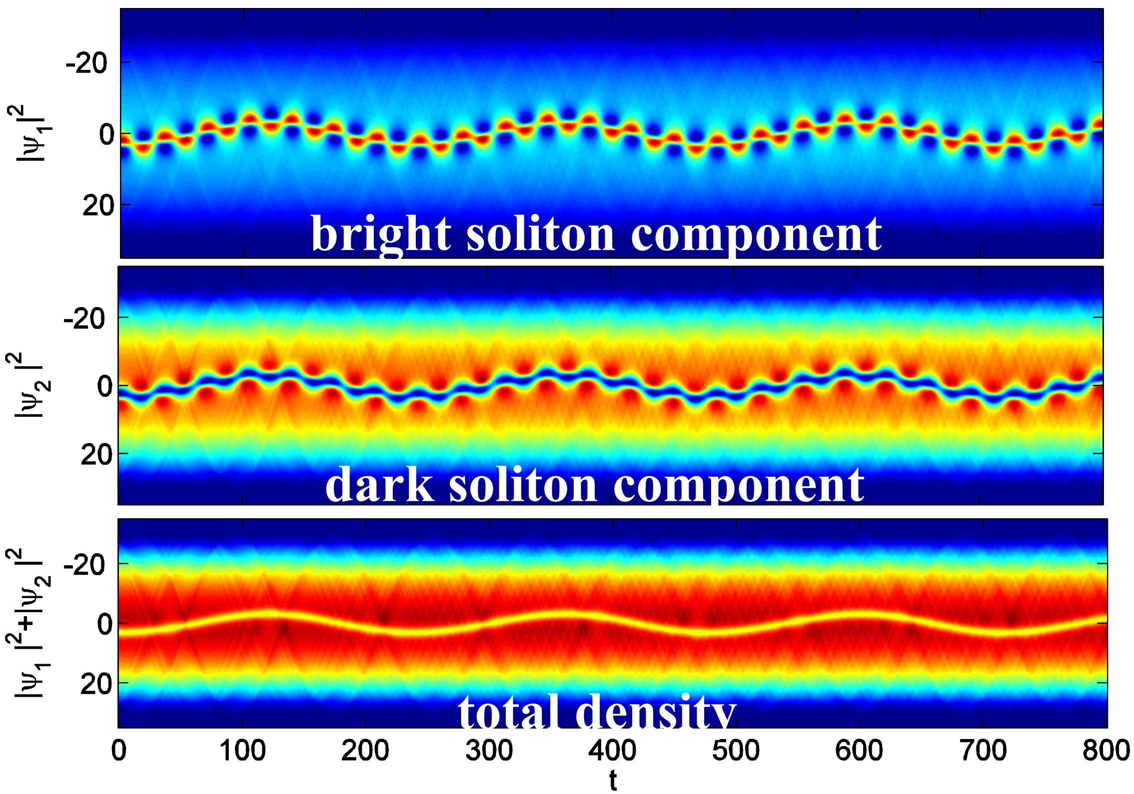


Beating dark-dark solitons: dynamics & stability

The system is invariant under SU(2) rotations (even in the presence of the trap) and so is the total energy *E. Thus, since*: $dE/dt = 0 \Rightarrow \omega_{osc}$ ⇨

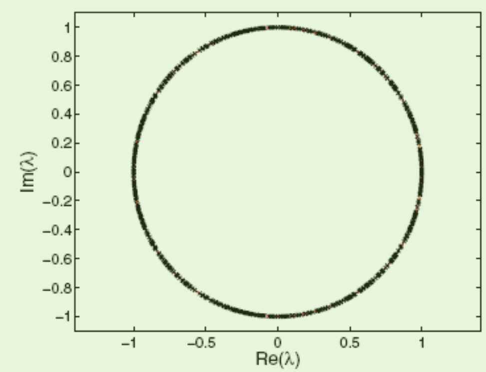
$$\omega_{osc} \; of \; beating \;\; dark-dark \; solitons \; = \; \omega_{osc} \; of \; unrotated \;\; dark-bright \;\; soliton$$

$$\omega_{osc} \;\; of \;\; regular \;\; dark-dark \;\; solitons \;\; = \;\; \Omega/\sqrt{2} \; (\text{as for a dark soliton in a single BEC})$$



Stability

Beating dark-dark solitons can be treated as periodic orbits and are found to be stable by means of a Floquet analysis



Dissipative dynamics at finite temperatures

Coupled dissipative Gross-Pitaevskii equation (DGPEs)

(Pitaevskii, Sov. Phys. JETP 1959)

$$(i-\gamma_j)\partial_t\psi_j = \left(-\frac{1}{2}\partial_z^2 + V(z) + \sum_{k=1}^{2}|\psi_k|^2 - \mu_j\right)\psi_j$$

$$\gamma_j \sim \mathrm{T}^{\alpha}; \quad \alpha = 4 \quad (k_B T \ll \mu), \quad \alpha = 1 \quad (k_B T \gg \mu)$$

Coupled DGPEs as coupled perturbed NLSEs

$$i\frac{\partial \upsilon}{\partial t} + \frac{1}{2}\frac{\partial^2 \upsilon}{\partial z^2} - \left(|\upsilon|^2 + |u|^2 - 1\right)\upsilon = Q_d(\upsilon) \equiv \frac{1}{2\mu^2}\left(2V\left(1-|\upsilon|^2\right)\upsilon + \frac{dV}{dz}\frac{\partial \upsilon}{\partial z} + \frac{\gamma_d}{\mu}\frac{\partial \upsilon}{\partial t}\right)$$

$$i\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial z^2} - \left(|\upsilon|^2 + |u|^2 - \mu\right)u = Q_b(\upsilon) \equiv \frac{1}{\mu^2}\left(V\left(1-|\upsilon|^2\right)u + \mu\gamma_b\frac{\partial u}{\partial t}\right)$$

Achilleos, Yan, Kevrekidis, Frantzeskakis, NJP 2012

Temperature-induced antidamping of solitons

For $V(z) = (1/2)\Omega^2 z^2$ adiabatic perturbation theory results in the following equation of motion (for sufficiently deep/slow solitons):

$$\ddot{z}_0 - a\dot{z}_0 + \omega_{osc}^2 z_0 = 0$$

$$a = \frac{2}{3}\mu\left(\gamma_d - \frac{N_b^2}{8\mu}\gamma_b\right) + \frac{N_b\mu}{6\sqrt{\mu + (N_b/4)^2}}\left(\gamma_b - \gamma_d + \frac{N_b^2}{8\mu}\gamma_b\right), \quad \omega_{osc} = \frac{\Omega}{\sqrt{2}}\left[1 - \frac{N_b}{4\sqrt{\mu + (N_b/4)^2}}\right]^{1/2}$$

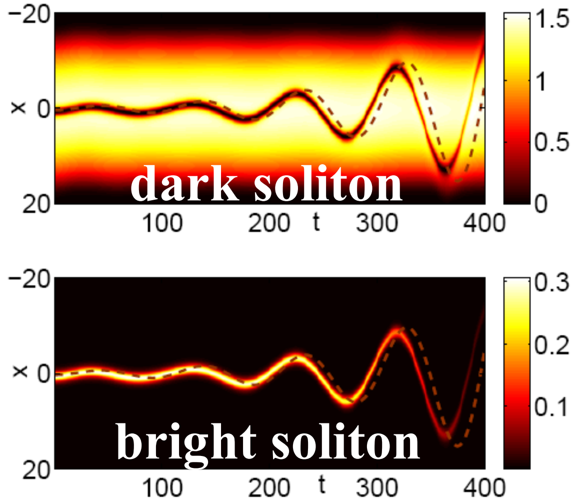
$$s^2 - as + \omega_{osc}^2 = 0 \;\Rightarrow\; s_{1,2} = \frac{1}{2}\left(a \pm \sqrt{a^2 - a_{cr}^2}\right), \quad a_{cr} = 2\omega_{osc}$$

- **Super-critical case :** $a > a_{cr} \Rightarrow s_{1,2} \in R^+$
- **Critical case :** $a = a_{cr} \Rightarrow s_1 = s_2 \in R^+$
- **Sub-critical case :** $a < a_{cr} \Rightarrow s_{1,2} \in C$

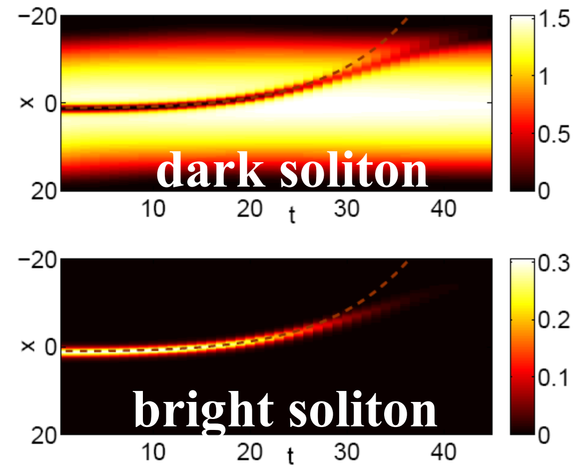
Cockburn, Nistazakis, Horikis, Kevrekidis, Proukakis, Frantzeskakis, PRL 2010; PRA2011
Achilleos, Yan, Kevrekidis, Frantzeskakis, NJP 2012

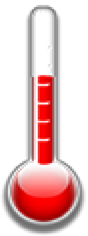
Dark and dark-bright soliton trajectories

Sub-critical case : $a < a_{cr}$

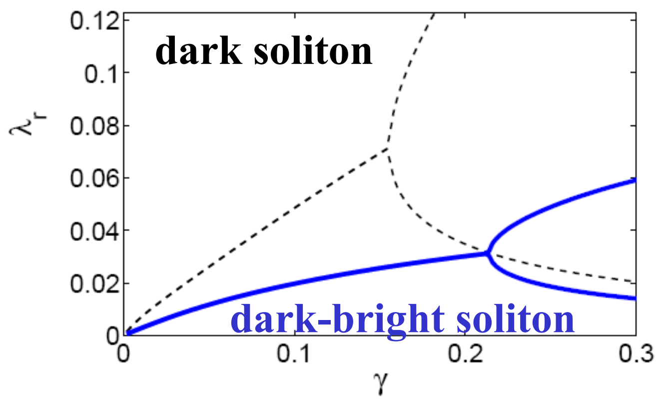


Super-critical case : $a > a_{cr}$





Temperature



The bifurcation diagram is "drifted" towards smaller values of γ $\Rightarrow$

dark-bright solitons are more robust than dark solitons

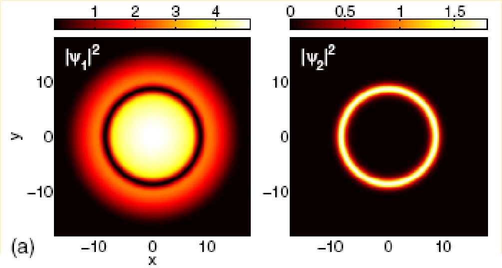
Conclusions

- **Dark-bright solitons in binary BECs:**

  - Single- and multiple-dark-bright solitons
  - SU(2) rotations: beating and regular dark-dark solitons
  - Dissipative dynamics of dark-bright solitons at finite temperatures
  - In all cases connection to *experiments* was provided

"Filled" (with the bright soliton) dark solitons are more robust than "bare" dark solitons in BECs against:

- **temperature-induced dissipation**
- **transverse instabilities (dark-bright rings)**



Current and future work

- **Vector solitons in multi-component BECs (including spinor BECs)**

  - Scattering of vector solitons at narrow barriers

  - Dissipative dynamics of vector solitons in various settings

  - Vector soliton dynamics in higher-dimensional settings

Some relevant references:

Physics Letters A 375 (2011) 642–646

Dynamics of dark–bright solitons in cigar-shaped Bose–Einstein condensates

S. Middelkamp[a], J.J. Chang[b], C. Hamner[b], R. Carretero-González[c,1], P.G. Kevrekidis[d,*], V. Achilleos[e], D.J. Frantzeskakis[e], P. Schmelcher[a], P. Engels[b]

PHYSICAL REVIEW A **84**, 053630 (2011)

Multiple dark-bright solitons in atomic Bose-Einstein condensates

D. Yan,[1] J. J. Chang,[2] C. Hamner,[2] P. G. Kevrekidis,[1] P. Engels,[2] V. Achilleos,[3] D. J. Frantzeskakis,[3] R. Carretero-González,[4,*] and P. Schmelcher[5]

J. Phys. B: At. Mol. Opt. Phys. **44** (2011) 191003 (5pp)

FAST TRACK COMMUNICATION

Dark–bright ring solitons in Bose–Einstein condensates

J Stockhofe[1], P G Kevrekidis[2], D J Frantzeskakis[3] and P Schmelcher[1]

PHYSICAL REVIEW A **84**, 053626 (2011)

Statics and dynamics of atomic dark-bright solitons in the presence of impurities

V. Achilleos,[1] P. G. Kevrekidis,[2] V. M. Rothos,[3] and D. J. Frantzeskakis[1]

J. Phys. B: At. Mol. Opt. Phys. **45** (2012) 115301 (11pp) doi

Beating dark–dark solitons in Bose–Einstein condensates

D Yan[1], J J Chang[2], C Hamner[2], M Hoefer[3], P G Kevrekidis[1], P Engels[2], V Achilleos[4], D J Frantzeskakis[4] and J Cuevas[5]

New Journal of Physics **14** (2012) 055006

Dark-bright solitons in Bose–Einstein condensates at finite temperatures

V Achilleos[1], D Yan[2], P G Kevrekidis[2] and D J Frantzeskakis[1,3]